Essays in Political Economy: Ambiguous parties and Conformist voters

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Chapter 1

General Introduction

The aims of this chapter are to describe the motivation, the goals and the findings pursuing this thesis. Two key components of the democracy have been analyzed: political parties with ambiguous features and voting behavior when agents have preferences affected by the influence of other agents when voting. Section 1.1 provides the motivation, some background information regarding the topic, and the main contributions for each topic. Section 1.2 presents each of the goals that are developed individually in Chapters 2 to 6. Finally, Section 1.3 presents a brief summary of the methodology used in each of the chapters.

1.1 Research Scope

This section introduces the scope of this thesis, which develops different scenarios individually regarding political parties and voters' behavior.

1.1.1 Political Parties

One of the main goals of this thesis is to demonstrate the strategies of two political parties that compete in order to win the elections, given that one party is viewed as traditional whilst the other party is perceived by the voters as being ambiguous. The traditional party is characterized by offering a unique platform and only being interested in winning the elections. Therefore, the strategy of this party is to find the location in which the party maximizes its probability of winning the elections. The ambiguous party also wants to win the elections, however, its political platform consists of several different policies. The motivation behind this study consists of the recent incorporation of new political parties within the political arena, that try to be perceived by voters as non-ideological and non-traditional, in order to attract the highest number of voters possible.

The existing literature already demonstrates what are the consequences of a two-party political competition when two traditional parties compete with the same intention of winning the elections. In this case, the Downsian equilibrium illustrates both parties locate at the median voter position (Downs, 1957), that is to say, the position at the middle of the policy space. On this basis, this thesis studies the case in which a traditional party and an ambiguous party compete to win the elections and studies the consequences in terms of political strategies taken from both parties. A political party is perceived as being ambiguous if voters cannot relate a specific policy to that party. This implies that the political party is evaluated by voters as a lottery, in which a positive probability is given to each policy announced by the ambiguous party.

In 1957, Anthony Downs states in his book "An Economic Theory of Democracy", that a political party can be interested in being ambiguous in order to win the elections. According to Downs (1957), *"Ambiguity thus increases the number of voters to whom a party may appeal. This fact encourages parties in a two party system to be as equivocal as possible about their stands on each controversial issue. And since both parties find it rational to be ambiguous, neither is forced by the other's clarity to take a more precise stand."*

This thesis analyzes two particular scenarios individually, in which one of the political parties represents an ambiguous party and the other party acts as a traditional one. The first scenario studies the case of an assembly party. An assembly party takes decisions according to what the affiliate members vote in an assembly prior to an election, and they can be represented by means of a lottery. This scenario is developed in Chapter 2.

The second scenario considers a political party composed of various internal factions: an ideological and an opportunist. This model follows the arguments of Roemer (1999) which characterizes political parties by obtaining several internal factions that take part in the design of the final policy implemented by the political party. The ideological faction is in charge of keeping the ideological principles of the founders of the party. This faction can also be interpreted as the "party label" or "party brand", or as the mean policy of the different political platforms that the party has implemented throughout its history. The opportunistic faction is in charge of maximizing the number of votes achieved by the party, that is to say, it proposes a platform that allows the party to win the elections. Given that both factions have the same weight within the party, the final political platform is estimated by voters as a lottery. This scenario is developed in Chapter 3.

The final issue is why political parties decide to be ambiguous, not as a political strategy, but due to their motivation to represent the highest number of voters possible. In this way, the theoretical setting is characterized by two parties with a two-fold objective: representing the electorate and winning the elections. This scenario is fully developed in Chapter 4.

1.1.2 Voters

The second goal of this thesis is to study a particular voting scenario in which agents not only consider their favorite option when voting, they also consider the favorite option of the other agents. In real life situations, it is common to find agents that, once they have to vote for an alternative, they misrepresent their opinions and vote for the alternative preferred by the group. The tendency of agents to adapt their votes to those of other agents is known in social psychology as *conformity.*

Conformity has been studied in social psychology since the 1950s. In 1951, social scientist Solomon Asch found the first evidence of this social attitude in a laboratory experiment. Later, the study of Deutsch and Gerard (1955) identified two types of social influences: informational and normative. Informational influence refers to updating an opinion taking into account the previous opinions of others, whereas normative influence describes the behavior of stating an opinion that fits with the group choice.

In a voting procedure, this behavior can determine that the chosen decision is not the one that represents the real preferences of the agents. Besides, the fact that they impose themselves the need to vote for the same option as some other group members, makes them vote for a option different from the preferred one. This can represent a serious problem in scenarios such as trials, boards of directors, quality committees, courts, among other committees in charge of taking decisions.

This thesis presents a model in which the discussion stage is not necessary since agents have their preferences well defined regarding two possible options: to remain with the current option or to change it. First, agents want their opinions to succeed. If their votes have no effect on the decision, conformity arises and they want their messages to coincide with the messages of other agents. In Dutta and Sen (2012), agents' preferences are also lexicographic, but in the sense that agents have a preference for honesty when announcing the true state does not change their welfare. Dutta and Sen (2012) call these agents "partially honest." However, agents affected by conformity are not necessarily partially honest. A conformist agent strictly prefers to conform to the opinion of certain reference group when her message does not have any influence on the decision, regardless of whether her message corresponds with her true opinion.

Thus, this thesis highlights the consequences of having agents with such preferences over the adopted decision, in an environment in which all agents vote simultaneously. Then, it proposes a mechanism that guarantees the outcome obtained when all agents vote truthfully. This case is studied in general, for any number of agents, for any threshold considered to accept the proposal and considering that agents want to coincide with any number of agents within the group. Finally, the same scenario is analyzed under a sequential setting. This model is fully developed in Chapter 5.

The fact that this procedure is binary – there are only two options, and there is complete information – all agents know the options preferred by the other agents, facilitates the use of a laboratory experiment to check whether theoretical results can be replicated empirically. The final contribution of this thesis is to induce conformity in the laboratory for groups of 5 agents for a particular preference profile in which, 3 agents prefer one option and the other 2 agents prefer the other. A theoretical and an experimental analysis is conducted in Chapter 6 showing these findings.

1.2 Research goals

The main objective of this thesis is the use of game theoretic tools in order to analyze the strategic behavior of two fundamental parts of democracy: political parties and voters. There are five specific goals which corresponds to each of the studies developed in this thesis:

1. Model of political competition where an assembly political party and a traditional party compete to win the elections. First, we present the political strategies that each of the parties consider in order to win the elections. We also demonstrate how the existence of an assembly party determines the political strategy that the traditional party can take.

2. Model of political competition between a traditional party and a party with two internal factions: an ideological and an opportunistic faction. We study the political strategies taken by both parties when there is uncertainty about the party which has valence advantage, and about the degree of such advantage when the two-factions party has a first mover advantage.

3. Model of political competition between two non-ideological political parties which are characterized by proposing ambiguous strategies. We study the political strategies that each party carries out in order to achieve the parties' two main goals: representing the electorate and winning the elections.

4. Binary-voting game in which agents not only consider the final decision when voting, they also consider the vote of the other agents. First, we identify the consequences of the conformity phenomenon over the voting behavior of the agents. Next, we propose a mechanism that guarantees that the decision obtained always coincides with the one obtained when all agents vote truthfully.

5. Finding empirical support for the theoretical predictions obtained in Objective 4 by means of a laboratory experiment applied to a particular situation with 5 group members and a particular preference profile.

Each of the goals previously mentioned are studied individually in chapters 2 to 6.

1.3 Methodology

In this section, the main characteristics of the methodology used in Chapters 2 to 6 are presented. They are divided into two parts: one part is dedicated to the methodology used in the three chapters that analyze political competition cases. The other presents the methodology used in the two chapters that analyze conformity in voting. Further details regarding the methodology are described throughout each chapter.

1.3.1 Political Parties

Two parties are considered. The strategies of each party are determined by the parties' utility function and by the benefits received in each of the strategies, meaning by strategies the locations each political party offers to the electorate within a unidimensional political space. Voters' utility depends on a valence characteristic β, associated to each of the parties, and the euclidean distance between the ideal policy of the voter and the final location of the political party. Therefore, voters support that party that derives a greater utility. In the case that voters are indifferent between both parties, they will abstain from voting. Each of the scenarios analyzed are characterized by a different timing that establishes which of the parties moves first in the case of a sequential competition game (Chapter 3) or, whether there are stages prior to the elections are held (Chapters 2 and 4). The equilibrium concept used in the abovementioned scenarios are those of game theory. In particular, we take the equilibrium concept of Nash equilibrium under weakly undominated strategies.

1.3.2 Voters

This part of the thesis concentrates on observing the consequences of voters who are affected by the conformity phenomenon upon voting. A group of n agents are considered (Chapter 5). They have to vote between accepting or rejecting a proposal taking their opinions of each option into consideration. Various thresholds are used to accept the proposal. Again, the equilibrium concept and the reasoning chosen to solve this game are those developed in game theory. In order to obtain the voters' strategies in equilibrium, the equilibrium concept used is of Nash equilibrium in weakly undominated strategies. The agents' best responses are studied for each of the thresholds used to accept the proposal, from where the results will determine whether the outcome is influenced by conformity or not. For the experimental scenario (Chapter 6), only groups of 5 agents are considered. First, the theoretical model is studied and then the experiment is conducted inducing conformity in the behavior of the agents. In the experiment, three scenarios are analyzed: when agents' payoffs only depend on the outcome, when agents' payoffs are induced by conformity, and when there is information about the vote of two agents prior to agents vote. Marginal effects are estimated using logit regressions.

Chapter 2

Downsian Competition with Assembly Democracy

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2.1 Introduction

In the last decade, small groups of citizens all over Europe and in the U.S. have spread their protests in demand for more civil participation in the process of policy decision making. In Spain, the so-called 15-M inspired in the Arab Spring and in the U.S. the Occupy Wall Street (OWS), are examples of social movements that are protesting against the current democratic systems. On the one hand, internet networks have facilitated the coordination in the action of these groups that have become stronger. On the other hand, the size of these groups does not seem to threaten, up to now, the stability of the current political systems neither in the U.S. nor in the European continent. While the media has widely covered the protests of these groups, politicians and the members of traditional political parties do not have attended these demands so far.

The social movements mentioned above do not agree with the power that political parties have acquired in representative democratic systems. They defend either independent candidates which are not tied by party discipline, or more direct participation of the citizens in the process of policy decision making.

In addition, there is a recent phenomenon in current western democracies by which the autonomy of states has reduced due to the development of supranational political institutions such as United Nations, European Union, IMF, NATO, among others (Held, 1991; Dahl 1994). Many countries in Europe and in the American continent have reduced their decision-making power whereas supranational institutions have increased their competencies. As a consequence, citizens find that the process of policy decision-making is increasingly moving out of their scope. This has generated an extra discontent over the traditional parties which have shown no clear opposition against the process of delegating state power. The pressure of the civil society to recover the state autonomy has become more intense (this is the case of many protests in European countries such as Greece, Italy, Portugal, Belgium, Spain and others across the Atlantic, Canada and U.S.).[1]

As a response to these protests, there is a number of new political parties in many European countries which incorporate, in their policy platforms, the proposals of these social movements. A key aspect of these parties' manifesto is the promotion of new forms of participatory democracy. The impact of these new parties will have to be tested in the ballot boxes. So far, however, they have shown to be quite successful. This was evidenced in the last 2014 European Elections in which political parties such as "Movimento 5 Stelle" obtained 17 seats out of 73 in Italy, and "Podemos", a three-month-old party in Spain, gained five seats in the European Parliament, being the fourth-largest representation for Spain.[2]

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[1] See http://www.cbsnews.com/2718-201_162-1290/occupy-wall-street-protests/ for a media coverage of these protests.

[2] http://www.nytimes.com/2014/05/29/world/europe/spanish-upstart-party-said-it-could-and-did-now-the-hard-part-begins.html?_r=0

In this paper, we propose a stylized model which tries to deduce the effects derived from the political competition between traditional parties and new parties which promote participatory democracy. Whereas Matsusaka (2005) suggests that assembly democracies has dwindled in importance, we find that, in the last decade, the media has taken the protests of social movements to the front page and voters are showing an increasing and non-negligible interest for alternative forms of democracy among which assembly democracy is one of them.

According to representative democracy, citizens vote to elect their representatives on whom they delegate political decisions. Representative democracy is the most widespread form of democracy. The essence of representative democracy is the competition among candidates which, in most cases, are affiliated to different political parties. Either a plurality system or a proportional system can lead to one or more representatives holding the ultimate power of policy decision making. In every legislature, citizens elect their representatives with their ballot and political accountability is guaranteed by the representatives' incentives to be reelected. Representative democracy is viewed as one of the most effective mechanisms to achieve political stability. This political stability, however, can be threatened when citizens perceive that the interests of the representatives are moving in opposite directions to their own interests (Kalt and Zupan 1984; Peltzman, 1984). As claimed by Budge (2001a, 2001b): "*Representative democracies are deficient in many respects, all of which fundamentally stem from the limited role they allow citizens in government. Most decisions are imposed on those affected without consulting them*".[3]

Assembly democracy is a form of direct democracy in which citizens in an assembly directly vote on initiatives. This type of democracy, that can be traced back to the Greek city of Athens, has scarcely been put into practice in our days. The most well-known experience is in Switzerland, in which popular assemblies in each of the cantons approve citizens' initiatives by popular vote. Assembly democracy is not exclusive of Switzerland, but also the towns of the states of New England in the U.S., are governed

[3] See also Buchanan and Tullock (1962).

by periodic meetings that discuss and vote their main issues (the term town corresponds to municipalities in other places).[4] There is no "pure" form of direct democracy as in both, Switzerland and New England, popular assemblies coexist with representative democracy at higher levels of government. Opponents to direct democracy claim that this procedure generates delays, conflicts, and even tyranny of the majority among others.

In this paper, we propose a theoretical exercise which combines elements from both, direct and representative democracy. Our simplified model tries to resemble as much as possible the well-known Downsian model of political competition (Downs, 1957; Hotelling, 1929). We consider a unidimensional policy space in which voters endowed with single-peaked preferences are identified with an ideal policy. A political party defends the principles of representative democracy (Party A) and another, defends assembly democracy (Party B). The degree of social protest against the traditional political party is introduced in the form of a valence characteristic. The two parties face each other at a general election that is solved by majority voting rule. Party A is a pure office seeking political party that selects a platform as to defeat its counterpart. Party B cannot commit to certain platforms given that the party manifesto contains those proposals decided in a pre-electoral assembly. In the case of winning the elections, Party B will implement the platform decided in a post-electoral assembly. Both assemblies, the pre-electoral and the post-electoral, we consider, are open to all who wish to take part.

At the pre-electoral assembly of Party B, citizens can launch and defend proposals. We follow the citizen-candidate approach as a rationale to deduce the endogenous location of the proposals at the pre-electoral assembly (Besley and Coate, 1997; Osborne and Slivinsky, 1996). According to this approach, every configuration of proposals at the pre-electoral assembly should be sustained as a Nash equilibrium outcome in which none of the citizens who have launched a proposal at the assembly can benefit from dropping it out, and no other citizen who has not launched a proposal can benefit from presenting one. For the sake of simplicity, we just consider pre-electoral assemblies in which just

[4] There are other experiencies of direct democracy in Italy (see Putnam et al., 1993).

two proposals are launched. Party A selects a platform as to maximize its chances of winning the elections given the common believe on the assembly outcome.

We consider that voters, when casting their ballots at the general election, do evaluate Party B in terms of the proposals launched at the pre-electoral assembly. As a result, voters evaluate Party B in terms of a lottery that assigns probabilities to the assembly-equilibrium proposals. That is, from the point of view of voters, Party B gathers certain degree of ambiguity (in line with Shepsle, 1972; Alesina and Cukierman, 1990), whereas Party A is characterized by a single policy.

Our results suggest that the assembly proposals (of Party B) should be sufficiently moderated as to defeat a traditional party (Party A). Interestingly, competition does not always result in a policy at the median voter's ideal point (similar result to Romer and Rosenthal, 1979). We find that extremist assembly parties induce the traditional party to locate at the median policy position, whereas centrist assembly parties move the traditional party away from the median just in the opposite direction of the assembly's median.

Ours is not the first contribution analyzing the impact of direct democracy. Matsusaka (2005) describes the practice and theory of direct democracy through referenda in some of the states of U.S., and shows that allowing the general public to participate in lawmaking seems to improve the performance of government. In the same line, Gerber (1996) compares states where referenda are available with those in which direct democracy is not available. She shows that the threat of a ballot proposition can cause the elected official to choose policies that more closely reflects the median's voter ideal policy. Maskin and Tirole (2004) highlights some of the negative side effects of direct democracy. They show that this may lead to a worse outcome due to the citizens' lack of access to the expert opinion that is just available to legislators.

Our proposal can also be related to the literature on endogenous selection of electoral rules. Barberà and Jackson (2004) explore this issue from a self-stable type of criteria and more closely related, Aghion et al. (2002) analyze how much society chooses to delegate power to its leaders. According to their approach, different constitutions establish the share of votes needed to block a leader, and this determines the level of "insulation" of a leader. In our simplified framework, voters face two options: delegation of power to a leader, or total insulation of the leader (i.e., assembly democracy). We show that leaders in our framework are constrained in their decision by the expected proposals at the assembly, that is, the assembly also has a relevant role in controlling political leaders.

The rest of the paper is organized as follows. In Section 2 we introduce the formal model. Section 3 provides the results for the case of full attendance at the assembly. Section 4 analyzes the case in which not all the citizens are expected to attend the assembly. Finally, Section 5 contains some concluding remarks.

2.2 Model

A general election is going to be held, in which voters will elect one out of two political parties. The two competing political parties are denoted by Party A and Party B. These parties differ in the constitutional structure they support. Party A defends representative democracy and Party B defends assembly democracy.

Let $[0,1]$ be the unidimensional policy space.[5] The continuum of voters have symmetric single-peaked preferences over the policy space. The ideal policies of voters are distributed over $[0,1]$ according to a strictly increasing distribution function F. Let $x_i \in [0,1]$ be the ideal policy of voter i and let $x_M \in [0,1]$ be the ideal policy of the median voter in the population. Preferences of voters over policies are represented by the following von Neumann-Morgenstern utility

$$u_i(x) = -|x - x_i|$$

where the absolute distance between the ideal point and the policy x measures the disu-

[5] All the results also hold if instead of taking $[0,1]$, we take the real line as the policy space.

tility for the agent.

The two political parties competing to win the elections are denoted by $j \in \{A, B\}$. Party A is a traditional party that offers a single policy $x_A \in [0,1]$. Party B, on the contrary, represents a party which decisions are taken in an assembly. Each party is associated to a characteristic $\beta_A, \beta_B > 0$, where β_A represents the social preference for a traditional structured political party and β_B represents the social preference for a new party which defends the participation of the civil society. Let $\Delta\beta = \beta_B - \beta_A$ be the difference in advantage between the two parties, which we interpret as a measure of the *degree of social protest* against the traditional parties. We assume that both parties are uncertain about the difference in the advantage $\Delta\beta$ and they both consider that the value of $\Delta\beta$ is distributed according to a strictly positive density function.

