# *On Augmented Lagrangeans in Linear Programming (LP) Phase I*

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## *Plan of the talk*

✓**Part I: Proposed framework**
0. LP Notation
1. Nonlinear Programming (NLP)

**Part II: Bound-constrained Dual efforts**
2. Dual penalty approaches
3. Dual augmented Lagrangean
4. Forrest-Galabova-Hall algorithm

**Part III: Unconstrained Pr(ox)imal formulations**
5. Main proposal
6. Antipin-Evtuchenko-Golikov-Mollaverdy algorithm
7. Inequality-form Netlib test problems

## 0. LP Notation

Unsymmetric primal-dual pair (non-standard notation):

$$(P) \quad \begin{array}{ll} \min & \ell(x) := c^T x \quad , \; x \in \mathbb{R}^n \\ \text{s.t.} & A^T x \geq b \end{array}$$

$$(D) \quad \begin{array}{ll} \max & \mathcal{L}(y) := b^T y \quad , \; y \in \mathbb{R}^m \\ \text{s.t.} & Ay = c \qquad\quad , \; y \geq 0 \end{array}$$

- $A \in \mathbb{R}^{n \times m}$, $m \geq n$, $\text{rank}(A) = n$, $\text{rank}(A_k) = m_k$
- $m_k := |\mathcal{B}_k|$, $m_k \leq n$, $\mathcal{N}_k := [1:m] \backslash \mathcal{B}_k$
- $A_k := A(:, \mathcal{B}_k) \in \mathbb{R}^{n \times m_k}$, $N_k := A(:, \mathcal{N}_k) \in \mathbb{R}^{n \times (m - m_k)}$
- $a_j := A(:, j)$, $a_j^T := A^T(j, :)$, $e_i^T A := A(i, :)$
- $\mathcal{R}(A)$ range of the columns in $A$, $\text{null}(A)$ its null space.
- $\mathcal{F} := \{x \in \mathbb{R}^n : A^T x \geq b\}$, $\mathcal{G} := \{y \in \mathbb{R}^m : Ay = c, y \geq 0\}$



## Primal (left) and dual (right) two-phase approaches

| Primal | Dual |
|---|---|
| [ Phase I: $(y \in \mathcal{G})$ or $(d$ of $\mathcal{F})$ ] | [ Phase I: $(x \in \mathcal{F})$ or $(\delta$ of $\mathcal{G})$ ] |
| $\downarrow$ | $\downarrow$ |
| $\langle$ *Keep basic set?* $\rangle$ | $\langle$ *Keep basic set?* $\rangle$ |
| $\downarrow$ | $\downarrow$ |
| [ Phase II: Primal-feasibility search loop ] | [ Phase II: Dual-feasibility search loop ] |
| (Primal) BDA $\equiv$ (Primal) NSA $\equiv$ $\equiv$ (Standard form) NSA | (Dual) BDA $\equiv$ (Dual) NSA $\equiv$ $\equiv$ (Inequality form) NSA |

## *Primal (left) and dual (right) two-phase approaches*

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## *1. Nonlinear Programming (NLP)*

Unsymmetric primal-dual pair (non-standard notation, $m \gg n$):

$$(P) \quad \begin{array}{ll} \min & f(x) := c^T x \\ \text{s.t.} & r(x) := A^T x - b \geq 0 \end{array} \quad , \; x \in \mathbb{R}^n$$

$$(D) \quad \begin{array}{ll} \max & g(y) := b^T y \\ \text{s.t.} & s(y) = c - Ay = 0 \end{array} \quad \begin{array}{l} , \; y \in \mathbb{R}^m \\ , \; y \geq 0 \end{array}$$

Constraints:

$$r_j(x) := a_j^T x - b_j, \quad \forall j \in \mathcal{I} := 1 \colon m$$

$$s_i(y) := e_i^T (c - Ay) = c_i - e_i^T Ay, \quad \forall i \in \mathcal{E} := 1 \colon n$$

Rewriting $(D)$ as a minimization problem:

$$(D) \quad \begin{array}{ll} \min & \gamma(y) := -b^T y := -g(y) \\ \text{s.t.} & s(y) = c - Ay = 0 \end{array} \quad \begin{array}{l} , \; y \in \mathbb{R}^m \\ , \; y \geq 0 \end{array}$$

$(P)$ is an all-inequalities NLP min problem, $(D)$ is an NLP min problem with only equalities and simple (non-negative) bound inequalities.

