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Abstract

Industries characterized by differentiated products are important contributors of greenhouse gases and currently subject to market-based policies such as emission taxes. In the context of developing countries, fears about foreign investment leaving the country is often times used as an argument not to address industry emissions through emission taxes. This paper develops a Cournot model with product differentiation in the presence of abatement efforts where host and foreign firms are subject to an emission tax. The analysis indicates that abatement efforts and differences in pollution intensity coefficients across firms may play a significant role in the characterization of optimal policy. The analysis also suggests that the government may opt to encourage foreign, less pollution-intensive, firms via higher taxation. Additionally, this paper examines how an optimal emission tax may be adjusted as products become more differentiated; industry emissions may fall/rise as a result of more differentiated products. One important contribution of this paper is that it emphasizes the role of abatement efforts, product differentiation and differences in pollution intensity coefficients across firms in the characterization of the optimal emission tax.

Keywords Environmental taxes· Free entry · Oligopoly ·Horizontal product differentiation

JEL Classification H23·L13·Q58

1. Introduction

There is a wide range of industries characterized by differentiated markets which are large contributors of greenhouse gases, and which are continuously evolving in order to differentiate their products (e.g., automobile and energy sectors are important examples).[1] Therefore, it is important to analyze environmental policy in the context of these industries in order to adjust current policy, if desirable, to varying degrees of product differentiation. Moreover, the potential role of incentive-based environmental policies in developing countries, where industries characterized by differentiated products may operate, has been recently getting some attention (e.g., Shan and Larsen 1992; Blackman and Harrington 2000; Tyler et al. 2013).[2] With these aspects in mind, this paper develops a Cournot model with product differentiation, cost asymmetries and abatement efforts, where host and foreign firms are subject to an emission tax, to examine, *inter alia*, how the optimal tax may be adjusted as products become more differentiated.

The literature on environmental taxation under imperfectly competitive markets is vast (see e.g., Requate (2006) for a survey). One important strand of the literature analyzes optimal environmental taxation and the impact of emissions taxes on industry emissions assuming product homogeneity under Cournot competition (e.g., Levine 1985; Ebert 1992; Simpson 1995; Lee 1999).[3] A second strand analyzes environmental taxation in the presence of vertically differentiated markets (e.g., Poyago-Theotoky and Teerasuwannajak 2002; Bansal and Gangopadhyay 2003; Rodríguez-Ibeas 2006) as well as horizontal product differentiation (e.g., Poyago-Theotoky 2003; Conrad 2005; Lahiri and Symeonidis 2007; Fujiwara 2009; McGinty and de Vries 2009; Gautier 2013). A third more recent strand on FDI and environmental policy has developed, which shows that even under stricter environmental policy foreign firms may reallocate to the host country under Cournot conditions (e.g., Dijkstra et al. 2011; Elliot and Zhou 2012; Sanna-Randaccio and Sestini 2012). The contribution of the present work is at the intersection of these three important strands.

The literature on optimal environmental taxation under Cournot oligopoly indicates that either in the long-run or short-run the optimal tax may be set below, or exceed, marginal damages; the literature has also studied cases in which taxes may raise emissions. These results depend, among others, on the assumptions made about the strategic interaction of firms, the degree of output distortion and cost asymmetries across firms. Part of the litera-

[1] The EPA (http://epa.gov/climatechange/ghgemissions/sources/industry.html) provides data on trends in greenhouse gas emissions for several industries in the US, and DOC (2010) shows evidence of industries where changes in emissions and pollution intensities have taken place. In many cases these are industries characterized by product differentiation. As indicated in Fujiwara (2009), the chemical industry exemplifies an industry characterized by a high degree of product differentiation, whereas the pulp and paper industry is less. See Conrad (2005) for more examples of industries characterized by product differentiation. OECD (2010) describes the adoption of an emission charges scheme on VOCs in Switzerland in the printing, metal cleaning and paint making industries, which exemplify industries with some degree of product differentiation.

[2] Tietenberg (2013) lists, in table 1, Taiwan, Korea and India as countries which have, or shortly will, adopt carbon pricing policies.

[3] There is also an extensive literature which analyzes the strategic interaction of governments when firms compete à la Cournot for the production of a homogeneous good e.g., Conrad (1993), Barrett (1994), Ulph (1996), Bayındır-Upmann (2003), Hamilton and Requate (2004), Requate (2006).

ture assumes away abatement efforts by firms, differences in pollution intensity coefficients across firms and the possibility of differentiated goods. For instance, Ebert (1992) assumes identical pollution intensity coefficients across firms and homogeneous goods, and allows for abatement efforts; Levine (1985) and Simpson (1995) allow for different pollution intensity coefficients across firms and assume homogeneous goods, but assume away abatement efforts by firms. Katsoulacos and Xepapadeas (1996), and Conrad and Wang (1993), for example, analyze the effect of free entry and exit of a set of identical firms on the optimal emission tax under Cournot competition and homogeneous goods. The present work contributes to this literature by incorporating abatement efforts, differentiated products and differences in pollution intensity coefficients into a unified framework of analysis. The case of free entry (with an emphasis on the role of abatement efforts) is analyzed in the context of two sets of firms, home and foreign, which operate in the host country and compete for the production of an imperfect substitute, where the latter is assumed to be endogenous. Arguably, this allows to analyze optimal policy from a developing country's perspective, where there might be concerns about foreign, less pollution-intensive, firms leaving as a result of stricter environmental policy.[4]

Furthermore, the literature has shown that product differentiation is an important aspect in the design of environmental policy (e.g., Fujiwara 2009; Espínola-Arredondo and Zhao 2012; Lambertini 2013).[5] Fujiwara (2009) allows for horizontal product differentiation, but assumes away abatement efforts and differences in pollution intensity coefficients. The author shows, *inter alia*, that in the case where the number of firms is exogenous optimal policy is taxation, if the government puts sufficient weight on environmental damages vis-à-vis the output distortion. The present analysis, in contrast, examines (i) the conditions under which it is optimal to tax firms even in the case where the government puts more weight on the output distortion, and (ii) the case where the government might promote relatively less pollution-intensive firms via higher taxation as products become differentiated, thus highlighting the potential role of differences in pollution intensity coefficients across firms.

A small theoretical literature, assuming Cournot competition, has recently emerged, which shows, among others, that stringer environmental regulation may not deter FDI (i.e., may not deter foreign firms from reallocating to the host country) and the conditions under which this result takes place (Dijkstra et al. 2011; Elliot and Zhou 2012; Sanna-Randaccio and Sestini 2012).[6] This result has been shown assuming either homogeneous goods, identi-

[4]Dardati and Saygili (2012) show that foreign firms are cleaner than domestic firms in the case of Chile.

[5]There is a small literature which incorporates horizontal product differentiation in an international context. Lahiri and Symeonidis (2007) examine the effects of taxation on industry emissions and output in the context of an international oligopoly model with product differentiation where firms exhibit cost asymmetries; the authors however do not present welfare analysis. Gautier (2013) expands Lahiri and Symeonidis' model and looks at the role of product differentiation in the context where governments choose an emission tax and abatement subsidy strategically; however, the case of free entry is not considered. The present work puts aside the strategic interaction of countries, considers an emission tax as the only policy and so contributes to this small literature by considering the case of free entry and exit of only one set of firms (i.e., foreign firms), and emphasizing the potential role of abatement efforts in the characterization of optimal policy as products become more differentiated. The cases under which the government might promote relatively less pollution-intensive firms through taxation, as products become differentiated, are analyzed and new to this branch of the literature.

[6]This literature adds to the earlier works of Markusen et al. (1993), Motta and Thisse (1994), Ulph and

cal pollution intensity coefficients or no abatement efforts. Although results here are in line with this literature, the present work, section 4., contributes to this strand by emphasizing the role of abatement efforts and differences in pollution intensities in the presence of product differentiation in the case where foreign firms can freely enter and exit the host country's market, as well as the analysis of how optimal policy may adjust as products become more/less differentiated. Dijkstra et al. (2011) look at reallocation decisions in a homogeneous good duopoly setting under Bertrand and Cournot behavior where pollution intensity coefficients are identical across firms and firms do not engage in pollution abatement efforts. Sanna-Randaccio and Sestini (2012) analyze location decisions of firms (partially and fully) in a two-country, two-firm oligopoly model with homogeneous goods, emissions are proportional to output and assume identical pollution intensities. Elliot and Zhou (2012) present a model where foreign (leader) and home firms make a strategic decision as to either export or establish operations in a different country in the presence of environmental permits; abatement efforts and differences in intensity coefficients are not explicitly included into the model and the authors assume homogeneous goods. It should be noted that the present work deviates from this line of research in that issues of strategic interaction of governments, trade effects and the strategic choice of foreign firms to either export or establish operations in the host country are assumed away. The focus here is on the welfare implications for the host country and the cases in which abatement efforts and differences in intensity coefficients may explain higher taxation to encourage foreign firms into the host country as products become more differentiated.

The analysis indicates, *inter alia*, that for the host country it may be optimal to tax firms even when more weight is put on the output distortion. The cases under which the government might promote relatively less pollution-intensive firms via higher taxation, as products become differentiated, are analyzed. These results are particularly relevant in countries (developing as well as developed) where concerns about competitiveness may preclude the government to raise the environmental tax in order to address the damage from emissions. This paper also looks at the case where pollution intensity coefficients depend exogenously on the degree of product differentiation, an aspect which seeks to partially capture the case of environmentally conscious firms and its effect on industry emissions. The idea of horizontal product differentiation is borrowed from the literature e.g., Lahiri and Symeonidis (2007), Fujiwara (2009). The idea is to capture the degree of product substitutability in the market and the degree of strategic behavior between two sets of firms. The inclusion of horizontal product differentiation allows to also think about how increasingly differentiated markets consists of firms which are looking to differentiate themselves as, for instance, environmentally conscious firms.

The framework of analysis consists of a host country where home (host) and foreign firms compete for the production of an imperfect substitute. Firms (both foreign and home) face an identical emission tax which is set optimally by the host government, but also engage in pollution abatement activities; I allow for cost asymmetries across firms. The case where the number of firms is exogenous is considered as well as the case where the number of foreign firms is endogenous. The reason for having the number of foreign firms being determined endogenously is that it allows to capture the case where countries, perhaps mainly developing

Valentini (2001), De Santis and Stahler (2009) to name a few.

countries, may be concerned about foreign investment leaving the country if environmental damages are addressed via higher taxation. Events unfold as follows. First, the government sets the optimal tax via welfare maximization; and second firms take policy as given and choose the level of emissions and output simultaneously in a Cournot fashion. The model is solved through backward induction.

The rest of the article is structured as follows. The next section presents the model followed by some of the results from the comparative statics exercise. Section three examines the relationship between policy and the degree of product differentiation when the number of foreign firms is exogenous; section four considers the case where the number of foreign firms is endogenous. The last section concludes.

2. The Model

In a formal sense the model is based on Lahiri and Symeonidis (2007). Consider a host country where there are n identical home (host) firms and m identical foreign firms, which compete à la Cournot for the production of an imperfect substitute. Inverse market demand faced by home and foreign firms, respectively, arise from preferences such that

$$p^h = \alpha^h - \beta^h(q_1^h + q_2^h + \ldots + q_n^h) - \gamma(q_1^f + q_2^f + \ldots + q_m^f) \tag{1}$$

$$p^f = \alpha^f - \beta^f(q_1^f + q_2^f + \ldots + q_m^f) - \gamma(q_1^h + q_2^h + \ldots + q_n^h) \tag{2}$$

where q_l^k denotes output of firm $l = i, j$ ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$) in country $k = h, f$, and γ the degree of product differentiation satisfying $\beta^k > \gamma > 0$. Each firm (foreign and home) generates pollution, incur in pollution abatement efforts and is subject to a per-unit (identical) emission tax, τ. Each firm maximizes profit, π_l^k, in a Cournot fashion by simultaneously choosing emissions, e_l^k, and output, q_l^k. In particular,

$$\max_{q_l^k e_l^k} \pi_l^k = p^k q_l^k - c_l^k(q_l^k, e_l^k) - e_l^k \tau - f_l^k \tag{3}$$

where f_l^k denotes fixed costs. Letting subscripts denote partial derivatives and dropping the subscript l for simplicity, the function $c^k(\cdot,\cdot)$ is assumed to satisfy $c_1^k > 0$, $c_2^k < 0$, $c_{11}^k > 0$, $c_{22}^k > 0$, $c_{12}^k < 0$ and $c_{22}^k c_{11}^k - (c_{12}^k)^2 > 0$.[7] Maximization of (3) yields two first-order conditions for firms in the home country, which under symmetry are given by

$$\alpha^h - c_1^h - \beta^h q^h(n+1) - \gamma m q^f = 0 \tag{4}$$

$$-c_2^h - \tau = 0 \tag{5}$$

Analogously,

$$\alpha^f - c_1^f - \beta^f q^f(m+1) - \gamma n q^h = 0 \tag{6}$$

$$-c_2^f - \tau = 0 \tag{7}$$

[7] The reader is referred to Requate (2006) for a discussion of the properties of the cost function. I follow Lahiri and Symeonidis (2007) in that the pollution intensity coefficient is equal to $-c_{12}/c_{22}$; see footnote 3, page 890.

