Horizontal product differentiation, pollution abatement and emission taxes under Cournot oligopoly

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Abstract

There is an increasing number of firms across a variety of industries, which are taking steps to differentiate themselves as environmentally conscious firms, both to achieve gains from product differentiation and to reduce production and pollution abatement costs. In light of this trend in many industries, this chapter considers a horizontally differentiated Cournot model to examine (i) the role of pollution abatement efforts by firms on the characterization of the optimal emission tax and (ii) the adjustment of optimal policy as products become differentiated. The analysis indicates that in the presence of abatement efforts by firms the government may afford a tax reduction to address the output distortion as products become more differentiated. Additionally, the analysis suggests that policies which may encourage firms to differentiate themselves as environmentally conscious firms may aid in the reduction of industry emissions, and at the same time allow a reduction in the emission tax to address the output distortion present in imperfectly competitive markets.

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1. Introduction

There is an increasing number of firms across a variety of industries, which are taking steps to differentiate themselves as environmentally conscious firms, both to achieve gains from product differentiation and to reduce production and pollution abatement costs (e.g., Eriksson, 2004; Conrad, 2005; Lyon and Maxwell, 2008; Portney, 2008; Lambertini, 2013). In many cases these firms operate in imperfectly competitive markets characterized by horizontal product differentiation and, at the same time, face carbon pricing policies such as emission taxes (e.g., Thollander and Ottosson, 2008; USDOC, 2010). As a result of the continued efforts by some of these firms to (i) differentiate themselves as environmentally conscious and (ii) engage in pollution abatement activities, traditional market-based environmental policies may need to account for these two aspects and adapt accordingly. This chapter considers a horizontally differentiated Cournot model, in the spirit of Fujiwara (2009), to examine the role of pollution abatement efforts by firms on the characterization of the optimal emission tax and the adjustment of optimal policy as products become differentiated. The analysis indicates that with sufficient abatement efforts by firms the government may lower the optimal emission tax in order to address the output distortion as products become more differentiated.

The literature of environmental policy under Cournot oligopolistic conditions is vast. In the context of a closed economy, one strand examines the characterization of optimal taxation assuming homogeneous goods but putting aside abatement efforts (e.g., Levine, 1985; Simpson, 1995; Lee, 1999), and including abatement efforts but ruling out the possibility of differentiated products (e.g., Ebert, 1992; Katsoulacos and Xepapadeas, 1995). A second strand incorporates horizontal product differentiation but assumes away abatement efforts by firms (e.g., Eriksson, 2004; Fujiwara, 2009; Espínola-Arrendondo and Zhau, 2012). The contribution of the present chapter is at the intersection of these two strands.

In a formal sense the present chapter is closest to Fujiwara (2009) where abatement efforts are assumed away and the conditions under which the optimal tax may fall/rise with more differentiated markets are derived. The author shows that in the case of exogenous number of firms the optimal tax may be positive or negative (subsidization), and that it may rise or fall as products become more differentiated depending on whether the government puts more weight on damages from pollution vis-à-vis the output distortion. Crucially, the present work differs from Fujiwara's in that abatement efforts are considered through two channels, namely, the direct reduction in marginal abatement costs (e.g., via cost-reducing environmental research and development or $R\&D$) and the ability of firms to differentiate themselves as environmentally conscious firms. These two important aspects, which are increasingly present in a number of industries, alter the characterization of optimal policy and the extent to which it falls or rises as products become more differentiated. It is noteworthy that these channels are assumed to be exogenous in the model since the strategic choice of abatement (e.g., cost-reducing environmental $R\&D$) has been analyzed elsewhere (e.g., Carlsson, 2000; Montero, 2002a, 2002b; Poyago-Theotoky, 2003; Ulph and Ulph, 1996, 2007; Yakita and Yamauchi, 2011).

Ebert (1992) points out in a rather general setting that, in the presence of abatement technologies and assuming away product differentiation, the optimal tax may be higher because of the abatement incentives provided by the tax; this result is important because it indicates the important role of abatement efforts in the characterization of policy. The present work builds from this important result by considering a more specific setting which allows to derive, among others, specific conditions under which the optimal policy is taxation and examine the role of product differentiation on abatement and, therefore, optimal policy, two aspects which have received little attention in the literature. Simpson (1995) and Levine (1985), among others, examine the effects of emission taxes on industry emissions assuming away abatement efforts and product differentiation, two important aspects in a number of industries. There is also a small literature which incorporates horizontal product differentiation, but differs from the present setting in that the analysis is undertaken in an international context (e.g, Lahiri and Symeonidis, 2007; Gautier, 2013) or in the presence of policies other than emission taxes (e.g., abatement subsidies) and assuming the strategic choice of $R\&D$ (e.g., Poyago-Theotoky, 2003). The present work assumes away the strategic interaction of the choice of $R\&D$ and the role of the abatement subsidy, and the focus is on how exogenous changes in the degree of product differentiation (e.g., firms differentiate themselves as environmentally conscious firms) may alter abatement efforts, the characterization of policy and consequently industry emissions.

