The intensive and the extensive margins: not only an international issue

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Abstract Empirical evidence shows that quantity (intensive margin) and variety availability (extensive margin) have effects of different magnitude on populations' welfare. Indeed, the pattern of a market dynamics may cause changes in welfare inequality. Low income consumers benefit more from quantity than high income consumers, who are more interested in enjoying variety.

These facts have been usually addressed as consequences of trade liberalization by international trade theory. However, market dynamics are also present within the borders of every country. It is important to understand what forces, unrelated with international trade, affect these dynamics. This paper explores the transmission of different real shocks into market dynamics in a new-Keynesian closed economy. Results show that the source of the shock is crucial to determine the magnitude and direction of the effects on each margin.

Keywords: extensive margin, innovation, market dynamics, endogenous entry, real shocks.

JEL classification: E32, E52

1 Introduction

A common view of theorists and applied economists is that welfare improves both with the quantity and the range of available varieties in the market. For instance, Funke and Ruhwedel (2003, 2008) run a quantitative assessment of welfare gains from variety for fourteen Central and Eastern Europe countries and for China and conclude that the magnitude of the improvement is relevant for all countries.[1] However, literature has mainly focused on the changes of diversity and quantities (which I call *market dynamics*) due to trade openness. As a consequence, almost all the existing literature develops long-run analyses linked to trade barriers removal and disregards the fact that the structure of the market (i.e., quantities and varieties available in the market) also varies for reasons other than international integration.

Some of the exceptions exploring close economies with firm entry are Bergin and Corsetti (2008) and Bilbiie et al. (2007). They analyse the influence of market dynamics on the transmission of the monetary policy and explore the potential optimal monetary policy rules for this scenario. Bilbiie et al. (2007) show that firms entry and exit has implications for inflation dynamics and for the equation of the New Keynesian Phillips Curve. Bilbiie et al. (2012) show that considering market dynamics improves the performance of standard real business cycle models.

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[1] Regarding theoretical literature, Krugman (1991), for example, has emphasized the role of product diversification on welfare in the context of spatial economics.

This paper aims to a better understanding of the goods market dynamics within national borders. To address this issue, I develop a model of a New-Keynesian closed economy and explore the responses of intensive (volume) and extensive (range of varieties) margins to real shocks. In so doing, the model takes into account the fact that household welfare depends both on the quantity and diversity of consumption, i.e., a household prefers to consume a bit of everything rather than a lot of one homogeneous good.

I distinguish between two technologies in the economy. The first determines the capacity to invent new varieties. Firms compete with their differentiated varieties in a monopolistic market. They devote one period to designing a variety and constructing a plant in which to produce it. Having done this, they then produce and sell this variety in the following period. However, the competition to offer new or improved varieties means that today's goods are obsolete after just one period of production. As such, firms are obliged to use their resources to invent new varieties every period. Thus, varieties should not be considered as haute couture fashion items but rather as manufactured goods that need to be constantly adapted and improved. New models of computers or cameras and the latest fashions for the forthcoming season are just some examples of the "butterfly" varieties that make up long-lived goods. This assumption, also used in Bergin and Corsetti (2008) simplifies the algebra considerably and allows to have an analytical solution. The second technology determines the method of production. Both technologies are homogeneous across firms and subject to external shocks. Here, the consideration of the two technologies has a considerable bearing on the results as each of them affects real variables, albeit in distinct ways.

Previous literature has established the linkages between market dynamics and monetary policy. This paper uses a microfounded quasi-dynamic framework similar to that in Bergin and Corsetti (2008) and study the response of intensive and extensive margins to different sources of real shocks. I refer to a quasi-dynamic framework because households take intertemporal decisions, although I focus my attention primarily on comparative statics. It is reasonable to believe that the effects of real shocks will differ when one accounts for the future expectations of agents. Likewise, uncertainty plays a crucial role in the maximizing behaviour of agents and, as a consequence, in the number of firms operating in a market. In order to capture this as simply as possible, I introduce risk to firms via price rigidities.

One of the main outcomes of the model is the fact that drawing a distinction between two kinds of productivity (of creation and of production) is crucial. The former increases production via the extensive margin and, as a consequence, reduces the size of firms. The latter has a negative impact on the number of varieties. A positive shock to the size of the population increases the number of firms, in a closed-economy "home market effect" equivalence. The presence of fixed costs generates dependence between the number of firms and household patience (measured by the subjective discount factor of future consumption). When households are more patient, they save more. As a result, the economy houses a larger number of acting firms. However, this cushions any shock on the ability to create new varieties.

Moreover, nominal rigidities place the economy in a suboptimal point in terms of welfare. In comparison with the flexible-price scenario, prices rise to higher levels and the number of firms falls. As such a result might justify intervention, I explore here the role of a central bank. The monetary authority is able to correct these imperfections by implementing a monetary policy that is tied to the level of productivity in production. However, if the central bank fails to choose the most appropriate monetary policy, this can destabilize the economy. The consequences of this are discussed. Finally, although the paper does not specifically analyze the role of fiscal policy as a tool for addressing market imperfections, a brief discussion is included in the section examining price stickiness.

This paper is organized as follows: Section 2 presents a brief overview of the literature that has considered the extensive margin in the open economy, while the sections that follow examine the possibilities of building analytical tools in a closed-economy setting. Section 3 establishes the general set-up for the basic model;

Section 4 develops a flexible-price version; Section 5 incorporates the rigidities in the prices chosen by the firms and examines the impact of monetary policy and the problems generated by insufficient stabilization. Finally, Section 6 concludes. An appendix containing the algebraic details is available at the end.

2 A look at open economy literature

With some relevant exceptions, the dynamics of markets that I aim to analyse have been mainly studied by trade theory. Nevertheless, even the vast body of "new-trade theory" literature often disregards the fact that the surge of trade has occurred in two dimensions, quantities and varieties, and abstracts from diversity in products. Hummels and Klenow (2005), analyzed the exports in 1995 from 110 countries to 59 importers and decomposed the greater trade of larger economies into contributions from intensive and extensive margins. Their main finding was that the extensive margin accounts for two-thirds of the greater exports of larger economies, and one-third of the greater imports of larger economies. Similar results are obtained by Funke and Ruhwedel (2001). Moreover, Hummels and Klenow (2005) greatly extend the results of previous studies regarding the relationship between the size of an economy, international trade, and product variety, i.e., they shed considerable light on the empirical aspects of the home market effect.[2] Broda and Weinstein's (2006) empirical analysis offers an extensive discussion of the dramatic consequences the traditional set-up, with a fixed number of products, has for research findings. By assuming that Krugman's (1980) model fits US data, they show the mismeasurement caused by the use of an incorrect price index that does not account for changes in the number of varieties. They conclude that US welfare increased by 2.83 percent solely as a result of the expansion in varieties from 1990-2001. These gains from variety are three to six times larger than the estimated gains from eliminating protectionism (e.g., Feenstra (1992) and Romer (1994)) and around ten times larger than the estimated gains from eliminating business cycles (Alvarez and Jermann (2004)).

From a theoretical perspective, however, the traditional literature has often assumed that the number of firms is given or fixed. This has prevented experts from accurately determining the implications of different shocks on the range of varieties available in different countries and, consequently, on national and international welfare. An exception to this rule are monopolistic competition models in the vein of Krugman (1980, 1981). They stress the "extensive" margin for exports (i.e., economies twice the size will produce and export twice as many goods).

All this literature concentrates in the international aspect of market dynamics and relies on trade liberalization to explain them. However, by doing so, it may disregard other forces that are relevant even at the national level and that help explaining changes in the structure of markets.

A number of "new open-economists" including Ghironi and Melitz (2005), Corsetti et al. (2007), Cavallari (2010) and Auray and Eyquem (2011) have recently begun to take this essential characteristic into account.[3] Ghironi and Melitz (2005) use a two-country heterogeneous firms model and explore changes in firm decisions to operate at the national and international level. They provide a microfounded explanation for a Harrod-Balassa-Samuelson effect in response to aggregate productivity differentials and deregulation. Corsetti et al. (2007) use homogeneous firms and analyze the international transmission and welfare implications of productivity gains and changes in market size, when the adjustment occurs in the intensive and the extensive margins. Ghironi and Melitz's (2005) approach differs from that adopted by Corsetti et al. (2007) in three

[2] In fact, they find that countries with more workers export greater quantities to each market, but at prices that are not lower. This is consistent with a model in which larger countries avoid terms of trade deterioration by enlarging the set and/or increasing the quality of the goods they produce.

[3] However, the idea is not new. Dixit and Stiglitz's (1977) model of industrial structure already considers the number of firms endogenous and determined by a free entry-exit condition.

main ways: a) they endogenize the tradability of goods instead of the creation of new varieties; b) they allow for heterogeneity of firms and, thus, are able to observe idiosyncratic shocks; and c) they incorporate dynamics on the understanding that firms must pay a sunk entry cost to start production. The two latter papers focus on the consequences of the application of different monetary policies on the structure of markets.

The present paper undertakes a closed-economy analysis with homogeneous firms. The model adopted incorporates quasi-dynamics a là Bergin and Corsetti (2008) and observes shocks to labour and innovation productivities. The objective is to understand the consequences of different real shocks on the *structure* of the market within a country's borders.

3 The Model

I model an autarchy where external trade in goods or assets is not possible. The economy consists of L_t infinitely-lived households, an endogenously determined number of varieties, and a government. The only source of investment is that involved in the start-up cost for each new variety produced by a monopolistic firm. This start-up cost is homogeneous to all new entrants.

3.1 Households

The economy is populated by L_t infinitely-lived households whose utility function is

$$E_t \sum_{t=0}^{\infty} \beta^t U_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - k\ell_t \right], \tag{1}$$

where C is the index of consumption defined below, ℓ is the labour supply which generates a constant disutility measured by k, ψ is the intertemporal elasticity of substitution, β is the subjective discount factor.

Households can be employed either in the creation and design of new varieties that will be supplied in the next period or in the production tasks of currently active firms. However, they are homogeneous and receive the same wage regardless of their occupation in the economy.

The index of consumption takes the usual form

$$C_t = \left[\int_{h=0}^{n_t} c_t(h)^{1-\frac{1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where σ is the elasticity of substitution between varieties, n_t is the number of varieties available for consumption in period t and $c_t(h)$ is the individual demand of variety h. Household preferences are assumed to be expressed for a very large set of goods, so that he utility is well defined (and increasing) for any new good introduced in the market, and for a given level of technology and labour endowment.

Consumers are restricted by their budget identity,

$$\int_{h=0}^{n_t} p_t(h)c_t(h)dh + s_t I_t + B_t = s_{t-1} \int_{h=0}^{n_t} \Pi_t(h)dh + (1+i_t) B_{t-1} + w_t\ell_t - T_t, \tag{3}$$

where s are savings expressed as a proportion of total investment, B are riskless bonds and $\Pi(h)$ are the profits generated by firm h. There are n monopolistic firms producing differentiated varieties, i is the nominal interest rate paid by the bonds, w represents the wage, $p(h)$ is the price for variety h, and T is a lump-sum tax paid each period. Finally, I_t denotes total investment, which is equal to

$$I_t = \left[\int_{h=0}^{n_{t+1}} q_t(h)dh \right] = q_t n_{t+1}, \tag{4}$$

where $q(h)$ is the cost incurred by a firm in order to create a new variety today that can be brought to the market tomorrow. Firms are homogeneous and $q(h)$ is the same for any new firm. So that, $s_t = \frac{1}{L_t}$ due to households homogeneity.

