Evolutionary Multiobjective Optimization (EMO) algorithms for fuzzy portfolio selection

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Abstract

In this paper, we consider a recently proposed model for portfolio selection, called Mean-Downside Risk-Skewness (MDRS) model. This modelling approach takes into account both the multidimensional nature of the portfolio selection problem and the requirements imposed by the investor. Concretely, it optimizes the expected return, the downside-risk and the skewness of a given portfolio, taking into account budget, bound and cardinality constraints. The quantification of the uncertain future return on a given portfolio is approximated by means of LR-fuzzy numbers, while the moments of its return are evaluated using possibility theory. The main purpose of this paper is to solve the MDRS portfolio selection model as a whole constrained three-objective optimization problem, what has not been done before, in order to analyse the efficient portfolios which optimize the three criteria simultaneously. For this aim, we propose new muta-

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tion, crossover and reparation operators for evolutionary multi-objective optimization, which have been specially designed for generating feasible solutions of the cardinality constrained MDRS problem. We incorporate the operators suggested into the evolutionary algorithms NSGAII, MOEA/D and GWASF-GA and we analyse their performances for a data set from the Spanish stock market. The potential of our operators is shown in comparison to other commonly used genetic operators and some conclusions are highlighted from the analysis of the trade-offs among the three criteria.

Keywords: efficient portfolio selection, evolutionary optimization, possibility distributions, LR-fuzzy numbers

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1. Introduction

Financial optimization models for allocating risky assets involve decision-making under uncertainty. Modern portfolio theory comes from the mean-variance model proposed by [1] and the nonlinear programming problem involved in this modelling approach for selecting portfolios has become into a classical optimization problem. From then, most authors attempt to build optimal portfolios applying a trade-off analysis between two criteria: expected outcome, which should be maximized while reducing a measure of the variability of outcomes, the risk of the investment. Alternative risk measures than variance has been considered in the literature in order to establish optimal portfolios [2, 3]. In recent years, there is a growing interest of including higher moments among the goals of the optimization problem, also taking into account trading and investors requirements, in such a way that using constrained multi-objective optimization techniques is needed for portfolio management [4, 5, 6, 7, 8].

Since [4] proposed to include skewness for selecting portfolios when symmetric behaviour of the returns is not achieved, there has been numerous papers examining the role of skewness in financial optimization. His proposal balanced the maximization of mean and skewness of portfolios and the minimization of their variance, and select optimal portfolios using goal programming. In addition, [9] showed that taking skewness as a goal in the portfolio model formulation causes major changes in the non-dominated frontier. The main difficulty of these modelling approaches relay on the estimation of the third moment, however some approximation models has been proposed. In particular, [10] proposed to replace variance by absolute deviation and approximate skewness by a piecewise linear function transforming the non-linear programming problem into a LP one. Recently, [8] presented a Mean-Downside Risk-Skewness model for portfolio selection in a fuzzy framework for the uncertainty quantification showing that the incorporation of skewness as a goal in a bi-objective optimization program provokes important changes in the patterns of investment.

Concerning investors preferences positive skewness is preferred because should provide a decreasing in the probability of large negative returns. In addition, building a portfolio selection model requires two basic components: (i) a suitable approach for quantifying uncertainty of future returns on a given portfolio, and (ii) an optimization procedure that provides a recommended portfolio under the investor's conditions of investment. Concerning the quantification of uncertainty of returns on a given portfolio we propose to apply possibility distributions of LR-type fuzzy numbers, whose membership functions are built using sample quantiles information from historical data. Derived from the distribution properties of the investment returns, we propose a multi-objective decision model which consider as objective functions the mean and the skewness of the return distribution, and the absolute semi-deviation below the mean, which is the risk measure. Our multi-objective portfolio selection model takes also into account the effect of using both the diversification of investment and the restriction of the number of assets that compose a portfolio. In this paper we analyse the performance of our proposed multi-objective approach with respect to some previously suggested multi-criteria selection methods. Concerning the techniques used to solve out multi-objective portfolio selection model, variants of well-known evolutionary multi-objective algorithms are considered.