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## *2.1 Dual quadratic penalty*

Quadratic penalty to deal with nonnegs (Nocedal & Wright, 06, §17.1):

$$\begin{array}{lll} \min & \gamma(y) & , \quad y \in \mathbb{R}^m, \\ \text{s.t.} & s_i(y) = 0 & , \quad \forall i \in \mathcal{E}, \\ & y_j \geq 0 & , \quad \forall j \in \mathcal{I}. \end{array}$$

Quadratic $\ell_2$ penalty is not twice continuosly differentiable:

$$\phi_2(y;\rho) := \gamma(y) + \frac{\rho}{2}\sum_{i\in\mathcal{E}} s_i^2(y) + \frac{\rho}{2}\sum_{j\in\mathcal{I}}(y_j^-)^2,$$

$$\nabla_y \phi_2(y;\rho) = \underbrace{\nabla\gamma(y)}_{-b} + \sum_{i\in\mathcal{E}} \rho s_i(y)\nabla s_i(y) + \ldots$$

where

$$y_j^- := \text{máx}(-y_j, 0) = \begin{cases} 0 & , \text{ if } y_j \geq 0 \\ -y_j & , \text{ if } y_j < 0. \end{cases}$$

Also, mín $\phi_2(y;\rho_k)$ increasingly more difficult as $\rho_k \uparrow \infty$ even if $\mathcal{I} = \varnothing$.

## 2.2 Dual $\ell_1$ non-smooth penalty

An alternative could be the $\ell_1$ non-smooth penalty function (Nocedal & Wright, 06, §17.2):

$$\phi_1(y;\rho) := \gamma(y) + \rho \sum_{i\in\mathcal{E}} |s_i(y)| + \rho \sum_{j\in\mathcal{I}} y_j^-,$$

where

$$y_j^- := \max(-y_j, 0) = \begin{cases} 0 & , \text{ if } y_j \geq 0 \\ -y_j & , \text{ if } y_j < 0, \end{cases}$$

but it's not differentiable due to $|\cdot|$ and $[\cdot]^-$. Thus, challenge is three-fold:

**(C1)** Can $y \in \mathcal{G}$ be obtained solving several bound-constrained subprobs?

**(C2)** Can $x \in \mathcal{F}$ be obtained by solving several unconstrained subprobs?

**(C3)** Can $y \in \mathcal{G}$ be obtained by solving several unconstrained subprobs?

Assuming $\mathcal{I} \neq \varnothing$ and $\mathcal{E} \neq \varnothing$, and that efficient subproblem solvers are available nowadays as well!!



## 3. Dual augmented Lagrangean

Modifying the quadratic penalty *without nonnegs*:

$$\mathcal{L}_D(y,x;\rho) := \gamma(y) - \sum_{i\in\mathcal{E}} x_i s_i(y) + \frac{\rho}{2}\sum_{i\in\mathcal{E}} s_i^2(y) =$$

$$= -b^T y - x^T \underbrace{(c - Ay)}_{s(y)} + \frac{\rho}{2}(c-Ay)^T(c-Ay),$$

and nonnegs are introduced in the subproblems of dimension $m \gg n$:

$$\min_{y\in\mathbb{R}^m} \quad \mathcal{L}_D(y,x;\rho) \quad \text{s.t} \quad y \geq 0.$$

Hence a sequence of simple bound-constrained subproblems to be solved (Nocedal & Wright, 06, §17.4.1) in Polyak-Treti'akov method (Güler, 92b):

Step 0: Choose a starting point $x^0 \in \mathbb{R}^n$.

