Rational Voting Behavior Accounting for Heterogeneous Ballots in British Elections

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Abstract: The most analyzed choice situation by electoral researchers is at the ballot box. In partially-contested elections, such as in Britain or Spain, voters cannot vote for all parties in every region or constituency. We present a modeling approach to study rational voting behavior that integrates such heterogeneous ballots and takes voting for the numerous parties therein seriously. The empirical application to the 2015 British Election demonstrates substantial insights not found when neglecting ballot composition heterogeneity or applying existing approaches.

Key words: discrete choice modeling; British elections; partially-contested elections;

rational choice behavior

1 Introduction

For electoral researchers, the most studied moment of choice is voting behavior at the ballot box, where voters choose based on the composition of ballots. In partially-contested elections, not all parties run nationwide. Thus, the party options voters face at the ballot box differ across regions, countries, and/or individual constituencies, which we call ballot composition heterogeneity. Partially-contested elections occur frequently, for instance, in Spain, Japan, Canada, and Britain. Table 1 presents a scenario of partially-contested elections where standard vote choice models fail because they do not capture the data structure.

Table 1: Data Structure of Partially-Contested Elections

Region	Constituency	Party 1	Party 2	Party 3	Party 4	Party 5	Party 6	Party 7
1	1	x	x	x		x		
1	2	x	x			x		
1	3	x	x	x			x	
	⋮							
2	1	x	x		x	x	x	
2	2	x		x	x		x	
2	3	x		x	x	x		
	⋮							
3	1	x	x	x	x			x
3	2	x		x				x
3	3	x	x	x	x	x		x
	⋮							
⋮								

Note: The x indicates the party is on the ballot.

Electoral research on rational voting behavior, where voters are assumed to vote for the party providing the highest expected utility, deals with the complex choice

situations in partially-contested elections in two ways. The first approach is to neglect ballot composition heterogeneity by simplifying the actual choice situation and thereby ignoring a substantial part of the electorate (e.g., Adams et al., 2005; Alvarez et al., 2000; Cho and Endersby, 2003; Endersby and Galatas, 1998; Quinn et al., 1999). These studies reduce the actual set of the competing parties to a simplified choice situation where a single ballot for all voters nationwide can be specified.

The second approach is to apply the varying choice set logit (VCL) model (e.g., Gallego et al., 2014; Labzina and Schofield, 2015; Labzina et al., 2017; Yamamoto, 2014), which acknowledges ballot composition heterogeneity. However, the VCL model, which is based on a random coefficient approach and accounts for ballot-specific effects, is demanding to use for applied researchers due to computational issues and the decisions involved and only yields limited substantive insights into rational voting behavior in partially-contested elections.

This contribution presents a third approach to studying rational vote choices in partially-contested elections. As the VCL model, our approach takes the actual composition of ballots seriously. However, instead of modeling effect heterogeneity across ballots, we propose a parameterization that offers a deeper understanding of how choice-specific criteria (e.g., ideological proximity or tactical considerations) impact voting for the numerous parties on the different ballots. As our proposed approach builds on the convenient multinomial logit model (also known as conditional logit model)[1], it is computationally straightforward, and applied researchers can easily use and interpret it.

[1]The term multinomial logit model is used differently across disciplines. As usual in the discrete choice literature, we use the term to refer to the model that contains both attributes of choice alternatives and decision makers as covariates.

We study up to seven parties spread across eight unique ballots in the 2015 British General Election, which was the most volatile British General Election since 1931 with substantial voter transitions (Thurner et al., 2020) and had the highest number of effective parties since 1918 (Green and Prosser, 2016). This complex choice situation and the available British election survey data (Fieldhouse et al., 2015b), which captures constituency-based realities of ballot compositions, makes it a good case to demonstrate the substantive insights that our modeling approach can obtain.

The following section describes the practical research problem this contribution addresses. Next, we outline our modeling approach to study partially-contested elections. Then, we present theoretical expectations and empirical results. Finally, we investigate differences with the VCL model that existing studies in the field apply to analyze partially-contested elections, and then conclude.

2 Practical Research Problem

Great Britain is a typical case of partially-contested elections where voters cast their votes based on different sets of parties that vary not only across but also within the three countries England, Scotland, and Wales. In line with previous studies in the field, we focus on Great Britain (referred to as Britain) and not the United Kingdom, thereby excluding Northern Ireland. Northern Ireland has a party system that bears very little resemblance to what is on offer in the rest of Britain, owing to the prominent cleavage between unionism and (Irish) nationalism.

Even though the particular set of parties that is actually on offer is of key importance in steering the voters' calculus and, therefore, in understanding electoral decisions,

existing studies on rational voting behavior in Britain do not give appropriate consideration to the actual choice situation British voters face (e.g., Adams et al., 2005; Alvarez et al., 2000; Cho and Endersby, 2003; Endersby and Galatas, 1998; Quinn et al., 1999).

These studies focus on the electoral choice between the three traditionally largest parties: Labour, Conservatives, and Liberal Democrats. As these three parties are on the ballot throughout Britain, disregarding other parties allows for modeling a single ballot composition for all British voters. However, such a simplification of the choice situation by excluding smaller national and regional parties limits our insights into voting behavior and party competition. It downplays the role of these parties as it pretends that voters do not consider them and ignores a substantial part of the electorate, as we demonstrate next for the 2015 British General Election.

Table 2: Vote Shares in the 2015 British General Election (in %)

	Great Britain	England	Scotland	Wales
Labour (Lab)	30.4	31.6	24.3	36.9
Conservatives (Cons)	36.9	41.0	14.9	27.2
Liberal Democrats (LD)	7.9	8.2	7.5	6.5
UK Independence Party (UKIP)	12.6	14.1	1.6	13.6
Greens	3.8	4.2	1.3	2.6
Scottish National Party (SNP)	4.7	–	50.0	–
Plaid Cymru (PC)	0.6	–	–	12.1
Others	3.1	0.9	0.4	1.1
Total	100.0	100.0	100.0	100.0

Note: The dash indicates the party does not compete in the country.

Table 2 reports the 2015 election results. The Scottish National Party (SNP) competes as an additional option in Scotland only and did so from a dominant position in the devolved Scottish Parliament. Only in Wales do candidates of Plaid Cymru compete, a nonnegligible force in the devolved Welsh Assembly. Even though the Conservatives and Labour were still by far the largest and politically most impor-

tant parties and the only ones with realistic hopes of leading the government, the remaining parties' electoral chances were substantial. Almost 33 percent of the votes were cast for parties other than these two major parties and more than 25 percent for parties other than the two major parties and the Liberal Democrats. The composition of the three largest parties in British politics also altered in 2015. The electoral support for the Liberal Democrats, which consistently had been the third party, collapsed from 23 to 7.9 percent. Instead, the UK Independence Party (UKIP) received the third-largest vote share by gaining 12.6 percent. The regional parties also significantly increased their electoral bases. The SNP defeated Labour and became the largest party in Scotland with 50 percent, receiving 30 percent more votes than in 2010.