The rest of the paper is organized as follows. Section 2 introduces the mean-downside risk-skewness portfolio selection problem that we are using for generate efficient portfolios and some general terminology of multi-objective optimization.

2. Multi-objective portfolio optimization model

2.1. Multi-objective optimization

Multi-objective optimization problems are mathematical programming problems with a vector-valued objective function, which is usually denoted by $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), ..., f_n(\mathbf{x}))$, where $f_j(\mathbf{x})$, for $j = 1, ..., n$, is a real-valued function defined on the feasible region $A \subseteq \Re^N$. Consequently, the decision space belongs to $\Re^N$ while the criterion space belongs to $\Re^n$, and the multi-objective optimization problem can be stated as follows:

$$\begin{array}{ll} \text{optimize} & [f_1(\mathbf{x}), ..., f_n(\mathbf{x})] \\ \text{s.t.} & \mathbf{x} \in A \end{array}$$

In the functional space of criterion some objective functions should be maximized ($j \in J_1$) while others should be minimized ($j \in J_2$), these subsets of indices verifying that $J_1 \cup J_2 = \{1, ..., n\}$. In this context, optimality is defined on the basis of the concept of dominance, in such a way that solving the above problem implies finding the subset of non-dominated solutions, that is those feasible solutions which are not dominated by any other feasible one.

A feasible solution $\mathbf{x}^0$ dominates another solution $\mathbf{x} \in A$ if and only if $f_j(\mathbf{x}^0) \geq f_j(\mathbf{x})$, for every $j \in J_1$ and $f_j(\mathbf{x}^0) \leq f_j(\mathbf{x})$, for every $j \in J_2$, with al least one strict inequality. The set of non-dominated solutions will also be referred by Pareto-optimal solutions and define the efficient frontier of the multi-objective optimization problem (see, for instance, [11]).

2.2. A Mean-Downside Risk-Skewness (MDRS) model

We consider a capital market with N financial assets offering uncertain rates of returns. An investor desires to know which is the optimal allocation of their wealth among the above N assets, looking for the maximization of the expected return of the investment at the end of period. Let us consider a portfolio $\mathbf{x}$ in which the total wealth is allocated, with the shares proportions devoted to each financial asset denoted by x_i, for $i = 1, ..., N$. Then, the allocation $\mathbf{x} = (x_1, ..., x_N)$ must verify:

$$\sum_{i=1}^{N} x_i = 1$$

and, the non-negative condition for every proportion: $x_i \geq 0$, when short selling is excluded. Additionally other constraints (trading requirements, investors preferences, transaction costs and so on) can be incorporated in order to define the feasible set of the multi-objective optimization problem.

To model uncertainty on future returns we work with a fuzzy set theory approach, in such a way that historical data information is used for building suitable membership functions for the returns of the investment. Since a fuzzy number induces a possibility distribution that coincides with its membership function, we consider power LR-type fuzzy numbers to approximate the uncertain return on a given portfolio [12, 13]. In addition, we directly approximate the possibility distribution of the return on a given portfolio $\mathbf{x}$ instead of aggregating the possibility distributions of the individual assets that compose a given portfolio, $i = 1, ..., N$. This modelling approach has been also used for fuzzy ranking risky portfolios and for obtaining efficient portfolios using bi-objective optimization procedures [14, 15].

Uncertainty regarding the future return on $\mathbf{x}$ is then approximated by the fuzzy quantity $\tilde{P}_{\mathbf{x}}$, whose membership function is built using the sample quantiles of the historical data set of the returns on a given portfolio $\mathbf{x}$. Let us consider the returns on $\mathbf{x}$ over T periods $\{r_t(\mathbf{x})\}_{t=1}^{T}$, their sample percentiles are denoted by q_j, being j the order of the percentile. The core and support of $\tilde{P}_{\mathbf{x}}$ are directly associated with suitable q_j's, while the shape parameters are evaluated using the reverse rating procedure deciding those percentiles with a 50% possibility of being realistic.

