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Next Reaction Method for Solving Dynamic Macroeconomic Models: A Growth Regressions Simulation

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Recent studies apply the Monte Carlo method to try to solve multiple data problems for dynamic macroeconomic models such as measurement errors, residue correlation, and omitted variables. This paper evaluates the estimate of economic growth regressions from the Solow model by applying the Next Reaction Method, similar to the Monte Carlo kinetic methods. Our results indicate that with the said algorithm the estimation of these models improves since they increase the levels of precision of the existing models simulated with Monte Carlo, achieving faster the convergence of the coefficients of the variables, reduces the possible measurement errors and the level of deviations. These results can be very useful in their application in dynamic macroeconomic models, which help the estimation challenges of policymakers and other related stakeholders.

Keywords: Monte Carlo algorithms, Next Reaction Method, Solow model, Growth regressions, Macroeconomic models

Introduction

Recently it has increased the importance of applying new computational methods for the estimation of macroeconomic models, mainly in economic growth models. These models are important, among other aspects, in the design of successful public policy for the economic development of countries, sustainable in the long run[1]. These models are being evaluated by the result obtained in the standard deviation of the regressors. The evaluation of the models built in the previous literature has yielded different adjustments. For example, models built with Monte Carlo have provided an adjustment of 0.12–0.47 of the square root of the mean error[1-3].In order to solve the problems of precision of the Monte Carlo method in estimating regressions of economic growth, this paper develops the Next Reaction Method algorithm, based on the Gillespie stochastic algorithm, similar to the Monte Carlo kinetic methods, which has already demonstrated in other scientific areas its methodological superiority in the uncertainty of estimation on aspects of measurement error, residual correlation and omitted variables[1,2], something required in economic growth works that suffers from studies with powerful computational simulations[4]. Our results show a more robust estimate in terms of accuracy and better behavior in the face of measurement problems[2]. These results can be very useful in their application in dynamic macroeconomic models, which help the estimation challenges of policymakers and other stakeholders.

Method

In this work, we have built a DSGE model using the Next Reaction Method, based on the stochastic simulation algorithms of Direct and First Reaction, which will be detailed in the Methods section. A sample of data has been prepared from which the variables representing the model have been extracted. This growth regression model used is the popular Solow model[1], to which the Next Reaction Method algorithm is applied for its resolution, obtaining the optimal values of the parameters of the equations that represent the model. Finally, we apply the root mean square error (RMSE) function to analyze the error level of our algorithm with the previous literature.

Stochastic Simulation Algorithm (SSA)

Assume the system imply N variables $\{S_1 ,...,S_N\}$, defined by the state vector $X(t) = [X1(t),...,XN(t)]$, where $X_i(t)$ is the number of variables S_i at time t. M reaction channels $\{R1,...,RM\}$ are involved in the system[5]. The dynamics of reaction channel R_j is represented by the propensity function a_j and by the state change vector $vj = (v_{1j} ,...,v_{Nj})$:$a_j(x)d_t$ shows the likelihood that given $X(t) = x$, one R_j reaction will

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happen in the next infinitesimal time interval [t,t+dt), and $v_{ij}$is the product of the change in the population of S_i induced by one R_j reaction.

The dynamics of the system is represented by the Eq. (1)

$$\frac{\partial P(x,t|x_0,t_0)}{\partial t} = \sum_{j=1}^{M}\left[\left(a_j(x-v_j)P(x-v_j,t|x_0,t_0) - a_j(x)P(x,t|x_0,t_0)\right)\right] \quad \ldots (1)$$

where the function $P(x,t|x_0,t_0)$ denotes the likelihood that $X(t)$ will be x, given that $X(t_0) = x_0$. The SSA is a popular stochastic simulation technique that is equivalent to Eq. (1). Starting from the initial states, the SSA simulates the trajectory trying to respond to the inquiries as to when time τ will be the next reaction, and which reaction channel indexμwill be the next[6,7]. The distributions of τ and μ are developed to respond to the two questions.

The time τ, given $X(t)=x$, that the reaction will be at $t+\tau$, is the exponentially distributed random variable with mean $[1/a_0(x)]$, $p(\tau=s) = a_0(x)\exp[-a_0(x)s]$, and the index μ of that firing reaction is the random variable with likelihood[5,8]

$$P(\mu = j)\frac{a_j(x)}{a_0(x)} \quad \ldots (2)$$

In every step, the SSA produces random numbers and estimates$\tau$and μ according to the probability distributions (2) and (3).

In every step, the direct method generates two random numbers r_1 and r_2 in $U(0,1)$. The time for the next reaction to happen is given by $t+\tau$[5,9]. The index μ of the happening reaction is determined by the smallest integer. The system states are upgraded by $X(t+\tau) = X(t)+v_\mu$. Then the simulation proceeds to the next occurring time[5,9,10].