These four equations determine the equilibrium level of emissions, e^k, output, q^k, and therefore abatement, a^k, for all $k = h, f$.[8]

3. Comparative Statics

This section examines the comparative static effects of the tax, τ, and degree of product differentiation (for given tax). Total differentiation of (4)-(7) yields the following system

$$\begin{bmatrix} -\beta^h(n+1)-c_{11}^h & -m\gamma & -c_{12}^h & 0 \\ -c_{21}^h & 0 & -c_{22}^h & 0 \\ -n\gamma & -\beta^f(m+1)-c_{11}^f & 0 & -c_{12}^f \\ 0 & -c_{12}^f & 0 & -c_{22}^f \end{bmatrix} \begin{bmatrix} dq^h \\ dq^f \\ de^h \\ de^f \end{bmatrix} = \begin{bmatrix} mq^f d\gamma \\ d\tau \\ nq^h d\gamma \\ d\tau \end{bmatrix}$$

where the determinant of the coefficient matrix is $\tilde{\Delta} < 0$.[9] This system of equations serves as the backbone for subsequent sections.

3.1. The Role of the Tax on Output and Emissions

The effect on output is given by (8) and (9). There are two opposing effects which explain the impact of the tax on output of foreign and home firms. On the one hand, an increase in the emission tax raises marginal costs to home firms which results in a reduction in output by those firms (direct effect); as a result, foreign firms become more cost competitive and thus react by increasing output due to the oligopolistic interdependence (indirect effect). On the other, the tax also raises marginal costs to foreign firms, which has the direct effect of lowering foreign firms' output, and as a result home firms react by increasing output as they become relatively more cost competitive (indirect effect). Therefore, the net effect on foreign and home firms' output is ambiguous and depends on the direct and indirect effects. If pollution intensity coefficients are identical across foreign and home firms, then the direct effect offsets the indirect effect and, as a result, home and foreign output falls with the tax. If the pollution intensity coefficient of home firms is sufficiently large so that the indirect effect in the case of foreign firms offsets the direct effect (i.e., $\delta^h n\gamma > \delta^f(\beta^h(n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h))$, then output by home (foreign) firms falls (rises), where the pollution intensity coefficient is given by $\delta^z = -c_{12}^z/c_{22}^z$ for $z = h, f$.[10] Analogously, if the pollution intensity coefficient of foreign firms is sufficiently large (i.e., $\delta^f m\gamma > \delta^h(\beta^f(m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f))$, then output by foreign (home) firms falls (rises).[11] It is noteworthy that if products are completely

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[8] I shall assume an interior solution throughout.

[9] $-\tilde{\Delta} = (c_{11}^h c_{22}^h - (c_{12}^h)^2)(c_{11}^f c_{22}^f - (c_{12}^f)^2 + c_{22}^f \beta^f(m+1)) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)\beta^h(n+1)c_{22}^h - c_{22}^f c_{22}^h(\beta^h\beta^f(m+1)(n+1) - nm\gamma^2 > 0$.

[10] I follow Lahiri and Symeonidis (2007), page 890, footnote 3, in the definition of the pollution intensity coefficient.

[11] However, it is not possible for the indirect effects on both home and foreign to offset the direct effects for a stable equilibrium to exist. To see this suppose that the indirect effects dominate: $\delta^h n\gamma > \delta^f(\beta^h(n+$

differentiated ($\gamma \simeq 0$) the oligopolistic interdependence effect vanishes and, as a result, home and, separately, foreign output falls (see appendix for proof). In particular,

$$\begin{aligned}\tilde{\Delta} dq^h &= c_{22}^h c_{22}^f \left[\frac{-c_{12}^h}{c_{22}^h} \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) + \frac{c_{12}^f}{c_{22}^f} m\gamma \right] d\tau \\ &\quad + m c_{22}^h c_{22}^f \left[q^f \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) - q^h n\gamma \right] d\gamma \end{aligned} \tag{8}$$

$$\begin{aligned}\tilde{\Delta} dq^f &= c_{22}^h c_{22}^f \left[\frac{-c_{12}^f}{c_{22}^f} \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) + \frac{c_{12}^h}{c_{22}^h} n\gamma \right] d\tau \\ &\quad + n c_{22}^h c_{22}^f \left[q^h \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) - q^f m\gamma \right] d\gamma \end{aligned} \tag{9}$$

where $\tilde{\Delta} < 0$, and $c_{22}^k > 0$, $c_{12}^k < 0$, $c_{22}^k c_{11}^k - (c_{12}^k)^2 > 0$ by the concavity of the cost function for $k = h, f$. Define industry emissions $E = ne^h + me^f$. Then, the effect of policy on emissions is given by[12]

$$\begin{aligned}\tilde{\Delta} n de^h &= \frac{-c_{12}^h}{c_{22}^h} c_{22}^h c_{22}^f n \left[\frac{-c_{12}^h}{c_{22}^h} \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) + \frac{c_{12}^f}{c_{22}^f} m\gamma + \frac{\tilde{\Delta}}{c_{12}^h} \right. \\ &\quad \left. - \frac{(c_{11}^h c_{22}^h - (c_{12}^h)^2)}{c_{12}^h c_{22}^h} \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) - \frac{\beta^h (n+1)(c_{11}^f c_{22}^f - (c_{12}^f)^2)}{c_{12}^f c_{22}^f} \right] d\tau \\ &\quad - \frac{c_{12}^h}{c_{22}^h} mn c_{22}^h c_{22}^f \left[q^f \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) - q^h n\gamma \right] d\gamma \end{aligned} \tag{10}$$

$$\begin{aligned}\tilde{\Delta} m de^f &= \frac{-c_{12}^f}{c_{22}^f} c_{22}^f c_{22}^f m \left[\frac{-c_{12}^f}{c_{22}^f} \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) + \frac{c_{12}^h}{c_{22}^h} n\gamma + \frac{\tilde{\Delta}}{c_{12}^f} \right. \\ &\quad \left. - \frac{(c_{11}^f c_{22}^f - (c_{12}^f)^2)}{c_{12}^f c_{22}^f} \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) - \frac{\beta^f (m+1)(c_{11}^h c_{22}^h - (c_{12}^h)^2)}{c_{12}^h c_{22}^h} \right] d\tau \\ &\quad - \frac{c_{12}^f}{c_{22}^f} nm c_{22}^h c_{22}^f \left[q^h \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) - q^f m\gamma \right] d\gamma \end{aligned} \tag{11}$$

In order to illustrate the effect on emissions, I shall use the change in emissions of home firms which is given by (10); an analogous analysis applies to changes in emissions of foreign firms.

$1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h)$ and $\delta^f m\gamma > \delta^h(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f)$. But this violates the stability condition for the Cournot equilibrium.

[12]The results for the tax obtained here are consistent with Lahiri and Symeonidis (2007).

The first two terms denote the change in emissions via changes in output, and the rest of the terms denote the change in emissions via pollution abatement. There are several effects at play here. First, an increase in the tax induces abatement by firms (home and foreign), which lowers home and foreign emissions. Second, the effect via output, as previously explained, depends upon whether the direct effect of the tax offsets the indirect effect which takes place via the oligopolistic interdependence. In the case of home firms, for instance, if the pollution intensity coefficient of home firms, δ^h, is sufficiently large (i.e., $\delta^h n\gamma > \delta^f(\beta^h(n+1)+(c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h))$, then home output falls and home emissions fall as a result; however, the effect on foreign emissions is ambiguous since foreign output rises (see appendix for proof).[13] Therefore, the effect on industry emissions, $E = ne^h + me^f$, is in general ambiguous.

However, it can be shown using (10) and (11) that in the case where the pollution intensity coefficient of home firms is large, the decrease in home emissions offsets the increase in foreign emissions and so industry emissions fall with the tax. Formally, if $\delta^h n\gamma > \beta^h(n+1)\delta^f$, then industry emissions fall.[14] It is noteworthy that in the case where (i) foreign and home firms are equally pollution intensive ($\delta^h = \delta^f$) or (ii) $\gamma \simeq 0$ industry emissions fall with the tax. The intuition for (i) is that with identical pollution intensities the direct effect of the tax via output completely offsets the indirect effect (i.e., the effect arising from the oligopolistic interdependence) and, as a result, emissions in each set of firms fall and thus industry emissions. The intuition for (ii) is straight forward: if $\gamma \simeq 0$, then the effect via the oligopolistic interdependence vanishes and as a result, output in each set of firms falls with the tax and so do emissions. Although these results are analogous to those in Lahiri and Symeonidis (2007) and Gautier (2013), they are important components in subsequent sections and, therefore, are stated as a remark (see appendix for proof).

Remark 1. *Industry emissions fall with the emission tax if (i) home firms exhibit a sufficiently large pollution intensity coefficient ($\delta^h n\gamma > \delta^f\beta^h(n+1)$), (ii) firms are equally pollution intensive ($\delta^h = \delta^f$) or (iii) products are very differentiated ($\gamma \simeq 0$).*

Combining (10) and (11) the change in industry emissions, $E = nde^h + mde^f$, is given by

$$\begin{aligned}\tilde{\Delta} dE \;=\; & c_{22}^h c_{22}^f \Bigg[\frac{c_{12}^h}{c_{22}^h} n \left(\frac{c_{12}^h}{c_{22}^h} \beta^f (m+1) - \frac{c_{12}^f}{c_{22}^f} m\gamma \right) + \frac{c_{12}^f}{c_{22}^f} m \left(\frac{c_{12}^f}{c_{22}^f} \beta^h (n+1) - \frac{c_{12}^h}{c_{22}^h} n\gamma \right) \\ & - \tilde{\Delta} \left(n/c_{22}^h + m/c_{22}^f \right) \\ & + \left(\frac{c_{11}^f c_{22}^f - (c_{12}^f)^2}{c_{22}^f} \right) \left(\beta^h (n+1)(n/c_{22}^h + m/c_{22}^f) + m \frac{c_{11}^h c_{22}^h - (c_{12}^h)^2}{c_{22}^h c_{22}^f} \right) \\ & + \left(\frac{c_{11}^h c_{22}^h - (c_{12}^h)^2}{c_{22}^h} \right) \left(\beta^f (m+1)(n/c_{22}^h + m/c_{22}^f) + n \frac{c_{11}^f c_{22}^f - (c_{12}^f)^2}{c_{22}^h c_{22}^f} \right) \Bigg] d\tau \end{aligned} \tag{12}$$

[13] By the same token, if the pollution intensity coefficient of foreign firms, δ^f, is sufficiently large in the sense that $\delta^f m\gamma > \delta^h(\beta^f(m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f)$, then foreign output falls and foreign emissions fall as a result; the net effect on home emissions in this case is ambiguous.

[14] Analogously, industry emissions fall if foreign firms exhibit a large intensity coefficient i.e., $\delta^f m\gamma > \beta^h(n+1)\delta^h$. It can also be shown that industry emissions fall either if $\delta^h\beta^f > 2\gamma\delta^f$ or if $\delta^f\beta^h > 2\gamma\delta^h$.

where $\tilde{\Delta} < 0$, $c_{12}^k/c_{22}^k = -\delta^k$, $c_{11}^k c_{22}^k - (c_{11}^k)^2 > 0$, $c_{12}^k < 0$, $c_{22}^k > 0$ for $k = h, f$. The first line in (12) captures the change in total emissions via changes in home and foreign output, and the remaining three lines (each of which is positive) capture the change in total emissions via the abatement induced by the tax.

3.2. The Role of Product Differentiation on Output and Emissions

This section examines how output and emissions change (for given tax) as products become more differentiated. The analysis relies on (8)-(11) and proceeds under the simplifying assumption of a cost function of the end-of-pipe. Specifically, $c = \tilde{c}q + (\delta q - e)^2/2$, $\tilde{c} > 0$, $a = \delta q - e$ where the first (second) term denotes production (abatement) costs.[15] One important result is that an exogenous change in the degree of product differentiation does not affect abatement and, as a result, changes in emissions work exclusively via changes in output. I shall first analyze the effect on output.