Modeling partially-contested elections requires identifying the different ballot compositions and correctly assigning voters to them. Modeling the individual choices on the ballots requires a sufficient number of voters for each party on the different ballots. Modeling rational voting behavior requires information about voter characteristics, perceptions, and attitudes. Data from the *British Election Study* (BES) meets all these requirements. To identify and construct the ballot compositions, we used the *2015 General Election Results Dataset* (Fieldhouse et al., 2015a), which reports the election results for each constituency drawn from the Electoral Commission. Then, we linked the results dataset with survey data from the *British Election Study Internet Panel*, BESIP (Fieldhouse et al., 2015b).

Let A_i denote the set of parties available for voter i. There is only a limited number of such sets that we call ballot compositions and denote by C_b, $b \in \{1, \ldots, B\}$. Focusing on the five main national parties (Labour, Conservatives, Liberal Democrats, UKIP,

Greens) and two regional parties, the SNP in Scotland and Plaid Cymru (PC) in Wales, we detected eight unique ballot compositions. Table 3 summarizes how they are distributed across the 631 constituencies.

Table 3: Ballot Compositions in the 2015 British General Election

		Ballot Compositions C_b							Constituencies	
		Lab	**Cons**	**LD**	**UKIP**	**Greens**	**SNP**	**PC**	**N**	**%**
England	C_1	x	x	x	x				31	4.91
	C_2	x	x	x	x	x			501	79.40
Scotland	C_3	x	x	x			x		8	1.27
	C_4	x	x	x	x		x		20	3.17
	C_5	x	x	x		x	x		10	1.58
	C_6	x	x	x	x	x	x		21	3.33
Wales	C_7	x	x	x	x			x	5	0.79
	C_8	x	x	x	x	x		x	35	5.55
Total									631	100

Source: 2015 General Election Results Dataset (Fieldhouse et al., 2015a). *Note:* Northern Ireland and the Speaker's constituency (108) in England are excluded. The x indicates the party was on the ballot.

Considering only the three traditional major parties (Labour, Conservatives and Liberal Democrats) ignores and does not model substantive variation in ballot compositions. Apart from the regional parties that do not compete nationwide, the national parties UKIP and Greens, which seek to represent all British voters, are not listed on the ballot in all constituencies in England, Scotland, and Wales. A party might decide not to put a candidate forward in a particular constituency as it lacks resources or estimates the chances of winning very small.

We merged the election results dataset with the BESIP survey data by using the Press Association Constituency ID that each respondent had been assigned (see Online Appendix A for details). The sample consists of 16,453 respondents. Table 4

Table 4: Distribution of Votes across Ballot Compositions in the BESIP Sample

		Party Votes by Ballot Compositions C_b							Votes	
		Lab	**Cons**	**LD**	**UKIP**	**Greens**	**SNP**	**PC**	N	%
England	C_1	202	215	38	105	–	–	–	560	(3.40)
	C_2	3,315	4,043	901	1,565	647	–	–	10,471	(63.64)
Scotland	C_3	109	37	6	–	–	192	–	344	(2.09)
	C_4	287	217	76	51	–	536	–	1,167	(7.09)
	C_5	145	76	31	–	18	274	–	544	(3.31)
	C_6	326	270	130	36	68	585	–	1,415	(8.60)
Wales	C_7	79	62	6	26	–	–	30	203	(1.23)
	C_8	603	466	141	249	95	–	195	1,749	(10.63)
Total		5,066	5,386	1,329	2,032	828	1,587	225	16,453	(100)

Source: 2015 BESIP Survey Data, Wave 5 (Fieldhouse et al., 2015b). *Note:* Northern Ireland and the Speaker's constituency (108) in England are excluded. Entries report absolute frequencies. Numbers in parentheses give relative frequencies. The dash indicates the party was not on the ballot.

summarizes how they are distributed across the seven parties in the eight ballot compositions. The BESIP survey data perfectly reflects the actual ballot compositions in the 2015 British General Election. It covers all 631 constituencies in England, Scotland, and Wales and provides sufficient votes for each party within the eight different ballot compositions. Restricting the analysis to the traditional three-party setting (Labour, Conservatives and Liberal Democrats) yields a single ballot composition for all voters nationwide and disregards more than a quarter of voters.

3 Vote Choice Model with Heterogeneous Ballots

The discrete choice framework has become well-established to analyze individual vote choices (Alvarez and Nagler, 1998) and to study rational voting behavior based on policy issues where voters and parties can take different stands (for an overview, see Mauerer, 2020). What distinguishes discrete choice models from regression for cate-

gorical responses (Agresti, 2013; Tutz, 2012) is that they account for both attributes of decision makers and choice alternatives. A detailed discussion of discrete choice modeling provides Hensher et al. (2015) or Train (2009).

The most established vote choice model is the standard multinomial logit model (for a recent review of the model, see Mauerer and Tutz, 2023). The model fails in partially-contested elections because it assumes that all party choices are available to each voter, thereby not capturing ballot composition heterogeneity. In the following, we outline our approach to studying rational vote choices in partially-contested elections by building on the standard multinomial logit model and drawing on the random utility framework to motivate discrete choice models.

Let $Y_i \in \{1, \ldots, J\}$ denote the nominal-scaled response that contains J finite and mutually exclusive party choices of which voters $i \in \{1, \ldots, n\}$ select. Partially-contested elections are characterized by different choice set types, which we call ballot compositions: not all parties run in every constituency nationwide. Again, C_b, $b \in \{1, \ldots, B\}$ indicates the set of party choices available for voter i (i.e., voters within each constituency are confronted with the same ballot). Thus, we do not model the set of parties voters consider as viable options, known as individual consideration sets (e.g., Moral and Zhirnov, 2018; Oscarsson and Rosema, 2019), but the actual, externally given, choice situation voters face in their constituencies.

Voters are assumed to follow the principle of maximum random utility. Let U_{ij} denote the latent utility voter i associates with party j. Thus, voters cast a ballot for the party that maximizes the random utility,

$$Y_i = j \quad \Leftrightarrow \quad U_{ij} = \max_{r \in A_i} U_{ir},$$

where the set of parties A_i in the random utility maximization process ensures that voters only evaluate those parties that run in their constituency.

