Analysis of complex amplitude modulation with checkerboard-type holograms in phase-only modulators

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***Abstract*—For the generation of optical propagation modes using a phase-only spatial light modulators, it is essential that both phase and amplitude components of their complex fields are encoded. In this paper, we obtain the complete expression of the complex field generated at Fresnel and Fraunhofer distances when applying a simple technique consisting of the use of checkerboard-type holograms to encode both phase and amplitude information. The obtained expression is verified with a developed simulator that calculates the complex field generated at any propagation distance, corroborating that the desired mode propagates in even diffraction orders, even in near field.**

I. Introduction

The efficient and precise generation of optical propagation modes, such as orbital angular momentum (OAM) modes, is essential for applications like wireless optical communications [1]. In the last decades, phase-only lyquid-cristal based spatial light modulators (SLM) have been accesible options to create these modes because of the ease and flexibility of use of these devices [2]. Generally, an optical propagation mode is defined by a complex amplitude field $\bar{u}$, with both amplitude and phase components. Therefore, to generate optical propagation modes with a phase-only SLM, it is necessary to apply techniques that allow to indirectly modulate also its amplitude.

Multiple techniques have been developed in the last decades to achieve both phase and amplitude modulation [3]. One of them is the so-called checkerboard approach, based on the decomposition of the desired complex field on the SLM screen into a sum of two constant amplitude fields $\bar{u}_1$ and $\bar{u}_2$ with opposite phase shifts, which is described in [4], [5] and in section II-A on this paper. To apply this decomposition for the discrete phase screen of an SLM, an alternated sampling of $\bar{u}_1$ and $\bar{u}_2$ must be performed as shown in Fig. 1, where the red circles are $\bar{u}_1$ samples and the blue crosses are $\bar{u}_2$ samples. In order to achieve the desired mode, it is assumed that, due to the proximity of the pixels (or samples), the superposition of $\bar{u}_1$ and $\bar{u}_2$ is going to happen for every pair of adjacent pixels at a very short propagation distance.

In [6], the application of checkerboard holograms is described from an energy distribution point of view, experimentally demonstrating the intensity received in different diffraction orders and generating the desired mode at the zero order. To the authors' knowledge, a complete expression of the complex field generated via this technique has not been proposed. Furthermore, the overall propagation of this complex field, from near to far field, has not been evaluated yet.

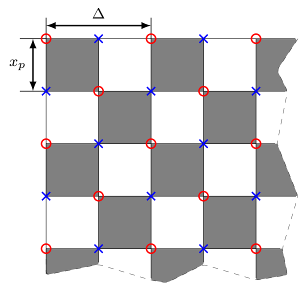


Fig. 1: Bidimensional sampling of $\bar{u}_1(x, y)$ and $\bar{u}_2(x, y)$ in checkerboard technique for pixelated SLM

In this paper, the analytical expression of the complex field generated at Fresnel and Fraunhofer distances via checkerboard holograms is obtained by applying the Fourier Transform (FT) of the sampled fields $\bar{u}_1$ y $\bar{u}_2$ [7]. The examination of the proposed expression reveals that the desired mode is generated in all even diffraction orders. This result is verified calculating and illustrating the complex field generated at a wide range of propagation distances by means of a developed simulator based on the Huygens-Fresnel principle [7]. The simulation results of encoding a fundamental Gaussian and a Laguerre-Gaussian (LG) mode [1] with the checkerboard technique confirm the analytical deductions, and imply that the desired mode is generated even in very near field. This outcome is not achieved with other techniques such as the phase-only "fork" approach, where a certain propagation distance is required [8].

II. Analytical expressions

A. Checkerboard technique

If the complex field of the desired mode is defined as $\bar{u}(x, y) = |\bar{u}(x, y)|\, e^{j\Phi(x,y)}$ in any plane transversal to the propagation axis z, being $|\bar{u}(x, y)|$ and $\Phi(x, y)$ the normalized amplitude and phase, respectively, then it can be decomposed as