Party B defends a new form of democracy in which their primary decision making body is an assembly open to all who wish to take part. This is, in fact, in the spirit of the global Occupy Movements. We consider that this party runs two assemblies, one before the general elections in which all those who wish to, can launch policy proposals, and another just after the elections in which all those who want to participate, vote over the pre-assembly proposals. The pre-electoral assembly aims at collecting information about those policy proposals with options to defeat any other proposal in a plurality vote election. The fact that the assembly party organizes two assemblies is inspired by the anecdotal evidence of the Spanish new left-wing party called "Podemos". For the first time participating in an election (European Parliament Elections), the party has organized its program around many assemblies, from which we outline the pre-electoral and the post-electoral assemblies. Besides, the Italian party "Movimento 5 Stelle" also organizes online referendums to take both pre-electoral and post-electoral decisions.

The timing of the proposed electoral game unfold as follows:

Stage 1: Party B organizes the pre-electoral assembly where all who wish to take part, can launch a proposal. Let X_B be the set of proposals made at the assembly.
Stage 2: Party A decides its political platform $x_A \in [0,1]$.

Stage 3: General elections are held.

Stage 4: If Party A wins, platform x_A is implemented. If Party B wins, there is a post-electoral assembly in which all who wish to take part vote over X_B and the policy obtaining more votes is implemented.

Observe that the proposals of Party B come from an assembly whereas the platform of Party A comes from the strategic decision of the members of Party A. In this way, there is an important difference between the two parties given that Party B limits its power to organizing the assembly and executing its decision. We next describe in more detail the stages of the electoral game.

Stage 1: The pre-electoral assembly

In Stage 1, citizens have the option of launching a proposal at the pre-electoral assembly. These proposals are defended by Party B during the electoral campaign and in the case of Party B winning the general election, the post-electoral assembly will select one of them.

Let $e_i \in \{0,1\}$ be citizen i's strategy where $e_i = 0$ means that agent i is not launching a proposal and $e_i = 1$ means that the citizen is launching a proposal. A profile of strategies e describes the strategy for each of the citizens. If a citizen makes a proposal, we consider that she cannot misrepresent her preferences so that the proposed policy is her ideal policy. Let $X_B = \{x_B^1, ..., x_B^m\}$ be the set of proposals such that each proposal x_B^i is the ideal policy of the citizen i who has launched it at the assembly. We assume that launching a proposal has a small cost $c > 0$. In this way, a citizen has only incentives to launch a proposal when either this has some chances of being selected at the post-electoral assembly, or when this can affect the policy that will be finally implemented if Party B wins the elections.

For each profile of entry strategies e, the *expected voting outcome* at the post-electoral assembly is represented by a lottery $L(e) = \{X_B, p\}$ where X_B is the set of proposals and $p = (p_1, ..., p_m)$ with $p_i \geqslant 0$ is the expected probability of each proposal being selected at the post-electoral assembly. For example, if there are two proposals $\{x_B^1, x_B^2\}$ and

$L(e) = \{\{x_B^1, x_B^2\}, (1,0)\}$, then x_B^1 is expected to win. However, if there are two proposals and $L(e) = \{\{x_B^1, x_B^2\}, (\frac{1}{2}, \frac{1}{2})\}$, the two proposals are expected to tie. Thus, $L(e)$ is a lottery that represents the expected voting outcome at the post-electoral assembly. The expected voting outcome is common knowledge.

Let e_{-i} be the entry strategies for all citizens except for i. We say that a profile of entry strategies e^* is a *pre-assembly equilibrium* if in expected utility terms,

$$Eu_i(L(e^*)) - ce_i^* \geq Eu_i(L(e_i', e_{-i}^*)) - ce_i' \text{ for all } i \text{ and all } e_i' \in \{0,1\}.$$

Hence, a pre-assembly equilibrium requires that, on the one hand, no citizen strictly improves launching a new proposal and, on the other hand, no candidate strictly benefits from dropping her proposal. Note that the pre-assembly equilibrium is a Nash equilibrium. For the sake of simplicity, we just consider pre-assembly equilibria in which just two proposals are launched.

Stage 2: Party A's election of platform

Party A is a pure office-seeking political party. This party selects a platform x_A as to win the general elections. Given the proposals of the pre-electoral assembly and its expected voting outcome, preferences of Party A are represented by:

$$v(x_A, L(e)) = \begin{cases} 1 \text{ if a strict majority of voters prefers } x_A \text{ over } L(e) \\ 0 \text{ otherwise} \end{cases} \tag{2.1}$$

The members of Party A are uncertain about the degree of social protest of the electorate. Therefore, they do not know whether they gather some advantage with respect to Party B. Their optimal decision, that we denote by x_A^*, maximizes their expected probability of winning:

$$x_A^* \in \arg\max Ev(x_A, L(e)).$$

Stage 3: General election

Given β_A and β_B, the platform of Party A and the expected voting outcome at the assembly $L(e)$, the optimal decision of a voter is the following:

$$\begin{array}{ll} \text{vote for Party A when} & \beta_A - u_i(x_A) > \beta_B - Eu_i(L(e)) \\ \text{vote for Party B when} & \beta_A - u_i(x_A) < \beta_B - Eu_i(L(e)) \\ \text{abstain from voting when} & \beta_A - u_i(x_A) = \beta_B - Eu_i(L(e)). \end{array} \tag{2.2}$$

Stage 4: Electoral outcome

If Party A wins, then the implemented policy is x_A. If Party B wins, then the post-electoral assembly takes place and, by plurality rule, one of the proposals in X_B is selected. Ties are broken at random.

Next, we introduce the equilibrium concept that accounts for the strategic behavior of Party A to select its platform, and for the strategic decision of the citizens to launch proposals at the pre-electoral assembly.

Definition: *A political equilibrium is a policy for Party A, x_A^*, and a lottery representing the expected voting outcome at the assembly, $L(e^*) = \{X_B, p\}$, such that:*
i) e^* is a pre-assembly equilibrium and
ii) given $L(e^*)$, policy x_A^* maximizes Party A's expected probability of winning.

Note that, given a pre-assembly equilibrium, the probability with which each pre-assembly proposal can be selected is directly derived from sincere voting behavior at the post-electoral assembly.[6]

[6] A similar analysis could be made in which participants at the post-electoral assembly vote strategically. None of our results rest on the sincere voting assumption.

2.3 The assembly with full attendance

We follow the citizen-candidate model proposed by Osborne and Slivinsky (1996) in order to define how endogenous political platforms can be proposed at the assembly. Osborne and Slivinsky (1996) consider a continuum of citizens with single-peaked preferences over the set of policy positions. Citizens can choose to enter or not and if they enter, they propose their ideal policy. After the citizens have made their entry decision, they vote over the proposals and under plurality rule one of them is selected. These authors do neither refer to an assembly, nor they consider that citizens belong to a political party. However, the result (in their Proposition 2) can be directly applied to our setting given that the entry stage in their model resembles our pre-electoral assembly stage. Their Proposition 2 can be rewritten as:

Lemma 1 *In every pre-assembly equilibrium with two proposals, these must be located symmetrically around the position of the median voter, i.e.* $x_B^1 = x_M - \varepsilon$ *and* $x_B^2 = x_M + \varepsilon$*, where* $\varepsilon \in (c, \overline{\varepsilon})$ *and the winning probabilities must coincide* $p_1 = p_2$*.*

Thus, in every pre-assembly equilibrium with two proposals, these should gather an equal probability of winning. The upper bound $\overline{\varepsilon}$ is defined as to avoid the entrance of a third proposal in between the two others. Thus, $\overline{\varepsilon}$ depends upon the distribution of voters and this is defined as to guarantee that for all $\varepsilon < \overline{\varepsilon}$, there is no citizen that proposing a policy in the interval $[x_M - \varepsilon, x_M + \varepsilon]$ can either defeat one of the policies x_B^1, x_B^2 at the post-electoral assembly, or can give the victory at the post-electoral assembly to one of the policies x_B^1 or x_B^2 that she prefers.[7]

At Stage 3, given a pre-assembly equilibrium with two proposals, the optimal decision of the voters with ideal policy $x_i \in [0, x_M - \varepsilon]$ and in the case that $x_A > x_M - \varepsilon$ is such that:

[7] Following Osborne and Slivinsky, this basically implies that there is no policy position in the interval $[x_M - \varepsilon, x_M + \varepsilon]$ such that either is strictly preferred by more than 1/3 of the electorate or that it can facilitate the victory of the closest proposal for the voter announcing this policy position.

$$\begin{array}{ll} \text{when } \Delta\beta < x_M - x_A & \text{they vote for Party A} \\ \text{when } \Delta\beta = x_M - x_A & \text{they abstain from voting} \\ \text{when } \Delta\beta > x_M - x_A & \text{they vote for Party B.} \end{array} \tag{2.3}$$

where $\Delta\beta = \beta_B - \beta_A$.

When $x_i \in [x_M + \varepsilon, 1]$ and in the case that $x_A < x_M + \varepsilon$ we have that:

$$\begin{array}{ll} \text{when } \Delta\beta < x_A - x_M & \text{they vote for Party A} \\ \text{when } \Delta\beta = x_A - x_M & \text{they abstain from voting} \\ \text{when } \Delta\beta > x_A - x_M & \text{they vote for Party B.} \end{array} \tag{2.4}$$

where $\Delta\beta = \beta_B - \beta_A$.

We refer to those voters whose ideal policy satisfies that $x_i \in [0, x_M - \varepsilon]$ as **low-outsiders** and to those for whom $x_i \in [x_M + \varepsilon, 1]$ as **up-outsiders**. We refer to the **insiders** as those voters such that $x_i \in (x_M - \varepsilon, x_M + \varepsilon)$.

Regarding the insiders, $Eu_i(L(e^*)) = \beta_B - \varepsilon$. Let $d_i = |x_i - x_A|$, then,

$$\begin{array}{l} \text{those insiders such that } \Delta\beta < \varepsilon - d_i \text{ vote for Party A.} \\ \text{those insiders such that } \Delta\beta > \varepsilon - d_i \text{ vote for Party B.} \end{array} \tag{2.5}$$

There is no abstention among insiders given that the probability for an agent to satisfy $d_i = \varepsilon - \Delta\beta$ is negligible. Figure 1 represents the provided classification of voters.

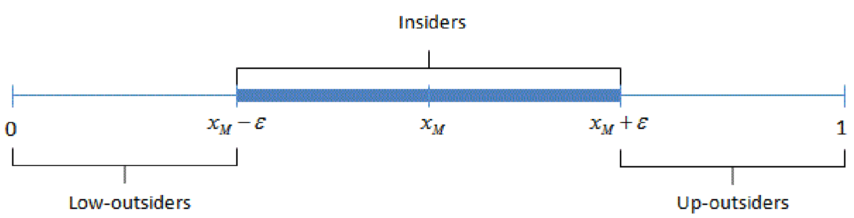

Figure 2-1: Location of the low-outsiders, the insiders and the up-outsiders with respect to the assembly.

Next, we derive the electoral result at the general election depending on the degree of social protest of the society, $\Delta\beta$. We describe which degree of discontent is favorable for Party B to win the elections.

Proposition 1 *In every political equilibrium with two proposals at the pre-electoral assembly, Party B wins the elections if and only if* $\Delta\beta \geq \frac{\varepsilon}{2}$.

Proof. First, we show that if $\Delta\beta \geq \frac{\varepsilon}{2}$ then, Party B always wins. We consider that $\Delta\beta = \frac{\varepsilon}{2}$.

If $x_A \in [0, x_M - \varepsilon]$, according to (4), the up-outsiders vote for Party B and by (5), for those insiders such that $x_i \in \left(x_M - \frac{\varepsilon}{2}, x_M + \varepsilon\right)$ we have $d_i > \frac{\varepsilon}{2}$, which implies that they also vote for Party B. Thus, Party B obtains a strict majority of votes. By a symmetric type of argument, if $x_A \in [x_M + \varepsilon, 1]$, Party B wins.

If $x_A \in \left(x_M - \varepsilon, x_M - \frac{\varepsilon}{2}\right)$, by (4), the up-outsiders vote for Party B and by (5), for those insiders such that $x_i \in [x_M, x_M + \varepsilon)$ we have $d_i > \frac{\varepsilon}{2}$, which implies that they vote for Party B. Thus, Party B obtains a strict majority of votes. By a symmetric type of argument, if $x_A \in \left(x_M + \frac{\varepsilon}{2}, x_M + \varepsilon\right)$, Party B wins.

If $x_A \in \left(x_M - \frac{\varepsilon}{2}, x_M\right]$, by (3) and (4) all the outsiders vote for Party B. Thus, Party A only obtains the vote of the insiders such that $d_i < \frac{\varepsilon}{2}$. However, in every pre-assembly equilibrium, no subinterval of size ε in between $(x_M - \varepsilon, x_M + \varepsilon)$ can contain more than $\frac{1}{3}$ of the votes and the rest of insiders vote for Party B.[8] Thus, Party B obtains a strict majority of votes. By a symmetric type of argument, if $x_A \in \left[x_M, x_M + \frac{\varepsilon}{2}\right)$, Party B wins.

In the last case, when $x_A = x_M - \frac{\varepsilon}{2}$ by (3), the low-outsiders abstain from voting. However, by (4) the up-outsiders vote for Party B and among the insiders, by (5), those with $x_i \in (x_M, x_M + \varepsilon]$ vote for Party B. Thus, even though Party B may not obtain a strict majority, Party A cannot obtain more than $\frac{1}{3}$ of the votes by the above argument

[8] Observe that for every $x_i \in (x_M - \varepsilon, x_M + \varepsilon)$, the size of the interval, according to sincere voting in the citizen candidate approach, is given by $\frac{x_i - [x_M - \varepsilon]}{2} + \frac{x_M + \varepsilon - x_i}{2} = \varepsilon$.

and Party B wins. By a symmetric type of argument, if $x_A = x_M + \frac{\varepsilon}{2}$, Party B wins. Given that Party B wins when $\Delta\beta = \frac{\varepsilon}{2}$, it also wins when $\Delta\beta > \frac{\varepsilon}{2}$.
Second, we show that when $\Delta\beta < \frac{\varepsilon}{2}$, Party B is defeated.
Suppose that $x_A = x_M - \frac{\varepsilon}{2}$ and let $\Delta\beta = \frac{\varepsilon}{2} - \gamma$ with $\gamma \to 0$. Then, by (3), the low-outsiders vote for Party A and by (5), those insiders such that $x_i \in (x_M - \varepsilon, x_M]$ also vote for Party A. Thus, Party A obtains a strict majority. Given that Party A wins when $\Delta\beta = \frac{\varepsilon}{2} - \gamma$, it also wins locating at $x_A = x_M - \frac{\varepsilon}{2}$ for every other case where $\Delta\beta \in \left[0, \frac{\varepsilon}{2}\right)$.

■

This result gives a clear prediction of the party winning at the general election as a function of the degree of social protest. If the degree of social protest is sufficiently high, we show, Party A must locate in one of the insiders positions as this will guarantee the votes of two different fractions of the electorate, some insiders and some outsiders. When $\Delta\beta \geq \frac{\varepsilon}{2}$, regardless of the location of Party A, there are no options for Party A to obtain a majority of votes.

Figure 2 shows that the smaller the value of the parameter that defines the proposals of the assembly ε, the higher the chances of Party B to win at the general election.[9] In the horizontal axis we represent the values of ε to which we refer as the degree of polarization within the assembly. We say that the assembly proposals are moderated when ε takes a small value. In the vertical axis we represent the degree of social protest. Thus, we can interpret the first result in Proposition 1 as one showing that the more moderated is the assembly, the higher the probability of the assembly party to win at the general election. Polarization of the assembly, on the other hand, reduces the set of values $\Delta\beta$ for which the assembly party can win at the general elections.

So far, we have paid attention to describing which party can win at the general election. Next, we describe the equilibrium location of Party A. In Proposition 1, we

[9] We take $c \to 0$ so that Figure 2 does not account for those values of $\varepsilon \to 0$ for which an equilibrium fails to exist.

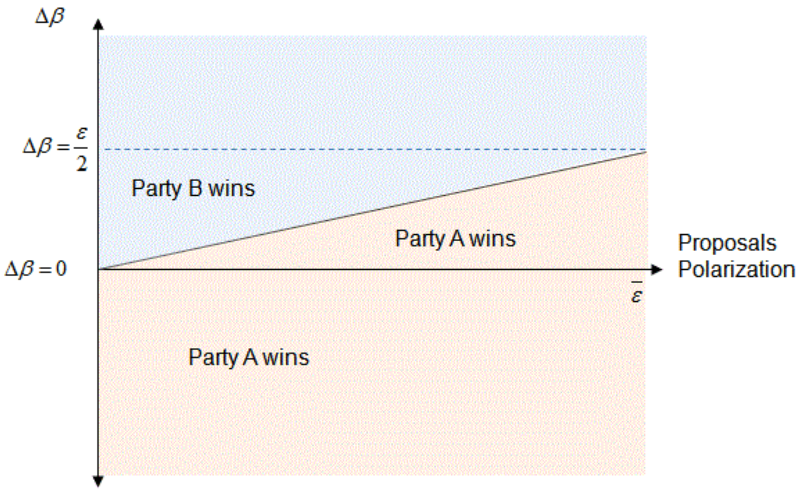

Figure 2-2: An illustration of Proposition 1.

showed that Party A can only win by supporting certain political positions. We next show that only two symmetric locations will be optimally selected by Party A in every political equilibrium.

Proposition 2 *In every political equilibrium with two proposals at the pre-electoral assembly, Party A sets its political platform either at* $x_A = x_M - \frac{\varepsilon}{2}$ *or at* $x_A = x_M + \frac{\varepsilon}{2}$.

Proof. The objective function of Party A is defined by Expression (3.1) hence, Party A only derives benefits from winning the elections. By Proposition 1, Party A cannot win the elections when $\Delta\beta \geq \frac{\varepsilon}{2}$. In this case, Party A is indifferent between every policy position. We analyze the case where $\Delta\beta = \frac{\varepsilon}{2} - \delta$, with $\delta \to 0$. As shown in Proposition 1, $x_A = x_M - \frac{\varepsilon}{2}$ guarantees the victory of Party A in this case (similar reasoning for $x_A = x_M + \frac{\varepsilon}{2}$). We proceed by showing the following statements:
i) extremist locations of Party A such that $x_A \in [0, x_M - \varepsilon]$ or $x_A \in [x_M + \varepsilon, 1]$ cannot guarantee the victory of Party A.
ii) every other location $x_A \in (x_M - \varepsilon, x_M + \varepsilon)$ such that $x_A \neq x_M - \frac{\varepsilon}{2}$ or $x_A \neq x_M + \frac{\varepsilon}{2}$ cannot guarantee the victory of Party A.
First, we show i). We consider that $x_A \in [0, x_M - \varepsilon]$. By (3), the low-outsiders vote for

Party A. Among the insiders, by (5), those agents with $d_i > \frac{\varepsilon}{2} + \delta$ vote for Party B. Thus, those insiders such that $x_i \in \left(x_M - \frac{\varepsilon}{2} + \delta, x_M + \varepsilon\right)$ vote for Party B. By (4), the up-outsiders vote for Party B. Therefore, Party B obtains a strict majority and wins. By a symmetric type of argument, if $x_A \in [x_M + \varepsilon, 1]$, Party B wins.

Next, we show ii). We distinguish two cases, when $x_A \in \left(x_M - \varepsilon, x_M - \frac{\varepsilon}{2}\right)$ and when $x_A \in \left(x_M - \frac{\varepsilon}{2}, x_M\right]$.

