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1 **Logic, Epistemology, and the Unity of Science**

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54 Claudia Fernández-Fernández

55 Awareness in Logic
56 and Epistemology

57 A Conceptual Schema and Logical Study
58 of The Underlying Main Epistemic Concepts

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1 *Contrariwise, continued Tweedledee,*
2 *if it was so, it might be; and if it were so,*
3 *it would be; but as it isn't, it ain't.*

4 *That's logic.*

5 *Carroll, Lewis (1920). Alice's Adventures in*
6 *Wonderland.*

7 *New York: Macmillan.*

1 **Preface**

2 I take the liberty of introducing the main goals of this book with a *metaphoric tale*
3 that will guide you through the different paths that will be covered.

4 Once upon a time, about thirteen years ago, in a highland, a woman named *Sofia* gave birth
5 to two beautiful twin girls. Sofia was very fond of Aristotle and Greek Philosophy in general,
6 so she named her girls *Episteme* and *Logos*.

7 Some weeks later, there was a terrible fire that burned down the house and took many lives
8 from the small town in the highland. The only survivors were the twin girls, whose destiny
9 was altered tragically. The State tried to find a new family for the girls, but they were forced
10 to be separated, since the only adoptive families they found could only take one of the girls,
11 respectively. Thus, Episteme stayed near the highland with a humble family, while Logos
12 was adopted by a family who lived in the big city.

13 Episteme grew up in harmony with nature. Her family were artists and writers, who inspired
14 her an interest for reading, writing and learning foreign languages. By the age of sixteen she
15 was already fluent in Spanish and French and had learned some German and Greek. When
16 she turned eighteen and finished high school she decided to move to the big city and start a
17 Philosophy degree at the university.

18 Logos, on the other hand, grew up in a technological environment. Her adoptive family
19 were engineers and technicians, who taught her, at a very young age, some programming
20 skills. Thereupon, Logos was attracted to logical puzzles, electrical circuits and computer
21 programming. During her teenage years she enjoyed developing software for making her
22 daily life easier. After finishing high school she decided to study Computer Science at the
23 university.

24 The degrees of Philosophy and Computer Science share a common subject at this university
25 in the first semester of the first year, namely, *Philosophical Logic*. The first day of class two
26 girls were rushing through the corridor to get to room 314, the Philosophical Logic class.
27 Right at the entrance, before even entering the room, they looked up at each other and their
28 faces went pale. It was as if they were looking at a mirror, but they were not. Their faces
29 looked exactly the same, though their hairs and make-up were different.

30 ‘Who are you?’, said Episteme after staring at Logos for a long time. ‘And who are *you*?’,
31 replied Logos. ‘Well, I am Episteme from the highland’, Episteme said. ‘And I am Logos
32 from the city. But how is this possible?’, answered Logos looking puzzled. From that moment
33 on, the twin girls began getting to know each other. They obviously did not know that they
34 had a twin sister and, as it turned out, neither did their adoptive families.

35 Episteme and Logos spent most of the following days together, learning about each other’s
36 life. They soon noticed that though they had different interests, their characters had many

37 similarities and they were both intellectually curious. But, when Logos started telling Epis-
 38 teme what her hobbies were, the communication began to fail since they did not share a
 39 common background knowledge.

40 Episteme was unable to understand, at first, all those strange programming languages and
 41 concepts that Logos tried to explain to her. And also the other way round, when Episteme
 42 told Logos about her interest in Philosophy and mentioned some authors and theories, Logos
 43 was astonished. She had no clue what her sister was talking about, though she was speaking
 44 in English.

45 This initial state of general confusion changed very quickly, once both caught up with
 46 each other's hobbies and its specific vocabularies. At this point, they were able not only to
 47 understand everything the other sister said, but also to find some common ground, thanks
 48 also to the subject of Philosophical Logic, which they enjoyed very much.

49 As they both grew older, finished their degrees and started their respective academic careers,
 50 they decided to collaborate and deepen into the connections between Epistemology and
 51 Epistemic Logic, which not only was a *hot topic* at the time, but also served them for giving
 52 a sense to their own life experiences.

53 This tale entails as its moral that though very different in its form, language and
 54 methods, the fields of Epistemology and Epistemic Logic share the same root in
 55 its foundations and can be reconnected at some point. This is precisely the main
 56 motivation for this book. When I realised that these fields of study, which I was
 57 aware-that shared a common ground, had been growing and developing in almost
 58 complete isolation from one another, I felt the need to explore its roots and develop
 59 a 'tool' for reconnecting them. With the creation of the conceptual schema, that
 60 this book provides, the two subjects will find that common ground, though only
 61 between two very specific areas (Awareness Justification Internalism and a dynamic
 62 interpretation of Awareness Logic).