Random utility models assume that there are utility sources determining the choice that can be observed and others that cannot, $U_{ij} = u_{ij} + \epsilon_{ij}$, where u_{ij} are fixed utilities and ϵ_{ij} are independent and identically distributed (i.i.d.) random variables. The fixed utilities $u_{i1}, \ldots, u_{iJ}$ contain two types of covariates: choice attributes and chooser attributes. First, the choice attributes (also known as choice-specific or category-specific variables) are covariates that take different values across parties j and voters i. We refer to them as *party-specific covariates* $z_{ijk}, j \in A_i, k \in \{1, \ldots, K\}$. The key party-specific covariate in the analysis of rational voting behavior is the ideological proximity between voters and parties on policy issue scales, such as the traditional left-right scale. Second, the chooser attributes (also known as chooser-specific or global variables) are covariates that vary across voters only, such as age, gender, or social status. We refer to them as *voter-specific covariates* $s_{im}, m \in \{1, \ldots, M\}$.

The resulting utility functions present linear predictors

$$u_{ij} = \beta_{j0} + \sum_{k=1}^{K} z_{ijk}\alpha_{jk} + \sum_{m=1}^{M} s_{im}\beta_{jm} = \beta_{j0} + \mathbf{z}_{ij}^T\boldsymbol{\alpha}_j + \boldsymbol{s}_i^T\boldsymbol{\beta}_j,$$

which are connected to the choice probabilities through a logistic response function by assuming that the unobserved utility sources $\epsilon_{i1}, \ldots, \epsilon_{iJ}$ follow a maximum extreme value distribution,

$$P(Y_i = j|\mathbf{z}_{ij}, \boldsymbol{s}_i, A_i) = \frac{\exp(u_{ij})}{\sum_{r\in A_i}\exp(u_{ir})} = \frac{\exp(\beta_{j0} + \mathbf{z}_{ij}^T\boldsymbol{\alpha}_j + \boldsymbol{s}_i^T\boldsymbol{\beta}_j)}{\sum_{r\in A_i}\exp(\beta_{r0} + \mathbf{z}_{ir}^T\boldsymbol{\alpha}_r + \boldsymbol{s}_i^T\boldsymbol{\beta}_r)}, \qquad (3.1)$$

where $\beta_{10}, \ldots, \beta_{J0}$ are intercepts, $\boldsymbol{\beta}_j$ is a coefficient vector related to the voter-specific

covariates $\boldsymbol{s}_i$, and $\boldsymbol{\alpha}_j$ are the parameters associated with the party-specific covariates $\mathbf{z}_{ij}$. As the voter-specific covariates do not vary across parties, not all corresponding parameters are identified. The same applies to the intercepts. A side constraint is necessary, such as selecting a reference choice alternative whose parameters are set to zero. The reference choice alternative must be chosen so that it is included in all ballots; selecting one of the parties with large vote shares is useful as they usually compete in all constituencies.

The distributional assumptions for $\varepsilon_{i1}, \ldots, \varepsilon_{iJ}$ yield a convenient closed-form solution for the choice probabilities, allow a straightforward effect interpretation based on (log) odds and relative odds, and result in the behavioral assumption of IIA (Independence from Irrelevant Alternatives), which implies proportional substitution patterns as it assumes that the ratio of the choice probabilities of any two alternatives depends only on the observed factors relating to those two alternatives. We come back to the IIA assumption when presenting our empirical results (Section 5) and discussing the VCL model (Section 6).

The model in Equation (3.1) differs from the standard multinomial choice model in two key aspects. First, the set A_i explicitly integrates information on the subset of party choices available to voters. The standard choice model does not account for this variation because it assumes the set of parties to be identical for all voters so that every voter can vote for any of the J parties.

Second, the effect parameterization of the party-specific covariates $\mathbf{z}_{ij}$ is *alternative-wise* $\boldsymbol{\alpha}_j$. In the standard specification, known as the *generic specification*, these parameters are constrained to be the same for all parties, $\boldsymbol{\alpha}_1 = \ldots = \boldsymbol{\alpha}_J := \boldsymbol{\alpha}$. The alternative-wise specification relaxes this equality assumption and allows detecting in-

sightful choice behavior. By allowing the parameters to differ across parties, we follow recent developments in rational choice modeling of voting behavior that demonstrate that voters do not assign the same importance to policy issues or valence aspects when they evaluate different parties (e.g., Mauerer et al., 2015b; Mauerer, 2016; Mauerer and Tutz, 2024). Mauerer et al. (2015a) present a regularized parameter selection approach that can handle the alternative-wise specification.

Next, we outline parameter estimation. Let the vector $\mathbf{y}_i = (y_{i1}, \ldots, y_{iJ})^T$ denote the choice outcome; it takes the value 1 when voter i casts a ballot for party j and 0 otherwise. Accordingly, if $Y_i = j$, one obtains $\mathbf{y}_i = (0, \ldots, 0, 1, 0, \ldots, 0)^T$. Let π_{ij} contain the choice probabilities $P(Y_i = j|\mathbf{z}_{ij}, \boldsymbol{s}_i, A_i)$, $\mathbf{x}_{ij}$ collects all covariates and $\boldsymbol{\delta}^T = (\beta_{10}, \ldots, \beta_{J0}, \boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_J^T, \boldsymbol{\alpha}_1^T, \ldots, \boldsymbol{\alpha}_J^T)$ all parameters to be estimated. Selecting the first party as the reference choice alternative to ensure identifiability by setting β_{10}, $\boldsymbol{\beta}_1^T = (0, \ldots, 0)$, the kernel of the log-likelihood is

$$\begin{aligned} l(\boldsymbol{\delta}) &= \sum_{i=1}^{n} \left(\sum_{j \in A_i} y_{ij} \log \left(\frac{\pi_{ij}}{1 - \pi_{i2} - \cdots - \pi_{iJ}} \right) + \log \left(1 - \pi_{i2} - \cdots - \pi_{iJ}\right) \right) \\ &= \sum_{i=1}^{n} \left(\sum_{j \in A_i} y_{ij} \mathbf{x}_{ij}^T \boldsymbol{\delta} - \log(1 + \sum_{r \in A_i} \exp\left(\mathbf{x}_{ir}^T \boldsymbol{\delta}\right)) \right). \end{aligned} \tag{3.2}$$

Using the score function for the maximization of the log-likelihood, one obtains

$$\frac{\partial l(\boldsymbol{\delta})}{\partial \boldsymbol{\delta}} = \sum_{i=1}^{n} \left(\sum_{j \in A_i} y_{ij} \mathbf{x}_{ij}^T \boldsymbol{\delta} - \frac{\sum_{r \in A_i} \mathbf{x}_{ir} \exp(\mathbf{x}_{ir}^T \boldsymbol{\delta})}{1 + \sum_{r \in A_i} \exp(\mathbf{x}_{ir}^T \boldsymbol{\delta})} \right). \tag{3.3}$$

4 Voter Considerations and Expectations

The BESIP survey data contains questions to study different considerations in the voters' calculus that are particularly relevant in the British majority electoral system. The first two are ideological proximity and tactical considerations, which are party-specific covariates $\mathbf{z}_{ij}$. We will outline theoretical expectations that justify the alternative-wise effect parameterization we apply to these covariates. The third are typical voter-specific covariates $\boldsymbol{s}_i$.

