AN EVOLUTIONARY ALGORITHM FOR PATH SYNTHESIS OF MECHANISMS

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Abstract

An algorithm for synthesis of mechanisms is described in this work. The algorithm is called MUMSA (Malaga University Mechanism Synthesis Algorithm). This algorithm is an improvement of one that was published in 2002. To check the performance of the algorithm, a comparative study of different strategies for six examples revealed in the bibliography for synthesis of four-bar and six-bar mechanisms is done. To that end, we have obtained the error between desired and target coupler curve in a four-bar mechanism and in a six-bar mechanism, showing that the found solutions by the MUMSA algorithm were accurate and valid for all cases.

Keywords- Mechanism synthesis. Genetic algorithms. Optimization. Differential Evolution.

1. Introduction

Different techniques have been used for mechanism synthesis, but during the last 40 years a great effort has been carried out in the computational synthesis of mechanisms. The great increase in computer power has permitted the recent development of routines that apply numerical methods to the minimization of a goal function. One of the first authors who studied these methods was Han [1], whose work was later improved by Kramer and Sandor [2] and Sohoni and Haug [3]. They optimized one of the most common goal functions, the error between the points tracked by the coupler and its desired trajectory.

Recently, an increasing number of works has used evolutionary strategies to solve mechanism synthesis problems [4-13]. The main advantages of these methods are their simplicity in implementing the algorithms and their low computational cost in some cases. In addition, there is no need for a deep knowledge of the searching space, such as whether or not it is continuous, presents local minimums or shows other mathematical characteristics demanded by traditional searching algorithms. One of the greatest challenges when these kinds of algorithms are used for synthesis of mechanisms is to find a good representation of the mechanisms. The first works using a genetic algorithm were carried out by Fang [4] and Ronston [5]. Their algorithms used a binary representation of the mechanisms, whose processing procedures were time consuming and computationally expensive. Kunjur [6] used a real number representation of the mechanisms and incorporated a guided genetic operator reducing the computing time and obtaining more accurate results. Cabrera [7, 8] used a new evolutionary technique called Differential Evolution to solve a four-bar path mechanism synthesis obtaining also very accurate results. The same way, Shiakolas [9, 10] used Differential Evolution and a novel technique called the Geometric Centroid of Precision Points for the optimum synthesis of four-bar and six-bar linkages. Laribi [11] combined genetic algorithms and fuzzy logic to solve the problem of path generation in mechanism synthesis. In this work, the fuzzy logic controlled the bounding interval of the design variables. Acharyya and Mandal [12] utilized three different evolutionary algorithms such as Genetic Algorithm, Particle Swarm Optimization and Differential Evolution to solve three examples of four-bar path synthesis and compared the results obtained for these three techniques. Smaili [13] combined an Ant Colony Optimization algorithm with gradient search to find optimum solutions for the dimensional synthesis of four-bar mechanisms.

As we have seen, evolutionary algorithms have been used in many works to solve the dimensional synthesis mechanism problem and the results obtained in each one have been accurate and better than the results obtained in other non-evolutionary techniques.

The approach presented in this paper to the synthesis of mechanisms deals with the Differential Evolution technique. Differential Evolution was firstly introduced by Storn and Price [14] and it has been extensively and successfully applied to different optimization problems.

The paper is organized as follows: section 2 deals with the development of an optimization algorithm for the synthesis of coupler trajectories of mechanisms based on Differential Evolution that we have called the MUMSA algorithm. In section 3 the goal functions

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for four-bar and six-bar planar mechanisms are defined. Section 4 analyzes the results found by the proposed method for six illustrative examples. Section 5 addresses the discussion of the results found by the proposed algorithm and validates its success by comparing it with other solutions available in literature. Finally, section 6 summarizes the conclusions of the paper.

2. The MUMSA algorithm

An optimization problem is given by:

$$
\begin{aligned}
&\min \quad f(p_1(X), p_2(X), \ldots, p_n(X)) \\
&\text{subject to:} \\
&\qquad g_j(X) \le 0 \quad j = 0,1,2,\ldots,m \\
&\qquad x_i \in [li_i, ls_i] \quad \forall x_i \in X
\end{aligned}
\tag{1}
$$

Where f is the goal function, which will be dealt with in section 3, where each individual X obtains a value, its fitness, $p_i(.)$ are functions of the properties that show the objectives of the system to be optimized and $g_j(.)$ are the constraints defining the searching space.

The strategy of evolutionary methods for optimization problems begins with the generation of a starting population. For synthesis of mechanisms, the starting population is sets of design variables, whose values are randomly generated within the searching space. Each individual (chromosome) of the population is a possible solution to the problem and it is formed by parameters (genes) that set the design variables of the problem. In this paper, the genes are expressed as real values. All genes are grouped in a vector that represents a chromosome.

$$X = [x_1 \; x_2 \ldots x_n] \quad \forall x \in \Re \tag{2}$$

Next the starting population has to evolve to populations where individuals are a better solution. This task can be reached by natural selection, reproduction, mutation or other genetic operators. In this work, selection and reproduction are carried out sequentially and mutation is used as an independent process.

In this paper, the best individual and two individuals randomly selected with uniform distribution create a disturbing vector, V. The scheme, known as Differential Evolution yields, Storn and Price [14]:

$$
\begin{aligned}
&X_i : i \in [1, NP] \\
&V = X_{best} + F \cdot (X_{r1} - X_{r2})
\end{aligned}
\tag{3}
$$

where X_{best} is the best individual of a population of NP individuals, X_{r1} and X_{r2} are two individuals randomly selected in the population, and F is a real value that controls the disturbance of the best individual.

The next step of the algorithm is reproduction, where V is crossed with individual i of the current population (X_i) to generate individual i of the next population (X_i^N). This operator is named crossover. If the new descendant, X_i^N, is better than its antecedent, X_i, it will replace it. Otherwise, X_i is retained and X_i^N is rejected. Therefore, population neither increases nor decreases. Crossover is carried out with a probability defined as CP∈[0, 1].