63 The core concept through which this bridge will be built and that connects the
 64 specific epistemological view with a concrete proposal of Epistemic Logic is that of
 65 *Awareness* (in its epistemic interpretations), as will be properly defined and justified.
 66 As well in Epistemology as in Epistemic Logic this concept will play a central role.
 67 The study of how we think or of how we come to think what we know has to take
 68 into account, in one sense or another, everything we consider or entertain. In other
 69 words, (i) Epistemology has to deal with Awareness.

70 In addition to that, the formal representation of our knowledge and informational
 71 attitudes would fall short in its connection to human knowledge should it not include
 72 Awareness in its system. And that means that (ii) Epistemic Logic has to deal with
 73 Awareness.

74 Claims (i) and (ii) should not be taken as dogmas, but rather as hopes or wishes
 75 that are fulfilled with the present research. I do not state that 'awareness' should be
 76 part of *every* epistemological view or system of Epistemic Logic. Instead, I claim
 77 that under the focus of this research, namely, that of reconnecting Epistemology and
 78 Epistemic Logic based on an intuitive understanding of the informational processes
 79 in human beings, the notion of 'Awareness' (or, at least, a similar concept) needs to
 80 be taken into account.

81 By analysing, classifying and combining the different interpretations of 'Aware-
 82 ness' I will build a theoretical bridge that serves Epistemic Logic as its philosophical

83 foundation and Epistemology as its logical correlate. The bridge will constitute a
84 conceptual schema that unifies, relates and connects the main topics of both fields.
85 It also serves to enrich these fields, creating thus a virtuous circle. Finally, I will
86 present a formal model that puts all these concepts together and formalises not only
87 the core notions, but also their dynamics, representing epistemic actions that change
88 the available information.

89 This book is the result of a revision of my Ph.D. Thesis, which I defended on
90 30th September 2019 in Málaga (Spain). My acknowledgement goes to every single
91 being that has ever crossed my path and left a mark or memory. I like to live every
92 life experience as an opportunity to learn and make it all worthwhile. Regarding the
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98 Málaga, Spain
November 2020

Claudia Fernández- Fernández

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1 About This Book

2 This book explores the current state of affairs of the relation between mainstream
3 Epistemology and Epistemic Logic. The apparent disconnection between these fields
4 of study is reverted with the creation of a conceptual schema that acts as a theoretical
5 bridge between both areas. With the concept of (epistemic) awareness as a guideline,
6 it offers a new understanding of the main epistemic concepts from Epistemology
7 and Epistemic Logic, where the notion of Explicit Aware Knowledge (EAK) plays
8 a central role.

9 EAK is presented as the (propositional) knowledge that Epistemology is interested
10 in, as well as the type of knowledge than an extension of Epistemic Logic should
11 represent. The relevance of the concept of ‘awareness’ is shown with respect to three
12 different applications: contemporary views on Epistemic Internalism, the dynamics
13 of information in logical proposals and a formal model that represents the different
14 informational attitudes of the agent, including the epistemic actions that formalise
15 the changes in information.

16 The interdisciplinary nature of this book makes it neither a book only on Epis-
17 temology, nor only about Epistemic Logic, nor about the dynamics of information,
18 though it is a book about all of these topics. The content can be classified between
19 Philosophy of Logic and mainstream Epistemology. This book discusses the epis-
20 temic concepts of *awareness*, *knowledge* and *justification*, and proposes some new
21 theoretical connections and re-definitions.

22 The structure of this book is the following one: Chap. 1 is devoted to an intro-
23 duction to the main topics in Epistemic Logic and Epistemology and reviews the
24 disconnection between these fields of study. Chapter 2 reviews the concept of ‘Epi-
25 stemic Awareness’, distinguishing between ‘Awareness-of’ and ‘Awareness-that’.
26 Awareness-of represents the attention of the agent, analogous to the language she has
27 at her disposal at a given moment and upon which she can create new information.
28 Awareness-that stands for the acknowledgement of the truth of some information,
29 being thus, a type of knowledge. Chapter 3 considers the notion of ‘Knowledge’
30 and analyses its different interpretations in Epistemology and some contemporary
31 developments of Epistemic Logic. It draws a distinction between implicit knowl-
32 edge, representing an ‘ideal’ understanding of knowledge, which is the result of the
33 closure under logical consequence of explicit knowledge; and the ‘real’ knowledge,

34 which Epistemology is about and that represents the human knowledge, which is
 35 named ‘Explicit Aware Knowledge’ (EAK). The EAK is formed due to a combi-
 36 nation of both types of awareness, such that for an agent to explicitly know some
 37 given information she needs to be aware-that it is the case and aware-of this infor-
 38 mation at a given moment. Hereafter, Chap. 4 reviews the different interpretations
 39 of the concept of ‘Justification’ and it is argued that justification will be defined as
 40 the process through which the agent obtains her EAK. This process is given by the
 41 different epistemic actions that are included in the conceptual schema and two of
 42 them will be able to provide a justification. Due to an act of deductive inference the
 43 agent will obtain a deductive or inferential justification, while an act of observation
 44 (or communication) will provide her with an evidential or observational justification.