There are two opposing effects on output as product become more differentiated (i.e., decrease in γ). On the one hand, as products become more differentiated home output increases (direct effect) and as a result foreign firms react by lowering output (indirect effect); on the other, foreign firms raise output as they differentiate themselves (direct effect), but home firms react by lowering output (indirect effect). The net effect on home and, separately, foreign output is thus ambiguous. Home (foreign) output rises (rises) if the direct effect, $q^f\beta^f(m+1)$ (resp. $q^h\beta^h(n+1)$), offsets the indirect effect, $\gamma q^h n$(resp. $\gamma q^f m$).[16] However, it can be shown that industry output rises unambiguously as products become more differentiated. In particular, combining (8) and (9) gives the effect on industry output, $Q = nq^h + mq^f$:

$$dQ = ndq^h + mdq^f = -nm\left[\frac{q^f(\beta^f(m+1) - m\gamma) + q^h(\beta^h(n+1) - n\gamma)}{\beta^h\beta^f(n+1)(m+1) - nm\gamma^2}\right]d\gamma \qquad (13)$$

where $dQ < 0$ and where in the case of end-of-pipe cost function $\tilde{\Delta} = -(\beta^h\beta^f(n+1)(m+1) - nm\gamma^2) < 0$ and $c_{11}^k c_{22}^k - (c_{12}^k)^2 = 0$, $c_{22}^k = 1$, for $k = h, f$. It is noteworthy that, in addition, $dQ < 0$ in the case of a general cost function.

The effect on industry emissions can be examined in light of changes in output and differences in pollution intensity coefficients. There is a myriad of cases where emissions may rise/fall so here I just point at a few using equation (14). If the pollution intensity coefficient across firms is the same ($\delta^h = \delta^f$), then industry emissions rise. This is because as products become more differentiated total output rises and, as result, emissions rise. Similarly, in the case where home and foreign output rise industry emissions also rise, albeit with differences in intensity coefficients ($\delta^h \neq \delta^f$); this is because the direct effect dominates

[15] This simplified functional form implies $c_{11}c_{22} - (c_{12})^2 = 0$. Results follow through with a more general end-of-pipe cost function, $c(q) + h(\delta(q) - e)$, where $\delta' > 0$, $\delta'' > 0$, $h' > 0$, $h'' > 0$, $c' > 0$, $c'' > 0$.

[16] Figure 1 illustrates the effect on home and foreign output as products become more differentiated in the case where the cost function is of the end-of-pipe i.e., $c = \tilde{c}q + (\delta q - e)^2/2$ where $\tilde{c} > 0$.

(meaning that foreign and home output, separately, rise) the indirect effect; that is, if $\delta^h \gamma n < \delta^f \beta^h(n+1)$ and $\delta^f \gamma m < \delta^h \beta^f(m+1)$, then $dE < 0$. In the case where the pollution intensity coefficient of home (foreign) firms is sufficiently large (small) and output by home (foreign) firms falls (rises) with a smaller γ, then industry emissions may fall as products become more differentiated products i.e., from (8) and (9) $q^f \beta^f(m+1) < n\gamma q^h$, $q^h \beta^h(n+1) > \gamma q^f m$ so that $|\delta^h[q^f \beta^f(m+1) - n\gamma q^h]| > |\delta^f[q^h \beta^h(n+1) - m\gamma q^f]|$. Alternatively, if home firms exhibit a sufficiently large pollution intensity coefficient, then emissions may rise if home output rises with more differentiated products (i.e., $q^f \beta^f(m+1) > n\gamma q^h$) even as foreign output falls (i.e., $q^h \beta^h(n+1) < \gamma q^f m$).[17] In particular, for a given tax the change in industry emissions is given by

$$\begin{aligned} \frac{-\tilde{\Delta}}{mn} dE &= \left[q^f(-\delta^h \beta^f(m+1) + \delta^f m\gamma) + q^h(-\delta^f \beta^h(n+1) + \delta^n n\gamma)\right] d\gamma \\ &= \left[-\delta^h(q^f \beta^f(m+1) - n\gamma q^h) - \delta^f(q^h \beta^h(n+1) - m\gamma q^f)\right] d\gamma \qquad (14) \end{aligned}$$

3.3. Optimal Policy and Product Differentiation

This section characterizes the optimal emission tax and examines the effect of product differentiation on optimal policy. One important contribution of the analysis is that the optimal tax, under certain conditions about pollution intensity coefficients, may rise in the case where products become very differentiated even when the government weighs the output distortion sufficiently.

The government in the home country solves the following maximization problem

$$\max_{\tau} W^h = CS(nq^h, mq^f) + n\pi^h + m\pi^f + E\tau - \varphi(E) \qquad (15)$$

where CS denotes consumer surplus and $\varphi(\cdot)$ damages from pollution satisfying $\varphi' > 0$, $\varphi'' > 0$ where φ' denotes marginal damages. Maximization yields (subscripts denote partial derivatives)

$$W^h_{\tau^h} = n\beta^h q^h q^h_\tau + m\beta^f q^f q^f_\tau + (\tau - \varphi')E_\tau = 0 \qquad (16)$$

This equation implicitly determines optimal policy τ^*; the following expression is obtained where the cost function is of the end-of-pipe as defined earlier:

$$\tau^* = -\frac{n\beta^h q^h(-\delta^h \beta^f(m+1) + \delta^f \gamma m) + m\beta^f q^f(-\delta^f \beta^h(n+1) + \delta^h \gamma n)}{n\delta^h(-\delta^h \beta^f(m+1) + \delta^f \gamma m) + m\delta^f(-\delta^f \beta^h(n+1) + \delta^h \gamma n) - (m+n)} + \varphi' \qquad (17)$$

where the numerator of the first term captures the output distortion and denominator the effect of the tax on industry emissions; the last term is simply marginal damages from

[17] These examples illustrate that there is a myriad of cases where emissions may rise/fall depending upon differences in intensity coefficients and whether foreign/home output rises/falls as products become more differentiated.

pollution.[18] Intuitively, the optimal tax balances pollution damages and the output distortion arising from the oligopolistic nature of the output market. If the direct effect of the tax on output dominates the indirect effect (i.e., $\delta^h\beta^f(m+1) > \delta^f\gamma m$ and $\delta^f\beta^h(n+1) > \delta^h\gamma n$) or if products are completely differentiated (i.e., $\gamma \simeq 0$), then the first term in (17) is negative and, as a result, the optimal tax is less than marginal damages; moreover, the tax is positive *if and only if* the government puts sufficient weight on addressing environmental damages (i.e., φ' is large), a result consistent with Fujiwara (2009).

Furthermore, it is possible to have a case where the term which captures the output distortion in (17) is positive. In other words, the numerator is positive because, for example, home output falls with the tax but foreign output rises in the sense that the indirect effect offsets the direct effect in the case of foreign output (i.e., $\delta^h n\gamma > \delta^f\beta^h(n+1)$ and $\delta^h\beta^f(m+1) > \delta^f\gamma m$) and this effect is strong (i.e.,$|m\beta^f q^f q^f_\tau| > |n\beta^h q^h q^h_\tau|$); in such a case the optimal tax is positive and greater than marginal damages.

The intuition is that the government sets a higher optimal tax since the output distortion is addressed via more output by less pollution-intensive foreign firms. The reason is that foreign firms are relatively more efficient (in the sense that they exhibit a smaller pollution intensity coefficient) and so a higher tax moves production towards the more efficient firms and away from the less efficient home firms. This possibility arises because of differences in cost functions across foreign and home firms. In this case the tax exceeds marginal damages because the output distortion arising from foreign firms, $\beta^f q^f$, is larger. The conditions considered in this case are consistent with the stability of the Cournot equilibrium.[19]

Proposition 1. *The optimal emission tax exceeds marginal damages if foreign firms are relatively less pollution-intensive ($\delta^h n\gamma > \delta^f\beta^h(n+1)$) and the government puts sufficient weight on addressing the output distortion via the less pollution-intensive, foreign firms ($|m\beta^f q^f q^f_\tau| > |n\beta^h q^h q^h_\tau|$).*

Proof. From remark 1 and since $\delta^h n\gamma > \delta^f\beta^h(n+1)$ it follows that $E_\tau < 0$. Since $\delta^h n\gamma > \delta^f\beta^h(n+1)$, then $q^f_\tau > 0$. Since $\delta^h n\gamma > \delta^f\beta^h(n+1)$, then $\delta^h\beta^f(m+1) > \delta^f\gamma m$ and so $q^h_\tau < 0$. Then, re-writing equation (16) gives $(\tau - \varphi')E_\tau = -\left(n\beta^h q^h q^h_\tau + m\beta^f q^f q^f_\tau\right)$. It follows then that $\tau > \varphi'$ as long as $|m\beta^f q^f q^f_\tau| > |n\beta^h q^h q^h_\tau|$. ∎

[18] It can be shown that the expression in (17) is equivalent to Fujiwara (2009) assuming away abatement efforts, $\beta^h = \beta^f$, $m = n$, $\delta^h = \delta^f = 1$, $\varphi' = sq$.

[19] The result where the optimal tax exceeds marginal damages is in line with Simpson (1995) in the context of a homogeneous good, 2-firm Cournot model, assuming away abatement efforts; thus, Simpson's result can be seen as a special case. The two conditions in this case are $\delta^h n\gamma > \delta^f\beta^h(n+1)$ and $\delta^h\beta^f(m+1) > \delta^f m\gamma$. Hence, $(\delta^h)^2 n\gamma\beta^f(m+1) > (\delta^f)^2 m\gamma\beta^h(n+1) \Rightarrow (\delta^h)^2\beta^h(n+1)\beta^f(m+1) > (\delta^f)^2 m\gamma n\gamma \Rightarrow (\beta^h(n+1)\beta^f(m+1))/(nm\gamma^2) > (\delta^f)^2/(\delta^h)^2 \Rightarrow (\beta^h(n+1)\beta^f(m+1))/(nm\gamma^2) > 1 > (\delta^f)^2/(\delta^h)^2 \Rightarrow \beta^h(n+1)\beta^f(m+1) > nm\gamma^2$. As stated in Lahiri and Symeonidis (2007) and Simpson (1995) this latter inequality is consistent with the stability condition for the Cournot equilibrium. It is noteworthy that by having less pollution-intensive foreign firms the gap between the optimal tax and marginal damages is reduced. Now, to ensure that the optimal tax exceeds marginal damages one needs *also* the output distortion arising from foreign firms, $q^f\beta^f$, to be larger. The literature points out cases where relatively efficient firms exhibit a larger output level; this result relies on symmetry assumptions across firms. However, in the present model this sort of result is not possible to obtain because of the asymmetries present in the model. Thus, the result where the tax exceeds marginal damages is stated from the viewpoint of output distortion and differences in pollution intensity coefficients.

Next, the effect of product differentiation on policy is examined. To make the analysis tractable I shall keep the assumption of an end-of-pipe cost function, $c = \tilde{c}q + (\delta q - e)^2/2$ where $\tilde{c} > 0$, and focus on the case where $0 < \tau < \varphi'$. Consider $W_\tau(\tau(\gamma), \gamma) = 0$ in (16). Then, total differentiation yields

$$
\begin{aligned}
-W_{\tau\tau}\tau^*_\gamma = W_{\tau\gamma} \quad = \quad & \frac{mn\beta^f}{\omega^2}\left(\delta^f\beta^h(n+1) - n\gamma\delta^h\right)\left[q^h\beta^h(m+n+2) - 2n\gamma q^f\right] \\
& \frac{nm\beta^h}{\omega^2}\left(\delta^h\beta^f(m+1) - m\gamma\delta^f\right)\left[\beta^f q^f(m+n+2) - 2m\gamma q^h\right] \\
& +(\tau - \varphi')E_{\tau\gamma} - \varphi'' E_\tau E_\gamma
\end{aligned} \tag{18}
$$

where $\omega = \beta^h\beta^f(n+1)(m+1) - nm\gamma^2 > 0$ and $W_{\tau\tau} < 0$ from the concavity assumption of the $W(\cdot)$ function. Moreover, $\omega E_\tau = n\delta^h(-\delta^h\beta^f(m+1) + \delta^f\gamma m) + m\delta^f(-\delta^f\beta^h(n+1) + \delta^h\gamma n) - \omega(m+n)$, $E_\gamma = (nm/\omega)\left(-q^f(\delta^h\beta^f(m+1) - m\gamma\delta^f) - q^h(\delta^f\beta^h(n+1) - n\gamma\delta^h)\right)$, $E_{\tau\gamma} = (2nm/\omega^2)\left(\delta^f\beta^h(n+1) - n\gamma\delta^h\right)\left(\delta^h\beta^f(m+1) - m\gamma\delta^f\right)$. The first term in (18) captures the adjustment of the tax as a result of the increased market power from foreign output as products become more differentiated; the second term captures the tax adjustment as a result of the increased market power from home output. The third and fourth terms capture the adjustment of the tax to address damages from pollution.