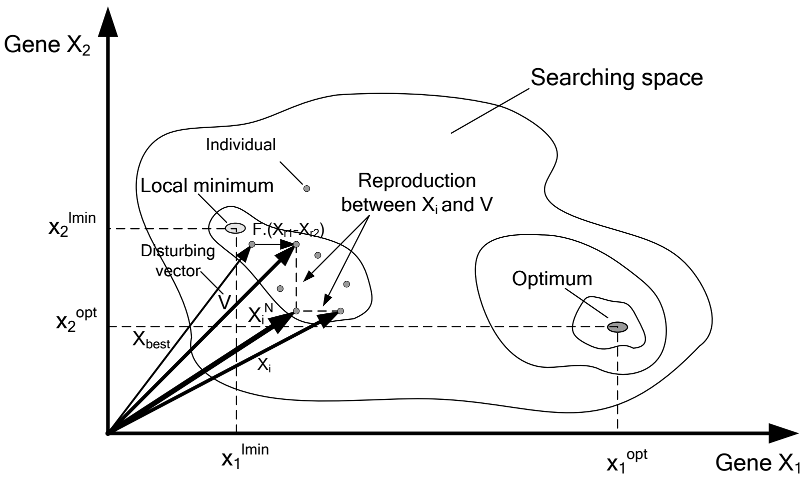

Figure 1.- Differential Evolution without mutation procedure.

A new mutation procedure of the parameters to be optimized is developed in this work. Mutation is an operator consisting of random change of a gene during reproduction. We have verified that this procedure is fundamental to obtain the optimum when the parameter range values are very different. The mutation procedure changes only some of these parameters allowing to find the correct optimum and not to stop in a local minimum. This problem was called stagnation in the work performed by Lampinen [15] and it is shown in Figure 1.

The whole procedure to obtain a new descendent in the original Differential Evolution algorithm is shown in Figure 1. In this case, there are two different parameters (genes) and the optimum has a very different value for these two parameters. As you can see in Figure 1, to obtain the new descendant, X_i^N, the reproduction operator takes gene x_1 from parent V and x_2 form X_i. The new descendant seems to be worse that X_i because there is more distance to the global optimum, but the algorithm does not reject it because it has a better value of the objective function as it is closer to a local minimum. For this reason, if the mutation procedure does not work properly, it is possible to drop in a local minimum. In Figure 1 and 2 the differences between both strategies with and without mutation procedure are shown.

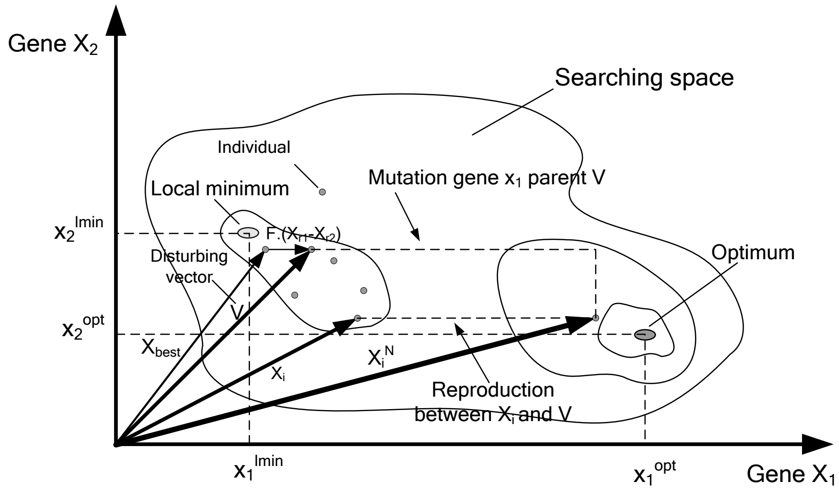

Figure 2.- Differential Evolution with mutation procedure.

The way to obtain a new descendent of the next population without mutation procedure is shown in Figure 1. In this case, the V and X_i couple generates the X_i^N descendent, but this new chromosome may not reach the global minimum due to the fact that the absolute values of the genes that compose it are very different, and the selection plus reproduction operations are not able to make the new descendent by themselves to overcome the valley of the local minimum.

With the mutation procedure, it is possible to solve the problem explained before. The generation of a new descendant using the mutation procedure is schemed in Figure 2. Here, the value of one or several of the genes of the V and X_i couple is changed in a range defined by the user, when the reproduction is taking place. This fact yields a new descendent, X_i^N, which has a different fitness from the X_i^N descendent studied in the previous case. This allows the algorithm to look for individuals with better fitness in the next generation.

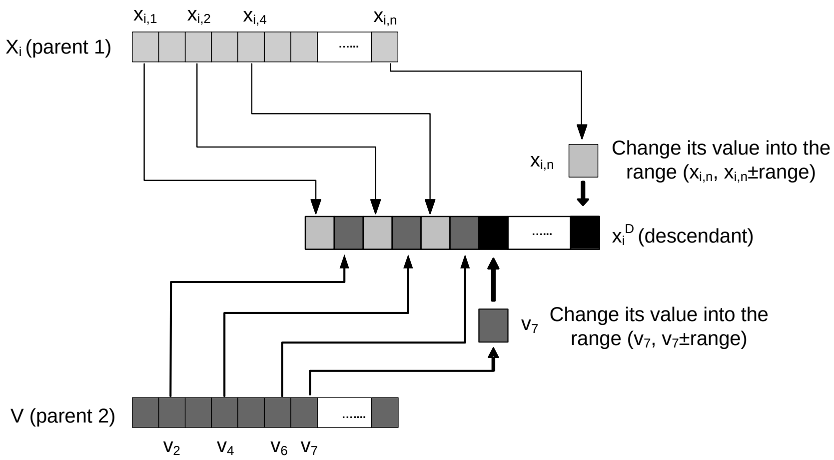

Figure 3.-Reproduction followed by mutation

In this work, mutation is defined as follows: depending on the mutation probability, genes of each parent can be chosen to mutate, for example if gene $x_{i,n}$ mutates, the operator randomly chooses a value within the interval of real values ($x_{i,n}$, $x_{i,n}$±range), which is added or subtracted from $x_{i,n}$, depending on the direction of the mutation. Mutation is carried out with a probability defined as MP$\in$[0, 1], much lower than *CP*. We can see how the reproduction and mutation operator work in Figure 3. Once the genetic operators are described, the optimization algorithm will be explained.

The proposed algorithm, which is defined as Mechanism Synthesis Algorithm (MUMSA), has the following steps:

1. The algorithm starts with the random generation of a starting population with *NP* individuals.
2. To create the new population, the selection of the couple, reproduction and mutation operators are used according to the definitions described above.
3. If the algorithm reaches the maximum number of iterations, it finishes; otherwise it returns to step 2.

A scheme of the proposed algorithm is shown in figure 4. First, we generate a parent for reproduction according to the Differential Evolution scheme, which has been defined above. Hence, the couple for reproduction is the actual population, X^G, and the disturbing vector population, *V*. As the reproduction and mutation operators are carried out, a new population is obtained, X^N. This one is compared with the actual population, X^G, to obtain the new population, X^{G+1}. As we can observe, the new population maintains the same number of individuals as the previous one, so this algorithm does not increase the number of individuals in the population.

