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Scoring markets: Theory and application in sports economics

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ABSTRACT

Some organizations can be analyzed as if they were markets, even if they do not precisely meet the canonical definition of one. In this article, we introduce and theoretically define the concept of scoring markets, referring to organizations in which outcomes are determined by a scoring system. We show that such organizations possess a distinct feature: the set of attainable configurations is discrete and discontinuous. This characteristic allows for the straightforward assignment of probabilities to feasible configurations, as well as the application of numerical methods to identify particular outcomes. Scoring markets arise in regulated markets, contests, elections, and sports events. Sports economics provides a compelling context for illustrating and advancing the understanding of scoring markets, especially through bilateral football competitions. Drawing from our theoretical framework, we identify potential levels of competitive balance in the group stage of the Union of European Football Associations Champions League. We compute the probabilities of these outcomes and compare the theoretical predictions to the actual results observed across each of the eight groups in every season from 1999/2000 to 2023/2024. This comparison reveals a general trend toward greater point concentration, corresponding to less probable theoretical configurations and indicating a broader decline in competitive balance.

1. Introduction

Markets consist of an organized set of institutional arrangements that facilitate exchanges among transacting parties. Therefore, a necessary first step in developing any theory of market solutions is to define the specific institutional arrangements that govern exchange behavior. In this article, we aim to apply concepts and analytical tools traditionally used to describe market dynamics to institutions or organizations (i.e., entities that may not formally qualify as markets) whose behavior can nevertheless be interpreted through a market-based analytical lens.

Competition is often considered a fundamental element in defining a market, frequently conceptualized as a process characterized by rivalry. Accordingly, the observed distribution of firm sizes within a market can be understood as the outcome of a competitive process, where each agent engages in multiple interactions—potentially with all rivals.

Conventional approaches to market analysis involving firm size typically assume that any outcome is feasible within standard economic constraints. As a result, any competitor can theoretically achieve any relative size. Thus, we assume that the vector of sizes of the n firms in a market exists in an n-dimensional real number space. Consequently, the range of potential outcomes (i.e., the sizes achieved by the firms) is defined over a continuous space. Each point in this space represents a possible market configuration, with each configuration mapped to a unique point.

However, some competitive processes feature a discontinuous configuration space that goes beyond merely assigning discrete values (i.e., values in the set of natural numbers rather than the real numbers). This discontinuity arises when the competition's outcome is determined by an externally imposed scoring or remuneration system that allocates a finite number of rewards to the participants. Such a system precludes an infinite range of potential outcomes and, therefore, a continuous configuration space. We refer to these settings as scoring markets. In scoring markets, the set of possible outcomes is finite, constrained, and forms a discrete space—unlike the continuous configuration space assumed in traditional market models. This study presents a novel and formal definition of what constitutes a scoring market.

Sports competitions—our primary focus—offer a clear illustration of this concept. In sports economics, teams resemble firms competing for consumer attention and satisfaction. League standings provide a dynamic measure of performance, with competition serving as the

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mechanism for point allocation. These competitions often aim to maximize total points rather than profits, ultimately seeking greater market share.

Although sports competitions are the most immediate example of scoring markets, similar mechanisms exist in various other domains. These include regulated markets, where firms submit technical and economic proposals for contracts that are evaluated by public authorities; competitive events in which participants are scored by judges; and democratic elections in which votes determine the size of each political party's representation.

To the best of our knowledge, this paper is the first to theoretically define the original concept of scoring markets. This definition, characterized by a finite and constrained set of possible outcomes or configurations that form a discrete space, enables the calculation of the probability of a given outcome as the ratio of favorable cases to total cases, without requiring the use of density functions. It also allows for the application of numerical methods to obtain specific solutions. These features are particularly useful for analyzing market phenomena through indices designed to measure inequality, market power, or concentration.

We demonstrate the relevance of our contribution by formally modeling three examples of bilateral competitions that correspond to actual sports championships or competition formats. Additionally, we compare our theoretical results for a competition that mirrors the group stage of the Union of European Football Associations (UEFA) Champions League (UCL) with actual outcomes observed in each season. This analysis covers all 25 seasons during which the UCL group stage comprised 32 teams, from 1999/2000 to 2023/2024. The UCL is Europe's most prestigious club football competition and features significant prize money. Within this framework, we pay particular attention to both the theoretical and actual values of a central concept in sports economics: competitive balance. Our comparison reveals right-skewed distributions and negative kurtosis in both the theoretical and actual datasets for the distance to competitive balance (*DCB*) index (Triguero-Ruiz and Avila-Cano, 2019). The sample mean approaches 60 % of the theoretical maximum, suggesting a growing trend in point concentration that reflects a decline in competitive balance over the 25 seasons. Several authors have noted this decline in the UCL's competitive balance (Ramchandani et al., 2023; Triguero-Ruiz and Avila-Cano, 2023). However, Csató and Petróczy (2025) found no such decline in the UCL group stage between the 2003/04 and 2023/24 seasons. They argued against using number of points as a measure of (*ex post*) competitive balance and instead propose an alternative metric.

Finally, we emphasize that our work offers a novel framework for future research into the effects of institutional changes in sports competitions on outcomes and on the level of competitive balance among teams.

The remainder of this paper is organized as follows. Section 2 defines the concept of scoring markets and reviews relevant literature on market structures and sports competitions. Section 3 introduces a theory of scoring markets with applications in sports economics, focusing on bilateral competitions involving two to four teams. Section 4 analyzes the probabilities of league configurations and their relationship to competitive balance, with a focus on the UCL group stage. Section 5 examines the actual competitive balance in the UCL from 1999/2000 to 2023/2024 and compares these results to the theoretical values. The conclusion follows. Supplementary material is provided in the annex to support replication of data and findings. The online appendix provides an example of forecasting final competitive balances for the 2022/23 season.

2. Related literature

The conceptualization of the term "market" has long been the subject of intense academic debate (Fernández-Huerga, 2013; Hodgson, 2008, 2019, 2020; Rosenbaum, 2000). Many economists use the term to describe a broad array of arrangements or processes observed in real-world contexts. Hodgson (2020) referred to this as market universalism, and Becker (1976) argued that family and intimate partner relationships are governed by market principles. Others emphasize organizations, viewing them as institutions that may not necessarily function as markets (Rosenbaum, 2000). The relationships and sometimes the blurred boundaries among the concepts of "institutions," "markets," and "organizations" have been examined in numerous studies (Hodgson, 2019; Ménard, 1995; Rosenbaum, 2000). As a result, it is not uncommon to encounter situations in which it is unclear whether a given institution qualifies as a market.

Weber (1978) asserted that a market exists wherever there is competition, even one-sided competition, for exchange opportunities among multiple potential participants. This framing places competition at the core of what defines a market, even though competition itself is a broader concept.

Competition is often described as a process of rivalry. Stigler (1987) defined it as "a rivalry between individuals (or groups or nations), and it arises whenever two or more parties strive for something that all cannot obtain." This definition implies that "competition" can apply to many domains where rivalry is present.

Stiglitz (1993) emphasized that a key characteristic of markets is that they are environments in which exchanges occur. That is, transactions between sellers and buyers. Not all exchanges, however, occur through markets. Nevertheless, when exchanges happen within a market framework, they imply a structured relationship among participants (Fourie, 1991; Jackson, 2007; Rosenbaum, 2000). Since Stigler (1966), there has been broad agreement that a market consists of the buyers and sellers of a particular good or service. In this view, supply, demand, and a market-clearing price collectively define the functioning of the market.

This direct connection between markets and the occurrence of exchanges was also highlighted by Plott and Smith (1978).[1] In such exchanges, both parties (i.e., firms and consumers) possess clearly defined property rights, respond to incentives, and make decisions within specific constraints. A market, in this institutional sense, is a system of rules encompassing behavioral norms, social conventions, and legal frameworks (Hodgson, 2019).

Although market outcomes such as prices are a central focus in standard economics textbooks, these texts often give little attention to the underlying rules and mechanisms that generate such outcomes. Moreover, the concept of the market itself frequently remains undefined (Hodgson, 2008).

As Simon (1991) reminded us, applying these considerations to real-world contexts may lead us to conclude that markets are not necessarily the primary institutions or governing mechanisms in society. In fact, a wide variety of organizations can be analyzed as if they were markets, even if they do not align perfectly with the canonical definition of a market. This is evident in contexts such as sports championships (e.g., a football league). In this regard, 2025 marks the 69th anniversary of Professor Simon Rottenberg's (Rottenberg, 1956) seminal article, "The Baseball Players' Labor Market." Focused on the structure of the professional baseball industry in the US, this work is widely recognized as the first serious attempt to apply economic analysis to sports (García, 2019)[2]. It introduced key concepts such as competitive balance and outcome uncertainty. Moreover, Rottenberg directly applied the concept

[1] These latter authors equate "modes of trading organization," "forms of market organization," and "exchange institutions."

[2] Statements such as "Two teams opposed to each other in play are like two firms producing a single product" (pp. 224–225); "The product is the game weighted by the revenues from its play; " "A baseball team, like other firm, produces its product by combining factors of production" and "The wealthy teams will usually prefer winning to losing" (p. 225) already suggest an analytical view that has become well-established over these years (Rottenberg, 1956).

of a market to sports in the title itself, analyzing the labor market for baseball players and highlighting features of imperfect competition. Since then, the sports labor market has been extensively examined by numerous scholars, including Berri et al. (2023), and Simmons (2022).

From the perspective of sports economics, teams compete with one another much like firms, offering a product or service to consumers—primarily fans. At the end of a competition, a team's "size" can be interpreted as the number of points it has accumulated from match outcomes. The final league standings thus reflect the distribution of team sizes. These competitions can also be understood as organizations that aim not to maximize profits, but rather to accumulate size, defined as the total point score. In some cases, this implies that the teams are effectively trying to maximize market share instead of profit. Késenne (2006) highlighted the long-standing debate over the appropriate objective function for clubs in a competitive setting, and Fort (2019) provided a comprehensive retrospective on this issue.

In general, the literature in sports economics identifies three primary maximization objectives for teams: profit, win, and fan welfare (Sloane, 1971). Rodríguez (2012) observed that while US clubs typically prioritize profit, European clubs more often focus on sporting success. Peeters (2015) explored this distinction through the lens of gate revenue sharing in US major leagues and European football. Madden (2012) examined the fan welfare perspective, and Dietl and Duschl (2012) emphasized that North American and European sports leagues exhibit profound structural differences. In North America, most teams are privately owned and operate with a clear profit motive. League expansion generally occurs through the addition of new teams rather than through full market saturation, and franchise relocations are relatively frequent. Salary caps are also common as a mechanism for controlling team expenditures. In contrast, European clubs are often structured as member-owned associations and prioritize winning over profitability (Garcia-del-Barrio & Szymanski, 2009). These leagues aim for comprehensive geographic market coverage, with team relocations being rare and salary caps typically absent. Perhaps the most fundamental structural distinction is organizational. That is, North American leagues are generally "closed" systems with stable membership, whereas European leagues operate under "open" systems that include promotion and relegation. This openness extends to club ownership, with members often playing a direct role in governance (Triguero-Ruiz and Avila-Cano, 2024). In practice, the structure and rules of competition may lead teams to deprioritize winning or total point accumulation (Csató, 2021; Kendall and Lenten, 2017). Csató (2023a) further showed that the revenue distribution mechanisms in UEFA club competitions can distort the incentives of some teams in their domestic leagues.

The business of sports competitions generates substantial annual revenue for clubs, although that revenue is distributed unequally. The allocation of revenues in sports competitions has received significant attention in the literature (Bergantiños and Moreno-Ternero, 2020, 2022, 2023; Dietzenbacher and Kondratev, 2023; Petróczy and Csató, 2021). We can envision this total revenue as a "cake" that is divided among clubs based on their final ranking in the championship. A higher placement in the standings corresponds to a larger number of tokens. In this framework, the championship itself functions as the governing mechanism or market institution that allocates unequal slices of the cake. These tokens can be seen as bids in a multi-item auction, where the portion of revenue (the cake) each club receives is proportional to the tokens it earns from its final standing.

This initial observation is relevant because this article focuses on organizations or institutions that might not, at first glance, meet the strict criteria to be classified as markets. Nevertheless, both theoretically and empirically, many of their characteristics can be analyzed using market-based tools.

A second key issue concerns the discrete nature of potential outcomes in a sports competition. When incorporating firm size into the conceptual framework of market theory, any outcome is generally assumed to be theoretically feasible within standard economic constraints, such as non-negativity. Accordingly, any competitor can, in principle, attain any relative size. In economic modeling, the inverse demand function is typically assumed to be continuous, and each firm produces output under a continuous cost function, although weaker regularity assumptions are sometimes accepted (Vives, 2000). Thus, we assume that the vector of sizes of the n firms making up a market is $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{R}_+^n$ and that any vector with its components $x_i \in \mathbb{R}_+$ is feasible. Therefore, the set of possible outcomes (i.e., firm sizes) occupies a continuous space, where each point represents a potential market configuration. If firm size is measured in terms of market share, then the outcome space becomes a simplex in which the shares of all firms sum to one.

One might argue that we cannot know with certainty whether the configuration space is continuous, but we assume continuity due to cognitive limitations that prevent us from identifying the full set of admissible configurations in advance. Nevertheless, we proceed as if the space were continuous, since this assumption cannot easily be relaxed unless the structure of the market explicitly indicates otherwise. In practice, microeconomics textbooks define and graph supply and demand as continuous functions, even though many of their illustrative examples involve inherently discrete environments.