Step $k \in \mathbb{N}$: Choose $\rho_k \geq \underline{\rho} > 0$ for some $\underline{\rho}$ with $\rho_k^{-1} \downarrow 0$ and calculate

$$y^k = \arg\min_{y\geq 0} \mathcal{L}_D(y, x^k; \rho_k), \qquad x^{k+1} = x^k + \rho_k s(y^k).$$

## *Primal (left) and dual (right) two-phase approaches*

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| (PRIMAL) BDA $\equiv$ (PRIMAL) NSA $\equiv$ $\equiv$ (STANDARD FORM) NSA | (DUAL) BDA $\equiv$ (DUAL) NSA $\equiv$ $\equiv$ (INEQUALITY FORM) NSA |



## *4. Forrest-Galabova-Hall algorithm*

Modify quadratic penalty *without nonnegs* for $(b \equiv 0) \wedge (x \equiv d)$ Phase I:

$$\mathcal{L}_D(y, d; \rho) := -0^T y - \sum_{i \in \mathcal{E}} d_i s_i(y) + \frac{\rho}{2} \sum_{i \in \mathcal{E}} s_i^2(y) =$$

$$= -0^T y - d^T \underbrace{(c - Ay)}_{s(y)} + \frac{\rho}{2}(c - Ay)^T (c - Ay),$$

and nonnegs **and** $\gamma(y)$ introduced in the subprobs of dimension $m \gg n$:

$$\min_{y \in \mathbb{R}^m} \quad \gamma(y) + \mathcal{L}_D(y, d; \rho) \quad \text{s.t} \quad y \geq 0.$$

Hence a sequence of simple bound-constrained subproblems to be solved:

Step 0: Choose $\omega > 1$ and starting points $d^0 := 0 \in \mathbb{R}^n$, $y^0 := 0 \in \mathbb{R}^m$.

Step $k \in \mathbb{N}$: Choose $\rho_k \geq \underline{\rho} > 0$ for some $\underline{\rho}$ with $\rho_k^{-1} \downarrow 0$ and calculate

$$y^{k+1} = \arg\min_{y \geq 0} \gamma(y) + \mathcal{L}_D(y, d^k; \rho_k), \quad \begin{cases} d^{k+1} = \rho_k^{-1} s(y^k) & , \text{ if } \rho_{k+1} = \rho_k \\ \rho_{k+1} = \omega \rho_k & , \text{ if } d^{k+1} = d^k \end{cases}$$

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## *5.1 Main proposal: theory (Nocedal & Wright, 06, §17.4.3)*

A sequence of unconstrained subprobs of dimension $n \ll m$ to be solved:

$$\min_{x\in\mathbb{R}^n} \quad \mathcal{L}_P(x;y,\rho) \qquad \equiv \qquad \min_{x\in\mathcal{F}} f(x), \qquad \text{where}$$

$$\mathcal{L}_P(x;y,\rho) := \max_{y\geq 0} \left\{ f(x) - \overbrace{\sum_{j\in\mathcal{I}} y_j r_j(x)}^{y^T r(x)} \right\} = \left\{ \begin{array}{ll} f(x) & , \text{ if } x \in \mathcal{F} \\ \infty & , \text{ if } x \notin \mathcal{F} \end{array} \right.$$

In practice, an iterative procedure is performed after approximating this pr(ox)imal augmented Lagrangean and explicitly doing the maximization:

$$\mathcal{L}_P(x;y,\rho) \approx \hat{\mathcal{L}}_P(x;y^k,\rho_k) := \max_{y\geq 0} \left\{ f(x) - y^T r(x) - \frac{1}{2\rho_k} \sum_{j\in\mathcal{I}} (y_j - y_j^k)^2 \right\} =$$

$$= f(x) + \sum_{j\in\mathcal{I}} \psi(r_j(x), y_j^k; \rho_k), \text{ where } y_j = \max(y_j^k - \rho_k r_j(x), 0) \text{ and}$$

$$\psi(r,y;\rho) := \left\{ \begin{array}{ll} -yr + \rho r^2/2 & , \text{ if } y - \rho r \geq 0 \\ -y^2/(2\rho) & , \text{ if } y - \rho r < 0 \end{array} \right\}, \text{ or } \begin{array}{l} \psi(r,y;\rho) := \frac{y(\mathrm{e}^{-\rho r}-1)}{\rho} \\ y_j = y_j^k \mathrm{e}^{-\rho_k r_j(x)} \end{array}$$

## 5.2 Main proposal: pseudocode

It's easy to see that the penalty-Lagrangean functions just given are just two ($(P_4, \theta_1)$ and $(P_5, \theta_4)$) of the 65 combinations of penalty functions $P$ and associated univariate function $\theta$ of (Birgin, Castillo & Martínez, 05).