By deciding how much to invest, households indirectly affect the amount of labour devoted to the creation of new equipment for the production of new or improved varieties in the next period. This amount of labour effort is, hence, prevented from participating in the production of today's consumption goods.

3.2 Firms

A continuum of n_t monopolistic firms acts in the economy at t. Investment appears in the form of an exogenous start-up cost, q_t, that entrepreneurs need to incur, at time t, in order to develop their new variety. These varieties are produced and sold in the market at $t+1$. This cost comprises the wages paid to the labour force allocated to the innovation tasks that have productivity ν_t. So that $q_t = \frac{w_t}{v_t}$.[4]

Investment is fully depreciated at the end of the productive period. As such, these two-period firms could be said to adhere to the "reinvent yourself or die" rule. Any firm wishing to survive in the market must necessarily incur constant costs of innovation. Successful firms, therefore, produce a renewed variety of their good and, in this model, they are considered "new firms/varieties" in the following period. Unsuccessful firms, by contrast, disappear and new firms enter the market during each period. The model does not, however, differentiate between old-renewed and brand-new firms. Simply, it provides us with the total number of varieties supplied in the market during each period, which is the measure that in essence affects welfare.

Once set up, the firms produce a differentiated variety with a homogeneous and linear technology that requires only labour,

$$y_t(h) = \alpha_t \ell_t(h), \tag{5}$$

where $y(h)$ is the production of variety h, α is the productivity parameter, which is completely exogenous and $\ell(h)$ is the amount of labour required for productive activities of variety h.

4 Equilibrium with flexible prices

Let us first focus on an environment without nominal frictions. In this case, all contracts and prices are written in nominal terms and are completely flexible. Hence, one can solve for the real variables only and resort to a cashless economy.

[4] It is arguably more natural to assume that the design of a new variety requires some final goods that would otherwise have been used as consumption. However, Bilbiie et al. (2007) discuss the implications of different start-up costs, i.e., labour or final goods as sunk costs to create a new firm in the extensions of their paper and they find that the requirement of labour for creation of firms produces the right sign in the business cycle of firms markups, while the requirement of some final goods does not. I follow them in choosing a labour start-up cost.

4.1 The Household's Problem

The representative household makes decisions regarding consumption, labour supply and savings. To do so, it maximizes (1) subject to (3), which produces the following first-order conditions:

$$C_t \; : \lambda_t = \frac{1}{P_t C_t^{\frac{1}{\psi}}}, \tag{6}$$

$$c_t(h) : c\,(h) = \left(\frac{p_t(h)}{P_t}\right)^{-\sigma} C_t, \tag{7}$$

$$s_t : \lambda_t q_t n_{t+1} = \beta E_t \left[\lambda_{t+1} \Pi_{t+1} n_{t+1}\right], \tag{8}$$

$$\ell_t : w_t = k P_t C_t^{\frac{1}{\psi}}, \tag{9}$$

$$B_t : \lambda_t = \beta\,(1+i_t)\, E_t \lambda_{t+1}, \tag{10}$$

where λ is the Lagrangian multiplier and P_t is the welfare-based consumption price index (defined below).

Observe that the first-order condition for s_t implies free entry in the goods market. This condition provides us with a result for n_{t+1}. It also informs us that new firms will be set up until the expected discounted profits of the marginal firm (i.e., the present value of the firm) are equal to the costs of creating it.

I define

$$\mu_t = P_t C_t^{\frac{1}{\psi}}, \tag{11}$$

to simplify notation, where μ, coincides with the inverse of the lagrange multiplier of the budget constraint. By using equation (11), the free entry condition can be rewritten as follows:

$$q_t = E_t \beta \frac{\mu_t}{\mu_{t+1}} \Pi_{t+1}, \tag{12}$$

As for portfolio choice, there is no borrowing or lending in the equilibrium, so that $B_t = 0 \; \forall t$. Moreover, each homogeneous household has an equal share of equities, $s_t = \frac{1}{L_t}$. Notice that in the case in which we allow for population growth, it has to be controlled for in order to keep homogeneity. The equal-share result can be retained by introducing a redistributive scheme with lump-sum taxes and transfers.[5]

4.2 The Firms' Problem

The productively homogeneous firms operating in this economy produce differentiated goods. They enjoy some monopolistic power and maximize their profits with respect to price, subject to the technology constraint (5). Profits are

$$\Pi_t\,(h) = p_t\,(h)\, y_t\,(h) - w_t \ell_t\,(h)\,.$$

[5] When population grows, every period we have households that are homogeneous in preferences but receive different levels of income. Assuming that newborns are able to work from birth, their first period income is $w_t \ell_t$. Older households receive income from three different sources, $s_{t-1} \int\limits_{h=0}^{n_t} \Pi_t(h)dh + (1+i_t)\, B_{t-1} + w_t \ell_t$. This would imply different decisions on investment and savings in bonds. One can control for it by imposing intergenerational transfers that ensure homogeneous incomes for every agent. Therefore, they also ensure homogeneous decisions in terms of investment and savings and allow us to go on using the representative agent for the analysis.

The first order condition for the maximization problem states the optimal price for the firm,

$$p_t(h) = \frac{\sigma}{\sigma - 1} \frac{k\mu_t}{\alpha_t}, \tag{13}$$

where $p_t(h)$ is the optimal price for variety h. It takes the usual form of a constant mark-up over the marginal cost of production.

Ex-post, once prices have been chosen, profits are a function of n_t via the expression of the consumer price index (CPI). That is,[6]

$$\Pi_t(h) = \frac{1}{\sigma} \frac{p_t(h)^{1-\psi} L_t \mu_t^{\psi}}{n_t^{\frac{\psi-\sigma}{1-\sigma}}} + \frac{1}{\sigma} p_t(h)\, n_t^{\frac{\sigma}{1-\sigma}} G_t. \tag{14}$$

The sign of the effects of the number of acting firms on profits depends on the level of substitutability or market power, determined by σ and the intertemporal elasticity of substitution ψ. In general, profits are not as high in markets in which competition is stronger (i.e., when n is larger). In equation (14) this condition holds unambiguously for imperfect substitutes and $\frac{\psi-\sigma}{1-\sigma} > 0$. I assume $\psi \leq 1 < \sigma$. Indeed, the literature typically parameterizes $1/2 \leqslant \psi \leqslant 1$. The most appropriate choice for σ is less obvious, although it is typically set above 1 and values between 2 and 10 are common.[7]

Under perfect foresight, n_{t+1} is the solution to the free entry condition in equation (8):

$$q_t = \beta \frac{\mu_t}{\mu_{t+1}} \left[\frac{1}{\sigma} \frac{p_{t+1}^{1-\psi}(h) L_{t+1} \mu_{t+1}^{\psi}}{n_{t+1}^{\frac{\psi-\sigma}{1-\sigma}}} + \frac{1}{\sigma} p_{t+1}(h) G_{t+1}(h) \right], \tag{15}$$

where household's demand, and ex-post profits at $t+1$ have been used. This depends on the exogenous technology processes, the size of the population, fiscal policy and some parameters,[8]

$$n_{t+1} = \left[\frac{\frac{k}{\nu_t} - \beta \frac{1}{\sigma-1} \frac{k}{\alpha_{t+1}} G_{t+1}(h)}{\beta \frac{1}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{k}{\alpha_{t+1}} \right]^{1-\psi} L_{t+1}} \right]^{\frac{\sigma-1}{\psi-\sigma}}. \tag{16}$$

The relationship between the number of active firms and the cost of creation lies in this equality. $\sigma > \psi$ is needed in order to ensure a decreasing effect of entry costs on the number of available varieties. This condition is required because an increase in the number of firms (which would crucially change expected profits) generates two effects on the consumption demand. First, the CPI falls, resulting in intertemporal substitution, represented by ψ, that consists in higher consumption today. Second, since there are more goods to consume, households split their income between them. This reallocation of private expenditure among all goods generates an intratemporal substitution away from existing goods, which is measured by σ. Notice that, in line with this reasoning, the inequality ($\sigma > \psi$) becomes a necessary condition for the steady state equilibrium to be stable.

[6] See the appendix for the derivation of $p_t(h)$ and ex-post profits.

[7] Estimates for ψ based on macro data range from near zero by Hall (1988), Campbell and Mankiw (1989) to near unity by Beaudry and van Wincoop (1995). Regarding σ, Broda and Weinstein (2006) estimates show that elasticities have been declining over time as goods become more and more differentiated. Bergin (2003) finds that the elasticity of substitution is slightly over 1 for Australia, slightly below 1 for United Kingdom and as high as 6 for Canada.

[8] Notice that I have also substitute $q_t = \frac{w_t}{v_t}$ in the following expression.

Due to the existence of in-advance investment, the subjective discount factor, β, affects the number of firms positively. When people are more patient they choose to save more and buy shares of the firms. As a consequence, an economy is supplied with a larger range of varieties. Obviously, the cost of creating new firms reduces the number of companies acting in the next period.

It is perhaps important at this juncture to stress the various implications of a change in the number of firms. First, consider that consumers have a love of variety. This means they prefer to consume a larger range of different goods rather than a large amount of just a few goods.[9] So, in principle, they should be better off with more firms in the market. However, a greater number of firms means that more labour will be used in non-production activities. Each new company incurs a prior fixed cost which requires labour. And as this new company will not supply a new good until the second period, the workers employed to innovate during the first period are kept away from the production of goods already available for consumption. This is a traditional trade-off. Households must renounce present consumption in order to invest (with the fixed cost) in innovation and enjoy consumption tomorrow.[10]

Hence, decisions regarding savings that are dedicated to innovation ultimately determine the size of a firm that I call Z.

$$\begin{aligned} Z_t &= \frac{L_t \ell_t - \frac{n_{t+1}}{v_t}}{n_t} \\ &= \frac{L_t C_t(\alpha_t) + G_t(\alpha_t)}{\alpha_t n_t}, \end{aligned} \tag{17}$$

where $n_t Z_t$ is the labour force available for the production of final goods at t. Equation (17) indicates that an increment in the overall number of currently active firms, *ceteris paribus,* reduces the number of workers in each one as the larger number of firms must share the same number of workers. An increase in expected profits or a reduction in the costs of creation makes innovation more attractive and n_{t+1} higher. Consequently, more labour force for the production of final goods is lost today in order to reach the new level of n. This causes a reduction in Z_t. Finally, if people experience a larger disutility for their labour effort and decide to increase their leisure time, either production per variety must be reduced or people must renounce future varieties. The second line of equation (17) shows that Z is a function of the total production. Finally, with flexible prices, an increase in α generates two off-setting effects on Z: First, for a given demand, an increase in labour productivity, $\Delta\alpha_t$, would generate a reduction on firm's size due to the fact that less labour is required to produce the same amount of goods. Second, the improvement in α_t causes a reduction of the price index, which results in an increase in the demand that completely offsets the first impact.

4.3 Government

The government controls both fiscal and monetary policy. The government cannot incur in deficit and its budget constraint is

$$T_t L_t + \int_{j=0}^{L_t} T_{a,t}(j)\, dj = G_t, \tag{18}$$

[9] There is a body of literature that deals with this consideration and which explicitly separates the love of variety from the elasticity of substitution. See, for example De Groot and Nahuis (1998).