45 Finally, Chap. 5 presents the EAK-Schema with the visual help of a diagram
 46 formed by three ellipses. The diagram starts out with a central small ellipse repre-
 47 senting the Awareness-that, those pieces of information the agent has acknowledged
 48 as true at any moment. From there, the Implicit Awareness-that can be deductively
 49 inferred, representing thus a bigger ellipse, containing the first one. As a third step,
 50 it incorporates the concept of Awareness-of as another ellipse, of the same size as
 51 the implicit information, overlapping the previous two ellipses, but not completely,
 52 creating thus three parts of Awareness-of. All these parts represent information the
 53 agent is considering at a given moment: pure Awareness-of (where there is space for
 54 any type of information, even false one), Implicit Knowledge and Explicit Aware
 55 Knowledge, EAK (as the only ‘real’ knowledge). EAK is located at the centre
 56 of the diagram, representing the kernel of it and corresponding also to the only
 57 type of knowledge that Epistemology will consider. With the visual help of the
 58 diagram and some arrows, epistemic actions that transform the information available
 59 for the agent (following the tradition of Dynamic Epistemic Logic) are presented.
 60 Acts of becoming aware-of or becoming unaware-of will change the information
 61 of the Awareness-of, modifying what the agent is paying attention to. The actions
 62 of performing a deductive inference or forgetting will either increase or decrease
 63 the information the agent has acknowledged, that is, her Awareness-that. Lastly, the
 64 action of observation (or communication in multi-agent settings) allows any piece of
 65 information to be directly part of the EAK. On the one hand, the action of deductive
 66 inference (transforming implicit information into EAK) constitutes the process for
 67 obtaining a deductive justification. An act of observation, on the other hand, will
 68 provide the agent’s EAK with an observational justification, indicating the source of
 69 its knowledge.

70 Chapter 6 presents a formal model, based on a neighbourhood-model structure,
 71 that shows one concrete application of the EAK-Schema into a logical structure. The
 72 Awareness-of is represented syntactically with a global awareness-function, while
 73 Awareness-that is depicted semantically due to the neighbourhood function. The
 74 concepts of Explicit Aware Knowledge and Implicit Knowledge are defined, and a
 75 list of their properties is provided. In addition, it includes a formalisation of the five
 76 epistemic actions defined in the EAK-schema and analyses some of its main features
 77 regarding the Explicit Aware Knowledge. To end this book, Chap. 7 is devoted to the
 78 conclusions and final remarks about the EAK-Schema, its uses and applications. The

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79 most important feature is that it serves both as a theoretical correlate to the dynamic
80 extensions of Awareness Logic, providing it with a philosophical background, and as
81 an abstract conceptual structure for a re-interpretation of Epistemology (specifically,
82 for the Awareness Justification Internalism).

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12 Quesada, F. R. (2020). Awareness-of and awareness-that their combination and
13 dynamics. *Logic Journal of the IGPL*, DOI: <https://doi.org/10.1093/jigpal/jzz043>;
14 Fernández-Fernández, C. (2019). Awareness Logic: an Epistemological Defence.
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18 and Collective Awareness. *Bulletin of the European Association for Theoretical*
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25 Republic) and *Third Lisbon International Conference on Philosophy of Science:*
26 *contemporary issues*, December 2016 in Lisbon (Portugal).

1 Abbreviations

2	AGM	Alchourrón, Gärdenfors and Makinson
3	AI	Artificial Intelligence
4	AJI	Awareness Justification Internalism
5	AL	Awareness Logic
6	ANM	Awareness Neighbourhood Model
7	DEL	Dynamic Epistemic Logic
8	EAK	Explicit Aware Knowledge
9	EL	Epistemic Logic
10	JL	Justification Logic
11	JTB	Justified True Belief
12	LP	Logic of Proof
13	MWD	Merriam-Webster Dictionary
14	OD	Oxford Dictionary
15	PAL	Public Announcement Logic

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Abstract	<p>This chapter summarizes the most relevant preliminary concepts from Epistemic Logic and Epistemology. Regarding the former, the problem of logical omniscience and its different solutions are revised, with Awareness Logic as the chosen solution for carrying out this research. With respect to Epistemology, a brief historical background is presented and the view of Awareness Justification Internalism is highlighted as the one that will serve as a theoretical background for the logical approach. To conclude, some aspects of the disconnection between both disciplines are pointed out and the concept of Epistemic Awareness is defined as the bridge-notion through which the theoretical re-connection will be developed.</p>	
Keywords (separated by '-')	Epistemic logic - - Awareness - Logical omniscience	