A second contribution of this chapter is that it presents a systematic analysis of how changes in the degree of product differentiation affect industry emissions via adjustments in the optimal tax as well as the direct effect (for given tax). To see this consider the case where firms differentiate their products. As a result, output in this market rises, thereby raising emissions, if abatement efforts are assumed away. If in order to tackle higher emissions the government raises the tax, then industry emission may fall and, as a result, the change in industry emissions, arising from an exogenous change in the degree of product differentiation may not be clear-cut. Additionally, the inclusion of abatement efforts into the model changes the characterization of policy and consequently the extent to which the government adjusts the tax with more differentiated products.

The analysis indicates that abatement efforts play an important role in the characterization of the optimal tax, particularly in the case where changes in the degree of horizontal product differentiation works via changes in the pollution intensity coefficient i.e., environmentally conscious firms. It is shown that policy induced abatement efforts may result in taxation by the government even in the case where sufficient weight is put on addressing the output distortion. Additionally, the analysis indicates that in the presence of abatement efforts the government can afford a tax reduction as products become more differentiated even when damages to the environment is a concern to the government. As a result, industry emissions are likely to increase even when abatement efforts by firms are sufficiently large. One policy implication from the analysis is that additional policies might be needed if a reduction in industry emissions is to be achieved as products become more differentiated.

The chapter is structured as follows. The next section spells out the model and section 3 presents some of the comparative static results. Section 4 characterizes the optimal emission tax and examines its adjustment as products become more differentiated. Section 5 analyzes the effect of policy adjustment on industry emissions. The last section concludes.

2. The Model

Consider an industry with a fixed number of firms $n > 1$ operating under Cournot oligopolistic conditions in the presence of horizontal product differentiation. Each firm i $(i = 1, 2, \ldots, n)$ engages in the production of an imperfect substitute or variety i, which is sold in the domestic market; each firm also generates pollution and consequently additional costs to society, but also engages in pollution abatement activities. We shall follow Fujiwara (2009) in the structure of the demand function (see also Cellini et al., 2004); in particular, inverse demand faced by each firm i comes from preferences such that $p^i = \alpha - (\beta - \gamma)q^i - \gamma \sum q^i$, satisfying $\beta > \gamma > 0$, $\alpha > 0\ \forall i$, where γ represents the degree of product differentiation. As it is standard in the literature, $\gamma \simeq 0$ captures the case where products become very differentiated, and $\gamma \simeq \beta$ the case of homogeneous goods. q_i denotes output of variety i produced by firm i. The government and firms play a two-stage game where the government sets policy optimally (i.e., per-unit emission tax) through social welfare maximization, and firms then take policy as given and maximize profits by simultaneously choosing the level of output and emissions. Throughout we shall analyze the symmetric equilibrium, assume interior solutions and that firms behave in a Cournot-Nash fashion.

To capture pollution abatement efforts by firms, we shall assume the end-of-pipe cost function is defined as $C^i = \tilde{c}^i q^i + z^i(\delta^i(\gamma)q^i - e^i)^2/2$ where $\tilde{c} > 0$, $z \geq 1$, $\delta' > 0$, $\delta'' = 0$. The first term captures production costs and the second abatement costs, where units of pollution abated are given by $a^i = \delta^i q^i - e^i$. Several remarks about the cost function are in order. First, it satisfies the usual concavity assumptions; and second it captures abatement efforts through two different channels.[1] The first channel consists of the pollution intensity coefficient, δ, which is assumed to depend positively and linearly on the degree of product differentiation, γ. This means that firms may seek to differentiate themselves by being environmentally conscious via a lower pollution intensity coefficient i.e., $\delta = \delta(\gamma)$ where $\delta'(\gamma) > 0$ (we shall touch on the effects of this on firm behavior later on). Important examples include companies like Patagonia and Whole Foods, which differentiate themselves as environmentally sustainable companies, and Wal-Mart and Costco have made the commitment to lower energy consumption through solar lighting and heating (Portney, 2008: 263). Additionally, companies may also differentiate themselves to avoid negative public opinion (Portney, 2008; Lyon and Maxwell, 2008). The second channel is via the direct reduction in pollution abatement costs; in particular, a large (small) value of z denotes less (more) abatement efforts by each firm since a large (small)

[1] See Requate (2006) for the properties of the cost function and Lahiri and Symeonidis (2007) for examples of end-of-pipe cost functions.