[10] A further consideration might be the existence of scale economies. In this case, the opportunity cost of enjoying a new variety would be much more important in the case of scale inefficiencies. But this analysis lies beyond the scope of the present paper.

where G_t is total public expenditure and $T_{a,t}(j)$ is a positive transfer for the new generation that owns no shares in the firms from the previous period and a negative transfer for the rest of households. This transfer program is self-financed, so that $\int_{j=0}^{L_t} T_{a,t}(j)\, dj = 0$. The subscript a refers to the age-group of the individual, young or old. Individuals may be ordered, $\int_{j=0}^{m_t} T_{old,t}(j)\, dj + \int_{j>m_t}^{L_t} T_{young,t}(j)\, dj = 0$.

The government chooses the interest rate by maximizing population's welfare subject to equation (10):[11]

$$\max_{i_t} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\frac{1}{1-\frac{1}{\psi}} \left[\frac{E_t(\mu_{t+s})}{\beta(1+i_t)P_t} \right]^{\psi\left(1-\frac{1}{\psi}\right)} - \left[\frac{E_t(\mu_{t+s})}{\beta(1+i_t)P_t} \right]^{\psi} \frac{k}{\alpha_t} \right].$$

The first order condition yields

$$1+i_t = \frac{E_t\left(P_{t+1}C_{t+1}^{\frac{1}{\psi}}\right)}{\beta P_t} \frac{\kappa}{\alpha_t}. \tag{19}$$

The optimal gross interest rate responds to productivity shocks. A positive productivity shock, i.e., higher α, makes suitable to reduce the interest rate. Therefore, population is encouraged to spend more in consumption and to absorb the potential excess of supply. An increase in the expected marginal utility of households income pushes i_t upwards. This higher interest rate implies a higher remuneration of bonds and, consequently, a higher income for next period. Finally, higher prices today reduce i_t. Similar to the case of a shock on productivity, the authority accommodates demand to the new situation. In this case, by maintaining households purchasing power. Notice that, once the monetary rule in equation (19) is applied, the Euler equation (10) implies,

$$\mu_t = \frac{\alpha_t}{\kappa} = P_t C_t^{\frac{1}{\psi}}. \tag{20}$$

Therefore, one may interpret μ as an indirect measure of the monetary stance, which is determined via the interest rate in (19).[12] Since final goods in the private sector are exclusively used for final consumption, C equals private aggregate demand in real terms, while PC is private aggregate nominal spending. Hence, μ synthesizes the effect of monetary policy (i_t, in our case) on PC. Equation (20) completes the information provided by (19). It shows that monetary authorities will act optimally by responding to contemporaneous productivity shocks. Since they are able to react after the shock occurs, they do not need to care about the past or future. This policy is discussed in Section 5.2. For simplicity, I refer to μ as the aggregate monetary stance of the country from now on. Notice that, given (9), the policymaker conditions wages by influencing the value of $P_t C_t^{\frac{1}{\psi}}$. As a consequence, $w_t = k\mu_t$.

In this context, the welfare-based price index is

$$P_t = n_t^{\frac{1}{1-\sigma}} p_t(h), \tag{21}$$

[11] Refer to the appendix for details.

[12] Other authors (see, for instance, Corsetti and Pesenti (2005, 2009)), directly assume this μ as the monetary instrument. We have shown this is an equilibrium result and exactly equivalent to the use of the standard interest rate as monetary instrument. For the rest of the paper, many discussions on the monetary policy refer to μ instead of i, for simplicity.

which is decreasing in the number of varieties. Finally, for the sake of simplicity we assume that the government's public demand is similar to the private demand derived in (7) for each specific variety. Hence,

$$G_t(h) = \left(\frac{p_t(h)}{P_t}\right)^{-\sigma} G_t, \tag{22}$$

where $G_t(h)$ is public consumption of variety h.

4.4 Market Clearing

The goods market clears when

$$y_t(h) = L_t c_t(h) + G_t(h), \forall h.$$

The labour supply equals labour demand for production and creation of new firms,

$$L_t \ell_t = \int_0^{n_t} \ell_t(h)\, dh + n_{t+1} \frac{1}{\nu_t},$$

where $n_{t+1}\frac{1}{\nu_t}$ is period t labour supply for innovation. Households choose it when they take the decision on the amount of investment, which determines n_{t+1}, once ν_t is known. Finally, the financial market clears when

$$L_t s_t = 1.$$

4.5 Consumer Price Index

As discussed above, household utility is derived from consumption; however, the household is concerned about both the quantity and variety of its basket of consumption. Having analyzed the factors that can affect the diversity of goods in the market, let us turn to examine how household purchasing power is affected by prices. The differentiation of equation (21), the welfare-based CPI, results in the following expression:[13]

$$\frac{dP}{P_t} = \frac{1}{1-\sigma}\frac{dn}{n_t} + \frac{d\mu}{\mu_t} - \frac{d\alpha}{\alpha_t}. \tag{23}$$

If we once again assume $\sigma > 1$, the more firms that operate in the market, the lower the price index P. That is, as the number of firms in the market increases, competition becomes fiercer, pushing prices down. Likewise, P will fall more rapidly, as the substitutability between varieties increases, i.e., as the monopolistic power of the firms is weakened. In this model, the elasticity of substitution among varieties and the taste for variety are captured by the same parameter. So, the fall in P is a direct consequence of love for variety. Therefore, one can also argue that an increase in n raises the value of consumption per unit of expenditure and, hence, P falls.

An improvement in productivity also reduces the CPI, since production costs are lower. The implications of this have been widely studied in open economy models as it means that countries with better technology (i.e., more developed countries) experience deteriorating terms of trade (defined as the ratio between import and export prices). Finally, consideration needs to be given to the effects of the monetary policy, which, as expected, are positive for monetary expansions.

[13] See the appendix for details.

4.6 Steady State Analysis

In order to define a steady state, let $\alpha = \nu = 1$, $L = 1$. These assumptions imply, once the optimal monetary policy is applied, $\mu = 1$ and, as a consequence, $w = k$. Thus, the free entry condition becomes

$$k = \beta \frac{1}{\sigma} \left[\left[\frac{\sigma}{\sigma - 1} k \right]^{1-\psi} \frac{1}{n^{\frac{\psi - \sigma}{1-\sigma}}} + \left[\frac{\sigma}{\sigma - 1} k \right] G(h) \right]. \tag{24}$$

Moreover, if I set $G = 0$ then,

$$n = \left[\beta \frac{1}{\sigma k} \left(\frac{\sigma}{\sigma - 1} k \right)^{1-\psi} \right]^{\frac{\sigma-1}{\sigma-\psi}}. \tag{25}$$

In the steady state, when firms are able to charge a high mark-up, entry is more attractive as they can expect higher profits. Given that investment is being made, in the form of the operating costs paid in advance, the subjective discount factor affects the number of varieties in equilibrium. The impact will be either positive or negative depending, again, on the relationship between σ and ψ. If $\psi > \sigma$ (i.e., goods are complements in the Edgeworth-Pareto sense), an increase in household patience leads to a reduction in the number of firms in equilibrium. However, if $\psi < \sigma$ (i.e., goods are substitutes), as assumed here, the greater the degree of household patience, the more likely people are to choose to save by investing in a firm's shares. This results in an increase in the number of varieties in equilibrium. By contrast, as the disutility of labour effort increases (Δk), consumers attach relatively more value to leisure than to consumption. Hence, the number of varieties is reduced as the labour supply decreases.

From equations (10), we find that

$$\beta = \frac{1}{1+i}. \tag{26}$$

For the rest of the endogenous variables we find:

$$p = \frac{\sigma}{\sigma - 1} k, \tag{27}$$

$$P = \left(\frac{\sigma k}{\beta} \right)^{\frac{1}{\sigma-\psi}} \left(\frac{\sigma k}{\sigma - 1} \right)^{\frac{\sigma-1}{\sigma-\psi}}. \tag{28}$$

In the steady state, individual prices depend positively on the monopolistic power of firms and their marginal costs, measured by $k = w$. The price index value is subject to the size of the mark-up and the level of patience of the households. The impact on the CPI of variations in these concepts differs depending on whether goods are substitutes or complements. If they are substitutes, a high level of patience reduces the price index. In this case, people work to generate more differentiated varieties, which is to the detriment of P. Moreover, and increase of σ, i.e., higher substitutability between varieties, reduces market power of firms and makes the basket of consumption cheaper ($\frac{\partial P}{\partial \sigma} < 0$).[14]

Consumption in equilibrium is

$$c = \frac{1}{\beta} (\sigma - 1),$$

$$C = \beta^{\frac{\psi}{\sigma-\psi}} (\sigma - 1)^{\frac{\psi(\sigma-1)}{\sigma-\psi}} (\sigma k)^{-\psi \frac{\sigma}{\sigma-\psi}}. \tag{29}$$

[14] The appendix offers the result of $\frac{\partial P}{\partial \sigma}$ and an analysis of changes on its value for different levels of σ.

Consumption per variety is only affected by patience and the elasticity of substitution. From (25) it is known that the elasticity of substitution has a negative relationship with the number of firms in equilibrium (when goods are substitutes). As varieties become more substitutable, people are more reluctant to work towards creating new firms so as to simply achieve more of the same after paying the fixed cost. Hence, they decide to invest less and consume a larger amount of each (already) available variety. For this reason, σ has a positive effect on c.[15] The effects of the parameters on C present exactly the opposite sign to those for CPI.

Finally,

$$Y = C, \tag{30}$$

$$\Pi = \frac{k}{\beta} n. \tag{31}$$

The first identity is a straightforward result of a closed economy without government consumption.[16] Notice the extremely simplified form found for aggregate profits. As households suffer more from their labour effort, they will renounce the creation of more firms and so profits per firm will increase. By contrast, if the patience levels of consumers are high, present consumption will have a lower value, savings will be higher and the economy will be able to support more producers in equilibrium. However, such an outcome reduces profitability per firm. Finally, the last steady state equation is indicative of the level of labour supply:

$$\ell = C + n = \left(\frac{\sigma}{\sigma-1}k\right)^{-\psi} n^{\frac{\psi}{\sigma-1}} + n; \tag{32}$$

$$= \beta^{\frac{\psi}{\sigma-\psi}} (\sigma-1)^{\frac{\psi(\sigma-1)}{\sigma-\psi}} (\sigma k)^{-\psi\frac{\sigma}{\sigma-\psi}} + \left(\frac{\beta}{\sigma k}\right)^{\frac{\sigma-1}{\sigma-\psi}} \left(\frac{\sigma}{\sigma-1}k\right)^{(1-\psi)\frac{\sigma-1}{\sigma-\psi}}. \tag{33}$$

As the population becomes more patient, the labour supply obviously undergoes an increment owing to the increase in n. Furthermore, an increase in the disutility of work reduces the labour supply. The effects of increases in σ on ℓ are ambiguous. The reason lays in the consequences of opposite sign of changes of σ on C (which is positive) and on n (which is negative). The sign of $\frac{\partial \ell}{\partial \sigma}$ depends on the values of ψ and σ.[17] In general, for $\sigma > 1$ and $\psi > \frac{1}{2}$, more substitutability causes a decrease in the steady state value of labour. Less firms are interested in entering the market (due to the lower expected markup) and, so, less investment is needed every period to cover the innovation cost. The consequent increase in $y(h)$ and, therefore in $\ell(h)$, does not compensate the loss of labour demand. The exception is found for low levels of both σ and ψ. When ψ is low, variations in consumption are costly, in terms of utility, for households and the consumption-smoothing is very relevant. As a result, they do not react so heavily to variations in the need of investment and the positive effect is stronger.

