Static Tip-Over Stability Analysis for a Robotic Vehicle with a Single-Axle Trailer on Slopes based on Altered Supporting Polygons

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***Abstract*— This paper analyzes the effect of towing a single-axle trailer on static tip-over stability for field mobile robots on slopes. For this purpose, this work defines Altered Supporting Polygons (ASP) for both tractor and trailer. These ASPs are based on a force-torque static equilibrium analysis that reshapes the corresponding ground contact supporting polygons. In this way, it is possible to estimate tip-over stability for each unit separately by projecting its center of gravity onto its ASP, even in the case that both bodies have different inclinations. Online implementation issues and simulation results are discussed. Moreover, a case study is presented for the Alacrane mobile manipulator, which carries a trailer for power generation.**

I. Introduction

Mobility on inclined terrain is a requirement for agricultural vehicles [1] and field robotics applications, such as search and rescue [2], where tip-over is a major concern [3]. In practice, overturn prevention can be realized through active suspension [4], a movable load [5] [6], reconfigurable traction mechanisms [7] [8], or appropriate control of motor torque [9]. Moreover, mobile manipulators allow controlling the position of the Center Of Gravity (COG) by moving the on-board arm [4] [8] [10] [11]. Nevertheless, these strategies rely on assessing tip-over stability.

Vehicle tip-over stability is affected when towing passive trailers for load transportation [12], operation devices such as crop sprayers [13], or power generators [14]. Nevertheless, even if tractor-trailer combinations have been widely studied as a non-holonomic control problem on the horizontal plane [13], tip-over stability on slopes has not received much attention. Heavy trailers can dominate the behavior of the entire system, so they have been analyzed on their own [15]. Other works have considered stability from the tractor standpoint, where the trailer would be treated as an external disturbance force [16] [17]. Interestingly, some authors have considered this problem for future work [1].

Arguably, the most influential tip-over stability criteria is the supporting polygon (SP) principle [18], which states that for static stability the COG must lie above the convex area spanned by the ground contact points. Several extensions of this principle have considered dynamic effects [5] [16] [19] [20] [21] [22] [23], but their implementation implies complex processing of inaccurate data from a heterogeneous sensor set-up and, frequently, only results from simulations are available.

For articulated vehicles on a plane, the SP has been considered for the whole system [24]. However, individual stability analysis is required when different units have different inclinations. Furthermore, tip-over can originate only in a specific unit and have consequences for the whole system. Nevertheless, the ground contact SP of a unit cannot be used to assess stability, as it does not account for interrelations with other units.

This paper proposes the definition of Altered SPs (ASPs) for both bodies of a tractor and trailer system. These polygons are different from the corresponding ground contact SP, as they consider force interactions from other units. Then, tip-over stability of each unit can be estimated separately by projecting its COG onto its ASP, even when they have a relative orientation. Based on this novel idea, the major contributions of this work are the following:

- The ASP concept is developed for a tractor and a single-axle trailer system based on a force-torque static equilibrium analysis.
- Solutions for online implementation of tip-over stability measurements based on ASP are discussed.
- The effects of the relative 3D orientation between units and of their COG positions are analyzed through simulations.
- A case study is presented for a tracked mobile manipulator towing a power generator on a single-axle trailer, as seen in Fig. 1.

The paper is organized as follows. After this introduction, section II presents the ASP concept. Section, III deduces the ASPs for a tractor and a single-axle trailer. Section IV presents the case study of the Alacrane mobile manipulator. Then, simulation results are analyzed in section V. Finally, section VI is devoted to conclusions and ideas for future work.

II. The ASP Concept

The ASP concept for bodies in tractor-trailer vehicles is developed upon the static SP stability principle, which is firstly reviewed. The SP of a single vehicle, as a rigid solid that is only subject to gravity, can be defined as the convex polygon whose vertexes are the outermost ground contact points [18]. On inclined terrain, the static tip-over stability condition can

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Fig. 1. The tracked mobile manipulator Alacrane (right) and its trailer (left) during an emergency response exercise.

be stated as follows: the weight vector from the vehicle's COG should intersect inside the SP. Therefore, a measure for static tip-over stability can be the shortest distance d_v between the intersection of the weight vector with the SP and the bounds of this polygon [25], which should be maintained as far from zero as possible.

Let the right-handed frame (O_v, X_v, Y_v, Z_v) be attached to the geometric center of the vehicle's SP. Then, terrain inclination can be specified by roll r_v and pitch p_v angles. These are defined as the angles between the local X_v and Y_v axes, respectively, and the horizontal plane (see Fig. 2). Let $(x_{cgv}, y_{cgv}, z_{cgv})$ be the coordinates of the COG. Then, the intersection of the weight vector with the X_vY_v plane is:

$$\begin{aligned} x_{proj} &= x_{cgv} + z_{cgv}\tan(r_v), \\ y_{proj} &= y_{cgv} - z_{cgv}\tan(p_v). \end{aligned} \tag{1}$$

The tip-over stability index d_v for a rectangular SP of width W along the X_v axis, and length L along the Y_v axis (see Fig. 2) can be computed as:

$$d_v = \min\left(\frac{W}{2} - |x_{proj}|, \frac{L}{2} - |y_{proj}|\right). \tag{2}$$

The ASP concept proposed in this work is based on the following tip-over condition: the body tips over any point defined by the intersection of the weight vector with the ground plane when the vertical component of the ground reaction force becomes null for at least one of the ground contact points. Then, the ASP of the body is defined by the set of all possible tip-over points.

This definition allows for consideration of external forces in addition to gravity. Thus, for bodies in tractor-trailer vehicles, the forces related to kingpin hitch interactions between units can be taken into account for measuring tipover stability. Note that only for a single vehicle subject uniquely to gravity, its ASP coincides with its ground contact SP.

The ASP concept allows that tip-over stability of each unit can be measured much in the same way as with the SP principle: the weight vector from its own COG should intersect inside its corresponding ASP. The ASP can be developed from a force-torque equilibrium analysis, and it can be obtained from the relative orientation between units.

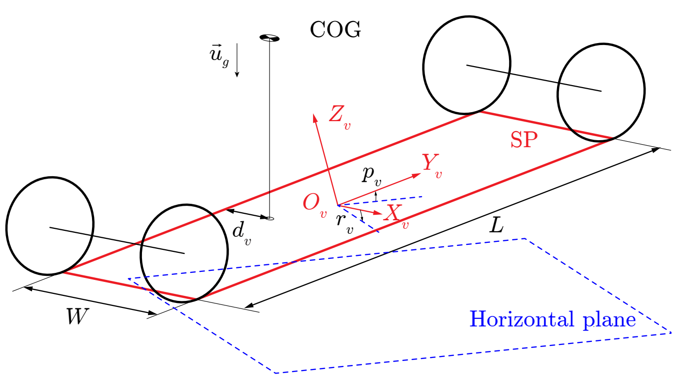

Fig. 2. Illustration of the SP concept for a four-wheel vehicle.