value of z translates into an increase (reduction) in pollution abatement costs. It is noteworthy that neither of these channels intends to capture the strategic decisions of firms to increase/decrease abatement efforts and so they are exogenous in the model. The reason for this assumption via a specific cost function is threefold. First, the strategic decision of environmental R&D and differentiated products have been examined elsewhere (e.g., Carlsson, 2000; Montero, 2000a, 2000b; Conrad, 2005; Ulph and Ulph, 2007). Second, specific functional forms allows to track the potential role of abatement efforts in the presence of product differentiation, one of the main objectives of this chapter, and analyze the effects of different pollution abatement efforts (channels) by firms and their impact on optimal policy. By modeling these two channels we are able to compare some results with the existing literature and at the same time derive some new results.

Each firm i maximizes profits by choosing e^i and q^i in a Cournot-Nash fashion:

$$\max_{q^i e^i} \pi^i = p^i q^i - C^i(q^i, e^i) - \tau e^i - f^i \tag{1}$$

where f^i denotes fixed costs and τ the emission tax set by the government. This yields two first-order conditions, which under symmetry are given by

$$p - \beta q - \tilde{c} - \delta z(\delta q - e) = 0 \tag{2}$$

$$z(\delta q - e) - \tau = 0 \tag{3}$$

where $p = \alpha - q(\beta + \gamma(n-1))$. These implicitly determine the equilibrium level of output, q, emissions, e, and therefore abatement, a, and have the usual interpretation. In particular, the tax is equal to marginal abatement costs, and marginal revenue equals marginal production costs. Since the number of firms is fixed, total output, $Q = nq$, total emissions, $E = ne$, and total abatement, $A = na$, are also implicitly determined. Notice that in (3) a small ratio $1/z$ (large z) simply says that, for given tax, the firm engages in less direct abatement efforts since a large z increases abatement costs thereby lowering equilibrium abatement; conversely, a large ratio $1/z$, captures the idea that the firm engages in more abatement efforts via lower abatement costs.[2] For completeness, we include the expression for the Cournot-Nash equilibrium level of output, q, which will aid in the analysis presented in subsequent sections. In particular, (2) and (3) yield $q = (\alpha - \tilde{c} - \delta\tau)/D$ where $D = 2\beta + \gamma(n-1) > 0$.

3. Comparative Statics

The comparative statics effects are obtained via differentiation of (2) and (3). In particular, the effects of the tax and degree of product differentiation (for given tax) on output and emissions are given by

$$Ddq = -\delta d\tau - \left[q(n-1) + \tau\delta'\right] d\gamma \tag{4}$$

$$Dde = -\left[\delta^2 + D/z\right] d\tau + \left[Dq\delta' - \delta\delta'\tau - \delta q(n-1)\right] d\gamma \tag{5}$$

[2]Formally, for $z \geq 1$ the ratio $1/z \in (0, 1]$.

where $D = 2\beta + \gamma(n-1) > 0$. There are two main effects at play. First, an increase in the tax raises production costs via higher tax payments and, as a result, output and emissions fall; the larger (smaller) the pollution intensity coefficient the more (less) firms will have to pay in taxes. Second, the tax raises the incentives for pollution abatement and, as a result, emissions fall via the term $1/z$. Notice that this effect becomes smaller (larger) as firms engage in less (more) direct abatement i.e., the ratio $1/z$ becomes smaller (larger).

As products become more differentiated (i.e., decrease in γ), output rises because, on the one hand, firms are differentiated and on the other a smaller δ' (i.e., as firms differentiate themselves and become more environmentally conscious) reduces production costs. In terms of emissions there are two opposing effects at play here. On the one hand, with more differentiated products output and therefore emissions rise; and on the other, the pollution intensity coefficient falls as firms seek to become more environmentally conscious. This has two effects in turn. A smaller pollution intensity coefficient results in (i) lower costs in the form of less tax payments and, as a result, higher output and therefore emissions; and (ii) lower marginal abatement costs (i.e., a downward shift in the marginal abatement cost function) and thus lower emissions. If firms are pollution-moderate and sufficiently environmentally conscious (i.e., δ small and δ' large), then emissions may fall via more abatement as a result of differentiated products; formally (subscript denotes partial derivative), $e_\gamma > 0$ *if and only if* $\delta' > \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$.[3] Intuitively, a lower intensity coefficient, δ, controls the increase in emissions (which takes place with higher output as products become more differentiated) and a relatively large δ' helps offset increased emissions via lower abatement costs. If δ' is somewhat small then environmentally conscious firms partially offset increased emissions, but emissions still rise as products become more differentiated i.e., $e_\gamma < 0$, if $\delta' \leq \delta(n-1)/D$.