4.7 Market Dynamics

If we differentiate the equilibrium condition (15) in the steady state defined above and take the assumption made in (22) the result is the following equation in differences:

$$\frac{\sigma-\psi}{\sigma-1}\frac{dn_{t+1}}{n} = \frac{d\nu_t}{\beta} + (\psi-1)\,d\alpha_{t+1} + dL_{t+1} + k\left(\frac{\sigma}{\sigma-1}\right)^{\psi} n^{\frac{\psi}{1-\sigma}} dG_{t+1}. \tag{34}$$

[15] See it in the appendix.

[16] Notice that Y is the total production of goods and it does not coincide with the Gross Domestic Product. GDP is consumption plus investment in new firms: $GDP_t = C_t + G_t + n_{t+1}q_t$. In this steady state, $GDP = C + n$, since $q = \frac{w}{\nu} = 1$.

[17] Refer to the appendix to see it.

In order to focus on the influence of the private sector behaviour, I set public expenditure to zero ($G = 0$). First of all, that the sign of the effects crucially depends on the relation between σ and ψ. Under the initial assumption, $\psi \leq 1 < \sigma$, if there is an increase in the efficiency with which firms are created, i.e., as v_t rises, the number of firms also increases. By contrast, when firms become more productive, i.e., each firm is able to supply a larger amount of variety h with the same quantity of inputs ($\Delta\alpha$), this productivity improvement discourages the creation of new firms. The reason for this is that a lower marginal cost forces firms to set prices accordingly, which is translated into smaller profits when the intertemporal elasticity of substitution is below 1 (i.e., $\psi < 1$).

Notice the consequences of an overall productivity change, i.e., $d\alpha = dv$. Although these two shocks act in opposite directions, in a comparative statics of the model, the dominant effect is unambiguously the positive impact of extra efficiency on the creation of new firms. The only exception is $\psi = 0$. In this case, one of the shocks balances the other exactly. However, when one considers the dynamic framework, this is not so clear. The relationship between the intertemporal elasticity of substitution ψ and the patience of the households β determines whether the total effect results in the entry or exit of firms. Based on the assumption $\psi < 1$ and assuming $0 < \beta \leq 1$, the dominant effect is, as in the static case, the improvement in v. In fact, this positive impact becomes stronger as β falls $(\forall \beta < 1)$. Hence, the greater the patience shown by the consumers, the smaller is the positive impact of a shock on the productivity of creation. The intuition behind this result is the following: on the one hand, when people are patient, they tend to invest more and, so, the economy produces a wider range of varieties than it does when consumers are impatient (notice that the derivative of (15) with respect to β is unambiguously positive). On the other hand, when consumers tend to save more, the demand for each variety falls and, so, the expected profits also fall. This serves as a disincentive to new market entrants. However, if $\psi < 1$ does not hold, the final result is uncertain.

Additionally, a larger market size, i.e., larger L, generates an increase in the number of firms.[18] More public expenditure means more total demand, which also leads to a greater range of varieties.

Finally, notice that the monetary stance μ is absent from the above equation, since monetary policy cannot have any impact on real variables when prices are completely flexible.

4.8 Analysis of the Macro-Dynamics

Let us log-linearize all the relevant equations: first-order conditions, budget constraints and any equilibrium condition for the study of the dynamics of the model. There are fourteen unknowns, i.e. endogenous variables: P, p, n, C, c, s, B, $G(h)$, ℓ, i, w, Y, I and Π, and fourteen linearized equations. This system can be reduced to the following rearranged expression for the free entry condition:

$$\left(\frac{1}{\psi} - \frac{1}{\sigma}\right) \frac{nk}{\varpi} \hat{n}_{t+2} = -\hat{v}_t - \eta \hat{\mathbf{P}}_{t+1} + \chi \hat{L}_{t+1} - \theta \hat{n}_{t+1} - \\ - \left(\frac{1}{\sigma} - \frac{1}{\psi}\right) \frac{k}{\varpi} \left[-\frac{n}{\beta} \hat{L}_t - \hat{\alpha}_{t+1} + n\hat{v}_{t+1}\right], \tag{35}$$

where $\varpi > 0$, $\theta > 0$, $\chi > 0$ for $0 < \beta$ and $\eta < 0$.[19]

Equation (35) is a first-order difference equation that depends on the endogenous variables n and P, the exogenous variables v, α and L and on the constant parameters contained also in ϖ, θ, η and χ.

[18] Recall that I control for growth in L with redistributive transfers.

[19] Refer to the appendix to see the linearized equations, the algebra to get equation (35) and the definitions of these aggregates of parameters.

Notice that, when the CPI deviates above its steady state value, it has a positive impact on the future number of firms. This is because higher prices are transformed into higher profits. Hence, firms will take advantage of this deviation and enter the market. Moreover, consumption becomes relatively expensive with respect to investment. People expect that P will return to its steady state value in the long term, so they prefer to wait for consumption. If n_{t+1} deviates from n (above), at $t+2$ the economy tends to correct that deviation by reducing the number of firms.

The effects of v and L are also worth observing. Both exogenous variables appear in two lagged periods (t and $t+1$) and their effects work in opposite directions. An improvement in v_t increases n_{t+1}, but by a smaller proportion.[20] Hence, at $t+1$ there is already a high range of varieties available, i.e. in deviations, n is at a higher level than that of the steady state value. Here again, the negative effect on $\hat{n}_{t+2}$ is the correction of this "excessive" value. $\hat{L}_t$ is derived from the substitution of $\hat{C}_{t+1}$ in the budget constraint. Therefore, the impact is generated by the deviations of C. If the population is low this period, people can expect to receive a larger proportion of the total profits generated by existing firms today. Hence, households can consume more. This higher demand acts as an incentive to businessmen and the number of firms created at $t+1$ rises. So, the deviation in $t+1$ is corrected in $t+2$.

To analyze the dynamic behaviour of the main variables, it is useful to organize the system so as to obtain a first-order difference equation. Notice that the unique endogenous variable with two-periods lag is the exogenous $\hat{v}_t$.[21][22]

$$\frac{kn}{\varpi}\hat{n}_{t+2} \simeq \vartheta_n \hat{n}_{t+1} + \gamma \hat{L}_{t+1} - \frac{kn}{\beta\varpi}\hat{L}_t + \frac{kn}{\varpi}\hat{v}_{t+1} + \Omega\frac{\sigma}{\sigma-1}\hat{v}_t + \vartheta_\alpha \hat{\alpha}_{t+1}. \tag{36}$$

For the accepted standard values of σ, ψ, β and k in the macro literature,[23] the intertemporal effects on n are the following: $\vartheta_n > 0$, i.e., positive deviations of n in t, increase the number of firms in the next period. The number of varieties in the market helps reduce the CPI, hence it is cheaper to consume and households agree to save more to create new firms. Variations in the size of the population have a longer term impact. First, since $\gamma > 0$, a larger population today provides more than one-to-one incentives for the creation of firms in the next period, since there is more labour available and expected profits are higher because of the extra demand. By contrast, $\hat{L}_t$ has a negative effect that is lower than one on $\hat{n}_{t+2}$, which helps correct the excess creation of firms. $\Omega < 0$ and $\varpi > 0$, so, as explained above, we find that once again the change in this variable has opposite intertemporal effects. Finally $\vartheta_\alpha > 0$ for a β extremely close to one and negative otherwise. A positive deviation in the productivity of production today will result in more varieties tomorrow, if the population is patient. If not, households will tend to consume more, since they expect to benefit, both in the current and in the following periods, from levels of α over its steady state. They over-consume and the number of varieties at $t+1$ decreases.

[20] Bear in mind that n_{t+1} is decided at t, therefore, the relevant productivity of creation is v_t.

[21] A further (numerical) analysis would require a study of the roots, considering L and ν as parameters of the characteristic polynomial, which would therefore depend on time. Here I adhere to a simple reasoning of the coefficients of the variables.

[22] See the appendix for detailed coefficients.

[23] The disutility of labor, k, in a linear technology is around 1.75. It is set in such a way that labour force participation matches the value of 66% reported by the BLS and the ILO (see, for example, Poschke (2009)). β is close to 1 (see Kollmann (2006)). The standard parameterization of σ and ψ has been discussed before in the text. The signs of the coefficients in the equation are stable for a large range of values above and below the standard values.

5 Equilibrium with nominal rigidities

The recent literature on macroeconomics and trade has often been concerned with nominal rigidities, seeing them as a possible, albeit partial, explanation for some of the more abstruse questions currently being addressed by the experts (the so called economic puzzles). In this section, I develop a version of the basic model which introduces rigidities in the prices of the varieties.

Corsetti and Pesenti (2009) and Obstfeld and Rogoff (2000) argue that sticky wages and flexible prices are a close reflection of reality. Yet despite this claim, if prices are set as a constant mark-up over marginal costs, for certain applications it would not matter whether one uses prices or wages as the sticky nominal variable.[24] The introduction of nominal rigidities means monetary policies can be analysed and the monetary authorities have the capacity to handle the distortions generated by such goods market frictions.

Although it lies beyond the scope of this paper, we cannot disregard completely the role of fiscal policies in the correction of the market imperfections that arise in this model: namely, those of monopolistic power and price stickiness.[25] A benevolent planner may give a per unit subsidy to firms to ensure they price at, and not above, their marginal cost. To address the optimality of the extensive margin (i.e., the number of varieties supplied to the market) from society's point of view, the government may apply a subsidy or a tax on the fixed costs. In fact, markets with monopolistic competition may suffer from either too many or too few varieties. An increase in competition does not always result in a rise in welfare because it generates two opposite externalities. On the one hand, each new firm obtains its profits by depriving their competitors of some of their sales and profits, i.e., a business-stealing effect coming from a reduction in $y(h)$. As a consequence, aggregate profits also reduce.[26] Since households are the owners of the firms and receive their dividends, a reduction in aggregate profits damages their income and, therefore, the utility that would have been derived from the lost consumption purchasing power. Moreover, they have less units of every product available in the market. On the other hand, it raises consumer surplus by reducing the price index, P, and by providing more diversity of products in the market. As it is shown in Mankiw and Whinston (1986), when firms have no fixed costs, the latter is always higher. However, when set-up costs are required, there is no guarantee that more firms will automatically generate more welfare.

Knowing that, what is the role of the monetary authority in the context of our closed economy? Firms can no longer flexibly alter their prices in the face of a shock. Rather, at the beginning of each period, they must sign contracts setting nominal prices for that period based on their expectations.[27] Prices crucially depend on expected μ. An expected monetary expansion raises the price level and nominal spending.

The government controls the path of short-term rates i, providing a nominal anchor for market expectations. A forward-looking monetary measure, μ is provided by equation (10) and the definition in (11). So

[24] Erceg, Henderson, and Levin (2000), however, provide an example of when it would matter - in a closed economy with both staggered price- and staggered wage-setting, the monetary authority can no longer replicate the flexible price equilibrium.

[25] Modern advanced textbooks on Industrial Organization and the creative-destruction models of Aghion and Howitt discuss these issues at length.

[26] One can see it from the partial derivative of the aggregate of equation (14):

$$\Pi_t = \int_0^{n_t} \Pi_t(h)\, dh = n_t \frac{1}{\sigma} \left[\frac{p_t(h)^{1-\psi} L_t \mu_t^{\psi}}{n_t^{\frac{\psi-\sigma}{1-\sigma}}} + p_t(h)\, n_t^{\frac{\sigma}{1-\sigma}} G_t \right]$$

which is,

$$\frac{\partial \Pi_t}{\partial n_t} = \frac{\psi-\sigma}{1-\sigma} n_t^{\frac{-\psi}{1-\sigma}} \left(\frac{1}{\sigma} p_t(h)^{1-\psi} L_t \mu_t^{\psi} \right) + \frac{1+\sigma}{1-\sigma} n_t^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{\sigma} p_t(h)\, G_t \right),$$

where $\frac{\psi-\sigma}{1-\sigma} < 0$ and $\frac{1+\sigma}{1-\sigma} < 0$ under the standard assumptions followed in the paper.