For a given portfolio $\mathbf{x}$, let us approximate its return $\tilde{P}_{\mathbf{x}} = (p_l, p_u, c, d)_{L_\pi R_\rho}$ as a LR-power fuzzy number, where $[p_l, p_u]$ is the core, c and d are the right and left spreads, respectively, and being π and ρ the positive shape parameters of the reference functions. Our proposed multi-objective approach takes into account possibilistic expected return, absolute semi-deviation and skewness in order to build efficient portfolios (see, for instance, [8]). Mathematically, we compute these possibilistic moments of $P_{\mathbf{x}}$ as follows:

- Possibilistic Mean Value

$$\bar{E}(\tilde{P}_{\mathbf{x}}) = \frac{p_u + p_l}{2} + \frac{d}{2}\frac{\rho}{\rho+1} - \frac{c}{2}\frac{\pi}{\pi+1}.$$

- Possibilistic Downside Risk

$$w(\tilde{P}_{\mathbf{x}}) = p_u - p_l + d\frac{\rho}{\rho+1} + c\frac{\pi}{\pi+1}$$

- Possibilistic Skewness

$$
\begin{aligned}
\mu_3(\tilde{P}_{\mathbf{x}}) \quad = \quad & \frac{1}{4}\left(d\frac{\rho}{\rho+1} - c\frac{\pi}{\pi+1}\right)^3 + \frac{1}{2}\left(d^3\frac{\rho}{\rho+3} - c^3\frac{\pi}{\pi+3}\right) + \\
& + \frac{3(p_u - p_l)}{4} \\
& \times \left[d^2\left(\frac{\rho}{\rho+2} - \frac{\rho^2}{(\rho+1)^2}\right) - c^2\left(\frac{\pi}{\pi+2} - \frac{\pi^2}{(\pi+1)^2}\right)\right] \\
& - \frac{3}{4}\left(d^2\frac{\rho}{\rho+2} + c^2\frac{\pi}{\pi+2}\right)\left(d\frac{\rho}{\rho+1} - c\frac{\pi}{\pi+1}\right)
\end{aligned}
$$

The above expression is the formula of the third possibilitic moment about possibilistic mean value $\bar{E}(\tilde{P}_{\mathbf{x}})$. In order to get a scale independent measure of the asymmetry of the portfolio returns, we consider the coefficient of possibilistic skewness which is defined as:

$$
S(\tilde{P}_{\mathbf{x}}) = \frac{\mu_3(\tilde{P}_{\mathbf{x}})}{w(\tilde{P}_{\mathbf{x}})^3}
$$

Note that the measure of risk, $w(\tilde{P}_{\mathbf{x}})$, is the amplitude of the interval-valued expectation of the absolute semi-deviation about possibilistic mean value. It is important to note that other definitions of possibilistic expected mean and risk are available in the literature but the choice does not affect our evolutionary scheme for building efficient portfolios.

In this paper we consider the limits on the budget to be invested in each asset i by using lower and upper bounds, l_i and u_i, respectively, which are associated with each asset $i = 1, ..., N$. In addition, in order to control the number of assets in the portfolio we incorporate an additional constraint by counting the number of non-negative components in the portfolio $\mathbf{x}$. The possibilistic mean-downside risk-skewness problem can then be formulated as follows:

$$
\begin{array}{lll}
(\mathbf{MDRS}) & \max & \bar{E}(\tilde{P}_{\mathbf{x}}) \\
& \min & w(\tilde{P}_{\mathbf{x}}) \\
& \max & S(\tilde{P}_{\mathbf{x}}) \\
& \text{s.t} & \sum_{i=1}^{N} x_i = 1 \\
& & 0 \le l_i \le x_i \le u_i \quad i = 1, 2, ..., N \\
& & k_l \le c(\mathbf{x}) \le k_u
\end{array}
$$

where $c(X) = rank(diag(\mathbf{x}))$ is the number of positive proportions in portfolio $\mathbf{x}$. Note that the cardinality constraint involves a quasi-concave function, $c(\mathbf{x})$, which implies that optimization problem with this feasible set are NP-hard.