By contrast, some competitive processes inherently operate within a discrete configuration space. This occurs when outcomes are governed by an externally imposed remuneration system (i.e., scoring system) which assigns a finite set of rewards to participants. The resulting outcomes are then restricted to a finite, discrete set rather than forming a continuum. Sports competitions (or leagues), as described earlier, are prime examples. We refer to these settings as scoring markets, where outcomes are finite and constrained, forming a discrete configuration space—unlike the continuous space assumed in conventional models. In this context, market configurations are defined as $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{N}^n$, where each $x_i \in \mathbb{N}_0$ (i.e., each component is a non-negative integer), provided that the values are consistent with the applicable scoring system. This constraint arises because participant size, represented by accumulated scores, follows specific, externally defined patterns rather than arbitrary values.

In a league format, the final standings are determined by aggregating the match outcomes for each team, using a scoring system that remains fixed for the duration of the championship. This applies across sports such as baseball, basketball, football, handball, hockey, rugby, soccer, and others. However, the concept of scoring markets extends well beyond sports. As mentioned earlier, scoring markets also arise in other contexts, including regulated markets where government agencies score technical and financial proposals submitted by competing firms,[3] contests where outcomes are based on evaluations by judges or audiences, and elections, where individual votes determine the size and representation of political parties in a legislative body.

Our characterization holds substantial relevance beyond the academic sphere. By enabling rigorous problem formulation across various scoring market contexts and by identifying feasible configurations, it enhances the standard control mechanisms employed by public administrations and, more broadly, by contest and tournament organizers, including in electoral systems. In sports economics, this distinction is especially useful for management decision-making and support systems, as well as for assessing competitive balance, estimating outcome probabilities, and informing sports betting strategies (e.g., Gasparetto et al., 2023; Gomez-Gonzalez et al., 2019; Owen et al., 2007; Pawlowski et al., 2010). Our framework also facilitates the comparison of theoretical and empirical results across numerous studies on the subject (e.g., Haan et al., 2012; Laica et al., 2021; Franck and Theiler, 2012; Jang et al., 2019).

[3] In this context, a market configuration could be of type $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{Q}^n$, with its components $x_i \in \mathbb{Q} \geq 0$.

In this article, we characterize football competitions as scoring markets, with a particular emphasis on the UCL. Our analysis focuses on measuring the competitive balance of the group stage throughout the entire period in which the competition has featured 32 teams. Triguero-Ruiz and Avila-Cano (2023) evaluated competitive balance in the UCL group stage before and after each season from 1999/2000 to 2017/2018. They found that *ex ante* competitive balance across groups is not homogeneous. Their results suggest that the group formation mechanism could be improved to ensure a more uniform distribution of team strengths. However, Avila-Cano and Triguero-Ruiz (2024) concluded that the initial group compositions have little to no bearing on which team ultimately becomes champion. Csató (2024) provided some evidence that the UCL group stage has become more predictable in recent years. Building on this, Csató et al. (2024) developed a simulation model to evaluate different scheduling arrangements for group stage matches. Additionally, Csató and Petróczy (2024) used bibliometric indices to assess the level of competition among clubs and national associations during the knockout stages from 2003 to 2023. Schokkaert and Swinnen (2016) found that although qualification rounds in the UCL have become more predictable compared to its predecessor, the outcomes in later stages have become less so.

3. Approach to a theory of scoring markets

We can understand competition as rivalry in the sense of Stigler (1966). In this regard, in any market typically analyzed, each firm competes multilaterally with the others, at least potentially, for the demand. Whether they obtain such demand, totally or partially, or fail to do so determines the final size of the firm. In other contexts, the development of rivalry may imply that the participants in each confrontation (sometimes bilateral and not necessarily multilateral) compete for a certain number of *points* that reward the outcome of the competition they have taken part in, according to a specific scoring system.

Definition 1. A scoring system is the point allocation mechanism in a competition, which integrates a scheme of potential results of the encounters, a scheme of point allocation based on the results (point award pattern) and a scoring function that converts the results into points.

For example, the feasible results pattern can be {win, tie, loss} and the corresponding point award pattern can be $\{3, 1, 0\}$; the scoring function transforms an outcome into a real number. Thus, the function assigns the value 7 to two wins and one tie $(3 + 3+1)$ (for a discussion on the optimal design of scoring systems, see Csató, 2023b; Kondratev et al., 2024; Sitarz, 2013).

The final *size* of each competitor is determined by the accumulation of points obtained from encounters with other rivals; these points assess the outcome of such confrontations. Therefore, the size of each participant in a competition is determined by the number of points obtained from the encounters with the other participants. Hence, we refer to these situations as *scoring markets*.

Definition 2. A scoring market is an organization or institution in which the participating agents' size outcomes are determined through the implementation of a point allocation mechanism or system (scoring system) associated with the characteristics, performance or behavior of these agents in their encounters.

Are we referring to extraordinary circumstances, or do scoring markets reflect situations of rivalry that have some presence in real life? To answer these questions, we can, for example, consider a regulated market in which a certain good or service is procured through a competitive bidding procedure under the conditions specified in the technical and economic specifications that have been established in advance. According to these conditions, each competitor is assigned a score based on their proposals, and a fixed number of firms are selected as suppliers of the product or service (think of highly regulated sectors, such as television licenses, mobile phone licenses or other telecommunications-related products). The possibility of a monopoly configuration can be eliminated if it is initially decided that there will be multiple providers. Additionally, the monopoly configuration can be ruled out if the remuneration system has a finite number, strictly greater than one, of evaluations (remuneration allocations). This prevents the final configuration from allowing one firm to accumulate all points while the rest are valued with zero points, as the bidding conditions indicate that all proposals submitted by the competing companies must be evaluated. The division of a public procurement into lots contributes to preventing the monopoly solution by facilitating access for small and medium-sized enterprises, as indicated in Directive (2014)/24/EU (European Union, 2014).

A second example of a scoring market is a contest in any field imaginable, such as the *Eurovision Song Contest* or a local literary competition, where the prospective winner achieves victory by winning a competition based on scores given by one or more external referees or evaluators. These scores are assigned by each member of a finite set of evaluators forming the jury: (i) either scores that reward a single contestant or (ii) a range of values sorted to reward multiple contestants. The number of possible outcomes is also finite, and the space is discrete. In the second case, the possibility of a single contestant accumulating all points (monopoly) is also excluded.

A third example is elections in a specific democratic country. Individual votes of citizens, through a transformation mechanism defined in electoral laws, become parliamentary seats (Balinski and Young, 1982). A vector representing the sizes of the political parties in the parliament is defined, where the discrete nature of possible outcomes is evident.

A fourth example of a scoring market, particularly relevant to us and what we will focus on in this article, is sports competitions. To understand how the competition takes place, i.e. how rivalry unfolds, regardless of whether the championship is individual or team-based (Szymanski, 2003), we can distinguish between two scenarios.

(i) Multilateral relationships among competing agents (as seen in cycling, automobile racing or motorcycling), which are similar to the three examples described earlier. Once again, if the reward system assigns a single value to the winner, a monopoly configuration becomes possible. However, if multiple evaluations are conducted (scoring the first, but also the second and third, fastest lap, reaching an intermediate goal, etc.), a monopoly configuration will not be admissible, since no one can achieve all possible points and thus no one can accumulate the entire size (see Peeters and Wesselbaum, 2023 for a study on competitiveness in racing sports and its measurement).

(ii) Bilateral relationships: All teams or competitors play against each other, but none can accumulate all points or victories as they do not participate in all the matches played by their rivals, thus preventing a monopoly configuration. The fact that the confrontations are bilateral limits the range of possible outcomes.

The theory of scoring markets can be implemented in both bilateral and multilateral competitions. Even though this article only focuses on the characterization of bilateral competitions as scoring markets, it is worth noting that analogous concepts can be applied to multilateral competitions as well.

In proper markets, the spaces over which possible equilibriums are defined are usually continuous. In other words, the configuration space is a continuous set in which any configuration is possible within the general limits or constraints of the problem. In scoring markets, to the extent that the set of possible outcomes that can be achieved in each confrontation and, therefore, the remuneration of each agent after each result (i.e. the points assigned for winning, drawing or losing) are finite (e.g., win, loss or, possibly, tie), the set of possible configurations that can be reached is discrete. In the previous examples of sports competitions, the result of each confrontation assigned a specific score to each team or participant within a range of possible scores. The number of possible score configurations is finite, and no matter how large it may be, it can also be calculated.

While '*some types of competition, such as wars, are obviously undesirable, and so are some forms of economic competition*' (Vickers, 1995, p. 4), this does not seem to apply to sports competitions. The interest and engagement of the contenders, combined with their professionalization and specialization, often transform these competitions into more of a spectacle than strictly sporting tournaments. In any case, we cannot question the existence of rivalry and, indeed, competition in most sports competitions. Hence, it becomes an appealing field of analysis on which we will focus. Next, we present a formalization of bilateral sports competitions before characterizing the discrete set of possible configurations that can be reached.

3.1. Formalization of bilateral competitions

We consider that a championship, $\mathbb{C}$, is defined by the scoring or reward system for agents based on the results they obtain, $\mathscr{O}$; the set of participating agents, $\mathscr{T}$; and the number of times they compete against each other, $\mathscr{S}$. The formalization is general enough that if the agents are sports teams, the competition can be understood as a championship or league. Thus,

Definition 3. A championship with bilateral confrontations is an organization identified by $\mathbb{C} = \{\mathscr{S};\mathscr{T};\mathscr{O}\}$.

In the case of a round robin tournament, $\mathscr{S}$ indicates the type of bilateral confrontation, with one or multiple rounds. Thus, $\mathscr{S} = 1$ indicates that the confrontations are single round, although they can be considered championships with a higher number of rounds (Rasmussen and Trick, 2008). The major European soccer leagues (the "Big Five"—English Premier League, Spanish La Liga, German Bundesliga, Italian Serie A, and French Ligue 1) use a double round-robin format, as does the UCL group stage. In contrast, the FIFA World Cup for national football teams uses single-round matches, while some major North American leagues (such as the NFL in American football or the NHL in hockey) feature more than two rounds. Overall, we consider the tournament of bilateral competitions to be balanced. The type of tournament may be of a type other than round robin, such as a knockout or Swiss system (Dong et al., 2023; Sauer et al., 2024). The definition also includes incomplete round-robin tournaments (Devriesere et al., 2025; Li et al., 2025), such as the new format of the UCL league phase (which replaced the former group stage) from the 2024/25 season.

The set of teams competing in the championship is denoted by $\mathscr{T} = \{t_i\}_{i=1}^{n}$, $n \in \mathbb{N}$, with $n \geq 2$. Therefore, the number of matches in a round robin tournament is given by $z = \mathscr{S}\cdot n\cdot(n-1)/2$; that is, each of the n teams plays $\mathscr{S}$ times against each of the remaining $n-1$ teams. The Big Five leagues are contested by 20 teams, except for the Bundesliga, which has 18 teams. None of the national championships in UEFA have more than 20 teams. In CONMEBOL competitions, Argentina has had up to 30 teams in some recent seasons.

The point award pattern is $\mathscr{O} = \{(P_w, P_t, P_l), \boldsymbol{b}\}$, where P_w, P_t and P_l are the rewards or points assigned to each team in each confrontation for winning (w), drawing (t) or losing (l), respectively. In football, the $\{3,1,0\}$ system predominates, having had the $\{2,1,0\}$ system as usual until the late 20th century. In rugby union, the incorporation of bonuses to some championships has been noticed recently. For instance, the Six Nations Championship (rugby union) has $\{(4,2,0),(3,1,1)\}$, which assigns a potential offensive or defensive bonus, in addition to 3 Grand Slam points, to the winner of all matches (see Avila-Cano et al., 2023).

These, in principle, are the three possible results that can be attained after each confrontation, although others could be articulated (e.g., victory or defeat after a draw, rewarded with fewer points than the initial victory and more than the initial defeat, leading to a differentiation between t_1 and t_2, with the former being the winner of the tiebreaker). For instance, in the NHL, tiebreakers are resolved with shootouts using a point award pattern $\{2,2,1,0\}$ or, in the Water Polo World League, with a pattern $\{3,2,1,0\}$. In turn, $\boldsymbol{b}$ is a vector of potential additional allocations to the score based on the result, which allows for its generalization (e.g., bonuses assigned to each team for demonstrating exceptional offensive or defensive ability, which are transformed into points and added to those obtained based on the result, as seen in rugby union). We assume that $P_w, P_t, P_l \in \mathbb{N}\cup\{0\}$ and, as a criterion for incentive consistency, $P_w > P_t \geq P_l$ or, as the case may be, $P_w > P_{t_1} > P_{t_2} \geq P_l$. It is usual to observe $P_l = 0$. Additionally, the components of $\boldsymbol{b}$ will be non-negative, such that $\boldsymbol{b} = \{b_j\}_{j=1}^{m}, b_j \geq 0$, where m is the type of potential bonuses (while the Six Nations Championship follows a pattern $\{(4,2,0),(3,1,1)\}$ that awards 3 Grand Slam points to the winner of all matches, most rugby union tournaments use bonus points: $\boldsymbol{b} = (0,1,1)$). Note that the scoring system is discrete and remains fixed throughout the competition.

There is no objection to considering that the scoring system does not allow for ties and requires tie-breaking rules in each match. However, in the formulation we are about to present, we do not consider this circumstance. Furthermore, the fact that tie-breaking rules exist for the classification is not relevant to the development we are undertaking (Berker, 2014).

Definition 4. Let $\Re$ denote the results pattern in which the cardinal number is $|\Re| = \tau$.