Proposition 1. *(a) Industry emissions fall as products become more differentiated if and only if firms are sufficiently environmentally conscious and pollution-moderate (i.e., $\delta' > \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$). (b) If δ' is somewhat small (i.e., $\delta' \leq \delta(n-1)/D$), then environmentally conscious firms partially offset increased emissions, but emissions still rise as products become more differentiated i.e., $e_\gamma < 0$.*

Proposition 1(a) suggests that as products become more differentiated industry emissions may fall by more for initially small values of γ, which are associated with small values of δ. Formally, under proposition 1(a) $e_\gamma > 0$ and $e_{\gamma\gamma} < 0$ where subscripts denote first and second order partial derivatives. The intuition is that for initially small values of γ (and therefore small values of δ) any increase in emissions, arising from higher output as products become more differentiated, is offset by a small intensity coefficient; at the same time, a large value of δ' indicates that emissions fall with more differentiated products since firms are sufficiently environmentally conscious. In the case where firms are not environmentally

[3] Analogously, $e_\gamma < 0 \Leftrightarrow \delta' < \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$, where it is assumed that $\alpha - \tilde{c} - 2\delta\tau > 0$. A sensitive δ with respect to γ (i.e., δ' large) captures the idea that firms are relatively more environmentally conscious, whereas a small (less sensitive) δ' captures the case where firms are less environmentally conscious. As firms differentiate themselves (this is captured via a decrease in γ) they become more environmentally conscious which results in a reduction in the pollution intensity coefficient, δ. Thus, the extent to which firms are less or more environmentally conscious is captured via a smaller or larger δ', respectively.

conscious (i.e., $\delta' \simeq 0$) emissions rise by more as products become more differentiated since (i) firms are not taking any steps to lower emissions via the intensity coefficient and (ii) output rises rapidly at initially small values of γ as product become more differentiated; formally, $e_\gamma < 0$ and $e_{\gamma\gamma} > 0$ (see the appendix for a derivation). These results illustrate the important role of abatement via changes in the degree of product differentiation.

Proposition 2. *If firms are sufficiently environmentally conscious (i.e., $\delta' > \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$), then emissions are more likely to fall by more at initially small values of γ. Alternatively, if firms are not sufficiently conscious ($\delta' \simeq 0$), then emissions are more likely to rise for initially small γ.*

4. Optimal Policy

The government chooses the emission tax,τ, through welfare maximization. In particular, the government solves

$$\max_{\tau} W = CS + ne\tau + n\pi - \varphi(en) \tag{6}$$

where CS denotes consumer surplus, and the function φ represents the damage from pollution satisfying $\varphi' > 0$ and $\varphi'' > 0$. Optimal policy is thus characterized by the following first-order condition (subscripts denote partial derivatives):

$$\begin{aligned} \frac{1}{n} W_\tau &= \beta q q_\tau - (\varphi' - \tau) e_\tau = 0 \\ &= -\delta\beta q/D + (\varphi'^2/D) = 0 \end{aligned} \tag{7}$$

There are two effects which the government needs to balance, namely, the output distortion and the damage from pollution. It is easy to show that the optimal tax is set below marginal damages in order to address the output distortion since $e_\tau < 0$ and $q_\tau < 0$. Additionally, evaluating (7) at $\tau = 0$ and using the strict concavity assumption of the $W(\cdot)$ function, indicates that the optimal tax is positive if the abatement effect is large; that is, if the incentives of the tax to induce abatement are sufficiently strong. Formally, at $\tau = 0$ $W_\tau > 0$ if φ'/z is large. In this case optimal policy is taxation even when the government cares about addressing the output distortion vis-à-vis damages to the environment, thereby ruling out the possibility of subsidizing output (i.e., negative optimal emission tax) because of the role of abatement.[4] To see this result consider, as in Fujiwara (2009), a damage function so that $\varphi' = en$.[5] The closed-form solution for the optimal emission tax is then given by

$$\tau^* = \frac{(\alpha - \tilde{c})\delta\left(Dn/z - \beta + \delta^2 n\right)}{\delta^2\left(Dn/z - \beta + \delta^2 n\right) + D^2/z + D\delta^2 + (Dn/z)(D/z + \delta^2)} \tag{8}$$

where $\tau^* > 0$ *if and only if* $Dn/z - \beta + n\delta^2 > 0$ and $D = 2\beta + \gamma(n-1) > 0$. The term Dn/z captures the abatement induced by the tax (see equation 5) and β the output

[4] A large optimal tax in the presence of abatement is also pointed out by Ebert (1992, p.161).