III. ASPs FOR A TRACTOR AND A SINGLE-AXLE TRAILER

This section develops the ASPs of a single axle trailer and its tractor, and discusses implementation issues. The following simplifying assumptions are adopted:

- The number of ground contact points for the tractor is $n > 2$.
- The trailer has two pure rolling passive wheels.
- Motion of the tractor (and its manipulator, if any) occurs at low speeds, so dynamic effects can be considered as minor disturbances.
- The tracks/wheels do not slip on slope.
- The hitch is an ideal spherical joint of the kingpin type, so moments are not transmitted between trailer and tractor.
- The hitch is placed on the respective longitudinal axes of the tractor and the trailer.

These are reasonable assumptions for many robotic vehicles towing a trailer.

A. Trailer analysis

The ground contact SP of an unhitched single-axle trailer is only the segment between the two wheel contact points. However, when hitched to a tractor, the ASP also depends on the kingpin position, which favors longitudinal stability.

Let (O_t, X_t, Y_t, Z_t) be a right-handed frame attached to the trailer at the middle point between wheel ground contacts, as shown in Fig. 3. These points have Cartesian coordinates ${}^t w_i = (x_{wi}, 0, 0)$ for $i = 1, 2$, where left superscript t indicates the trailer frame. Considering the trailer's free body diagram, the static equilibrium equations are:

$${}^t\vec{F}_{w1} + {}^t\vec{F}_{w2} + {}^t\vec{F}_{ht} + {}^t\vec{F}_{cg_t} = 0, \tag{3}$$

$${}^t\vec{M}_h = \overrightarrow{hw_1} \wedge {}^t\vec{F}_{w1} + \overrightarrow{hw_2} \wedge {}^t\vec{F}_{w2} + \overrightarrow{hcg_t} \wedge {}^t\vec{F}_{cg_t} = 0, \tag{4}$$

where $\wedge$ represents the vector product, and ${}^t\vec{F}_{wi}$ are the ground reactions at wheel contact points:

$${}^t\vec{F}_{wi} = \begin{pmatrix} F_{wix} & F_{wiy} & F_{wiz} \end{pmatrix}^T, \tag{5}$$

where superscript T means the transpose operator, ${}^t\vec{F}_{ht}$ is the reaction from the tractor at the kingpin hitch position ${}^t h =$

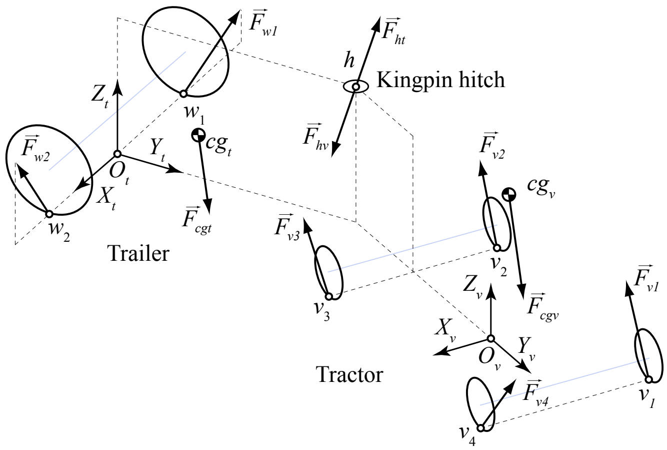

Fig. 3. Forces acting on a four-wheel tractor with a single-axle trailer.

$(0,\, y_{ht},\, z_h)$, ${}^t\vec{F}_{cg_t}$ is the trailer weight at its center of gravity ${}^t cg_t = (x_{cgt},\, y_{cgt},\, z_{cgt})$, and ${}^t\vec{M}_h$ is the sum of moments with respect to ${}^t h$. Note that ground reaction forces in the motion direction are not transmitted to the trailer from its wheels due to the pure rolling assumption, i.e., $F_{wiy} = 0$.

The weight force is known and given by

$$ {}^t\vec{F}_{cg_t} = m_t\, g\, {}^t\vec{u}_g, \tag{6} $$

where m_t is the trailer mass, g the gravitational acceleration, and ${}^t\vec{u}_g$ is a unit vector with the orientation of the gravity force, which depends on ground inclination:

$$ {}^t\vec{u}_g = \begin{pmatrix} u_{gtx} & u_{gty} & u_{gtz} \end{pmatrix}^T. \tag{7} $$

The moment equation with respect to axis $\overrightarrow{w_1 w_2}$ is defined as:

$$ {}^t\vec{M}_{w_1} \cdot \overrightarrow{w_1 w_2} = \left(\overrightarrow{w_1 h} \wedge {}^t\vec{F}_{ht} + \overrightarrow{w_1 cg_t} \wedge {}^t\vec{F}_{cg_t} \right) \cdot \overrightarrow{w_1 w_2} = 0, \tag{8} $$

where $\cdot$ represents the scalar product. Then, substituting (6) in (3) and (4) results in an equation system, which can be used with (8) to obtain the force at the kingpin hitch:

$$ {}^t\vec{F}_{ht} = m_t\, g \begin{pmatrix} (x_{cgt}\, u_{gty} - y_{cgt}\, u_{gtx})/y_{ht} \\ -u_{gty} \\ (z_{cgt}\, u_{gty} - z_h\, u_{gty} - y_{cgt}\, u_{gtz})/y_{ht} \end{pmatrix}. \tag{9} $$

Two trailer tip-over axes are defined by ${}^t h$ and each wheel contact point: $\overrightarrow{h w_2}$ and $\overrightarrow{h w_1}$. The line $\overrightarrow{w_1 w_2}$ is not a tip-over axis because the kingpin hitch reaction can pull the trailer either down or up. This is different from ground reaction forces, whose vertical component is always positive.