Chapter 1

Introduction to Epistemic Logic and Epistemology



Abstract This chapter summarizes the most relevant preliminary concepts from Epistemic Logic and Epistemology. Regarding the former, the problem of logical omniscience and its different solutions are revised, with Awareness Logic as the chosen solution for carrying out this research. With respect to Epistemology, a brief historical background is presented and the view of Awareness Justification Internalism is highlighted as the one that will serve as a theoretical background for the logical approach. To conclude, some aspects of the disconnection between both disciplines are pointed out and the concept of Epistemic Awareness is defined as the bridge-notion through which the theoretical re-connection will be developed.

Keywords Epistemic logic · Epistemology · Epistemic internalism · Awareness logic · Logical omniscience

1.1 Epistemic Logic: Main Concepts

This section provides the reader with a brief and sometimes schematic presentation of the main logical concepts that will be employed during the following chapters. While introducing these main notions some of them will be highlighted as significantly relevant; and in those cases where it comes down to different interpretations some choices will be made and justified.

Epistemic Logic. Epistemic Logic (EL) was ‘born’ in von Wright (1951). Von Wright took the first steps towards a formal study of knowledge and belief. Some years later, in Hintikka (1962), the author transforms von Wright’s ideas into more semantic concepts and EL was established as what we know today. EL has been developed and revisited by many authors since then.

Epistemic Logic¹ is an extension of Modal Logic, in which the necessity operator (\Box) is interpreted as knowledge and ranges over a set of epistemic agents (Ag),²

¹What follows is one possible contemporary presentation of EL.

²In *doxastic logic*, the logic of belief, the necessity operator is interpreted as what the agent believes.

26 such that the formula $\Box_i \varphi$ is read as *agent i knows φ* (or ‘agent i knows the truth of
27 φ ’ or ‘agent i knows that φ is the case’). The semantic structure of EL, like Modal
28 Logic, is based on the *possible worlds model* (also called Kripke model), where every
29 epistemic possibility (possible world) an agent may consider is represented via the
30 epistemic accessibility relation of each agent to those possible worlds she considers.

31 An **epistemic model** is the structure in which the knowledge of the agents is
32 represented. Let \mathbb{P} be a set of atomic propositions and let Ag be a set of epistemic
33 agents. Then an epistemic model is a tuple $\langle W, \{R_i\}_{i \in \text{Ag}}, V \rangle$, such that:

- 34 • $W \neq \emptyset$ is a non-empty set of possible worlds,
- 35 • $R_i \subseteq (W \times W)$ is the epistemic accessibility relation of agent i (indicating the
36 worlds each agent considers possible from each one of them), and
- 37 • $V : W \rightarrow \wp(\mathbb{P})$ is the atomic valuation function (indicating the atomic proposi-
38 tions in \mathbb{P} that are true at each possible world).

39 The pair (M, w) , with $w \in W$, is called an **epistemic state in model M** and
40 corresponds to the evaluation point, that is, the world in which the given formula for
41 the given agent will be assessed based on the other worlds that are accessible from
42 w for agent i . Concretely, from each world $w \in W$, the agent considers possible all
43 those worlds u that she can R -access from w .

44 The epistemic model M is described by the **language** of epistemic logic, $\mathcal{L}(\Box)$,
45 whose formulas φ, ψ are given by:

$$46 \quad \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box_i \varphi, \quad \text{where } p \in \mathbb{P} \text{ and } i \in \text{Ag}.$$

47 Recall that $\Box_i \varphi$ is read as *agent i knows that φ is the case*. The other Boolean
48 connectives can be defined from negation (\neg) and conjunction (\wedge) in the standard
49 way.

50 The **semantic interpretation** of the formulas is such that for (M, w) being an epis-
51 tic state in $M = \langle W, \{R_i\}_{i \in \text{Ag}}, V \rangle$ and φ and ψ any formulas in $\mathcal{L}(\Box)$; $(M, w) \Vdash \varphi$
52 indicates that φ is true at w in M (and $(M, w) \not\Vdash \varphi$ indicates that it is not true). Then,

- 53 • $(M, w) \Vdash p$ iff $w \in V(p)$
- 54 • $(M, w) \Vdash \neg\varphi$ iff $(M, w) \not\Vdash \varphi$
- 55 • $(M, w) \Vdash \varphi \wedge \psi$ iff $(M, w) \Vdash \varphi$ and $(M, w) \Vdash \psi$
- 56 • $(M, w) \Vdash \Box_i \varphi$ iff for every $u \in W$, $R_i w u$ implies $(M, u) \Vdash \varphi$.