First, we suppose that $x_A \in \left(x_M - \varepsilon, x_M - \frac{\varepsilon}{2}\right)$. If $x_i = x_M$, by (5), the median agent prefers Party B over Party A when

$$\Delta\beta > \varepsilon - (x_M - x_A) \tag{2.6}$$

Given that $x_M - x_A > \frac{\varepsilon}{2}$, we have that the second term of Expression 2.6 is smaller than $\frac{\varepsilon}{2}$. If we take $\Delta\beta = \frac{\varepsilon}{2} - \delta$ where $\delta \to 0$, we can always define δ sufficiently close to 0 such that $\frac{\varepsilon}{2} - \delta > \varepsilon - (x_M - x_A)$. Then, an agent located at $x_i = x_M$ votes for Party B as they do those agents located at $x_i \in (x_M, x_M + \varepsilon)$. Besides, by (4), the up-outsiders also vote for Party B.

Second, we suppose that $x_A \in \left(x_M - \frac{\varepsilon}{2}, x_M\right]$. By (3) and (4) and given that $\Delta\beta > |x_M - x_A|$ the outsiders vote for Party B. By (5), Party A only obtains the vote of those insiders such that $d_i < \frac{\varepsilon}{2} + \delta$. In other words, the votes of Party A are those contained in an interval of size $\varepsilon + 2\delta$. Given that in every interval of size ε, there is strictly less than 1/3 of the votes, for δ close to 0, Party A derives strictly less than 1/3 of the votes.

Thus, when $\Delta\beta = \frac{\varepsilon}{2} - \delta$ with $\delta \to 0$ there are only two policies that guarantee the victory of Party A ($x_A = x_M - \frac{\varepsilon}{2}$ and $x_A = x_M + \frac{\varepsilon}{2}$) besides, these two policies also guarantee the victory of Party A when $\Delta\beta < \frac{\varepsilon}{2} - \delta$. Then, this proves that these two policies are the only ones that maximize Party A's probability of winning. This completes the proof.

■

We have shown that locating in a platform too close to the median voter does not allow Party A to defeat Party B. The main argument for this is that in order to obtain votes from insiders as well as from outsider voters, Party A must set its platform at one of the sides of the median voter. In particular, we find that when $\Delta\beta = \frac{\varepsilon}{2} - \delta$, with $\delta \rightarrow 0$, the only two locations that guarantee the victory of Party A are $x_A = x_M - \frac{\varepsilon}{2}$ and $x_A = x_M + \frac{\varepsilon}{2}$. Besides, for every other degree of social protest below $\Delta\beta = \frac{\varepsilon}{2} - \delta$, these locations also guarantee the victory of Party A.

Our analysis reveals that Party A must differentiate its policy from the median voter position to attract a majority of the electorate. In a similar vein, but in a different setting, Ansolabehere and Snyder (2000) and Groseclose (2001) show that when a candidate has an advantage over another, the weaker candidate moves away from the center.[10] In Figure 3, we represent both winning strategies of Party A. The strategy $x_A = x_M - \frac{\varepsilon}{2}$ gives Party A the support of those voters located in the interval $[0, x_M]$ whereas the strategy $x_A = x_M + \frac{\varepsilon}{2}$ assures Party A the votes of those located in the interval $[x_M, 1]$.

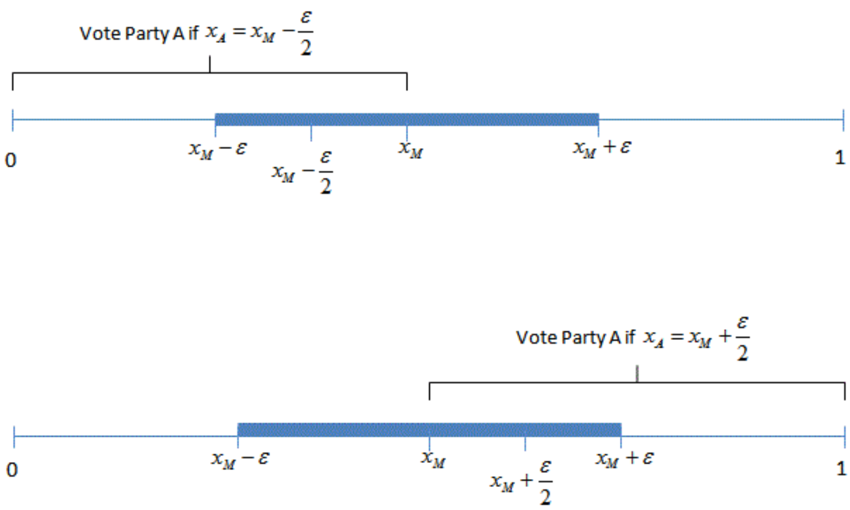

Figure 2-3: An illustration of Proposition 2.

[10] Observe that our result differs from the one of Shepsle (1972) who shows that when a party stands at the median, the other has incentives to take a lottery stand. See also Page (1976).

2.4 The assembly with partial attendance

In the previous section, we analyzed the case in which two policies symmetrically located around the median voter are proposed at the pre-electoral assembly. In addition, we took for granted that all the citizens attended the pre-electoral and the post-electoral assemblies of Party B.

Now, we want to consider a scenario where just a fraction of voters with close policy positions attend the assembly. We still maintain the pre-assembly equilibrium concept. Hence, the previous section is a particular case in which the median voter position of both the assembly and the electorate coincide. We define the *assembly* median voter position as x_M^a and from now on, x_M is the *electorate* median.

Following the pre-assembly equilibrium concept, in every political equilibrium with two expected proposals at the assembly, these are symmetrically located around the assembly median, i.e., $x_M^a - \varepsilon$ and $x_M^a + \varepsilon$, where $\varepsilon \in (c, \overline{\varepsilon})$ and the winning probabilities coincide $p_1 = p_2$.[11]

Next, we derive the electoral result at the general election depending on the location of the *assembly* median voter position x_M^a with respect to x_M. We distinguish two scenarios: a **centrist assembly**, which occurs when the electorate median voter is an *insider*, i.e. $x_M^a - \varepsilon < x_M < x_M^a + \varepsilon$; and a **non-centrist assembly**, which implies that the assembly is either to the left or to the right of the electorate median voter, i.e. $x_M \leq x_M^a - \varepsilon$ or $x_M \geq x_M^a - \varepsilon$. In each situation, we take into account the degree of social protest. Voting decisions as described by (3), (4) and (5) do not change except for substituting x_M by x_M^a.

[11] Where the bounds $(c, \overline{\varepsilon})$ should be recalculated accounting for the truncated distribution of voters that attend the assembly.

2.4.1 The centrist assembly

We study the case where the electorate median voter x_M is among the bounds of the assembly, i.e. $x_M^a - \varepsilon < x_M < x_M^a + \varepsilon$. In Figure 4, we show different locations of the centrist assembly with respect to x_M. The first case shows a centrist-left assembly where $x_M^a < x_M$. The second case shows a centrist assembly where both the assembly and the electorate median voter coincide. This case is similar to the one we have analyzed in the previous section. The last case shows a centrist-right assembly where $x_M < x_M^a$.

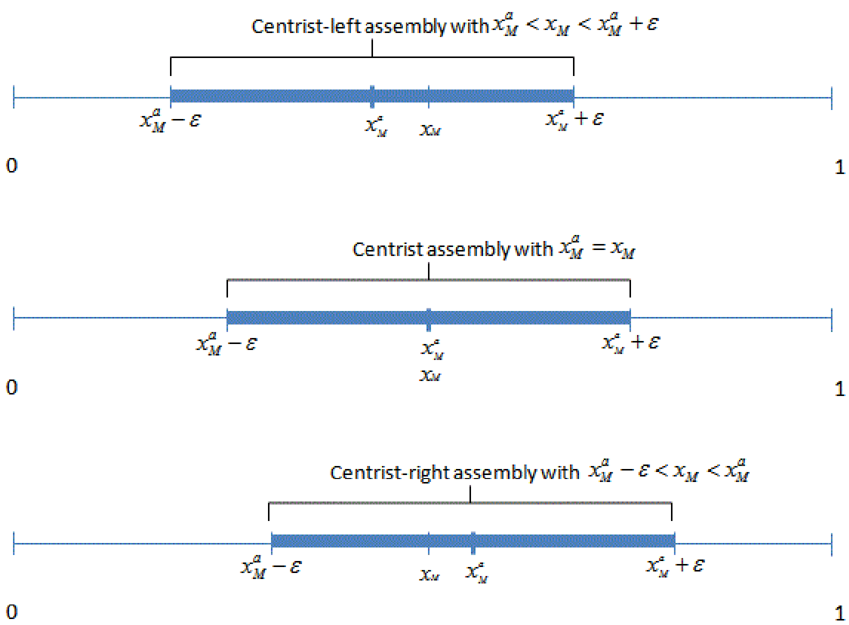

Figure 2-4: Centrist assemblies' location with respect to x_M.

The party winning the elections in the case of a centrist assembly also depends on the degree of social protest. As we next show, the optimal position of the traditional party is not the electorate median but it is the midpoint between the electorate median and one out of the two proposals of the assembly.

Proposition 3 *Consider that the assembly median differs from the electorate median and that the assembly is centrist. Then, Party B wins the elections if and only if* $\Delta\beta \geq$

$\frac{\varepsilon}{2}+\frac{\gamma}{2}$ where $\gamma=\left|x_M-x_M^a\right|$. Besides, in every political equilibrium with two proposals at the pre-electoral assembly, Party A locates at:
i) $x_A=\frac{x_M+x_M^a+\varepsilon}{2}$ in the case of a centrist-left assembly ($x_M^a<x_M$)
ii) $x_A=\frac{x_M+x_M^a-\varepsilon}{2}$ in the case of a centrist-right assembly ($x_M^a>x_M$).

Proof. Following Proposition 1, we know that there is a threshold value $\Delta\beta$ above which Party B always wins the elections. Besides, by Proposition 2, we know that just below the threshold there are two symmetric strategies for Party A that guarantee its victory, these strategies clearly reduce to one when the assembly moves either to the right or to the left of the median. We calculate the corresponding threshold and the corresponding unique location of x_A in the case of a centrist-left assembly where $x_M^a<x_M$. When the assembly is centrist-left, Party A cannot achieve equal votes locating at symmetric positions around the electorate median. In fact, in this case, it is easier for Party A to achieve a majority of votes among those located to the right of the electorate median, that is, those voters in the interval $[x_M,1]$. First, we study the agents with ideal policy $x_i=x_M$ and $x_i=x_M^a+\varepsilon$. For agent $x_i=x_M$, she votes for Party A when

$$x_A<-\Delta\beta+\varepsilon+x_M \tag{2.7}$$

For agent $x_i=x_M^a+\varepsilon$, the utilities derived from voting Party A and Party B are $\beta_A-\left|x_M^a+\varepsilon-x_A\right|$ and $\beta_B-\varepsilon$, respectively. Then, she votes for Party A when $\beta_A-\left|x_M^a+\varepsilon-x_A\right|>\beta_B-\varepsilon$ which implies that:

$$x_A>\Delta\beta+x_M^a \tag{2.8}$$

The values $\Delta\beta$ for which Party A can obtain the votes in the interval $[x_M,x_M^a+\varepsilon]$ is deduced from the above two equations and it yields $\Delta\beta<\frac{\varepsilon}{2}+\frac{\gamma}{2}$. Moreover, the only strategy that guarantees that Party A obtains all the votes in the interval $[x_M,x_M^a+\varepsilon]$ when $\Delta\beta=\frac{\varepsilon}{2}+\frac{\gamma}{2}-\delta$ where $\delta\to0$ is deduced by substituting the value $\Delta\beta=\frac{\varepsilon}{2}+\frac{\gamma}{2}$ in Expression (2.7) or (2.8). We deduce that $x_A=\frac{x_M+\left(x_M^a+\varepsilon\right)}{2}$. By (2.4), this value of

x_A also guarantees that the up-outsiders vote for Party A. Finally, if $x_A = \frac{x_M + \left(x_M^a + \varepsilon\right)}{2}$ guarantees the victory of Party A when $\Delta\beta = \frac{\varepsilon}{2} + \frac{\gamma}{2} - \delta$, it also guarantees the victory of Party A for smaller values of $\Delta\beta$. This implies that this strategy of Party A maximizes its expected probability of winning. The symmetric case in which there is a centrist-right assembly follows a similar reasoning. ■

We find that when there is a centrist-right assembly and $\Delta\beta < \frac{\varepsilon}{2} + \frac{\gamma}{2}$ where $\gamma = x_M - x_M^a$, Party A locating at $x_A = \frac{x_M + \left(x_M^a - \varepsilon\right)}{2}$ obtains the support of those voters whose ideal policy is in the interval $(0, x_M^a - \varepsilon)$ plus a fraction of the voters which ideal policy is in the interval $(x_M^a - \varepsilon, x_M^a + \varepsilon)$. Symmetrically, when there is a centrist-left assembly and $\Delta\beta < \frac{\varepsilon}{2} + \frac{\gamma}{2}$ where $\gamma = x_M - x_M^a$, Party A wins the elections locating at $x_A = \frac{x_M + \left(x_M^a + \varepsilon\right)}{2}$ given that voters to the right of the electorate median vote for Party A.

2.4.2 The non-centrist assembly

We study the case in which the electorate median x_M is either to the left of the assembly $x_M \leq x_M^a - \varepsilon$ or to the right of the assembly $x_M \geq x_M^a + \varepsilon$. We can interpret the assembly in these cases as left-extremist or right-extremist. Figure 5 shows the intervals in which a right-extremist assembly and a left-extremist assembly can be located.

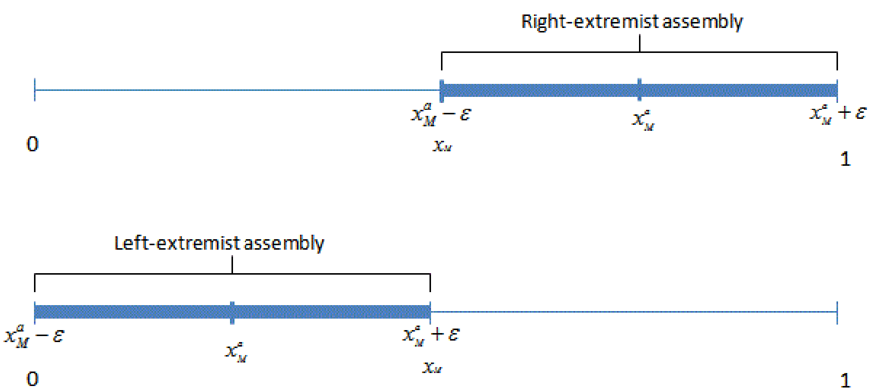

Figure 2-5: Location of the non-centrist assemblies with respect to x_M.

In both cases, we show that for every degree of social protest, Party A wins the elections and besides, its strategy consists of locating at the electorate median.

Proposition 4 *Consider that the assembly median differs from the electorate median and that the assembly is non-centrist. Then, Party B wins the elections if and only if* $\Delta\beta \geq \gamma$ *where* $\gamma = |x_M - x_M^a|$ *. Besides, in every political equilibrium with two proposals at the pre-electoral assembly, Party A locates at* $x_A = x_M$*.*

Proof. We calculate the greatest degree of social protest for which Party A can defeat Party B. Consider the case of a right-extremist assembly where $x_M < x_M^a$. The easiest way for Party A to win the elections is by obtaining the votes of those located to left of the policy space, that is those in the interval $[0, x_M]$. If voter $x_i = x_M$ votes for Party A, all the other voters in this interval also vote for Party A. In the best scenario for voter $x_i = x_M$, Party A locates at $x_A = x_M$. Following Expression (3), all the voters in $[0, x_M]$ vote for Party A when $\Delta\beta < x_M^a - x_A$ and substituting $x_A = x_M$ we obtain that $\Delta\beta < \gamma$. Besides, when $\Delta\beta = \gamma - \delta$ where $\delta \to 0$ there is no other value $x_A \neq x_M$ that guarantees a majority of votes for Party A. Thus, $x_A = x_M$ is the unique strategy of Party A that maximizes its expected probability of winning. Finally, if $\Delta\beta \geq \gamma$ the strategy $x_A = x_M$, cannot guarantee a majority of votes for Party A, and it is in fact Party B which wins with the votes of the agents in the interval $[x_M, 1]$. The case of a left-extremist assembly follows a similar reasoning. ■

We have shown that for every $\Delta\beta < \gamma$ where $\gamma = |x_M - x_M^a|$, Party A can always guarantee a majority of voters locating at the electorate median. Thus, the electorate median is the policy that maximizes the expected probability of winning of Party A.

In Figure 6, we summarize the obtained results regarding the optimal location of Party A as a function of the location of the assembly median voter along the policy space. Interestingly, the presence of an assembly party makes the traditional party to move along the policy space.

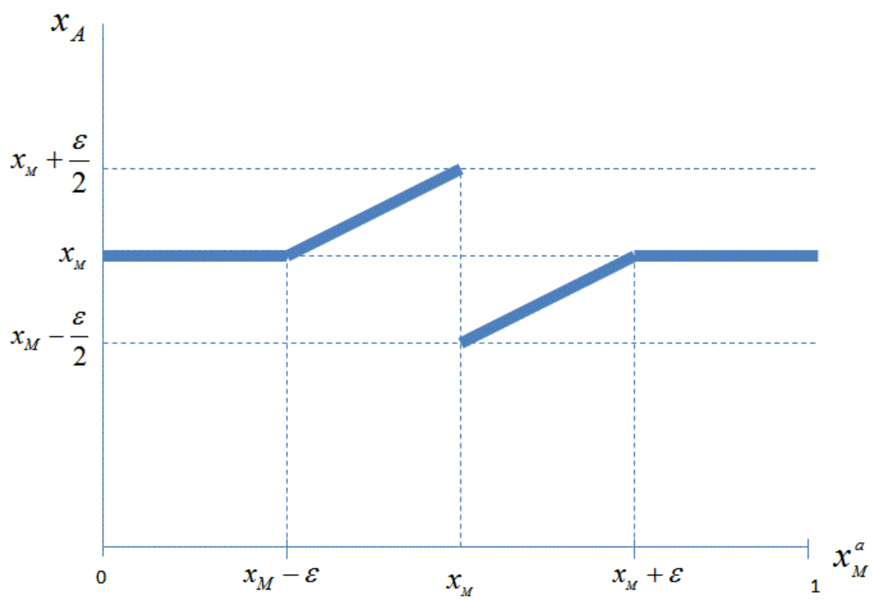

Figure 2-6: Strategies of Party A with respect to the location of the assembly median voter x_M^a.

On the one hand, in the case of an extremist assembly, regardless of the ideology of the assembly, Party A moderates its policy and it locates at the median voter position, i.e. $x_A = x_M$. On the other hand, in the case of a centrist assembly, Party A locates either to the left or to the right of the median voter location, just in the opposite direction of the assembly median location. This is due to the fact that Party A needs to differentiate from the assembly proposals in order to attract not only centrist voters but also voters to one of the sides of the median. As we have shown, this is the type of strategy that guarantees the victory of Party A when the victory is possible.