Ideological Proximity. In rational choice explanations of voting behavior, the ideological proximity between voters and parties is the primary heuristic voters employ; when the ideological proximity to parties increases, the utility voters perceive increases. Several studies found that ideological proximity is an important predictor of vote choice in Britain (e.g., Adams et al., 2005; Alvarez et al., 2000; Cho and Endersby, 2003; Endersby and Galatas, 1998; Quinn et al., 1999). We focus on the traditional 11-point Left-Right scale (1 extreme left, 11 extreme right) and define ideological preferences by the proximity between the self-placement of voter i and the placement of party j as perceived by voter i.

Tactical Considerations. The British electoral system of First-Past-The-Post (FPTP) with single-member districts (SMD) gives voters strong incentives for tactical considerations. In the context of FPTP, voters have to consider at the national and the constituency level what they want the electoral outcome to be and how their vote might contribute to that outcome. Each election at the constituency level bears the risk of a wasted vote if the party voted for has no chance of winning. Voters face the dilemma of casting a ballot for a party they prefer based on national considerations

or a party that can win the constituency. Despite the importance of tactical considerations of voters, only a few studies relying on rational voting theories to explain electoral choices in Britain include them (exceptions are Alvarez et al., 2006; Alvarez and Nagler, 2000).

We operationalize tactical considerations, which we label win probabilities, by individual perceptions of the parties' chances of gaining the constituency. They are measured by the stated probability of voter i that party j, which competes in the constituency the voter resides, wins the constituency. To ensure comparability of effect sizes between these win probabilities (from 0 to 100 percent) and ideological proximities (11-point scales), we standardized both variables by subtracting the mean and dividing by the empirical standard deviation.

Expectations on Ideological Proximity and Tactical Considerations. Our general expectation is that ideological proximity and tactical considerations positively affect vote choice. However, we do not suppose the impact of both considerations to be the same for all the seven British parties. We expect ideological proximity to strongly influence voting for large parties and tactical considerations to considerably impact voting for smaller parties. As the alternative-wise effect parameterization we apply to these party-specific covariates relaxes the assumption of equal effects on all party choices, we can empirically test our expectations we lay out in more detail next.

Although more than two parties are competing, Britain still has an FPTP electoral system with SMD that discourages voting for third parties, and we expect this factor to play a role in the voters' decision-making process. Ideological proximity might substantially impact voting for large parties because voters need to worry less that their vote will not matter. Therefore, they can freely engage in expressing their

ideological preferences. When voters perceive a party's chance to win the constituency to be large, this is more likely to affect vote choices for the smaller parties, as no tactical considerations deter them from following their ideological preferences. Voters who prefer a small party based on ideological proximity are less likely to follow up on this when they perceive the win probabilities for that party to be low and, as a result, might engage in tactical voting for a lesser preferred party. The lower the chances of winning, the higher the incentives to not vote for the party for tactical reasons.

Voter Demographics. Similar to other Western countries, voting behavior in Britain was, for a long time, primarily explained by sociological approaches and social cleavage structures. Whereas most scholars argue that traditional social cleavages, especially along the lines of social class, have transformed and weakened as predictors of electoral choice (e.g., Evans and Tilley, 2012), others contest the decline of cleavage-based voting in Britain (e.g., Andersen and Heath, 2002; Elff, 2009). We consider the following voter demographics to account for these explanations: Homeownership (1 homeowner, 0 otherwise), union membership (1 union member, 0 otherwise), subjective social class (working class and middle class with reference category no class affiliation), age (centered around the sample mean of 50.12 and measured in decades), gender (1 female, 0 male), and education (age completed formal education).

5 Estimation Results

We present the estimation results as follows. First, we focus on the party-specific covariates $\mathbf{z}_{ij}$, ideological proximity and tactical considerations. We compare the estimates of the traditional three-party setting, which is restricted to Labour, Conser-

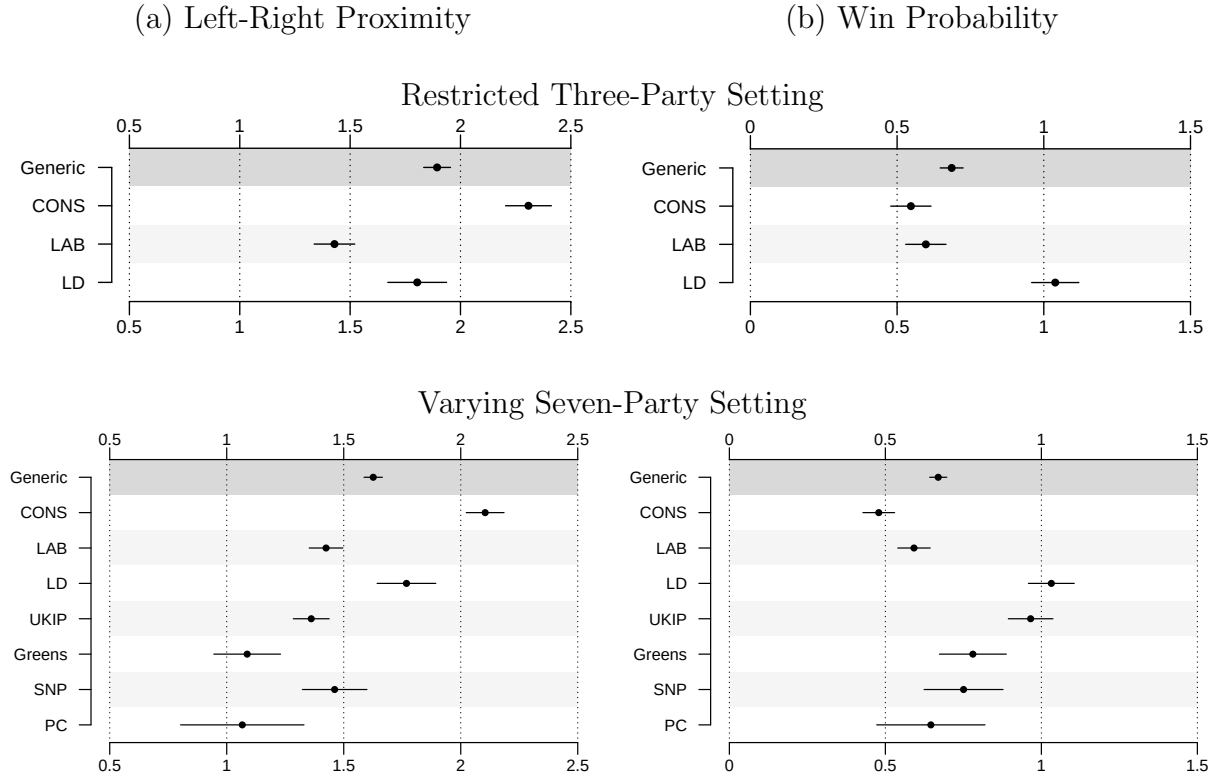
vatives and Liberal Democrats, and the seven-party setting with heterogeneous ballot compositions. Second, we demonstrate how voter demographics $\boldsymbol{s}_i$ affect vote choice in the seven-party setting. The models are estimated with the software LIMDEP/NLOGIT. The code to run the models is available in Mauerer and Walter (2024).