57 Note how the semantic interpretation of $\Box_i \varphi$ says that agent i knows φ at w if
58 and only if φ is true at all worlds u that are accessible for i from w due to R_i .

59 The concepts of **satisfiability** and **validity** are defined in a standard way. Let φ
60 be a formula in $\mathcal{L}(\Box)$. A formula is said to be *satisfiable* if and only if it is true in at
61 least one world w of at least one model M (that is, if and only if there is one M such
62 that $(M, w) \Vdash \varphi$). A formula is said to be *valid* if and only if it is true at all worlds
63 w of all models M . The valid formulas are represented as follows: $\vdash \varphi$.

64 The **knowledge operator** (\Box_i) stems from the universal necessity modal operator.
65 As such, it has the same properties as the modal operator. This means that the knowl-
66 edge operator satisfies both the *rule of necessity (Nec)*, $\vdash \varphi$ implies $\vdash \Box_i \varphi$, and the

67 K -axiom, $\vdash \Box_i(\varphi \rightarrow \psi) \rightarrow (\Box_i\varphi \rightarrow \Box_i\psi)$. The first one states that if a formula is
 68 true, then agent i knows it. The second property makes the knowledge closed under
 69 logical consequence, meaning that if agent i knows φ and $\varphi \rightarrow \psi$ at world w , then
 70 she automatically knows ψ via the *Modus Ponens* rule. And this, conversely, means
 71 that now ψ is also true at every u accessible from w . This feature of Epistemic Logic
 72 has been given the name of the *problem of logical omniscience*.

73
 74 **The problem of logical omniscience.** The fact that the knowledge of the epistemic
 75 agents that EL proposes is closed under logical consequence should not be a problem
 76 by itself. The controversy arises when the notion of ‘epistemic agents’ is applied to
 77 human beings, instead of abstract entities that possess knowledge (like some ideal
 78 agents from Artificial Intelligence (AI)). But, of course, when one tries to understand
 79 the concept of ‘epistemic agent’, the most common and intuitive interpretation is to
 80 relate it to ourselves.

81 This was also the case of Hintikka, who in his seminal work argued in favour of
 82 Epistemic Logic as a way of representing human knowledge. He was already aware
 83 of the difficulties that this would generate and stated that: “[t]he logical implications
 84 of what we know do not come to us without any work on our own part; they are truths
 85 which we can extract, often with considerable labor, from whatever information we
 86 already have” (Hintikka 1962, p. 37). He continued advocating that although the
 87 laws of logic are not equivalent to the laws of thought, they could be the “laws of
 88 the sharpest possible thought” (Hintikka 1962, p. 37). A few years later, in (Hintikka
 89 1975) he proposed one strategy to solve the logical omniscience of epistemic agents
 90 as will be explained below.

91 Human subjects, though rational, are not omniscient (we could leave this adjective
 92 for the knowledge of some God(s)). In fact, humans, as computing machines, have
 93 *limited reasoning abilities*. Though the rationality of human subjects has not been
 94 questioned, what is clear is that it is limited, as our lives are.

95 In *Reasoning about Knowledge* Fagin et al. (1995), the authors devote a chapter
 96 to the problem of logical omniscience (LO) and its possible solutions.³ They state
 97 that the problem as defined above corresponds to *Full Logical Omniscience*. There
 98 are also weaker forms of omniscience derived from the Full LO, such as the closure
 99 under logical implication or closure under logical equivalence. Other types of LO,
 100 not derivable from the Full LO are also mentioned, like closure under conjunction or
 101 closure under valid implication. As may already be clear, all forms of LO are related
 102 to some kind of closure property that is imposed on the knowledge of the agents in
 103 Epistemic Logic.

104 As the authors wisely conclude, the different strategies out of LO pass through
 105 a reconsideration of the conditions for knowledge or the definition of ‘knowledge’
 106 itself. In the mentioned chapter (Fagin et al. 1995, pp. 309–362) they make a thorough
 107 analysis of the existing solutions to the problem of LO by that time, classifying them
 108 with respect to the semantic or syntactic structure that is imposed and the different
 109 concepts that are being redefined. A complete overview of these solutions may be

³For an updated overview and contemporary proposal about this topic see Hawke et al. (2019).

found in the given reference and would lie beyond the limits of this chapter. Some key notions for a general glimpse should thus suffice.

The **solutions to the problem of LO** need to reconsider the conditions for knowledge to be true. In standard EL knowledge is true at a given world if it is true in all worlds accessible from the given one, and that was exactly the cause of the problem. It is an ideal representation of knowledge. One way of solving the problem would be to establish a *different definition of knowledge*, from a syntactic or a semantic point of view. Though attractive as a solution to the problem, these strategies may also leave knowledge without other properties that were important; they are useful for defining knowledge, but not for modelling it formally.