Regarding the possibilistic measures that we are using to define non-dominated portfolios, we say that a feasible portfolio $\mathbf{x}^*$ is Pareto-optimal if there exists no other feasible portfolio $\mathbf{x}$ such that: $\bar{E}(\tilde{P}_{\mathbf{x}}) \ge \bar{E}(\tilde{P}_{\mathbf{x}^*})$, $w(\tilde{P}_{\mathbf{x}}) \le w(\tilde{P}_{\mathbf{x}^*})$ and $S(\tilde{P}_{\mathbf{x}}) \ge S(\tilde{P}_{\mathbf{x}^*})$ with at least one strict inequality. Since the explicit values of the three objective functions involves the quantiles of the return possibility distribution, they can lead to non-convex programs due to the nature of quantiles calculation. Therefore, for solving the above MOP problem the use of metaheuristic procedures is highly recommended.

2.3. EMO in Portfolio Selection

Habría que hacer referencia a los métodos EMO usados en Portfolio Selection, incluido el vuestro.

3. EMO algorithms used for the model MDRS

3.1. New evolutionary operators: mutation, crossover and reparation

A population in an Evolutionary Algorithm is composed of individuals. An individual $\mathbf{x}$, can be considered like a vector of N variables. In our case, they are assets and indicate the percent of the total capital inverted in each asset i. This is the reason why each variable i must verify $x_i \geq 0$.

Given that we need to distinguish assets regarding the inversion, let us set $I^+_{\mathbf{x}} = \{i | x_i > 0, \forall i = 1, ..., N\}$. It contains the indexes of the assets (variable values) where the inversion is not null. $I^0_{\mathbf{x}}$ is the complementary of the $I^+_{\mathbf{x}}$ set, and it contains the indexes of the assets where the inversion is null.

3.1.1. Mutation Operator

The mutation operator proposed is an unary operator acting in a single individual $\mathbf{x}$.

Let us set C and $P_m = 1/C$, where C is the cardinality of the problem and P_m the mutation probability, respectively. According to the P_m value, in average, only one asset (with not null inversion) is mutated.

For each asset in $I^+_{\mathbf{x}}$ an uniform random number is generated. If it is lower than P_m, that asset is exchanged with other randomly selected asset in $I^0_{\mathbf{x}}$.

If the original solution $\mathbf{x}$ was feasible, the mutated solution $\mathbf{x}'$ will be feasible too.

Figure 1 shows an example generation of the mutated solution $\mathbf{x}'$ from the previous solution $\mathbf{x}$. For simplicity, $C = 4$ and $N = 9$ are considered. The symbol ◯ are components of $I^+_{\mathbf{x}}$ and $I^+_{\mathbf{x}'}$

3.1.2. Crossover Operator

The crossover operator proposed, is a binary operator acting in two solutions. Given two parent solutions, $\mathbf{x}^1$ and $\mathbf{x}^2$, this operator generates two offspring solutions, $\mathbf{y}^1$ and $\mathbf{y}^2$. The role of crossover operator is to inherit some characteristics of the two parents to generate the offsprings. Concretely, the offsprings will have, initially, a null inversion in all of the N assets, but they will inherit C positive assets from the parents.

Depending of the variable values of the parent solutions, the crossover operator is divided in three cases:

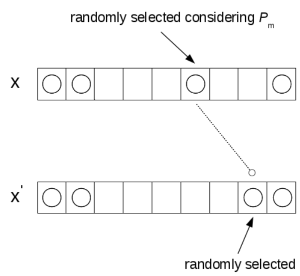

Figure 1: Mutation operator example.