In their notation, we use $\Omega \equiv \mathbb{R}^n$ (hence identity as Euclidean projection operator $\mathcal{P}$ onto $\Omega$ and unconstrained subprobs); using also same penalty parameter for all components (i.e., $\rho_j^k \equiv \rho_k, \ \forall j \in \mathcal{I}$) just to be clearer:

*Step* 0: Choose $\omega > 1$ and starting points $x^0 \in \mathbb{R}^n, \ y^0 \in \mathbb{R}^m$.

*Step* $k \in \mathbb{N}$: Choose $\rho_k \geq \underline{\rho} > 0$ for some $\underline{\rho}$ with $\rho_k^{-1} \downarrow 0$ and calculate

$$x^{k+1} = \arg\min_{x \in \mathbb{R}^n} \mathcal{L}_P(x; y^k, \rho_k), \quad y_j^{k+1} = -\frac{\partial \psi}{\partial r}(r_j(x); y_j^k, \rho_k), \ \forall j \in \mathcal{I},$$

with $\rho_{k+1} = \rho_k$ if $\|r(x)\|$ and $|y^T r(x)|$ improves, or $\rho_{k+1} = \omega \rho_k$ if not.

They proved $\{x^k\}$ converges to either a KKT point of $(P)$ or a *degenerate point* (one for which a direction $\delta$ of $\mathcal{G}$ exists, in LP), even if $\mathcal{L}_P(x; y, \rho)$ not guaranteed to be quadratic for general all-inequalities NLPs like $(P)$.



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| (Primal) BDA ≡ (Primal) NSA ≡ ≡ (Standard form) NSA | (Dual) BDA ≡ (Dual) NSA ≡ ≡ (Inequality form) NSA |

## 6. Antipin-Evtuchenko-Golikov-Mollaverdy algorithm

A sequence of unconstrained subprobs of dimension $n \ll m$ to be solved:

$$\min_{x\in\mathbb{R}^n} \mathcal{L}_P(x;y,\rho) \equiv \min_{x\in\mathcal{F}} f(x), \text{ where } \psi(r,y;\rho) := \begin{cases} y^2/(2\rho), & \text{if } y-\rho r > 0 \\ 0, & \text{if } y-\rho r \le 0 \end{cases}$$

$$\mathcal{L}_P(x;y,\rho) \approx \rho c^T x + \frac{1}{2} y^T y, \text{ where } y_j = \max(y_j^k - \rho_k r_j(x), 0),$$

or equivalently, using $z := \rho x$ new variable and $r(x) := A^T x - b$ definition,

$$\mathcal{L}_P(z;y,\rho) \approx c^T z + \frac{1}{2} y^T y, \text{ where } y_j = \max(y_j^k - a_j^T z + \rho_k b_j, 0) :$$

*Step* 0: Choose starting points $x^0 \in \mathbb{R}^n$, $y^0 \in \mathbb{R}^m$.

*Step* $k \in \mathbb{N}$: Set $\rho_k \ge \underline{\rho} > 0$ for $\underline{\rho} = 10^{-1}$ with $\rho_k^{-1} \downarrow 0$ and calculate

$$x^{k+1} = \arg\min_{x\in\mathbb{R}^n} \mathcal{L}_P(x;y^k,\rho_k), \quad y_j^{k+1} = \max(y_j^k - \rho_k r_j(x^k), 0), \ \forall j \in \mathcal{I}.$$

Subprobs using Mangasarian's generalized Newton because there's no ordinary Hessians; generalized Hessian is $AD_kA^T \ge 0$ with $D_k \in \mathbb{R}^{m\times m}$ diag Boolean matrix, hence regularization (with $10^{-4}$) and Armijo stepsize.