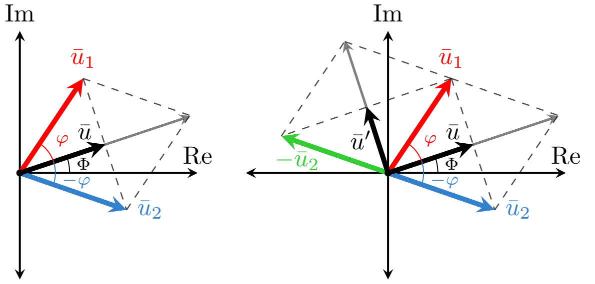
$$\bar{u}(x, y) = \frac{1}{2}[\bar{u}_1(x, y) + \bar{u}_2(x, y)] \,, \tag{1}$$

where

$$\bar{u}_1(x, y) = e^{j\Phi(x,y)} e^{j\varphi(x,y)}, \tag{2}$$

$$\bar{u}_2(x, y) = e^{j\Phi(x,y)} e^{-j\varphi(x,y)}, \tag{3}$$

and $\varphi(x,y) = \arccos(|\bar{u}(x,y)|)$ [4], [5]. For better comprehension, the vector representation of this decomposition is represented in Fig. 2a. Because of the expressions in Eq. (2) and (3), the adjacent pixels will present opposite phase shifts. This phase gap is illustrated by the checkerboard appearance of the hologram generated at the SLM screen, as shown in Fig. 1. Hence the name commonly given to this technique [2].



(a) (b)

Fig. 2: (a) Complex field $\bar{u}$ as superposition of fields $\bar{u}_1$ and $\bar{u}_2$. (b) Complex field $\bar{u}'$ as superposition of fields $\bar{u}_1$ and $-\bar{u}_2$.

B. Combined unidimensional sampling

To encode $\bar{u}_1(x,y)$ and $\bar{u}_2(x,y)$ as shown in Fig. 1, both fields must be sampled in two dimensions. However, for better understanding, we are going to consider first $\bar{u}_1$ and $\bar{u}_2$ as one dimensional complex fields. This way, the sampled fields would be, respectively

$$\bar{u}_{1s}(x) = \text{comb}\left(\frac{x}{\Delta}\right)\bar{u}_1(x) \tag{4}$$

$$\bar{u}_{2s}(x) = \text{comb}\left(\frac{x - \frac{\Delta}{2}}{\Delta}\right)\bar{u}_2(x). \tag{5}$$

where $\text{comb}\left(\frac{x}{\Delta}\right)$ is a spatial domain impulse train function of period Δ, and $\text{comb}\left(\frac{x-\frac{\Delta}{2}}{\Delta}\right)$ is the same function with a spatial shift of $\frac{\Delta}{2}$. If a uniform unidimensional wave is modulated by $\bar{u}_{1s}(x)$ and $\bar{u}_{2s}(x)$, then the complex fields generated at Fraunhofer distance coincide, except for a constant and a phase distortion, with the Fourier Transform (FT) [7] of $\bar{u}_{1s}(x)$ and $\bar{u}_{2s}(x)$, and therefore, are defined as

$$U_{1s}(\omega) = \frac{1}{\Delta}\sum_{k=-\infty}^{\infty} U_1(\omega - \omega_k) \tag{6}$$

$$U_{2s}(\omega) = \frac{1}{\Delta}\sum_{k=-\infty}^{\infty} \mathrm{e}^{-j\pi k} U_2(\omega - \omega_k), \tag{7}$$

where $U_1(\omega - \omega_k)$ and $U_2(\omega - \omega_k)$ are the spatially shifted FT of $\bar{u}_1(x)$ and $\bar{u}_2(x)$, being $\omega = \frac{2\pi}{\lambda z}x$ and $\omega_k = \frac{2\pi}{\Delta}k$ the scaled version of the coordinate x as a result of the Fraunhofer approximation [7] and the spatial shift, respectively. The phase factor $\mathrm{e}^{-j\pi k}$ is due to the shift in the sampling of $\bar{u}_2(x)$. Thus, the total complex field generated at Fraunhofer distance is

$$U_s(\omega) = \frac{1}{2\Delta}\sum_{k=-\infty}^{\infty}[U_1(\omega - \omega_k) + (-1)^k U_2(\omega - \omega_k)]. \tag{8}$$

This expression shows that the resulting complex field is a superposition of infinite components because of the sampling of $\bar{u}_1(x)$ and $\bar{u}_2(x)$. Each of these components, whose spatial shift from the propagation axis depends on the index k, is known as k diffraction order.