Then, $\Re = \{win, tie, loss\}$ denotes the results pattern whose cardinal number is $\tau = 3$. If there are no ties, it will be $\tau = 2$. If there are tiebreakers (by penalty shootouts or overtime) with different rewards, then $\Re = \{win, tie_{win}, tie_{loss}, loss\}$ and $\tau = 4$. If the point award pattern includes potential additional allocations to the score, for example, bonuses, the cardinal number of $\Re$ will be increased by the number of potential bonuses (m). Nevertheless, for simplicity and without loss of generality, henceforth, we consider $\tau = 3$.

Definition 5. Let Y^z denote the set of possible results of the matches obtained in the competition.

For example, in a competition with four teams ($n = 4$) in a single round-robin format and a results pattern $\Re = \{win, tie, loss\}$, there are six matches played, resulting in a vector $\mathbf{y} \in Y^6$ as the final competition outcome. Thus, a vector such as $\boldsymbol{y} = (w, w, w, t, l, w)$ means that the first team considered has won against the second, third, and fourth teams (first three components: w, w, w); the second team has tied with the third (fourth component, t) and lost to the fourth (fifth component, l) and the third team has won against the fourth (sixth component, w).

Definition 6. Let Ψ^z denote the set of different results of the matches obtained in the competition when the name of the teams does not matter.

For instance, in a league with three teams in a single round-robin format, where there will be three matches, the vector (w, w, t) does not indicate different competition outcomes from (w, t, w) and (t, w, w).

Under these conditions.

(i) The number of elements of Y^z is determined by the number of $\tau-$ element variations of $z-$ elements with repetition allowed: $V_{\tau,z} = \tau^z = \tau^{\mathscr{S}\cdot n\cdot(n-1)/2}$.

(ii) The number of elements of Ψ^z is equal to the number of different elements of Y^z (anonymity). It is determined by the number of $\tau-$ element combinations of $z-$ elements, with repetition: $C_{\tau,z} = \frac{(\tau+z-1)!}{z!\ (\tau-1)!}$.

Therefore, because the cardinal number of the results pattern is finite, the number of possible results is finite. Triguero-Ruiz and Avila-Cano (2019) provided a table that presents the characterization of the spaces for different n participants.

Definition 7. Let $\theta : Y^z \rightarrow \Lambda^{n-1} \subset \mathbb{R}_+^n$ denote a scoring function that calculates the points earned by each team at the end of the competition. Note that, in Λ^{n-1}, the points awarded to each team are determined by those awarded to the remaining $(n-1)$ teams. Thus, the set Λ^{n-1} is discrete.

The scoring function allows (i) to obtain the aggregate matrix of results by teams, $M \in \mathbb{M}_{n\times\tau}$, which records in its columns the number of wins, ties, and losses, respectively, for each of the n teams, represented in the rows, and (ii), given $\Re$, to assign the number of points achieved by each team according to the point award pattern, $\mathscr{O}$. Thus, for every $\mathbf{y} \in Y^z$, $M\cdot(P_w\ P_t\ P_l)^t = \theta(\mathbf{y})$.

In a competition without bonuses, $b_j = 0$ for all j, the score of each team at the end of the championship is the aggregation of the scores achieved in each match. Each team, i, has a number of points obtained at the end of the competition, $\rho_i \in \mathbb{N} \cup \{0\}$, where $\sum_{i=1}^n \rho_i = P$ is the total number of points awarded in the competition. Therefore, this competition can be shown as a vector of points $\boldsymbol{\rho} = \theta(\mathbf{y}) = (\rho_1, ..., \rho_n) \in \Lambda^{n-1}$, which represents the final results. The subscript i indicates the position of each team in the final results table.

Definition 8. Given the vector $\boldsymbol{\rho}$, a competition configuration is a vector of teams' point shares, denoted as $\mathbf{s} = (s_1, s_2, ..., s_n)$, in a competition of n teams, where $s_i = \frac{\rho_i}{\sum_{i=1}^n \rho_i} \in [0,1]$ represents the point share of team i and the vector $\mathbf{s}$ belongs to the $(n-1)-$ dimensional unit simplex, $S^{n-1} = \left\{ \mathbf{s} = (s_1, s_2, ..., s_n) \in \mathbb{R}^n, 0 \le s_i \le 1 \text{ for every } i, \sum_{i=1}^n s_i = 1 \right\}$.

Given n, the simplex represents the space of all possible configurations that the competition could have. All possible configurations that the competition can have are in the simplex. However, the set of admissible competition configurations will be a subspace of S^{n-1}. Indeed, the admissible solutions form a discrete subspace included in the simplex, $X^{n-1} \subset S^{n-1}$.

On the one hand, a monopoly configuration (or vertex of the simplex S^{n-1}) would be a competition configuration in which $\mathbf{s} \in S^{n-1}$, such that $s_i = 1$ for some agent i. For each agent i, s_i^m would be the monopoly configuration in which i is the monopolist. On the other hand, the barycenter is a competition configuration $\mathbf{s}^b \in S^{n-1}$ such that $s_i^b = 1/n$ for each agent i. Note that, in a championship with bilateral matches, the monopoly configuration cannot be reached; it is not admissible. In contrast, the barycenter corresponds to a configuration in which all participants have achieved the same final score after their matches.

How does each agent achieve its final size? In a well-defined market, through a competition of 'everyone against everyone' to capture the demand, firm size is measured, for example, by the volume of production or the number of jobs attained at the end of a specific period. In a sports championship where the competition is bilateral and may take place over one or more rounds, the size is measured by the points accumulated at the end of the tournament. Note that the point score encompasses the possibility of no ties (with nonzero rewards), and the size can be measured by the number of victories.

Under these conditions, we can return to Definitions 1 and 2 to recall that a scoring market is a competition in which the size of the agents is defined based on a scoring system. The set of all vectors of admissible results is not infinite but limited. In fact, note that, given a competition, not any vector of results is possible: given the number of teams, there are scoring patterns fixed with limited rewards. All simplex points are not possible competition configurations because, in each competition, there is a point award pattern fixed with limited rewards. The possible results of each match (win, loss or, when applicable, a tie) are a finite number, and each result has a reward, fixed or variable, but this is limited. Thus, we have a discrete subspace that is the set of admissible competition configurations: $X^{n-1} \subset S^{n-1}$.

In addition, there are admissible configurations whose elements are permutations.

Definition 9. Given a competition configuration, $\mathbf{s}$, an analogous competition configuration is $\mathbf{s_p} = \Pi\mathbf{s}$, where Π is an $n \times n$ permutation matrix.

Notice that permutation anonymizes the outcome. Any measurement to be performed based on the competition configuration (e.g., measuring competitive balance) will assign the same value to all analogous configurations. Note that the concept of analogous configuration may not be shared by fans of each team, but it allows identifying the relationship that exists between configurations and measured concepts (concentration, inequality, competitive balance, etc.).

We now present several examples to illustrate and analyze the discrete nature of the configuration space.

3.2. Discrete nature of the set of configurations in bilateral competitions

In this section, we address three cases that constitute, in principle, the simplest situations that can arise. In all three cases, the competition is bilateral. This will allow us to utilize graphical tools while easily identifying all possible scenarios. The first case is a final one, which involves a single match between two rivals. The second case is a league with three competitors playing in a single round-robin format. This would be, for example, the case in the last qualifying round for *EuroBasket*. In the third case, the four teams compete in a double round-robin league format, as seen, for example, in the group stage of the UCL.

3.2.1. Case of a final

Consider a competition that is final and is played as a single-legged match.

(i) $n = 2$; that is, $\mathscr{T} = \{t_i\}_{i=1}^2$.
(ii) $\mathscr{S} = 1$; that is, $z = 1$.
(iii) $\Re = \{win, tie, loss\}$; that is, $|\Re| = \tau = 3$.

Then, $|Y^z| = 3$ and the possible competition outcomes are $Y^z = \{w, t, l\}$. For convenience, each element of Y^z indicates the result achieved by the same team, and this will be the 'home team.' As it is a final, the other team, the 'away team,' will have the complementary result.

In contrast, the set of different competition outcomes is given by Ψ^z, and its cardinality is $|\Psi^z| = 3$, with the potential different competition outcomes being $\Psi^z = \{w, t, l\}$. In this case, $Y^z \equiv \Psi^z$.

Therefore, if the home team is the winner after the match, we will have $\mathbf{y} = (w)$, if it has lost, we will have $\mathbf{y} = (l)$, and if it is a tie, then we will have $\mathbf{y} = (t)$. In other words, the spaces Y^z and Ψ^z are not numerical but categorical.[4]

Given that there is only one match, the point award pattern $\mathscr{O} = \{P_w, P_t, P_l\}$ is irrelevant for determining the final configuration. Under these conditions, it is trivial that the possible competition configurations (in terms of the teams' point shares after the final) belong to the one-dimensional simplex, and they would be $\left\{(1,0), (0,1), \left(1/2, 1/2\right)\right\}$, representing the two vertices, s_1^m and s_2^m, (in each one a team wins and obtains all the points) and the barycenter, s^* (it ties). Fig. 1 shows that, with the segment $\overline{s_1^m s_2^m}$ as the unit simplex, the three possible results are the extreme points, s_1^m and s_2^m, and the barycenter, s^*.

3.2.2. Case of a league with three teams

Consider a competition with three teams in a single round-robin format.

[4] Thus, just as we can represent the win, tie, or loss of the first team we consider, denoted as the set of possible outcomes of the match, as (w, t, l), we could also do it using $(1, X, 2)$, for example, which are the symbols used in football betting in some countries to represent the home team's win (1), tie (X) or loss (2).

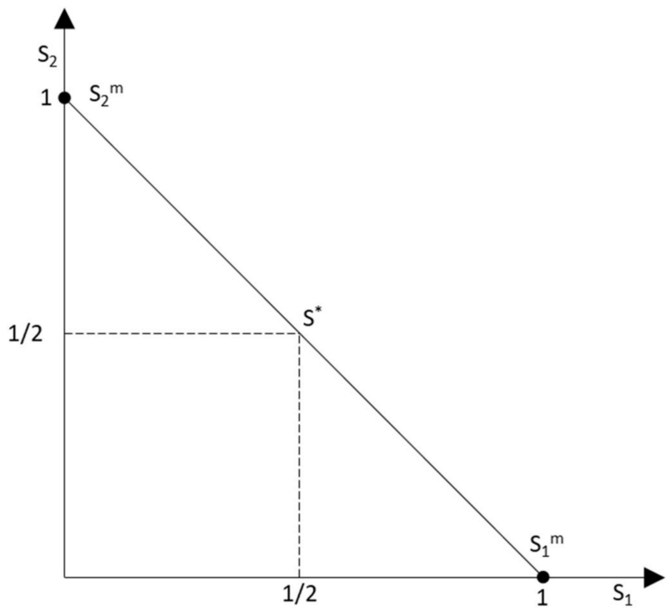

Fig. 1. Three possible competition configurations in a final: one team wins, s_1^m; the other team wins, s_2^m; or they tie, s^*.

(i) $n = 3$; that is, $\mathscr{T} = \{t_i\}_{i=1}^3$.
(ii) $\mathscr{S} = 1$; that is, $z = 3$.
(iii) $\Re = \{win, tie, loss\}$; that is, $|\Re| = \tau = 3$.

Then, $|Y^z| = 27$ and $|\Psi^z| = 10$. The details of the elements in sets Y^z and Ψ^z are shown in Table 1, which also displays the corresponding permutations, the points assigned to each team in each scenario, the total points obtained collectively by the teams, the corresponding competition configurations and the Herfindahl–Hirschman index (*HHI*) for each of them, which indicates the concentration level observed in each configuration (Herfindahl, 1950; Hirschman, 1945, 1964). The *HHI* is defined as the square of the sum of point shares, s_i, achieved by the n teams in the championship: $HHI = \sum_{i=1}^{n} s_i^2$. Since the minimum and maximum values may vary between competitions, it is convenient to normalize the *HHI* when making comparisons (see Section 4). Notice that, given a scoring system, a normalized *HHI* is a linear function of *HHI* if the number of players is fixed. In our case, the raw *HHI* values are shown in Table 1.

This tournament can be useful in illustrating how the scoring function operates. Let us assume that the competition has taken place among three teams named A, B and C with the following results: team A has won both of its matches, team B has won against team C and team C has lost both of its matches. By combining the order of the matches sequentially, the vector of results for the three played matches, $\mathbf{y} \in Y^3$, is $\mathbf{y} = (w, w, w)$.

The element (w, w, w) indicates that, in the A-B match, team A (the home team, mentioned first) wins, corresponding to the first component (w); in the A-C match, team A wins (second component); and in the remaining match, B-C, team B wins (third component). Recall that the first team in each match is considered the home team. Similarly, the element (w, w, l) would mean that team A has won against B and C, and team B has lost to C in the third match. Therefore, once the teams are initially ordered, the elements of Y^z have a sequence: the first team plays against all others sequentially, the second team plays against the rest, and so on, until the penultimate team competes against the last team in the initial ranking. The procedure restarts for each additional round.

Set Λ^{n-1} is defined by the scores obtained by the teams. Thus, (w, w, w) corresponds to $(6, 3, 0)$ for $\mathscr{Q} = \{(P_w = 3, P_t = 1, P_l = 0), \boldsymbol{b} = \mathbf{0}\}$, and (w, w, l) corresponds to $(6, 0, 3)$. For each of the 27 elements of Y^z, we will have a vector of scores, although one of them is repeated—$(w, l, w) \rightarrow (3, 3, 3)$ and $(l, w, l) \rightarrow (3, 3, 3)$—so the cardinality of Λ^{n-1} is 26. At the end of the competition, comprising these three matches, the teams can collectively obtain 6, 7, 8 or 9 points. In addition, there can only be 25 different competition configurations, as the configuration $(1/3, 1/3, 1/3)$ appears three times, once for a total score of 6 points (t, t, t) and twice for 9 points (the two previously mentioned).