There are also many *non-standard logics* that redefine knowledge or the whole structure of Epistemic Logic. These non-standard approaches commonly change the notion of truth by extending some operator or some part of the semantic structure with a counterpart where not every property or validity holds. Though interesting and applicable to concrete problems in AI and other fields, the authors do not highlight any of them as a complete solution to the problem of LO (if such ‘complete solution’ even exists).

The two most promising strategies they present, from an intuitive and theoretical point of view, are *impossible worlds* and *awareness logic*. The impossible worlds approach was proposed in Hintikka (1975). He assumes the problem of LO and proposes to change the semantic structure, inspired by contemporary ideas from Veikko Rantala on surface models in Rantala 1975. Hintikka establishes a difference between an *epistemic alternative* and a *logical possibility*, where the former are all the alternatives the agent may consider (including impossible worlds in which, for example, contradictory information may hold), and the latter are maximally consistent descriptions of the world. Using a complex structure he manages to adapt the semantics of possible worlds for including also impossible worlds, having now the possibility of evaluating formulas in both types of worlds. It can be considered a good solution for the problem of LO, since agents are not logically omniscient with respect to the impossible worlds, but, on the other hand, they can ‘know’ everything in those worlds, making their knowledge in impossible worlds quite trivial.

I left **Awareness Logic (AL)** until last, since it is the strategy I consider most suitable for the purpose of solving the problem of LO in EL. The main idea behind this approach, already proposed in Fagin and Halpern (1988), is that in order to know something, it is necessary to be aware of it. They state it in the following terms:

The underlying idea is that it is necessary to be *aware* of a concept before one can have beliefs about it. One cannot know something of which one is unaware. Indeed, how can someone say that he knows or doesn’t know about p if p is a concept of which he is completely unaware? (Fagin et al. 1995, p. 337. Original italics).

With this view in mind, they propose to add awareness to the standard system of EL as a syntactic operator, acting as a filter on the knowledge of the agent. They also incorporate a new operator for the *explicit knowledge*, representing the ‘real’ knowledge the agent possesses. The explicit knowledge of the agent is then defined

153 by the conjunction of both implicit knowledge (standard knowledge in EL) and
154 awareness.

155 Awareness is defined as an arbitrary list of formulas and only those that coincide
156 with the ones implicitly known, will be the explicitly known formulas. Formally, they
157 modify the original language $\mathcal{L}(\Box)$, being now $\mathcal{L}^A(\Box)$, formed by $\mathcal{L}(\Box)$ and two
158 new operators: the *awareness operator*, A , such that formulas of the form $A_i \varphi$ are
159 read as *agent i is aware of φ* ; and the *explicit knowledge operator*, X , with formulas
160 of the form $X_i \varphi$ read as *agent i knows φ explicitly*.

161 The semantic model M is modified, adding the *awareness function*, \mathfrak{A} , that assigns
162 a list of formulas, of which each agent is aware of, to each world. Then M^A is an
163 *awareness model*: $M^A = \langle W, \{R_i\}_{i \in \text{AG}}, V, \mathfrak{A}_i \rangle$.

164 The *semantic interpretations* for the two new operators are the following ones: let
165 (M^A, w) be an awareness epistemic state with $w \in W$ and φ any formula in $\mathcal{L}^A(\Box)$,
166 then

- 167 • $(M^A, w) \Vdash A_i \varphi$ iff $\varphi \in \mathfrak{A}_i(w)$
- 168 • $(M^A, w) \Vdash X_i \varphi$ iff $(M^A, w) \Vdash A_i \varphi$ and $(M^A, w) \Vdash \Box_i \varphi$

169 From the latter, the authors conclude the following validity, which is the key of
170 this proposal: $\vdash X_i \varphi \leftrightarrow (A_i \varphi \wedge \Box_i \varphi)$, stating the logical equivalence between agent
171 i having explicit knowledge about φ and her fulfilling both implicitly knowing φ and
172 being aware of φ .

173 The original proposal leaves the awareness function without any restrictions, but
174 they argue that different closure properties might easily be achieved, such as closure
175 under subformulas or self-reflection. This proposal does overcome the problem of
176 LO and maintains the original structure of EL. It is also compatible with the previous
177 proposal of impossible worlds and with some other semantic structures. Probably the
178 stroke of genius that comes with this proposal, is that Fagin and Halpern managed
179 to formalize a common sense intuition that was already present in many other fields
180 of study.