- Case 1: $I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2} = \varnothing$

  The offsprings inherit some assets of each parent. Concretely, the offspring $\mathbf{y}^1$ inherits $\frac{C}{2}-1$ assets from $\mathbf{x}^1$ and $\frac{C}{2}+1$ assets from $\mathbf{x}^2$ if $C \pmod 2 = 0$. In other case, it inherits $\lfloor \frac{C}{2} \rfloor$ assets from $\mathbf{x}^1$ and $\lceil \frac{C}{2} \rceil$ assets from $\mathbf{x}^2$. The offspring $\mathbf{y}^2$ inherits $\frac{C}{2}+1$ assets from $\mathbf{x}^1$ and $\frac{C}{2}-1$ assets from $\mathbf{x}^2$ if $C \pmod 2 = 0$. In other case, it inherits $\lceil \frac{C}{2} \rceil$ assets from $\mathbf{x}^1$ and $\lfloor \frac{C}{2} \rfloor$ assets from $\mathbf{x}^2$. Anyway, the inherited assets are selected randomly.

  The cardinality constraint and the variables bounds are verified. However, it is necessary repair the new solutions to verify the total inversion constraint.

  Figure 2 shows the generation of the two offsprings from the parents $\mathbf{x}^1$ and $\mathbf{x}^2$. For simplicity, $C = 4$ and $N = 9$ are considered.

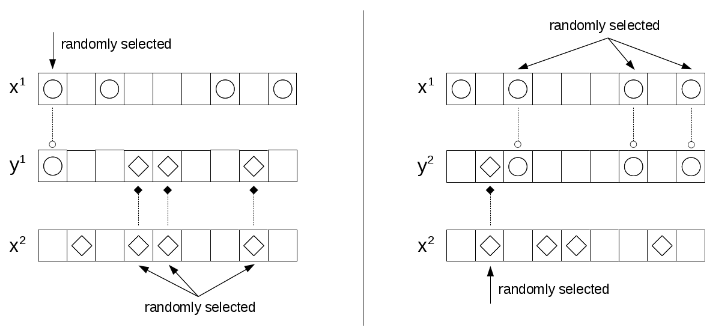

Figure 2: Crossover operator example in the Case 1.

- Case 2: $0 < \#(I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2}) < C$

  The offsprings inherit the matching assets of the parents. The other assets are exchanged in the offsprings.

  Concretely, the offspring $\mathbf{y}^1$ inherits the $I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2}$ assets of the parent $\mathbf{x}^1$ and the assets $I^+_{\mathbf{x}^2} \setminus (I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2})$ of the parent $\mathbf{x}^2$. The offspring $\mathbf{y}^2$ inherits the $I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2}$ assets of the parent $\mathbf{x}^2$ and the assets $I^+_{\mathbf{x}^1} \setminus (I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2})$ of the parent $\mathbf{x}^1$.

The cardinality constraint and the variables bounds are verified. However, it is necessary repair the new solutions to verify the total inversion constraint.

Figure 3 shows the generation of the two offsprings from the parents $\mathbf{x}^1$ and $\mathbf{x}^2$. For simplicity, $C = 4$ and $N = 9$ are considered. The dark grey boxes represent the matching assets in $\mathbf{x}^1$ and $\mathbf{x}^2$.

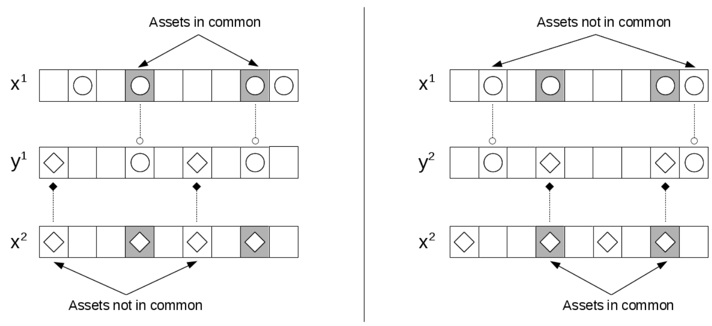

Figure 3: Crossover operator example in the Case 2.