The equilibrium of moments with respect to these axes is defined as:

$$ {}^t\vec{M}_h \cdot \overrightarrow{h w_1} = 0, \qquad {}^t\vec{M}_h \cdot \overrightarrow{h w_2} = 0. \tag{10} $$

Using (4) in (10) gives two equations that only depend on the weight force and the vertical components of the ground reaction forces F_{w1z} and F_{w2z}, because:

$$ {}^t\vec{M}_h \cdot \overrightarrow{h w_1} = (x_{w2} - x_{w1}) y_{ht} F_{w2z} + (\overrightarrow{h cg_t} \wedge {}^t\vec{F}_{cg_t}) \cdot \overrightarrow{h w_1}, \tag{11} $$

$$ {}^t\vec{M}_h \cdot \overrightarrow{h w_2} = (x_{w1} - x_{w2}) y_{ht} F_{w1z} + (\overrightarrow{h cg_t} \wedge {}^t\vec{F}_{cg_t}) \cdot \overrightarrow{h w_2}. \tag{12} $$

By definition, the weight force orientations that determine the ASP are obtained by setting F_{w1z} and F_{w2z} to zero, which yields two plane equations. When translated to contain ${}^t cg_t$, they intersect with the trailer ground plane ($u_{gtz} = 0$) in two lines that define an open ASP:

$$ x - x_{wi} + \frac{(z_h - z_{cgt})\, x_{wi} - z_h\, x_{cgt}}{(z_h\, y_{cgt} - y_{ht}\, z_{cgt})}\, y = 0, \tag{13} $$

where $i = 1,\, 2$, and x, y represent $X_t Y_t$ coordinates. These equations indicate that trailer stability only depends on ${}^t cg_t$ and ${}^t h$.

B. Tractor analysis

The tractor SP is defined by n ground contact points ${}^v v_i = (x_{vi},\, y_{vi}, 0)$, where $i = 1, \ldots, n$, and left superscript v indicates the $(O_v,\, X_v,\, Y_v,\, Z_v)$ frame. This is illustrated in Fig. 3 for $n = 4$. In this frame, the kingpin hitch position is ${}^v h = (0,\, y_{hv},\, z_h)$.

With the exception of the ground contact force ${}^v\vec{F}_{vi}$:

$$ {}^v\vec{F}_{vi} = \begin{pmatrix} F_{vix} & F_{viy} & F_{viz} \end{pmatrix}^T, \tag{14} $$

for $i = 1, \ldots, n$, all forces acting on the tractor are known. The trailer reaction on the tractor ${}^v\vec{F}_{hv}$ is obtained from (9) as:

$$ {}^v\vec{F}_{hv} = -{}^v R_t\, {}^t\vec{F}_{ht}, \tag{15} $$

where ${}^v R_t$ is the rotation matrix that relates trailer and tractor frames. Also, the tractor weight force is

$$ {}^v\vec{F}_{cg_v} = m_v\, g\, {}^v\vec{u}_g, \tag{16} $$

where m_v is the tractor mass, and ${}^v\vec{u}_g$ is a unit vector with the orientation of the gravity force:

$$ {}^v\vec{u}_g = \begin{pmatrix} u_{gvx} & u_{gvy} & u_{gvz} \end{pmatrix}^T. \tag{17} $$

Note that the unit vectors ${}^t\vec{u}_g$ and ${}^v\vec{u}_g$ are also related by ${}^v R_t$:

$$ {}^v\vec{u}_g = {}^v R_t\, {}^t\vec{u}_g. \tag{18} $$

Then, equilibrium of moments with respect to tip-over axes $\overrightarrow{v_i v_{i+1}}$, $\overrightarrow{v_n v_1}$ can be defined as:

$$ {}^v\vec{M}_{vi} \cdot \overrightarrow{v_i v_{i+1}} = 0, \qquad {}^v\vec{M}_{vn} \cdot \overrightarrow{v_n v_1} = 0, \tag{19} $$

for $i = 1, \ldots, n-1$, where

$$ {}^v\vec{M}_{vi} = \sum_{j=1\ldots n}^{j \neq i} \left(\overrightarrow{v_i v_j} \wedge {}^v\vec{F}_{vj} \right) + \overrightarrow{v_i h} \wedge {}^v\vec{F}_{hv} + \overrightarrow{v_i cg_v} \wedge {}^v\vec{F}_{cg_v}. \tag{20} $$

Setting $F_{viz} = 0$, where $i = 1, \ldots, n$, gives n plane equations. After translating them to pass through the tractor center of gravity ${}^v cg_v = (x_{cgv},\, y_{cgv},\, z_{cgv})$, their intersection with the ground plane defines the tractor ASP. Unlike the trailer case in (13), the relative orientations of both units have to be considered through ${}^v\vec{F}_{hv}$. Consequently, the tractor ASP is not constant. Furthermore, in the case of a mobile manipulator, ${}^v cg_v$ is not constant but varies with the manipulator posture.

For a tractor with four symmetric contact points (i.e., $x_{v1} = x_{v2} = -W/2$, $x_{v3} = x_{v4} = W/2$, $y_{v1} = y_{v4} = L/2$, and

$y_{v2} = y_{v3} = -L/2$), and a co-planar trailer, the relative orientation between units is:

$$ {}^{v}R_t = \begin{pmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{21} $$

where γ is the relative yaw angle, and C and S denote the cosine and sine functions, respectively. The corresponding line equations that define its closed ASP are given by (22). These equations indicate that the tractor ASP depends on COG positions of both units, and also on the position and yaw angle of the hitch.