[27] Today, a number n_{t+1} of firms (matching condition 12) has been created. These firms will start producing tomorrow only

that,

$$\frac{1}{\mu_t} = \beta \left(1 + i_t\right) E_t \frac{1}{\mu_{t+1}},$$

where $\frac{\mu_{t+1}}{\mu_t}$ represents the (gross) inflation target in a non-stochastic steady state.

5.1 Firms' Problem

Firms maximize their expected profits with respect to labour, ℓ, and the price, p. So that,

$$\underset{\ell,\, p}{Max}\ E_t \left[Q \left(p_{t+1} \left(L_{t+1} c_{t+1}(h) + G_{t+1}(h) \right) - \frac{w_{t+1}}{\alpha_{t+1}} \left(L_{t+1} c_{t+1}(h) + G_{t+1}(h) \right) \right) \right],$$

where Q is the discount factor. Thus, the first-order conditions yield the following expression for the optimal price of the differentiated variety:

$$p_{t+1} = \frac{\sigma}{\sigma - 1} \frac{E_t \left(\frac{\beta k}{\alpha_{t+1}} \mu_{t+1}^{\psi} \right)}{E_t \left(\beta \mu_{t+1}^{\psi - 1} \right)}. \tag{37}$$

Considering the optimal choice of prices and assuming that public expenditure is zero, the free entry condition (FEC) becomes

$$\frac{k}{v_t} = \beta E_t \left(\frac{p_{t+1}}{\mu_{t+1}} - \frac{k}{\alpha_{t+1}} \right) L_{t+1} c_{t+1}(h). \tag{38}$$

Equation (38) provides us with the number of varieties exchanged in the consumer goods market at time $t+1$ when prices are rigid, $n_{r,t+1}$. This is

$$n_{r,t+1}^{\frac{\sigma-\psi}{\sigma-1}} = \frac{v_t}{k} \beta L_{t+1} \left(\frac{\sigma}{\sigma - 1} \right)^{-\psi} k^{1-\psi} * \tag{39}$$

$$* \, E_t \left[\begin{array}{c} \mu_{t+1}^{\psi-1} \frac{\sigma}{\sigma-1} \left(\frac{E_t \left(\frac{\mu_{t+1}^{\psi}}{\alpha_{t+1}} \right)}{E_t \left(\mu_{t+1}^{\psi-1} \right)} \right)^{1-\psi} - \\ - \frac{\mu_{t+1}^{\psi}}{\alpha_{t+1}} \left(\frac{E_t \left(\frac{\mu_{t+1}^{\psi}}{\alpha_{t+1}} \right)}{E_t \left(\mu_{t+1}^{\psi-1} \right)} \right)^{-\psi} \end{array} \right].$$

---

if the price they have fixed in the previous period is at least as high as their marginal cost, i.e.,

$$\begin{aligned} p_t - MC_t &= \frac{\sigma}{\sigma - 1} \frac{k P_t C_t^{\frac{1}{\psi}}}{\alpha_t} - \frac{w_t}{\alpha_t} \geq 0 \\ &=> \frac{\sigma k}{\sigma - 1} P_t C_t^{\frac{1}{\psi}} \geq w_t. \end{aligned}$$

In what follows, we need not concern ourselves with this condition as it is never violated under this current framework.

Hence, households consume

$$c_{r,t}\left(h\right)=n_t^{\frac{\sigma-\psi}{1-\sigma}}\left(\frac{\mu_t}{\frac{\sigma}{\sigma-1}\frac{\beta k E_{t-1}\left(\frac{\mu_t^{\psi}}{\alpha_t}\right)}{\beta E_{t-1}\left(\mu_t^{\psi-1}\right)}}\right)^{\psi} \tag{40}$$

of each variety when prices are rigid. In this scenario, the real variables n_t and c_t are tied to the expected behaviour of the monetary authorities and the expected shocks to productivity of processes of production. Thus, the credibility of the government becomes a relevant factor.

5.2 The monetary policy

Economic policies may seek to stabilize economic cycles and correct market imperfections. This paper restricts itself to comparative statics, so that this section examines the capability of monetary policy to solve the imperfection generated by nominal stickiness. Here a government can aim to close the output gap and replicate the flexible-price situation. The flexible-price scenario is better in terms of welfare given that it has only to suffer the monopolistic power imperfection. The monetary authority should commit itself to a monetary policy μ, whereby the number of firms in the case of rigidities, $n_{r,t+1}$, equals the number of firms in the flexible-price situation, $n_{f,t+1}$. Once again, the authorities may have to deal with an important trade-off: it may not be true that by simply closing the output gap the economy will simultaneously attain the consumption of the flexible set-up. In other words, although the output gap might be narrowed, the consumption gap could well remain.

To achieve the same number of firms as in the flexible-price scenario, the monetary authority would need to set a monetary policy $\mu=\frac{\alpha}{\kappa}$, taking the CPI as given. This policy rule involves committing itself to providing a nominal anchor for the economy and deviating from such a stance only when productivity shocks shake the economy and destabilize marginal costs. By so doing, the policy eliminates uncertainty in marginal costs and in profits. Therefore, knowing the optimal monetary rule in equation (19), which implies $\mu=\frac{\alpha}{\kappa}$, policymakers may choose any κ, not necessarily the disutility of labour, but any constant or evolutionary process that all private agents know and apply the Monetary Rule:

$$1+i_t=\frac{E_t\left(\mu_{t+1}\right)}{\mu_t\beta}=\frac{1}{\beta}\frac{b_{t+1}}{b_t}\frac{E_t\left(\alpha_{t+1}\right)}{\alpha_t}, \tag{41}$$

where b is the publicly known nominal anchor, either constant or not.

Let us verify the implications of this monetary policy on output. After plugging the optimal policy in (39), $n_{r,t+1}$ becomes exactly equal to $n_{f,t+1}$:

$$n_{r,t+1}^{\frac{\sigma-\psi}{\sigma-1}}=\frac{v_t}{k}\beta L_{t+1}\left(\frac{\sigma}{\sigma-1}\right)^{-\psi}\left(\frac{1}{\sigma-1}\right)k^{1-\psi}E_t\left[\mu_{t+1}^{\psi-1}\right]= \tag{42}$$
$$=\left[\frac{\nu_t}{k}\beta k^{1-\psi}L_{t+1}\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\psi}\left[\frac{1}{\alpha_{t+1}}\right]^{1-\psi}\right]=n_{f,t+1}^{\frac{\sigma-1}{\sigma-\psi}}.$$

Notice that the policymaker is able to fix both the output and consumption levels of the flexible-prices regime by using the defined policy. Consumption, (40), becomes $c_{r,t}\left(h\right)=n_t^{\frac{\sigma-\psi}{1-\sigma}}\left(\frac{k\sigma}{\sigma-1}\right)^{-\psi}\alpha_t^{\psi}$, which is the same as it was in the flexible version. Moreover, in the steady state,

$$n_r = \left(\beta\left(k\sigma\right)^{-\psi}\left(\sigma-1\right)^{\psi-1}\right)^{\frac{\sigma-1}{\sigma-\psi}}.$$

5.2.1 The Costs of Insufficient Stabilization

Although benevolent, the monetary authority may fail to choose the correct policy. What, therefore, would be the consequences of adopting a sub-optimal monetary policy? Logically, we can predict that welfare would be damaged by such macroeconomic uncertainty, since insufficient stabilization results in suboptimal prices and a suboptimal number of varieties.

Let us assume that the policymaker incorrectly sets a monetary policy $\mu = \alpha^{\Gamma}$ where $0 \leq \Gamma \leq 1$ ($\Gamma = 1$ would be the suitable policy for a flexible-price replication). For any value of Γ different from one, the policy response to shocks will be inefficient in relation to the government's target. Thus, now

$$p_{t+1} = \frac{\sigma k}{\sigma-1}\frac{E_t\left(\alpha_{t+1}^{\Gamma\psi-1}\right)}{E_t\left(\alpha_{t+1}^{\Gamma(\psi-1)}\right)}.$$

Applying Jensen's inequality, it is known that $E_t\left(\alpha_{t+1}^{\Gamma\psi-1}\right) \geq E_t\left(\alpha_{t+1}\right)^{\Gamma\psi-1}$ and $E_t\left(\alpha_{t+1}^{\Gamma(\psi-1)}\right) \geq E_t\left(\alpha_{t+1}\right)^{\Gamma(\psi-1)}$. Without any loss of generality, I choose $\Gamma = 0$ and verify the effect of insufficient stabilization. The consequence is undoubtedly higher prices:

$$p = \frac{\sigma k}{\sigma-1}E_t\left(\frac{1}{\alpha}\right) > \frac{\sigma k}{\sigma-1}. \tag{43}$$

Uncertainty about marginal costs tends to reduce expected discounted profits. This is because people are risk averse and prefer a certain amount of income x rather than an expected average income of x. In order to make discounted profits less sensitive to shocks, firms raise their preset prices. In this way, the economy suffers higher price levels and, consequently, lower levels of consumption.[28]

Moreover, if the government tends to adopt monetary policies that do not completely offset the productivity shock, investors, who are risk averse, will tend to feel uncertain about their future profits and so create fewer firms:

$$\begin{aligned}
&n_{r,t+1}^{\frac{\sigma-\psi}{\sigma-1}}\left(\mu=\alpha^{\Gamma}\right) - n_{r,t+1}^{\frac{\sigma-\psi}{\sigma-1}}\left(\mu=\alpha\right) = \\
&\delta\left[\frac{\sigma}{\sigma-1}E_t\left(\alpha_{t+1}^{\Gamma(\psi-1)}\right) - E_t\left(\alpha_{t+1}^{\Gamma\psi-1}\right)\right] - \delta\left[\frac{\sigma}{\sigma-1}E_t\left(\alpha_{t+1}^{(\psi-1)}\right) - E_t\left(\alpha_{t+1}^{\psi-1}\right)\right] \\
&= \frac{\sigma}{\sigma-1}\left[\underbrace{E_t\left(\alpha_{t+1}^{\Gamma(\psi-1)}\right) - E_t\left(\alpha_{t+1}^{(\psi-1)}\right)}_{(-)}\right] + \left[\underbrace{E_t\left(\alpha_{t+1}^{\psi-1}\right) - E_t\left(\alpha_{t+1}^{\Gamma\psi-1}\right)}_{(-)}\right],
\end{aligned}$$

where $\delta = \frac{v_t}{k}\beta L_{t+1}\left(\frac{\sigma}{\sigma-1}\right)^{-\psi}k^{1-\psi}$. To sum up, when monetary authorities fail to use the replication policy,[29] the economy is characterised by prices that are too high and a number of varieties that is too low.

[28] This result coincides with the conclusion reached in related studies. See, for instance, Corsetti and Pesenti (2009).

[29] In other words, the monetary policy that matches the solution in a flexible-price framework.

Moreover, the surplus of consumers is seriously damaged in its two dimensions: first, the level of consumption is affected by reduced purchasing power, and, second, there is very little variety of differentiated goods, i.e., people are unable to satisfy their love of variety.

As discussed above, as regards its fiscal policy, the government could intervene to remove the imperfections of monopoly power. For instance, it could use a subsidy to reduce the marginal costs of production.