5.1 Ideological Proximity and Tactical Considerations

Figure 1 depicts the estimates of the effects of ideological proximity and win probabilities for the three and seven-party settings (Online Appendix B reports complete estimation tables). For each setting, we specified two models. In the first specification, the parameters are constrained to be identical across parties, $\boldsymbol{\alpha}_1 = \ldots = \boldsymbol{\alpha}_J := \boldsymbol{\alpha}$ (generic specification). The second specification is alternative-wise $\boldsymbol{\alpha}_j$ and relaxes the assumption of equal reactions. The upper part shows the estimates for the restricted three-party setting where all voters stating a choice in favor of UKIP, the Greens, SNP, or Plaid Cymru (PC) are excluded, resulting in only one single ballot composition and neglecting 28.4 percent of the sample (see Table 4). These models are based on 11,781 respondents and drop 4,672 voters of our total sample size of 16,453. The lower part depicts the estimates based on eight unique ballots containing up to seven parties.

Comparing the generic and the alternative-wise specifications provides strong empirical evidence that ideological proximity and win probabilities unequally influence voting for different parties. The generic specification is misleading as it averages out differing effects on party preferences, which equally applies to ideological and tactical considerations and is observed for both the three and the seven-party setting. If we base our conclusions on the generic specification, we would overpredict their impact

Figure 1: Generic vs. Alternative-wise Effects of Ideological Proximity and Win Probability in the Three and Seven-Party Settings



Source: 2015 BESIP (Fieldhouse et al., 2015b). *Note:* Maximum likelihood estimates with 95% CIs. The dark gray shaded areas show the generic parameters, the remaining areas the alternative-wise parameters. Online Appendix B reports complete tabled estimation results.

for some parties and underpredict the effects for others.

Likelihood ratio tests support this finding. The models with alternative-wise specification fit significantly better than the ones with generic specification (three-party setting: $\chi^2(4)= 265.61$, p=0.00; seven-party setting: $\chi^2(12)=570.15$, p=0.00). Thus, the alternative-wise specification more accurately reflects the behavior of voters and the assumption of identical effects for all parties does not hold.

The comparison also demonstrates that splitting the generic parameters into alternative-wise ones leaves the parameters for the three main parties largely unaffected. Al-

though there is some variation, the tendency is the same: The estimates do not systematically differ when including the four usually neglected parties, which indicates that there might be no strong violation of IIA. However, it should be clear that IIA is an underlying assumption of the model.

In the following, we discuss the substantive insights that can be obtained by the alternative-wise estimates in the seven-party setting, but one would miss when neglecting the effect heterogeneity and restricting the analysis to the three-party setting. The estimates indicate that the ideological proximity decision criterion is especially relevant for the Conservative vote, whose parameter is partly twice as large as the ones for the remaining parties. Particularly interesting is the large impact of ideological proximity on voting for the Liberal Democrats. By contrast, ideological proximity only marginally influences the Greens and PC votes and moderately affects Labour, UKIP, and SNP.

The overall impact of win probabilities is smaller than ideological proximities but still substantial and again with great variation across parties. Voters' expectations of the parties' chances to win the constituency are only of some relevance for the large parties, Conservatives and Labour. By contrast, the estimates suggest that when the Liberal Democrats or UKIP are perceived to win the constituency, tactical considerations strongly affect voting for them. Tactical considerations also influence voting for the Greens, SNP, and PC. However, these effects are significantly smaller than the ones for the Liberal Democrats and UKIP.

The analysis demonstrates the practical relevance of the proposed parameterization of effect heterogeneity and moving beyond the restricted three-party setting. In accordance with our expectations, the estimates indicate that voting for the two large

parties is less affected by tactical considerations and more by ideological proximity. In contrast, voting for smaller parties is influenced more by tactical considerations and less by ideological proximity.

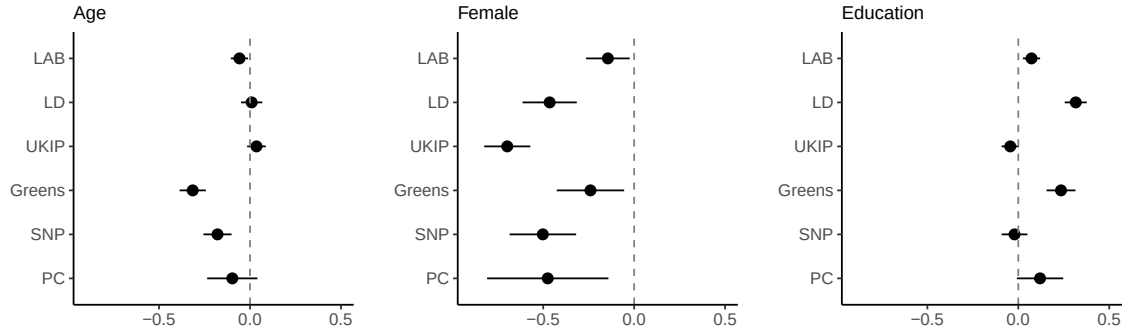
5.2 Voter Demographics

This section discusses the estimates related to the voter-specific covariates $\boldsymbol{s}_i$ in the seven-party setting, which tell us how socioeconomic factors affect voting for the smaller national and regional parties in addition to the usually investigated parties Labour, Conservatives, and Liberal Democrats. To ensure identifiability, we selected the Conservative party, which has the largest vote share and competes in all constituencies, as the reference choice alternative.

Figure 2 depicts standard voter demographics age, gender, and education. The estimates for age suggest that the Conservatives (compared to Labour, Greens, and SNP) attract older voter segments and the Greens are particularly strong among younger voter segments. By contrast, age is not decisive in voting for the Liberal Democrats, UKIP, and PC (compared to Conservatives). The estimates for gender indicate that females are less likely to vote for all parties (compared to Conservatives), and males are especially more likely to support UKIP; the odds of voting UKIP instead of Conservatives are remarkably 50 percent lower among females, ceteris paribus; $100 \cdot [\exp(-0.698 \cdot 1) - 1] = -50.24\%$. Finally, we find that education has a substantial positive effect on voting for the Liberal Democrats and the Greens and only marginally impacts the Labour vote (compared to Conservatives).