## 7. Inequality-form NETLIB test problems

| N° | Name | Optimal value | $m$ | $n$ | nnz($A$) | nnz($b$) | nnz($c$) | nnz($LP$) |
|---|---|---|---|---|---|---|---|---|
| 1 | AFIRO | $.46475314286E+3$ | 51 | 27 | 102 | 5 | 7 | 114 |
| 2 | SC50B | $.70000000000E+2$ | 78 | 50 | 148 | 1 | 5 | 154 |
| 3 | SC50A | $.64575077059E+2$ | 78 | 50 | 160 | 1 | 10 | 171 |
| 4 | SC105 | $.52202061212E+2$ | 163 | 105 | 340 | 1 | 20 | 361 |
| 8 | STOCFOR1 | $.41131976219E+5$ | 165 | 117 | 501 | 27 | 8 | 536 |
| 6 | ADLITTLE | $-.22549496316E+6$ | 138 | 56 | 424 | 82 | 37 | 543 |
| 9 | BLEND | $.30812149846E+2$ | 114 | 74 | 522 | 30 | 8 | 560 |
| 7 | SCAGR7 | $.23313898243E+7$ | 185 | 129 | 465 | 133 | 53 | 651 |
| 10 | SC205 | $.52202061212E+2$ | 317 | 205 | 665 | 1 | 38 | 704 |
| 12 | SHARE2B | $.41573224074E+3$ | 162 | 96 | 777 | 36 | 24 | 837 |
| 14 | LOTFI | $.25264706062E+2$ | 366 | 153 | 1136 | 8 | 49 | 1193 |
| 15 | SHARE1B | $.76589318579E+5$ | 253 | 117 | 1179 | 31 | 103 | 1313 |
| 17 | SCORPION | $-.18781248227E+4$ | 466 | 388 | 1534 | 282 | 76 | 1892 |
| 22 | BRANDY | $-.15185098965E+4$ | 303 | 220 | 2202 | 2 | 54 | 2258 |
| 19 | SCAGR25 | $.14753433061E+8$ | 671 | 471 | 1725 | 475 | 179 | 2379 |
| 20 | SCTAP1 | $-.14122500000E+4$ | 660 | 300 | 1872 | 360 | 154 | 2386 |
| 16 | DUISRAEL | $-.89664482186E+6$ | 316 | 142 | 2411 | 171 | 89 | 2671 |
| 23 | ISRAEL | $.89664482186E+6$ | 316 | 174 | 2443 | 89 | 171 | 2703 |
| 29 | BANDM | $.15862801845E+3$ | 472 | 305 | 2494 | 165 | 118 | 2777 |
| 31 | SCFXM1 | $-.18416759028E+5$ | 600 | 330 | 2732 | 23 | 116 | 2871 |
| 30 | E226 | $.18751929066E+2$ | 472 | 223 | 2768 | 189 | 99 | 3056 |
| 26 | SCSD1 | $-.86666666743E+1$ | 760 | 77 | 2388 | 760 | 1 | 3149 |
| 28 | AGG | $.35991767287E+8$ | 615 | 488 | 2862 | 131 | 432 | 3425 |

## *Conclusions and future work*

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## *Addendum: Where were we 25 years ago?*