Table 1
Tree of *possible* and distinct results and configurations in a single round-robin competition with ties and with three teams.

Match 1	Match 2	Match 3	Elements of $\boldsymbol{Y^z}$	Elements of $\boldsymbol{\Psi^z}$	Analogous results (permutations)	Score points (ρ)	Total points	Configuration	***HHI***
w	w	w	www	www	www	(6, 3, 0)	9	(2/3, 1/3, 0)	5/9
		t	wwt	wwt	wwt-wtw-tww	(6, 1, 1)	8	(3/4, 1/8, 1/8)	**19/32**
		l	wwl	wwl	wwl-wlw-lww	(6, 0, 3)	9	(2/3, 0, 1/3)	5/9
	t	w	wtw			(4, 3, 1)	8	(1/2, 3/8, 1/8)	13/32
		t	wtt	wtt	wtt-twt-ttw	(4, 1, 2)	7	(4/7, 1/7, 2/7)	3/7
		l	wtl	wtl	wtl-wlt-twl-tlw-lwt-ltw	(4, 0, 4)	8	(1/2, 0, 1/2)	1/2
	l	w	wlw			(3, 3, 3)	9	(1/3, 1/3, 1/3)	1/3
		t	wlt			(3, 1, 4)	8	(3/8, 1/8, 1/2)	13/32
		l	wll	wll	wll-lwl-llw	(3, 0, 6)	9	(1/3, 0, 2/3)	5/9
t	w	w	tww			(4, 4, 0)	8	(1/2, 1/2, 0)	1/2
		t	twt			(4, 2, 1)	7	(4/7, 2/7, 1/7)	3/7
		l	twl			(4, 1, 3)	8	(1/2, 1/8, 3/8)	13/32
	t	w	ttw			(2, 4, 1)	7	(2/7, 4/7, 1/7)	3/7
		t	ttt	ttt	ttt	(2, 2, 2)	6	(1/3, 1/3, 1/3)	1/3
		l	ttl			(2, 1, 4)	7	(2/7, 1/7, 4/7)	3/7
	l	w	tlw			(1, 4, 3)	8	(1/8, 1/2, 3/8)	13/32
		t	tlt			(1, 2, 4)	7	(1/7, 2/7, 4/7)	3/7
		l	tll			(1, 1, 6)	8	(1/8, 1/8, 3/4)	**19/32**
l	w	w	lww			(3, 6, 0)	9	(1/3, 2/3, 0)	5/9
		t	lwt			(3, 4, 1)	8	(3/8, 1/2, 1/8)	13/32
		l	lwl			(3, 3, 3)	9	(1/3, 1/3, 1/3)	1/3
	t	w	ltw			(1, 6, 1)	8	(1/8, 3/4, 1/8)	**19/32**
		t	ltt	ltt	ltt-ttl-tlt	(1, 4, 2)	7	(1/7, 4/7, 2/7)	3/7
		l	ltl			(1, 3, 4)	8	(1/8, 3/8, 1/2)	13/32
	l	w	llw			(0, 6, 3)	9	(0, 2/3, 1/3)	5/9
		t	llt	llt	llt-ltl-tll	(0, 4, 4)	8	(0, 1/2, 1/2)	1/2
		l	lll	lll	lll	(0, 3, 6)	9	(0, 1/3, 2/3)	5/9

Note: The minimum and maximum *HHI* values are highlighted by underlining and bold, respectively.

Table 1 shows that some competition configurations are less probable; for example, the aforementioned $(1/3, 1/3, 1/3)$, $(3/4, 1/8, 1/8)$ and $(1/2, 1/2, 0)$ are repeated three times, while the configurations $(2/3, 1/3, 0)$, $(1/2, 3/8, 1/8)$ and $(4/7, 2/7, 1/7)$ are repeated six times each, all of them alternating the teams' point shares as analogous competition configurations.

In any case, there are only six different competition configurations, that is, six vectors of point shares that make up the subspace $X^{n-1} \subset S^{n-1}$. Note that there are only six possible values of *HHI* corresponding to the different competition configurations:

(i) $HHI = 1/3$, which is repeated three times, once for a total of six points in the competition and twice for nine points. This is the minimum possible value and is highlighted in the table: each team wins only one match, or all three matches end in a tie.
(ii) $HHI = 1/2$, with a frequency of three for 8 total points.
(iii) $HHI = 19/32$, the same as the previous one. It is the maximum possible value and is highlighted in the table: one team wins its two matches, and the other two teams end up in a tie in the remaining match.
(iv) $HHI = 3/7$, which is repeated six times for 7 points.
(v) $HHI = 13/32$, which is repeated six times for 8 points.
(vi) $HHI = 5/9$, which is repeated six times for 9 points.

Graphically, the possible competition configurations belong to a two-dimensional simplex, which is represented in Fig. 2, which also depicts 25 different competition configurations using balls. The purpose of these balls is to identify where the only different and possible configurations are located within the entire space. Note that 33 % (1/3, green mark) corresponds to the minimum theoretical *HHI* value and 59 % (19/32, red mark) to the maximum theoretical *HHI* value.

The procedure for moving from the match results to the competition configurations is carried out through the scoring function. The results matrix $M \in \mathbb{M}_{3\times 3}$ records in its three columns the number of wins, ties and losses, respectively, for each of the three teams represented in the rows. In the example of a vector of results (w, w, w), the results matrix is given by

$$M = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

Assume that the point award pattern is given by $\mathscr{Q} = (P_w\ P_t\ P_l) = (3, 1, 0)$. Thus, the scoring function assigns the following vector of final scores to the teams:

$$\rho = \theta(\mathbf{y}) = M \cdot \mathscr{Q}^t = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = (6, 3, 0)$$

where $(6, 3, 0) \in \Lambda^2 \subset \mathbb{R}^3_+$. Therefore, the competition configuration is $\mathbf{s} = (2/3, 1/3, 0) \in X^{n-1} \subset S^{n-1}$.

3.2.3. *Case of a league with four teams*

Now consider a competition with four teams in a single round-robin format.

(i) $n = 4$; that is, $\mathscr{T} = \{t_i\}_{i=1}^4$.
(ii) $\mathscr{S} = 1$; that is, $z = 6$.
(iii) $\Re = \{win, tie, loss\}$; that is, $|\Re| = \tau = 3$.

Therefore, $|Y^z| = 729$ and $|\Psi^z| = 28$. It would be excessive to detail both Y^z and Ψ^z sets here; hence, in the Supplementary Material, an Excel spreadsheet is provided that incorporates this information.

To further illustrate the development, let us assume that the competition has unfolded in a way in which team A has won all three matches, team B has won against team C and team D, team C has won against team D and team D has lost all three matches. Consequently, the vector of results for the six played matches $\mathbf{y} \in Y^6$ is $\mathbf{y} = (w, w, w, w, w, w)$. Thus, the results matrix is given by

$$M = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

In addition, if, as previously assumed, $\mathscr{Q} = (P_w\ P_t\ P_l) = (3, 1, 0)$, the scoring function assigns the following vector of final results to the teams:

$$\rho = \theta(\mathbf{y}) = M \cdot \mathscr{Q}^t = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = (9, 6, 3, 0)$$

where $(9, 6, 3, 0) \in \Lambda^3 \subset \mathbb{R}^4_+$ and $\mathbf{s} = (1/2, 1/3, 1/6, 0) \in S^3$.

If the previous tournament is played in a double round-robin format ($S = 2$), that is, $z = 12$ and $\tau = 3$, then $|Y^z| = 531{,}441$ and $|\Psi^z| = 91$. In this case, we can clearly see the increasing difference between the number of possible competition configurations and the number of different configurations. The construction of all match outcomes following the nomenclature used is provided in the Supplementary Material.

4. Probability of competition configurations and competitive balance

In a sports competition, we may be interested in understanding the extent to which a few competitors are able to deploy their strengths and grab maximum points against their competitors. If this ability is high, we can say that the championship exhibits a low competitive balance, which implies a significant concentration of strengths among a few competing teams. In contrast, if there is no concentration of strengths or positive outcomes around a few competitors—that is, any participant has a similar probability of winning any of their matches—we can say that the competition shows a high level of competitive balance (Andreff, 2015; Kringstad and Gerrard, 2004; Szymanski, 2003; Zimbalist, 2002).

The levels of competitive balance achieved in sporting competitions can provide valuable insights into the predictability or uncertainty of potential outcomes attainable in these competitions and the existing level of competitiveness. These factors can affect the interest and engagement of fans in these sports and competitions, as well as the participants' access to resources (Guironnet, 2023; Lenten, 2011; Schmidt and Berri, 2001; Szymanski, 2003). Understanding and analyzing competitive balance not only has implications for the overall success and sustainability of the sports industry but also plays a crucial role in shaping the opportunities available to those involved in these pursuits.

Different indices are used to measure the competitive balance of a championship. It seems reasonable to focus on the measures that, when available, use all the information about the distribution of point shares rather than just a part of it (excluding concentration ratios). Inequality indices provide information on the dispersion of points and the variations concerning the mean point value (Humphreys, 2002). However, these are not measures of concentration, as they do not provide information about the degree to which a specific group of teams within the distribution gathers points in relative terms. Therefore, it is reasonable to use concentration indices, although they need to be adapted to certain specificities of sports economics. The *HHI* is widely used in practice. Depken (1999) proposed a correction to this index, accounting for the effect of the number of participants on *HHI*'s lower bound. The adjustment involves subtracting the lower bound—namely, the inverse of the number of teams—from *HHI*: $dHHI = HHI - \frac{1}{n}$. An alternative adjusted index was also provided by Lenten (2008) or by Pawlowski et al. (2010) to address this concern. They proposed dividing *HHI* by its lower bound:

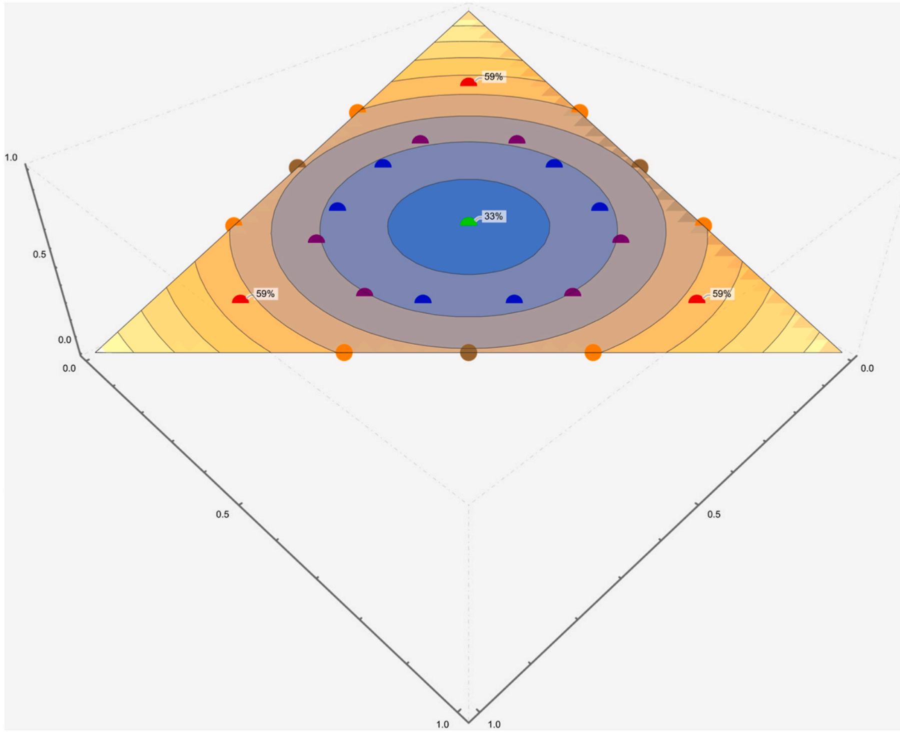

Fig. 2. Representation of possible and distinct competition configurations for $\mathscr{Q} = \{3,1,0\}$ and $\mathscr{S} = 1$.

$HHICB = \frac{HHI}{1/n}$. Triguero-Ruiz et al. (2023) indicated that the number of teams and the point award pattern influence the minimum *HHI* value. Their findings question the notion that the minimum *HHI* value can always reach the inverse of '*n*' and show that the minimum *HHI* value may not equal the inverse of '*n*' in sports leagues without ties.

In any case, the number of teams also affects the upper bound of *HHI*, which, due to the bilateral nature of the confrontations, is less than unity and needs to be calculated. Consequently, Owen et al. (2007) defined a normalized *HHI* as the ratio of *dHHI* to the range of values that *HHI* can take (i.e. the difference between the upper and lower bounds):

$$HHI_{norm} = \frac{HHI - HHI_{min}}{HHI_{max} - HHI_{min}}$$

Avila-Cano et al. (2021) and Avila-Cano et al. (2023) identified the value of HHI_{max} in championships with varying number of teams and different scoring systems. Furthermore, Triguero-Ruiz and Avila-Cano (2019) demonstrated that the square root of the normalized *HHI* is a mathematical distance that fulfils the cardinality property; they proposed this new index for measuring the competitive balance and called it the 'distance to competitive balance' (*DCB*) index[5]:

$$DCB = \sqrt{HHI_{norm}}$$

Considering these metrics within the framework of scoring markets provides valuable insight into both the potential and actual levels—and dynamics—of competitive balance across different competitions.