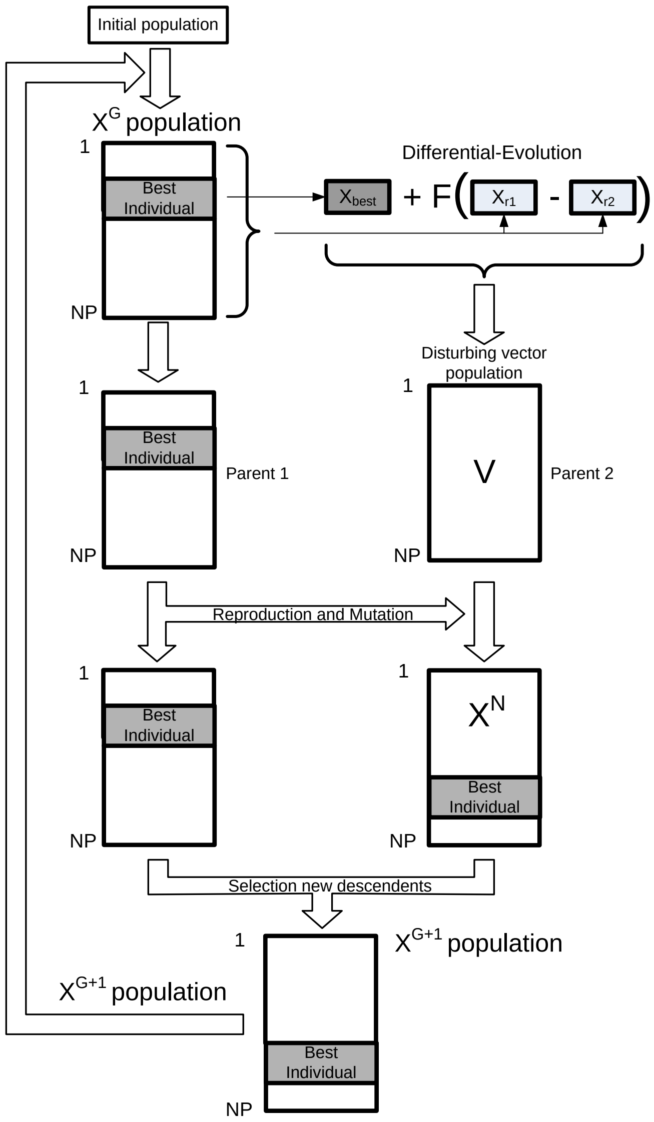

Figure 4.- Scheme algorithm.

3. Goal functions

Two goal functions have been used in this work. The first one computes the position error between a set of target points indicated by the designer that should be met by coupler *C* of a four-bar mechanism and the set of positions of the coupler of the designed four-bar mechanism (see Figure 5):

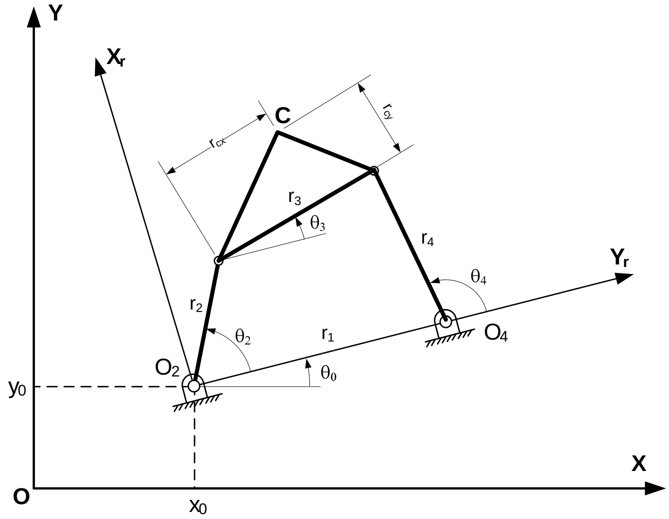

Figure 5.-Four-bar mechanism for the first objective function

$$f(\cdot) = \sum_{i=1}^{N}\left[\left(C^i_{Xd} - C^i_X\right)^2 + \left(C^i_{Yd} - C^i_Y\right)^2\right] \quad (1)$$

where *N* is the required number of target points, C^i_{Xd}, C^i_{Yd} and C^i_X, C^i_Y are the coordinates of the desired and generated precision points respectively. The coordinates of the generated precision points are calculated using the Freudenstein equation [16] and it has been indicated in [7]. Also, three constraints have been used in the optimization problem: the satisfaction of the Grashof criterion, the sequence of the crank angle and the range of the design variables. To define the complete optimization problem, the two first constraints were introduced as penalty functions, so the problem is defined as:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C^i_{Xd}(X) - C^i_X\right)^2 + \left(C^i_{Yd}(X) - C^i_Y\right)^2\right] + M_1 \cdot h_1(X) + M_2 \cdot h_2(X)\right\}$$

$$\textit{subject to}: \quad (2)$$

$$x_i \in \left[li_i, ls_i\right] \quad \forall x_i \in X \quad \textit{where} \quad X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1, \theta_2^2 ......\theta_2^N\right]$$

where $h_1(X)$ and $h_2(X)$ evaluate the Grashof condition and the sequence condition for the crank angle respectively, M_1 and M_2 are the penalty factors for those functions and *X* are the design variables.

The second objective function consists of two different parts and it is used to optimize a six-bar mechanism that will pass through the precision points (coupler *C*) while meeting the coordinated requirement between input and output angles in the dwell portion with the desired accuracy level (see Figure 6). Therefore, the objective function has a first part which is formulated in the same way as equation (1):

$$f_1(\cdot) = \sum_{i=1}^{N}\left[\left(C^i_{Xd} - C^i_X\right)^2 + \left(C^i_{Yd} - C^i_Y\right)^2\right] \quad (3)$$

where *N* is the required number of target points, C^i_{Xd}, C^i_{Yd} and C^i_X, C^i_Y are the coordinates of the desired and generated precision points of coupler *C* respectively. The second part defines the error at the output angle for the circular arc (dwell) and it can be formulated as:

$$f_2(\cdot) = \sum_{i=1}^{Nd}\left(\theta^i_{6d} - \theta^i_{6g}\right)^2 \quad (4)$$

where *Np* is the required number of target points on dwell period, θ_{6d} and θ_{6g} are the desired and generated output angles respectively. We use the Freudenstein equation [16] to obtain the coordinates of coupler C as it has been indicated in [7] and the study of the output dyad to calculate the θ_6 angle.