C. Combined bidimensional sampling

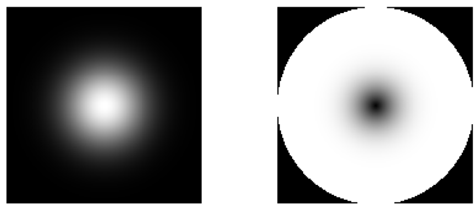
With a combination of comb functions thoughtfully designed to sample $\bar{u}_1$ and $\bar{u}_2$ in two dimensions as shown in Fig. 1, and following the calculation process of section II-B, the total bidimensional complex field generated at Fraunhofer distance is characterized as

$$U_s(\omega_x,\omega_y) = \frac{1}{2\Delta^2}[\sum_{k,l}[1 + (-1)^{k+l}]U_1(\omega_x - \omega_k, \omega_y - \omega_l) + \\ + \sum_{k,l}[(-1)^k + (-1)^l]U_2(\omega_x - \omega_k, \omega_y - \omega_l)]. \tag{9}$$

Analyzing the complex field for different values of the indexes k and l, it can be concluded that

- if k and l are even numbers,
$$U_s^{k,l}(\omega_x - \omega_k, \omega_y - \omega_l) = \frac{2}{\Delta^2}\mathcal{F}[\bar{u}(x,y)]. \tag{10}$$
- if k and l are odd numbers,
$$U_s^{k,l}(\omega_x - \omega_k, \omega_y - \omega_l) = \tag{11}$$
$$= \frac{2}{\Delta^2}\mathcal{F}[\exp(j\Phi(x,y) + \frac{\pi}{2}) \cdot \sqrt{1 - |\bar{u}(x,y)|^2}] \tag{12}$$
- and if k is even and l is odd or vice versa
$$U_s^{k,l}(\omega_x - \omega_k, \omega_y - \omega_l) = 0. \tag{13}$$

Therefore, in even diffraction orders, the resulting complex field is the FT of $\bar{u}(x,y)$, meaning that the desired optical propagation mode is obtained [7]. Nonetheless, in odd diffraction orders, the result is the FT of a complex field $\bar{u}'(x,y)$ associated with $\bar{u}(x,y)$ by the Pythagorean relation $|\bar{u}'(x,y)|=\sqrt{1 - |\bar{u}(x,y)|^2}$. This outcome is due to the superposition of $\bar{u}_1(x,y)$ and $-\bar{u}_2(x,y)$, represented in Fig. 2b. For example, if $|\bar{u}(x,y)|$ is the amplitude of the fundamental gaussian mode from Fig. 3a, then $|\bar{u}'(x,y)|$ is the amplitude represented in Fig 3b. Finally, if the parity of k and l is different, then the complex field is canceled.



(a) (b)

Fig. 3: Amplitudes of (a) a fundamental gaussian mode, and (b) a mode in Pythagorean relation with (a).

To entirely describe the complex field generated, the Sample and Hold procedure, required to forge the checkerboard hologram of pixels of size x_p shown in Fig. 1, must be taken into account [9]. Therefore, the total complex field is actually

$$U_s'(\omega_x,\omega_y) = U_s(\omega_x,\omega_y)\left(\frac{1 - \mathrm{e}^{-jx_p\omega_x}}{j\omega_x}\right)\left(\frac{1 - \mathrm{e}^{-jx_p\omega_y}}{j\omega_y}\right). \tag{14}$$

This analysis can be adapted for shorter distances (Fresnel approximation), if the quadratic phase factor $e^{j\frac{\pi}{\lambda z}(x^2+y^2)}$ [7] is added previous to the FT calculations of $\bar{u}_1$ and $\bar{u}_2$.

III. Propagation simulator

A MATLAB script has been implemented to verify the obtained analytical results of section II-C. It is based on the application of the Huygens-Fresnel principle, which defines the complex field $\bar{u}(x, y, z)$ generated at any propagation distance z when a light beam of wavelength λ illuminates an aperture (z=0), as

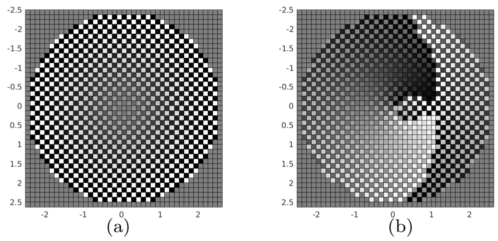
$$\bar{u}(x, y, z) = \frac{1}{j\lambda} \iint_{\Sigma} \bar{u}(\xi, \eta) \frac{\mathrm{e}^{-j\frac{2\pi}{\lambda} r_{01}}}{r_{01}} \cos(\bar{n}, \bar{r}_{01}) \mathrm{d}\xi \mathrm{d}\eta, \quad (15)$$

where $\bar{u}(\xi, \eta)$ is now the complex field at the aperture (SLM screen) at (ξ, η), and r_{01} is the distance between the coordinates $(\xi, \eta, 0)$ and (x, y, z) [7].