If the set of possible outcomes is discrete, then the set of possible values for a championship's competitive balance is also discrete. Assuming we can compute these values, a number of questions arise: Are they equally probable? How are they distributed? What can we learn from our ability to calculate and understand them? These are some of the questions we will explore below through an example.

Consider a competition defined by

$$\mathbb{C} = \left\{\mathscr{S} = 2;\ \mathscr{T} = \{t_i\}_{i=1}^{4};\ \mathscr{Q} = \{(3,1,0),\mathbf{0}\}\right\}$$

This setup resembles the group stage of the UCL, as will be discussed in Section 5. Once the competition concludes, each of the four teams will have played six matches. Given the scoring system, the maximum number of points a team can earn is 18 (by winning all six matches), while the minimum is zero (by losing all matches). Collectively, the total number of points distributed among the four teams will range from 24 to 36, depending on the combination of match outcomes. Consequently, there are 13 distinct total point sums that correspond to different competition configurations—no more, no fewer.

The maximum level of competitive balance occurs when all teams earn the same number of points. This perfectly balanced distribution is achievable in four distinct scenarios.

(i) All teams tie in all their matches. In this scenario, each team would have 6 points at the end of the competition, and a total of 24 points would have been distributed.
(ii) Each team wins one match, loses another and ties the remaining four, obtaining 7 points. In total, 28 points would have been distributed.
(iii) Each team wins two matches, loses two, and ties the remaining two, obtaining 8 points. A total of 32 points would have been distributed.
(iv) Each team wins one match (e.g., the first leg) and loses the other (e.g., the second leg) against each opponent. In this case, each team would have 9 points at the end of the competition, and a total of 36 points would have been distributed.

In all four scenarios, each team holds a point share of 1/4. This

[5] The calculation of the theoretical values that the *DCB* can attain is done by identifying all possible result combinations. Please note that the calculation of the *DCB* values is based on the normalization of the *HHI*. For the calculation of the maximum values, Avila-Cano et al. (2021) and Avila-Cano et al. (2023) provided Excel spreadsheets as supplementary material, which allow for calculations across different scoring or remuneration patterns of results, including the possibility of bonuses. To calculate the minimum values, refer to Triguero-Ruiz et al. (2023).

perfectly balanced distribution generates the minimum level of concentration, yielding the corresponding value of the *HHI*: $HHI_{min} = {}^{1}/_{n} = 1/4$, where n is the number of teams in the competition. Accordingly, $DCB = 0$.

By contrast, the least balanced distribution of scores, which corresponds to the highest point concentration, aligns with a *truncated cascade distribution* in the first team (Avila-Cano et al., 2021). In this scenario, the first team would have won all of their matches (and obtained 18 points), while each of the other three teams would have lost both matches against the first team and tied with the other two teams. Consequently, each of these other three teams would have a total of 4 points. The total points amount to 30, and the distribution of point shares would be (3/5, 2/15, 2/15, 2/15). This distribution generates the maximum point concentration, leading to the corresponding value of the *HHI* in this distribution: $HHI_{max} = 0.413$. The *DCB* index has a unitary value: $DCB = 1$.

Other situations that also fall within the high end of the point concentration and thus are close to the minimum competitive balance are as follows.

(i) (18, 12, 6, 0), known as the *complete cascade distribution*, which generates HHI $= 0.389$ and $DCB = 0.922$.
(ii) (18, 12, 2, 2), referred to as the *truncated cascade distribution* in $q = 2$, which generates $HHI = 0.412$ and $DCB = 0.995$, where q is the number of teams winning in the cascade.
(iii) Additionally, (14, 14, 2, 2), which generates $HHI = 0.391$ and $DCB = 0.929$.

As seen in Section 3, as the number of possible outcomes is finite, so is the number of result combinations that can take place. Thus, the outcomes space is discrete, meaning that the set of all league configurations derived from the set of possible outcomes is finite. In Section 3.2.3, we showed that the set of possible results from the matches, denoted as Y^z, obtained in this specific competition consists of 531,441 elements. Hence, we can calculate the probability of each potential league configuration by determining its relative frequency—this is the result of dividing the number of favorable cases by the total number of possible cases.

Under these conditions, it is reasonable to consider the corresponding concentration index values for all possible point distributions that may arise, along with the probabilities of their occurrence. As previously noted, we use the *DCB* index (see Triguero-Ruiz and Avila-Cano, 2019). Table A.1 (see Appendix A) presents the calculated theoretical levels of competitive balance. Each column displays the theoretical *DCB* values based on the total points awarded in a double round-robin league competition with four teams, using the point award pattern $\{3,1,0\}$. The table also provides the probability (as a percentage) of each *DCB* value, which reflects the likelihood of that specific competition configuration occurring.

For clarity, Table A.1 is divided into two panels: the first displays the *DCB* values and their associated probabilities for championships, where the total points fall between 24 and 31. The second panel provides the same information for championships, with total points ranging from 32 to 36. If the possible values are numerous, two columns with respective headers (a) and (b) appear, as is the case for scores between 31 and 34.

For example, if 24 total points are distributed (first column), each team would have reached 6 points, and the level of competitive balance measured by the *DCB* index will be null. This configuration has a probability of 0.0002 %, understood as the number of favorable cases relative to the number of possible cases. Note that, as previously indicated, the same null value of the *DCB* index can be achieved with 28, 32, or 36 points. In all cases, the resultant distribution of point shares assigns 1/4 to each team, implying that they possess a zero value for the *DCB* index.

The least probable outcome corresponds to the maximum concentration (minimum competitive balance): $DCB = 1$. This occurred in the truncated cascade distribution in $q = 1$, where the league-winning team won all its matches, while the remaining teams tied the matches they did not lose against the league-winning team. This implies that the four teams accumulated a total of 30 points. The probability of this outcome occurring is 0.0008 % (four possible cases out of 531,441).

The most probable outcome is obtained with a frequency of 12,912 times (2.4296 %) when the total sum of points is 32, yielding a $DCB = 0.28932$. How is this obtained? In other words, what is the corresponding distribution of points? This most probable outcome is generated by the point distribution (6, 7, 8, 11) 50.55 % of the time (6,528) and by the distribution (5, 8, 9, 10) the remaining 49.45 % (6384 times).

In Table A.1, there are 393 pairs of *DCB* values and probabilities, although they are different in 387 cases. The zero value of *DCB* can be obtained for competitions played with 24, 28, 32 and 36 points; the value 0.4124 with 30 and 36 points and the value 0.7143 with 30, 33 and 36 points. Fig. 3 displays the theoretical *DCB* values arranged in ascending order.

The plotted *DCB* values follow a nearly linear trend. A linear least-squares fit yields $R^2 = 0.982$ with values lying above the trend line for the first two-thirds and below it for the final third. A cubic polynomial fit provides an even better approximation with $R^2 = 0.997$: $DCB = 2E^{-8}x^3 - E^{-5}x^2 + 0.0044x + 0.0605$, $x = 1,2,3,\ldots,393$. The mean of the theoretical *DCB* is 0.5549.

Fig. 4 illustrates the frequency with which each theoretical *DCB* value occurs. The reduced levels of competitive balance (higher *DCB* index values) are individually improbable, yet they exhibit numerous combinations to be achieved. Similarly, values indicating high competitive balance (lower *DCB* index values) are also improbable, albeit less numerous. Meanwhile, moderate levels of competitive balance exhibit higher frequencies. Note that the envelope of maximum probabilities for each *DCB* index level resembles a distribution with positive skewness (the right tail is longer, resulting in more values deviating from the mean toward the right).

Fig. 5 compiles the relative frequencies (probabilities) from Fig. 4 and theoretical *DCB* values from Fig. 3, indicating that *DCB* values are more likely to occur slightly below the median. In other words, globally, starting from a certain value, as the *DCB* value increases, its relative frequency decreases.

Therefore, we can now address the following questions: If we consider the seasons in which the UCL has taken place, what are the probabilities of the attained *DCB* index values? Do the most probable ones actually occur? Is the development of competitive balance in the UCL aligned with what we can theoretically anticipate? The subsequent section delves into these issues.

5. Application to the group stage of the UCL

In Sections 3.2.3 and 4, we characterized and analyzed a competition configuration similar to that of the group stage of the UCL. In this phase of the premier global club football championship, there are 32 teams divided into eight groups of four players each. Within each group, the four teams engage in a double round-robin league using a point award pattern $\mathscr{O} = \{(3,1,0),\mathbf{0}\}$. The top two teams from each group, determined by the total points attained after the aforementioned league, proceed to the subsequent round of 16 (the third-placed team qualified to the UEFA Cup/UEFA Europa League in each season). From this point onwards, there are double-legged knockout matches, except for the final, which is a single-legged match. Nevertheless, we focus only on the group stage of this competition (the 1999/2000–2002/2003 editions contained two subsequent group stages but there has been only one group stage between 2003/2004 and 2023/2024). We calculate the levels of competitive balance using the *DCB* index for each of the eight groups in each season between 1999/2000 and 2023/2024. In the Appendix B, we also conduct a forecasting exercise on competitive balance

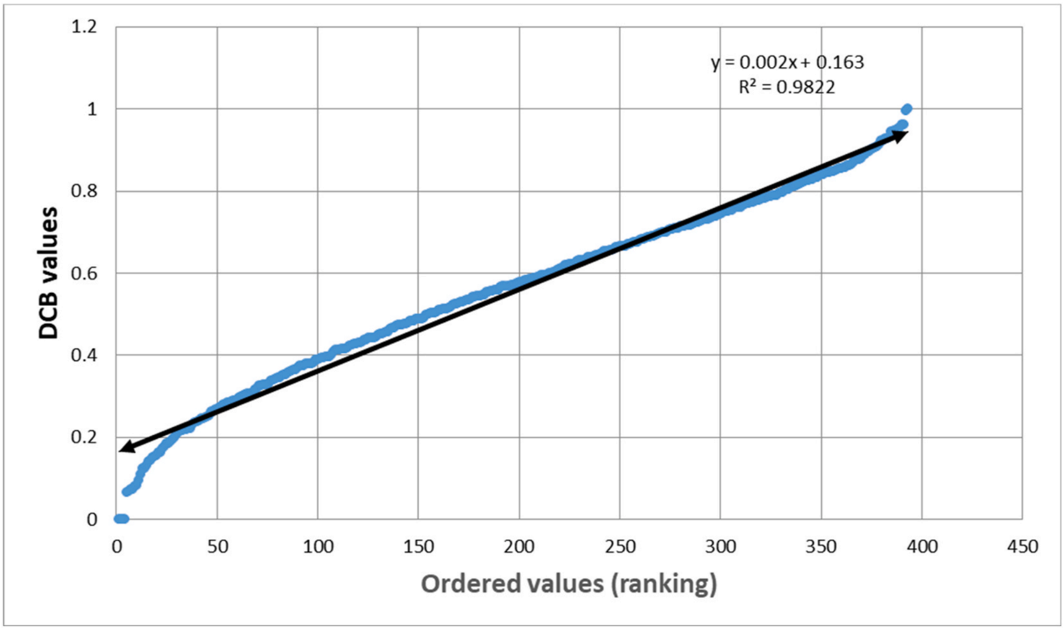

Fig. 3. Theoretical *DCB* values and linear least-squares fit.

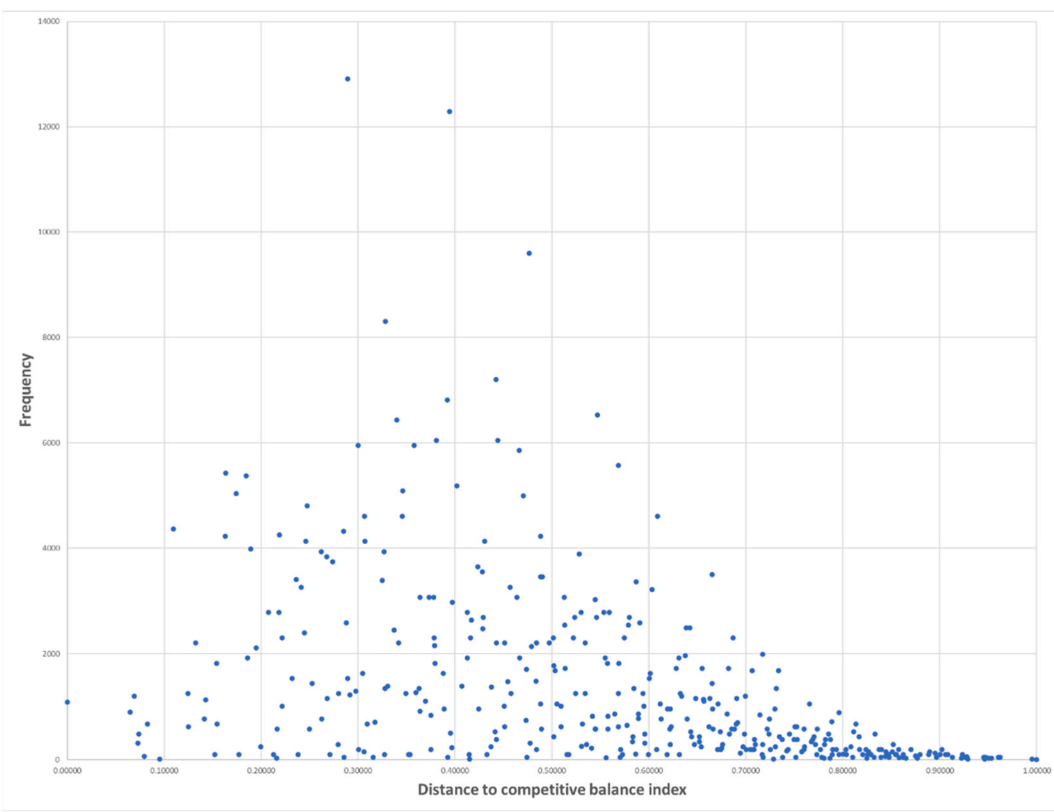

Fig. 4. Frequency distribution for each theoretical *DCB* value.

levels for the final matchday of the 2022/23 season, using that season and matchday exclusively as an example.