In the important case where products are very differentiated ($\gamma \simeq 0$), the indirect effect via the oligopolistic interdependence is small, thereby inducing a tax increase if environmental damages are relatively more important i.e., the terms $\beta^h[\cdot]$ and $\beta^f[\cdot]$ are small. With very differentiated products foreign and home output fall with a tax increase and so the first two terms in (18) are positive thus suggesting a reduction in the tax as products become more differentiated in order to address the increased market power. In addition, the tax is increased to address higher emissions and the optimal tax is set below marginal damages i.e., last two terms in (18) are negative. Therefore, if environmental damages are relatively more important then the tax increases with more differentiated products.

As a second case if products are homogeneous ($\beta^h = \beta^f = \gamma = 1$), the number of foreign and home firms are equal and pollution intensity coefficients are identical across firms, then the tax rises *if and only if* the government puts more weight on environmental damages. Intuitively, with sufficient symmetry across firms the cross effects on foreign and home output, arising from a reduction in the tax in order to address the output distortion, partially offset each other. Thus the tax rises because the government puts more weight on damages to the environment. These results are analogous to results in the literature (e.g., Fujiwara 2009). For completeness, in the case of perfect symmetry in the sense that $q^h = q^f$, $n = m$, $\delta^h = \delta^f$, and $\beta^h = \beta^f$ the tax rises with more differentiated products if damages to the environment are sufficiently important.

Remark 2. *With sufficient symmetry across foreign and home firms or if the degree of oligopolistic interdependence between foreign and home firms is small, then the optimal tax rises as products become more differentiated as long as the government puts sufficient weight on environmental damages.*

Proof. Firstly, it is shown that if $\gamma = 0$, and $\beta^h[\cdot]$, $\beta^f[\cdot]$ are sufficiently small, then $W_{\tau\gamma} < 0$ and so the tax rises with a decrease in γ as long as marginal damages are large. Since $\gamma = 0$, $q^h_\tau < 0$, $q^f_\tau < 0$, $E_\tau < 0$, $q^h_\gamma < 0$, $q^f_\gamma < 0$, $E_\gamma < 0$ and $E_{\tau\gamma} > 0$. Moreover, from (16) $\tau < \varphi'$ because $q^h_\tau < 0$, $q^f_\tau < 0$ and $E_\tau < 0$. Hence, the last two terms in (18) are negative and since the first two terms, $\beta^h[\cdot]$ and $\beta^f[\cdot]$ though positive, are sufficiently small by assumption the expression in (18) is negative.

Secondly, it is shown that if $\beta^h = \beta^f = \gamma = 1$, $\delta^h = \delta^f$, $n = m$, then $W_{\tau\gamma} < 0$ and so the tax rises with a decrease in γ as long as marginal damages are large. Since in this case $E_\tau < 0$, $E_\gamma < 0$, $E_{\tau\gamma} > 0$ and $\tau < \varphi'$, then $(\tau - \varphi')E_{\tau\gamma} < 0$ and $\varphi'' E_\tau E_\gamma > 0$ i.e., the last two terms in (18) are negative. Moreover, the first two terms reduce to $(2n^2/\omega^2)(q^h + q^f) > 0$. If this term is small vis-à-vis $(\tau - \varphi')E_{\tau\gamma} - \varphi'' E_\tau E_\gamma$, then $W_{\tau\gamma} < 0$.

Thirdly, it is shown that if $q^h = q^f = q$, $n = m$, $\delta^h = \delta^f = \delta$, and $\beta^h = \beta^f = \beta$, then the tax rises with more differentiated products as long as marginal damages are sufficiently large. Under these conditions $W_{\tau\gamma} = (2n^2\delta(\beta(n+1) - n\gamma)^2(2\beta q + \tau - \varphi'))\,/((\beta^2(n+1)^2 - n^2\gamma^2)) - \varphi'' E_\tau E_\gamma$ where the second term is negative since $E_\tau < 0$, $E_\gamma < 0$, but the sign of the first term is ambiguous since $\tau - \varphi' < 0$ (because $q^h_\tau <$, $q^f_\tau < 0$ and $E_\tau < 0$) and $2\beta q > 0$. Hence, $W_{\tau\gamma} < 0$, if $|2\beta q| < |\tau - \varphi'|$. □

As a third case suppose that the pollution intensity of home firms is large in the sense that $n\gamma\delta^h > \beta^h(n+1)\delta^f$ and the government puts sufficient weight on addressing the output distortion via more foreign output so that $|n\beta^h q^h q^h_\tau| < |m\beta^f q^f q^f_\tau|$. In this case home output falls and foreign output rises with the tax, the optimal tax exceeds marginal damages and industry emissions fall with the tax. In addition, suppose that $q^f\beta^f > \gamma n q^h$ and $\beta^h q^h > \gamma q^f m$, which implies that foreign and home output rise with more differentiated products (decrease in γ) and, as a result, emissions rise with a smaller γ. Therefore, with more differentiated products the government raises the tax as long as it puts sufficient weight on addressing the increased market power and output distortion by encouraging output by foreign, less pollution-intensive firms.

There are several offsetting effects at play here. First, to address the increased market power resulting from differentiated products the government raises the tax to promote less pollution-intensive foreign output (first term in 18), but at the time it lowers the tax to encourage output by home firms (second term in 18). Second, as a result of increases in output via a higher degree of product differentiation higher pollution arises, which induces the government to raise the tax. Therefore, the government raises the tax as products become more differentiated as long as foreign output is encouraged sufficiently (equivalently, second term in 18 is small) and higher emissions are addressed.

Proposition 2. *Let home firms be pollution-intensive and assume that the number of foreign and home firms is exogenous. Then, as products become differentiated the government may address the output distortion and damage from pollution by promoting the less pollution-intensive foreign firms sufficiently via higher taxation.*

Proof. Since $n\gamma\delta^h > \beta^h(n+1)\delta^f$, $q^h_\tau < 0$, $q^f_\tau > 0$, $E_{\tau\gamma} < 0$ and by remark 1 $E_\tau < 0$. Since $|n\beta^h q^h q^h_\tau| < |m\beta^f q^f q^f_\tau|$, $\tau > \varphi'$. Moreover, assume $q^f\beta^f > q^h\gamma n$ and $q^h\beta^h > q^f\gamma m$ which implies $q^f_\gamma < 0$, $q^h_\gamma < 0$ and so $E_\gamma < 0$ and $\beta^h \left(\delta^h\beta^f(m+1) - m\gamma\delta^f\right)\left[\beta^f q^f(m+n+2) - 2m\gamma q^h\right] > 0$ and $\beta^f \left(\delta^f\beta^h(n+1) - n\gamma\delta^h\right)\left[q^h\beta^h(m+n+2) - 2n\gamma q^f\right] < 0$. Hence, $W_{\tau\gamma} < 0$ as long as the second term in (18), $\beta^h(\cdot)[\cdot]$, is small. □

3.4. When firms are environmentally conscious

In this section I consider the case where the pollution intensity coefficients, δ^h and δ^f, depend positively on the degree of product differentiation, γ.[20] The idea here is to capture the notion that firms may differentiate themselves by being more environmentally conscious through a lower pollution intensity coefficient (see e.g., Lambertini (2013) for a discussion on the literature on environmentally conscious firms). More formally, let $\delta^k = \delta^k(\gamma)$ so that $\delta^{k\prime} \geq 0$ for $\beta^k \geq \gamma \geq 0$; $k = h, f$. Notice that this structure does not endogenize the ability of firms to become more or less pollution intensive, and in so doing I assume away issues about the strategic decision of firms on pollution abatement or environmental $R\&D$, which the literature has touched on (e.g., Montero 2002a, 2002b; Ulph 2006; Ulph and Ulph 2007; Gautier 2013). However, by assuming pollution intensity coefficients to depend positively on γ I introduce a new channel whereby an exogenous change in the degree of product differentiation alters emissions. It is noteworthy that in this case optimal policy will also be affected through these new channels (I touch on this issue later on), but results presented in the previous section, along with the intuition, do not change substantially. As a result, the analysis presented in this section limits itself to looking at how changes in the degree of product differentiation (for given tax) alter industry emissions. The analysis proceeds under the assumption of an end-of-pipe cost function. In particular, for home emissions differentiation of (4)-(7) gives

$$\begin{aligned}\tilde{\Delta} de^h &= \Big[\delta^h m\left(\beta^f(m+1)q^f - nq^h\gamma\right) + \delta^{h\prime}\delta^h\beta^f(m+1)a^h - \delta^{f\prime}\delta^h m\gamma\delta^{f\prime}a^f \\ &\quad - q^h\delta^{h\prime}(\beta^h\beta^f(n+1)(m+1) - nm\gamma^2)\Big]\, d\gamma \qquad (19)\end{aligned}$$

where $\tilde{\Delta} < 0$, and (19) becomes (10), if $\delta^{k\prime} = 0$. As before, the first two terms denote the interaction between foreign and home output as products become more differentiated: on the one hand, home output rises as products become more differentiated and on the other it falls via the oligopolistic interdependence as foreign output rises with more differentiated products. The third term captures the increase in home emissions as home firms lower their pollution intensity coefficient via more differentiated products: a reduction in δ^h results in a decrease in marginal costs of home firms which in turn results in higher output and thus emissions by home firms. The fourth term captures the reduction in home emissions through the oligopolistic interdependence nature of the model as foreign firms lower their pollution intensity coefficient via more differentiated products: a reduction in δ^f results in an increase in foreign output and , as a result, a decrease in home output via the oligopolistic interdependence which results in lower emissions by home firms. The last term captures the reduction in home emissions (at each level of output) as home firms exhibit a smaller pollution intensity coefficient.

[20] Analogous results are obtained when the number of foreign firms is endogenous.

There are many cases under which emissions in the home country may fall or rise as products become differentiated so here I just point at a few. In particular, from (19) it can be shown that in the case where products are very differentiated home emissions rise *if and only if* the reduction via δ^h for given level of output is small. This is because with $\gamma \simeq 0$ the effects via the oligopolistic interdependence vanish. Analogously, in the case where all direct effects dominate (i.e.,$q^f\beta^f(m+1) > n\gamma q^h$ and $\delta^{h\prime}\beta^f(m+1) > m\gamma\delta^{f\prime}$), then a similar result arises. In contrast, in the case where all indirect effects dominate (i.e., $nq^h\gamma > q^f\beta^f(m+1)$ and $\delta^{f\prime}\gamma > \delta^{h\prime}\beta^f(m+1)$), then home emissions fall both via a reduction in home output and a decrease in pollution intensity for given level of output. A similar analysis applies to foreign emissions.

In terms of industry emissions, combining (19) and an analogous expression for foreign emissions gives,

$$\begin{aligned}\tilde{\Delta}dE &= \Big[nm\delta^h\left(q^f\beta^f(m+1) - q^h\gamma n\right) + nm\delta^f\left(q^h\beta^h(n+1) - q^f m\gamma\right) \\ &\quad \delta^{h\prime}a^h n\left(\delta^h\beta^f(m+1) - \delta^f\gamma m\right) + \delta^{f\prime}a^f m\left(\delta^f\beta^h(n+1) - \delta^h\gamma n\right) \\ &\quad - \left(\beta^h\beta^f(n+1)(m+1) - nm\gamma^2\right)\left(nq^h\delta^{h\prime} + mq^f\delta^{f\prime}\right)\Big]\, d\gamma \qquad (20)\end{aligned}$$

where in the benchmark case where $\delta^h = \delta^f$, or $\delta^h = \delta^f$ and $\delta^{h\prime} = \delta^{f\prime}$, $dE < 0$ *if and only if* the reduction in emissions via δ^h and δ^f (for given output q^h, q^f) is small i.e., last term in (20) is small. The intuition here is that with equal pollution intensity coefficients and equally environmentally conscious firms the direct effect completely offsets the indirect effects and, consequently, with a relatively small reduction in emissions (for given output) industry emissions rise as a result. An analogous result applies in the case where products are independent ($\gamma \simeq 0$) since in this case the effects via the oligopolistic interdependence vanish.