[5] Strictly speaking Fujiwara's model assumes $\delta = 1$, $1/z = 0$ and $\varphi\hat{a} = sne$. Equation (8) is equivalent to Fujiwara's under these two assumptions, plus $s = 1$.

distortion. Intuitively, the tax is positive to tackle the damage from pollution via higher induced abatement and production costs, and set below marginal damages to address the output distortion. This result, however, contrasts with Fujiwara (2009) where abatement efforts are assumed away. In the present analysis Fujiwara's result is a special case and so it is obtained if $1/z \simeq 0$ i.e., abatement efforts are negligible. In the special case where $z = 1$, then the optimal tax is unambiguously positive.

Proposition 3. *Let $\varphi' = en$. In the presence of sufficiently large induced pollution abatement, (i.e., $Dn/z > \beta - n\delta^2$), optimal policy is taxation.*

Next, the effect of product differentiation on policy is examined. As a first case, it is noteworthy that if $\delta = 1$, $\delta' = 0$ and $1/z \simeq 0$, then identical results to Fujiwara (2009) are obtained through differentiation of (8) with respect to γ. In order to examine the role of abatement efforts it is useful to differentiate $W_\tau(\tau(\gamma), \gamma) = 0$, which yields $-W_{\tau\tau} d\tau/d\gamma = W_{\tau\gamma}$ where $W_{\tau\tau} < 0$:

$$n^{-1}W_{\tau\gamma} = (\varphi' - \tau)\frac{(n-1)}{zD} + \beta q_\tau q_\gamma - \varphi'' e_\tau \delta q_\gamma$$

$$+ \delta' \left(\frac{\delta(\varphi' - \tau)}{D} - \varphi'' e_\tau q - \frac{(\varphi' - \tau)}{z\delta} \right) \quad (9)$$

where from (7) $W_{\tau\gamma}$ is the direct effect of γ (i.e., for given tax) and $\beta q - \delta(\varphi' - \tau) = (\varphi' - \tau)D/z\delta > 0$, and $e_\tau < 0$, $q_\tau < 0$, $q_\gamma < 0$. The first term in (9) is positive because of the presence of pollution abatement; that is, because of pollution abatement by firms the government can afford a reduction in the optimal tax as products become differentiated to address the output distortion arising from the increased market power by firms. The second term,$\beta q_\tau q_\gamma$, is the profit effect which is positive since a lower tax raises profits; and the third term, $-\varphi''\delta e_\tau q_\gamma$, is negative thus indicating that the government needs to address higher pollution arising from the increase in output as products become more differentiated. The last three terms (i.e., the terms in $\delta'(\cdot)$) capture the effects via firms differentiating themselves as environmentally conscious firms. In particular, the first two terms are positive which indicates that the government can afford a reduction in the tax due to the reduction in emissions by firms via δ' i.e., lower emissions via abatement from environmentally conscious firms. The last term is negative and captures the increased emissions arising from lower production costs and therefore higher output as firms differentiate themselves.

As a benchmark case where $\delta' = 0$ and $1/z \simeq 0$ (i.e., no abatement) the tax falls *if and only if* the government puts more weight on the output distortion (i.e., $\beta - \varphi''\delta > 0$); this is analogous to results in the literature. In the case where $1/z \neq 0$ and $\delta' = 0$, the tax falls with more differentiated products (decrease in γ) *if and only if* $1/z$ is large even if the government puts enough weight on environmental damages ($\beta - \varphi''\delta < 0$).[6] This result

[6]Since $z > 1$, $1/z \in (0, 1)$ and so we shall assume that the condition of large $1/z$ is consistent with these bounds. Formally, from (9) simplification in this case yields a polynomial of degree two in $1/z$. In any case, the presence of $1/z > 0$ ensures that the government is likely to increase the tax by less as products become more differentiated.

illustrates the role of abatement, in particular the incentives for abatement the tax induces, which affords the government a tax reduction, to address the output distortion arising from more differentiated markets, even when damages to the environment are sufficiently important. Indeed, even if $1/z$ is not large enough the government still gains some room to lower the tax to address the output distortion and so the emission tax is likely to increase by less in the case where the government cares about damages to the environment.

Next, consider the case where $1/z \simeq 0$, $\delta' \neq 0$. In this case the optimal tax falls via firms being more environmentally conscious and the promotion of profits, but rises as more emissions take place as products become more differentiated. If the effect which takes place via firms being more environmentally conscious is large (i.e., $\delta' > \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$), then the tax falls even as the government cares a lot about environmental damages; this is because the reduction in emissions, which takes place as firms become environmentally conscious, affords the government a reduction in the tax to encourage output and profits.