6 Conclusions

The two dimensions of the goods market, the volume of production and the range of available varieties, change over time and across countries due to the ongoing globalization process, but not only. The model presented here has showed how two different real shocks (in innovation and in labour productivity) affect both dimensions in a closed economy. The results suggest to policymakers that trade liberalization policies, although necessary, may not be sufficient to bring the goods market to the structure (in number of varieties and in volumes) that maximizes population welfare.

This paper has presented a quasi-dynamic general-equilibrium model of a closed economy and has analysed full price flexibility and the simplest case of nominal price rigidities. It shows the relevance of intertemporal decisions and uncertainty in determining the level of economic activity. Thus, on the one hand, a change in the degree of patience manifested by the consumer moves the economy to a new steady state, so that the greater the consumer's patience, the greater the product diversity enjoyed by that society (i.e., higher β, which means a lower subjective discount factor of future consumption). On the other hand, when a society faces nominal rigidities, firms tend to set higher prices to (partially) offset the losses they can expect to suffer in the case of a negative shock to productivity (of production). At the same time, managers tend to create fewer firms as profits are no longer certain. Based on the evidence in Funke and Ruhwedel (2003) that shows the welfare gains of diversity, a straightforward conclusion for policymakers is the need of correcting market rigidities, when possible, and of controlling inflation to prevent firms to set high prices to cover themselves for the inflation risk.

Moreover, an increase in market size has a positive impact on the number of varieties. And, finally, the different types of productivity shock have effects of opposite signs on the extensive margin of production: an improvement to the technology of creation increases the number of varieties, while an increase in the productivity of operational firms reduces the number of goods supplied.

Although this is beyond the scope of this paper, the latter result may suggest some policy intervention based on Tarasov (2009) empirical result. He concludes that the dynamics of the goods market generate different welfare effects for relatively poor and relatively rich populations. Whereas the former benefits relatively more from larger quantities of basic goods in the market, the latter gains relatively more welfare from larger diversity in the market. Therefore, specially in developed countries, where the gains from more diversified goods market are larger, policymakers may consider to subsidize innovation, i.e., the development of new products.

In the present paper, as in a large part of the literature, the elasticity of substitution between goods and the love of variety are perfectly tied together (in the set-up presented here, love of variety equals the firms' mark-up). It may, however, be of interest to relax this assumption and to consider their explicit separation. De Groot and Nahuis (1998) claim that the disentanglement of the two elasticities might have important implications for economic growth and welfare.

Finally, it is worth stressing the current dearth of empirical research. Establishing a separation between the two kinds of productivity empirically is by no means a straightforward task, and while Debaere and Lee (2004) report an initial attempt at identifying them separately, there remains much work to be done.

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Appendix

This appendix provides some details on the algebra of the model and shows how the monetary stance used in the text is a simplification of specific and more standard policy instruments.

The monetary policy

Interest rate: Policymakers control the interest rate. They maximize population welfare subject to the Euler equation in the household's problem. For simplicity, let us assume population size is 1. One can substitute the Euler equation in the authority maximization problem in the following way

$$\max_{i_t} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\frac{1}{1-\frac{1}{\psi}} \left[\frac{E_t\left(\mu_{t+s}\right)}{\beta\left(1+i_t\right) P_t} \right]^{\psi\left(1-\frac{1}{\psi}\right)} - \left[\frac{E_t\left(\mu_{t+s}\right)}{\beta\left(1+i_t\right) P_t} \right]^{\psi} \frac{k}{\alpha_t} \right].$$

The first order condition is

$$1+i_t = \frac{E_t\left(\mu_{t+1}\right)}{\beta} \frac{k}{\alpha_t}.$$

Recalling the Euler equation found:

$$1+i_t = \frac{E_t\left(\mu_{t+1}\right)}{\mu_t \beta}. \tag{44}$$

The later two equations are equilibrium conditions that must both hold. Thus,

$$\frac{E_t\left(\mu_{t+1}\right)}{\beta} \frac{k}{\alpha_t} = \frac{E_t\left(\mu_{t+1}\right)}{\mu_t \beta},$$

$$\mu_t = \frac{\alpha_t}{k}. \tag{45}$$

Notice that we could have found the equilibrium condition (45) by directly choosing μ in the maximization problem. Using

$$\left[\frac{\mu_t}{P_t}\right]^{\psi} = C_t \text{ and } \ell_t = \frac{Y_t}{\alpha_t} = \left[\frac{\mu_t}{P_t}\right]^{\psi} \frac{1}{\alpha_t}, \tag{46}$$

the first order condition is

$$\frac{\partial W_t}{\partial \mu_t} = 0 : \psi \left[\frac{\mu_t}{P_t}\right]^{\psi-1-1} \frac{1}{P_t} = \frac{\kappa}{\alpha_t} \psi \left[\frac{\mu_t}{P_t}\right]^{\psi-1},$$

$$\mu_t = \frac{\alpha_t}{\kappa}, \tag{47}$$

where W is population's welfare.

CPI total differentiation

Equation (23) is the result of totally differentiate equation (21). Recall equation (21)

$$P_t = n_t^{\frac{1}{1-\sigma}} p_t. \tag{48}$$

Substitute p_t by the optimal price,

$$P_t = n_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \frac{w_t}{\alpha_t}. \tag{49}$$

Use the households first order condition for labour,

$$P_t = n_t^{\frac{1}{1-\sigma}} \frac{\sigma k}{\sigma - 1} \frac{P_t C_t^{\psi}}{\alpha_t},$$

$$P_t = n_t^{\frac{1}{1-\sigma}} \frac{\sigma k}{\sigma - 1} \frac{\mu_t}{\alpha_t}. \tag{50}$$

Now, apply logarithms and totally differentiate it,

$$\ln P_t = \ln \frac{\sigma k}{\sigma - 1} + \frac{1}{1-\sigma} \ln n_t + \ln \mu_t - \ln \alpha_t, \tag{51}$$

$$\frac{dP}{P_t} = \frac{1}{1-\sigma} \frac{dn}{n_t} + \frac{d\mu}{\mu_t} - \frac{d\alpha}{\alpha_t}. \tag{52}$$

Optimal pricing and ex-post profits

The optimal price for the flexible pricing regime is the first order condition from profit maximization subject to the linear technology of production. Profits can be rewritten from

$$\Pi_t(h) = p_t(h)\, y_t(h) - w_t \ell_t(h), \tag{53}$$

by substituting the technology,

$$\Pi_t(h) = p_t(h)\, y_t(h) - w_t \frac{y_t(h)}{\alpha_t},$$

$$\Pi_t(h) = \left[p_t(h) - \frac{w_t}{\alpha_t} \right] y_t(h). \tag{54}$$

Use optimal demands from households (7) and the government (22) by splitting $y_t(h)$ into $L_t c_t(h)$ and $G_t(h)$,

$$\Pi_t(h) = \left[p_t(h) - \frac{w_t}{\alpha_t} \right] L_t \left(\frac{p_t(h)}{P_t} \right)^{-\sigma} C_t + \tag{55}$$

$$\left[p_t(h) - \frac{w_t}{\alpha_t} \right] \left(\frac{p_t(h)}{P_t} \right)^{-\sigma} G_t.$$

The first order condition is exactly that of equation (13),

$$p_t(h) = \frac{\sigma}{\sigma - 1} \frac{w_t}{\alpha_t}, \tag{56}$$

$$p_t(h) = \frac{\sigma}{\sigma - 1} \frac{k\mu_t}{\alpha_t}. \tag{57}$$

Since households and the government bundles of G and C are structured equally and have identical elasticities of substitution, firms charge the same price to both consumers and government. Therefore, we can write the ex-post profits (once prices have been chosen) with the following expression, after using equation (13) to substitute $\frac{w_t}{\alpha_t}$:

$$\Pi_t(h) = \left[p_t(h) - p_t(h)\frac{\sigma - 1}{\sigma}\right] y_t(h),$$

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h) y_t(h). \tag{58}$$

Let us separate profits coming from sales to households and to the government and substitute the optimal demand, so we can get an expression where profits depend on the monetary policy:

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h) L_t \left(\frac{p_t(h)}{P_t}\right)^{-\sigma} C_t + \frac{1}{\sigma} p_t(h) G_t(h).$$

Use equation (11) to substitute C_t and rearrange terms,

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h) L_t \left(\frac{p_t(h)}{P_t}\right)^{-\sigma} \left(\frac{\mu_t}{P_t}\right)^{\psi} + \frac{1}{\sigma} p_t(h) G_t(h), \tag{59}$$

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h)^{1-\sigma} L_t \frac{\mu_t^{\psi}}{P_t^{\psi-\sigma}} + \frac{1}{\sigma} p_t(h) G_t(h). \tag{60}$$

Use government's optimal demand (22), the CPI, (21) and rearrange to get equation (14) in the text,

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h)^{1-\sigma} L_t \frac{\mu_t^{\psi}}{n_t^{\frac{\psi-\sigma}{1-\sigma}} p_t(h)^{\psi-\sigma}} + \frac{1}{\sigma} p_t(h) \left(\frac{p_t(h)}{P_t}\right)^{-\sigma} G_t, \tag{61}$$

$$\Pi_t(h) = \frac{1}{\sigma} p_t(h)^{1-\psi} L_t \frac{\mu_t^{\psi}}{n_t^{\frac{\psi-\sigma}{1-\sigma}}} + \frac{1}{\sigma} p_t(h) n_t^{\frac{\sigma}{1-\sigma}} G_t. \tag{62}$$

It has been discussed in the text that, assuming imperfect substitutability between varieties and a standard intertemporal elasticity, profits reduce when more firms are operative.

Steady state analysis

In the text, we have defined a steady state following these assumptions: $\alpha = \nu = 1$, $L = 1$. Thus, $\mu = 1$ and $w = k$. This implies a free entry condition that depends on n, public expenditure and some parameters:

$$k = \beta \frac{1}{\sigma} \left[\left[\frac{\sigma}{\sigma - 1} k \right]^{1-\psi} \frac{1}{n^{\frac{\psi - \sigma}{1-\sigma}}} + \left[\frac{\sigma}{\sigma - 1} k \right] G(h) \right]. \tag{63}$$

Moreover, I set $G = 0$. Then,

$$n = \left[\beta \frac{1}{\sigma k} \left(\frac{\sigma}{\sigma - 1} k \right)^{1-\psi} \right]^{\frac{\sigma - 1}{\sigma - \psi}}. \tag{64}$$

From equations (10) and (8) it is found that,

$$\beta = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{1 + i_t} = \frac{P_{t+1} C_{t+1}^{\frac{1}{\psi}}}{P_t C_t^{\frac{1}{\psi}}} \frac{1}{1 + i_t} = \frac{\mu_{t+1}}{\mu_t} \frac{1}{1 + i_t}.$$

In equilibrium, the standard result for the interest rate holds:

$$\beta = \frac{1}{1 + i}. \tag{65}$$

For the rest of the endogenous variables I find:

$$p = \frac{\sigma}{\sigma - 1} k,$$

$$P = \left(\frac{\sigma k}{\beta} \right)^{\frac{1}{\sigma - \psi}} \left(\frac{\sigma k}{\sigma - 1} \right)^{\frac{\sigma - 1}{\sigma - \psi}}. \tag{66}$$

The consumption in equilibrium is

$$C = P^{-\psi} = \beta^{\frac{\psi}{\sigma - \psi}} (\sigma - 1)^{\frac{\psi(\sigma - 1)}{\sigma - \psi}} (\sigma k)^{-\psi \frac{\sigma}{\sigma - \psi}}. \tag{67}$$