The way to qualify for this group stage of the UCL and the mechanism used by UEFA to form the groups (UEFA, 2009) determine that, in principle, the level of competitive balance in each group should be low. The group draw is organized around different pots with teams distributed according to similar strengths defined by the coefficients assigned to them by UEFA and their national federations. In this way, each of the eight groups, considered independently, is theoretically characterized by a notable concentration of strengths in one or two teams (Triguero-Ruiz and Avila-Cano, 2023). The seeding policy of the UCL is discussed in Corona et al. (2019); Csató (2020); Dagaev and Rudyak (2019); and Engist et al. (2021). Additionally, regarding competitive balance of the UCL group stage, a limitation is that some matches played in the last two rounds may be 'stakeless' as the final ranking of one or both teams is already known. These matches may not reflect the true

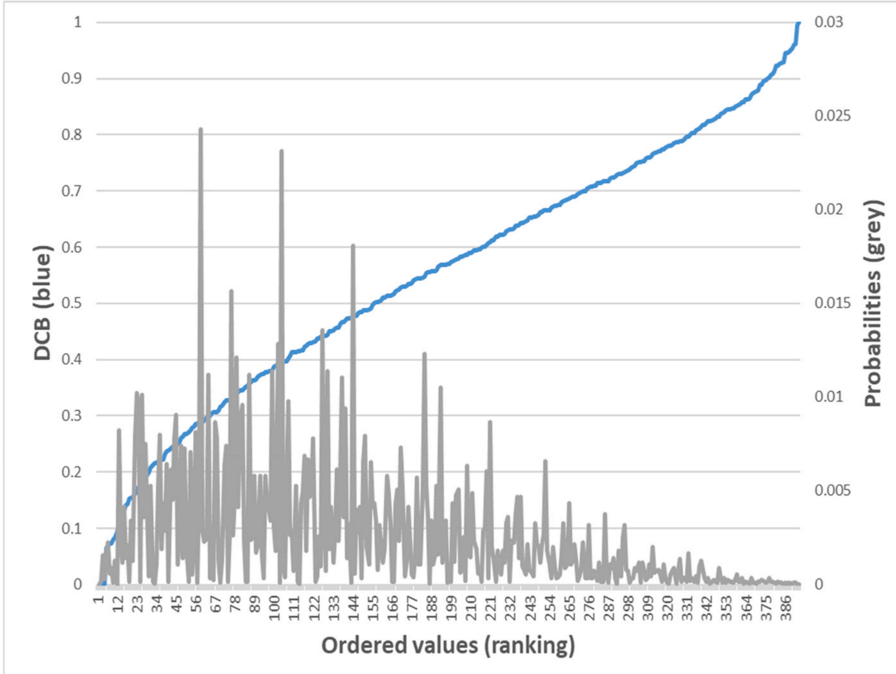

Fig. 5. Theoretical *DCB* values ordered from lowest to highest with corresponding probabilities.

abilities of the playing teams (Csató et al., 2024). In any case, given the theoretical values of *DCB* in this competition (Section 4), what are the actual *DCB* values achieved in recent seasons?

This information is provided in Table A.2, which displays the actual *DCB* values (eight groups per twenty-four seasons), along with the probability of their occurrence and the frequencies of the results that lead to them. Note that this analysis covers all 25 seasons in which the UCL group stage has consisted of 32 teams. The format of the UCL has seen a fundamental reform from 2024/25 season, replacing the group stage with a single league format in which each of the 36 teams plays eight matches (Csató, 2024). Gyimesi (2024) analyzed the new UCL format in relation to competitive balance.

Table 2 provides the main descriptive statistics of the *DCB* index at a theoretical level (i.e. population-based) and with respect to the actual data corresponding to the seasons of the last quarter century (a sample comprising the 1999/2000–2023/2024 seasons). To calculate the *DCB* values, *HHI* normalization is required. The normalization of *HHI* is done using the theoretical maximum and minimum values corresponding to the analyzed seasons, given the number of teams and the scoring system. In our case, since neither the number of teams nor the scoring system changes, the theoretical maximum and minimum values are the same for all seasons $HHI_{min} = 0.250$ and $HHI_{max} = 0.413$.

In these 25 seasons, a substantial portion of the theoretical *DCB* range is covered: only 18 theoretical values are not observed below the minimum actual value achieved (0.163) in Group D of the 2000/01 season and Group B of the 2003/04 season, and 4 above the maximum (0.951), from Group H of the 2017/18 season.

At both theoretical and actual levels, the median is slightly higher than the mean, indicating right-skewed distributions. The negative kurtosis indicates a peak around the mean, with more values clustered near it than in the tails. The sample representing these 25 seasons is quite similar in its statistical characterization to the population.

Note that the sample mean surpasses the theoretical mean and is situated at elevated levels, close to 60 % of the theoretical maximum concentration. Furthermore, the empirical mean (median) value is greater than the theoretical mean (median) by 7 % (5.9 %), suggesting a higher concentration of points in the UCL group stage. This fact is not unrelated to the increasing trend toward point concentration in the UCL (Ramchandani et al., 2023; Triguero-Ruiz and Avila-Cano, 2023). Fig. 6 represents the evolution of the average *DCB* indices across the eight groups per season. The increasing trend of concentration and, consequently, the decline of competitive balance in the UCL over the past quarter century is evident, although other analyses do not confirm this decline (Csató and Petróczy, 2025).

Likewise, in Fig. 6, we represent the five-year moving averages that illustrate this decline in competitive balance. If, at the turn of the millennium (1999/2000–2003/2004), the *DCB* index is below 48 %, in the subsequent five-year periods, it will rise to 57 %, 62 %, 64 % and 66 %.

Previously, Fig. 4 depicted the population values of the *DCB* index. Since we now also have the actual values observed in the analyzed seasons (Table A.2), Fig. 7 combines both sets of information and displays the frequencies of potential theoretical *DCB* values and those of the *DCB* values that have actually occurred in the considered seasons.

Among the potential values, in the 25 represented seasons, the actual values in each of the 8 groups tend to be located along the right side of the distribution. That is, for each frequency band, the highest *DCB* values are achieved, and for each *DCB* band, the highest frequencies are achieved.

At this point, it is appropriate to recap the key contributions. Defining a bilateral competition as a scoring market implies that the set of potential outcomes is finite, countable, and identifiable. In measuring competitive balance in the UCL group stage, this means that the seem-

Table 2
Descriptive statistics of theoretical and actual *DCB* indices for the 1999/2000–2023/2024 seasons.

	Theoretical ***DCB***	Actual ***DCB***
Observations	393	200
Minimum	0	0.163
Maximum	1	0.951
Mean	0.555	0.593
Standard error	0.012	0.013
Median	0.571	0.605
Standard deviation	0.228	0.188
Kurtosis	−0.712	−0.831
Skewness coefficient	−0.275	−0.241

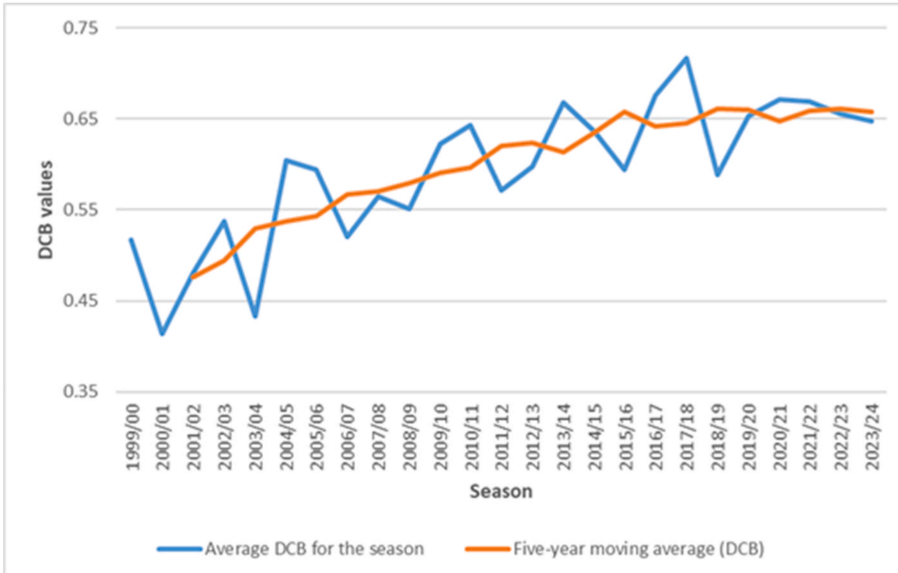

Fig. 6. Seasonal evolution of *DCB* averages and five-year moving averages.

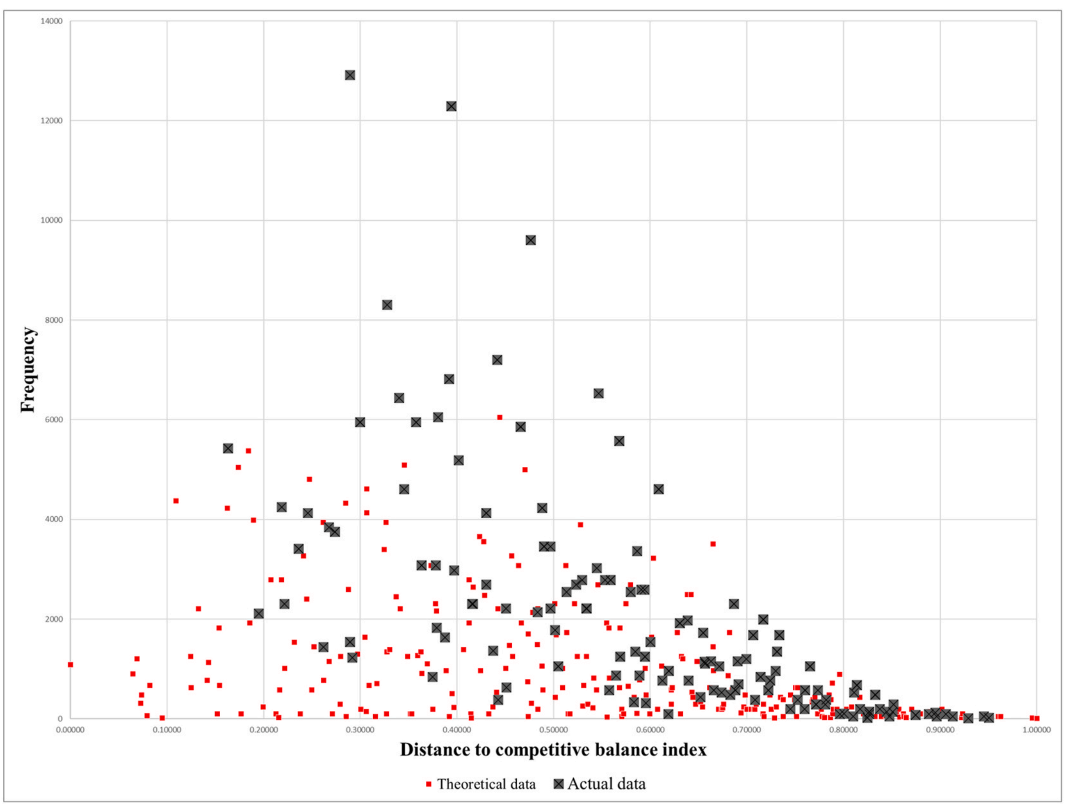

Fig. 7. Comparison of theoretical and actual *DCB* index values in UCL group stage, 1999/2000–2023/2024.

ingly “infinite” spectrum of outcomes along the unit interval of the *DCB* index is actually constrained to just 393 distinct values. This fact is important when making predictions about the range of possible competitive balance values—essentially, all theoretically viable outcomes lie within this discrete set. However, its utility for making predictions about specific outcomes is limited.

Krumer (2021) found that a stronger team (the favorite) regarding UEFA rankings achieves significantly fewer points in the 4th round than in any other round. In addition, the favorite achieved significantly fewer points in rounds 5 and 6 if it did not win in the first leg (rounds 1–3). Thus, it is a strong assumption to consider that the match outcomes within a specific group on a matchday during a season are events that are statistically independent from those preceding or following. However, we can consider, for example on the last matchday (the final two matches involving the four teams in their sixth match), the events do not follow, at least, a previously predetermined course. We do know that out of all the potential outcomes in terms of competitive balance, only 9 are possible: the *DCB* values corresponding to the three potential results (win, loss or tie) of each of the two matches. The example in the Appendix B may help to clarify this point.

6. Conclusions

A wide range of organizations can be analyzed "as if" they were markets, even if they do not strictly conform to a canonical definition of a market. The application of concepts and tools from economics can contribute to a better understanding and explanation of how these organizations operate. Their outcomes arise from actual or potential competition among participants, ultimately determining their relative positions or sizes within the organization.

We are particularly interested in organizations where competition outcomes are determined by an externally assigned scoring or remuneration system (referred to as a scoring system) with a finite number of rewards for participating competitors. These were termed "scoring markets."