Proposition 4. *Let $1/z \simeq 0$, $\delta' \neq 0$. Then, if firms are sufficiently environmentally conscious (i.e., $\delta' \geq \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$), then the optimal tax falls with more differentiated products even as the government cares a lot about environmental damages.*

If the effect via δ' is rather small (i.e., firms are not necessarily very environmentally conscious in the sense that $\delta' < \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$) and the government cares more about the output distortion vis-à-vis damages from pollution (i.e., $\varphi''\delta$ is small), then the tax falls as γ falls; the tax rises if the government cares sufficiently about the environment (i.e., $\varphi''\delta$ is large). In particular, if $1/z \simeq 0$, $\delta' \neq 0$ then using (7) the expression in (9) becomes

$$Dn^{-1}W_{\tau\gamma} = -\delta q_\gamma(\beta - \varphi''\delta) + \delta' q(\beta + \varphi''\delta)$$

$$= \delta\varphi'' e_\gamma + \beta(\delta' q - \delta q_\gamma) \tag{10}$$

where $q_\gamma < 0$ and as indicated in proposition 1(a) $e_\gamma = \delta q_\gamma + \delta' q < 0$ *if and only if* $\delta' < \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$ where $\alpha - \tilde{c} - 2\delta\tau > 0$.

Proposition 5. *In the absence of abatement efforts, the optimal tax rises if and only if the government cares sufficiently about the environment. However, in the presence of sufficient pollution abatement efforts by firms either directly (i.e., via $1/z$) or by differentiating themselves as environmentally conscious (i.e., via δ'), then the optimal tax may fall as products become differentiated even as the government cares sufficiently about environmental damages (i.e., $\beta < \varphi''\delta$).*

In the case where there is some form of abatement both directly and via differentiation (i.e., $1/z \neq 0$ and $\delta' \neq 0$) there is a myriad of cases so here we point a few. For a range of δ' the extent to which the optimal tax may decrease or increase as products become more differentiated depends on whether the government puts sufficient weight on environmental damages. In particular, (9) can be re-written as follows:

$$Dn^{-1}W_{\tau\gamma} = \frac{\varphi' - \tau}{\delta}\left(-\delta'/z + \frac{\delta(n-1)}{zD} + \delta^2\delta'\right) + \beta q_\gamma q_\tau - \varphi'' e_\tau e_\gamma \tag{11}$$

where the first two terms vanish if $\delta(\alpha-\tilde{c}-\delta\tau)(n-1)/D(\alpha-\tilde{c}-2\delta\tau) > \delta(n-1)/D = \delta'$, the last term is negative since $e_\tau < 0$ and $e_\gamma < 0$, and the remaining terms are positive. Intuitively, for small δ' abatement efforts by firms are partially offset (i.e., $e_\gamma < 0$) and so the government still needs to balance damages to the environment. If φ'' is small then the tax falls as products become more differentiated.

If $\delta' \geq \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$, then the last term in (11) vanishes as firms differentiate themselves sufficiently in the sense that there is no need to adjust the tax upwards via this effect (i.e., $e_\gamma \geq 0$); the first and third terms in parentheses are negative and positive, respectively, thus indicating that the increase in emissions is tackled via a higher tax but also output is encouraged through less taxation. If this latter effect is large enough($1/z \leq \delta^2$), then the optimal tax falls. Intuitively, on the one hand, as firms take on sufficient abatement via product differentiation, δ', emissions do not rise as much so there is less need for a tax increase with more differentiated products; on the other, with insufficient direct abatement, $1/z \leq \delta^2$, the increase in emissions, arising from the room the government has to afford a tax reduction due to abatement efforts, $1/z$, is less which in turn suggests that there is less need for a tax increase.

Proposition 6. *Let $1/z \neq 0$, $\delta' \neq 0$. Then, if firms take on sufficient abatement as environmentally conscious firms (i.e., $\delta' \geq \delta(\alpha - \tilde{c} - \delta\tau)(n-1)/D(\alpha - \tilde{c} - 2\delta\tau)$) and relatively less direct abatement (i.e., $1/z \leq \delta^2$), then the optimal tax falls with more differentiated products.*

5. Product differentiation and industry emissions

This section looks at the change in industry emissions as products become more differentiated both via the tax adjustment and the direct effect. One of the main policy implications derived in this section is that industry emissions may still rise even as the government adjusts the optimal tax in order to balance the output distortion and damages from pollution. It is also shown that in some cases industry emission may fall even as the government lowers the tax since, among others, firms become sufficiently environmentally conscious. We shall consider some of the results from previous sections and differentiation of $e(\tau(\gamma), \gamma)$, which gives