A. Near field

At propagation distances where the Fraunhofer approximation is not valid yet, the expressions of section II-C can not be initially applied. However, since the simulator allows to calculate the complex field at any propagation distance, it may be interesting to examine its evolution at near field.

In the script, a circular aperture of diameter D=5 mm is considered to emulate the SLM plane where the checkerboard hologram is encoded. The aperture is going to be illuminated by a uniform plane wave of λ= 633 nm. A gaussian fundamental mode is encoded via checkerboard technique, generating the hologram shown in Fig. 4a, but with a higher density of pixels to emulate a commercial SLM.



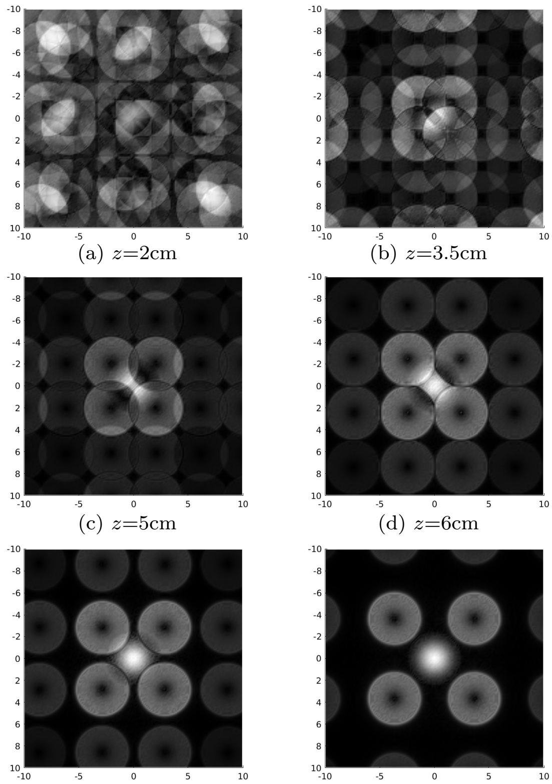
(a) (b)

Fig. 4: Checkerboard holograms for generation of (a) fundamental gaussian mode, and (b) LG_{10} mode, in a 5 mm diameter aperture.

The resulting complex fields at propagation distances from 2 to 9 cm are represented in Fig. 5. These distances are far below the Fraunhofer distance $z = \frac{\pi}{\lambda}\frac{D^2}{2}$, which in this case is 62 meters. At z =2cm, diffraction orders are overlapping, but as the complex field propagates, diffraction orders begin to separate. At approximately 9 cm, the amplitudes represented in Fig. 3a and 3b are recognized in the even (0,0) order and in all odd diffraction orders, respectively. This outcome is significant, because it means that the desired mode is obtained long before the Fresnel and Fraunhofer approximations. That the desired mode only seems to appear at the (0,0) order is due to the severe attenuation and distorsion caused by the impact of the required Sample and Hold procedure described in Eq. 14.

B. Fraunhofer Approximation

When calculating the complex field for propagation distances higher than 62 meters, the Fraunhofer approximation is valid. Therefore, the analytic expressions defined in section II-C can be verified. Applying the hologram in Fig. 4a, the



(a) z=2cm (b) z=3.5cm (c) z=5cm (d) z=6cm (e) z=7cm (f) z=9cm

Fig. 5: Near field propagation in a 20 mm length plane

amplitude of the complex field generated at z=65>62m is the one represented in Fig. 6. Because of the Fraunhofer approximation, the spatial separations of the diffraction orders to the propagation axis are given by $\Delta x = \frac{\lambda z}{2\pi}\omega_k$ and $\Delta y = \frac{\lambda z}{2\pi}\omega_l$ [7]. In this case, the separation for k=1 and l=1 is 164 mm in both dimensions. Considering this and Fig. 6, it is confirmed that odd diffraction orders present a different amplitude to the even ones, and that the field is canceled in diffraction orders with different parity.

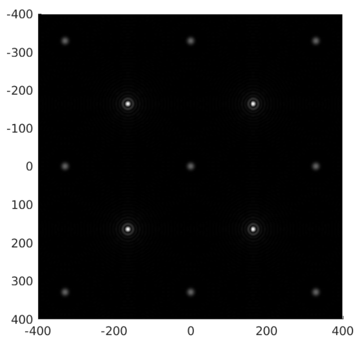


Fig. 6: Amplitude of complex field generated in a 400 mm length plane at z=65 m by encoding a fundamental Gaussian mode.