2.5 Conclusions

In this paper, we have studied the consequences of political competition between a party implementing assembly democracy (Party B) and a traditional Downsian party (Party A). We have introduced, in terms of a valence characteristic, the social preferences in favor or against new forms of democracy. Citizens when participating at the assembly are strategic and they want their proposals to achieve a majority at the assembly. Party A is a pure office seeking party which selects its platform as to maximize its probability of winning the general election. We have compared different scenarios regarding the location of the assembly party.

We find that the more centrist the assembly party is, the more chances it has of winning the elections. Interestingly, we also find that the location of the assembly party induces Party A to locate at different platforms.

Surprisingly, due to the competition with an assembly party, when the assembly is centrist, the traditional party moves its platform away from the median voter location in order to attract a larger fraction of voters. In particular, we find that a centrist assembly party located to the left of the overall median, moves the traditional party to the right, whereas a centrist assembly party located to the right of the overall median, moves the traditional party to the left. The centrist assembly party, therefore, generates a centrifugal effect over the traditional party, which moves it in the opposite direction. However, when the assembly party is non-centrist (or extremist), we find that the traditional party moves towards the median of the electorate. In this case, the extremist assembly party leaves an empty center which can be occupied by a traditional party.

Our main message is that extremist assembly parties may have no effect regarding the location of a traditional office-seeking party, whereas moderated assembly parties have an impact by moving away from the median the traditional political party. In equilibrium, the traditional party moves in the opposite direction of the assembly proposals but within the bounds of the proposals made by the assembly. As a result, the assembly party generates divergence between the platforms of the parties which is in close contrast to the convergence prediction of the pure Downsian model.

We have shown that new assembly parties may not only have a direct effect when winning the elections and taking the assembly as their policy making body, but also an indirect effect by affecting the policy of its competing parties. This is a testable prediction that is open to empirical scrutiny.

In this study, we only include the results for assemblies with two proposals. We leave the analysis with more than two proposals for further research.

Chapter 3

Political Competition between parties with heterogeneous factions

3.1 Introduction

In many democracies parties' elite do not always delegate authority into new party candidates. As an example, Spanish Socialist Party (PSOE) has recently approved primary elections to nominate candidates and in the meanwhile, the party-elite – composed by the party's most experienced members – still defines the party's manifesto.

It is not uncommon to observe democracies in which different practices are used by different political parties to elect its candidates. Many countries follow the described pattern by which primary elections have just been adopted by some (but not all) political parties: Denmark, France Finland, Greece, Italy, Japan, Israel or the United Kingdom are additional examples (Kenig, 2009). Renewed parties in these countries usually compete against traditional parties which candidates are selected by former party leaders.

We propose a game theoretical setting to analyze political competition between two heterogeneous parties. One of the parties has two factions, the elite faction and the opportunistic faction (represented by the nominated candidate). The elite faction defends the ideology of the founders of the party. The opportunistic faction sets another platform with the aim of winning the election. From the voters' viewpoint there is uncertainty regarding which of the two proposals will prevail. The other party sets a single platform to win the election. We propose a sequential game in which the two-faction party decides first. This sequentiality captures that the two-faction party celebrates its convention and announces its nominee (possibly elected by primaries), whereas the other party has no convention and therefore can set its platform afterwards. We analyze the resulting political and electoral outcome when there is uncertainty about the parties' valence advantage.

Models of political competition typically consider parties that only differ in some parameters such as incumbent versus challenger positioning (Bevia and Llavador, 2009), valence characteristic (Ansolabehere and Snyder, 2000), or ideological cleavages (Wittman, 1983), among others. From the well-known Downsian model (Downs, 1957) to the Wittman model, or the Citizen-candidate model (Osborne and Slivinski, 1996), homogeneous parties compete in different political environments. Uncertainty about the median voter's position (in Wittman model) or strategic candidacy (in the Citizen-candidate model) are key assumptions to predict divergent political platforms. A new and different scenario arises when parties are considered to be composed by different factions (Roemer, 2001). In contrast to our framework, in Roemer's model each faction struggles to achieve a different objective and factions have to agree on a unique platform.

This note proposes an alternative framework in which parties with different constituencies compete against each other. As far as we know, ours is a first contribution to analyze competition between heterogeneous parties from this perspective. Our results are in coherence but cannot be directly derived from the literature on political ambiguity (Shepsle, 1972; Page, 1976).[1]

[1]The two-faction party launches two policy proposals. In terms of Shepsle (1972), this party is perceived by voters as a lottery over the two proposals.

3.2 Model

An election is going to be held in which voters elect by majority voting either a leftist (Party L) or a rightist (Party R) political party.

Party L has two factions: one representing the party's elite with fixed platform $x_L^E \in [0,1]$, and another representing an opportunistic party faction that seeks to win the election by proposing a platform $x_L^O \in [0,1]$ where $x_L^O \geq x_L^E$. Party R has a single faction that seeks to win the election by proposing a platform $x_R \in [0,1]$.

Each party is associated to a valence characteristic $\beta_L, \beta_R \in \mathbb{R}$, on which all agents are in agreement. Valence can measure the retrospective judgement on economic issues, or the parties' perceived fight against corrupt practices. Let $\Delta\beta = \beta_L - \beta_R$ be the differential valence. Parties are uncertain about the differential valence $\Delta\beta$.

There is a continuum of voters whose ideal policies are distributed over the unidimensional policy space. Let $x_i \in [0,1]$ be the ideal policy of voter i and let x_M be the ideal policy of the voter in the median of the distribution. Preferences of voters over Party R are measured by valence minus the absolute distance between the party's platform and the voter's ideal policy

$$u_i(R) = \beta_R - |x_R - x_i|.$$

Preferences of voters over Party L are measured by the vNM utility function defined over the platforms x_L^E and x_L^O, when assigned equal probability

$$Eu_i(L) = \tfrac{1}{2}(\beta_L - \left|x_L^E - x_i\right|) + \tfrac{1}{2}(\beta_L - \left|x_L^O - x_i\right|) = \beta_L - \frac{(\left|x_L^E - x_i\right| + \left|x_L^O - x_i\right|)}{2}.$$

Preferences of the opportunistic faction of Party L and the single faction of Party R over the policy proposals $\left(x_L^E, x_L^O\right)$ and x_R, are measured by the following vNM utility function

$$v_j((x_L^O, x_L^E), x_R) = \begin{cases} 1 \text{ if Party } j \text{ wins} \\ 0 \text{ otherwise} \end{cases} \quad \text{for all } j \in \{L, R\}.$$

The timing of the proposed electoral game unfolds as follows.

Stage 1: Given the fixed proposal of the elite faction, where w.l.g. $x_L^E < x_M$, the opportunistic faction of Party L sets its political platform x_L^O as to maximize the expected payoff given its uncertainty about $\Delta\beta$

$$x_L^{*O} \in \arg\max Ev_L((x_L^O, x_L^E), x_R).$$

Stage 2: Party R observes the announced policy of Party L and sets its platform x_R as to maximize the expected payoff given its uncertainty about $\Delta\beta$

$$x_R^* \in \arg\max Ev_R((x_L^O, x_L^E), x_R).$$

Stage 3: The election is held and voters opt for one of the two political parties

$$\begin{array}{ll} \text{if } \beta_L - \frac{(|x_L^E - x_i| + |x_L^O - x_i|)}{2} > \beta_R - |x_R - x_i| & \text{voter } i \text{ opts for Party L,} \\ \text{if } \beta_L - \frac{(|x_L^E - x_i| + |x_L^O - x_i|)}{2} < \beta_R - |x_R - x_i| & \text{voter } i \text{ opts for Party R.} \end{array}$$

When a voter is indifferent between Party L and Party R, we consider that he/she abstains from voting.

3.3 The competition game

We solve the proposed game by backward induction. First, we describe voting decisions at Stage 3. Second, we describe Party R's best response. Finally, we solve for Party L's optimal decision. We only account for those equilibria in which parties preserve their ideological cleavages and therefore, the mean platform of Party L, $x_L = \frac{x_L^E + x_L^O}{2}$, is more leftist than the platform of Party R, i.e., $x_L \leq x_R$.

Stage 3:

First, we analyze the voting decision of the agents with ideal policies $x_i \in [0, x_L^E)$. For these agents, $Eu_i(L) = \beta_L - (x_L - x_i)$ and since $x_L \leq x_R$,

$$\begin{aligned} \text{when } \Delta\beta &> x_L - x_R, \text{ they vote for Party L,} \\ \text{when } \Delta\beta &< x_L - x_R, \text{ they vote for Party R.} \end{aligned} \tag{3.1}$$

Second, for those voters such that $x_i \in \left[x_L^E, x_L^O\right]$, $Eu_i(L) = \beta_L - \frac{(x_L^O - x_L^E)}{2}$. When comparing this term with the utility derived from voting for R,

$$\begin{aligned} \text{when } \Delta\beta &> \tfrac{x_L^O - x_L^E}{2} - |x_R - x_i|, \text{ they vote for Party L,} \\ \text{when } \Delta\beta &< \tfrac{x_L^O - x_L^E}{2} - |x_R - x_i|, \text{ they vote for Party R.} \end{aligned} \tag{3.2}$$

Third, when $x_i \in \left(x_L^O, 1\right]$ then $Eu_i(L) = \beta_L - (x_i - x_L)$. Regarding the location of Party R with respect to voter i, there are two possibilities, either $x_R < x_i$ or $x_R > x_i$. In the first case,

$$\begin{aligned} \text{when } \Delta\beta &> x_R - x_L, \text{ they vote for Party L,} \\ \text{when } \Delta\beta &< x_R - x_L, \text{ they vote for Party R.} \end{aligned} \tag{3.3}$$

In the second case,

$$\begin{aligned} \text{when } \Delta\beta &> 2x_i - x_L - x_R, \text{ they vote for Party L,} \\ \text{when } \Delta\beta &< 2x_i - x_L - x_R, \text{ they vote for Party R.} \end{aligned} \tag{3.4}$$

Stage 2:

For every location of the opportunistic faction of Party L, we solve for the platform that maximizes the probability of Party R winning a majority of votes. Since the party is uncertain about the differential valence $\Delta\beta = \beta_L - \beta_R$, the optimal policy has to guarantee a majority of votes up to some maximal value of the differential valence.

First, suppose that $x_L^O < x_M$. If Party R sets its platform too close to Party L's mean platform x_L then, the two political parties offer too similar positions and the party with the highest valence obtains a majority. Therefore, the best location for Party R consists of preserving certain distance with respect to the policy x_L, while achieving a majority of votes. If Party R offers the median voter's ideal policy $x_R = x_M$ then, by (3.3) all the voters with ideal policy $x_i \in [x_M, 1]$ vote for Party R when the differential valence is such that

$$\Delta\beta < x_R - x_L = x_M - x_L. \tag{3.5}$$

Therefore, Party R wins the election for every $\Delta\beta < x_M - x_L$. If instead $x_R < x_M$, by (3.5) there are less values $\Delta\beta$ for which Party R guarantees a majority. And if $x_R > x_M$, by (3.4) those voters such that $x_i \in [x_M, x_R]$ vote for Party R when $\Delta\beta < 2x_M - x_R - x_L$. When comparing this expression with (3.5), we find that $2x_M - x_R - x_L < x_M - x_L$, i.e., there are less values $\Delta\beta$ for which Party R guarantees a majority. We deduce that when $x_L^O < x_M$, the optimal policy of Party R is $x_R^* = x_M$.

Second, suppose that $x_L^O > x_M$. For Party R to guarantee that every voter such that $x_i \in [x_M, 1]$ votes for Party R, there are two conditions that should hold. On the one hand, if $x_i \in [x_M, x_L^O]$, condition (3.2) implies that

$$\Delta\beta < \tfrac{x_L^O - x_L^E}{2} - |x_R - x_i|\,. \tag{3.6}$$

On the other hand, by (3.3) and (3.4), if $x_i \in [x_L^O, 1]$

$$\Delta\beta < x_R - x_L \text{ when } x_i > x_R \tag{3.7}$$

$$\Delta\beta < 2x_i - x_L - x_R \text{ when } x_i < x_R. \tag{3.8}$$

If $x_R \in [x_M, x_L^O]$, we check for the maximum value $\Delta\beta$ below which Party R guarantees a majority. By expression (3.6), $\frac{x_L^O - x_L^E}{2} - |x_R - x_i|$ is decreasing in x_R when $x_i \in [x_M, x_R]$ and it is strictly increasing in x_R when $x_i \in [x_R, x_L^O]$, and by (3.7) $x_R - x_L$ is always strictly increasing in x_R. When (3.6) is decreasing, the most adverse case for Party R corresponds to the minimal value of $\Delta\beta$ which is achieved when $x_i = x_M$. When (3.6) is increasing, the most adverse case for Party R corresponds to the minimal value of $\Delta\beta$ which is achieved when $x_i = x_L^O$. In this last case, however, substituting $x_i = x_L^O$ in (3.6) yields $\Delta\beta < \frac{x_L^O - x_L^E}{2} + x_R - x_L^O = x_R - x_L$ which is equivalent to (3.7). Thus if the inequalities $\Delta\beta < \frac{x_L^O - x_L^E}{2} - x_R + x_M$ and $\Delta\beta < x_R - x_L$ hold, Party R achieves a majority of votes. Figure 1 represents the shaded area in which both conditions simultaneously hold. Solving for x_R in the intersection and substituting that $\frac{x_L^O - x_L^E}{2} = x_L^O - x_L$, yields $x_R = \frac{x_L^O + x_M}{2}$. In addition to (3.6) and (3.7), expression (3.8) should also hold if $x_R > x_L^O$ for those voters such that $x_i \in [x_L^O, x_R]$. Since (3.8) is not embedded neither in condition (3.6) nor in (3.7), an additional restriction over $\Delta\beta$ needs to be satisfied if $x_R > x_L^O$. This implies that the optimal policy has to be $x_R^* = \frac{x_L^O + x_M}{2}$ when $x_L^O > x_M$ and $x_L \leq x_M$.[2]

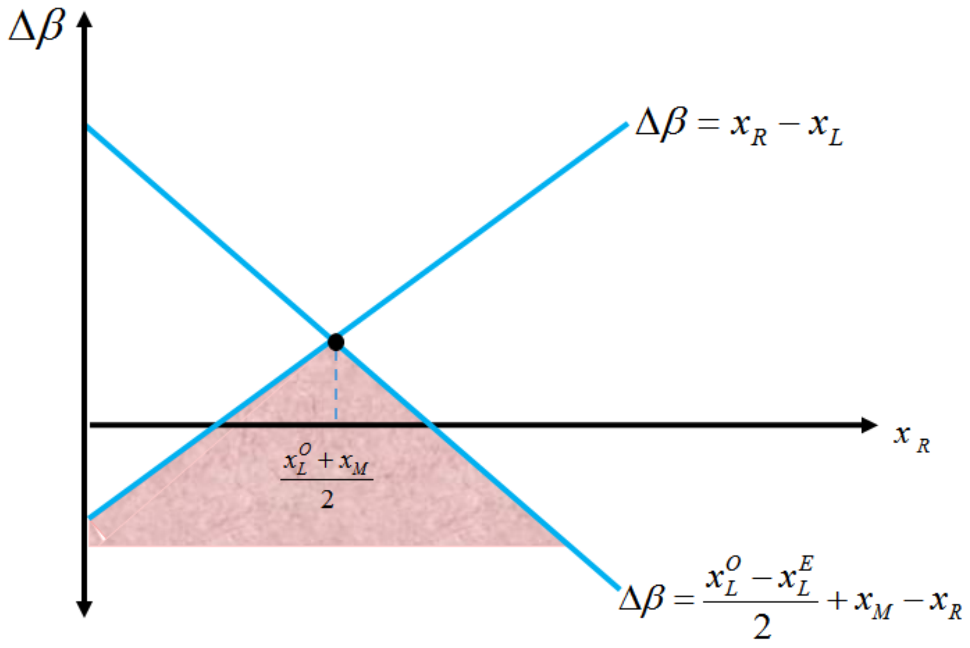

Figure 3-1: Optimal platform of Party R.

[2]If $x_L > x_M$, a symmetric argument shows that $x_R^* = \frac{x_L^E + x_M}{2}$ and so, parties would not preserve their ideological cleavages since $x_R^* \leqslant x_L$.

Since $x_L \leq x_M$ implies $x_L^O \leq 2x_M - x_L^E$, the best response function of Party R to x_L^O is

$$x_R^*(x_L^O) = \begin{cases} x_M & \text{if } x_L^O \leq x_M \\ \frac{x_L^O + x_M}{2} & \text{if } x_M < x_L^O \leq 2x_M - x_L^E \end{cases}. \tag{3.9}$$

Substituting x_R^* in either (3.6) when $x_i = x_M$ or in (3.7), we deduce the values of $\Delta\beta$ for which Party R achieves a majority of votes

$$\Delta\beta < \frac{x_L^O + x_M}{2} - x_L = \frac{x_M - x_L^E}{2}. \tag{3.10}$$

Stage 1:

We finally solve for the best platform of the opportunistic faction. This faction maximizes the probability of achieving a majority of votes.

When $x_L^O \leq x_M$ then $x_R^* = x_M$. By (3.5), Party L obtains a majority of votes for every value of the differential valence $\Delta\beta > x_M - x_L$. Since $x_L = \frac{x_L^E + x_L^O}{2}$, the strategy x_L^O with which Party L maximizes its probability of winning (and therefore, minimizes the value $x_M - x_L$) is $x_L^O = x_M$. By doing so, Party L guarantees its victory for every $\Delta\beta > \frac{x_M - x_L^E}{2}$.

When $x_L^O > x_M$ then $x_R^* = \frac{x_L^O + x_M}{2}$. By (3.10), Party L obtains a majority of votes for every value of the differential valence $\Delta\beta > \frac{x_M - x_L^E}{2}$, which is independent of x_L^O.

Therefore, every $x_L^O \in [x_M, 2x_M - x_L^E]$ is equivalent in terms of the expected payoff that Party L obtains. We conclude that backward induction displays the following equilibrium predictions:

Proposition 1: *Every pair of policy positions* (x_L^{*O}, x_R^*) *such that*

$$x_L^{*O} \in \left[x_M, 2x_M - x_L^E\right] \text{ and } x_R^* = \frac{x_L^{*O} + x_M}{2}$$

is an equilibrium of the competition game. Besides, in every equilibrium, if the valence advantage of Party L over Party R satisfies that $\Delta\beta > \frac{x_M - x_L^E}{2}$ *then Party L wins, if*

$\Delta\beta < \frac{x_M - x_L^E}{2}$ then Party R wins, and in the remaining case there is a tie.

Our result shows that there are multiple equilibria and in all of them the opportunistic faction of Party L sets its platform at the median x_M or above the median. Party R sets its platform in the midpoint between the median x_M and x_L^{*O}. Besides, Party L needs a positive differential valence to defeat Party R even though Party L has a first mover advantage.

3.4 Conclusion

We have showed that there are no electoral benefits associated to a two-faction party when its opponent has a single leader. Surprisingly, the Downsian equilibrium in which both parties offer the same policy position is not an equilibrium in this framework, and parties have incentives to offer divergent platforms.[3] In every equilibrium, the expected platform of the two-faction party, and the platform of its opponent locate at a different side of the median voter's ideal policy.

From the voters' viewpoint, competition between heterogeneous parties generates more pluralistic political platforms. From the parties' viewpoint, however, the ballot box punishes the multi-faction party when competing against a united opponent.

[3]If Party L had a single leader, in the unique equilibrium of the proposed game, both parties would propose the median's ideal policy.