$$de/d\gamma = e_\tau \tau_\gamma + e_\gamma \tag{12}$$

where the first term captures the effect via the tax adjustment and $e_\tau < 0$, and the second term, e_γ, the direct effect (i.e., for given tax). Let $1/z \simeq 0$ and $\delta' = 0$ as a first benchmark case (i.e., no abatement). In this case the last term in (12) is negative and $\tau_\gamma > 0$ *if and only if* the government weighs the output distortion sufficiently, and as a result industry emissions rise with more differentiated products. Intuitively, since the government lowers the tax to address the output distortion emissions rise via the tax and, additionally, for a given tax emissions also rise because of an increase in output as product become differentiated. This result suggests that in the absence of abatement efforts an additional policy (e.g., output subsidy) may be needed to address the output distortion and at the same time tackle higher emissions. In contrast, if the government cares about damages to the environment then the tax rises with more differentiated products and, as a result, emissions fall via the tax; emissions still rise via the direct effect. Thus, industry emissions fall *if and only if* the effect via the tax is sufficiently strong (i.e., $-\delta q_\tau(\beta-\delta^2\varphi'')/nW_{\tau\tau} > 1$ where $\beta < \delta^2\varphi''$).[7] In particular, in the benchmark case where $1/z \simeq 0$ and $\delta' = 0$ total change in emissions is given by

$$\frac{de}{d\gamma} = \frac{-q_\gamma\delta}{D}\left(\frac{-\delta q_\tau(\beta-\delta^2\varphi'')}{nW_{\tau\tau}} - 1\right) \tag{13}$$

where $W_{\tau\tau} < 0$, $q_\tau < 0$, $q_\gamma < 0$, $D = 2\beta + \gamma(n-1) > 0$. As a second case, if $1/z \neq 0$ and $\delta' = 0$ then the tax falls as products become more differentiated with sufficient abatement (i.e., $1/z$ large) and emissions still rise via the direct effect. As a result, industry emissions rise as products become more differentiated. If instead the abatement effect is small analogous results to (13) are obtained.

Next, suppose that $1/z \simeq 0$, $\delta' \neq 0$, $\delta' < \delta(\alpha-\tilde{c}-\delta\tau)(n-1)/D(\alpha-\tilde{c}-2\delta\tau)$. In this case emissions rise via the direct effect, the tax may rise or fall and so emissions may rise or fall via the tax. If damages are not important (i.e., $\varphi''\delta$ is small) then the tax falls as products become more differentiated thereby raising emissions. As a result, industry emissions rise with more differentiated products. This result indicates that if firms are not very environmentally conscious, then industry emissions rise if the government does not care sufficiently about pollution damages. In particular, using (12) and (10) gives

$$\frac{de}{d\gamma} = -e_\gamma\left(\frac{\delta q_\tau\varphi''}{W_{\tau\tau}} - 1\right) - e_\tau\frac{\beta(\delta' q - \delta q_\gamma)}{W_{\tau\tau}} < 0 \;\; if \;\; \delta q_\tau\varphi''/W_{\tau\tau} < 1 \tag{14}$$

where $e_\gamma < 0$. In the special case where $\delta' = \delta(\alpha-\tilde{c}-\delta\tau)(n-1)/D(\alpha-\tilde{c}-2\delta\tau)$ and $1/z \simeq 0$ the direct effect is negligible and the tax falls with more differentiated products and, as a result, industry emissions rise unambiguously. If $\delta' > \delta(\alpha-\tilde{c}-\delta\tau)(n-1)/D(\alpha-\tilde{c}-2\delta\tau)$, then the direct effect is positive and the tax falls with more differentiated products; therefore, industry emissions may fall if the adjustment in the tax is small.

Proposition 7. *Industry emissions rise if firms take on sufficient abatement effort directly i.e., $1/z$ is large. If sufficient abatement efforts are taken via differentiation i.e., δ' is large, then industry emissions may fall if the tax adjustment is small.*

Proposition 7 suggests that the presence of abatement increases the likelihood of increased industry emissions in a number of cases; this is because in the presence of abatement the government is more likely to afford a tax reduction to address the output distortion as products become more differentiated. This in turn indicates the need for additional policies, even in the presence of abatement efforts, if lower industry emissions are to be achieved in the case where changes in the degree of product differentiation take place. Additionally, policies to encourage/discourage either type of abatement may also aid to achieve the objective of lowering industry emissions.

[7] Industry emissions are given by $E = ne$. Since n is fixed, results in this section apply to industry emissions.

6. Conclusion

This chapter examines the role of abatement efforts by firms in the characterization of the optimal emission tax as products become more differentiated. The analysis becomes relevant particularly since across a variety of industries firms are taking steps to differentiate themselves as environmentally conscious. As a result, policymakers should adapt environmental policy to this changing business environment in order to address environmental damages and potential market distortions. Surprisingly, the literature on environmental policy in the presence of horizontal product differentiation and abatement efforts has put only partial attention to this issue. The analysis indicates that in the presence of abatement efforts by firms the government may afford a tax reduction as products become more differentiated. This result is important because it illustrates the role abatement efforts may have in allowing the government to address the output distortion, an issue which is prevalent in a number of imperfectly competitive industries. This adjustment in the tax, however, may lead to an increase in industry emissions, which in turn raises the question of potential policies which may be implemented along with the emission tax. In this regard, the analysis indicates that policies which may encourage firms to differentiate themselves as environmentally conscious firms may aid in the reduction of industry emissions and at the same time allow a reduction in the emission tax.