To precisely analyze the resulting even and odd diffraction orders, the normalized amplitude of the (0,0) and (1,1) orders are represented in Fig. 7 and 8. Undoubtedly, the fundamental gaussian mode is generated in order (0,0), while a different mode with visible side lobes is generated in order (1,1). This is because the amplitude in Pythagorean relation with the amplitude of a gaussian mode, as the one shown in Fig.3b, presents high values at the edges, resulting in side lobes when calculating its FT.

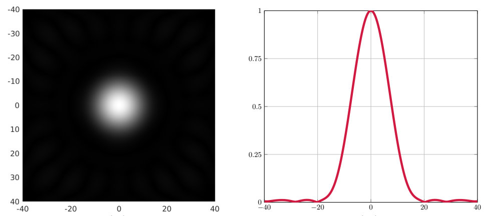


Fig. 7: Amplitude obtained in order (0,0) when encoding a gaussian fundamental mode. (a) In bidimensional plane. (b) In axis x=0 mm.

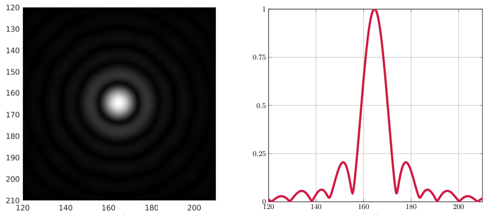


Fig. 8: Amplitude obtained in order (1,1) when encoding a gaussian fundamental mode. (a) In bidimensional plane. (b) In axis x=164 mm.

The defined expressions in section II-C are suitable for any desired mode generated with the checkerboard technique. Therefore, an LG_{10} mode has also been encoded, with the hologram of Fig. 4b. The amplitude of the calculated complex field at z =65 m is represented in Fig. 9. Since the sampling frequency is the same as for the fundamental gaussian mode, the spatial separations have remained the same as in Fig. 9. Again, orders (0,0) and (1,1) are represented en Fig. 10 and 11, respectively, and the same conclusions as when encoding the gaussian fundamental mode are derived.

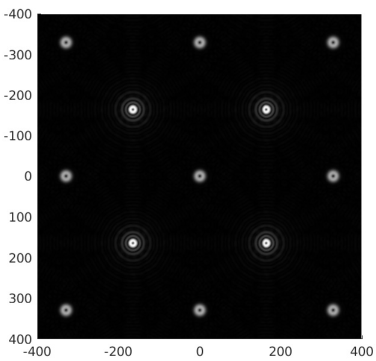


Fig. 9: Amplitude of complex field generated in a 400 mm length plane at z=65 m by encoding a LG_{10} mode.

IV. Conclusions

In this paper, a complete expression of the complex field generated with checkerboard-type holograms in SLMs is obtained. The expression, calculated by applying the Fraunhofer approximation to the sampled complex field at the SLM screen, reveals that the desired mode is generated in all even diffraction orders, while the FT of a complex field with an amplitude related to the desired one by the pythagorean theorem is generated in the odd orders. With a simulator, the propagation of the complex field generated at any distance has been examined, verifying the obtained expression and concluding that the desired mode is generated even in near field. In both theoretical analysis and simulation, the distortion caused by the required Sample and Hold procedure has been considered to accurately describe the results of applying the checkerboard technique.

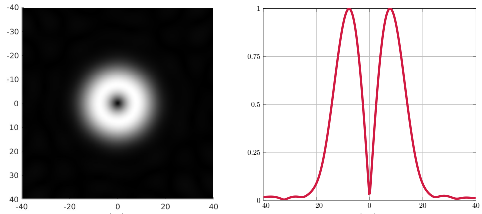


Fig. 10: Amplitude obtained in order (0,0) when encoding a LG_{10} mode. (a) In bidimensional plane. (b) In axis x=0 mm.

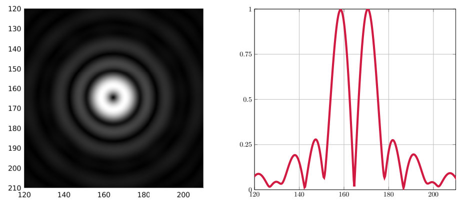


Fig. 11: (Amplitude obtained in order (1,1) when encoding a LG_{10} mode. (a) In bidimensional plane. (b) In axis x=164 mm.

Acknowledgements

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