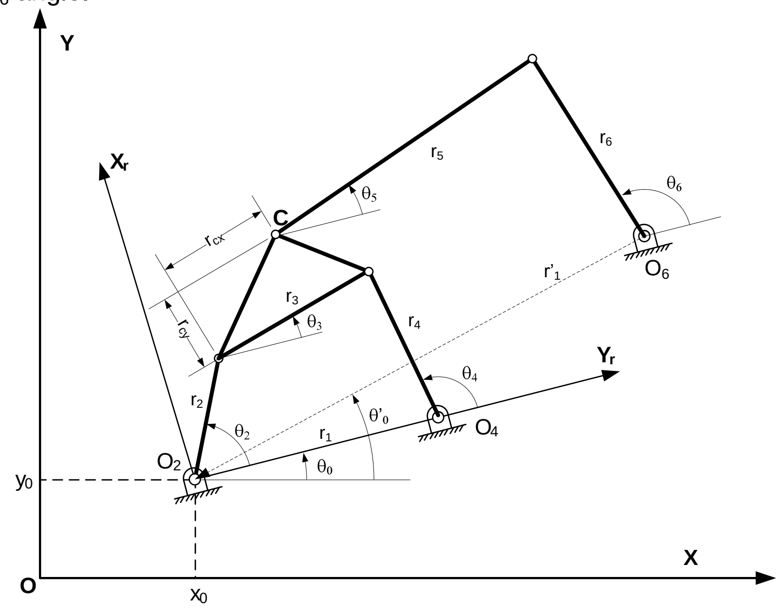

Figure 6.- six-bar mechanisms for the second objective function

Four constraints have been used in this optimization problem: the satisfaction of the Grashof criterion, the sequence of the crank angle, the range of the design variables and the non-violation of the transmission angle (The transmission angle is defined as an acute angle between the coupler and output links [17]). The first three constraints are the same as in the first objective problem defined above. The last constraint is verified at each target point and the goal is to keep the minimum transmission angle of the mechanism larger than the desired value when the mechanism passes through those target points. To define the complete optimization problem, the first two and the last constraints were introduced as penalty functions, so the problem is defined as:

$$\min\left\{\begin{array}{l}\sum_{i=1}^{N}\left[\left(C_{Xd}^{i}(X)-C_{X}^{i}\right)^{2}+\left(C_{Yd}^{i}(X)-C_{Y}^{i}\right)^{2}\right]+\sum_{i=1}^{Nd}\left(\theta_{6d}-\theta_{6g}(X)\right)^{2}+\\ +M_{1}\cdot h_{1}(X)+M_{2}\cdot h_{2}(X)+M_{3}\cdot h_{3}(X)\end{array}\right\}$$

$$\textit{subject to}: \qquad (5)$$

$$x_i\in\left[li_i, ls_i\right]\quad \forall x_i\in X \quad \textit{where} \quad X=\left[r_1,r_2,r_3,r_4,r_5,r_6,r_{cx},r_{cy},r_1^{'},\theta_0^{'},\theta_0,x_0,y_0,\theta_2^{1},\theta_2^{2}......\theta_2^{N}\right]$$

where $h_1(X)$, $h_2(X)$ and $h_3(X)$ evaluate the Grashof condition, the sequence condition for the crank angle and the non-violation of the transmission angle respectively, M_1, M_2 and M_3 are the penalty factors for those functions and X are the design variables.

4. Results

This section analyzes a set of results found when applying the algorithm developed in the previous sections. The entire algorithm proposed, including the genetic operators, have been run sequentially following the scheme of Figure 4, where its simplicity is observed.

As described, a set of target points is first put in by the designer to define the problem. The cases discussed here are path synthesis problems with and without prescribed timing and a combination of path and function synthesis with prescribed timing. For a comparison in the same frame of initial conditions like other optimization methods, the same conditions in relation to the number of goal function evaluations are used.

4.1.- Case-1: Path generation without prescribed timing

The first case of this section is a four-bar path synthesis problem with all target points aligned which traces a vertical straight line and without prescribed timing. The problem was proposed by Cabrera et al. [7]. The final error is computed by equation (2) for this problem and is defined as:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1, ..., \theta_2^6 \right]$$

- Target points:

$$C_d^i \;=\; 20,20 \;,\; 20,25 \;,\; 20,30 \;,\; 20,35 \;,\; 20,40 \;,\; 20,45$$

- Limits of the variables:

$$r_1, r_2, r_3, r_4 \in \; 0,60 \qquad r_{cx}, r_{cy}, x_0, y_0 \in \; -60,60 \qquad \theta_0, \theta_2^1, ..., \theta_2^6 \in \; 0,2\pi$$

- Parameters of the algorithm:

NP=100, CP=0.9, MP=0.1, range= 0.1, F=0.6, itermax=1000

The best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 1 along with the best results found by Cabrera et al. [7] and Acharyya and Mandal [12].

Table 1
Comparative results for case 1

	Cabrera [7]	*GA [12]*	*PSO [12]*	*DE [12]*	***MUMSA***
r_1	39.46629	28.77133	31.15501	35.02074	**31.788264**
r_2	8.562912	5.00000	5.00000	6.404196	**8.2046468**
r_3	19.09486	35.36548	23.84561	31.60722	**24.932131**
r_4	47.83886	59.13681	45.80352	50.59949	**31.385926**
r_{cx}	13.38556	0.00000	39.00066	20.80324	**34.193719**
r_{cy}	12.211961	14.85037	18.50846	41.54364	**14.415668**
x_0	29.7225	29.91329	59.99999	60.00000	**-6.366519**
y_0	23.4545	32.60228	17.91696	18.07791	**56.83676**
θ_0	6.20163	5.287474	0.419837	0.000000	**4.015959**
θ_2^1	6.11937	6.283185	4.842412	6.283185	**1.366547**
θ_2^2	0.19304	0.318205	0.404684	0.264935	**2.330773**
θ_2^3	0.44083	0.638520	0.657415	0.500377	**2.871039**
θ_2^4	0.68467	0.979950	0.922086	0.735321	**3.394591**
θ_2^5	0.95835	1.412732	1.247066	0.996529	**3.97096**
θ_2^6	1.35533	2.076254	2.298727	1.333549	**4.96349**
evaluations	100000	200000	100000	100000	**100000**
error	0.035142	1.21216	0.29862	0.0178414	**0.0002057**

The results obtained by the MUMSA algorithm (error=0.0002057) improve the previous achieved by Cabrera [7] and Acharyya and Mandal [12]. We can see how the algorithm uses the same number of function evaluations as the original problem proposed. Also, we show desired coupler points and the path traced by the coupler for the best mechanism obtained by the MUMSA algorithm and Cabrera [7] in Figure 7.