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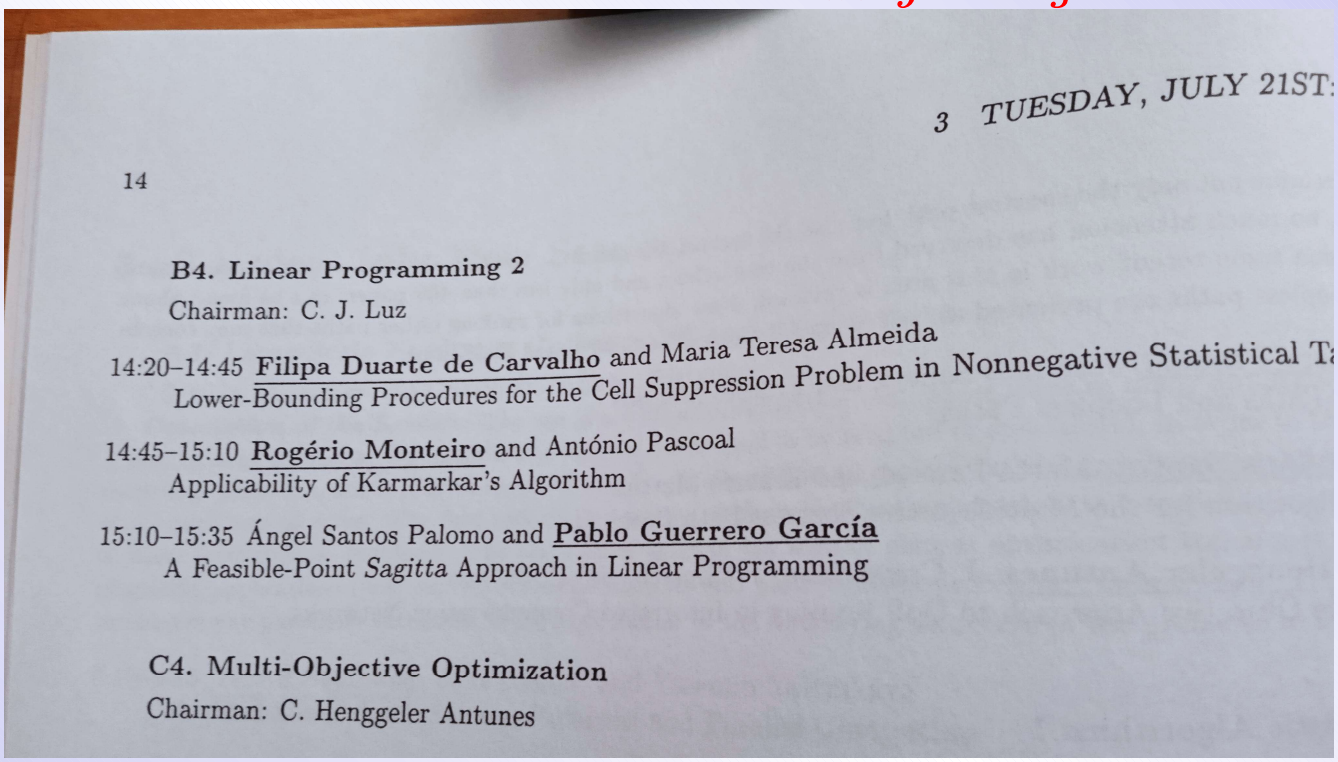



**B4. Linear Programming 2**

Chairman: C. J. Luz

14:20–14:45 **Filipa Duarte de Carvalho** and Maria Teresa Almeida
Lower-Bounding Procedures for the Cell Suppression Problem in Nonnegative Statistical T

14:45–15:10 **Rogério Monteiro** and António Pascoal
Applicability of Karmarkar's Algorithm

15:10–15:35 Ángel Santos Palomo and **Pablo Guerrero García**
A Feasible-Point *Sagitta* Approach in Linear Programming