The last equation of the steady state shows the labour supply level:

$$\ell = C + n = \left(\frac{\sigma}{\sigma - 1} k \right)^{-\psi} n^{\frac{\psi}{\sigma - 1}} + n,$$

$$\ell = \beta^{\frac{\psi}{\sigma - \psi}} (\sigma - 1)^{\frac{\psi(\sigma - 1)}{\sigma - \psi}} (\sigma k)^{-\psi \frac{\sigma}{\sigma - \psi}} + \left(\frac{\beta}{\sigma k} \right)^{\frac{\sigma - 1}{\sigma - \psi}} \left(\frac{\sigma}{\sigma - 1} k \right)^{(1 - \psi) \frac{\sigma - 1}{\sigma - \psi}}. \tag{68}$$

The text discusses equations (64), (66), (67) and (68). Specially, it focuses on the effects of the degree of substitutability between varieties on each of these steady state values. Here I present the partial derivatives for the four equations with respect to σ. Figure (1) shows the value of the partial derivative for different σ and, in the case of ℓ, also for different ψ.

$$\frac{1}{n} \frac{\partial n}{\partial \sigma} = \frac{\sigma - 1}{\sigma - \psi} \left[\ln \beta - \ln(\sigma \kappa) + (1 - \psi) \ln \left(\frac{\sigma \kappa}{\sigma - 1} \right) \right] - \frac{\sigma - 1}{\sigma(\sigma - \psi)} - \frac{1}{\sigma \kappa} \frac{1 - \psi}{\sigma - \psi} \tag{69}$$

$$\frac{1}{P}\frac{\partial P}{\partial \sigma} = \frac{\ln\beta - \psi\ln(\sigma\kappa) - (1-\psi)\ln(\sigma-1)}{(\sigma-\psi)^2} + \frac{\sigma-1}{\sigma(\sigma-\psi)}. \tag{70}$$

$$\frac{1}{C}\frac{\partial C}{\partial \sigma} = \left[\beta^{\frac{\psi}{\sigma-\psi}}(\sigma-1)^{\frac{\psi(\sigma-1)}{\sigma-\psi}}(\sigma\kappa)^{-\frac{\psi\sigma}{\sigma-\psi}}\right] * \left[2+\ln\beta+\frac{\psi(\sigma-1)}{\sigma-\psi}\ln(\sigma-1)-\frac{\psi\sigma}{\sigma-\psi}\ln(\sigma\kappa)\right]. \tag{71}$$

The partial derivative for ℓ is extremely long and complex. It has been solved exclusively with matlab and Figure (2) evaluates it for different levels of ψ and σ. Remember that, a relatively high ψ means high intertemporal elasticity of consumption. The standard interpretation for ψ works as follows: if the elasticity is high large changes in consumption are not very costly to consumers and as a result if the real interest rate is high they will save a large portion of their income. If the elasticity is low the consumption smoothing motive is very strong and, because of this, consumers will save a little and consume a lot if the real interest rate is high.

In the text, nothing has been mentioned about the fact that partial derivatives have a different sign for very low levels of substitutability. In Figure (1) one can see that $\frac{\partial n}{\partial \sigma}$ and $\frac{\partial P}{\partial \sigma}$ are positive and $\frac{\partial C}{\partial \sigma}$ is negative for σ close to 1.[30] In equation (69) every term on the right hand side is unambiguously negative, except $(1-\psi)\ln\left(\frac{\sigma\kappa}{\sigma-1}\right)$, which is positive under our assumptions. This term captures the effect on n of changes in consumption caused by changes in prices. Again, it is the combination of ψ and σ values what determines when this term overcomes the rest. For larger σ and, hence, lower $p(h)$, demand per variety increases. When substitutability increases, departing from a low level of σ, the increase in $c(h)$ compensates firms for the loss in the markup and profits per firm increase, encouraging entrance. The smaller is ψ the larger the term. I.e., when the intertemporal elasticity of consumption is low, the indicated term is stronger and counteracts the rest until a relatively higher σ.

The reasoning is similar for equation (70). Every term is negative except $-(1-\psi)\ln(\sigma-1)$, which is unambiguously negative for $\sigma > 2$. However, for $1 < \sigma < 2$ it is positive and may offset the rest of terms for certain ranges of values of ψ and σ. P is negatively affected by increases in n and by decreases in $p(h)$. For the range of values in which $p(h)$ decreases whereas n increases when substitutability goes up, the relative strength of the two effects depend on ψ. As said in the text, any effect on C goes in the opposite direction in P because, in the steady state, $C = P^{-\psi}$. Finally, labour has several terms which sign depend on the relative values of ψ and σ. It incorporates the effects coming from the extensive margin, summarized in n, via the labour force operating in innovation activities and the effects coming from prices that influence the intensive margin, via the labour force which produces the final good. Figure (2) evaluates the partial derivatives $\frac{\partial n}{\partial \sigma}$, $\frac{\partial P}{\partial \sigma}$, $\frac{\partial C}{\partial \sigma}$ and $\frac{\partial \ell}{\partial \sigma}$ for combinations of ψ and σ.

Dynamic analysis

The fourteen log-linearized equations are:

$$0 \simeq \hat{r}_t - \frac{1}{\psi}\left(\hat{C}_{t+1}-\hat{C}_t\right) - \left(\hat{\mathbf{P}}_{t+1}-\hat{\mathbf{P}}_t\right), \tag{72}$$

[30] The parameterization for the three subplots is: $\beta = 0.99$, $\psi = 0.6$, $\kappa = 1.75$.

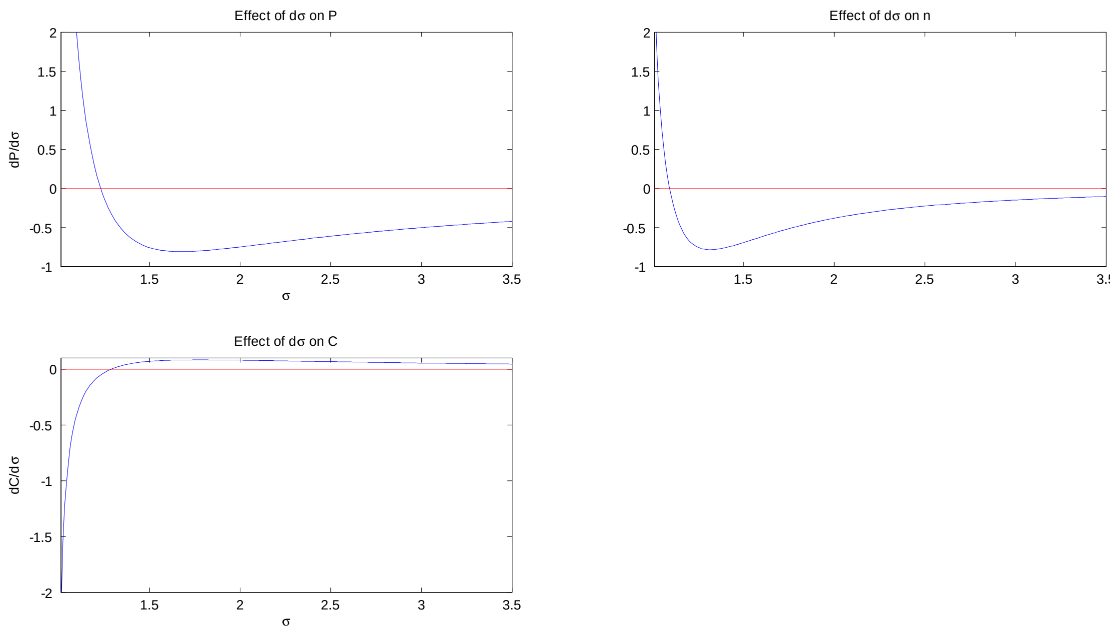

Figure 1: Partial derivatives evaluated for σ.

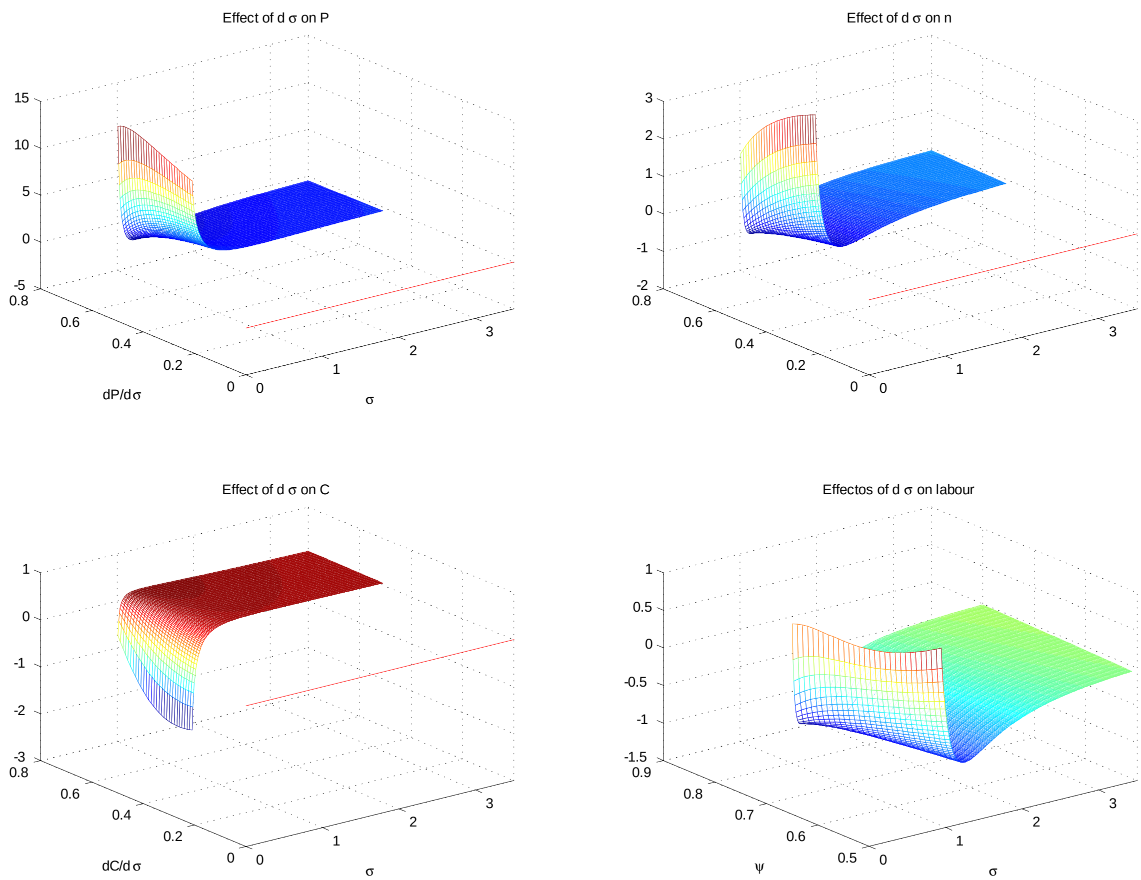

Figure 2: Partial derivatives evaluated for σ and ψ.