Chapter 4

When Parties want to be Ambiguous but they can't

Coauthored with Socorro Puy. Submitted.

4.1 Introduction

Ambiguity as a political party strategy was first mentioned by Downs (1957), assuring that in a two-party competition,"it increases the number of voters to whom a party may appeal." In the same vein, some authors refer to "catch-all parties" (Kirchheimer, 1966; Williams, 2009) as those that try to appeal to a broad group of voters by being vague about the preferences of the party leadership.

One of the advantages of ambiguous policy platforms, as opposed to a single policy, is that it can facilitate intra-party politics. That is, political parties may find it easier to agree on a wider set of close policies instead of reaching an agreement on a concrete policy platform. Moreover, a party can address a greater fraction of voters when proposing an ambiguous platform.

We propose a theoretical model in which political parties are office-seeking and have a taste for ambiguity within the framework of the Downsian model of political competition

(Downs, 1957).[1] We solve the proposed model using the concept of Nash equilibrium in weakly undominated strategies and analyze the extent to which ambiguous strategies are sustained in equilibrium.

Political parties in our model simultaneously propose ambiguous policy platforms in the form of lottery stands which assign a probability distribution over certain set of policies (see also Zeckhauser, 1969; Fishburn, 1972; Page, 1976; Mckelvey, 1980). We account for discrete and continuous lottery stands, where the former are probability distributions over a discrete set of policies, and the latter are continuous probability distributions over an interval of the policy space. In contrast to Berger et al. (2000), parties propose not only a mean policy but also the variance of its policy platform.

Our main result is that office-seeking incentives discard the existence of equilibria with ambiguous strategies. We find that when office-seeking incentives are high in comparison to parties' incentives for ambiguity, there is a unique equilibrium in which parties converge to the ideal policy of the median voter. That is, political parties sacrifice ambiguity when competing for votes. When office-seeking incentives are low in comparison to parties' incentives for ambiguity, there is no equilibrium. The intuition for this last result is that, on the one hand, when parties propose ambiguous platforms with an equal expected value, the less ambiguous proposal gets more votes. On the other hand, when parties converge to a tying situation in which both propose the ideal policy of the median voter, there are incentives to deviate to an ambiguous platform. Only when the benefits from holding office are null, we find that in the unique equilibrium, parties propose ambiguous platforms.

Previous literature in strategic ambiguity is also related to our results. In particular, voters with risk acceptance attitudes is a key assumption which induces equilibria with ambiguous platforms (Shepsle, 1972; Aragonès and Postlewaite, 2002). The seminal contribution by Shepsle (1972) studies the case in which an incumbent that proposes a

[1] According to Downs (1957), if two office-seeking parties compete in the one-dimensional policy space, in the unique equilibrium, they locate at the median voter position.

single policy competes against a challenger that proposes a lottery stand. He finds that when a majority of voters is risk-acceptant, the challenger chooses a lottery stand when the incumbent locates at the median. Aragonès and Postlewaite (2002) show equilibrium strategies in which parties are ambiguous. Their results rest on two assumptions: intensity of voters preferences over candidates, and some restrictions over the set of lottery stands. They show that when a Condorcet winner is not the first choice for a majority, it can be defeated by an ambiguous strategy. As shown by Shepsle (1972), such result is not possible when voters' preferences are risk-averse.[2]

There are other related contributions (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000), in which equilibria with ambiguous platforms are deduced when combining two assumptions, parties' uncertainty about the median voter and parties' preferences for ambiguity. Alesina and Cukierman (1990) introduce a dynamic electoral model characterized by policy-motivated parties that are uncertain about the location of the median voter and by voters which are not fully informed about the preferences of the incumbent. Politicians, in this case, face a trade-off between the policies that maximize their choices of re-election and the party ideology. Aragonès and Neeman (2000) propose a two-stage political competition model. In the first stage, candidates choose ideologies and in the second, they choose their levels of ambiguity. As in our model, candidates benefit from winning the elections and from proposing ambiguous platforms. In contrast to our model, candidates are uncertain about the median voter position. When office-seeking incentives are sufficiently high, candidates choose unambiguous policies and locate at the median. When candidates assign a greater weight to ambiguity, they choose different ideologies and equal levels of ambiguity.

Our model neither accounts for parties' uncertainty about voters nor for voters' risk-acceptance attitudes, but we account for parties with a taste for ambiguous platforms. In this way, we predict the extent to which the combination of parties' taste for ambigu-

[2] There is recent empirical evidence that tries to estimate voters' risk attitudes (see Tomz and Van Houweling (2009), Berinsky and Lewis (2007) and Morgenstern and Zeckmeuster (2011), among others).

ity and office-holding incentives induce parties to propose ambiguous policy strategies. While most of the contributions to the literature consider continuous lotteries (see, e.g., Aragonès and Neeman, 2000; Berger et al. 2000; Laslier, 2006), we additionally allow for discrete lotteries. As far as we know, our proposal is the first which accounts for these two types of ambiguous strategies – discrete and continuous lottery stands –. Regarding voters' attitude towards risk, voters in our model are indifferent or strictly prefer the certain expected value of a lottery, to the lottery itself, that is, we account for both types, risk-neutral and risk-averse voters.

The rest of the paper is organized as follows. In the next section we describe the model. Section 3 analyzes voters' preferences over lottery stands. Section 4 describes equilibrium strategies. Section 5 concludes. We relegate all the proofs to the Appendix.

4.2 Model

A general election is going to be held in which, by majority voting, voters will elect one out of two political parties, Party A and Party B. There is a continuum of voters. Each voter i has an ideal policy $x_i \in [0,1]$. The ideal policies of the voters are distributed over the interval $[0,1]$ according to a continuous and strictly increasing distribution function, which is common-knowledge. We denote the ideal policy of the median voter by $x_M \in [0,1]$.

The platform of each political party can be a single policy $x \in [0,1]$ or a lottery stand, which consists of a probability distribution over some policies in $[0,1]$. For the sake of simplicity, we assume that lottery stands are uniform distributions.

Lottery stands are interpreted by voters as ambiguous policy proposals and every voter equally interprets the probability with which each party will implement each of its proposals once in office. Parties can propose two types of lottery stands: continuous and discrete.

Continuous lottery stands are continuous uniform distributions over subintervals of the policy space. Continuous lottery stands are characterized by two parameters (x, ε), where $x \in [0,1]$ is the mean of the lottery interval and ε is the level of ambiguity that defines the interval $[x-\varepsilon, x+\varepsilon]$ of possible implemented policy. The maximum level of ambiguity is denoted by $\varepsilon^{\max} = \frac{1}{2}$, which corresponds to a lottery stand centered in the midpoint of the unit interval $x = \frac{1}{2}$ and such that its extreme policies coincide with the upper and lower bound of the unit interval. Therefore, the level of ambiguity that a party $j \in \{A, B\}$ can propose is contained in the interval $\varepsilon_j \in \left[0, \frac{1}{2}\right]$.

Discrete lottery stands are discrete uniform distributions over a finite number of policy stands in the interval $[0,1]$, with the particular simplifying assumption that every two adjacent policies gather the same distance. Thus, discrete lottery stands are characterized by a set of m policies $X = \{x_1, x_2, ..., x_m\}$ where each single policy belongs to the policy space $[0,1]$ and where policies can be ordered as follows: $x_1 < x_2 < ... < x_m$. The smallest proposal (or the leftist) is x_1 and, in the opposite side, the greatest (or rightist) proposal is x_m. As already mentioned, every two adjacent policies gather the same distance, which implies that $x_2 - x_1 = x_3 - x_2 = ... = x_m - x_{m-1}$.[3] Thus, if the number of policies m is an odd number, the mean policy is one of the polices in X (for example, if $m = 3$, then the mean policy is x_2). However, if the number of policies is even then, the mean policy does not belong to the set X. In order to characterize a discrete lottery stand, we use three parameters (m, x, ε) where m is an integer number which indicates the number of policies, policy x is the mean policy which coincides with one of the policies in the lottery when m is odd and it does not coincide when m is even, and finally ε is the level of ambiguity which satisfies that $x - x_1 = x_m - x = \varepsilon$.

We use notation $L_m(x, \varepsilon)$ to represent a discrete lottery stand with m proposals centered around policy x and where ε measures the distance between the mean and the extreme policies in the lottery stand. For example, the lottery stand $L_2(x, \varepsilon)$ consists of two policies $\{x-\varepsilon,\ x+\varepsilon\}$, where each of them gathers equal probability. The lottery

[3] Related to this assumption, Aragonès and Xefteris (2014) propose a policy space with equidistant policies.

stand $L_3(x, \varepsilon)$ consists of three policies $\{x - \varepsilon, x, x + \varepsilon\}$ where each of them gathers equal probability, and so on and so forth. A particular case is $m = 1$ where the lottery stand is a degenerate distribution, $\varepsilon = 0$, and this is denoted by $L_1(x, 0)$. Note that when a party proposes a single policy, there is no uncertainty regarding the policy that this party will implement in the case of holding office. When $m \to \infty$, the proposed lottery is continuous. In this case, we use notation $L_C(x, \varepsilon)$ to represent the continuous lottery stand centered in x and with a level of ambiguity ε. To simplify notation, when possible, we write L_m when referring to discrete lottery stands and L_C when referring to continuous lottery stands.

Political parties do not only care about winning the elections but also about their proposed level of ambiguity ε. When proposing a lottery stand instead of a single policy, the political party represents a wider range of the policy space and this entails certain benefits in terms of internal stability among its members (Alesina and Cukierman, 1990; Aragonès and Neeman, 2000).[4] Besides, parties derive benefits from holding office when winning the elections.

Let $\mathbb{L}$ denotes the space containing every lottery stand or degenerate lottery that a party can propose, i.e., in terms of a game form, $\mathbb{L}$ is the strategy space of Party A and Party B. Both parties simultaneously announce their strategies, after which elections are held and voters cast their ballots. The party holding office is the one achieving a majority of votes and, in the case of a tie, both parties face an equal probability of holding office. Once in office, the party can implement one of the policy proposals included in its announced lottery.

Given two lottery stands, the one proposed by Party A and that of Party B, $(L^A, L^B) \in \mathbb{L}^2$, the preferences of each party $j \in \{A, B\}$ over these proposals are represented by the following utility function v_j:

[4] Our results are robust to the case of including the number of policies m as an extra-benefit in the parties' utility function.

$$v_j(L^A, L^B) = \begin{cases} \varepsilon_j + \alpha & \text{if Party } j \text{ wins} \\ \varepsilon_j + \frac{\alpha}{2} & \text{if parties tie} \\ \varepsilon_j & \text{if Party } j \text{ loses} \end{cases} \tag{4.1}$$

where $\varepsilon_j \in \left[0, \frac{1}{2}\right]$ is the level of ambiguity proposed by Party j and $\alpha \geq 0$ represents the benefits derived from holding office. In the case of winning and when $\alpha > 0$, the party derives benefits not only from winning but also from its level of ambiguity. In the case of a tie, both political parties gather an equal probability of holding office and therefore, parties just account for the value $\frac{\alpha}{2}$ plus their proposed level of ambiguity. In the case of losing the elections, the parties also derive a positive payoff when holding certain level of ambiguity.

The preferences of each voter i over single policies are represented by the negative absolute distance between the policy proposals and the ideal policy of the voter, x_i. In this way, the preferences of voters are single-peaked over the policy space and can be represented by the utility function $u_i(x) = -|x - x_i|$. Preferences of voters over discrete lottery stands are measured by the von Neumann-Morgenstern utility representation so that the expected utility is

$$U_i(L_m) = E_{L_m}[u_i(x)] = \frac{1}{m} \sum_{x_n \in X_m} u_i(x_n) = -\frac{1}{m} \sum_{n=1}^{m} |x_n - x_i| \tag{4.2}$$

where every policy is weighted by voters with an equal probability.

In the limit, when $m \to \infty$, the lottery stand is the uniform distribution over the policies in the interval $[x - \varepsilon, x + \varepsilon]$, characterized by the uniform density function $f(x_n) = \frac{1}{2\varepsilon}$ for every $x_n \in [x - \varepsilon, x + \varepsilon]$. In this case, the von Neumann-Morgenstern utility representation over the continuous lottery yields the following expected utility

$$U_i(L_C) = E_{L_C}[u_i(x)] = \int_{x-\varepsilon}^{x+\varepsilon} u_i(x_n) f(x_n) dx_n = -\int_{x-\varepsilon}^{x+\varepsilon} \frac{|x_n - x_i|}{2\varepsilon} dx_n. \tag{4.3}$$

Given the strategy of the parties $\left(L^A, L^B\right)$, agent i votes for Party A when $U_i(L^A) > U_i(L^B)$, votes for Party B when $U_i(L^A) < U_i(L^B)$ and abstains from voting when $U_i(L^A) = U_i(L^B)$. Another interpretation of this latter case is that citizens are not motivated to vote when parties' platforms are not substantially different (Hortala-Vallve and Esteve-Volart, 2011).

When elections are held, the party holding office is the one achieving a majority of votes. In the case of a tie, both parties face an equal probability of winning. When both parties propose the same strategy $L^A = L^B$, we assume that there is a tie.

We say that (L^A, L^B) is an equilibrium when none of the political parties can benefit from unilateral deviations of its strategy and moreover, both strategies, L^A and L^B, are weakly undominated.

Formally, the pair of strategies $(L^A, L^B) \in \mathbb{L}^2$ is an *equilibrium* when i) and ii) are satisfied:

i) $v_A(L^A, L^B) \geq v_A(L^{A'}, L^B)$ for every $L^{A'} \in \mathbb{L}$ and $v_B(L^A, L^B) \geq v_B(L^A, L^{B'})$ for every $L^{B'} \in \mathbb{L}$,

ii) $\nexists L^{A'} \in \mathbb{L}$ such that $v_A(L^{A'}, L) \geq v_A(L^A, L)$ for every $L \in \mathbb{L}$ with strict inequality for some L, and $\nexists L^{B'} \in \mathbb{L}$ such that $v_B(L, L^{B'}) \geq v_B(L, L^B)$ for every $L \in \mathbb{L}$ with strict inequality for some L.

Requirement (i) implies that (L^A, L^B) is a Nash Equilibrium, and requirement (ii) implies that neither L^A nor L^B are weakly dominated strategies. That is, the proposed equilibrium concept is Nash equilibrium in weakly undominated strategies.

4.3 Voting over lottery stands

In this section, we evaluate the preferences of voters over different lottery stands. As an example, consider the comparison between two lotteries: $L_2(x, \varepsilon)$ and $L_3(x, \varepsilon)$. These lotteries are equal in terms of the level of ambiguity ε and the mean policy x, and they only differ in the number of policies, $m = 2$ against $m = 3$. Whereas in the lottery

$L_2(x, \varepsilon)$ the set of policies is $X_2 = \{x - \varepsilon, x + \varepsilon\}$, in the lottery $L_3(x, \varepsilon)$ the set of policies is $X_3 = \{x - \varepsilon, x, x + \varepsilon\}$. Evaluating the preferences of voters with ideal policy $x_i \in [0, x - \varepsilon]$ over $L_2(x, \varepsilon)$,

$$U_i(L_2) = -\frac{1}{2}\left(|x - \varepsilon - x_i| + |x + \varepsilon - x_i|\right) = -|x - x_i|.$$

The peak of the voters with ideal policy in the interval $[0, x - \varepsilon]$ is out of the range of the lottery L_2 (or coincides with the lottery's lower bound), then both policies $\{x - \varepsilon, x + \varepsilon\}$ are in the decreasing side of the single-peaked shape of the preferences of voters. Therefore, each of these voters is risk-neutral with respect to this lottery stand and $U_i(L_2) = u_i(x)$, i.e., the utility of the lottery is equal to the lottery's expected value.

A similar reasoning follows for every voter with ideal policy in the interval $[x + \varepsilon, 1]$ and for whom $U_i(L_2) = u_i(x)$.

For every voter i with ideal policy in the open interval $(x - \varepsilon, x + \varepsilon)$, we have that

$$U_i(L_2) = -\frac{1}{2}\left(|x - \varepsilon - x_i| + |x + \varepsilon - x_i|\right) = -\varepsilon. \tag{4.4}$$

Regarding $L_3(x, \varepsilon)$, for every voter i such that $x_i \in [0, x - \varepsilon]$ or $x_i \in [x + \varepsilon, 1]$, we can also apply risk neutrality by which $U_i(L_3) = u_i(x) = -|x - x_i|$. And for every other voter i such that $x_i \in (x - \varepsilon, x + \varepsilon)$, we have that

$$U_i(L_3) = -\frac{1}{3}\left(|x - \varepsilon - x_i| + |x - x_i| + |x + \varepsilon - x_i|\right) = -\frac{2\varepsilon}{3} - \frac{|x - x_i|}{3}. \tag{4.5}$$

As a result, all the voters with ideal policy in the intervals $[0, x - \varepsilon]$ and $[x + \varepsilon, 1]$ derive an equal utility from the two lottery stands given that $U_i(L_2) = U_i(L_3) = -|x - x_i|$ and they are indifferent between the two lotteries. For the remaining voters, we compare Expressions (4.4) and (4.5). Since $|x - x_i| < \varepsilon$, for every voter with ideal policy $x_i \in (x - \varepsilon, x + \varepsilon)$, we deduce that $U_i(L_2) < U_i(L_3)$.

Thus, we have showed that $L_3(x, \varepsilon)$ weakly dominates $L_2(x, \varepsilon)$: both strategies propose an equal level of ambiguity, but $L_3(x, \varepsilon)$ provides greater utility to a fraction of voters which translates into additional situations in which a party can win when proposing $L_3(x, \varepsilon)$ instead of $L_2(x, \varepsilon)$.[5]

Given a lottery stand $L \in \mathbb{L}$, we refer to the *outsider voters* as those whose ideal policies are out of the bounds of the lottery, that is, those for whom $x_i \in [0, x - \varepsilon]$ and $x_i \in [x + \varepsilon, 1]$. We refer to *insider voters* as those whose ideal policies are inside the bounds of the lottery, that is, $x_i \in (x - \varepsilon, x + \varepsilon)$.

In general, given a lottery $L_m \in \mathbb{L}$, its corresponding set of policies X_m can be calculated as a sequence of policies defined by

$$X_m = \left\{ x - \frac{m - 1 - 2j}{m - 1} \varepsilon \right\}_{j=0}^{m-1}. \tag{4.6}$$

From Expression (4.6), we can describe the set of policies for every value $m \geq 2$, for example $m = 4$ yields $X_4 = \left\{ x - \varepsilon, x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3}, x + \varepsilon \right\}$.