**C4. Multi-Objective Optimization**

Chairman: C. Henggeler Antunes



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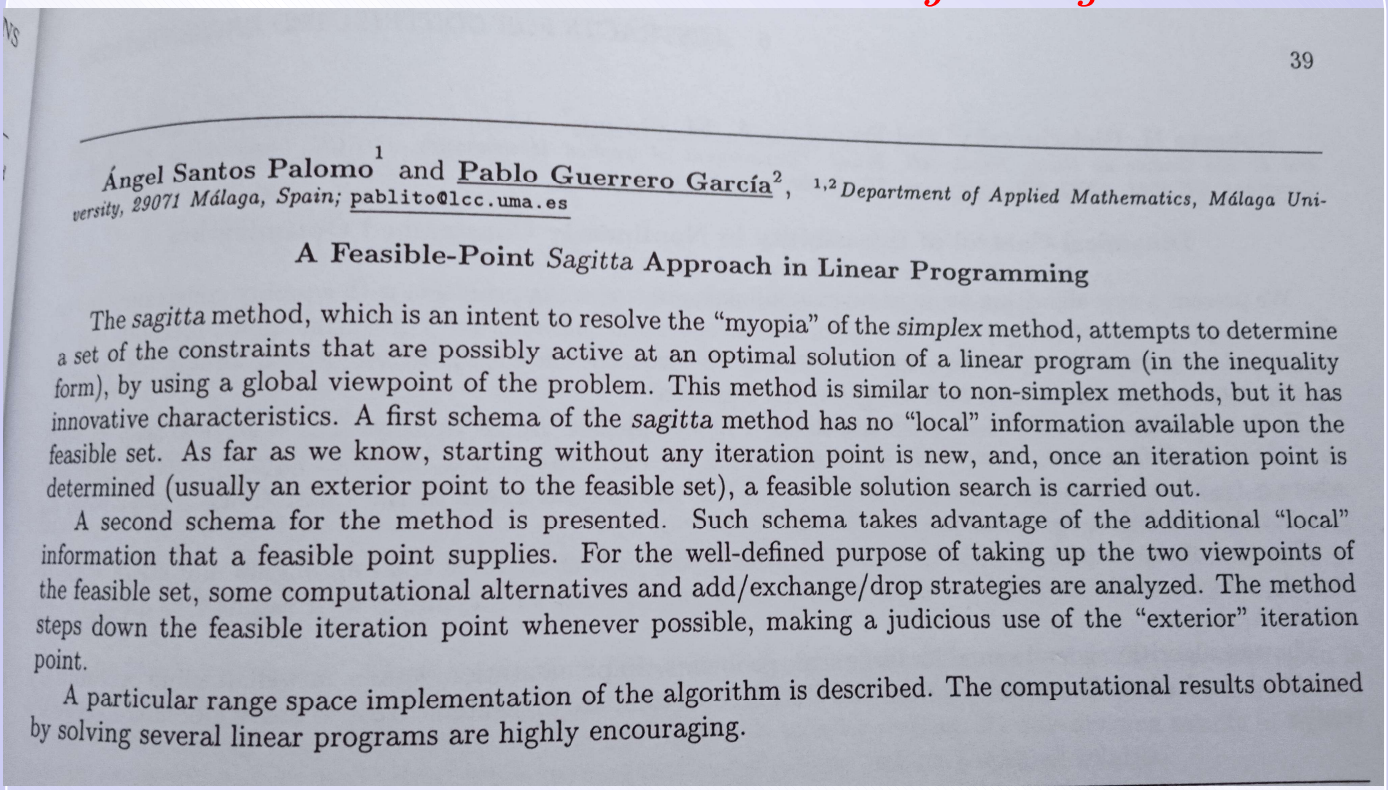



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**A Feasible-Point *Sagitta* Approach in Linear Programming**

The *sagitta* method, which is an intent to resolve the "myopia" of the *simplex* method, attempts to determine a set of the constraints that are possibly active at an optimal solution of a linear program (in the inequality form), by using a global viewpoint of the problem. This method is similar to non-simplex methods, but it has innovative characteristics. A first schema of the *sagitta* method has no "local" information available upon the feasible set. As far as we know, starting without any iteration point is new, and, once an iteration point is determined (usually an exterior point to the feasible set), a feasible solution search is carried out.

A second schema for the method is presented. Such schema takes advantage of the additional "local" information that a feasible point supplies. For the well-defined purpose of taking up the two viewpoints of the feasible set, some computational alternatives and add/exchange/drop strategies are analyzed. The method steps down the feasible iteration point whenever possible, making a judicious use of the "exterior" iteration point.

A particular range space implementation of the algorithm is described. The computational results obtained by solving several linear programs are highly encouraging.

# PTIMIZATION 98

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July 20 – 22, 1998

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