The estimates for the covariates capturing traditional cleavage structures along the line of social class also reveal interesting patterns (see Figure 3). Homeownership

Figure 2: Impact of Standard Voter Demographics in the Seven-Party Setting with Heterogeneous Ballot Compositions



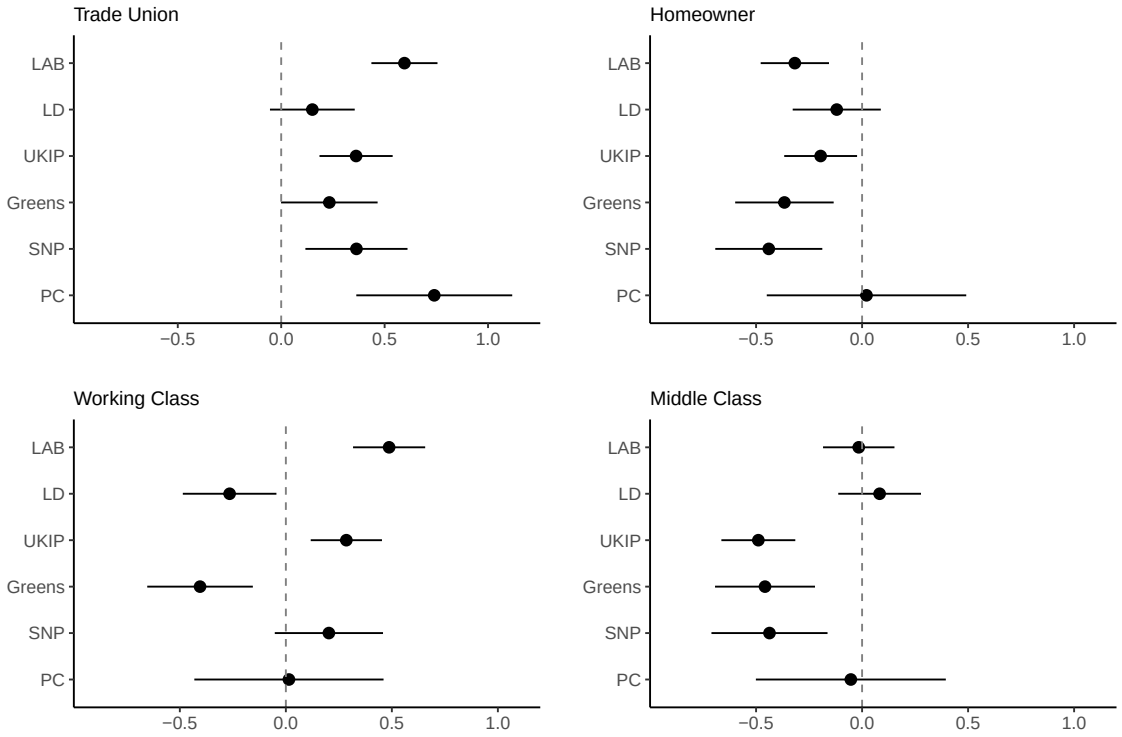
Source: 2015 BESIP (Fieldhouse et al., 2015b). *Note:* Maximum likelihood estimates with 95% CIs. Interpretation refers to the Conservatives (Cons).

has not only a substantial negative effect on voting for Labour but also on UKIP, SNP, and the Greens (compared to Conservatives). The strong positive impact of union membership on the Labour vote aligns with conventional sociological voting approaches. Our results indicate that also UKIP, SNP and PC attract union members. As expected, working-class voters are more likely to vote for Labour than the Conservatives. However, our model also uncovers a positive effect of working-class members supporting UKIP and a negative impact on voting for the Liberal Democrats and the Greens (compared to the Conservatives). A middle-class affiliation negatively impacts voting for UKIP, Greens, and SNP (compared to Conservatives).

6 The Varying Choice Set Logit Model: A Discussion

Existing studies on rational voting behavior that do not ignore ballot composition heterogeneity tend to apply the varying choice set logit model (VCL), which has been used to study partially-contested elections in Japan (Yamamoto, 2014), Canada (Gallego et al., 2014), Spain (Labzina et al., 2017), or Britain (Labzina and Schofield,

Figure 3: Impact of Voter Demographics related to Traditional Cleavage Structures in the Seven-Party Setting with Heterogeneous Ballot Compositions



Source: 2015 BESIP (Fieldhouse et al., 2015b). *Note:* Maximum likelihood estimates with 95% CIs. Interpretation refers to the Conservatives (Cons).

2015). The VCL model draws on a random coefficient approach, has no closed-form solution, and requires (Bayesian or frequentist) simulation-based estimation methods. A discussion of the VCL model will help illustrate its drawbacks and the advantages of the approach we propose to analyze vote choices in partially-contested elections.

We note that the VCL model is not the only existing model that can handle ballot composition heterogeneity. In practice, any choice model can be extended to account for varying choice sets, such as the mixed logit model (Glasgow, 2001; Glasgow and Golder, 2015) or the conditional binary quantile model (Lu, 2020). We discuss the VCL model because we are not aware of any contribution that applies these alternative models to the study of partially-contested elections in the context of rational vote choices. As the models can be embedded into the random coefficient framework, the major points we discuss here also apply to them. For a detailed discussion of the mixed logit model, see Mauerer and Tutz (2023). A major objective of these alternative models is to capture (unobserved) heterogeneity related to voter-specific covariates and not party-specific covariates as our vote choice model does. Another objective is to relax the IIA assumption, a point we return to at the section's end.

According to Yamamoto (2014, p. 8), the VCL model can be derived from the latent utilities

$$U_{ijb} = \beta_{j0} + \boldsymbol{s}_i^T \boldsymbol{\beta}_j + \mathbf{z}_{ij}^T \boldsymbol{\alpha} + \mathbf{w}_{ij}^T \boldsymbol{\tau}_b + \varepsilon_{ij}, \tag{6.1}$$

where $\mathbf{w}_{ij}$ is a subset of the party-specific covariates contained in $\mathbf{z}_{ij}$, and b indicates the choice set types that we call ballot compositions. The VCL model differs from the standard multinomial choice model in the term $\mathbf{w}_{ij}^T \boldsymbol{\tau}_b$, which captures ballot-specific effects as an interaction between covariates $\mathbf{w}_{ij}$ and ballot compositions b as effect modifiers. The parameters $\boldsymbol{\tau}_b$ are specified as random effects and assumed to follow

an i.i.d. multivariate normal distribution.