The utility of an outsider voter over a lottery stand is equal to the utility of the lottery's expected value, that is $U_i(L) = -|x - x_i|$ for all $L \in \mathbb{L}$. The utility of an insider voter depends on the number of policies m. According to Expressions (4.2) and (4.6), we deduce that the expected utility of an insider voter over every discrete lottery stand is measured by

$$U_i(L_m) = -\frac{1}{m} \sum_{j=0}^{m-1} \left| x - \frac{m - 1 - 2j}{m - 1} \varepsilon - x_i \right|. \tag{4.7}$$

When the lottery stand is a continuous lottery L_C over the interval $[x - \varepsilon, x + \varepsilon]$, the corresponding density function is $f(x) = \frac{1}{2\varepsilon}$. Expression (4.3) evaluates the expected utility of an insider voter over L_C. Decomposing the integral and substituting the absolute

[5] The fact that the lottery $L_3(x, \varepsilon)$ defeats the lottery $L_2(x, \varepsilon)$ does not contradict the result by Zeckhauser (1969). This author shows that there is always a lottery stand with two policies that can defeat a lottery stand with three policies. The range of the lotteries in the comparison proposed by Zeckhauser, however, is different.

values we deduce that

$$U_i(L_C) = -\frac{1}{2\varepsilon}\left[\int_{x-\varepsilon}^{x_i} (x_i - x_n)\, dx_n + \int_{x_i}^{x+\varepsilon} (x_n - x_i)\, dx_n\right] \tag{4.8}$$

where note that $x_i - x_n \geq 0$ for every $x_n \in [x - \varepsilon, x_i]$ and $x_n - x_i \geq 0$ for every $x_n \in [x_i, x + \varepsilon]$. Solving for the integrals and simplifying we derive the following expression[6]

$$U_i(L_C) = -\frac{(x - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}. \tag{4.9}$$

Now, we compare the expected utility over the continuous lottery $L_C(x, \varepsilon)$ with the expected utility over the discrete lottery $L_m(x, \varepsilon)$ where $m > 1$. Both lotteries gather an equal level of ambiguity and an equal mean policy position x. These two lotteries are equivalent for the outsider voters given that they provide an equal expected value x. For the insider voters, however, these two lotteries are not equivalent. Given Expression (4.7) and Expression (4.9), the differential utility for every insider voter i over these two lotteries is defined by

$$F_i = U_i(L_C) - U_i(L_m). \tag{4.10}$$

If $L_m = L_2(x, \varepsilon)$, we have that $U_i(L_2) = -\varepsilon$ and comparing this utility level with that of Expression (4.9), we deduce that condition $-\frac{(x-x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2} > -\varepsilon$ implies that $F_i > 0$. Simplifying yields, $x - x_i < \varepsilon$, and given that the insider voters are those for whom $x_i \in (x - \varepsilon, x + \varepsilon)$, this condition holds.

We have then shown that, for every voter, when comparing the lotteries $L_C(x, \varepsilon)$ and $L_2(x, \varepsilon)$, $U_i(L_C) \geq U_i(L_2)$ and where this inequality is strict for the insider voters. In addition, the lotteries $L_C(x, \varepsilon)$ and $L_2(x, \varepsilon)$ gather an equal level of ambiguity and therefore, according to the utility function of the parties in Expression (4.1), strategy

[6]Expression (8) yields $-\frac{1}{2\varepsilon}[(x_i x_n - \frac{x_n^2}{2})\Big|_{x-\varepsilon}^{x_i} + (\frac{x_n^2}{2} - x_i x_n)\Big|_{x_i}^{x+\varepsilon}] = -\frac{1}{2\varepsilon}[x_i^2 - 2x_i x + x^2 + \varepsilon^2] = -\frac{(x-x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$.

$L_2(x,\varepsilon)$ is weakly dominated by $L_C(x,\varepsilon)$ (i.e., whatever the strategy of the opponent political party, the strategy $L_C(x,\varepsilon)$ always provides equal or higher payoffs for the political party than the strategy $L_2(x,\varepsilon)$). We deduce that there cannot be an equilibrium in which one or both parties propose the strategy $L_2(x,\varepsilon)$.

The following proposition analyzes the extent to which every discrete lottery stand $L_m(x,\varepsilon)$ is weakly dominated by the continuous lottery stand $L_C(x,\varepsilon)$.

Proposition 5 *According to the preferences of the political parties, every discrete lottery $L_m(x,\varepsilon)$ with $m>1$ is weakly dominated by the continuous lottery $L_C(x,\varepsilon)$ with equal mean policy x and equal level of ambiguity ε.*

Proof. For every insider voter, the utility over $L_m(x,\varepsilon)$ is measured by (4.7). We can decompose and substitute the absolute values of Expression (4.7) so that

$$U_i(L_m) = -\frac{1}{m}\left[\sum_{j=0}^{j^*}\left(-\gamma_i + \frac{(m-1-2j)}{m-1}\varepsilon\right) + \sum_{j^*+1}^{m-1}\left(\gamma_i - \frac{(m-1-2j)}{m-1}\varepsilon\right)\right]$$

where $\gamma_i = x - x_i$ and $j^* = k-1$ with k being the number of policy proposals $x_n \in X_m$ such that $x_n < x_i$. The above expression is equivalent to

$$\begin{aligned} U_i(L_m) \quad = \quad & -\frac{1}{m}\Bigg[-(j^*+1)\gamma_i + (j^*+1)\varepsilon - \tfrac{2\varepsilon}{m-1}\sum_{j=0}^{j^*} j + (m-1-j^*)\gamma_i \\ & -(m-1-j^*)\varepsilon + \tfrac{2\varepsilon}{m-1}\sum_{j^*+1}^{m-1} j\Bigg]. \end{aligned}$$

Substituting that $\sum_{j=0}^{j^*} j = \frac{j^*(j^*+1)}{2}$, $\sum_{j^*+1}^{m-1} j = \frac{(n-1)n}{2} - \frac{j^*(j^*+1)}{2}$ and simplifying yields

$$U_i(L_m) = -\frac{1}{m}\left[(m-2-2j^*)\gamma_i + \frac{2\varepsilon(j^*+1)(m-1-j^*)}{m-1}\right].$$

Substituting the above expression and Expression (4.9) into (4.10) we deduce

$$F_i = -\frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2-2j^*)\,\gamma_i}{m} + \frac{2\varepsilon\,(j^*+1)\,(m-1-j^*)}{m\,(m-1)}.$$

Next, we show that $F_i > 0$ for all $\gamma_i \in (-\varepsilon, \varepsilon)$. When $\gamma_i \to \varepsilon$, the value of $j^* = 0$ so that $F_i(\gamma_i) = \frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2)\gamma_i}{m} + \frac{2\varepsilon}{m}$. This function is continuous in γ_i from where

$$\lim_{\gamma_i \to \varepsilon} F_i(\gamma_i) = F_i(\varepsilon) = 0.$$

The value of the parameter j^* changes with γ_i. Expression (4.6) defines the values x_n in X_m. Then, we can define the values γ_i for which j^* changes, as the ones in the sequence $\left\{\frac{m-1-2j}{m-1}\varepsilon\right\}_{j=0}^{m-1}$. In particular, for every agent i such that $\gamma_i \in \left[\frac{m-1-2(j^*+1)}{m-1}\varepsilon, \frac{m-1-2j*}{m-1}\varepsilon\right]$, the value of j^* is the same (e.g., $j^* = 0$ for all $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$, and $j^* = 1$ for all $\gamma_i \in \left[\frac{m-5}{m-1}\varepsilon, \frac{m-3}{m-1}\varepsilon\right]$). We show that in each of these intervals, F_i' is strictly increasing when evaluated in its lower bound and strictly decreasing when evaluated in its upper bound. Solving for the derivative evaluated in its lower bound

$$\frac{\partial F_i(\frac{m-1-2(j^*+1)}{m-1}\varepsilon)}{\partial \gamma_i} = -\frac{m-1-2(j^*+1)}{m-1} + \frac{m-2-2j^*}{m} = \frac{2(j^*+1-m)}{(m-1)m}$$

where given that $0 \leq j^* \leq m-2$ then, $j^*+2 \leq m$, from where $j^*+1 < m$ and the above derivative is strictly negative. Solving for the derivative evaluated in its upper bound

$$\frac{\partial F_i(\frac{m-1-2j^*}{m-1}\varepsilon)}{\partial \gamma_i} = -\frac{m-3-2j^*}{m-1} + \frac{m-2-2j^*}{m} = \frac{2+2j^*}{(m-1)m} > 0.$$

Given that $\frac{\partial^2 F_i}{\partial \gamma_i^2} = -\frac{1}{\varepsilon} < 0$, F_i is concave in each of the proposed intervals. Besides, $\frac{\partial^3 F_i}{\partial \gamma^3} = 0$ implies that in each of the intervals, function F_i is symmetric around its maximum value. Let $\gamma_0^{\max}$ be the agent with maximum F_i in the interval where $j^* = 0$. Starting from the interval where $j^* = 0$ and therefore, $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$, we first show that the agent in the upper bound, $\gamma_i = \frac{m-3}{m-1}\varepsilon$ is closer to $\gamma_0^{\max}$ than the agent in the lower bound, $\gamma_i = \varepsilon$. This feature, together with the symmetry of F_i and the fact that $F_i(\varepsilon) = 0$ implies that

$F_i > 0$ in the interval $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$. Second, we show that in every subsequent interval, where $j^* > 0$, the agent in the upper bound is not further from $\gamma_{j^*}^{\max}$ than the agent in the lower bound. By continuity of F_i this implies that $F_i > 0$ in the interval $\gamma_i \in (-\varepsilon, \varepsilon)$. In each interval, defined by a value of j^* where $0 \leq j^* \leq m-2$, the lower and upper bounds are defined by $\gamma_L = \frac{m-1-2(j^*+1)}{m-1}\varepsilon$ and $\gamma_U = \frac{m-1-2j^*}{m-1}\varepsilon$, respectively. We want to show that $\gamma_{j^*}^{\max} - \gamma_L \leq \gamma_U - \gamma_{j^*}^{\max}$ for every j^*, which is equivalent to showing that $\gamma_{j^*}^{\max} \leq \frac{\gamma_U+\gamma_L}{2}$. The maximum in each interval is deduced from the first order condition $\frac{\partial F_i}{\partial \gamma_i} = -\frac{\gamma_i}{\varepsilon} + \frac{(m-2-2j^*)}{m} = 0$ which yields $\gamma_{j^*}^{\max} = \varepsilon\frac{m-2-2j^*}{m}$. Calculating the mean value $\frac{\gamma_U+\gamma_L}{2} = \varepsilon\frac{m-2j^*-2}{(m-1)}$. Substituting in condition $\gamma_{j^*}^{\max} \leq \frac{\gamma_U+\gamma_L}{2}$ yields $(m-1) \leq m$ which always holds. This shows that $F_i > 0$ for every insider voter, which implies that $L_C(x, \varepsilon)$ weakly dominates $L_m(x, \varepsilon)$ for every $m > 1$. ■

In the proof we show that for every insider voter, the utility derived from the continuous lottery is greater than the utility derived from every other lottery stand with the same mean policy and with the same level of ambiguity. Besides, given that outsider voters are indifferent between these two lottery stands, each political party derives a greater utility when proposing the continuous lottery than when proposing every other discrete lottery with equal mean policy and equal level of ambiguity.

For a given value $\varepsilon > 0$ and a fixed policy x, Figure 1 illustrates the value F_i in Expression (4.10) as a function of x_i, that is, it indicates the differential utility between the continuous lottery L_C and the discrete lottery L_m when $m = 2$ up to $m = 10$. We show how the greater m, the smaller the differential utility and, in all the cases, $F_i > 0$ for all $x_i \in (x - \varepsilon, x + \varepsilon)$.

Finally, we argue why the lottery $L_C(x, \varepsilon)$ and the single policy $L_1(x, 0)$ cannot be compared in terms of domination. Note that in those situations in which both proposals, $L_C(x, \varepsilon)$ and $L_1(x, 0)$, display the same electoral result, the party strictly prefers the continuous lottery over the single policy since it has a positive level of ambiguity $\varepsilon > 0$. However, in some situations $L_1(x, 0)$ can guarantee the electoral victory but $L_C(x, \varepsilon)$

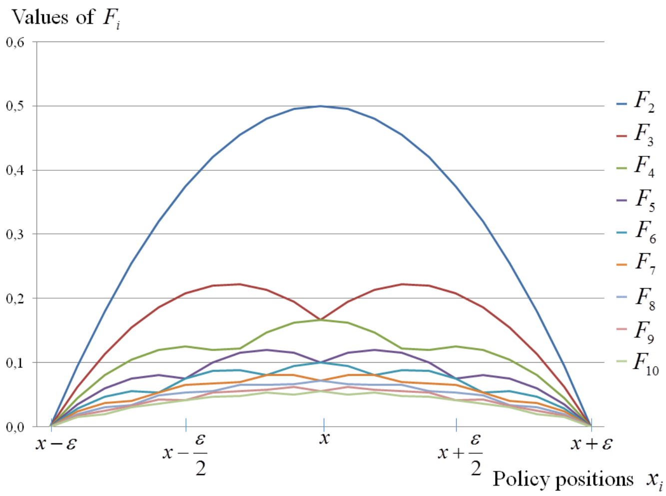

Figure 4-1: Values of the differential utility when $m = 2$ up to $m = 10$.

cannot. This is the case, for instance, when one party proposes $L_1(x, 0)$ and the other $L_C(x, \varepsilon)$. Then, for those agents with ideal policy close to x, they strictly prefer $L_1(x, 0)$ over $L_C(x, \varepsilon)$. If a majority of voters has their ideal policies around x, this shows that the single policy can guarantee the victory over the continuous lottery. Consequently, in this situation, the party strictly prefers $L_1(x, 0)$ over $L_C(x, \varepsilon)$ if the benefits from holding office are sufficiently high. Thus, the non-weakly dominated set of strategies in our setting contains continuous lotteries and single policies.[7]

4.4 Equilibrium analysis

In this section we analyze equilibrium existence. We first discard non-equilibrium strategies and we second analyze lottery stands that either differ in the proposed mean policies or in the proposed level of ambiguity.

Our first result shows that there cannot be an equilibrium in which one party wins and the other loses the elections.

[7] Thus, the non-weakly dominated set of strategies in our setting is equivalent to the set of strategies proposed by Aragonès and Neeman (2000).

Lemma 1 *If an equilibrium exists then, there is a tie.*

Proof. When $\alpha > 1$, the minimum payoff that a political party derives from tying $\frac{\alpha}{2}$ is always greater than the maximum payoff associated to losing $\varepsilon^{\max}$ and therefore, a party always strictly prefers tying to losing. When $\alpha \leq 1$, however, this is not the case. Suppose to the contrary, that there is an equilibrium in which one party loses and the other wins. We distinguish two cases depending on the value of α.

When $\alpha > 1$, the losing party can deviate by selecting the same strategy as the winning party, which guarantees a tie. This is a profitable deviation in contradiction with an equilibrium situation.

When $\alpha \leq 1$, consider *wlog* that Party A wins and Party B loses. Then, because Party B cannot improve deviating, its unique optimal strategy is $L^B = L_C(\frac{1}{2}, \varepsilon^{\max})$ with $v_B = \varepsilon^{\max} = \frac{1}{2}$ since for every other lottery stand such that $\varepsilon < \varepsilon^{\max}$, $v_B = \varepsilon < \frac{1}{2}$. Given that Party A is winning, its strategy has to be different from that of Party B. Let $L^A = L_C(x, \varepsilon)$ with $\varepsilon \in [0, \varepsilon^{\max})$, then, Party A's utility is $v_A = \alpha + \varepsilon$. We find, however, that $L^A = L_C(x, \varepsilon)$ with $\varepsilon \in [0, \varepsilon^{\max})$ and $L^B = L_C(\frac{1}{2}, \varepsilon^{\max})$ is not a Nash equilibrium: if Party A deviates to $L^{A'} = L_C\left(\frac{1}{2}, \varepsilon'\right)$ where $\varepsilon < \varepsilon' < \varepsilon^{\max}$, it still wins (as we show next) and its payoffs increase $v_A(L^{A'}, L^B) = \alpha + \varepsilon' > v_A(L^A, L^B) = \alpha + \varepsilon$. We show that Party A wins when $L^{A'} = L_C\left(\frac{1}{2}, \varepsilon'\right)$ and $L^B = L_C(\frac{1}{2}, \varepsilon^{\max})$. By Expression (4.9), $\frac{\partial U_i}{\partial \varepsilon} = \frac{(\frac{1}{2} - x_i)^2}{\varepsilon^2} - 1$ where the fact that $(\frac{1}{2} - x_i)^2 < \varepsilon^2$ implies that $\frac{\partial U_i}{\partial \varepsilon} < 0$, that is, insider voters prefer the lottery with smaller ε whereas outsider voters remain indifferent. Thus, voters with ideal policy $x_i \in (\frac{1}{2} - \varepsilon', \frac{1}{2} + \varepsilon')$ prefer the lottery with smaller ε, i.e., $L^{A'} = L_C\left(\frac{1}{2}, \varepsilon'\right)$. Then, for ε' sufficiently close to $\varepsilon^{\max}$, Party A obtains a majority of votes and wins, in contradiction with an equilibrium situation. ■

Lemma 1 shows that every pair of strategies in which one party wins and the other loses cannot be sustained as a Nash equilibrium. The proof distinguishes between two cases: when $\alpha > 1$ (the benefits from holding office are more than two times the maximum benefits derived from ambiguity $\varepsilon^{\max} = \frac{1}{2}$), and when $\alpha \leq 1$. When $\alpha > 1$, the losing party improves inducing a tie by selecting the same strategy as its opponent. When

$\alpha \leq 1$, there can be situations in which the losing party has no incentives to induce a tie, but in those cases, the winning party improves by proposing a more ambiguous lottery stand. In particular, in the hypothetical case of an equilibrium in which a party wins, the losing party proposes the maximum level of ambiguity $\varepsilon^{\max}$. We then show that the winning party always has a profitable deviation in which it increases its proposed level of ambiguity (up to some level below $\varepsilon^{\max}$). Interestingly, this deviation guarantees the electoral victory due to the fact that insider voters, when comparing lotteries with equal mean policies, strictly prefer the less ambiguous one.

From Lemma 1 we deduce that the search for equilibrium strategies can be restricted to tying situations. The following lemma discards certain lottery stands as equilibrium strategies in a tying situation. In particular, we show that in the case of an equilibrium in which parties tie, parties cannot hold different levels of ambiguity.

Lemma 2 *If there is an equilibrium in which parties tie then, parties propose an equal level of ambiguity,* $\varepsilon_A = \varepsilon_B$.

Thus, in a tying situation, parties' taste for ambiguity makes them propose equal ε.

Some additional strategies can also be discarded as equilibrium strategies when the proposed mean policies do not satisfy certain conditions.

Lemma 3 *If there is an equilibrium in which parties tie then, parties' mean policies are either equal* $x_A = x_B$, *or equidistant to the ideal policy of the median voter,* $|x_A - x_M| = |x_B - x_M|$.

The fact that in a tying equilibrium both parties propose an equal level of ambiguity implies that parties mean policies have to be either symmetric around the median or equal. The proof shows that in any other case, there is a party which achieves a majority of votes, in contradiction with a tying equilibrium. When parties propose mean policies that are symmetric around the median voter, all the voters with ideal policy to one of the sides of the median (say right) vote for one of the parties, the voters with ideal policy to the other side of the median voter (left) vote for the other party and the median voter is indifferent between the two proposals. When both parties propose the same mean policy and given that their proposed levels of ambiguity have to be equal, we also obtain a tying situation.

According to the results obtained so far, we next show that we can discard all but one of the parties' strategies in the case of a tying situation. The strategy that is not discarded is the degenerate lottery in which the party proposes the ideal policy of the median voter.