$$
\begin{aligned}
0 =& - z_{cgv}\left(\frac{W\,m_v g}{2} - m_v x_{cgv} g + \frac{W\,m_t\,y_{cgt}\,g}{2\,y_{ht}}\right)\\
&-(y-y_{cgv})\left(\frac{2z_h\,C_\gamma\,\left(m_t\,x_{cgt}\,g\,C_\gamma - m_t\,y_{cgt}\,g\,S_\gamma\right) - W\,\left(m_t\,z_h\,g\,C_\gamma - m_t\,z_{cgt}\,g\,C_\gamma\right)}{2y_{ht}} + m_t\,z_h\,g\,C_\gamma\,S_\gamma\right)\\
&+(x-x_{cgv})\left(\frac{2z_h\,C_\gamma\,\left(m_t\,y_{cgt}\,g\,C_\gamma + m_t\,x_{cgt}\,g\,S_\gamma\right) - W\,\left(m_t\,z_h\,g\,S_\gamma - m_t\,z_{cgt}\,g\,S_\gamma\right)}{2y_{ht}} + m_t\,z_h\,g\,S_\gamma^2 + m_v z_{cgv} g\right),\\
0 =&(x-x_{cgv})\left(\frac{y_{hv}\,\left(m_t\,z_h\,g\,S_\gamma - m_t\,z_{cgt}\,g\,S_\gamma\right) + z_h\,S_\gamma\,\left(m_t\,y_{cgt}\,g\,C_\gamma + m_t\,x_{cgt}\,g\,S_\gamma\right)}{y_{ht}} - m_t\,z_h\,g\,C_\gamma\,S_\gamma\right)\\
&-(y-y_{cgv})\left(\frac{y_{hv}\,\left(m_t\,z_h\,g\,C_\gamma - m_t\,z_{cgt}\,g\,C_\gamma\right) + z_h\,S_\gamma\,\left(m_t\,x_{cgt}\,g\,C_\gamma - m_t\,y_{cgt}\,g\,S_\gamma\right)}{y_{ht}} - m_t\,z_h\,g\,C_\gamma^2 - m_v z_{cgv} g\right)\\
&+z_{cgv}\left(m_v y_{cgv} g + \frac{m_t\,y_{hv}\,y_{cgt}\,g}{y_{ht}}\right),\\
0 =&(x-x_{cgv})\,\left(m_t\,z_h\,g\,C_\gamma\,S_\gamma\right)\\
&+(x-x_{cgv})\left(\frac{L\,(m_t\,z_h\,g\,S_\gamma - m_t\,z_{cgt}\,g\,S_\gamma) - y_{hv}\,(m_t\,z_h\,g\,S_\gamma - m_t\,z_{cgt}\,g\,S_\gamma) - z_h S_\gamma\,(m_t\,y_{cgt}\,g\,C_\gamma + m_t\,x_{cgt}\,g\,S_\gamma)}{y_{ht}}\right)\\
&-(y-y_{cgv})\left(\frac{L\,(m_t\,z_h\,g\,C_\gamma - m_t\,z_{cgt}\,g\,C_\gamma) - y_{hv}\,(m_t\,z_h\,g\,C_\gamma - m_t\,z_{cgt}\,g\,C_\gamma) - z_h\,S_\gamma\,(m_t\,x_{cgt}\,g\,C_\gamma - m_t\,y_{cgt}\,g\,S_\gamma)}{y_{ht}}\right)\\
&-(y-y_{cgv})\,\left(m_t\,z_h\,g\,C_\gamma^2 + m_v z_{cgv} g\right) + z_{cgv}\,\left(L\,m_v g - m_v y_{cgv} g + \frac{L\,m_t\,y_{cgt}\,g - m_t\,y_{hv}\,y_{cgt}\,g}{y_{ht}}\right),\\
0 =& - z_{cgv}\left(\frac{W\,m_v g}{2} + m_v x_{cgv} g + \frac{W\,m_t\,y_{cgt}\,g}{2\,y_{ht}}\right)\\
&+(y-y_{cgv})\left(\frac{W\,\left(m_t\,z_h\,g\,C_\gamma - m_t\,z_{cgt}\,g\,C_\gamma\right) + 2z_h\,C_\gamma\,\left(m_t\,x_{cgt}\,g\,C_\gamma - m_t\,y_{cgt}\,g\,S_\gamma\right)}{2y_{ht}} + m_t\,z_h\,g\,C_\gamma\,S_\gamma\right)\\
&-(x-x_{cgv})\left(\frac{W\,\left(m_t\,z_h\,g\,S_\gamma - m_t\,z_{cgt}\,g\,S_\gamma\right) + 2z_h\,C_\gamma\,\left(m_t\,y_{cgt}\,g\,C_\gamma + m_t\,x_{cgt}\,g\,S_\gamma\right)}{2y_{ht}} + m_t\,z_h\,g\,S_\gamma^2 + m_v z_{cgv} g\right).
\end{aligned} \tag{22}
$$

For the non co-planar case, the relative orientation between units can be specified using Euler angles rotating with respect to moving frames in the order XYZ as:

$$ {}^{v}R_t = \begin{pmatrix} C_\beta\,C_\gamma & -C_\beta\,S_\gamma & S_\beta \\ C_\alpha\,S_\gamma + S_\alpha\,C_\gamma\,S_\beta & C_\alpha\,C_\gamma - S_\alpha\,S_\gamma\,S_\beta & -C_\beta\,S_\alpha \\ S_\alpha\,S_\gamma - C_\alpha\,C_\gamma\,S_\beta & S_\alpha\,C_\gamma + C_\alpha\,S_\gamma\,S_\beta & C_\alpha\,C_\beta \end{pmatrix}, \tag{23} $$

where α, β, γ are the relative pitch, roll and yaw angles between tractor and trailer, respectively. With this rotation matrix, a symbolic mathematical package can be used to derive the line equations. The resulting expressions are omitted because they are longer than the coplanar case (22). In spite of their length, these equations remain easy to evaluate.

C. Implementation issues

For online tip-over stability assessment with ASPs, specific onboard sensors are needed in the tractor-trailer system.

Firstly, to calculate the ASPs, the relative orientation between units (23) can be obtained by using optical spherical encoders [26] [27] mounted on the hitch. Note that for the co-planar case (21) a conventional angle sensor can be employed instead (as in the case study of section IV).

Secondly, terrain inclination must be known for both units to project each COG on its ASP. This can be obtained with inclinometers mounted on both tractor and trailer. Alternatively, it is possible to deduce trailer inclination from p_v and r_v measures in the tractor. With these values, the following rotation matrix can be calculated:

$$ R_v = \begin{pmatrix} C_{r_v} & 0 & S_{r_v} \\ S_{p_v}\,S_{r_v} & C_{p_v} & -C_{r_v}\,S_{p_v} \\ -C_{p_v}\,S_{r_v} & S_{p_v} & C_{p_v}\,C_{r_v} \end{pmatrix}. \tag{24} $$

Then, the rotation matrix of the trailer with respect to the horizontal plane can be obtained as:

$$R_t = R_v\,{}^vR_t = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}. \quad (25)$$

From R_t, trailer roll r_t and pitch p_t angles are:

$$r_t = \arctan\left(\frac{e_{13}}{\sqrt{e_{23}^2 + e_{33}^2}}\right), \quad p_t = \arctan\left(\frac{-e_{23}}{e_{33}}\right). \quad (26)$$

IV. Case Study: The tracked mobile manipulator Alacrane

Alacrane is a mobile manipulator for exploration and rescue missions (see Fig. 4). It consists of a rugged tracked vehicle with a main articulated arm [14]. Fourteen hydraulic actuators provide powerful traction and heavy load manipulation. The weight of this mobile robot is $m_v = 568\,\mathrm{kg}$, and its dimensions are 600 mm width, 1200 mm length and 940 mm height.