Proposition 3. *In the case where home and foreign firms are equally environmentally conscious ($\delta^{h\prime} = \delta^{f\prime}$) and equally pollution intensive ($\delta^h = \delta^f$) industry emissions rise (fall) if the induced effect for given level of output is small (large) as products become differentiated.*

Proof. Let $\delta^h = \delta^f = \delta$, $a^h = a^f = a$ and $\delta^{h\prime} = \delta^{f\prime} = \delta^{\prime}$. Factoring out common terms in (20) gives

$$\begin{aligned}\tilde{\Delta}dE/d\gamma &= \Big[nm\delta\left(q^f(\beta^f(m+1) - \gamma m) + q^h(\beta^h(n+1) - \gamma n)\right)\Big] \\ &\quad +\delta^{\prime}\delta a\left[n\left(\beta^f(m+1) - \gamma m\right) + m\left(\beta^h(n+1) - \gamma n\right)\right] \\ &\quad -\delta^{\prime}\left(\beta^h\beta^f(n+1)(m+1) - nm\gamma^2\right)\left(nq^h + mq^f\right)\end{aligned}$$

Hence, $dE < 0 \Longleftrightarrow \delta^{\prime}\omega\left(nq^h + mq^f\right) > \Big[nm\delta\left(q^f(\beta^f(m+1) - \gamma m) + q^h(\beta^h(n+1) - \gamma n)\right)\Big] +$ $\delta^{\prime}\delta a\left[n\left(\beta^f(m+1) - \gamma m\right) + m\left(\beta^h(n+1) - \gamma n\right)\right]$. □

As a final case, let home firms be relatively more pollution-intensive ($\delta^h n\gamma > \delta^f\beta^h(n+1)$) and assume $q^f\beta^f > \gamma nq^h$, $q^h\beta^h > \gamma mq^f$ so that $q^h_\gamma < 0$ and $q^f_\gamma < 0$. Additionally, let home firms be relatively less environmentally conscious so that $\delta^{f\prime}$ is large. In this case, then, industry emissions fall as products become more differentiated. This is because home emissions fall via the oligopolistic interdependence effect and foreign emissions fall as their pollution intensity coefficient become smaller; these effects offsets any increase in output and therefore emissions. Since home firms are relatively more pollution-intensive and home emissions fall as foreign firms become more environmentally conscious, any increase in foreign output (and emissions) is completely offset.

Proposition 4. *In the case where foreign firms are relatively less pollution-intensive (i.e., $\delta^h n\gamma > \delta^f\beta^h(n+1)$) and relatively more environmentally conscious (i.e., $\delta^{f\prime}$ large) industry emissions fall as products become more differentiated.*

Proof. Need to show that (20) is positive when $\delta^h n\gamma > \delta^f\beta^h(n+1)$ and $\delta^{f\prime}$ large. Assume $q^f\beta^f > \gamma nq^h$, $q^h\beta^h > \gamma mq^f$ so that $q^h_\gamma < 0$ and $q^f_\gamma < 0$. Thus, the first three terms in (20) are positive and the last two terms with the $\delta^{f\prime}$ term are negative. If these two last terms are sufficiently large, then $\tilde{\Delta}dE > 0$ since $\tilde{\Delta} < 0$. □

Propositions 3 and 4 present at least two implications. First, they indicate that for a given tax industry emissions may fall as firms differentiate their products and, therefore, suggest a channel whereby reductions in environmental damages can be achieved. Second, if the induced effect for given level of output is large as firms become environmentally conscious, then not only do industry emissions fall but also the incentives for the government to lower the optimal tax as products become more differentiated rise, thereby leaving more room to address the output distortion.

4. Free Entry and Exit of Foreign Firms

This section examines the case where foreign firms can freely enter and exit the market, a condition which is characterized by the zero-profit condition, $\pi^f = 0$. The analysis proceeds under the assumption of an end-of-pipe cost function. In particular, using (4)-(7) and $\pi^f = 0$ yields the free-entry equilibrium which is obtained sequentially

$$
\begin{aligned}
q^f &= \sqrt{\left(f^f - (\tau)^2/2\right)/\beta^f} && (21)\\
\tilde{\omega}q^h &= \left(\beta^f(\alpha^h - \tilde{c}^h - \delta^h\tau) - \gamma(\alpha^f - \tilde{c}^f - \delta^f\tau - \beta^f q^f)\right) && (22)\\
\tilde{\omega}q^f m &= \left(\beta^h(n+1)(\alpha^f - \tilde{c}^f - \delta^f\tau - \beta^f q^f) - n\gamma(\alpha^h - \tilde{c}^h - \delta^h\tau)\right) && (23)
\end{aligned}
$$

where $\tilde{\omega} = \beta^h\beta^f(n+1) - n\gamma^2 > 0$. The effect of the tax is given by

$$
\begin{aligned}
\partial q^f/\partial\tau &= -\tau/2\beta^f q^f < 0 && (24)\\
q^f\tilde{\omega}\partial m/\partial\tau &= -\beta^h(n+1)\delta^f + n\gamma\delta^h + (\beta^h\beta^f(n+1)(m+1) - mn\gamma^2)\tau/2\beta^f q^f && (25)\\
\tilde{\omega}\partial q^h/\partial\tau &= -\beta^f\delta^h + \gamma\delta^f - \tau\gamma/2q^f && (26)\\
\tilde{\omega}\partial Q^f/\partial\tau &= \beta^h\beta^f(n+1)\tau/2\beta^f q^f - \beta^h(n+1)\delta^f + n\gamma\delta^h && (27)
\end{aligned}
$$

There are several effects at play here. First, an increase in the tax induces foreign firms to increase abatement and, as a result, profits rise which results in more foreign firms entering the market (last term in equation 25, $\tau/2\beta^f q^f$). As a result, total foreign output increases (first term in 27) but less output is produced by each foreign firm (equation 24). The increase in foreign output resulting from the abatement induced by the tax induces home firms to react strategically by lowering output, $\tau\gamma/2q^f$ in equation (26); this is because of the oligopolistic interdependence nature of the model. Second, tax payments exhibit offsetting effects as discussed in section 3. On the one hand, an increase in the tax results in home firms paying more in taxes for each additional unit of output produced, thereby lowering home output and, as a result, foreign firms react strategically and enter the market (foreign output rises) since, *ceteris paribus*, foreign firms are relatively more cost competitive. On the other, foreign firms also pay more in taxes given the tax increase, which results in lower foreign profits, number of foreign firms and total output by foreign. As a result, home firms react strategically by increasing output. If home firms exhibit a relatively large pollution intensity coefficient (i.e., $\beta^f\delta^h > 2\gamma\delta^f$), then home output falls because home firms are less cost competitive. If the strategic effect is small, particularly if products are very differentiated ($\gamma \simeq 0$), then foreign total output and home output fall with the tax since the strategic effect vanishes. In either case, however, the extent to which the number of foreign firms rises or falls depends upon whether the abatement effect is large vis-à-vis the direct effect of the tax; specifically, a sufficiently large abatement effect (i.e., $\tau/2\beta^f q^f$ large) raises the number of foreign firms.

Next, I touch on the effect of the tax on emissions using (24)-(27) and the definition of industry emissions, $E = ne^h + me^f$, where by assumption $e^h = \delta^h q^h - a^h$ and $e^f = \delta^f q^f - a^f$, and $a^h = \tau$, $a^f = \tau$ from the first-order conditions. In particular,

$$\begin{aligned} dE &= mde^f + e^f dm + nde^h = m\left(\delta^f dq^f - d\tau\right) + n\left(\delta^h dq^h - d\tau\right) + e^f dm \\ &= \Bigg[m\left(-\delta^f \frac{\tau}{2\beta^f q^f} - 1\right) + n\left(\frac{\delta^h}{\tilde{\omega}}(-\beta^f\delta^h + \gamma\delta^f - \gamma\tau/2q^f) - 1\right) \\ &+ \frac{e^f}{q^f\tilde{\omega}}\left(-\beta^h(n+1)\delta^f + \gamma n\delta^h + \tau\hat{\omega}/2\beta^f q^f\right)\Bigg] d\tau \end{aligned} \tag{28}$$

where $\tilde{\omega} = \beta^h\beta^f(n+1) - n\gamma^2 > 0$, $\hat{\omega} = \beta^h\beta^f(n+1)(m+1) - mn\gamma^2 > 0$. The first term captures the change in industry emissions via foreign per-firm emissions i.e., via reductions in foreign per-firm output, $-m\delta^f\tau/2\beta^f q^f$, and higher abatement induced by the tax, $-m$. The second term captures the change in industry emissions via home per-firm emissions i.e., reductions in home per-firm output, $-n\beta^f\delta^h$, a higher abatement induced by the tax, $-n$, and increases in home output via the oligopolistic interdependence, $\gamma\delta^f - \gamma\tau/2q^f > 0$. The third term captures changes via changes in the number of foreign firms i.e., reductions in the number of firms, $-\beta^h(n+1)\delta^f$, and increases via the oligopolistic interdependence, $\gamma n\delta^h$, and higher abatement induced by tax, $\tau\hat{\omega}/2\beta^f q^f$.

Industry emissions fall if the reduction in home emissions (arising from lower home output and higher abatement by home firms) and the reduction in foreign emissions (arising from lower per-firm foreign output and higher abatement by foreign firms) offsets any increase in emissions arising from the number of foreign firms that enter the market as a result of a higher tax. If home firms are relatively more pollution-intensive (i.e., $\beta^f\delta^h > 2\gamma\delta^f$), then home emissions, via reductions in home output, fall sufficiently so that industry emissions fall. In the case where products are very differentiated ($\gamma \simeq 0$), industry emissions fall since in this case the strategic effect vanishes and so home output and total foreign output fall with the tax, thereby lowering home and foreign emissions.

Proposition 5. *In the case where the number of foreign firms is endogenous, industry emissions fall if home firms are sufficiently more pollution-intensive ($\beta^f\delta^h > 2\gamma\delta^f$) or if products are very differentiated ($\gamma \simeq 0$).*

Proof. First, suppose $\gamma \simeq 0$. Then, equation (28) becomes

$$dE/d\tau = \left[m\left(-\frac{\tau^2}{2\beta^f(q^f)^2} - 1\right) + n\left(-\frac{(\delta^h)^2}{\beta^h(n+1)} - 1\right) + \frac{e^f}{q^f\beta^f}\left(-\delta^f + \tau/2q^f\right)\right] < 0$$

where $-\delta^f + \tau/2q^f = -\delta^f + \tau/q^f - \tau/2q^f = -e^f/q^f - \tau/2q^f < 0$. As a second case, suppose $\beta^f\delta^h > 2\gamma\delta^f$. Then, re-writing equation (28) gives

$$\begin{aligned} dE/d\tau &= \left[m\left(-\frac{\tau^2}{2\beta^f(q^f)^2} - 1\right) + n\left(\frac{\delta^h}{\tilde{\omega}}(-\beta^f\delta^h + 2\gamma\delta^f - \gamma\tau/2q^f) - 1\right)\right. \\ &\quad \left. + \frac{e^f\beta^h(n+1)}{q^f\tilde{\omega}}\left(-\delta^f + \tau/2q^f\right) - \tau n\gamma\delta^h/q^f\tilde{\omega}\right] < 0 \end{aligned}$$

□

4.1. Optimal Policy and Product Differentiation under Free Entry and Exit of Foreign Firms

In this section I characterize the optimal tax and examine the extent to which it rises or falls as products become more differentiated. One key feature is that the incentives to abate pollution become prominent via the free entry and exit of foreign firms. Similar to the case where the number of foreign firms is exogenous, the extent to which the government lowers the tax depends on how it weighs environmental damages.