As a preliminary step in formulating any theory concerning potential solutions in scoring markets, we specify the precise set of institutional arrangements that govern scoring competition, considering the detailed rules and mechanisms that shape the outcomes. Consequently, this study first provides a novel and formal characterization of what constitutes a scoring market.

One of the first examples that might come to mind in practical terms regarding this concept is sports competitions, which are the focus of our analysis. Within the realm of sports economics, sports teams can be likened to firms competing to satisfy their consumer base. The final league standings serve as a dynamic reflection of how teams are distributed based on their performance. Thus, the competition itself serves as the governing mechanism (i.e., the market institution) to allocate the points. These competitions can also be seen as organizations that prioritize expanding their size (total point score) over maximizing profits, possibly aiming to increase their market share instead of pursuing profit maximization.

Although sports are the most intuitive domain for applying the scoring market framework, the concept also extends to other settings, such as various contests, bidding systems, and parliamentary elections.

A defining feature of scoring markets and sports competitions in particular is that the set of potential outcomes or configurations is finite, bounded, and discrete. This discrete outcome space allows us to compute the probability of a given configuration as the ratio of favorable cases to the total number of possible cases. Because this total is finite, the use of density functions is unnecessary. In a discrete space, we can also count potential configurations, simplifying the application of numerical methods to achieve specific solutions. Furthermore, it allows for the characterization of various phenomena, such as inequality, market power, concentration and their implications in sports economics, particularly in relation to competitive balance.

We demonstrated the value of this approach by formalizing bilateral competitions as scoring markets. Three illustrative examples were presented, each corresponding to a real-world sports format: a single-match final, a league with three teams in a single round-robin format, and a four-team league in a double round-robin format.

The present article also examines the implications of characterizing scoring markets for a central concept in sports economics: competitive balance. The level of competitive balance in sports may offer insights into outcome predictability and competitiveness and its influence on fan engagement and resource access. Understanding competitive balance is relevant for the sports industry's success and shaping opportunities for those involved. In this work, we use the distance to competitive balance (*DCB*) index to measure competitive balance.

In scoring markets, the space of potential theoretical *DCB* values for measuring competitive balance is also discrete, which simplifies collecting them and the process of calculating their probabilities, as well as determining how they are distributed. These properties become evident when we consider a competition configuration that matches the group stage of the UCL. We show the possible theoretical league configurations and, according to them, all potential levels of competitive balance and their respective probabilities that can be achieved by characterizing a competition matching the stage of this championship. The plot of the frequency of each potential value of the *DCB* index indicates that extreme levels of competitive balance (both very high and very low) are less likely, while moderate levels are more common. The distribution of probabilities for each *DCB* level shows positive skewness, with more values deviating from the mean toward higher levels of *DCB* index (lower levels of competitive balance).

Building on this, we address the comparison of our theoretical results with those that actually occurred in the UCL group stage during the 1999/2000–2023/2024 seasons. An initial statistical descriptive analysis of theoretical (i.e., population-based) and actual (i.e., sample-based) *DCB* values reveals right-skewed distributions in both theoretical and actual data and a negative kurtosis indicating a peak around the mean. There is a rising trend toward point concentration in the UCL group stage, with the sample mean approaching 60 % of the theoretical maximum concentration. This actual trend, spanning a quarter century, shows a decline in competitive balance. The plot of the frequencies of actual *DCB* values tends to cluster on the right side of the distribution, consistently achieving higher *DCB* levels (lower competitive balance) within each frequency band.

We should emphasize the relevance of our contribution to the *ex ante* analysis of potential outcomes for the operation and solutions in the design of sports competitions or modifications in the mechanisms underlying existing sports competitions. Regarding the *ex post* analysis, this work enhances our understanding of the dynamics and outcomes observed through its comparison with those theoretically predicted by our scoring markets theory, assessing the positive and normative implications.

As possible extensions of this study, the application of the scoring markets theory to other sports competitions besides football or other domains is worth considering in order to explore new research areas and, if applicable, to compare the conclusions drawn with our theory with those that may have been reached by other authors using different approaches.

Author contributions

Antonio Avila-Cano, Francisco Triguero-Ruiz and José Manuel Ordóñez-de-Haro contributed equally to this work.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2025.107236.

Appendix A. Tables

Table A.1
DCB index values and their probabilities.

Panel A. Total points between 24 and 31

24	25	26	27	28	29	30	31	
							a	b
0.0000 (2E-6%)	0.2157 (5E-5 %)	0.0952 (2E-5 %)	0.0794 (0.01 %)	**0.0000 (0.02 %)**	0.0739 (0.09 %)	0.0825 (0.13 %)	0.0691 (0.23 %)	0.6993 (0.23 %)
		0.2128 (0.02 %)	0.1520 (0.02 %)	0.1250 (0.12 %)	0.1415 (0.14 %)	0.1429 (0.21 %)	0.1324 (0.42 %)	0.7083 (0.04 %)
		0.2855 (9E-5 %)	0.1997 (0.05 %)	0.1767 (0.02 %)	0.1860 (0.36 %)	0.1844 (1.01 %)	0.1740 (0.95 %)	0.7173 (0.05 %)
		0.3156 (9E-5 %)	0.2381 (0.02 %)	0.2165 (0.11 %)	0.2217 (0.19 %)	0.2474 (0.90 %)	0.2074 (0.52 %)	0.7348 (0.08 %)
		0.3924 (9E-5 %)	0.2711 (0.02 %)	0.2500 (0.11 %)	0.2524 (0.27 %)	0.2736 (0.70 %)	0.2361 (0.64 %)	0.7434 (0.04 %)
		0.4148 (2E-5 %)	0.3005 (0.04 %)	0.2795 (0.05 %)	0.2798 (0.23 %)	0.2974 (0.24 %)	0.2617 (0.74 %)	0.7604 (0.05 %)
			0.3272 (0.02 %)	0.3061 (0.03 %)	0.3047 (0.31 %)	0.3401 (1.21 %)	0.2850 (0.81 %)	0.7687 (0.07 %)
			0.3520 (0.02 %)	0.3307 (0.26 %)	0.3277 (0.25 %)	0.3595 (0.24 %)	0.3066 (0.87 %)	0.7770 (0.04 %)
			0.3751 (0.04 %)	0.3535 (0.02 %)	0.3492 (0.23 %)	0.3780 (0.58 %)	0.3267 (0.74 %)	0.7851 (0.09 %)
			0.3968 (0.04 %)	0.3749 (0.16 %)	0.3695 (0.21 %)	**0.4124 (0.28 %)**	0.3456 (0.87 %)	0.7932 (0.04 %)
			0.4371 (0.05 %)	0.3952 (0.09 %)	0.3887 (0.18 %)	0.4286 (0.47 %)	0.3636 (0.58 %)	0.8169 (0.08 %)
			0.4740 (9E-5 %)	0.4145 (0.02 %)	0.4070 (0.26 %)	0.4442 (1.14 %)	0.3807 (1.14 %)	0.8324 (9E-5 %)
			0.5556 (6E-5 %)	0.4329 (0.02 %)	0.4245 (0.18 %)	0.4738 (0.32 %)	0.3971 (0.56 %)	0.8475 (9E-5 %)
			0.5705 (9E-5 %)	0.4506 (0.19 %)	0.4413 (0.10 %)	0.4880 (0.65 %)	0.4128 (0.52 %)	0.8550 (0.02 %)
				0.4840 (0.04 %)	0.4575 (0.23 %)	0.5017 (0.08 %)	0.4280 (0.67 %)	0.8624 (9E-5 %)
				0.5153 (0.02 %)	0.4731 (0.14 %)	0.5281 (0.73 %)	0.4426 (0.42 %)	0.9603 (9E-5 %)
				0.5302 (0.05 %)	0.4883 (0.20 %)	0.5409 (0.04 %)	0.4568 (0.61 %)	
				0.5448 (0.11 %)	0.5030 (0.32 %)	0.5533 (0.52 %)	0.4705 (0.94 %)	
				0.5727 (0.02 %)	0.5172 (0.02 %)	0.5774 (0.12 %)	0.4839 (0.42 %)	
				0.5862 (0.02 %)	0.5311 (0.13 %)	0.5890 (0.15 %)	0.4969 (0.42 %)	
				0.5994 (0.02 %)	0.5447 (0.11 %)	0.6005 (0.29 %)	0.5095 (0.19 %)	
				0.7179 (9E-5 %)	0.5579 (0.15 %)	0.6227 (0.12 %)	0.5219 (0.43 %)	
				0.7287 (2E-5 %)	0.5708 (0.04 %)	0.6335 (0.23 %)	0.5340 (0.42 %)	
					0.5834 (0.08 %)	0.6442 (0.08 %)	0.5458 (0.51 %)	
					0.5957 (0.09 %)	0.6650 (0.27 %)	0.5573 (0.34 %)	
					0.6078 (0.04 %)	0.6751 (0.04 %)	0.5686 (0.23 %)	
					0.6313 (0.02 %)	0.6851 (0.11 %)	0.5797 (0.51 %)	
					0.6540 (0.05 %)	0.7047 (0.04 %)	0.5906 (0.49 %)	
					0.6759 (0.05 %)	**0.7143 (0.02 %)**	0.6013 (0.31 %)	
					0.6971 (0.05 %)	0.7238 (0.09 %)	0.6118 (0.20 %)	
					0.7075 (0.04 %)	0.7423 (0.03 %)	0.6221 (0.18 %)	
					0.7377 (9E-5 %)	0.7514 (9E-5 %)	0.6323 (0.23 %)	
					0.8649 (5E-5 %)	0.7781 (9E-5 %)	0.6423 (0.47 %)	
						0.7868 (5E-5 %)	0.6521 (0.06 %)	
						0.8207 (0.02 %)	0.6618 (0.12 %)	
						0.8289 (0.02 %)	0.6714 (0.20 %)	
						0.8452 (0.02 %)	0.6808 (0.16 %)	
						1.0000 (8E-6 %)	0.6901 (0.13 %)	

Panel B. Total points between 32 and 36

32		33		34		35	36
a	b	a	b	a	b		
0.0000 (0.15 %)	0.6916 (0.13 %)	0.0649 (0.17 %)	0.6569 (0.21 %)	0.0728 (0.06 %)	0.8330 (0.09 %)	0.1541 (0.34 %)	**0.0000 (0.03 %)**
0.1094 (0.82 %)	0.7002 (0.04 %)	0.1243 (0.23 %)	0.6654 (0.18 %)	0.1627 (0.79 %)	0.8393 (9E-5 %)	0.2318 (0.29 %)	0.2916 (0.23 %)
0.1547 (0.13 %)	0.7087 (0.07 %)	0.1634 (1.02 %)	0.6738 (0.10 %)	0.2183 (0.52 %)	0.8518 (0.05 %)	0.2893 (0.29 %)	**0.4124 (0.08 %)**
0.1894 (0.75 %)	0.7171 (0.37 %)	0.1948 (0.40 %)	0.6821 (0.33 %)	0.2414 (0.61 %)	0.8580 (7E-5 %)	0.3372 (0.46 %)	0.5051 (0.20 %)
0.2187 (0.80 %)	0.7254 (0.04 %)	0.2218 (0.43 %)	0.6903 (0.22 %)	0.2624 (0.14 %)	0.8763 (9E-5 %)	0.3791 (0.34 %)	0.5832 (0.06 %)
0.2445 (0.45 %)	0.7336 (0.32 %)	0.2458 (0.78 %)	0.6984 (0.09 %)	0.3001 (1.12 %)	0.8883 (0.02 %)	0.4168 (0.50 %)	0.6521 (0.08 %)
0.2679 (0.22 %)	0.7497 (0.07 %)	0.2677 (0.72 %)	0.7064 (0.32 %)	0.3172 (0.13 %)	0.9002 (0.02 %)	0.4513 (0.12 %)	**0.7143 (0.02 %)**
0.2893 (2.43 %)	0.7576 (0.03 %)	0.2880 (0.49 %)	**0.7143 (0.12 %)**	0.3639 (0.17 %)	0.9060 (0.02 %)	0.4834 (0.28 %)	0.7715 (0.05 %)
0.3093 (0.13 %)	0.7655 (0.20 %)	0.3069 (0.78 %)	0.7221 (0.11 %)	0.3782 (0.43 %)	0.9234 (0.02 %)	0.5135 (0.33 %)	0.8248 (5E-5 %)
0.3281 (1.56 %)	0.7732 (0.02 %)	0.3247 (0.64 %)	0.7299 (0.18 %)	0.3919 (1.28 %)	0.9291 (2E-5 %)	0.5419 (0.15 %)	0.8748 (0.01 %)
0.3458 (0.96 %)	0.7809 (0.05 %)	0.3416 (0.42 %)	0.7375 (0.07 %)	0.4305 (0.78 %)	0.9461 (2E-5 %)	0.5689 (0.34 %)	0.9221 (5E-5 %)
0.3627 (0.25 %)	0.7886 (0.14 %)	0.3576 (1.12 %)	0.7451 (0.09 %)	0.4427 (0.07 %)	0.9952 (2E-5 %)	0.5946 (0.19 %)	
0.3788 (0.41 %)	0.7961 (0.02 %)	0.3730 (0.58 %)	0.7526 (0.12 %)	0.4660 (1.10 %)		0.6194 (0.18 %)	
0.3943 (2.31 %)	0.8036 (0.02 %)	0.3878 (0.31 %)	0.7601 (0.11 %)	0.4772 (0.06 %)		0.6431 (0.10 %)	
0.4235 (0.69 %)	0.8110 (0.10 %)	0.4020 (0.98 %)	0.7674 (0.06 %)	0.4882 (0.79 %)		0.6660 (0.11 %)	
0.4374 (0.26 %)	0.8256 (0.02 %)	0.4158 (0.43 %)	0.7819 (0.07 %)	0.5094 (0.12 %)		0.6882 (0.11 %)	
0.4509 (0.42 %)	0.8400 (0.04 %)	0.4291 (0.51 %)	0.7891 (0.02 %)	0.5298 (0.52 %)		0.7096 (0.05 %)	
0.4639 (0.58 %)	0.8470 (0.02 %)	0.4420 (1.35 %)	0.7962 (0.17 %)	0.5590 (0.52 %)		0.7304 (0.05 %)	