By allowing the parameters to vary randomly across ballot compositions, the VCL model is designed to detect how the effects of the covariates in $\mathbf{w}_{ij}$ differ among ballot compositions. From an electoral research perspective, one would be interested in understanding what causes heterogeneous effects across ballot compositions. However, the VCL model does not include covariates that characterize the ballot compositions and, therefore, cannot uncover what explains ballot-specific effects.

A closer inspection of the empirical applications of the VCL model raises concerns about the substantial insights into effect heterogeneity across ballot compositions gained and suggests that one can dispense this complexity. The analysis of Japanese elections in Yamamoto (2014, Figure 1, p. 18) shows almost no differences between the point estimates obtained from the standard multinomial choice model and the VCL model. However, the VCL model comes with much larger standard errors. Inspecting the ballots-specific effects for eleven different ballot compositions (see Figure 2, p. 20) reveals that these estimates are very similar across ballot compositions, with overlapping confidence intervals and large standard errors in most cases. Similar results are presented in Gallego et al. (2014) or Labzina et al. (2017).

When applying the VCL model, the researcher needs to decide which covariates exhibit heterogeneous effects across ballot compositions (enter $\mathbf{w}_{ij}$) and which ones are assumed to impact vote choice homogeneously, independent from the ballot composition (enter $\mathbf{z}_{ij}$). As the VCL model can be embedded into varying-coefficient framework (see Fan and Zhang, 1999; Hastie and Tibshirani, 1993; Park et al., 2015), identifiability issues occur if $\mathbf{w}_{ij}$ and $\mathbf{z}_{ij}$ are not different (Mauerer and Tutz, 2023).

The specification of random effects also involves the need to use simulation-based methods to obtain choice probabilities (see, e.g., Train, 2009), which are typically computationally demanding. Yamamoto (2014, p. 17) reports a total computation time of 40 hours in his empirical application, which is enormous for applied researchers. Furthermore, it typically comes with a large number of parameters that tend to be unstable without careful variable selection (see, e.g., Mauerer and Tutz, 2023).

The definition of random parameters also requires selecting a specific distribution that appropriately approximates the underlying behavioral process. The VCL model assumes a normal distribution. However, such an assumption is inappropriate in some choice situations, and alternative specifications are needed. Ideological proximities are an example because they are expected to exhibit only positive effects on voting.

The main difference between the VCL model and the choice model we propose lies in the specification of effect heterogeneity. The VCL model estimates ballot-specific effects but constrains the impact of party-specific covariates $\mathbf{z}_{ij}$ to be the same across parties. By contrast, we do not allow the effects to vary across ballot compositions but estimate for the party-specific covariates $\mathbf{z}_{ij}$ parameters $\boldsymbol{\alpha}_j$ that are specific to each party.

The parameterization of effect heterogeneity across parties permits analyzing how the party-specific covariates impact voting for each party, thereby offering a deeper understanding of their differential effects on voting for the numerous parties on heterogeneous ballots. The more parties are considered by modeling heterogeneous ballots, the more unrealistic the assumption becomes that party-specific covariates have the same effect on the choice of all these parties. In line with previous research (Mauerer

et al., 2015b; Mauerer and Tutz, 2024), our empirical application shows considerable effect heterogeneity for the party-specific covariates, yielding substantial insights not found when neglecting this effect heterogeneity.

Compared to the VCL model, our approach comes with convenient assumptions and properties. Our model assumes that party choices are independent across ballot compositions. Thus, complexity (in terms of the number of parameters) does not increase in dependence on the number of ballot compositions. However, it is a strong assumption, and the reference choice alternative must be chosen so that it is included in all ballots. On the other side, the assumption also makes sense when covariates specific to ballot compositions, which need to be motivated by theoretical expectations, are not given. This could be solved by interactions and fixed effects, which cause parameter inflation and no additional benefit is expected, given the results of VCL applications.

Another key benefit of our model is that applied researchers can easily use it as it provides a closed-form solution for calculating the choice probabilities. Parameter estimates can be obtained by the well-known maximum likelihood estimation technique without simulating choice probabilities with complex maximization methods. It also frees the researcher from selecting an appropriate distribution for the random effects. Therefore, our approach is computationally straightforward, analytically solvable, and easy to apply and interpret.

Several studies in the field perceive the IIA assumption as very restrictive and argue that it often needs to be relaxed in empirical applications (e.g., Glasgow, 2001; Glasgow and Golder, 2015; Lu, 2020). However, the IIA assumption is a convenient property as it allows a straightforward effect interpretation based on relative odds. It

is also always an empirical question of whether IIA holds in the respective research setting. Relaxing the IIA assumption by moving toward more complex models, such as the VCL model, raises other methodological concerns, such as the "Invariant Proportion of Substitution Property" (Steenburgh, 2008). It implies that the substitution ratio is independent of which choice attribute is improved, which can cause similar counterintuitive choice behavior as under IIA when the assumption is violated.

7 Concluding Remarks

In partially-contested elections, voters are not confronted with the same ballot nationwide. Instead, voters are offered different sets of parties depending on their region or constituency of residence. Building on the convenient multinomial choice model, we presented a modeling strategy to study rational choice behavior that takes such heterogeneous ballots and voting for the numerous parties therein seriously.

Our empirical findings for the 2015 British General Election, where we modeled the full range of heterogeneous ballots at the constituency level, demonstrate the substantial insights that our approach can obtain. Using British election survey data, we studied vote choices for up to seven parties spread across eight unique ballot compositions. The proposed parameterization of effect heterogeneity across parties uncovers substantial effect differences. Ideological proximity tends to affect voting for Conservatives and Labour more than for smaller parties. By contrast, tactical considerations exhibit a more considerable impact on voting for the smaller parties. These insightful results would remain hidden under previous approaches.

This contribution enables electoral researchers to take the actual composition of bal-

lots into account when studying the unique choices voters make at the polls, and we see great potential to advance our knowledge of rational voting behavior in other countries with partially-contested elections, such as Spain, Canada or Japan.

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Supplementary materials

Supplementary materials for this article can be found in the Online Appendix. Section A describes the data management. Section A.1 provides details on the construction of the ballot compositions. Section A.2 describes how we arrive at the BESIP subsample we use. Section B reports tabled estimation results. The code and data to reproduce the results of this article can be found in Mauerer and Walter (2024).

References

Adams, J., Merrill III, S., and Grofman, B. (2005). *A Unified Theory of Party Competition: A Cross-national Analysis Integrating Spatial and Behavioral Factors.*

Cambridge University Press, New York, NY.

Agresti, A. (2013). *Categorical Data Analysis*. Wiley, Hoboken, NJ, 3 edition.

Alvarez, R. M. and Nagler, J. (1998). When politics and models collide: Estimating models of multiparty elections. *American Journal of Political Science*, **42**(1), 55–96.