In the case of free entry the government solves the following maximization problem

$$\max_{\tau} W = CS(nq^h, mq^f) + n\pi^h + E\tau - \varphi(E) \tag{29}$$

where profits of foreigns firms are assumed away because of the free-entry condition. From (29) the following first-order condition is obtained:

$$W_\tau = Q^f(\beta^f Q^f_\tau + n\gamma q^h_\tau) + me^f + nq^h\beta^h q^h_\tau + (\tau - \varphi')E_\tau \tag{30}$$

Setting (30) equal to zero the following expression for the tax is obtained:

$$\tau = \varphi' - \left(Q^f\left(\beta^f Q^f_\tau + n\gamma q^h_\tau\right) + me^f + nq^h\beta^h q^h_\tau\right)/E_\tau = \varphi' - \left(-m\tau/2 + nq^h\beta^h q^h_\tau\right)/E_\tau \quad (31)$$

where using (26), (27), $Q^f = mq^f$ and $e^f = \delta^f q^f - \tau$ gives $Q^f\left(\beta^f Q^f_\tau + n\gamma q^h_\tau\right) + me^f = -m\tau/2$. There are several effects at play here which the government needs to balance. The first term captures damages from pollution and the second and third terms consumer surplus gains and tax revenue, respectively. The fourth term captures the output distortion. The consumer surplus effect adjusts the tax downwards, meaning that the government sets a lower tax in order to achieve those gains, but the tax revenue induces the government to raise the tax; the former effect dominates the latter i.e., $Q^f\left(\beta^f Q^f_\tau + n\gamma q^h_\tau\right) + me^f = -m\tau/2 < 0$. By adjusting the tax downwards the government also controls the entry of firms, for a lower tax reduces the abatement incentives from the tax and thus the entry of firms. Evaluating (30) at $\tau = 0$ gives

$$W_\tau|_{\tau=0} = n\beta^h q^h q^h_\tau - \varphi' E_\tau \quad (32)$$

where, assuming strict concavity of the $W(\cdot)$ function, optimal policy is to tax emissions if the government cares sufficiently about environmental damages, provided that $\beta^f\delta^h > 2\gamma\delta^f$ or $\gamma \simeq 0$. Notice that either of these conditions ensure both a reduction in industry emissions and home output via the tax, and so the extent to which the optimal tax is positive depends on whether the government puts more weight on the environment i.e., $-n\beta^h q^h q^h_\tau < -\varphi' E_\tau$. This result can also be interpreted as follows: if the government cares sufficiently about the environment and home firms are relatively more pollution-intensive, then the optimal policy is to tax emissions.

Proposition 6. *Let home firms be pollution-intensive ($\beta^f\delta^h > 2\gamma\delta^f$) so that home output and emissions fall with the tax . In the case where the number of foreign firms is endogenous the optimal emission tax is (i) set below marginal damages, and (ii) positive if and only if the government cares sufficiently about the environment.*

Proof. Part (i) from proposition is shown first. Since $\beta^f\delta^h > 2\gamma\delta^f$, $q^h_\tau < 0$ and $E_\tau < 0$ by proposition 5. From (31) we have that $\tau = \varphi' - (-m\tau/2 + nq^h\beta^h q^h_\tau)/E_\tau < \varphi'$. To show part (ii) of proposition 6 equation (31) is simplified to $(-\varphi' E_\tau + n\beta^h q^h q^h_\tau)/(m/2 - E_\tau)$. Hence, the denominator is positive and so $\tau > 0 \Longleftrightarrow -\varphi' E_\tau + n\beta^h q^h q^h_\tau > 0$. □

A note on the optimal tax vis-à-vis marginal damages is in order. In (31) the optimal tax may or may not exceed marginal damages depending upon whether the entry of foreign firms is large (strong free entry effect), the degree of damages from pollution and the output distortion. This result is in line with the literature e.g., Requate (1997) and Katsoulacos and Xepapadeas (1995).[21] I consider several cases in turn to illustrate the key results.

[21] Requate (1997) assumes, in a special case, that both sets of firms are endogenous, while in Katsoulacos and Xepapadeas (1995) there is only one set of identical firms. Even with these differences the underlying results here are consistent with the literature. Although mentioned in Requate (1997), pp. 273-274, one contribution of the present analysis is to specifically identify the role of abatement and differences across pollution intensities in explaining the case for an optimal tax exceeding marginal damages. The analysis in Katsoulacos and Xepapadeas (1995) is a special case of Requate (1997).

First, if the incentives to abate via the tax are sufficiently large, then the entry of new firms into the market results in sufficiently more total abatement and thus lower total emissions; as a result, the optimal tax is less than marginal damages: the second term in (31) is negative because by setting the optimal tax less than marginal damages the government reduces the abatement incentive arising from the tax, thereby controlling the entry of foreign firms. This case is characterized by a small foreign pollution intensity coefficient so that (i) emissions do not rise with more firms entering the market, (ii) foreign output rises as more firms enter the market, and (iii) home output falls with the entry of foreign firms; that is, $E_t < 0$ due to the large abatement effect, $Q^f_\tau > 0$ and $q^h_t < 0$ due to the small pollution intensity coefficient of foreign firms. As a second case, consider a strong free entry of foreign firms but a large pollution intensity coefficient of foreign firms so that total emissions rise with the entry of firms; in this case the second term in (31) is positive and thus the optimal tax exceeds marginal damages in order to control (mitigate) the excessive entry of foreign firms. Because the pollution intensity coefficient of foreign firms is large then, in this case, raising the tax controls the number of foreign firms by raising payments of emissions per unit of output. Formally, this case is characterized by $E_\tau > 0$ due to the large entry effect and large foreign pollution intensity coefficient, $Q^f_t < 0$ due to the large foreign pollution intensity coefficient, and $q^h_\tau < 0$. It is noteworthy that it is possible to have a case where $E_\tau > 0$, $Q^f_t < 0$, but $q^h_\tau > 0$; in this case the government needs to balance two opposing effects: on the one hand, as before, it raises the tax to control the entry of firms because of the large pollution intensity coefficient of foreign firms (in this case the increase in the tax raises home output since $q^h_\tau > 0$), and on the other it lowers the tax to control emissions via lower home output. The extent to which the tax exceeds or falls short of marginal damages depends upon which effect is relatively large.

In contrast, the free entry effect of foreign firms may not be sufficiently strong; this case may be characterized by the abatement incentive from the tax not being large enough, where the increase in the number of foreign firms does not result in increased total emissions either when the pollution intensity coefficient of foreign firms is small ($Q^f_\tau > 0$) or large ($Q^f_\tau < 0$). In these cases total emissions do not increase as a result of entry of foreign firms; this is captured by $E_\tau < 0$ and $q^h_\tau < 0$. As a result, the optimal tax falls short of marginal damages i.e., because of the "weak" entry effect the government has room to address the output distortion, $\beta^h q^h$, by setting a tax less than marginal damages. Another possibility when the entry effect is not strong is that $E_\tau < 0$ and $q^h_\tau > 0$, where $Q^f_\tau < 0$. In this case there are two opposing effects the government needs to balance; on the one hand, the output distortion, $\beta^h q^h$, is addressed via a higher tax, but on the other a lower tax controls the number of firms by reducing the abatement incentive, $-m\tau/2$. If the former effect is large, then the optimal tax exceeds marginal damages. [22]

Next, I look into the issue raised earlier, viz., how the government adjusts the optimal

[22] In principle it is possible to have a case where $q^h_\tau > 0$ and $Q^f_\tau > 0$, but this case is ruled out because the inequalities $Q^f_\tau > 0$ and $q^h_\tau > 0$ are not consistent with each other. To see this let $q^h_\tau > 0$ and $Q^f_\tau > 0$. Applying these inequalities to the expressions in (27) and (26) yields $\gamma > \beta^h$, which contradicts one of the assumptions of the model. Additionally, when the free entry effect is strong and δ^f is large the case where $E_\tau > 0$ and $Q^f_\tau > 0$ is ruled out. This is because with a large foreign pollution intensity coefficient consistent with $E_\tau > 0$ (i.e., $\tau(\delta^f(\beta^h(n+1) - \delta^h n\gamma) - \tau^2 2/3\beta^f q^f > 0)$ it can be shown that $E_\tau > 0 \Rightarrow m_\tau < 0$ and thus $Q^f_\tau < 0$; in this case the condition for large δ^f is obtained from E_τ.

tax as products become more differentiated. Changes in output with respect to γ (for given tax) follow a similar analysis to that in section 3.2.; in particular, the extent to which foreign total output, $Q^f = mq^f$, and home total output, $Q^h = nq^h$, rise as products become more differentiated depends upon whether the effect via the oligopolistic interdependence is small. It is noteworthy that in the case where the number of foreign firms is endogenous, however, changes in foreign output take place exclusively via changes in the number of foreign firms.

As products become more differentiated the government needs to balance (i) the gains from product variety and entry of firms, (ii) the output distortion and (iii) the damages from pollution. From (30), $W_\tau = -m\tau/2 + n\beta^h q^h q^h_\tau + (\tau - \varphi')E_\tau$, and so differentiation of $W_\tau(\tau(\gamma), \gamma) = 0$ yields $\tau_\gamma = -W_{\tau\gamma}/W_{\tau\tau}$, where $W_{\tau\tau} < 0$ from the concavity assumption of the $W(\cdot)$ function and

$$W_{\tau\gamma} = -m_\gamma\tau/2 + n\beta^h q^h_\tau q^h_\gamma + n\beta^h q^h_{\tau\gamma} + (\tau - \varphi')\, E_{\tau\gamma} - \varphi'' E_\tau E_\gamma \tag{33}$$

where E_γ is given by (14); E_τ is given by (28); q^h_τ is given by (26) and

$$\tilde{\omega} q^f m_\gamma = -n\beta^h(n+1)q^h + n\gamma q^f m \tag{34}$$

$$\tilde{\omega} q^h_\gamma = -m\beta^f(m+1)q^f + m\gamma q^h n \tag{35}$$

$$\begin{aligned}(\tilde{\omega})^2 q^h_{\tau\gamma} &= \beta^f\left(\delta^f\beta^h(n+1) - n\gamma\delta^h\right) + \gamma n\left(\gamma\delta^f - \beta^f\delta^h\right) \\ &\quad -\tau\left(\beta^h\beta^f(n+1) + n\gamma^2\right)/2q^f\end{aligned} \tag{36}$$

$$\begin{aligned}(\tilde{\omega})^2 q^f m_{\tau\gamma} &= n\beta^h(n+1)\left(\beta^f\delta^h - \gamma\delta^f\right) + n\gamma\left(n\gamma\delta^h - \beta^h(n+1)\delta^f\right) \\ &\quad +\tau n\beta^h(n+1)\gamma/q^f + m_\gamma\tau(\tilde{\omega})^2/2\beta^f q^f\end{aligned} \tag{37}$$

$$\begin{aligned}E_{\tau\gamma} &= nq^h_{\tau\gamma} + e^f m_{\tau\gamma} \\ &= \frac{2n}{(\tilde{\omega})^2}\left(n\gamma\delta^h - \beta^h(n+1)\delta^f\right)\left(\gamma\delta^f - \beta^f\delta^h\right) \\ &\quad + \frac{n\tau}{(\tilde{\omega})^2 2q^f}\Big[\beta^h(n+1)\left(-\delta^h\beta^f + 2\gamma\delta^f\right) - \delta^h n\gamma^2 \\ &\quad + \frac{\tilde{\omega}}{\beta^f q^f}\left(-\beta^h(n+1)q^h + \gamma q^f m\right)\Big]\end{aligned} \tag{38}$$

$$\begin{aligned}n\beta^h q^h_\tau q^h_\gamma + n\beta^h q^h_{\tau\gamma} &= \frac{n\beta^h}{(\tilde{\omega})^2}\Big[\beta^f q^h\left(\beta^h(n+1)\delta^f - n\gamma\delta^h - \tau\beta^h(n+1)/2q^f\right) \\ &\quad + \left(-\beta^f\delta^h + \gamma\delta^f - \tau\gamma/2q^f\right)\left(-\beta^f q^f m + \gamma q^h n\right)(m+1)\Big]\end{aligned} \tag{39}$$

The sign of (33) dictates the sign of τ_γ i.e., the adjustment of the tax with respect to the degree of product differentiation. The first term in (33) captures the control for the entry of foreign firms. The second and third terms capture the adjustment in the tax to address the output distortion. The fourth and fifth terms capture the adjustment in the tax to address changes in industry emissions.

There are several effects at play here. Firstly, as products become more differentiated foreign firms exhibit increased market power and, as a result, the government lowers the tax to encourage foreign output. By lowering the tax the government also achieves gains from product variety and controls the number of firms via reduced abatement (recall that a lower tax reduces abatement and thus profits). Now, because of the oligopolistic interdependence output by home firms increases and this partially offsets the need to lower the tax. Analogously, as products become more differentiated home firms exhibit increased market power, which induces a tax reduction by the government, but at the same time, via the oligopolistic interdependence, foreign output rises which partially offsets the need to lower the tax. Therefore, if the oligopolistic interdependence effect is small then the government lowers the tax to encourage output to address the increased market power. Secondly, the government needs to account for changes in industry emissions. On the one hand, as products become differentiated emissions rise, thereby inducing a tax increase. But at the same time via the oligopolistic interdependence the need for a higher tax diminishes. If the effect via the oligopolistic interdependence is small then the tax increases to address higher emissions. As a result, the government raises the tax if the damages from pollution are relatively more important vis-à-vis the gains from product variety and the increased market power distortion. It is noteworthy that the extent of the need for a higher tax to address emissions is somewhat offset by the abatement induced by the tax; this is because a lower tax controls abatement, the number of firms and emissions.