During navigation, the hydraulic circuit can only power the arm base rotation θ. The other arm articulations adopt a navigation posture (as shown in Fig. 4) that keeps the COG as low as possible and around the geometric center of the vehicle [10].

An onboard IMU provides the roll and pitch angles of Alacrane with respect to the horizontal plane. Assuming that tractor and trailer are co-planar, the yaw angle of the hitch is measured by an absolute encoder.

The robot is powered by a three-phase AC power petrol-fed generator carried on a single-axle trailer for untethered operation. The trailer dimensions are 550 mm width, 800 mm length and 540 mm height. When its 5 L petrol tank is full, the trailer weights $m_t = 117\,\mathrm{kg}$. The articular limits of the yaw angle are $\gamma_{min} = -85.6°$ and $\gamma_{max} = 81.2°$, which are asymmetric due to the power plug. The COGs of both Alacrane and its trailer have been obtained by following the experimental procedure given in [10].

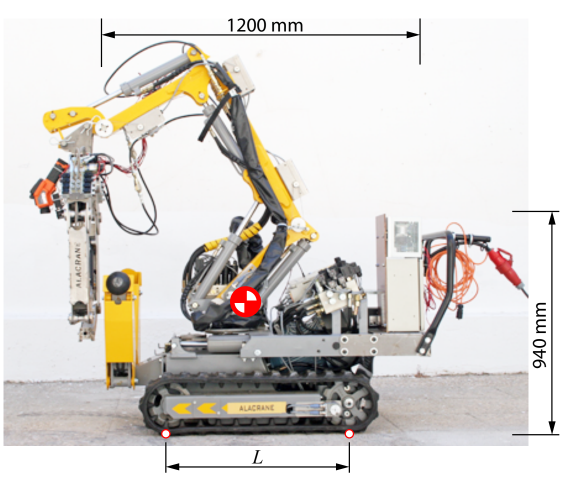

Fig. 4. Lateral view of the Alacrane mobile robot with its arm in navigation posture. The Alacrane COG position and some dimensions are overprinted.

A. Trailer ASP

The COG for the trailer has been estimated as ${}^tcg_t = (-39,\ 32,\ 390)\,\mathrm{mm}$. Wheel contact points are $x_{w2} = -x_{w1} = 330\,\mathrm{mm}$, and the hitch position is ${}^th = (0,\ 820,\ 380)\,\mathrm{mm}$. The X_tY_t coordinates of these points are shown in Fig. 5 along with the ASP of the trailer, as computed from (13). This open ASP indicates good longitudinal stability of the hitched trailer.

B. Alacrane's COG

Alacrane can be assumed to have $n = 4$ ground contact points (v_1, v_2, v_3, and v_4), as the pressure of each track is concentrated mainly onto two points (see Figs. 4 and 7). This assumption is based on the flexible nature of the rubber track that lies between the ground and only two rigid wheels, i.e., the sprocket and the idler, without intermediate rollers [10]. Thus, the lateral and longitudinal distances between these points are $W = 470\,\mathrm{mm}$ and $L = 735\,\mathrm{mm}$, respectively.

The COG positions are distributed on a circumference arc parallel to the X_vY_v plane with $z_{cgv} = 530\,\mathrm{mm}$. The values obtained for (x_{cgv}, y_{cgv}) are depicted in Fig. 7 for the full articular range of the arm base. Articular limits $\theta_{min} = -108.6°$ and $\theta_{max} = 130°$ are asymmetrical due the construction of the arm base actuator. Note that the COG does not lie on the Y_v axis for $\theta = 0°$, but for $\theta_0 = 2.63°$, which is the best arm base position for the horizontal plane.

The circumference arc provides COG configurations for all four quadrants of the X_vY_v plane. This arc can be fitted, in a least-squares error sense, by:

$$\begin{aligned} x_{cgv} &= R\,\cos\left(1.016\,\theta + 87.33°\right) + x_c, \\ y_{cgv} &= R\,\sin\left(1.016\,\theta + 87.33°\right) + y_c, \end{aligned} \quad (27)$$

where $R = 126\,\mathrm{mm}$ and $(x_c, y_c) = (1, -71)\,\mathrm{mm}$ are the radius and the local coordinates of the center of the circumference, respectively, and θ is expressed in degrees.

C. Alacrane ASP

The hitch position of the Alacrane robot is ${}^vh = (0,\ -767.5,\ 380)\,\mathrm{mm}$. Assuming that the trailer is coplanar, the tractor SP shown in Fig. 7 has been compared with the ASPs resulting from different combinations of γ and θ. These

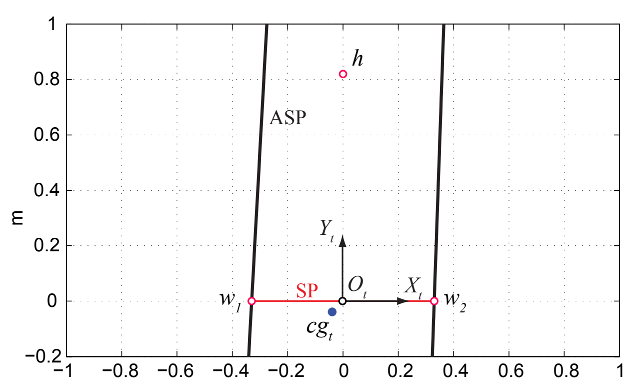

Fig. 5. Trailer ASP (thick black), SP (red), COG, and hitch position in the X_tY_t plane.

results, as well as the ratio (in %) between the ASP and SP areas, are presented in Fig. 6.

It can be observed that towing a trailer reduces the ASP area to about 86.4%. This reduction mainly occurs in the direction of the Y_v axis when the trailer is aligned to the tractor (i.e., $\gamma = 0$), and in the X_v axis when the trailer is almost perpendicular (i.e., γ near the articular limits). Moreover, the ASP is shifted along the X_v axis following tractor COG displacements provoked by changes in the arm base angle θ.

The shaded area in Fig. 7 has been obtained by overlapping the edges of tractor ASPs (see Fig. 6) for the whole ranges of θ and γ using (22). A rectangular polygon can be inscribed within all of these ASPs that could be used independently of θ and γ. All in all, the inscribed ASP has a width of $W_i = 342\,\mathrm{mm}$ and a length of $L_i = 594\,\mathrm{mm}$ with an area ratio of 58.8% with respect to SP.

D. COG control strategy

Based on IMU measures of pitch (p_v) and roll (r_v) angles, the controller goal is to avoid tip-over during navigation by actuating over the arm base rotation θ. The primary loop of a cascade COG control (see Fig. 8) issues an angle set-point θ_{sp} every 1 s. The secondary loop is performed with a faster relay controller that limits arm base velocity to $\pm 6°/\mathrm{s}$ to reduce dynamical effects [10].