$$\hat{w}_t \simeq \frac{1}{\psi}\hat{C}_t + \hat{\mathbf{P}}_t = \hat{\mu}_t, \tag{73}$$

$$\hat{\mathbf{P}}_t \simeq \frac{1}{1-\sigma}\hat{n}_t + \hat{p}_t, \tag{74}$$

$$\hat{c}_t \simeq -\sigma\hat{p}_t + \sigma\hat{\mathbf{P}}_t + \hat{C}_t, \tag{75}$$

$$\hat{G}_t(h) \simeq -\sigma\hat{p}_t + \sigma\hat{\mathbf{P}}_t + \hat{G}_t, \tag{76}$$

$$\hat{p}_t \simeq \hat{w}_t - \hat{\alpha}_t, \tag{77}$$

$$\begin{aligned} sqn\left(\hat{s}_t + \hat{q}_t + \hat{n}_{t+1}\right) \simeq\ & s\pi n\left(\hat{s}_{t-1} + \hat{\pi}_t + \hat{n}_t\right) + \\ & + k\mathbf{P}C^{\frac{1}{\psi}}\left(\hat{w}_t + \hat{\ell}_t\right) - npc\left(\hat{n}_t + \hat{p}_t + \hat{c}_t\right), \end{aligned} \tag{78}$$

$$\begin{aligned} \hat{q}_t = \hat{\mathbf{P}}_t - \hat{\mathbf{P}}_{t+1} + \frac{1}{\psi}\left(\hat{C}_t - \hat{C}_{t+1}\right) + \hat{\pi}_{t+1} \simeq \\ \hat{\mu}_t - \hat{\mu}_{t+1} + \hat{\pi}_{t+1}, \end{aligned} \tag{79}$$

$$\hat{\mu}_t \simeq \hat{\mathbf{P}}_t + \frac{1}{\psi}\hat{C}_t, \tag{80}$$

$$\hat{\pi}_{t+1} \simeq \frac{1}{\sigma}\left(\hat{p}_{t+1} + \hat{L}_{t+1} + \frac{\sigma}{1-\sigma}\hat{n}_{t+1} + \hat{C}_{t+1}\right), \tag{81}$$

$$\hat{Y}_t \simeq \hat{\alpha}_t + \hat{\ell}_t = \hat{L}_t + \hat{C}_t, \tag{82}$$

$$\hat{C}_t \simeq \frac{\sigma}{\sigma-1}\hat{n}_t + \hat{c}_t, \tag{83}$$

$$\hat{s}_t = -\hat{L}_t, \tag{84}$$

$$\hat{I}_t = \hat{q}_t + \hat{n}_{t+1}, \tag{85}$$

where $r_t = 1 + i_t$. Equations (72), (73), (75), (79) and (80) are the log-linearized first order conditions from the household's problem in the following order: Euler equation, labour, demand for variety h, the free entry condition and demand of consumption, C. (76) is the first order condition for the public demand of variety h; (77) is the optimal price from the firm's problem and (74) the CPI implied by the structure of our basket of consumption. Equations (78), (81), (82), (84) and (85) are household's budget constraint, profits, technology function that equals goods market clearing, equities market clearing and investment respectively. Notice that, in the equilibrium with homogeneous agents, debt must be zero, $B_t = 0$. That is the reason B is not present in the linearized equations.

Combining the equations in the above system one can rearrange the expressions for the free entry condition, in (87), and household's budget constraint, in (86):

$$\begin{aligned} \hat{C}_t \simeq \frac{s\pi n}{\varpi}\left(-\hat{L}_{t-1} + \frac{1}{\sigma}\left(\hat{\mathbf{P}}_t + \hat{L}_t\right) + \left(1 - \frac{1}{\sigma}\right)\hat{n}_t\right) + \\ \frac{k\mathbf{P}C^{\frac{1}{\psi}}}{\varpi}\left(+\hat{\mathbf{P}}_t + \hat{L}_t - \hat{\alpha}_t\right) - \frac{npc}{\varpi}\hat{\mathbf{P}}_t - \frac{sqn}{\varpi}\left(-\hat{L}_t + \hat{\mathbf{P}}_t - \hat{v}_t + \hat{n}_{t+1}\right), \end{aligned} \tag{86}$$

$$-\hat{v}_t = \left(\frac{1}{\sigma}-1\right)\hat{\mathbf{P}}_{t+1} + \left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\hat{C}_{t+1} + \tag{87}$$
$$+\frac{1}{\sigma}\hat{L}_{t+1} + \left(\frac{1}{1-\sigma}-\frac{1}{\sigma(1-\sigma)}\right)\hat{n}_{t+1},$$

where $\varpi = sqn\frac{1}{\psi} - s\pi n\frac{1}{\sigma} - kPC^{\frac{1}{\psi}}\frac{1+\psi}{\psi} + npc = k\left[\left(\frac{1}{\psi}+\frac{\sigma}{\beta}-\frac{1}{\sigma\beta}\right)n - \frac{1+\psi}{\psi}\right] > 0$. Once you substitute the latter expression for the budget constraint into the free entry condition you get equation (35) in the text of the paper:

$$\left(\frac{1}{\psi}-\frac{1}{\sigma}\right)\frac{nk}{\varpi}\hat{n}_{t+2} = -\hat{v}_t - \eta\hat{\mathbf{P}}_{t+1} + \chi\hat{L}_{t+1} - \theta\hat{n}_{t+1} - \tag{88}$$
$$-\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\frac{k}{\varpi}\left[-\frac{n}{\beta}\hat{L}_t - \hat{\alpha}_{t+1} + n\hat{v}_{t+1}\right],$$

where $\theta = \left(\frac{1}{\sigma} - \left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\frac{kn}{\beta\varpi}\frac{\sigma-1}{\sigma}\right) > 0$ and $\chi = \frac{k}{\varpi}\left(\frac{1}{\psi}-\frac{1}{\sigma}\right)\left(\frac{n}{\beta\sigma}+n+1\right) - \frac{1}{\sigma} > 0$ because $0 > \frac{1+\sigma(1+n)}{n} + \frac{1-\beta}{\beta} - \left(1+\frac{\sigma}{\beta}\right)\psi$ for $0<\beta$ and finally,

$$\eta = \left(\frac{1-\sigma}{\sigma} + \frac{n}{\varpi}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\left[\frac{\pi}{\sigma} + \frac{k\mathbf{P}C^{\frac{1}{\psi}}}{n} - pc - k\right]\right) =$$
$$= \frac{1-\sigma}{\sigma} + \frac{k}{\varpi}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right) + \frac{nk}{\varpi\beta}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\left[\frac{1}{\sigma}-\sigma+\beta\right] < 0,$$

because $\sigma^2 > \frac{n-1}{n}\beta\sigma$.

Now there are one equation and two (endogenous) unknowns n and P. So, a system is needed to work it out. Remember that $\hat{C}_t \simeq \left[\hat{\alpha}_t + \frac{1}{\sigma-1}\hat{n}_t\right]\psi$. Therefore, we can rewrite the budget constraint, in (89), and the free entry condition, in (90), as follows:

$$\left[\hat{\alpha}_{t+1} + \frac{1}{\sigma-1}\hat{n}_{t+1}\right]\psi \simeq \frac{kn}{\beta\varpi}\left(-\hat{L}_t + \frac{1}{\sigma}\left(\hat{\mathbf{P}}_{t+1}+\hat{L}_{t+1}\right) + \left(1-\frac{1}{\sigma}\right)\hat{n}_{t+1}\right) + \tag{89}$$
$$+\frac{k}{\varpi}\left(+\hat{\mathbf{P}}_{t+1}+\hat{L}_{t+1}-\hat{\alpha}_{t+1}\right) - \frac{n\sigma k}{\varpi\beta}\hat{\mathbf{P}}_{t+1} - \frac{kn}{\varpi}\left(-\hat{L}_{t+1}+\hat{\mathbf{P}}_{t+1}-\hat{v}_{t+1}+\hat{n}_{t+2}\right);$$

$$-\hat{v}_t = \left(\frac{1}{\sigma}-1\right)\hat{\mathbf{P}}_{t+1} + \left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\left[\hat{\alpha}_{t+1}+\frac{1}{\sigma-1}\hat{n}_{t+1}\right]\psi + \tag{90}$$
$$+\frac{1}{\sigma}\hat{L}_{t+1} + \left(\frac{1}{1-\sigma}-\frac{1}{\sigma(1-\sigma)}\right)\hat{n}_{t+1}.$$

Now, substitute P in both equations and rearrange terms:

$$\frac{kn}{\varpi}\hat{n}_{t+2} \simeq -\frac{kn}{\beta\varpi}\hat{L}_t + \frac{k}{\varpi}\left[\frac{n}{\sigma\beta}+1\right]\hat{L}_{t+1} + \Omega\hat{\mathbf{P}}_{t+1} + \tag{91}$$
$$+\frac{kn}{\varpi}\left(\hat{L}_{t+1}+\hat{v}_{t+1}\right) - \left[\psi+\frac{k}{\varpi}\right]\hat{\alpha}_{t+1} - \left[\frac{\psi}{\sigma-1} - \frac{kn}{\beta\varpi}\frac{\sigma-1}{\sigma}\right]\hat{n}_{t+1};$$

$$\hat{\mathbf{P}}_{t+1} = \frac{\sigma}{\sigma-1}\hat{v}_t + \psi\frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)\hat{\alpha}_{t+1} + \tag{92}$$
$$+\frac{\sigma}{(\sigma-1)^2}\left[\psi\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)+\frac{1-\sigma}{\sigma}\right]\hat{n}_{t+1}+\frac{1}{\sigma-1}\hat{L}_{t+1},$$

where $\Omega = \frac{nk}{\varpi}\left[\frac{1}{n}+\frac{1}{\sigma\beta}-\frac{\sigma}{\beta}-1\right]$. Now put (92) into (91):

$$\frac{kn}{\varpi}\hat{n}_{t+2} \simeq -\frac{kn}{\beta\varpi}\hat{L}_t + \left(\frac{k}{\varpi}\left[\frac{n}{\sigma\beta}+1\right]+\frac{\Omega}{\sigma-1}+\frac{kn}{\varpi}\right)\hat{L}_{t+1} + \tag{93}$$
$$+\Omega\frac{\sigma}{\sigma-1}\hat{v}_t + \frac{kn}{\varpi}\hat{v}_{t+1} + \left(\Omega\psi\frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)-\left[\psi+\frac{k}{\varpi}\right]\right)\hat{\alpha}_{t+1}+$$
$$+\left(\Omega\frac{\sigma}{(\sigma-1)^2}\left[\psi\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)+\frac{1-\sigma}{\sigma}\right]-\left[\frac{\psi}{\sigma-1}-\frac{kn}{\beta\varpi}\frac{\sigma-1}{\sigma}\right]\right)\hat{n}_{t+1}.$$

Finally, group parameters to simplify the expression:

$$\frac{kn}{\varpi}\hat{n}_{t+2} \simeq -\frac{kn}{\beta\varpi}\hat{L}_t + \gamma\hat{L}_{t+1} + \tag{94}$$
$$+\Omega\frac{\sigma}{\sigma-1}\hat{v}_t + \frac{kn}{\varpi}\hat{v}_{t+1} + \vartheta_\alpha\hat{\alpha}_{t+1} + \vartheta_n\hat{n}_{t+1},$$

where $\vartheta_\alpha = \left(\Omega\psi\frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)-\left[\psi+\frac{k}{\varpi}\right]\right)$, $\vartheta_n = \left(\Omega\frac{\sigma}{(\sigma-1)^2}\left[\psi\left(\frac{1}{\sigma}-\frac{1}{\psi}\right)+\frac{1-\sigma}{\sigma}\right]-\left[\frac{\psi}{\sigma-1}-\frac{kn}{\beta\varpi}\frac{\sigma-1}{\sigma}\right]\right)$ and $\gamma = \left(\frac{k}{\varpi}\left[\frac{n}{\sigma\beta}+1\right]+\frac{\Omega}{\sigma-1}+\frac{kn}{\varpi}\right)$.

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