Structure and stability of rotor generated vortices

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We study the structure and stability of the vortical wake generated by a two-blade Joukowski rotor of radius R_b, rotating at angular velocity Ω in an axial wind of velocity V_∞. In this rotor model, each blade is assumed to emit a free tip vortex of circulation Γ and a hub vortex of opposite circulation centered on the rotor axis. Considering a fixed vortex core size a and neglecting viscosity, the vortex dynamics is computed by a free vortex method[1] using the Biot-Savart law.

The problem is defined by three non-dimensionalized parameters: the tip speed ratio $\lambda = R_b\Omega/V_\infty$, the vortex strength $\eta = \Gamma/\Omega R_b^2$ and the vortex thickness $\varepsilon = a/R_b$. For a small and fixed vortex thickness $\varepsilon = 0.01$, we first show that stationary solutions (in the rotating frame) can be obtained for almost all values of λ and η except in a small parameter region indicated in gray in figure 1(a). In this figure are indicated the different flight regimes of a helicopter associated with each solution. Of particular interest is the so-called Vortex Ring State (VRS) occurring during rapid descent that cannot be described by the general momentum theory[2]. In this regime, vortices are present on both sides of the rotor plane as illustrated in figure 1(b).

The stability of the various solutions is also addressed by analyzing the linear response to a Dirac perturbation applied at the rotor tip. For most flight regimes, the flow is found to be convectively unstable: the perturbation grows but is advected away from the rotor plane. The property of the wave packet far away has been compared to the theoretical predictions for uniform helices[3] and a very good agreement has been observed [see figure 1(c)]. In the VRS regime, a different behavior is observed: the perturbation continues to grow in the neighborhood of the rotor and a well-defined global mode emerges everywhere. The rotor wake has become globally unstable.

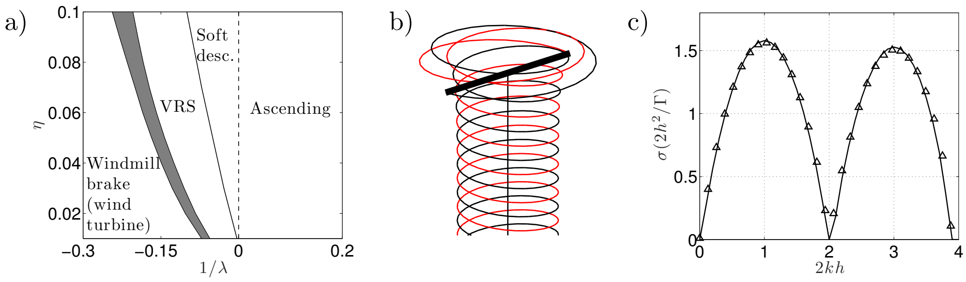

Figure 1: (a) Helicopter flight regimes in the $(1/\lambda, \eta)$ plane for a two-blade rotor for $\varepsilon = 0.01$. (b) Vortex-Ring-State structure for $\lambda = -15$, $\eta = 0.04$ and $\varepsilon = 0.01$. (c) Growth rate versus wavenumber normalized by the far-wake helix pitch h for $\lambda = \infty$, $\eta = 0.02$ and $\varepsilon = 0.01$. Symbols: numerics. Line: Gupta & Loewy theory.

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[1] Leishman, *Principles of Helicopter Aerodynamics*, Cambridge University Press (2006).

[2] Sørensen, *General momentum theory for horizontal axis wind turbines*, Springer (2016).

[3] Gupta and Loewy, *AIAA journal* **12**(10),1381 (1974).