As a simple strategy, the inscribed ASP can be employed for a worst case assessment of static stability. As shown in Fig. 7, the center of the inscribed ASP $(x_i,\, y_i) = (-1.5,\, -16.2)\,\mathrm{mm}$ does not coincide with O_v. To take into account this asymmetry, the following equation should be used instead of (2):

$$d_v = \min\left(\frac{W_i}{2} - |x_{proj} - x_i|, \frac{L_i}{2} - |y_{proj} - y_i|\right). \quad (28)$$

The primary COG control strategy consists mainly on searching around θ, with constant increments $\Delta\theta$, for a set-point θ_{sp} that has an index value d_{sp} that improves the current d_v [10].

E. Experimental results

This section discusses an experiment where Alacrane, with COG control and towing its trailer, was guided manually on a

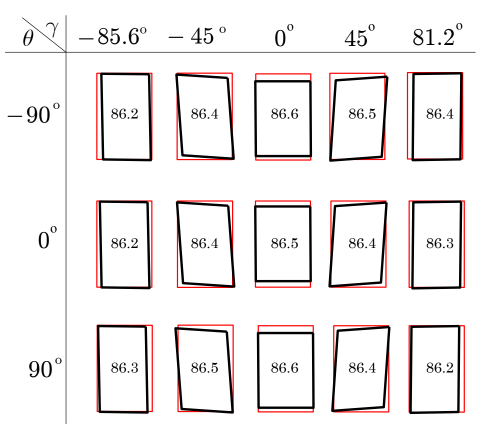

Fig. 6. Alacrane ASP (thick black), SP (thin red), and area ratio (in %) for different combinations of the arm base angle θ and hitch angle γ.

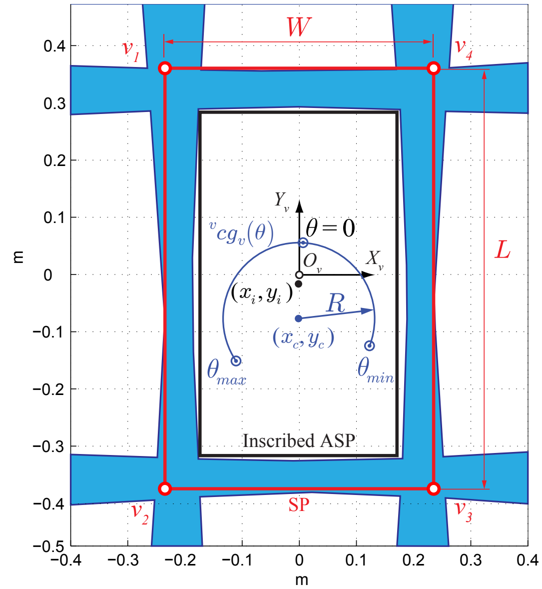

Fig. 7. Alacrane SP (red line) defined by the ground contact points, and effect of the arm base angle θ on the X_vY_v coordinates of the cg_v position. Inscribed ASP (black line) inside the overlap of the ASPs (blue area) for all combinations of the arm base angle θ and hitch angle γ.

5° slope. Fig. 9 shows the evolution with time of the hitch, roll and pitch angles, as well as the values of θ and θ_{sp}. Moreover, the figure shows tip-over indices d_v for the vehicle and d_t for the trailer. The comparison of these indices clearly reveals that the trailer is farther from tip-over than Alacrane. As for the tractor, the controller maintains a distance d_v near 165 mm, which is close to its maximum value of 178.5 mm.

The vehicle index has also been calculated as if the arm base angle was fixed with θ_0. The comparison of index d_v with COG control and with fixed $\theta = \theta_0$ shows that the former usually keeps a better stability margin.

V. Simulation results

In addition to the case study, simulation tests have been performed to evaluate non co-planar cases and a heavier trailer. Particularly, the simulated trailer has the same weight as the tractor (i.e., $m_t = m_v$). Moreover, the local XY coordinates for the tractor and trailer COGs have been considered null constants (i.e., $x_{cgt} = x_{cgv} = y_{cgt} = y_{cgv} = 0$) unless indicated otherwise. No mechanical hitch limits have been considered. The rest of model parameters have been set to the same values as in the case study.

Simulation results for the tractor ASP are summarized in Fig. 10, where variation of γ is combined with changes in other parameters. The figure presents the resulting ASPs against the

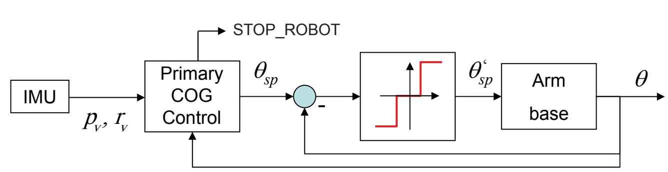

Fig. 8. COG control block diagram.

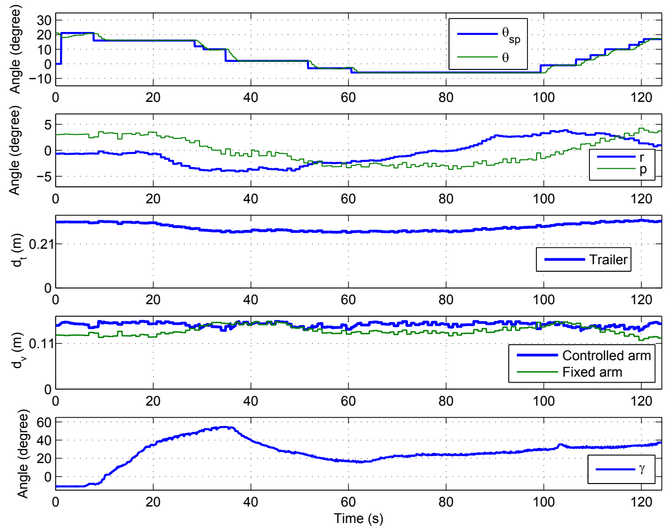

Fig. 9. Control of the COG with the trailer while turning on a 5° ramp.

ground contact SP and the corresponding area ratio. In general, all these experiments show a greater reduction of the ASP with respect to the SP than in the case study, which is due to a heavier trailer. When $\gamma = 0°$, the ASP is shrunk in the longitudinal direction, whereas a transversal shrinkage occurs for $\gamma = \pm 90°$. This is because the tractor is affected by the trailer in the Y_t direction.