- Case 3: $\#(I^+_{\mathbf{x}^1} \cap I^+_{\mathbf{x}^2}) = C$

  The assets where the inversion is not null are the same for the two parents. We take into account the inversion for each asset to classify this case in two sub-cases:

  - Case 3.1: $\mathbf{x}^1 \neq \mathbf{x}^2$

    The inversion in the C assets of $\mathbf{x}^1$ is different, at least in one asset, to the corresponding inversion in the assets of $\mathbf{x}^2$.

    The offspring $\mathbf{y}^1$ inherits $\frac{C}{2} - 1$ assets from $\mathbf{x}^1$ and $\frac{C}{2} + 1$ assets from $\mathbf{x}^2$ if $C \pmod 2 = 0$. In other case, it inherits $\lfloor \frac{C}{2} \rfloor$ assets from $\mathbf{x}^1$ and $\lceil \frac{C}{2} \rceil$ assets from $\mathbf{x}^2$. The inherited assets from $\mathbf{x}^1$ are selected randomly, but from $\mathbf{x}^2$ are selected the assets not selected from $\mathbf{x}^1$. The offspring $\mathbf{y}^2$ inherits $\frac{C}{2} + 1$ assets from $\mathbf{x}^1$ and $\frac{C}{2} - 1$ assets from $\mathbf{x}^2$ if $C \pmod 2 = 0$. In other case, it inherits $\lceil \frac{C}{2} \rceil$ assets from $\mathbf{x}^1$ and $\lfloor \frac{C}{2} \rfloor$ assets from $\mathbf{x}^2$. The inherited assets from $\mathbf{x}^1$ and $\mathbf{x}^2$ are the previously selected assets in the generation of $\mathbf{y}^1$, from $\mathbf{x}^2$ and $\mathbf{x}^1$, respectively.

Figure 4 shows the generation of the two offsprings from the parents $\mathbf{x}^1$ and $\mathbf{x}^2$. For simplicity, $C = 4$ and $N = 9$ are considered.

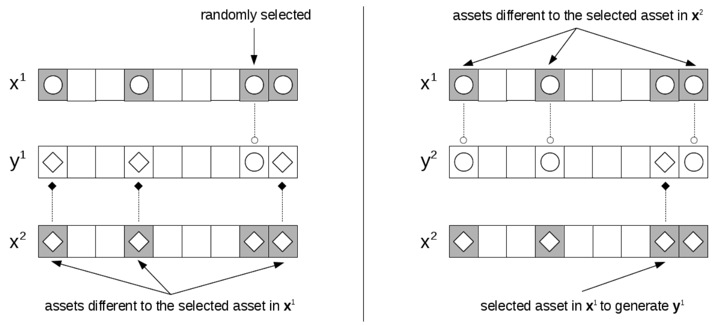

Figure 4: Crossover operator example in the Case 3.1.

– Case 3.2: $\mathbf{x}^1 = \mathbf{x}^2$
The inversion in the C assets is the same for $\mathbf{x}^1$ and $\mathbf{x}^2$.

The offspring $\mathbf{y}^1$ inherits the $I^+_{\mathbf{x}^1}$ assets from $\mathbf{x}^1$, but the inversion x of each asset i is incremented a randomly value v, where $v \in [x_i, ub_i]$. In the same way, the offspring $\mathbf{y}^2$ inherits the $I^+_{\mathbf{x}^2}$ assets from $\mathbf{x}^2$, but the inversion x of each asset i is decremented a randomly value v, where $v \in [lb_i, x_i]$. ub_i and lb_i are the upper bound and lower bound of the asset i, respectively.

The cardinality constraint and the variables bounds are verified. However, it is necessary repair the new solutions to verify the total inversion constraint.

3.1.3. Reparation of solutions to verify the total inversion constraint

When the crossover operator is used, it is necessary repair the new solutions to satisfy the total inversion constraint. To get it, the reparation of a solution $\mathbf{x}$ requires the normalization of its assets. It is done through (1).

$$x_i = \frac{x_i}{\sum_{i \in I^+_{\mathbf{x}}} x_i}, \forall i \in I^+_{\mathbf{x}} \tag{1}$$

Using the previous normalization, the value of some assets can be greater than the corresponding upper bound. This is the reason why each asset is repaired if its new value is incorrect. The reparation is explained in the Algorithm 1. Basically, for each asset i in $I^+_{\mathbf{x}}$, if its value x_i is greater than the upper bound ub_i, the excess is randomly distributed in $I^+_{\mathbf{x}} \setminus \{i\}$.