181 The potential that AL has, was fruitfully explored in the following decades and
182 incorporated to the new paradigm of the *studies of language and information*. There
183 are also other proposals that include an external factor to the main structure of EL
184 in order to divide knowledge into implicit and explicit and hence prevent the agents
185 from being logically omniscient, but I will not review them here for lack of interest
186 to this research. Though the authors present it as one possible solution among oth-
187 ers, they had already explored this approach (as mentioned before) and continued
188 afterwards this line of research.

189 **New approaches of knowledge and information.** The last decades have been very
190 prolific in the studies of language and information from the logical point of view. One
191 could speak of a new *paradigm* that has been settled regarding the logical interactions
192 of language and information (and also computation).

194 The different ways in which information is represented will give rise to the dif-
195 ferent logics and semantics that one finds nowadays. The milestone that turned the

196 concepts from Epistemic Logic into a whole new paradigm was probably the con-
 197 cept of *information change*. In contemporary approaches it is hard to find the mere
 198 concepts of ‘knowledge’ and ‘belief’ as such. Instead, one finds the notion of ‘infor-
 199 mation’ and its various interpretations will turn it into either knowledge, or belief,
 200 or other types of epistemic attitudes. Of course the concept of ‘information change’
 201 has its origins in the studies of communication, that are seen as multi-agent scenar-
 202 ios in Epistemic Logic, where the given formulas or propositions change during the
 203 communication process.

204 Epistemic Logic represents a simple semantic interpretation of information. The
 205 new approaches to the notion of information (change) have been made from a syn-
 206 tactic and semantic point of view. The purely *syntactic representations* understand
 207 ‘information’ as a list of formulas, or propositions, and sometimes add some prop-
 208 erties to the set of information that is assigned to an agent for achieving (or coming
 209 closer to) omniscience. The interactions between these formulas or propositions are
 210 then deductive. There are different proposals, with *Belief Revision* being one of the
 211 most relevant ones (for a general overview see Gärdenfors (1992) and Williams and
 212 Rott (2001), for example).

213 In **Belief Revision Theory** one could highlight the *AGM-model* (named after
 214 the authors Alchourrón, Gärdenfors and Makinson that published the originating
 215 paper Alchourrón et al. 1985). This proposal of representation of belief establishes a
 216 distinction between the *belief set* (consistent set of formulas closed, normally, under
 217 logical consequence) and the *belief base* (simple set of formulas acting as a basis
 218 for the belief set, without closure properties). The AGM-model provides tools for
 219 formalizing the different types of changes (e.g., revisions and updates) there might be
 220 in the agent’s belief set, but from a purely syntactic point of view. As pointed out in
 221 Velázquez-Quesada (2011), p. 9, “syntactic approaches have been criticized as being
 222 too fine-grained, making differences in meaning where there seems to be none”.
 223 However, Belief Revision Theory had great influence in the forthcoming approaches
 224 and is still very useful in different fields of Artificial Intelligence.

225 The *semantic approaches* in this new paradigm are also based on the idea that
 226 information changes, and so do the assigned truth-values; and this change needs to be
 227 reflected by the formal structure. The *dynamics of information* is the most important
 228 concept that arises with this considerations. The fact that the new approaches adopt
 229 a dynamic perspective implies that information is not a static concept any more,
 230 instead it is in constant motion. What brings the informational entities to move are
 231 the *epistemic actions*, that are formalized as operators that change the model. This
 232 branch of logics classifies the actions that promote the information to change.

233
 234 **Dynamic Epistemic Logic.** The designation of *Dynamic Epistemic Logic* (DEL)⁴
 235 includes different approaches, sharing as a common ground Epistemic Logic as
 236 their foundation and epistemic actions as its modification. One of the most salient
 237 handbooks regarding the dynamics of information is van Ditmarsch et al. (2008). In

⁴The first research that considers the appearance of DEL a change of paradigm is Gochet (2002). DEL as it is considered here is best developed in van Benthem (2011).

238 there, the authors define very precisely in which sense the concept of ‘information’
239 is now interpreted.

240 We regard information as something that is relative to a subject who has a certain perspective
241 on the world, called an *agent*, and the kind of information we have in mind is meaningful
242 as a whole, not just loose bits and pieces. This makes us call it *knowledge* and, to a lesser
243 extent, *belief*. This conception of information is due to the fields known as *epistemic* and
244 *doxastic logic* (van Ditmarsch et al. 2008, p. 1. Original italics).

245 As mentioned above, DEL serves as a designation for different approaches and
246 logics, inspired partly by EL and partly by Belief Revision theory. The different
247 logics and their interactions that have been proposed in the last decades conform a
248 wide range of possibilities. I will mention the most important notions that appear in
249 those proposals and that make DEL constitute a new paradigm.