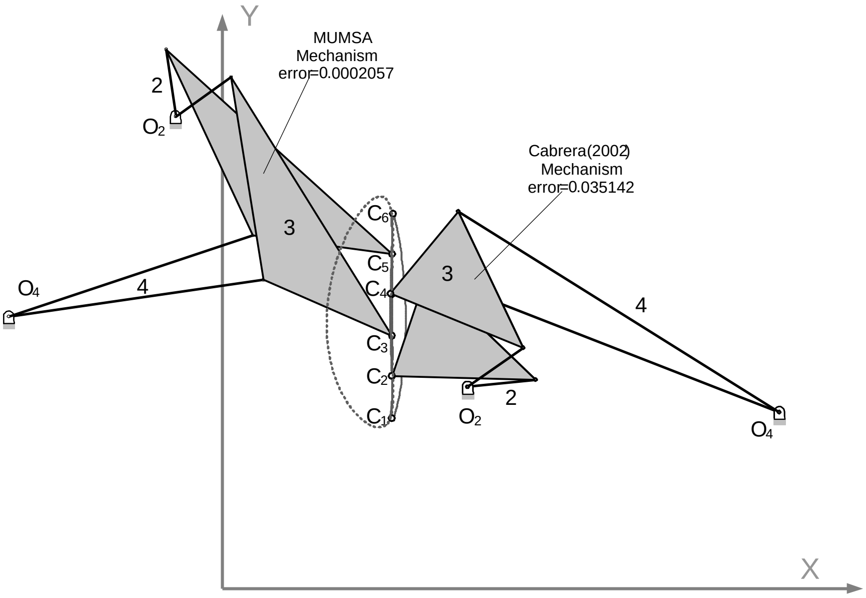

Figure 7.-The best mechanism obtained by MUMSA and Cabrera (2002)

4.2.- Case-2: Path generation with prescribed timing

The problem deals with a dimensional synthesis of a four-bar mechanism with prescribed timing whose coupler point must trace a path with five non-aligned points. The four-bar mechanism has its crank fixed point in the origin of the coordinate system and the fixed link is parallel to the X-axis, that is, *[θ_0, x_0; y_0]=[0, 0, 0]*. The final error is also computed by equation (2). The problem is defined by:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy} \right]$$

- Target points:

$$C_d^i = \quad 3,3 \ , \ 2.759,3.363 \ , \ 2.372,3.663 \ , \ 1.890,3.862 \ , \ 1.355,3.943$$

$$\theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5 \ = \ \pi/6, \pi/4, \pi/3, 5\pi/12, \pi/2$$

- Limits of the variables:

$$r_1, r_2, r_3, r_4 \in \ 0,5 \qquad r_{cx}, r_{cy} \in \ -5,5$$

- Parameters of the algorithm:

NP=50, CP=0.9, MP=0.1, range= 0.1, F=0.6, itermax=100

The best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 2 along with the best results found by Cabrera et al. [7] and Kunjur [6].

Table 2

Comparative rusults for case 2

	Cabrera [7]	*Kunjur [6]*	***MUMSA***
r_1	3.0630424	3.509643	**3.77326856**
r_2	1.9959624	1.857606	**2.00000403**
r_3	3.305823	4.725835	**4.11697104**
r_4	2.524706	3.518721	**2.74615671**
r_{cx}	1.645158	1.959538	**1.67848787**
r_{cy}	1.708959	1.558898	**1.67098007**
evaluations	5000	5000	**5000**
error	$4.075\text{x}10^{-6}$	$9.429\text{x}10^{-4}$	$\mathbf{1.7678x10^{-6}}$

The results obtained by the MUMSA algorithm (error=1.7678×10^{-6}) improve the previous achieved by Cabrera [7] and Kunjur and Krishnamurty [6]. We can see how the algorithm uses the same number of function evaluations as the original problem proposed. Also, we show desired coupler points and the path traced by the coupler for the best mechanism obtained by the MUMSA algorithm in two positions of the crank link in Figure 8.

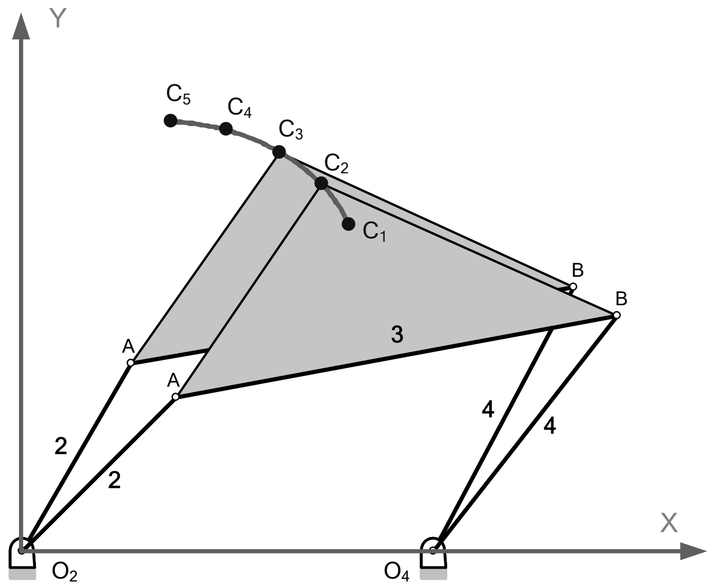

Figure 8.- The best mechanism obtained by the MUMSA algorithm.

4.3.- Case-3: Path generation with prescribed timing

In this dimensional synthesis problem, the coupler point of a four-bar mechanism must trace a close loop path generation with eighteen coupler points and with prescribed timing. The original problem was presented by Kunjur and Krishnamurty [6] and they defined the problem as:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1 \right]$$

- Target points:

$$C_d^i = \left\{ \begin{array}{l} 0.5,1.1 \ , \ 0.4,1.1 \ , \ 0.3,1.1 \ , \ 0.2,1.0 \ , \ 0.1,0.9 \ , \ 0.05,0.75 \ , \\ 0.02,0.6 \ , \ 0,0.5 \ , \ 0,0.4 \ , \ 0.03,0.3 \ , \ 0.1,0.25 \ , \ 0.15,0.2 \ , \\ 0.2,0.3 \ , \ 0.3,0.4 \ , \ 0.4,0.5 \ , \ 0.5,0.7 \ , \ 0.6,0.9 \ , \ 0.6,1.0 \end{array} \right\}$$

$$\theta_2^i = \theta_2^1, \theta_2^1 + 20 \cdot i \quad i:1..17$$

- Limits of the variables:

$$r_1, r_2, r_3, r_4 \in 0,50 \qquad r_{cx}, r_{cy} \in -50,50 \qquad \theta_0, \theta_2^1 \in 0,2\pi$$

- Parameters of the algorithm:

  NP=50, CP=0.85, MP=0.1, range= 0.5, F=0.5, itermax=100

Again, the best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 3 along with the best results found by Cabrera et al. [7] and Kunjur and Krishnamurty [6].