The first reaction method produces a τ_k for each reaction channel R_k defined as follows

$$\tau_k = \frac{1}{a_k(x)}\ln\left(\frac{1}{r_k}\right)(k = 1,\ldots,M) \quad \ldots (3)$$

where $r_1, \ldots, r_M$ are M statistically independent samplings of $U(0,1)$. Then τ and μ are selected as $\tau=min\{\tau_1,\ldots,\tau_M\}$ and μ = the index of $min\{\tau_1,\ldots,\tau_M\}$.

The direct and the first reaction methods are equivalent to each other although they look different[5,8,10]. The random pairs (τ, μ) generated by both methods follow the same distribution. The first reaction method eliminates$M-1$ unused reaction times. The reasoning for the advantage of the next reaction method over the direct method is based mainly on two observations: First, in each step, the next reaction method generates only one random number while the direct method needs two. Second, the search for the index μ of the next reaction channel takes $O(M)$ time for the direct method, while the cost produced for the next reaction method is on the upgrade of the indexed priority queue which is $O[ln(M)]$[5].

Next reaction method

Cao, Li and Petzold[5] show the transformation of the first reaction method into an equivalent but a more efficient new algorithm. The next reaction method is considerably faster than the first reaction method. It is widely known to be more efficient than the direct method when the system involves many variables and freely coupled reaction channels. The next reaction method can be considered as an extension of the first reaction method in which the $M-1$ unused reaction times are suitably changed for reuse.

Specifications of growth regression

Ditzen and Gundlach[2] use a dynamic panel specification in discrete-time from the Solow model in continuous time to perform the simulation. This specification is explained by equation (4):

$$\begin{aligned}&\ln y_{i,t} = c + b_1 \ln y_{i,t-\tau} + b_2 \ln s^k_{i,t-\tau} + b_3 ln\, s^h_{i,t-\tau} + \\ &b_4 \ln(n_{i,t-\tau} + g + \delta) + \mu_i + v_{i,t} ln_{i,t} = c + \\ &b_1 \ln y_{i,t-\tau} + b_2 \ln s^k_{i,t-\tau} + b_3 ln\, s^h_{i,t-\tau} + \\ &b_4 \ln(n_{i,t-\tau} + g + \delta) + \mu_i + v_{i,t} ln_{i,t} = c + \\ &b_1 lny_{i,t-\tau} + b_2 lns^k_{i,t-\tau} + b_3 ln\, s^h_{i,t-\tau} + b_4 \ln(n_{i,t-\tau} + \\ &g + \delta) + \mu_i + v_{i,t} ln_{i,t} = c + b_1 lny_{i,t-\tau} + \\ &b_2 lns^k_{i,t-\tau} + b_3 ln\, s^h_{i,t-\tau} + b_4 ln_{(n_{i,t-\tau}+g+\delta)} + \mu_i + v_{i,t}\end{aligned} \quad \ldots (4)$$

where all variables are cross-sectionally downgraded to eliminate time fixed effects, c is a regression constant, $y_{i,t}$ is per capita income of country i at time t, τ is the selected time interval, $s^k_{i,t-\tau}$ and $s^h_{i,t-\tau}$ are the investment rates in physical and human capital, $n_{i,t-\tau}$is the rate of population growth, g is the rate of technical change, δ is the depreciation rate, μ_i is a country fixed effect, and $v_{i,t}$ is a zero-mean error term that may be correlated with the regressors[1,2]. The explanatory variables s^k, s^h, and n are measured as τ-year averages, and it is assumed that $g + \delta = 0.07$.

All parameters b_j (j = 1,…, 4) of equation (13) have well-defined interpretations that can be derived from the Solow model[1,2]. For example, the true parameter

b_1 in the Eq. (1) is defined as follows: $b_1 = e^{-\lambda t} = e^{-[(1-\alpha-\beta)(n_{i,t-\tau}+g+\delta)]\tau}$, where $\lambda = (1 - \alpha - \beta)(n_{i,t-\tau} + g + \delta)$ is the rate of convergence to the steady-state. The rate of convergence can be derived from a first-order Taylor approximation of the steady-state production function with α and β as the capital coefficients, such as the production elasticities of physical and human capital. Applying a steady-state parameterization of $n = 0.01$, $g = 0.02$, $\delta = 0.05$, and supposing that $\alpha = \beta = 0.27$, which is rigorously linked with standard parameterizations of factor shares[2], derive a true value of $b_1 = 0.832$ for $\lambda = 0.0368$ and $\tau = 5$.