To illustrate these effects and as a benchmark case, suppose the effect via the oligopolistic interdependence is small (both via changes in the tax and degree of product differentiation). This assumption implies that foreign (home) output rise (rise) as products become more differentiated and that foreign (home) output fall (fall) with an increase in the emission tax. As a result, industry emissions rise and fall, respectively, as products become more differentiated and with the emission tax, and the emission tax is set below marginal damages. More formally, in the benchmark case $m_\gamma < 0$, $q^h_\gamma < 0$, $E_\gamma < 0$, $E_\tau < 0$, $m_\tau < 0$, $\tau - \varphi' < 0$. In this case the government weighs environmental damages vis-à-vis gains from product variety and the output distortion; if the former is relatively more important then the tax rises with more differentiated products. It is noteworthy that via the abatement induced by the tax the government, one the one hand, controls the number of foreign firms via a tax reduction; this puts a downward pressure on the tax as products become more differentiated i.e., lowering the tax controls the entry of foreign firms. One the other, as emissions are addressed via a higher tax, there is entry of foreign firms via the abatement induced by the tax. If these effects are small, then τ rises as long as damages to the environment are relatively more important.

In the case where products are very differentiated (i.e., $\gamma \simeq 0$) the oligopolistic interdependence effect vanishes and so the tax rises if the government puts more weight on addressing environmental damages and the entry of foreign firms via the abatement induced by the tax is small. In contrast, in the case where products are homogeneous in the sense that $\beta^h = \beta^f = \gamma = 1$, the incentives to lower the tax to achieve gains from product variety are less, a point made in Fujiwara (2009). The government in this case still needs to balance environmental damages vis-à-vis the output distortion.

Proposition 7. *In the case where the number of foreign firms is endogenous, the effect via the oligopolistic interdependence is small and the entry of foreign firms via the abatement induced by the tax is small, the optimal tax rises as products become more differentiated if and only if the government puts sufficient weight on environmental damages.*

Proof. Suppose $\gamma = 0$. Then, $q^h_{\tau\gamma} > 0$, $q^h_\tau < 0$, $m_\gamma < 0$, $q^h_\gamma < 0$, $E_\tau < 0$, $E_\gamma < 0$, $\tau - \varphi' < 0$ and so

$$W_{\tau\gamma}|_{\gamma=0} = [-m_\gamma \tau/2 + n\beta^h q^h_\tau q^h_\gamma + n\beta^h q^h_{\tau\gamma}] + [(\tau - \varphi')(n\delta^h q^h_{\tau\gamma} + e^f m_{\tau\gamma}) - \varphi'' E_\tau E_\gamma]$$

where

$$\begin{aligned}\left(n\beta^h q^h_\tau q^h_\gamma + n\beta^h q^h_{\tau\gamma}\right)|_{\gamma=0} &= \frac{n\beta^h\beta^f}{(\tilde{\omega})^2}\left(q^h\beta^h(n+1)\left(\delta^f - \tau/2q^f\right) + \delta^h\beta^f q^f m(m+1)\right) > 0 \\ \beta^h\beta^f(n+1)E_{\tau\gamma}|_{\gamma=0} &= \delta^h n\left(\delta^f - \tau/2q^f\right) + n\left(\delta^h\delta^f - \frac{\tau\beta^h(n+1)q^h}{2\beta^f(q^f)^2}\right)\end{aligned}$$

The first term in squared brackets in $W_{\tau\gamma}|_{\gamma=0}$ is positive (i.e., tax falls to achieve gains from product variety, control the entry of firms via abatement and address output distortion) and the second negative (i.e., tax increases to address damages from emissions) as long as the last term in $E_{\tau\gamma}|_{\gamma=0}$ is small i.e., entry of foreign firms via a higher tax (abatement) is small. Thus $W_{\tau\gamma} < 0$ as long as the second term in squared brackets is relatively large i.e., sufficient weight on environmental damages. □

Next, I shall assume home firms to be relatively more pollution-intensive ($\delta^h\gamma > \beta^h(n+1)\delta^f$) so that foreign (home) output and firms rise (fall) with a tax increase, industry emissions fall with a tax increase, and the optimal tax is set below marginal damages. Further, suppose that home output, number of foreign firms and therefore emissions rise with more differentiated products. In this case there is an incentive to raise the tax in order to increase foreign output and achieve gains from product variety; a higher tax addresses damages from emissions. But also there is an incentive to reduce the tax to control for the entry of foreign firms (via reduced abatement) and thus emissions, and encourage home output. If the government puts relatively more weight on promoting home output, controlling emissions and entry of foreign firms, then the tax falls as products become differentiated as long as the environmental damage arising from increased emissions is small. A second possibility is that the government may put a sufficiently large weight on promoting foreign less pollution-intensive firms; if the need to lower the tax to control for emissions and entry of firms is small, then the tax may increase as products become more differentiated.

Proposition 8. *In the case where the number of foreign firms is endogenous the government may promote foreign, less pollution-intensive, firms by raising the optimal tax as products become more differentiated as long as the need to control for the entry of foreign firms is small.*

Proof. Suppose (i) $m_\gamma < 0$, (ii) $q_\gamma < 0$, and (iii) $\delta^h\gamma > \beta^h(n+1)\delta^f$. From (i) and (ii) it follows that $E_\gamma < 0$. From (iii) it follows that a) $n\gamma\delta^h > \beta^h(n+1)\delta^f$ and b) $\beta^f\delta^h > 2\gamma\delta^f$. In turn, from a) it follows that $m_\tau > 0$ and $Q^f_\tau > 0$; and from b) it follows that $q^h_\tau < 0$ and, from proposition 5, $E_\tau < 0$. Moreover, $\tau - \varphi' < 0$ from proposition 6. Then, inspection of (34) - (39) indicates that $q^h_{\tau\gamma} < 0$, $E_{\tau\gamma} < 0$ and so

$$W_{\tau\gamma}|_{\gamma\delta^h > \beta^h(n+1)\delta^f} = \left[-m_\gamma\tau/2 + n\beta^h q^h_\tau q^h_\gamma + (\tau - \varphi')E_{\tau\gamma}\right] - \varphi'' E_\tau E_\gamma + n\beta^h q^h q^h_{\tau\gamma}$$

where the term in squared brackets (which is positive) captures the reduction in the tax to control for the entry of foreign firms and emissions, and promote home output; the second term captures the tax increase to address higher emissions, $-\varphi'' E_\tau E_\gamma < 0$; and the third term captures the stimulus to foreign output, $\beta^h q^h q^h_{\tau\gamma} < 0$. If the government puts sufficient weight on foreign output and gains from product variety, then $W_{\tau\gamma} < 0$. □

In contrast to the case where the number of foreign firms is exogenous, here the policy of promoting foreign, less pollution-intensive, firms via taxation, as products become more differentiated, is more restrictive since now the government needs to control for the entry of foreign firms.

5. Conclusion

Industries characterized by differentiated products are important contributors of greenhouse gases and currently subject to market-based policies such as emission taxes; there are cases of this type of industries operating in developing countries. This paper examines how an optimal emission tax may be adjusted as products become more differentiated. The analysis indicates that abatement efforts and differences in pollution intensity coefficients across firms may play a significant role in the characterization of optimal policy. Industry emissions may fall/rise as a result of more differentiated products. The analysis also suggests that the government may opt to encourage foreign, less pollution-intensive, firms via higher taxation, a result which is particularly important in the context of developing countries where there might be concerns about foreign investment leaving the country. The chief contribution of this paper is that it emphasizes the role of abatement efforts and differences in pollution intensity coefficients across firms, two important aspects with limited attention in the literature.

In drawing policy implications it should be noted some of the limitations about the present work. Firstly, aspects of strategic interaction of governments (e.g., between a foreign and host government), trade issues and the strategic decision of foreign firms to either export or establish operations in the host country are assumed away, all of which have been studied in the literature, but nevertheless play a key role in designing policy. The focus here, instead, is on the welfare implications for the host country and the cases in which abatement efforts and differences in intensity coefficients may explain higher taxation to encourage foreign firms into the host country. Secondly, it should be kept in mind that many of the results are derived under the assumption of an end-of-pipe cost function. Thus, a potential line of research would be to relax this assumption and observe whether results change.

The present work can be extended to study several policy issues. Firstly, additional policies may be introduced into the model. This is a valid extension because in many cases firms are subject to a set of policies, rather than just emission taxes. Relevant policies to consider may include lump-sum, output/abatement subsidies or standards, three policies which may capture cases where firms are subject to emission taxes, different funding schemes and technology standards. Secondly, the analysis could be easily extended to an international context, which would in turn open the door for the analysis of policy reform (e.g., as in Lahiri and Symeonidis 2007) and issues of transboundary pollution. Thirdly, in an international context the case where the degree of product differentiation is endogenous might provide insights about the strategic interaction of firms and governments as firms become more environmentally conscious. Fourthly, in the context of developing countries the present model set-up could be extended to account for employment considerations in order to see how the incentives to attract foreign firms vis-à-vis domestic firms may change. Finally, it might be constructive to investigate the extent to which results may change in a price-setting model set-up as opposed to a quantity-setting model.

Appendix

Derivation of conditions where e^h, e^f, q^h, q^f, E fall/rise.

(a) Conditions under which q^h falls with the tax (these are mentioned on page 5); an analogous analysis applies to q^f:

Equation 8, specifically the change as a result of the tax, has been re-written here for completeness

$$\tilde{\Delta} dq^h = c_{22}^h c_{22}^f \left[\frac{-c_{12}^h}{c_{22}^h} \left(\beta^f (m+1) + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right) + \frac{c_{12}^f}{c_{22}^f} m\gamma \right] d\tau$$

where $\tilde{\Delta} < 0$, $-c_{12}^h/c_{22}^h > 0$, $-c_{12}^f/c_{22}^f > 0$, $c_{22}^f > 0$, $c_{22}^h > 0$, $(c_{11}^f c_{22}^f - (c_{12}^f)^2) > 0$ by the concavity assumption of the $c(\cdot,\cdot)$ function.

(i) First, consider the case where pollution intensities are identical: $-c_{12}^f/c_{22}^f = -c_{12}^h/c_{22}^h = -c_{12}/c_{22} > 0$. Want to show that under this condition $dq^h < 0$.

Imposing the condition of identical pollution intensity coefficients the change in q^h can be re-written as

$$\tilde{\Delta} dq^h = \frac{-c_{12}}{c_{22}} c_{22}^h c_{22}^f \left[\beta^f (m+1) - m\gamma + (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right] d\tau$$

Thus, q^h falls with a tax increase; this is because the right-hand-side of the equation is positive since $-c_{12}/c_{22} > 0$, $c_{22}^h > 0$, $c_{22}^f > 0$, $c_{11}^f c_{22}^f - (c_{12}^f)^2 > 0$, $\beta^f > \gamma > 0$ and so $dq^h < 0$ since $\tilde{\Delta} < 0$. By the same token $dq^f < 0$ in equation (9).

(ii) Second, consider the condition $\delta^h n\gamma > \delta^f \beta^h (n+1)$; or equivalently, $-c_{12}^h/c_{22}^h n\gamma > -c_{12}^f/c_{22}^f (n+1)\beta^h$. Again, want to show that under this condition $dq^h < 0$.

Notice that from this condition $-c_{12}^h/c_{22}^h n\gamma > -c_{12}^f/c_{22}^f (n+1)\beta^h > -c_{12}^f/c_{22}^f n\gamma \Rightarrow -(c_{12}^h/c_{22}^h) > -(c_{12}^f/c_{22}^f)$ since $\beta^h > \gamma$.

The change in q^h can be re-written as

$$\tilde{\Delta} dq^h = c_{22}^h c_{22}^f \left[\frac{-c_{12}^h}{c_{22}^h} \beta^f (m+1) + \frac{c_{12}^f}{c_{22}^f} m\gamma - \frac{c_{12}^h}{c_{22}^h} (c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f \right] d\tau$$

Thus, q^h falls with a tax increase. This is because the right-hand-side of the equation is positive (and $\tilde{\Delta} < 0$) since $-(c_{12}^h/c_{22}^h)(c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f > 0$; and $-(c_{12}^h/c_{22}^h)\beta^f(m+1) + (c_{12}^f/c_{22}^f)m\gamma > 0$ because $\beta^f > \gamma$ and $-(c_{12}^h/c_{22}^h) > -(c_{12}^f/c_{22}^f)$. In this case, however, the sign of dq^f is ambiguous.