Algorithm 1 Individual reparation after normalization

Require: An individual $\mathbf{x}$

for all $i \in I_{\mathbf{x}}^{+}$ **do**
  while $x_i > ub_i$ **do**
    repeat
      $index$ = randomly selected index in $I_{\mathbf{x}}^{+} \setminus \{i\}$
    until $x_{index} < ub_{index}$
    $quantity$ = random number in $(0, ub_{index} - x_{index}]$
    $x_{index} = x_{index} + quantity$
    $x_i = x_i - quantity$
  end while
end for

3.2. Adapted NSGAII

One of the most known EMO algorithm is the *Non-dominated Sorting Genetic Algorithm* (NSGA-II) [16]. It is an EMO algorithm based on the Pareto dominance principle, which uses an elite-preserving strategy and an explicit diversity preserving mechanism. It has stood out by its fast nondominated sorting procedure for ranking solutions into several Pareto nondominated fronts for the selection of the best individuals. In general, at each generation of NSGA-II, the populations of parents and offsprings are joined and a new population is created. From the resulting population, the individuals which are not dominated by any solution constitute the so-called first nondominated front. These individuals are temporarily discarded from the population and, subsequently, the second nondominated front is formed by the next individuals which are not dominated by any solution. This process continues until every individual has been included in some front. Afterwards, the solutions in the lower level nondominated fronts are considered the best solutions and they passed to the next generation. If there are more solutions in the last allowed front than the remaining space in the new population, the individuals in that front are sorted according to a crowding distance, which, somehow, measures the objective space around each solution which is not occupied by any other solution in the population. Then, the individuals with the least crowding distance complete the new population. It has provided well-distributed sets of nondominated solutions for different types of real-life problems [17, 18].

We propose an adapted version of NSGAII, called A-NSGAII, to solve the MDRS model. Concretely, the crossover operator and the mutation operator are modified, using the operators described in Section 3.1.

3.3. Adapted MOEA/D

A recent group of EMO algorithms is based on decomposition. These algorithms decompose the multiobjective optimization problem into a set of scalar (single-objective) optimization subproblems and optimize them, simultaneously or subsequently, following a population evolution based rule. One of them is the *Multiobjective Evolutionary Algorithm Based on Decomposition* (MOEA/D) [19]. Using a set of evenly distributed weight vectors, a number of subproblems are optimized at each generation of MOEA/D relying on the assumption that neighbour weighting vectors will produce neighbour solutions. To be more precise, a newly generated offspring is compared with its neighbours for local replacement, and the fitness evaluation of the individuals is done through the scalarizing function used with different weight vectors.

We propose an adapted version of MOEA/D, called A-MOEA/D, to solve the MDRS model. Concretely, the mutation operator is modified using the operator described in Section 3.1. In addition to this, we apply, in the Step 2.2 of the algorithm [19], the reparation method proposed in Section 3.1. The differential evolution operator [20] is used like crossover operator.

4. Numerical Results

To analyze the effectiveness of the operators described in Section 3.1, we have carried out a set of computational tests over the three objectives MDRS model. The computational implementation of the algorithms has been made in Java and incorporated to the jMetal framework [21], where the original implementations of NSGAII and MOEA/D algorithms are available. This framework is an object-oriented Java-based framework for multiobjective optimization using metaheuristic algorithms.