Results and conclusions

Table 1 shows our results for the averages of the BE estimates of the four coefficients of equation (13) over 1000 replications, dependent on varying the size of the coefficient b_1 of the lagged endogenous variable. The first row in Table 1 provides our simulation results for an initial parameterization of Eq. (1) with $b_1 = 0.832$, $b_2 = b_3 = 0.099$, and $b_4 = -0.197$, following the work of Ditzen and Gundlach[2]. Our estimates of $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ and $\hat{b}_4$ are trying to replicate the Ditzen and Gundlach' scheme[2]. In the same way, Table 2 shows the estimates of the capital coefficients α and β of the production function, and the coefficients of the two investment variables s^k and s^h. Unbiased estimates of the individual coefficients $\hat{b}_j$ (j = 1, . . . , 4) would suggest capital coefficients in the range of $\hat{\alpha} = \hat{\beta} = 0.27$. The comparison of the

Table 1 — Variation of b_j (j = 1, . . . , 4) with alternative τ

τ	True	Estimated			True	Estimated		
	*b*1	SM	ESE	RMSE	*b*2	SM	ESE	RMSE
	λ	*λ*	ASE	Bias			ASE	Bias
5	0.832	0.998	0.012	0.126	0.099	0.09	0.021	0.018
	0.037	0.001	0.012	13.3			0.02	-4.2
10	0.692	0.924	0.024	0.152	0.181	0.179	0.029	0.031
	0.037	0.011	0.025	21.8			0.029	-0.4
20	0.479	0.697	0.041	0.157	0.306	0.323	0.047	0.048
	0.037	0.013	0.038	32.5			0.048	3.1
40	0.229	0.386	0.052	0.103	0.452	0.512	0.072	0.092
	0.037	0.024	0.053	40.4			0.072	8.4
τ	True	Estimated			True	Estimated		
	b3	SM	ESE	RMSE	b4	SM	ESE	RMSE
			ASE	Bias			ASE	Bias
5	0.099	0.109	0.019	0.017	-0.197	-0.118	0.142	0.142
			0.020	7			0.138	-31.5
10	0.181	0.203	0.027	0.033	-0.361	-0.323	0.221	0.185
			0.026	8.2			0.223	-6.8
20	0.306	0.331	0.038	0.042	-0.612	-0.659	0.384	0.379
			0.038	5.4			0.378	4.7
40	0.452	0.485	0.051	0.059	-0.905	-1.357	0.514	0.664
			0.050	3.9			0.511	42.6

SM is the sample mean of the estimated coefficients over 1,000 replications. ESE is the empirical standard error, i.e., the standard deviation of the estimated coefficients. RMSE is the root-mean-squared error of the estimated coefficients. ASE is the average of the standard errors of the estimated coefficients. Bias is the deviation of the sample mean from the true coefficient in percent (in italics). λ is the convergence rate implied by the estimated b^1. Avg. abs. bias gives the arithmetic average of the absolute biases of the sample means in percent (in italics).

Table 2 — Long-run coefficients and capital coefficients for alternative values of τ

τ	$\hat{b}_2/(1-\hat{b}_1)$	$\hat{\alpha}$ *bias*	$\hat{b}_3/(1-\hat{b}_1)$	$\hat{\beta}$ *bias*	Average absolute bias
5	21.043	0.383	25.386	0.462	72.3
		54.8		87.9	
10	1.315	0.271	1.475	0.336	28.4
		26.5		38.6	
20	0.924	0.268	0.873	0.274	14.8
		16.7		18.7	
40	0.739	0.245	0.688	0.229	7.2
		11.2		5.2	

empirical (ESE) and the average (ASE) standard error suggests that the simulation results can be considered as robust. The comparisons of the ESEs and the individual root-mean-squared errors (RMSE), both Table 1 and Table 2, conclude that our estimation obtains smaller deviations of the estimate by Ditzen and Gundlach[2].

Therefore, the results of the average errors estimated from the Next Reaction Method are much smaller than those obtained by Monte Carlo in the previous works[1,2].

This study presents an alternative simulation technique to estimate dynamic macroeconomic models. We demonstrate with the example of the economic growth regressions, when it adapts properly to this type of model, the Next Reaction technique is more efficient than other algorithms commonly used as Monte Carlo. Our results show high robustness of the Next Reaction algorithm, improving the convergence ratios of the coefficients, showing a smaller level of errors and biases, a concern shown by the previous literature. Finally, the great precision shown by this new algorithm also means an improvement in the optimization of the calculation of economic projections without using a large number of available resources, nor having to make a broad specification of the dynamic models to be used, being of vital importance for public institutions and other interest groups in macroeconomic analysis.

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