In this case, the results obtained by the MUMSA algorithm (error=0.0196) improve the previous achieved by Cabrera [7] and Kunjur and Krishnamurty [6]. We can see how the algorithm uses the same number of function evaluations as the original problem proposed. Also, we show desired coupler points and the path traced by the coupler for the best mechanism obtained by the MUMSA, Kunjur and Cabrera (2002) algorithms in Figure 9.

Table 3
Comparative rusults for case 3

	Cabrera [7]	*Kunjur [6]*	***MUMSA***
r_1	3.057878	1.879660	**4.453772**
r_2	0.237803	0.274853	**0.297057**
r_3	4.828954	1.180253	**3.913095**
r_4	2.056456	2.138209	**0.849372**
r_{cx}	0.767038	-0.833592	**-2.067338**
r_{cy}	1.850828	-0.378770	**1.6610626**
θ_0	1.002168	4.354224	**2.7387359**
x_0	1.776808	1.132062	**-1.309243**
y_0	-0.641991	0.663433	**2.806964**
θ_2^1	0.226186	2.558625	**4.853543**
evaluations	5000	5000	**5000**
error	0.0337	0.043	**0.0196**

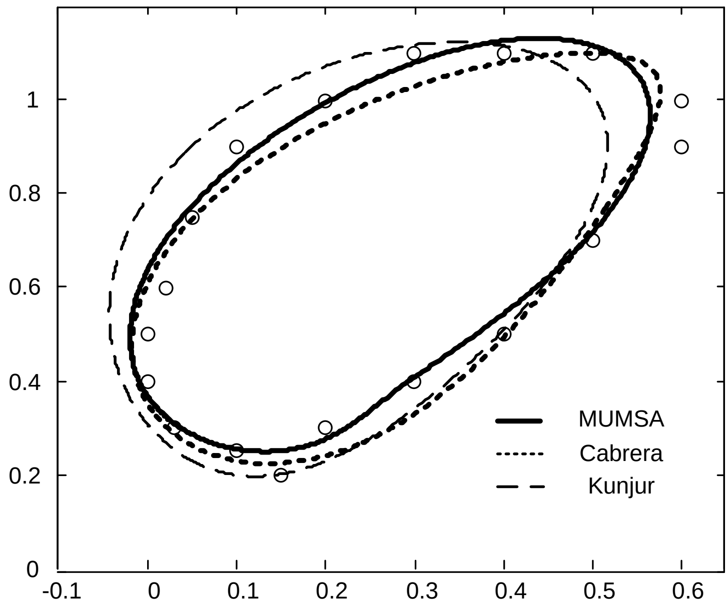

Figure 9.- The best path traced by the coupler for the three algorithms.

Case 4.- Path generation with prescribed timing

The fourth problem chosen here is a problem of path generation with prescribed timing. Six coupler points are chosen to find out an optimal solution that traces a semi-circular arc. This problem is solved by Acharyya and Mandal [12] and they defined it as:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0 \right]$$

- Target points:

$$C_d^i = \left\{ \begin{array}{l} 0,0 \ , \ 1.9098,5.8779 \ , \ 6.9098,9.5106 \ , \\ 13.09,9.5106 \ , \ 18.09,5.8779 \ , \ 20,0.0 \end{array} \right\}$$

$$\theta_2^i = \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi \right\}$$

- Limits of the variables:

$$r_1, r_2, r_3, r_4 \in \ 0,50 \qquad r_{cx}, r_{cy}, x_0, y_0 \in \ -50,50 \qquad \theta_0 \in \ 0,2\pi$$

- Parameters of the algorithm:
  NP=100, CP=0.85, MP=0.1, range= 0.5, F=0.45, itermax=1000

The best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 4 along with the best results found by Acharyya and Mandal [12].

Table 4
Comparative results for case 4

	PSO [12]	*DE [12]*	***MUMSA***	***MUMSA****
r_1	49.994859	50	**50**	**49.999217**
r_2	5	5	**5**	**1.3476638**
r_3	5.915643	5.905345	**7.031047**	**1.3476825**
r_4	49.994867	50	**48.134183**	**49.999235**
r_{cx}	18.925715	18.819312	**16.976687**	**11.383058**
r_{cy}	0	0	**12.952139**	**4.4426855**
x_0	14.472475	14.373772	**12.197494**	**10.1944353**
y_0	-12.494409	-12.444295	**-15.998203**	**-3.6925458**
θ_0	0.467287	0.463633	**0.0428247**	**6.2157809**
evaluations	100000	100000	**100000**	**100000**
error	5.547239	5.52068	**2.58035**	**1.21622**

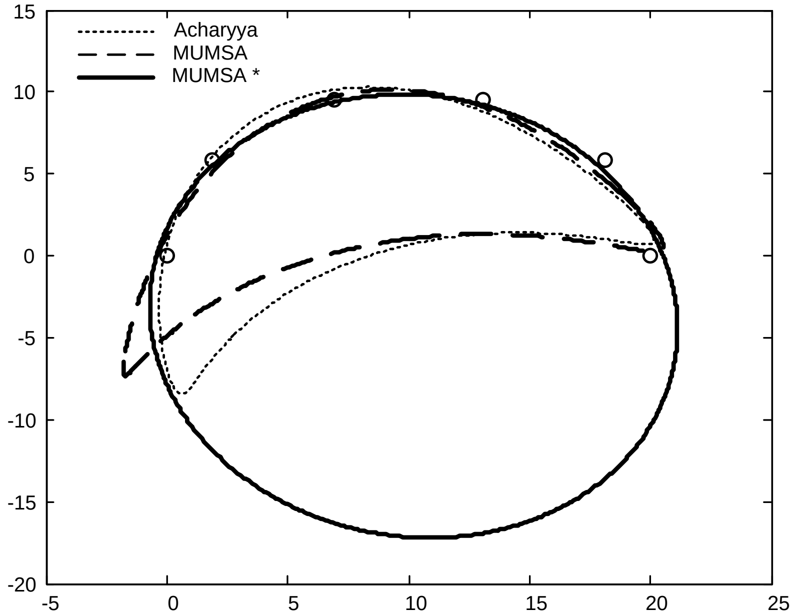

Figure 10.- The best path traced by the coupler for the three algorithms in case 4.