(*continued on next page*)

Table A.1 (*continued*)

Panel B. Total points between 32 and 36							
32		33		34		35	36
a	b	a	b	a	b		
0.4767 (1.81 %)	0.8541 (0.02 %)	0.4546 (0.28 %)	0.8032 (0.04 %)	0.5684 (0.12 %)		0.7507 (0.12 %)	
0.4890 (0.11 %)	0.8951 (0.02 %)	0.4668 (0.36 %)	0.8102 (9E-5 %)	0.5867 (0.63 %)		0.7704 (0.08 %)	
0.5011 (0.43 %)	0.9084 (0.02 %)	0.4786 (0.40 %)	0.8171 (0.04 %)	0.5957 (0.06 %)		0.7896 (0.04 %)	
0.5129 (0.48 %)	0.9279 (0.01 %)	0.4903 (0.65 %)	0.8239 (0.04 %)	0.6218 (0.05 %)		0.8084 (0.05 %)	
0.5244 (0.23 %)	0.9470 (9E-5 %)	0.5016 (0.33 %)	0.8375 (0.04 %)	0.6386 (0.47 %)		0.8267 (0.03 %)	
0.5357 (0.05 %)	0.9533 (5E-5 %)	0.5127 (0.58 %)	0.8442 (0.01 %)	0.6550 (0.33 %)		0.8447 (0.03 %)	
0.5468 (1.23 %)		0.5235 (0.51 %)	0.8508 (0.03 %)	0.6630 (0.22 %)		0.8622 (0.02 %)	
0.5576 (0.11 %)		0.5342 (0.23 %)	0.8574 (0.04 %)	0.6710 (0.04 %)		0.8795 (0.02 %)	
0.5682 (1.05 %)		0.5446 (0.57 %)	0.8704 (0.04 %)	0.6866 (0.43 %)		0.8963 (9E-5 %)	
0.5786 (0.48 %)		0.5548 (0.36 %)	0.8768 (5E-5 %)	0.6942 (0.02 %)		0.9129 (9E-5 %)	
0.5889 (0.16 %)		0.5649 (0.16 %)	0.8896 (0.03 %)	0.7168 (0.02 %)		0.9452 (9E-5 %)	
0.6089 (0.87 %)		0.5747 (0.43 %)	0.9021 (0.04 %)	0.7241 (0.14 %)			
0.6186 (0.02 %)		0.5844 (0.25 %)	0.9267 (9E-5 %)	0.7314 (0.25 %)			
0.6282 (0.33 %)		0.5940 (0.23 %)	0.9507 (5E-5 %)	0.7528 (0.07 %)			
0.6376 (0.37 %)		0.6034 (0.61 %)	0.9624 (9E-5 %)	0.7598 (0.04 %)			
0.6469 (0.05 %)		0.6126 (0.14 %)		0.7736 (0.11 %)			
0.6561 (0.21 %)		0.6217 (0.11 %)		0.7804 (5E-5 %)			
0.6652 (0.66 %)		0.6307 (0.36 %)		0.7872 (0.07 %)			
0.6741 (0.04 %)		0.6395 (0.14 %)		0.8005 (0.02 %)			
0.6829 (0.09 %)		0.6483 (0.22 %)		0.8137 (0.13 %)			

Note: Probabilities are presented in parentheses, distributed according to attainable total points in the competition. Repeated *DCB* values are highlighted in bold.

Table A.2
Actual ***DCB*** values in the UCL during the seasons 1999/2000–2023/2024, with probabilities and occurrence frequencies.

Seasons	Group	*DCB*	Probabilities	Frequencies	Seasons	Group	*DCB*	Probabilities	Frequencies	Seasons	Group	*DCB*	Probabilities	Frequencies
1999/ 2000	**A**	0.56821	0.01048	5568	**2007/ 2008**	**A**	0.30006	0.01120	5952	**2015/ 2016**	**A**	0.82672	0.00027	144
	B	0.71707	0.00375	1992		**B**	0.39428	0.02312	12,288		**B**	0.43054	0.00777	4128
	C	0.37796	0.00578	3072		**C**	0.41579	0.00434	2304		**C**	0.56821	0.01048	5568
	D	0.46599	0.01102	5856		**D**	0.37906	0.00343	1824		**D**	0.44267	0.00072	384
	E	0.56887	0.00343	1824		**E**	0.58673	0.00632	3360		**E**	0.57971	0.00506	2688
	F	0.45087	0.00415	2208		**F**	0.85181	0.00054	288		**F**	0.58321	0.00063	336
	G	0.69926	0.00226	1200		**G**	0.75279	0.00072	384		**G**	0.73138	0.00253	1344
	H	0.28932	0.02430	12,912		**H**	0.83295	0.00090	480		**H**	0.59464	0.00190	1008
2000/ 2001	**A**	0.63860	0.00470	2496	**2008/ 2009**	**A**	0.46599	0.01102	5856	**2016/ 2017**	**A**	0.84754	0.00009	48
	B	0.61935	0.00181	960		**B**	0.23610	0.00641	3408		**B**	0.34562	0.00867	4608
	C	0.45129	0.00117	624		**C**	0.66601	0.00108	576		**C**	0.70868	0.00072	384
	D	0.16342	0.01021	5424		**D**	0.72211	0.00108	576		**D**	0.75980	0.00036	192
	E	0.27355	0.00704	3744		**E**	0.34007	0.01210	6432		**E**	0.49686	0.00415	2208
	F	0.35763	0.01120	5952		**F**	0.76547	0.00199	1056		**F**	0.72211	0.00108	576
	G	0.41675	0.00497	2640		**G**	0.58442	0.00253	1344		**G**	0.74510	0.00090	480
	H	0.39428	0.02312	12,288		**H**	0.62818	0.00325	1728		**H**	0.78192	0.00072	384
2001/ 2002	**A**	0.55759	0.00108	576	**2009/ 2010**	**A**	0.83295	0.00090	480	**2017/ 2018**	**A**	0.77152	0.00054	288
	B	0.43741	0.00257	1368		**B**	0.48819	0.00795	4224		**B**	0.82479	0.00005	24
	C	0.29161	0.00230	1224		**C**	0.49025	0.00650	3456		**C**	0.59060	0.00488	2592
	D	0.30006	0.01120	5952		**D**	0.78093	0.00054	288		**D**	0.72986	0.00181	960
	E	0.39191	0.01283	6816		**E**	0.82672	0.00027	144		**E**	0.55328	0.00524	2784
	F	0.77152	0.00054	288		**F**	0.38071	0.01138	6048		**F**	0.65205	0.00081	432
	G	0.36359	0.00578	3072		**G**	0.60885	0.00867	4608		**G**	0.65688	0.00208	1104
	H	0.73356	0.00316	1680		**H**	0.56821	0.01048	5568		**H**	0.95066	0.00005	24
20002/ 2003	**A**	0.26773	0.00723	3840	**2010/ 2011**	**A**	0.39428	0.02312	12,288	**2018/ 2019**	**A**	0.76005	0.00108	576
	B	0.85079	0.00027	144		**B**	0.46599	0.01102	5856		**B**	0.65611	0.00215	1140
	C	0.37492	0.00158	840		**C**	0.76547	0.00199	1056		**C**	0.38780	0.00307	1632
	D	0.39709	0.00560	2976		**D**	0.69160	0.00131	696		**D**	0.77361	0.00108	576
	E	0.43054	0.00777	4128		**E**	0.59464	0.00190	1008		**E**	0.81017	0.00009	48
	F	0.56887	0.00343	1824		**F**	0.77152	0.00054	288		**F**	0.60045	0.00289	1536
	G	0.59569	0.00059	312		**G**	0.68656	0.00434	2304		**G**	0.28934	0.00289	1536
	H	0.81365	0.00126	672		**H**	0.77152	0.00054	288		**H**	0.43054	0.00777	4128
2003/ 2004	**A**	0.19481	0.00397	2112	**2011/ 2012**	**A**	0.73138	0.00253	1344	**2019/ 2020**	**A**	0.89509	0.00023	120
	B	0.16342	0.01021	5424		**B**	0.23610	0.00641	3408		**B**	0.84466	0.00027	144
	C	0.54458	0.00569	3024		**C**	0.73356	0.00316	1680		**C**	0.54676	0.01228	6528
	D	0.49025	0.00650	3456		**D**	0.92913	0.00002	12		**D**	0.68815	0.00108	576
	E	0.68815	0.00108	576		**E**	0.47666	0.01806	9600		**E**	0.71429	0.00159	844
	F	0.71707	0.00375	1992		**F**	0.39191	0.01283	6816		**F**	0.65688	0.00208	1104
	G	0.44200	0.01355	7200		**G**	0.26170	0.00741	3936		**G**	0.24584	0.00777	4128
	H	0.22180	0.00434	2304		**H**	0.81098	0.00099	528		**H**	0.63068	0.00361	1920
2004/ 2005	**A**	0.55900	0.00524	2784	**2012/ 2013**	**A**	0.83295	0.00090	480	**2020/ 2021**	**A**	0.79610	0.00018	96
	B	0.63068	0.00361	1920		**B**	0.56485	0.00163	864		**B**	0.21870	0.00799	4248
	C	0.72410	0.00145	768		**C**	0.39428	0.02312	12,288		**C**	0.82672	0.00027	144
	D	0.66301	0.00217	1152		**D**	0.71707	0.00375	1992		**D**	0.63068	0.00361	1920

(*continued on next page*)

Table A.2 (*continued*)

Seasons	Group	*DCB*	Probabilities	Frequencies	Seasons	Group	*DCB*	Probabilities	Frequencies	Seasons	Group	*DCB*	Probabilities	Frequencies
	E	0.53395	0.00415	2208		**E**	0.63954	0.00145	768		**E**	0.81708	0.00036	192
	F	0.46599	0.01102	5856		**F**	0.61935	0.00181	960		**F**	0.68290	0.00090	480
	G	0.81365	0.00126	672		**G**	0.52981	0.00524	2784		**G**	0.89633	0.00009	48
	H	0.44200	0.01355	7200		**H**	0.48338	0.00280	1488		**H**	0.50508	0.00199	1056
2005/2006	**A**	0.82672	0.00027	144	**2013/2014**	**A**	0.70637	0.00316	1680	**2021/2022**	**A**	0.46599	0.01102	5856
	B	0.83747	0.00036	192		**B**	0.69027	0.00217	1152		**B**	0.81365	0.00126	672
	C	0.65498	0.00325	1728		**C**	0.65498	0.00325	1728		**C**	0.87482	0.00014	72
	D	0.27355	0.00704	3744		**D**	0.82479	0.00005	24		**D**	0.68656	0.00434	2304
	E	0.40204	0.00975	5184		**E**	0.50159	0.00334	1776		**E**	0.88834	0.00018	96
	F	0.72410	0.00145	768		**F**	0.71429	0.00159	844		**F**	0.39428	0.02312	12,288
	G	0.53415	0.00235	1248		**G**	0.71707	0.00375	1992		**G**	0.39428	0.02312	12,288
	H	0.49686	0.00415	2208		**H**	0.53415	0.00235	1248		**H**	0.83295	0.00090	480
2006/2007	**A**	0.73138	0.00253	1344	**2014/2015**	**A**	0.51346	0.00325	1728	**2022/2023**	**A**	0.87482	0.00014	72
	B	0.47666	0.01806	9600		**B**	0.81365	0.00126	672		**B**	0.49025	0.00650	3456
	C	0.48819	0.00795	4224		**C**	0.35763	0.01120	5952		**C**	0.91291	0.00009	48
	D	0.58888	0.00163	864		**D**	0.76005	0.00108	576		**D**	0.30006	0.01120	5952
	E	0.73356	0.00316	1680		**E**	0.61260	0.00145	768		**E**	0.52352	0.00506	2688
	F	0.28934	0.00289	1536		**F**	0.83295	0.00090	480		**F**	0.67378	0.00099	528
	G	0.49025	0.00650	3456		**G**	0.60885	0.00867	4608		**G**	0.67137	0.00199	1056
	H	0.36359	0.00578	3072		**H**	0.59396	0.00235	1248		**H**	0.80053	0.00020	108
										2023/2024	**A**	0.70637	0.00316	1680
											B	0.60885	0.00867	4608
											C	0.90604	0.00018	96
											D	0.61859	0.00018	96
											E	0.55900	0.00524	2784
											F	0.32806	0.01563	8304
											G	0.94519	0.00009	48
											H	0.50508	0.00199	1056

Data availability

Data and codes are available: Avila-Cano, Antonio; Triguero-Ruiz, Francisco; Ordoñez-de-Haro, Jose Manuel (2025), "Scoring Markets", Mendeley Data, V1, doi: 10.17632/w7v9k9s7cp.1

Avila-Cano, Antonio; Triguero-Ruiz, Francisco; Ordoñez-de-Haro, Jose Manuel (2025), "Scoring Markets", Mendeley Data, V1, doi: 10.17632/w7v9k9s7cp.1 (Original data) (Mendeley Data)

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