Lemma 4 *If there is an equilibrium in which parties tie then, parties propose the unambiguous ideal policy of the median voter, x_M.*

In the proof we show that equilibrium strategies in which parties propose equal mean policies and certain positive level of ambiguity $\varepsilon > 0$ can be discarded. In fact, when parties propose equal mean policies, one of the parties has incentives to deviate by proposing less ambiguity. We show that the lottery with smaller level of ambiguity ε is more preferred for those insider voters (of one or the other lottery), whereas outsider voters (of both lotteries) remain indifferent between them. This is very intuitive given that once a voter is within the bounds of a lottery stand, the smaller the level of ambiguity, the higher the probability assigned to policy proposals which are closer to the voter's ideal policy. From the outsiders' viewpoint, the two lottery stands yield an equal expected value and these voters are immune to different levels of ambiguity. Therefore, this result reveals that voters dislike ambiguity and that the smaller the range over which the lottery stand is distributed, the greater the electoral support. In every other situation in which the parties propose different mean policies we find that, independent of the level of ambiguity ε, a profitable deviation always exists in which a party moves closer to the median voter.

In our last result, we show that equilibrium existence crucially depends on the magnitude of the office-holding benefits. We distinguish three scenarios: one in which the benefits from holding office are above or equal to one $\alpha \geq 1$, another in which $0 < \alpha < 1$, and the one in which $\alpha = 0$ (i.e., where there are no office-holding benefits).

Proposition 6 *When $\alpha \geq 1$, there is a unique equilibrium (L^A, L^B) in which the two parties propose, with no ambiguity, the ideal policy of the median voter, i.e., $L^A = L^B = L_1(x_M, 0)$. When $0 < \alpha < 1$, there is no equilibrium. When $\alpha = 0$, there is a unique equilibrium (L^A, L^B) in which the two parties propose the maximal level of ambiguity, i.e., $L^A = L^B = L_C\left(\frac{1}{2}, \varepsilon^{\max}\right)$.*

When office-holding incentives are sufficiently rewarding (in comparison to ambiguity), there is a unique equilibrium prediction in which parties converge to the ideal policy of the median voter. Surprisingly, once the benefits from holding office are not high enough (in comparison to the extra-benefits derived from ambiguity), an equilibrium fails to exist. When office-holding benefits are not present, the only prediction consists of both parties proposing the maximal level of ambiguity.

From a theoretical perspective, the proposed payoff function is discontinuous. For example, when parties' strategies are the same and equal to $L_C(\frac{1}{2}, \varepsilon^{\max})$, one of the parties strictly improves announcing $L_C(\frac{1}{2}, \varepsilon)$ where ε is slightly smaller than $\varepsilon^{\max}$, which induces a winning situation. In fact, the discontinuity of the payoff function implies that the standard result by Glicksberg (1952) on existence of mixed-strategy equilibria does not apply to our model. However, according to Dasgupta and Maskin (1986), we cannot discard existence of other type of equilibrium in mixed-strategy.[8] An equilibrium in mixed strategies in which parties do not propose continuous or degenerate lottery stands would imply mixing over different levels of ambiguity or/and mixing over non-connected intervals of the policy space. In both cases, interpretation of such strategies are difficult and this would only apply to a scenario in which $0 < \alpha < 1$ which means that office-holding benefits are less than two times the benefits derived from being ambiguous.

[8] Dasgupta and Maskin (1986) maintain that mixed-strategy equilibria can exist when the set of discontinuities is of (Lebesgue) measure zero.

4.5 Concluding remarks

Our main result states that office-seeking incentives mitigate parties' taste for ambiguity or, in other words, when office-seeking incentives are present, ambiguous policy platforms cannot be sustained as (Nash) equilibrium strategies. When the intensity of parties' preferences over ambiguity is low (with respect to winning the elections), parties converge to the ideal policy of the median voter. However, an equilibrium fails to exist when the intensity of preferences over ambiguity is high. Convergence to the median voter is not any more an equilibrium prediction in this last case since parties have incentives to deviate to an ambiguous platform even when this implies losing the elections.

Our proposal accounts for two types of ambiguous strategies – discrete and continuous lottery stands –. Continuous lotteries, we show, weakly dominate discrete lotteries. Among the continuous lottery stands, when comparing lotteries with equal mean policy, we find that the less ambiguous lottery attracts more votes. Intuitively, once the preferred policy of a voter is within the bounds of a lottery stand, the smaller the level of ambiguity, the higher the probability assigned to policy proposals which are not far from the voter's ideal policy. This result derives from the single-peaked shape of preferences, by which those voters whose ideal policies are within the bounds of a lottery stand behave as risk-averse voters.

According to our results and the existing literature, we conclude that there are two key features that induce ambiguous strategies – voters with risk-acceptance attitudes and parties with uncertainty about voters' preferences –. In particular, in a setting with uncertainty about voters' preferences and a taste for ambiguity, parties may sacrifice some probability of winning in exchange for the extra-benefits derived from proposing an ambiguous platform. When there is no uncertainty about voters' preferences, we show, both parties perfectly predict the election result and this eliminates those profitable trade-offs between ambiguity and votes.

Our theoretical prediction, by which uncertainty about voters' preferences generates ambiguous platforms whereas ambiguous strategies vanish when there is no uncertainty, is open to empirical scrutiny. We propose a two-round election as a suitable scenario for this purpose. In this setting, the first election round can be interpreted as one in which parties are more uncertain about voters' preferences, and the second election round as one with less uncertainty. The variance and location of parties' policy proposals, as perceived by voters in election surveys, reveal information about parties' strategies. This a natural experiment which, we believe, can provide additional insights into the electoral use of strategic ambiguity.

There are several directions in which our proposed theoretical model can be extended. Among others, we can analyze strategic ambiguity in a setting with more political parties and different electoral rules, from majority rule to proportional representation. The analysis of these scenarios is left for further research.

4.6 Appendix

This Appendix presents the proofs of Propositions 1 and 2 and Lemmas 1 to 4.

Proof of Proposition 1. For every insider voter, the utility over $L_m(x,\varepsilon)$ is measured by (4.7). We can decompose and substitute the absolute values of Expression (4.7) so that

$$U_i(L_m) = -\frac{1}{m}\left[\sum_{j=0}^{j^*}\left(-\gamma_i + \frac{(m-1-2j)}{m-1}\varepsilon\right) + \sum_{j^*+1}^{m-1}\left(\gamma_i - \frac{(m-1-2j)}{m-1}\varepsilon\right)\right]$$

where $\gamma_i = x - x_i$ and $j^* = k-1$ with k being the number of policy proposals $x_n \in X_m$ such that $x_n < x_i$. The above expression is equivalent to

$$\begin{aligned} U_i(L_m) \quad = \quad & -\frac{1}{m}\Bigg[-(j^*+1)\gamma_i + (j^*+1)\varepsilon - \tfrac{2\varepsilon}{m-1}\sum_{j=0}^{j^*} j + (m-1-j^*)\gamma_i \\ & -(m-1-j^*)\varepsilon + \tfrac{2\varepsilon}{m-1}\sum_{j^*+1}^{m-1} j\Bigg]. \end{aligned}$$

Substituting that $\sum_{j=0}^{j^*} j = \frac{j^*(j^*+1)}{2}$, $\sum_{j^*+1}^{m-1} j = \frac{(n-1)n}{2} - \frac{j^*(j^*+1)}{2}$ and simplifying yields

$$U_i(L_m) = -\frac{1}{m}\left[(m-2-2j^*)\gamma_i + \frac{2\varepsilon(j^*+1)(m-1-j^*)}{m-1}\right].$$

Substituting the above expression and Expression (4.9) into (4.10) we deduce

$$F_i = -\frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2-2j^*)\gamma_i}{m} + \frac{2\varepsilon(j^*+1)(m-1-j^*)}{m(m-1)}.$$

Next, we show that $F_i > 0$ for all $\gamma_i \in (-\varepsilon, \varepsilon)$. When $\gamma_i \to \varepsilon$, the value of $j^* = 0$ so that $F_i(\gamma_i) = \frac{\gamma_i^2}{2\varepsilon} - \frac{\varepsilon}{2} + \frac{(m-2)\gamma_i}{m} + \frac{2\varepsilon}{m}$. This function is continuous in γ_i from where

$$\lim_{\gamma_i \to \varepsilon} F_i(\gamma_i) = F_i(\varepsilon) = 0.$$

The value of the parameter j^* changes with γ_i. Expression (4.6) defines the values x_n in X_m. Then, we can define the values γ_i for which j^* changes, as the ones in the sequence $\{\frac{m-1-2j}{m-1}\varepsilon\}_{j=0}^{m-1}$. In particular, for every agent i such that $\gamma_i \in \left[\frac{m-1-2(j^*+1)}{m-1}\varepsilon, \frac{m-1-2j*}{m-1}\varepsilon\right]$, the value of j^* is the same (e.g., $j^* = 0$ for all $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$, and $j^* = 1$ for all $\gamma_i \in \left[\frac{m-5}{m-1}\varepsilon, \frac{m-3}{m-1}\varepsilon\right]$). We show that in each of these intervals, F_i' is strictly increasing when evaluated in its lower bound and strictly decreasing when evaluated in its upper bound. Solving for the derivative evaluated in its lower bound

$$\frac{\partial F_i(\frac{m-1-2(j^*+1)}{m-1}\varepsilon)}{\partial \gamma_i} = -\frac{m-1-2(j^*+1)}{m-1} + \frac{m-2-2j^*}{m} = \frac{2(j^*+1-m)}{(m-1)m}$$

where given that $0 \leq j^* \leq m-2$ then, $j^*+2 \leq m$, from where $j^*+1 < m$ and the above derivative is strictly negative. Solving for the derivative evaluated in its upper bound

$$\frac{\partial F_i(\frac{m-1-2j^*}{m-1}\varepsilon)}{\partial \gamma_i} = -\frac{m-3-2j^*}{m-1} + \frac{m-2-2j^*}{m} = \frac{2+2j^*}{(m-1)m} > 0.$$

Given that $\frac{\partial^2 F_i}{\partial \gamma_i^2} = -\frac{1}{\varepsilon} < 0$, F_i is concave in each of the proposed intervals. Besides, $\frac{\partial^3 F_i}{\partial \gamma^3} = 0$ implies that in each of the intervals, function F_i is symmetric around its maximum value. Let $\gamma_0^{\max}$ be the agent with maximum F_i in the interval where $j^* = 0$. Starting from the interval where $j^* = 0$ and therefore, $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$, we first show that the agent in the upper bound, $\gamma_i = \frac{m-3}{m-1}\varepsilon$ is closer to $\gamma_0^{\max}$ than the agent in the lower bound, $\gamma_i = \varepsilon$. This feature, together with the symmetry of F_i and the fact that $F_i(\varepsilon) = 0$ implies that $F_i > 0$ in the interval $\gamma_i \in [\frac{m-3}{m-1}\varepsilon, \varepsilon)$. Second, we show that in every subsequent interval, where $j^* > 0$, the agent in the upper bound is not further from $\gamma_{j^*}^{\max}$ than the agent in the lower bound. By continuity of F_i this implies that $F_i > 0$ in the interval $\gamma_i \in (-\varepsilon, \varepsilon)$. In each interval, defined by a value of j^* where $0 \leq j^* \leq m-2$, the lower and upper bounds are defined by $\gamma_L = \frac{m-1-2(j^*+1)}{m-1}\varepsilon$ and $\gamma_U = \frac{m-1-2j^*}{m-1}\varepsilon$, respectively. We want to show that $\gamma_{j^*}^{\max} - \gamma_L \leq \gamma_U - \gamma_{j^*}^{\max}$ for every j^*, which is equivalent to showing that $\gamma_{j^*}^{\max} \leq \frac{\gamma_U+\gamma_L}{2}$. The maximum in each interval is deduced from the first order condition $\frac{\partial F_i}{\partial \gamma_i} = -\frac{\gamma_i}{\varepsilon} + \frac{(m-2-2j^*)}{m} = 0$ which yields $\gamma_{j^*}^{\max} = \varepsilon\frac{m-2-2j^*}{m}$. Calculating the mean value

$\frac{\gamma_U+\gamma_L}{2} = \varepsilon\frac{m-2j^*-2}{(m-1)}$. Substituting in condition $\gamma_{j^*}^{\max} \leq \frac{\gamma_U+\gamma_L}{2}$ yields $(m-1) \leq m$ which always holds. This shows that $F_i > 0$ for every insider voter, which implies that $L_C(x,\varepsilon)$ weakly dominates $L_m(x,\varepsilon)$ for every $m > 1$. ■

Proof of Lemma 1. When $\alpha > 1$, the minimum payoff that a political party derives from tying $\frac{\alpha}{2}$ is always greater than the maximum payoff associated to losing $\varepsilon^{\max}$ and therefore, a party always strictly prefers tying to losing. When $\alpha \leq 1$, however, this is not the case. Suppose to the contrary, that there is an equilibrium in which one party loses and the other wins. We distinguish two cases depending on the value of α.

When $\alpha > 1$, the losing party can deviate by selecting the same strategy as the winning party, which guarantees a tie. This is a profitable deviation in contradiction with an equilibrium situation.

When $\alpha \leq 1$, consider *wlog* that Party A wins and Party B loses. Then, because Party B cannot improve deviating, its unique optimal strategy is $L^B = L_C(\frac{1}{2},\varepsilon^{\max})$ with $v_B = \varepsilon^{\max} = \frac{1}{2}$ since for every other lottery stand such that $\varepsilon < \varepsilon^{\max}$, $v_B = \varepsilon < \frac{1}{2}$. Given that Party A is winning, its strategy has to be different from that of Party B. Let $L^A = L_C(x,\varepsilon)$ with $\varepsilon \in [0,\varepsilon^{\max})$, then, Party A's utility is $v_A = \alpha + \varepsilon$. We find, however, that $L^A = L_C(x,\varepsilon)$ with $\varepsilon \in [0,\varepsilon^{\max})$ and $L^B = L_C(\frac{1}{2},\varepsilon^{\max})$ is not a Nash equilibrium: if Party A deviates to $L^{A'} = L_C\left(\frac{1}{2},\varepsilon'\right)$ where $\varepsilon < \varepsilon' < \varepsilon^{\max}$, it still wins (as we show next) and its payoffs increase $v_A(L^{A'},L^B) = \alpha+\varepsilon' > v_A(L^A,L^B) = \alpha+\varepsilon$. We show that Party A wins when $L^{A'} = L_C\left(\frac{1}{2},\varepsilon'\right)$ and $L^B = L_C(\frac{1}{2},\varepsilon^{\max})$. By Expression (4.9), $\frac{\partial U_i}{\partial \varepsilon} = \frac{(\frac{1}{2}-x_i)^2}{\varepsilon^2} - 1$ where the fact that $(\frac{1}{2}-x_i)^2 < \varepsilon^2$ implies that $\frac{\partial U_i}{\partial \varepsilon} < 0$, that is, insider voters prefer the lottery with smaller ε whereas outsider voters remain indifferent. Thus, voters with ideal policy $x_i \in (\frac{1}{2}-\varepsilon', \frac{1}{2}+\varepsilon')$ prefer the lottery with smaller ε, i.e., $L^{A'} = L_C\left(\frac{1}{2},\varepsilon'\right)$. Then, for ε' sufficiently close to $\varepsilon^{\max}$, Party A obtains a majority of votes and wins, in contradiction with an equilibrium situation. ■

Proof of Lemma 2. Suppose that there is an equilibrium in which both parties tie and where $\varepsilon_A < \varepsilon_B$. Then, $v_A(L^A,L^B) = \varepsilon_A + \frac{\alpha}{2} < v_B(L^A,L^B) = \varepsilon_B + \frac{\alpha}{2}$. In this case, Party A can benefit by selecting the same strategy as Party B, which also guarantees a tie and besides, this increases its payoffs. This contradicts an equilibrium situation. ■

Proof of Lemma 3. Let x_A and x_B be the mean policies of the lottery stands proposed by Party A and Party B, respectively. Suppose to the contrary that there is an equilibrium in which parties tie and where x_A and x_B are neither equidistant to x_M nor equal. We take, *wlog*, the case $x_B < x_A$, where x_A is closer to x_M than x_B, i.e. $(|x_A - x_M| < |x_B - x_M|)$. By Proposition 1, equilibrium strategies if they exist, can only be continuous lotteries or single policy stands. Moreover, by Lemma 2, there cannot be a tying situation in which one party proposes a single policy and another a continuous lottery. Therefore, either both parties propose a single policy or both parties propose a continuous lottery with an equal level of ambiguity. In the former case, Party A wins the election in contradiction with a tying situation. Thus, we just account for tying situations in which both parties propose continuous lottery stands with equal ε. Then, the group of voters with ideal policy $x_i \in [x_M, 1]$ fits into one of the three following cases:
- *Case 1.* Agents with $x_i \in [x_M, 1]$ are all outsider voters of the lottery stands of Party A and Party B. This can only occur in a situation where $x_B < x_A < x_M$. For these agents, $U_i(L^A) = -|x_A - x_i|$ and $U_i(L^B) = -|x_B - x_i|$ and they prefer the party which mean policy is closer to their ideal policy. Given that $x_B < x_A < x_M$, all outsider agents prefer x_A. Therefore, a majority votes for Party A, in contradiction with a tying situation.
- *Case 2.* Agents with $x_i \in [x_M, 1]$ are of two types: agents with $x_i \in [x_A + \varepsilon, 1]$ are outsider voters of Party A and Party B, and agents with $x_i \in [x_M, x_A + \varepsilon]$ are insider voters of Party A. This can occur either in a situation where $x_B < x_A < x_M$ or $x_B < x_M < x_A$. For every outsider voter, $U_i(L^A) = -|x_A - x_i|$ and $U_i(L^B) = -|x_B - x_i|$ and they all prefer Party A over Party B since their ideal policies are closer to x_A than to x_B. Insiders of Party A evaluate L^A according to Expression (4.9), i.e. $U_i(L^A) = -\frac{(x_A - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$. By definition, an insider voter satisfies that $|x_A - x_i| < \varepsilon$ and so, $-(x_A - x_i)^2 > -\varepsilon^2$, from where $U_i(L^A) > -\frac{\varepsilon^2}{2\varepsilon} - \frac{\varepsilon}{2} = -\varepsilon$. For these voters $U_i(L^B) = -|x_B - x_i|$ and, given that they are outsider voters of Party B, then $-|x_B - x_i| < -\varepsilon$. We deduce that $U_i(L^B) < -\varepsilon < U_i(L^A)$, i.e. insider voters of Party A strictly prefer Party A over Party B. Thus,

a majority votes for Party A, in contradiction with a tying situation.