Again, the results obtained by the MUMSA algorithm (error=2.58035 and error=1.21622) improve the previous achieved by Acharyya and Mandal [12]. In this case, the MUMSA algorithm obtains two different results. The first one is achieved if the limits of the variables are: $r_1, r_2, r_3, r_4 \in 5,50$ as they were defined in [12], but if the limits of the variable are defined as: $r_1, r_2, r_3, r_4 \in 0,50$, we obtain a second result with a lower error. We can check how the second solution has two singularities when the crank link angles are 0º and 180º.

For this problem, the algorithm uses the same number of function evaluations as the originally proposed problem and we show desired coupler points and the path traced by the coupler for the best mechanism obtained by the MUMSA and Acharyya and Mandal algorithms in Figure 10. It is necessary to point out that the result obtained by Acharyya and Mandal, the first two columns in Table 4 are shown with the same trace in Figure 10, which is due to the fact that they have a very similar trajectory.

Case 5.- Path generation without prescribed timing

This case is an elliptical path generation synthesis problem without prescribed time in which the trajectory is defined by ten points. The elliptical path has a major axis of 20 units and a minor one of 16 units. The centre is kept at (10,10) and the major axis is kept horizontal. The problem is defined by:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1, ..., \theta_2^{10} \right]$$

- Target points:

$$C_d^i = \left\{ \begin{array}{l} 20,10 \;,\; 17.66,15.142 \;,\; 11.736,17.878 \;,\; 5,16.928 \;,\; 0.60307,12.736 \;, \\ 0.60307,7.2638 \;,\; 5,3.0718 \;,\; 11.736,2.1215 \;,\; 17.66,4.8577 \;,\; 20,10 \end{array} \right\}$$

- Limits of the variables:

$$r_1, r_2, r_3, r_4 \in 0,80 \qquad r_{cx}, r_{cy}, x_0, y_0 \in -80,80 \qquad \theta_0 \in 0, 2\pi$$

- Parameters of the algorithm:

NP=100, CP=0.7, MP=0.1, range= 1, F=0.4, itermax=1000

The best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 5 along with the best results found by Acharyya and Mandal [12]. We can observe that the results obtained by the MUMSA algorithm (error=0.0047) improves the previous achieved by Acharyya and Mandal [12].

Table 5

Comparative results for case 5

	PSO [12]	*DE [12]*	***MUMSA***
r_1	52.535162	54.360893	**79.516068**
r_2	8.687886	8.683351	**9.723973**
r_3	36.155078	34.318634	**45.842524**
r_4	80	79.996171	**51.432848**
r_{cx}	0	0.000187	**8.213922**
r_{cy}	1.481055	1.46525	**-2.9539575**
x_0	11.002124	10.954397	**2.021109**
y_0	11.095585	11.074534	**13.2165878**
θ_0	1.403504	2.12965	**5.5969445**
θ_2^1	6.282619	6.283185	**0.6376873**
θ_2^2	0.615302	0.616731	**1.3255329**
θ_2^3	1.305421	1.310254	**2.0080339**
θ_2^4	2.188053	2.19357	**2.6955659**
θ_2^5	2.913049	2.91717	**3.3845794**
θ_2^6	3.499313	3.490746	**4.08293762**
θ_2^7	4.125586	4.132017	**4.79845482**
θ_2^8	4.919977	4.922075	**5.51170565**
θ_2^9	5.685021	5.695372	**6.2127919**
θ_2^{10}	6.282323	6.28297	**0.6371866**
evaluations	100000	100000	**100000**
error	1.971004	1.952326	**0.0047**

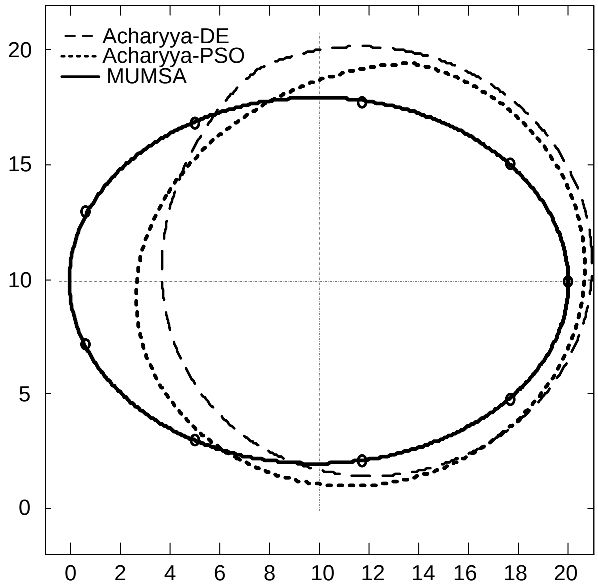

Figure 11.- The best path traced by the coupler for the three algorithms in case 5

We show desired coupler points and the path traced by the coupler for the best mechanism obtained by the MUMSA and Acharyya and Mandal algorithms in Figure 11.

Case 6.- Path generation and function synthesis with prescribed timing

In this case, we synthesize a six-bar mechanism where its coupler has to pass through the precision points and its output link has to maintain a desired accuracy angle in the dwell portion defined by the input link angles (see Figure 6). This problem was defined by Shiakolas et al. [10]. The final error is computed by equation (5) for this problem and is defined as:

- Design variables:

$$X = \left[r_1, r_2, r_3, r_4, r_5, r_6, r_{cx}, r_{cy}, r_1^{'}, \theta_2, \theta_0^{'}, \theta_0, x_0, y_0 \right]$$

- Target points for coupler:

$$C_d^i = \left\{ \begin{array}{l} (-0.5424, 2.3708), (0.2202, 2.9871), (0.9761, 3.4633), \\ (1.0618, 3.6380), (0.8835, 3.7226), (0.5629, 3.7156), \\ (0.1744, 3.6128), (-0.2338, 3.4206), (-0.6315, 3.1536), \\ (-1.0, 2.8284), (-1.3251, 2.4600), (-1.5922, 2.0622), \\ (-1.7844, 1.6539), (-1.8872, 1.2654), (-1.8942, 0.9448), \\ (-1.8096, 0.7665), (-1.6349, 0.8522), (-1.1587, 1.6081) \end{array} \right\}$$

$$\left\{ \delta_2^i \right\} \left\{ \begin{array}{l} 0^o\, 15^o\, 40^o\, 60^o\, 80^o\, 100^o\, 120^o\, 140^o\, 160^o\, 180^o \\ 200^o\, 220^o\, 240^o\, 260^o\, 280^o\, 300^o\, 320^o\, 345^o \end{array} \right\} \rightarrow \theta_2^i = \theta_2 + \delta_2^i$$