(iii) Third, consider the condition that if pollution intensity coefficient of home firms is sufficiently large so that, in the case of foreign firms, the indirect effect offsets the direct effect (i.e., from equation (9) $\delta^h n\gamma > \delta^f(\beta^h(n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h)$, or equivalently $-c_{12}^h/c_{22}^h n\gamma > -c_{12}^f/c_{22}^f((n+1)\beta^h + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h)$, then $dq^h < 0$ and $dq^f > 0$.

Want to show that under this condition home output falls unambiguously and foreign output rises unambiguously i.e., $dq^h < 0$ and $dq^f > 0$.

So, from condition

$$\frac{-c_{12}^h}{c_{22}^h} n\gamma > \frac{-c_{12}^f}{c_{22}^f}\left((n+1)\beta^h + \frac{c_{11}^h c_{22}^h - (c_{12}^h)^2}{c_{22}^h}\right)$$

One gets

$$\frac{-c_{12}^h}{c_{22}^h} n\gamma > \frac{-c_{12}^f}{c_{22}^f}(n+1)\beta^h$$

Hence,

$$\frac{-c_{12}^h}{c_{22}^h} > \frac{-c_{12}^f}{c_{22}^f}$$

Thus, $dq^h < 0$ i.e., q^h falls with an increase in the tax. To see this consider the change in q^h, which can be re-written as follows

$$\tilde{\Delta} dq^h = c_{22}^h c_{22}^f \left[\frac{-c_{12}^h}{c_{22}^h}\beta^f(m+1) + \frac{c_{12}^f}{c_{22}^f} m\gamma - \frac{c_{12}^h}{c_{22}^h}(c_{11}^f c_{22}^f - (c_{12}^f)^2)/c_{22}^f\right] d\tau$$

where $\tilde{\Delta} < 0$, the first two terms are positive since $\beta^f > \gamma$ and $-c_{12}^h/c_{22}^h > -c_{12}^f/c_{22}^f$; and the third term is positive as before.

Now, q^f rises with an increase in the tax (i.e., $dq^f > 0$). The change in foreign output is given by equation 9:

$$\tilde{\Delta} dq^f = c_{22}^h c_{22}^f \left[\frac{-c_{12}^f}{c_{22}^f} \left(\beta^h (n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h \right) + \frac{c_{12}^h}{c_{22}^h} n\gamma \right] d\tau$$

where $\tilde{\Delta} < 0$ and $-c_{12}^h/c_{22}^h n\gamma > -c_{12}^f/c_{22}^f((n+1)\beta^h + (c_{11}^h c_{22}^h - (c_{12}^h)^2)/c_{22}^h)$.

It is important to note that in the case of an end-of-pipe cost function as defined on page 8, section 3.2, the term $c_{11}c_{22} - c_{12}^2 = 0$. Therefore, the results for dq^h in (i)-(iii) above hold as before. Now, the results for dq^f are as follows: in (i) result holds and so $dq^f < 0$; in (ii) we now have that the sign for the change in q^f is unambiguous i.e., $dq^f > 0$; and in (iii) result holds and so $dq^f > 0$.

(b) Conditions under which e^h and e^f change with the tax as described in bottom of page 6 and top of page 7. Want to show that under the condition $(-c_{12}^f/c_{22}^f)((\beta^h(n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2))/c_{22}^h)) < -(c_{12}^h/c_{22}^h)n\gamma$ home emissions, e^h, unambiguously fall with a tax increase but the sign of de^f is ambiguous. The condition$(-c_{12}^f/c_{22}^f)\beta^h(n+1) < -(c_{12}^h/c_{22}^h)n\gamma$ is also considered here.

(i) First, results associated with e^h are derived. By inspection of equations (8) and (10) the first two terms in (10), $(-c_{12}^h/c_{22}^h)((\beta^f(m+1)+(c_{11}^f c_{22}^f - (c_{12}^f)^2))/c_{22}^f)) + (c_{12}^f/c_{22}^f)m\gamma$, denote changes in emissions via changes in output, q^h. As a result, the conditions which induce a reduction in output, q^h, result in a reduction in emissions, e^h. The rest of the terms in (10) are positive and denote changes in emissions, e^h, via changes in abatement; since $\tilde{\Delta} < 0$ emissions of home firms always fall via abatement. Thus, if $\delta^h n\gamma > \delta^f \beta^h(n+1)$, and so $\delta^h > \delta^f$, then the first two terms (terms associated with changes in output of home firms) in (10) are positive and so emissions, e^h, fall with the tax since $\tilde{\Delta} < 0$. The same result holds if the condition is $\delta^h n\gamma > \delta^f(\beta^h(n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2))/c_{22}^h)$. As a result, under this condition e^h unambiguously falls with a tax increase.

(ii) Second, the result associated with e^f is shown next. Using (11) (and analogous to the analysis using equation (10)), the first two terms of (11) denote the change in emissions of foreign firms via changes in output of foreign firms, q^f; this is clear by inspection of (9) and (11). The rest of the terms denote changes via abatement, which always reduce emissions of foreign firms. So, if $\delta^h n\gamma > \delta^f(\beta^h(n+1) + (c_{11}^h c_{22}^h - (c_{12}^h)^2))/c_{22}^h)$, then on the one hand foreign output rises with a tax increase and so do foreign emissions, but on the other hand foreign emissions fall via abatement i.e., the last three terms in (11). As a result, under this condition the change in e^f is ambiguous.

It is important to note that in the case of an end-of-pipe cost function as defined on page 8, section 3.2, the term $c_{11}c_{22}-c_{12}^2=0$. Therefore, the results for de^h in (b)(i) above holds and results for de^h in (b)(ii) above holds.

(c) Conditions under which total emissions, E, change with the tax. Want to show that under the conditions in remark 1 E falls with a tax increase.

Combining (10) and (11) the change in total emissions is given by equation (12):

$$\begin{aligned}\tilde{\Delta} dE &= c_{22}^h c_{22}^f \left[\frac{c_{12}^h}{c_{22}^h} n \left(\frac{c_{12}^h}{c_{22}^h} \beta^f (m+1) - \frac{c_{12}^f}{c_{22}^f} m\gamma \right) + \frac{c_{12}^f}{c_{22}^f} m \left(\frac{c_{12}^f}{c_{22}^f} \beta^h (n+1) - \frac{c_{12}^h}{c_{22}^h} n\gamma \right) \right. \\ & -\tilde{\Delta}\left(n/c_{22}^h + m/c_{22}^f\right) \\ & + \left(\frac{c_{11}^f c_{22}^f - (c_{12}^f)^2}{c_{22}^f} \right) \left(\beta^h (n+1)(n/c_{22}^h + m/c_{22}^f) + m \frac{c_{11}^h c_{22}^h - (c_{12}^h)^2}{c_{22}^h c_{22}^f} \right) \\ & \left. + \left(\frac{c_{11}^h c_{22}^h - (c_{12}^h)^2}{c_{22}^h} \right) \left(\beta^f (m+1)(n/c_{22}^h + m/c_{22}^f) + n \frac{c_{11}^f c_{22}^f - (c_{12}^f)^2}{c_{22}^h c_{22}^f} \right) \right] d\tau\end{aligned}$$

where $\tilde{\Delta} < 0$, $c_{12}^k/c_{22}^k = -\delta^k$, $c_{11}^k c_{22}^k - (c_{11}^k)^2 > 0$, $c_{12}^k < 0$, $c_{22}^k > 0$ for $k = h, f$.

Since the last three lines are positive total emissions fall with the tax since $\tilde{\Delta} < 0$; these represent the abatement induced by the tax which reduces home and foreign emissions. So, the focus on the following three cases is on the sign of the first line:

$$\frac{c_{12}^h}{c_{22}^h} n \left(\frac{c_{12}^h}{c_{22}^h} \beta^f (m+1) - \frac{c_{12}^f}{c_{22}^f} m\gamma \right) + \frac{c_{12}^f}{c_{22}^f} m \left(\frac{c_{12}^f}{c_{22}^f} \beta^h (n+1) - \frac{c_{12}^h}{c_{22}^h} n\gamma \right) \tag{A.1}$$

Want to show it is positive which in turn indicates a reduction in total emissions since $\tilde{\Delta} < 0$.

(i) Consider condition $\gamma \simeq 0$. In this case one obtains the following expression for (A.1)

$$\left(\frac{c_{12}^h}{c_{22}^h} \right)^2 n\beta^f (m+1) + \left(\frac{c_{12}^f}{c_{22}^f} \right)^2 m\beta^h (n+1) > 0$$

(ii) Consider identical pollution intensities $c_{12}^f/c_{22}^f = c_{12}^h/c_{22}^h = c_{12}/c_{22}$. Hence, (A.1) becomes

$$(c_{12}/c_{22})^2 \left[n(\beta^f (m+1) - m\gamma) + m\left(\beta^h (n+1) - n\gamma\right) \right] > 0$$

(iii) Consider $\delta^h n\gamma > \delta^f \beta^h(n+1)$ where $\delta^h = -c_{12}^h/c_{22}^h$ and $\delta^f = -c_{12}^f/c_{22}^f$. So, from this condition it follows that $\delta^h > \delta^f$. Then,

$$\frac{c_{12}^h}{c_{22}^h} n \left(\frac{c_{12}^h}{c_{22}^h} \beta^f(m+1) - \frac{c_{12}^f}{c_{22}^f} m\gamma \right) + \frac{c_{12}^f}{c_{22}^f} m \left(\frac{c_{12}^f}{c_{22}^f} \beta^h(n+1) - \frac{c_{12}^h}{c_{22}^h} n\gamma \right)$$

$$\begin{aligned}
&= -\delta^h n \left(-\delta^h \beta^f(m+1) + \delta^f m\gamma\right) - \delta^f m \left(-\delta^f \beta^h(n+1) + \delta^h n\gamma\right) \\
&= \delta^h n \left(\delta^h \beta^f(m+1) - \delta^f m\gamma\right) + \delta^f m \left(\delta^f \beta^h(n+1) - \delta^h n\gamma\right) \\
&> \delta^f n \left(\delta^h \beta^f(m+1) - \delta^f m\gamma\right) + \delta^f m \left(\delta^f \beta^h(n+1) - \delta^h n\gamma\right) \\
&= \delta^f \left[n \left(\delta^h \beta^f(m+1) - \delta^f m\gamma\right) + m \left(\delta^f \beta^h(n+1) - \delta^h n\gamma\right)\right] \\
&= \delta^f \left[n\delta^h \left(\beta^f(m+1) - m\gamma\right) + \delta^f m \left(\beta^h(n+1) - n\gamma\right)\right] \\
&> 0.
\end{aligned}$$

As a result, (A.1) is positive and so $dE < 0$ since $\tilde{\Delta} < 0$.

It is important to note that in the case of an end-of-pipe cost function as defined on page 8, section 3.2, the term $c_{11}c_{22} - c_{12}^2 = 0$. Therefore, the results for dE above in (c)(i) - (c)(iii) hold.

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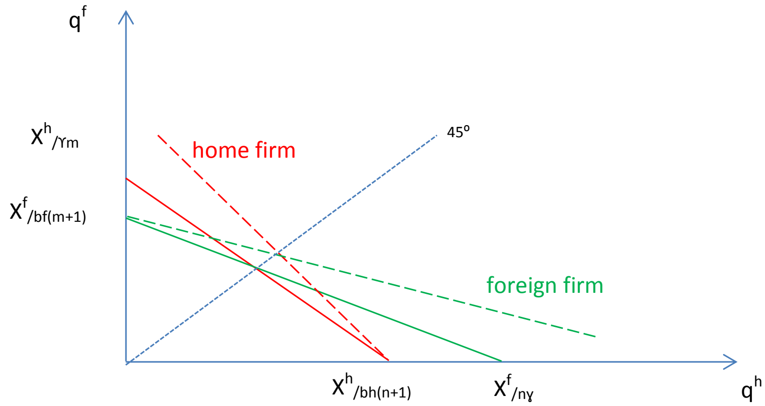
Figure 1: Reaction Functions and Product Differentiation



Remarks: $X^h = \alpha^h - c^h - \delta^h\tau$ and $X^f = \alpha^f - c^f - \delta^f\tau$