Fig. 10(a) shows the effect of a displacement of y_{cgt} values. It can be observed that the ASP area increases and that ASP moves forward in the Y_v direction as y_{cgt} values change from negative to positive. This is because the trailer produces a torque that either lifts or presses the hitch when y_{cgt} is negative or positive, respectively.

Fig. 10(b) shows results from combinations of roll and yaw hitch angles. Note that when $\gamma = \alpha = 0°$, the roll angle has no influence on the tractor's ASP. For positive and negative values of β, it can be observed that the ASP is shifted to the left and to the right, respectively.

Fig. 10(c) offers different combinations of pitch and yaw hitch angles. When $\gamma = \pm 90°$ and $\beta = 0°$, α has no influence on the tractor's ASP because it is acting as a roll angle. It can also be observed that with negative α values, the ASP area is significantly increased in the Y_v direction. However, with positive α values, the ASP area is slightly decreased and shifted in the forward Y_v direction. This asymmetry is because of the different values for z_{cgt} and z_{cgv} employed in simulation.

Fig. 11 shows the trailer ASP for different values of x_{cgt}. The two lines that define this polygon intersect at a far point ahead of the trailer, at a distance that varies with y_{cgt} and is open in the rear side. It can be observed that each line contains the corresponding wheel contact point, and that line orientation is affected by the x_{cgt} coordinate.

VI. Conclusions

This work has addressed the effect of towing a single-axle trailer on static tip-over stability for field mobile robots on slopes. Since an articulated vehicle is not a rigid solid, each unit has its own ground contact Supporting Polygon (SP). However, these SPs cannot be directly employed to assess stability as they do not account for interrelations with other units.

The paper proposes the definition of Altered SPs (ASPs) for the tractor and the trailer through a force-torque static equilibrium analysis that considers the kingpin hitch interaction. Then, tip-over stability of each unit can be estimated separately by projecting its Center of Gravity (COG) onto its ASP. The

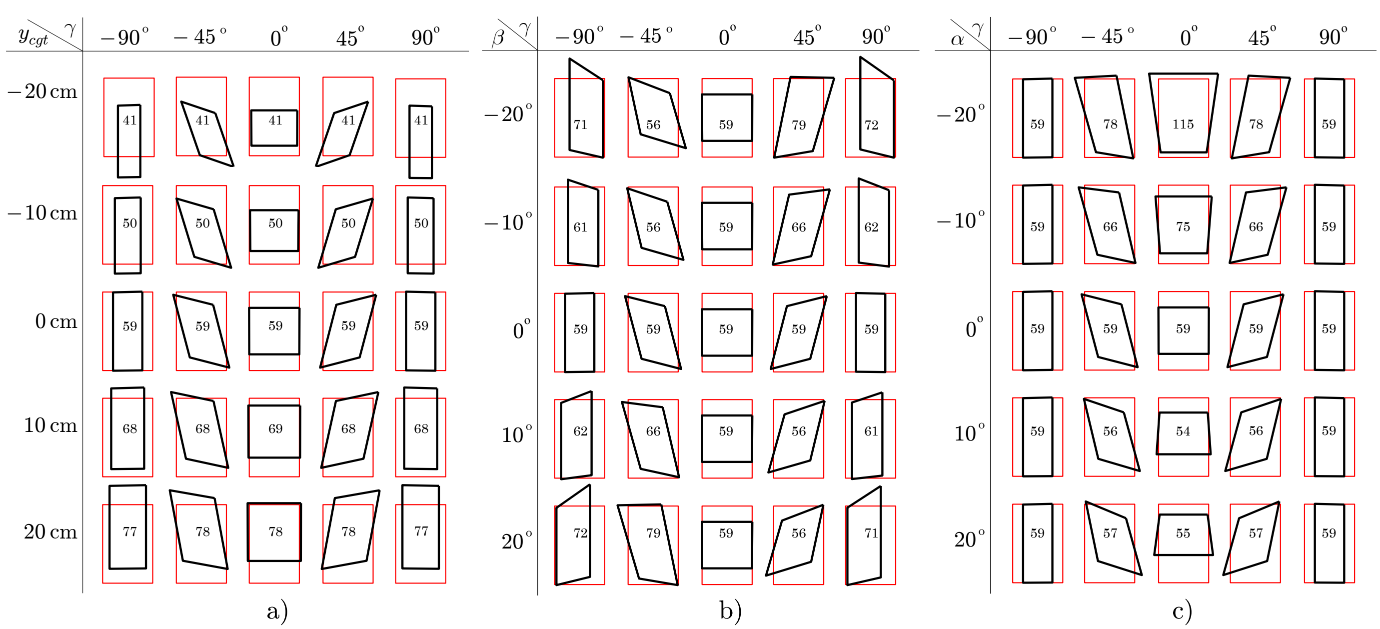

Fig. 10. Simulated tractor ASP (thick black), SP (thin red), and area ratio (in %) for different combinations of a) y_{cgt} positions and yaw γ angles, with null pitch α and roll β angles (i.e., the co-planar case), b) roll β and yaw γ relative angles ($\alpha = 0°$) and c) pitch α and yaw γ relative angles ($\beta = 0°$)

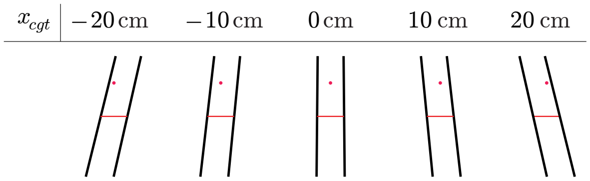

Fig. 11. Simulated trailer ASPs (thick black), SP (thin red segment), and X_tY_t hitch position (red dot) for different x_{cgt} values ($y_{cgt} = 0\,$cm).

proposed approach can be implemented online by measuring hitch angles and terrain inclination.

The analysis of the trailer ASP confirms that, through the kingpin hitch, static stability is greatly increased in its longitudinal axis. Moreover, the trailer ASP is independent of hitch angles and of the tractor's COG. On the other hand, the tractor ASP is reshaped with changes of the relative orientation between units, and of the position of both tractor and trailer COGs.

A case study is presented for the Alacrane mobile manipulator, which carries a passive trailer for power generation. A reduced tractor ASP has been inscribed into all possible ASPs obtained by varying the hitch and manipulator base angles. A simple control approach, based on repositioning the arm base, has been used to avoid tractor tip-over, and to monitor trailer stability.

Future work includes online use of the ASPs for the Alacrane COG control instead of its inscribed ASP, and to consider non-null relative pitch and roll angles between Alacrane and its trailer. It is also of interest to extend the ASP approach to take into account dynamic effects, such as high accelerations or trailed tankers.