Alvarez, R. M. and Nagler, J. (2000). A new approach for modelling strategic voting in multiparty elections. *British Journal of Political Science*, **30**(1), 57–75.

Alvarez, R. M., Nagler, J., and Bowler, S. (2000). Issues, economics, and the dynamics of multiparty elections: The British 1987 general election. *American Political Science Review*, **94**(1), 131–149.

Alvarez, R. M., Boehmke, F. J., and Nagler, J. (2006). Strategic voting in British elections. *Electoral Studies*, **25**(1), 1–19.

Andersen, R. and Heath, A. (2002). Class matters: The persisting effects of contextual social class on individual voting in Britain, 1964-97. *European Sociological Review*, **18**(2), 125–138.

Cho, S. and Endersby, J. W. (2003). Issues, the spatial theory of voting, and British general elections: A comparison of proximity and directional models. *Public Choice*, **114**(3), 275–293.

Elff, M. (2009). Social divisions, party positions, and electoral behaviour. *Electoral Studies*, **28**(2), 297–308.

Endersby, J. W. and Galatas, S. E. (1998). British parties and spatial competition: Dimensions of party evaluation in the 1992 election. *Public Choice*, **97**(3), 363–382.

Evans, G. and Tilley, J. (2012). The depoliticization of inequality and redistribution: Explaining the decline of class voting. *The Journal of Politics*, **74**(4), 963–976.

Fan, J. and Zhang, W. (1999). Statistical estimation in varying coefficient models. *Annals of Statistics*, **27**(5), 1491–1518.

Fieldhouse, E., Green., J., Evans, G., Schmitt, H., van der Eijk, C., Mellon, J., and Prosser, C. (2015a). British Election Study, 2015: General Election Results Dataset. Data File Version 2.2. DOI: 10.13140/RG.2.1.1162.1844. Version 2.2.

Fieldhouse, E., Green., J., Evans, G., Schmitt, H., van der Eijk, C., Mellon, J., and Prosser, C. (2015b). British Election Study Internet Panel 2014-2015 (Waves 1-6). Data File Version 1.2. The University of Manchester. DOI: 10.15127/1.293723.

Gallego, M., Schofield, N., McAlister, K., and Jeon, J. S. (2014). The variable choice set logit model applied to the 2004 Canadian election. *Public Choice*, **158**(3), 427–463.

Glasgow, G. (2001). Mixed logit models for multiparty elections. *Political Analysis*, **9**(2), 116–136.

Glasgow, G. and Golder, S. N. (2015). A new approach to the study of parties entering government. *British Journal of Political Science*, **45**(4), 739–754.

Green, J. and Prosser, C. (2016). Party system fragmentation and single-party government: The British general election of 2015. *West European Politics*, **39**(6), 1299–1310.

Hastie, T. and Tibshirani, R. (1993). Varying-coefficient models. *Journal of the Royal Statistical Society*, **B 55**(4), 757–796.

Hensher, D. A., Rose, J. M., and Greene, W. H. (2015). *Applied Choice Analysis*. Cambridge University Press, Cambridge, England, 2 edition.

Labzina, E. and Schofield, N. (2015). Application of the variable choice logit model to the British general election of 2010. In Schofield, N. and Caballero, G., editors, *The Political Economy of Governance. Institutions, Political Performance and Elections*, pages 313–334. Springer, New York, NY.

Labzina, E., Barceló, J., and Schofield, N. (2017). Valence and ideological proximity in the rise of nationalist parties: Spanish general elections, 2008 and 2011. In Schofield, N. and Caballero, G., editors, *State, Institutions and Democracy*, pages 105–142. Springer, Cham, Switzerland.

LIMDEP/NLOGIT. Statistical & Data Analysis Software LIMDEP/NLOGIT, Econometric Software, INC., by William H. Greene.

Lu, X. (2020). Discrete choice data with unobserved heterogeneity: A conditional binary quantile model. *Political Analysis*, **28**(2), 147167.

Mauerer, I. (2016). A party-varying model of issue voting. A cross-national study. Doctoral dissertation, University of Munich (LMU), Germany.

Mauerer, I. (2020). The neglected role and variability of party intercepts in the spatial valence approach. *Political Analysis*, **28**(3), 303–317.

Mauerer, I. and Tutz, G. (2023). Heterogeneity in general multinomial choice models. *Statistical Methods & Applications*, **23**, 129–148.

Mauerer, I. and Tutz, G. (2024). Vote choices and valence: Intercepts and alternate specifications. *Political Analysis*, **FirstView**, 1–18.

Mauerer, I. and Walter, A. (2024). Replication Data for: Rational Voting Behavior Accounting for Heterogeneous Ballots in British Elections. DOI: 10.7910/DVN/AV8RZI, Harvard Dataverse, V1.

Mauerer, I., Pößnecker, W., Thurner, P. W., and Tutz, G. (2015a). Modeling electoral choices in multiparty systems with high-dimensional data: A regularized selection of parameters using the lasso approach. *Journal of Choice Modelling*, **16**(3), 23–42.

Mauerer, I., Thurner, P. W., and Debus, M. (2015b). Under which conditions do parties attract voters' reactions to issues? Party-varying issue voting in German elections 1987-2009. *West European Politics*, **38**(6), 1251–1273.

Moral, M. and Zhirnov, A. (2018). Issue voting as a constrained choice problem. *American Journal of Political Science*, **62**(2), 280–295.

Oscarsson, H. and Rosema, M. (2019). Consideration set models of electoral choice: Theory, method, and application. *Electoral Studies*, **57**, 256–262.

Park, B. U., Mammen, E., Lee, Y. K., and Lee, E. R. (2015). Varying coefficient regression models: A review and new developments. *International Statistical Review*, **83**(1), 36–64.

Quinn, K. M., Martin, A. D., and Whitford, A. B. (1999). Voter choice in multiparty democracies: A test of competing theories and models. *American Journal of Political Science*, **43**(4), 1231–1247.

Steenburgh, T. J. (2008). The invariant proportion of substitution property (IPS) of discrete-choice models. *Marketing Science*, **27**(2), 300–307.

Thurner, P. W., Mauerer, I., Bort, M., Klima, A., and Küchenhoff, H. (2020). Integrating large-scale online surveys and aggregate data at the constituency level:

The estimation of voter transitions in the 2015 British General Elections. *Survey Research Methods*, **14**(5), 461–476.

Train, K. E. (2009). *Discrete Choice Methods with Simulation*. Cambridge University Press, New York, 2 edition.

Tutz, G. (2012). *Regression for Categorical Data*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, New York.

Yamamoto, T. (2014). A multinomial response model for varying choice sets, with application to partially contested multiparty elections. Working Paper, Corpus ID: 42230431.