250 The most relevant notion is that of *epistemic actions*, understood as the fulfilment
251 of an epistemic act that causes the existing information of the agent to change by
252 losing information, gaining new pieces of information or transforming it. The first
253 and most simple action that was formalised is the act of communicating new infor-
254 mation, captured in **Public Announcement Logic** (PAL) (see Plaza 1989, 2007). I
255 called it ‘simple’ since this logic formalises an idealisation of the process of commu-
256 nication, i.e. it represents informational changes “per the occurrence of completely
257 trustworthy, truthful announcements” (Baltag and Renne 2016). This logic deals with
258 how the announcement of new information alters the existing one, changing part of
259 what the agent had and creating new information derived from the old one.

260 Different logics that include different types of information loss or wins and updates
261 have been developed. The variety of epistemic actions correspond to a huge classifica-
262 tion that keeps on growing with every new research. Of course, as happened with
263 standard EL, one also finds here the problem of logical omniscience and some actions
264 are designed to prevent the agents from different types of omniscient properties, while
265 other maintain some closure properties and have been applied to Computation, Arti-
266 ficial Intelligence or Economics.

267 The main feature that defines non-omniscient agents is to have **limited reasoning**
268 **abilities**. It is not intended that agents cannot reason at all, but to prevent full logi-
269 cal omniscience the agents each logic presents need to have some kind of limiting
270 mechanism that makes their inferential processes not infinite and provides them with
271 just enough rationality for creating new information by themselves, but at the same
272 time prevents them from being able to be informed about everything there is.

273 In line with this, there have been some very interesting proposals that, based on
274 Awareness Logic and adopting a dynamic perspective, incorporate the actions of
275 *becoming aware* (and also unaware) or *performing a deductive inference*. Both are
276 individual actions (only affecting one agent) that help limit the deductive possibilities
277 of the agent, while reconciling itself with the common sense. These proposals mix
278 different syntactic and semantic structures in order to be able to capture the intended
279 consequences of each action.

280 To sum up, one could understand the new paradigm of the dynamics of informa-
281 tion as an interdisciplinary field of study, where logicians, linguists, computational

282 engineers and philosophers have established as a common ground the concept of
 283 ‘information change’, and constructed a new branch of knowledge. Contemporary
 284 advances in the field are constant and new interpretations of the basic concepts come
 285 with almost every new proposal that is conceived.

286 1.2 Epistemology: An Overview and a Choice

287 This section starts by introducing the concept of *Epistemology* and its historical ori-
 288 gins. After describing some of its basic ideas it explores the contemporary picture of
 289 Epistemology and reviews some classifications. Then, one specific approach, Aware-
 290 ness Justification Internalism, is highlighted as the chosen view for this research.

291 **Definition of the term ‘Epistemology’.** The term *Epistemology*⁵ has suffered from
 292 several semantic changes. Nowadays, Epistemology is the branch of Philosophy that
 293 studies knowledge. There is no consensus for a commonly shared definition. Going
 294 through the most common dictionaries, one finds the following interpretations: the
 295 *Oxford Dictionary* defines it as “[t]he theory of knowledge, especially with regard
 296 to its methods, validity, and scope, and the distinction between justified belief and
 297 opinion.”⁶ While the *Merriam-Webster Dictionary* says that it is “the study or a
 298 theory of the nature and grounds of knowledge especially with reference to its limits
 299 and validity”.⁷ Turning to specialised literature, Hendricks says that “[t]he systematic
 300 and detailed study of knowledge, its criteria of acquisition and limits and models of
 301 justification is known as epistemology” (Hendricks 2005, p. 1).

302 Rivers of ink have already flown regarding the different interpretations of the
 303 term and in which sense it is related to all types of knowledge or information. For the
 304 present purpose I will stick to the first and simple definition I gave above: *branch of*
 305 *Philosophy that studies knowledge*. As such, there are many different aspects regard-
 306 ing the notion of knowledge that may fall under the epistemological investigations.
 307 Of course not every approach to knowledge is suitable of being considered Epis-
 308 temology, but nowadays this field of study has widen its range in such a way, that
 309 aspects like perception, communication, social factors or cognitive processes are also
 310 part of it.
 311

312 **Origins of Epistemology.** The topic of Epistemology has been one of the main
 313 themes in philosophical discussion since its very beginning in Ancient Greece. One
 314 could establish its origin in Plato’s dialogues. It is said that Plato states that knowledge
 315 is equivalent to *justified true belief* (JTB) in the *Meno* and the *Theatetus*. Though this
 316 claim is not accurate and those who state it in these terms are accused of anachronism,
 317

⁵Despite of the fact that the topic is very old, the term itself was coined for the first time in Ferrier (1854).

⁶In <https://www.lexico.com/definition/epistemology>, accessed 01/02/2020.

⁷In <https://www.merriam-webster.com/dictionary/epistemology>, accessed 01/02/2020.

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