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References

[1] M. Kise and Q. Zhang, "Sensor-in-the-loop tractor stability control: Look-ahead attitude prediction and field tests," *Computers and Electronics in Agriculture*, vol. 52, pp. 107–118, 2006.

[2] M. Guarnieri, I. Takao, P. Debenest, K. Takita, E. Fukushima, and S. Hirose, "HELIOS IX tracked vehicle for urban search and rescue operations: Mechanical design and first tests," in *Proc. IEEE/RJS International Conference on Intelligent Robots and Systems*, Nice, France, 2008, pp. 1612–1617.

[3] P. R. Roan, A. Burmeister, A. Rahimi, K. Holz, and D. Hooper, "Real-world validation of three tipover algorithms for mobile robots," in *Proc. IEEE International Conference on Robotics and Automation*, Anchorage, USA, 2010, pp. 4431–4436.

[4] K. Iagnemma, A. Rzepniewski, S. Dubowsky, and P. Schenker, "Control of robotic vehicles with actively articulated suspensions in rough terrain," *Autonomous Robots*, vol. 14, pp. 5–16, 2003.

[5] A. Diaz-Calderon and A. Kelly, "On-line stability margin and attitude estimation for dynamic articulating mobile robots," *The International Journal of Robotics Research*, vol. 24, no. 10, pp. 845–866, 2005.

[6] S. Nakamura, M. Faragalli, N. Mizukami, I. Nakatani, Y. Kunii, and T. Kubota, "Wheeled robot with movable center of mass for traversing over rough terrain," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems*, San Diego, USA, 2007, pp. 1228–1233.

[7] Y. Liu and G. Liu, "Track-stair interaction analysis and online tipover prediction for a self-reconfigurable tracked mobile robot climbing stairs," *IEEE/ASME Transactions on Mechatronics*, vol. 14, no. 5, pp. 528–538, 2009.

[8] ——, "Interaction analysis and online tip-over avoidance for a reconfigurable tracked mobile modular manipulator negotiating slopes," *IEEE/ASME Transactions on Mechatronics*, vol. 15, no. 4, pp. 623 – 635, 2010.

[9] S. Oh, N. Hata, and Y. Hori, "Integrated motion control of a wheelchair in the longitudinal, lateral, and pitch directions," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 4, pp. 1855–1862, 2008.

[10] J. Morales, J. L. Martínez, A. Mandow, J. Serón, A. García-Cerezo, and A. Pequeño-Boter, "Center of gravity estimation and control for a field mobile robot with a heavy manipulator," in *Proc. IEEE International Conference on Mechatronics*, Málaga, Spain, 2009.

[11] C. Beck, J. Valls-Miró, and G. Dissanayake, "Trajectory optimisation for increased stability of mobile robots operating in uneven terrains," in *Proc. IEEE International Conference on Control and Automation*, Christchurch, New Zealand, 2009, pp. 1913–1919.

[12] E. Gartley and D. Bevly, "Online estimation of implement dynamics for adaptive steering control of farm tractors," *IEEE/ASME Transactions on Mechatronics*, vol. 13, no. 4, pp. 429 –440, 2008.

[13] J. L. Martínez, J. Morales, A. Mandow, and A. García-Cerezo, "Steering limitations for a vehicle pulling passive trailers," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 4, pp. 809–818, 2008.

[14] A. García-Cerezo, A. Mandow, J. L. Martínez, J. Gómez-de-Gabriel, J. Morales, A. Cruz, A. Reina, and J. Serón, "Development of ALACRANE: A mobile robotic assistance for exploration and rescue missions," in *Proc. IEEE International Workshop on Safety, Security and Rescue Robotics*, Rome, Italy, 2007.

[15] A. G. M. Hunter, "Some stability and control problems with trailed farm tankers on slopes," *Journal of Terramechanics*, vol. 21, no. 3, pp. 273–282, 1984.

[16] E. G. Papadopoulos and D. A. Rey, "A new measure of tipover stability margin for mobile manipulators," in *Proc. IEEE International Conference on Robotics and Automation*, Minneapolis, USA, 1996, pp. 3111–3116.

[17] N. H. Abu-Hamdeh and H. F. Al-Jalil, "Computer simulation of stability and control of tractor-trailed implement combinations under different operating conditions," *Bragantia*, vol. 63, no. 1, pp. 149–162, 2004.

[18] R. B. McGhee and A. A. Frank, "On the stability properties of quadruped creeping gaits," *Mathematical Biosciences*, vol. 3, no. 3, pp. 331–351, 1968.

[19] R. F. Abo-Shanab and N. Sepehri, "Dynamic modeling of tip-over stability of mobile manipulators considering the friction effects," *Robotica*, vol. 23, pp. 189–196, 2005.

[20] Y. Li and Y. Liu, "Kinematics and tip-over stability analysis for the mobile modular manipulator," *Proc. of the Institution of Mechanical Engineers, Part C, Journal of Mechanical Engineering Science*, vol. 219, no. 3, pp. 331–342, 2005.

[21] A. Meghdari, D. Naderi, and M. R. Alam, "Neural network based observer for real time tipover estimation," *Mechatronics*, vol. 15, pp. 989–1004, 2005.

[22] S. A. A. Moosavian and K. Alipour, "Moment-height tip-over measure for stability analysis of mobile robotic systems," in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Beijing, China, 2006, pp. 5546–5551.

[23] C. Qiu, Q. Cao, L. Yu, and S. Miao, "Improving the stability level for on-line planning of mobile manipulators," *Robotica*, vol. 27, pp. 389–402, 2009.

[24] M. Tanaka and F. Matsuno, "A study on sinus-lifting motion of a snake robot with switching constraints," in *Proc. IEEE International Conference on Robotics and Automation*, Kobe, Japan, 2009, pp. 2270–2275.

[25] R. B. McGhee and G. I. Iswandhi, "Adaptive locomotion of a multi-legged robot over rough terrain," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 9, no. 4, pp. 176–182, 1979.

[26] D. Stein, E. R. Scheinerman, and G. S. Chirikjian, "Mathematical models of binary spherical-motion encoders," *IEEE/ASME Transactions on Mechatronics*, vol. 8, no. 2, pp. 234–244, 2003.

[27] K. M. Lee and D. Zhou, "A real-time optical sensor for simultaneous measurement of three-DOF motions," *IEEE/ASME Transactions on Mechatronics*, vol. 9, no. 3, pp. 499–507, 2004.