The results and policy implications presented in this chapter are inevitably limited to the assumptions made about the functional forms of the cost function, fixed number of firms and strategic behavior of firms. As a result, there are several lines of research which may be explored. First, the case of free entry and exit of firms may be considered as a potential interesting case, particularly in industries where entry costs are relatively small. Second, the assumption about exogenous changes in the degree of product differentiation could be relaxed. Although the literature has explored this issue in the context of vertically differentiated markets (e.g., Bansal and Gangopadhyay, 2003; Conrad, 2005; Rodríguez-Ibeas, 2006), the present model set-up could be extended to include consumer heterogeneity. As mentioned earlier, the analysis indicates that additional policies may be needed to complement the emission tax and, as a result, the model could incorporate additional policies such as subsidies (as in e.g., Poyago-Theotoky, 2003; McGinty and de Vries, 2009). Since a key aspect of the chapter is the presence abatement efforts, an extension of the model may include more general functional forms for the cost function as long as the abatement component remains tractable throughout the model.

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A Appendix

Here we derive equation (7). The welfare function, $W(\cdot)$, is given in equation (6) i.e., $W = CS(Q) + ne\tau + n\pi - \varphi(en)$. Then, the derivative of W with respect to τ is given by

$$W_\tau = -nqp_\tau + n\pi_\tau + (\tau - \varphi')ne_\tau + ne$$

where it is well known that $dCS = -Qdp$. Factoring out n gives

$$\frac{1}{n}W_\tau = -qp_\tau + \pi_\tau + (\tau - \varphi')e_\tau + e \tag{A.1}$$

Next, we find the expression for π_τ. Profits are given by

$$\pi = pq - \tilde{c}q - z(\delta q - e)^2/2 - e\tau - f \tag{A.2}$$

where $\delta q - e = a = \tau/z$ from the first order condition in equation (3). Using $e_\tau = \delta q_\tau - a_\tau = \delta q_\tau - 1/z$, the derivative of (A.2) with respect to τ yields

$$\begin{aligned}\pi_\tau &= (p - \tilde{c} - \delta\tau)q_\tau - \tau/z - e + \tau/z + qp_\tau \\ &= (p - \tilde{c} - \delta\tau)q_\tau - e + qp_\tau\end{aligned} \tag{A.3}$$

where $p - \tilde{c} - \delta\tau = \beta q$ from the first order conditions given by equations 2 and 3. Hence, (A.3) can be re-written as

$$\pi_\tau = \beta q q_\tau - e + qp_\tau \tag{A.4}$$

Substituting (A.4) into (A.1) and simplifying gives equation (7).

Next, we derive the condition for e_γ to be negative or positive. In particular, from (5)

$$\begin{aligned}e_\gamma &= \frac{\delta'}{D}(Dq - \delta\tau) - \frac{\delta q(n-1)}{D} \\ &= \frac{\delta'(\alpha - \tilde{c} - 2\delta\tau)}{D} - \frac{\delta(\alpha - \tilde{c} - \delta\tau)(n-1)}{D^2}\end{aligned} \tag{A.5}$$

Hence,

$$e_\gamma > 0 \Leftrightarrow \delta' > \frac{\delta(n-1)(\alpha - \tilde{c} - \delta\tau)}{(\alpha - \tilde{c} - 2\delta\tau)D} \tag{A.6}$$

where $D = 2\beta + \gamma(n-1)$ and by assumption $(\alpha - \tilde{c} - 2\delta\tau) > 0$. If $\delta' \simeq 0$, then $e_\gamma < 0$. Next, we derive the expression for $e_{\gamma\gamma}$. Differentiation of e_γ in (5) w.r.t. γ gives

$$e_{\gamma\gamma} = \frac{2\delta(n-1)^2 q}{D^2} + \frac{2\delta'}{D}\left(-q(n-1) - \tau\delta' + \frac{\delta\tau(n-1)}{D}\right) \tag{A.7}$$

If δ is small and δ' large (i.e., $\delta' > \delta(n-1)(\alpha - \tilde{c} - \delta\tau)/(\alpha - \tilde{c} - 2\delta\tau)D$), then $e_{\gamma\gamma} < 0$. If $\delta' \simeq 0$ then $e_{\gamma\gamma} > 0$. □