- Input-output angle correlation during dwell period:

$$\theta_2^i = \theta_2 + \{160^o\, 180^o\, 200^o\, 220^o\} \rightarrow \theta_{6d}^i = 210^o$$

$$\theta_2^i = \theta_2 + \{345^o\, 0^o\, 15^o\} \rightarrow \theta_{6d}^i = 225^o$$

- Parameters of the algorithm:

NP=100, CP=0.9, MP=0.1, range= 0.5, F=0.5, itermax=934

The best mechanism found by the MUMSA algorithm in the last iteration is the one listed in Table 6 along with the two best results found by Shiakolas et al. [10]. The first two columns show case A and case B indicated in the work by Shiakolas. The optimization process is performed in two stages in case A, stage one considers the synthesis of a four-bar mechanism and the second stage extends the four-bar mechanism to a six-bar one. In return, case B was performed in one stage considering the direct synthesis of a six-bar mechanism without first synthesizing a four-bar mechanism. The numbers of evaluations were 53310 for case A and 93405 for case B. The MUMSA algorithm carried out the optimization process in the same way as case B and with 93405 numbers of evaluations. The minimum transmission angle was 20º in all cases. We can observe that the results obtained by the MUMSA algorithm (error=0.0014) improve the previous achieved by Shiakolas [10].

Table 6

Comparative results for case 6

	Case A [10]	*Case B [10]*	***MUMSA***
r_1	0.9911	0.9826	**1.713529**
r_2	1.9995	2.0177	**0.92602**
r_3	2.0315	2.0009	**1.991373**
r_4	1.8145	1.8065	**1.848672**
r_{cx}	2.00770842	2.033594	**2.0047582**
r_{cy}	1.976575	2.026749	**2.19935**
θ_0	6.269879	0.011582	**0.067677**
x_0	0.0115	0.0415	**0.175257**
y_0	0.01577	-0.0377	**-0.1187032**
r_5	4.367	5.7769	**5.35498**
r_6	2.4924	2.5296	**2.54979**
$r_1^{'}$	4.4158	5.2817	**4.87374**
$\theta_0^{'}$	0.235595	0.0170309	**0.0967027**
θ_2	0.00052	0.003648	**6.222361**
evaluations	53310	93405	**93405**
error	0.01076	0.0074	**0.0014**

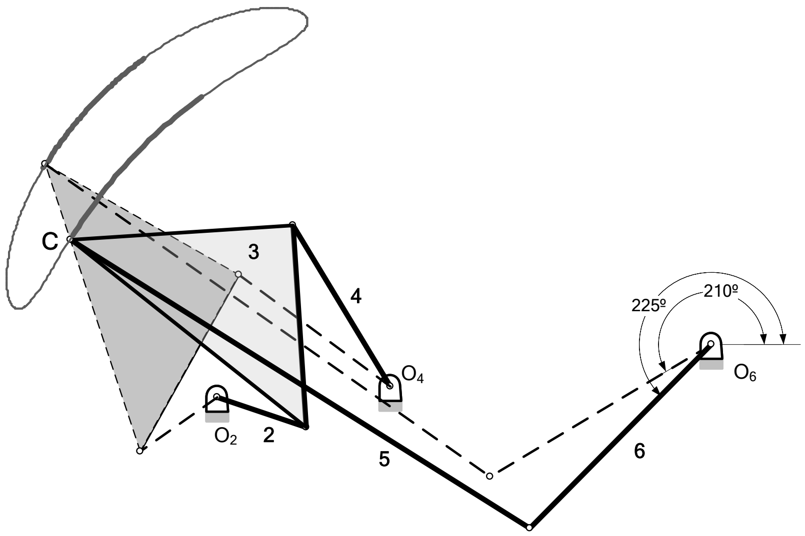

Figure 12.-Coupler curve and dwells pictorial representation for the best MUMSA mechanism.

We show the path traced by the coupler for the best six-bar mechanism obtained by the MUMSA algorithm in Figure 12. Also, two positions of the mechanism inside the dwell trajectory are drawn. We can observe how the mechanism maintains the output link angle in 210º and 225º in the dwell portions of the coupler curve.

5. Discussion about the method

The first five examples deal with the path synthesis of four-bar linkages with and without prescribed timing and the last case is applied to the synthesis of a six-bar mechanism capable of generating dwells with prescribed timing relative to the motion of the input crank. All these cases have been frequently studied in bibliography using different methodologies.

Case 1 works with a medium-sized design domain, i.e. medium number of target points, 6 points aligned. In return, the number of design variables is elevated because it is a problem without prescribed timing The best solution (error= 0.0002057) found by MUMSA had an error around 100 times lower than the best error obtained by the other algorithms using the same number of evaluations of the objective function (evaluations=100000). Case 2 to Case 4 are problems with prescribed timing, thus the number of design variables is reduced. Case 2 has a small number of precisions points and design variables, so the error found by the MUMSA algorithm is low with a lower number of evaluations (error= $1.7678x10^{-6}$, evaluations=5000). Case 3 and Case 5 are problems with a closed coupler curve with an elevated number of precision points. Case 5 has the greatest number of design variables because it is a problem without prescribed timing and ten precisions points. The error found by MUMSA for this problem again improves the previous error obtained by other methodologies and algorithms. Finally, Case 6 is a synthesis problem for a six-bar mechanism with prescribed timing. A coupler closed curve consisting of 18 precisions points with two circular arcs is used for the synthesis of the six-bar mechanism. For this problem, the objective function is composed of two parts. The first one calculates the summation of the square of the error at each precision point and the second part evaluates the summation of the square of the error at the output angle in the dwell portion.

In all cases studied, the error obtained by the MUMSA algorithm improves significantly the accuracy of the solutions of any of the other methods. The results are compared with the mechanisms found by those algorithms in tables 1 to 6.

It is necessary to point out that the initial values of the design variables used by the algorithm are randomly chosen by the computer code. But it has been observed that the solution reached is always very similar for different runs of the algorithm. This fact points out that the method escapes local minimums when the number of evaluations is large enough.

6. Conclusions

This paper approaches a modified method based on evolutionary techniques for path and function synthesis of planar mechanisms. An algorithm to optimize the position error between the target points selected by the designer for the coupler point and the points reached by the resulting mechanism, subject to different constraints, is presented. Constraints are satisfied by means of penalty terms in the objective function. The same algorithm has been applied to optimize a different goal function composed of path and function synthesis. The algorithm is fully developed for synthesis of four-bars and six-bar mechanisms, but it can be used for any other mechanism type.

The developed methodology was successfully applied to six cases of path and function synthesis of four-bar and six-bar mechanisms. A comparative study of the results based on different evolutionary techniques is also presented. It was observed that the algorithm shows fast convergence to the optimal result and very low error of adjustment to target points. The results presented verify the validity of the developed algorithm.

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