4.1. Parameters settings

Next, we summarize the parameter setting used for the experiments:

- We have set the population size to 220 individuals.
- 500 generations have been executed.
- The crossover operator defined in Section 3.1 has been used in NSGA-II, with a probability $P_c = 0.9$. Since we have considered the version of MOEA/D using the differential evolution operator, we have set the crossover ratio and the scale factor to the default values described in [20, 22], that is, $CR = 1.0$ and $F = 0.5$.
- The mutation operator described in Section 3.1 has been considered in all of the algorithms, with a mutation probability $P_m = 1/n$, where n is the number of variables.
- The stopping criterion used is the maximal number of generations.
- The vectors of weights used in MOEA/D have been generated according to [19] and we use the scheme proposed by the authors for the subproblems formulation, based on the Tchebychev approach. Additionally, the following control parameters of MOEA/D have been used:

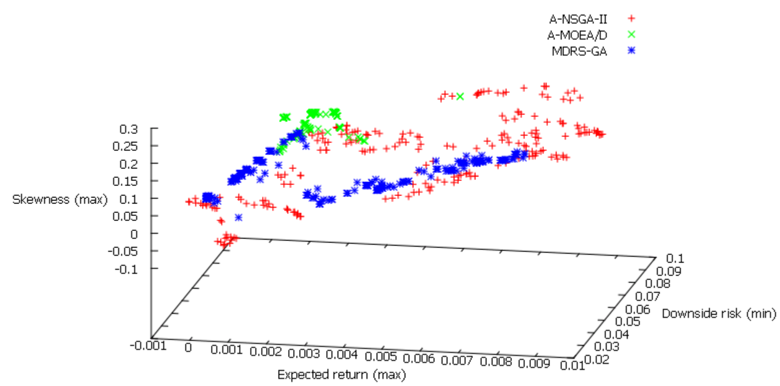

Figure 5: MDRS-MOEAD, A-MOEA/D and A-NSGAII: skewness, expected return and downside risk.

– Number of weight vectors in the neighbourhood of each weight vector: $T = 20$.

– Probability that parent solutions are selected from the neighbourhood: $\delta = 0.9$.

– Maximal number of solutions replaced by each child solution: $n_r = 2$.

4.2. Possibilistic MDRS MOEA vs Adapted MOEA/D and Adapted NSGA-II

The Pareto optimal fronts of the MDRS-MOEA, A-NSGAII and the A-MOEA/D algorithms for the three objectives MDRS problem is showed in Figure 5.

MDRS-MOEA, AMOEA/D and A-NSGAII contain 229, 220 and 191 non-dominated solutions.

Due to the nature of multi-objective optimization problems, multiple performance metrics should be used for comparing the performances of different algorithms [23]. In our experiments, the performance metric based in set coverage called $C - metric$ [19] is used. Let A and B be two approximations to the Pareto optimal front of a multi-objective optimization problem, $C(A, B)$ is defined as the percentage of the solutions in B that are dominated by at least one solution in A.

- C(A-NSGAII,MDRS-MOEA) = 78.166
- C(MDRS-MOEA,A-NSGAII) = 0.454
- C(A-MOEAD,MDRS-MOEA) = 48.035
- C(MDRS-MOEA,A-MOEAD) = 0.0

- C(A-NSGAII,A-MOEAD) = 0.0
- C(A-MOEAD,A-NSGAII) = 7.272

Regarding the $C - metric$, the A-NSGAII and A-MOEA/D algorithms generate better solutions than MDRS-MOEA. No solutions of A-MOEA/D are dominated for the other algorithms but it has a poor diversity. To measure and compare the diversity of the approximations of the Pareto optimal front for each algorithm, the hypervolume based metric has been used. The hypervolume [24] of a population of solutions, denoted by HV, can be defined as the hypervolume of the portion of the objective space that is dominated by the population and is bounded by a reference point **r**, which is dominated by all the solutions in the population. In our computational tests, we have selected the worst objective value for each objective function in the union of the Pareto optimal front obtained for each algorithm. This point has been used as reference point **r**. Obviously, it is better to have a hypervolume indicator as higher as possible.

- HV(A-NSGAII) = 0.560
- HV(A-MOEAD) = 0.422
- HV(MDRS-MOEA) = 0.395

Take into account the previous values of the HV metric, A-NSGAII obtains a better approximation of the Pareto optimal front for the MDRS model.

5. Conclusion

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