- *Case 3.* Agents with $x_i \in [x_M, 1]$ are of three types: agents with $x_i \in [x_A + \varepsilon, 1]$ are outsider voters of Party A and Party B, agents with $x_i \in [x_B + \varepsilon, x_A + \varepsilon]$ are insider voters of Party A and agents with $x_i \in [x_M, x_B + \varepsilon]$ are insider voters of both parties. This can occur either in a situation where $x_B < x_A < x_M$ or $x_B < x_M < x_A$. For every outsider voter, $U_i(L^A) = -|x_A - x_i|$ and $U_i(L^B) = -|x_B - x_i|$ and they all prefer Party A since x_A is closer to their ideal policy than x_B. Insider voters of Party A evaluate L^A according to Expression (4.9), i.e. $U_i(L^A) = -\frac{(x_A - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$, and evaluate Party B by $U_i(L^B) = -|x_B - x_i|$. Since $|x_A - x_i| < \varepsilon$ and $-(x_A - x_i)^2 > -\varepsilon^2$, then $U_i(L^A) > -\frac{\varepsilon^2}{2\varepsilon} - \frac{\varepsilon}{2} = -\varepsilon$. Given that $-|x_B - x_i| < -\varepsilon$ we deduce that $U_i(L^B) < -\varepsilon < U_i(L^A)$, i.e. insider voters of Party A strictly prefer Party A over Party B. Insider voters of Party A and Party B evaluate L^A and L^B according to Expression (4.9) so that $U_i(L^j) = -\frac{(x_j - x_i)^2}{2\varepsilon} - \frac{\varepsilon}{2}$, for $j = A, B$. Given that ε is equal in L^A and L^B, these voters prefer the party which mean policy is closer to their ideal policy, which is x_A. Thus, a majority votes for Party A in contradiction with a tying situation. ■

Proof of Lemma 4. Consider to the contrary a tying equilibrium in which the strategies of the parties differ from $L_1(x_M, 0)$. By Proposition 1, Lemma 2 and Lemma 3 the possible equilibrium strategies in a tie situation are restricted to one of the following three cases:

- *Case 1.* Suppose that parties gather equal mean policies in their lotteries and that $L^A = L^B = L_C(x, \varepsilon)$ where $\varepsilon > 0$. Then, parties tie and obtain the payoffs $\varepsilon + \frac{\alpha}{2}$. Consider that Party A deviates to $L^A = L_C(x, \varepsilon')$ where $\varepsilon' < \varepsilon$. According to Expression (4.9), $\frac{\partial U_i}{\partial \varepsilon} = \frac{(x_A - x_i)^2}{\varepsilon^2} - 1$ where the fact that $(x_A - x_i)^2 < \varepsilon^2$ implies that $\frac{\partial U_i}{\partial \varepsilon} < 0$, that is, insider voters prefer the lottery with smaller ε, while outsider voters remain indifferent. Thus, for $\varepsilon' < \varepsilon$ sufficiently close to ε, Party A can win the elections and obtain a payoff $\varepsilon' + \alpha$, that is greater than $\varepsilon + \frac{\alpha}{2}$, in contradiction with an equilibrium situation.

- *Case 2.* Suppose that parties gather equal mean policies in their lotteries and that $L^A = L^B = L_1(x, 0)$ where $x \neq x_M$. Then, parties tie and obtain the payoffs $\frac{\alpha}{2}$. Suppose that Party A deviates to $L^A = L_1(x', 0)$ where $|x_M - x'| < |x_M - x|$. Then, Party A can win with this strategy since the median voter now strictly prefers Party A over Party B. This implies that Party A obtains a greater payoff with this deviation, in contradiction with an equilibrium situation.

- *Case 3.* Suppose that parties mean policies are different and equidistant to x_M so that $L^A = L_C(x_A, \varepsilon)$ and $L^B = L_C(x_B, \varepsilon)$ where, *wlog* $x_B < x_M < x_A$ with $x_A - x_M = x_M - x_B$, and where $\varepsilon \geq 0$. In this tying situation each of the parties obtains the payoff $\varepsilon + \frac{\alpha}{2}$. Suppose that Party A deviates to $L_C(x'_A, \varepsilon)$, where $x_M < x'_A < x_A$, reducing the distance to the median voter location. Then, Party A can win with this strategy since the median voter now strictly prefers Party A over Party B: if the median voter is an outsider voter, then x'_A is closer to x_M than x_B, which implies that Party A is more preferred than Party B; if x_M is an insider voter of both lottery stands, by Expression (4.9), $U_i = -\frac{(x_j - x_M)^2}{2\varepsilon} - \frac{\varepsilon}{2}$, $j = A, B$, so that the fact that x'_A is closer to x_M than x_B, together implies that Party A is more preferred than Party B for the median voter. Party A wins the elections and obtain the payoff $\varepsilon + \alpha$, that is greater than $\varepsilon + \frac{\alpha}{2}$, in contradiction with an equilibrium situation. ■

Proof of Proposition 2. *First*, we show existence of equilibrium when $\alpha \geq 1$. By Lemma 1, if there exist some equilibrium strategies, these induce a tie. Moreover, by Lemma 4, all the strategies but $L_1(x_M, 0)$ are discarded as equilibrium strategies in a tying situation. Thus, it remains to show that $L^A = L^B = L_1(x_M, 0)$ are equilibrium strategies. Suppose to the contrary that (L^A, L^B) is not an equilibrium and consider, *wlog,* that Party A has a profitable deviation. There are up to three possible deviations: i) $L_1(x'_A, 0)$ where $x'_A \neq x_M$; ii) $L_C(x_M, \varepsilon_A)$ where $\varepsilon_A\,(0, \varepsilon^{\max}]$ and iii) $L_C(x'_A, \varepsilon_A)$ where $x'_A \neq x_M$ and $\varepsilon_A \in (0, \varepsilon^{\max}]$. This is sufficient to analyze these deviations given that, by Proposition 1, a discrete lottery is weakly dominated by a continuous lottery.

i) Suppose that Party A deviates to $L^{A'} = L_1(x'_A, 0)$, where, *wlog,* $x'_A < x_M$. Given that Party B's platform is the median voter location x_M, those voters with ideal policy $x_i \in [x_M, 1]$ vote for Party B. Thus, Party A loses the election and its utility is

$v_A(L^{A'}, L^B) = 0$, whereas $v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2}$, in contradiction with a profitable deviation.

ii) Suppose that Party A deviates to $L^{A'} = L_C(x_M, \varepsilon_A)$ where $\varepsilon_A \in (0, \varepsilon^{\max}]$. Those voters with ideal policy $x_i \in [0, x_M - \varepsilon_A]$, $x_i \in [x_M + \varepsilon_A, 1]$ are outsider voters so that $U_i(L^{A'}) = U_i(L^B) = -|x_M - x_i|$ and they abstain from voting. Insider voters of Party A compare $U_i(L^B) = -|x_M - x_i|$ with the utility of the lottery stand which, according to Expression (4.9), is $U_i(L^{A'}) = -\frac{(x_M - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2}$. We want to show that $U_i(L^{A'}) < U_i\left(L^B\right)$ for every insider voter. Take, *wlog*, $x_i \in (x_M - \varepsilon_A, x_M]$ and let $\beta = |x_M - x_i|$. Then, condition $U_i(L^{A'}) < U_i\left(L^B\right)$ can be rewritten as $-\frac{\beta^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} + \beta < 0$, from where $-\beta^2 + 2\varepsilon_A\beta - \varepsilon_A^2 < 0$. We define $F_i(\beta) = -\beta^2 + 2\varepsilon_A\beta - \varepsilon_A^2$. Since $F_i'(\beta) = -2\beta + 2\varepsilon_A > 0$ for $\beta \in (0, \varepsilon_A)$, $F_i(0) = -\varepsilon_A^2$ and $F_i(\varepsilon_A) = 0$, we deduce that $F_i(\beta) < 0$ for every $\beta \in (0, \varepsilon_A)$. This implies that $U_i(L^{A'}) < U_i\left(L^B\right)$ for every voter such that $x_i \in (x_M - \varepsilon_A, x_M]$ and, in an equivalent way, for those voters such that $x_i \in (x_M, x_M + \varepsilon_A)$. Thus, Party A loses the elections with this deviation and its utility is $v_A(L^{A'}, L^B) \leq \varepsilon^{\max} = \frac{1}{2}$, whereas $v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2}$, in contradiction with a profitable deviation.

iii) Suppose that Party A deviates to $L^{A'} = L_C(x'_A, \varepsilon_A)$ where $x'_A \neq x_M$ and $\varepsilon_A \in (0, \varepsilon^{\max}]$. Take, *wlog*, $x'_A > x_M$. Then, those agents with ideal policy $x_i \in [0, x'_A - \varepsilon_A]$ are outsider voters and for them, $U_i(L^{A'}) = -|x'_A - x_i| < U_i(L^B) = -|x_M - x_i|$, so they vote for Party B. For those agents such that $x_i \in [x'_A - \varepsilon_A, x_M]$, their utility over Party A is measured by Expression (4.9) so that $U_i(L^{A'}) = -\frac{(x'_A - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2}$ and $U_i(L^B) = -|x_M - x_i|$. We want to show that $U_i(L^{A'}) < U_i\left(L^B\right)$ for every insider voter. In the analysis of deviation (ii), we have shown that $-\frac{(x_M - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -|x_M - x_i|$ for every agent with $x_i \in (x_M - \varepsilon_A, x_M]$. Since $(x'_A - x_i) > (x_M - x_i)$ then, $(x'_A - x_i)^2 > (x_M - x_i)^2$, which implies that $-(x'_A - x_i)^2 < -(x_M - x_i)^2$ and $-\frac{(x'_A - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -\frac{(x_M - x_i)^2}{2\varepsilon_A} - \frac{\varepsilon_A}{2} < -(x_M - x_i)$. Then, those voters to the left of the median (including the median) prefer Party B over Party A. Thus, Party A loses the elections and its utility is $v_A(L^{A'}, L^B) \leq \varepsilon^{\max} = \frac{1}{2}$, whereas $v_A(L^A, L^B) = \frac{\alpha}{2} \geq \frac{1}{2}$, in contradiction with a profitable deviation.

We have showed that all the deviations from $L_1(x_M, 0)$ (when the strategy of the opponent is $L_1(x_M, 0)$), are such that the party deviating loses the election. This also proves that $L_1(x_M, 0)$ is not weakly dominated by any other strategy and this completes the proof.

Second, we show non existence of equilibrium when $0 < \alpha < 1$.

By Lemmas 1 and 4, this is sufficient to show that $L^A = L^B = L_1(x_M, 0)$ is not an equilibrium in this case. Consider that Party A deviates to $L^{A'} = L_C(\frac{1}{2}, \varepsilon^{\max})$. As shown above, in deviations (ii) and (iii), Party A cannot defeat Party B when proposing $L_C(\frac{1}{2}, \varepsilon^{\max})$ where either $\frac{1}{2} \leq x_M$ or $\frac{1}{2} > x_M$. However, when $\alpha < 1$, Party A derives more utility when proposing $L^{A'} = L_C(\frac{1}{2}, \varepsilon^{\max})$ than when proposing $L^A = L_1(x_M, 0)$: $v_A(L^{A'}, L^B) = \varepsilon^{\max} = \frac{1}{2}$ and $v_A(L^A, L^B) = \frac{\alpha}{2} < \frac{1}{2}$. Therefore, $L^A = L^B = L_1(x_M, 0)$ is not a Nash equilibrium and this completes the proof.

Third, we show that $L^A = L^B = L_C\left(\frac{1}{2}, \varepsilon^{\max}\right)$ is an equilibrium when $\alpha = 0$.

In this case, parties only derive benefits from being ambiguous and therefore, the strategy $L_C\left(\frac{1}{2}, \varepsilon^{\max}\right)$ is a strictly dominant strategy for each party and the unique Nash equilibrium. ■

Chapter 5

Conformity in voting

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"*Sooner or later if a guy sits in the General Assembly for ten years, I think it is crazy not to think that he is gonna make at least one judgement on, maybe, his principle…[But] what good is it for me to sit there and vote what I feel would be my principle - in terms of the philosophy that I would have on how government ought to be run relating to an issue - and I voted against my constituency and voted my political philosophy, and then still when they took the tally, I was still on the losing side?…When you are really in a position where you can make it happen, then it would be rewarding enough to say, 'I'll see you guys later; beat me in an election!' I don't care whatever it is, that is where it makes it worthwhile. Otherwise you are crazy, in my estimation.*"
A pro-Equal Rights Amendment legislator in *Why we lost the ERA,* Mansbridge (1986, p. 162).

5.1 Introduction

A group of agents has to decide whether to accept or reject a proposal. Each agent is either in favor of or against the proposal. Agents vote and the proposal is accepted if the number of agents in favor of the proposal is greater than or equal to a certain threshold. There are real life situations in which agents misrepresent their preferred options when voting (as shown in the abovementioned quotation from the pro-Equal Rights Amendment legislator, these agents may adopt similar reasoning when voting: why confront others by voting following my philosophy if I cannot obtain any gain by doing so?). The tendency of agents to adapt their votes to those of other agents is known as *conformity.*

In this study, we analyze the consequences of agents voting by considering not only their true opinions about the proposal but also the votes of the rest of the voters. We refer to these agents as *conformists.* By contrast, when voters consider only their own opinions when voting, we call them *independents.* Our objective is to implement the $q-$*truthful social choice function*, which takes as messages the true opinion of the agents and accepts the proposal if the number of those in favor is greater than or equal to certain threshold.

First, we consider a situation in which agents vote simultaneously. When all agents are independents, the unique weakly undominated Nash equilibrium announces their true opinions. That is, independent agents always vote truthfully. The question that arises is as follows: when all agents are conformists, is announcing the true opinion also the unique equilibrium strategy?[1] Do new equilibrium strategies emerge? In such a case, we check whether the decision associated with any equilibrium when all agents are conformists coincides with that obtained if all agents vote truthfully.

Let h be any integer number from 1 to $n-1$. We interpret h as the minimal number of agents that any conformist voter wants to coincide with in situations in which her vote does not determine the decision.

When all agents are conformists, we show that for any required threshold to accept the proposal, there are undominated Nash equilibria in which the decision does not

[1]In the sequential version of the game, in Section 5, the equilibrium concept used is that of subgame perfect Nash equilibrium.

coincide with that obtained when all agents vote truthfully. This result holds regardless of the minimal number of agents that each conformist voter wants to coincide with. In particular, for any threshold, there are profiles of true opinions in which the decision in some equilibria is different from that obtained when all agents vote truthfully. We refer to such profiles as *problematic.* To compare preference profiles, a possible criterion is referring to a preference profile as more problematic than another if the set of $q-$truthful social choice functions that cannot be implemented is greater in the sense of set inclusion for the former than for the latter. Following this criterion, the most problematic profiles of true opinions are either all agents preferring to accept the proposal or all agents rejecting it. In addition, we use the word "problematic" when referring to $q-$truthful social choice functions. That is, a $q-$truthful social choice function is problematic if there are profiles of true opinions in which the decision in some equilibrium is different from that obtained when all agents vote truthfully. To compare $q-$truthful social choice functions, we can also use a similar criterion and referring to a $q-$truthful social choice function as more problematic than another if the set of preference profiles in which the former $q-$truthful social choice function cannot be implemented is greater in the sense of set inclusion than the latter $q-$truthful social choice function. Those $q-$truthful social choice functions in which either all agents (unanimity to accept the proposal) or only one agent (unanimity to reject the proposal) are required to accept the proposal are equally problematic among them and they are the least problematic among all $q-$truthful social choice functions. Surprisingly, for certain $q-$truthful social choice functions and preference profiles, announcing the true opinion is not even an equilibrium.

In the corporate world, committees are concerned about the possibility of agents misrepresenting their opinions when taking decisions. One way to face this problem is to replace some committee members by external advisors who provide unbiased and independent opinions about the decision-making procedure.[2] With this idea in mind, our

[2] We assume that external advisors or consultants have incentives only for being sincere and representing the interests of their employers; they receive no gain by deviating from that behavior.

next step is to ask whether there is any partition of independent and conformist agents that, in any equilibrium, yields the decision obtained when all agents vote truthfully for any profile of true opinions. We offer a positive answer assuming uniformity of h. We show that the minimum number of independent agents depends on the number of agents, the required threshold to accept the proposal, and h.

In addition, the number of independent agents allows us to identify which thresholds are more problematic, that is, the greater the number of independent agents, the more problematic a threshold is. Interestingly, the least problematic are the unanimous thresholds, either to accept or to reject the proposal, which is similar to the case in which all agents are conformist.

Other versions of conformity can be studied. Among those that are anonymous, an intuitive one is to consider that agents want to conform to as many agents as possible. We show that this alternative way of modeling conformity is equivalent to considering that agents in our case want to conform to at least a majority of voters. This case is very demanding in the sense that almost all agents must be independent to guarantee that the decision coincides with that obtained when agents vote truthfully.

Finally, we consider a situation in which agents vote sequentially. We show that for any threshold and any subgame perfect Nash equilibrium, the decision obtained is not affected by the conformist behavior of the agents. This result holds regardless of the number of conformist and independent agents, h, and the sequence in which agents vote.

Section 2 outlines a summary of the literature regarding the conformity phenomenon. Section 3 presents the model and an impossibility result when the equilibrium concept used is that of weakly undominated Nash equilibrium. In Section 4, we provide the number of independent agents that is sufficient to obtain, in any equilibrium, the decision obtained when all agents vote for their own opinions. Section 5 solves the sequential version of the problem. Some concluding remarks are given in Section 6. We relegate the proofs of the results of Sections 4 and 5 to the Appendix.

5.2 Previous Research

Conformity is first analyzed by Asch (1951), showing how agents misrepresent their true opinions when they are persuaded by the vote of a majority within a group. In Asch's (1951) experiment, agents have to match a line of a specific length to one of three lines. All members of the group except one are accomplices of Asch (1951). The aim of the experiment is to observe whether agents conform to the wrong answer of the rest of the agents, despite the correct answer being obvious. On average, about one-third of the tested subjects conform to the wrong option. Critics of this study have arisen owing to the open ballot nature of the experiment. However, Deutsch and Gerard (1955) show evidence of conformity, even when agents report their vote secretly (see also Bernheim and Exley, 2015). In addition, Deutsch and Gerard (1955) identify two types of social influence: informational and normative. Informational influence refers to updating an opinion taking into account others' previous opinions, whereas normative influence describes the behavior of stating an opinion that fits with the group choice.

In this study, we analyze conformity from a normative perspective. Under normative conformity, it is common to use the social distance approach, which is characterized by a penalty term that accounts for the distance between the preferred option of the agent and the group option (see also Jones, 1984; Bernheim 1994; Akerlof, 1997; Luzzati, 1999). Bernheim (1994) endogenizes conformity assuming that agents' preferences are based on an intrinsic utility and a preference for status, which is not observed by the rest of agents (see also Akerlof, 1997). Multiple equilibria exist and, in particular, people conform to a social norm when status is sufficiently important. Luzzati (1999) investigates how to account for conformity within economic models. Following Jones (1984), Luzzati (1999) models conformity exogenously and shows that, when the production of a public good depends on voluntary contributions by agents, the introduction of conformism allows for multiple equilibria, including agents contributing with positive amounts. According to Luzzati (1999, p. 130), "what social psychology seems rather to suggest is that agents, although they sometimes consciously use conformism as a strategy, are more truly (un-consciously) conformists" and therefore, it should be considered as a model of social interaction. In addition, Postlewaite (1998) highlights that traditional economic modeling is sufficient to explain agents' behavior when accounting for conformity. In fact, he suggests that social environments are crucial in determining agents' behavior.

We opt to study conformity by introducing it in the preferences of the agents. First, agents want their opinions to prevail. If their votes have no effect on the decision, conformity arises and they want their messages to coincide with the messages of other agents. In Dutta and Sen (2012), agents' preferences are lexicographic, but in the sense that agents have a preference for honesty when announcing the true state does not change their welfare. Dutta and Sen (2012) call agents "partially honest." Our agents, however, are not necessarily partially honest. A conformist agent strictly prefers to conform to the opinion of some reference group when her message does not have any influence on the decision, regardless of whether her message corresponds with her true opinion.[3] Our approach, although different, has some degree of similarity with the literature of interdependent preferences (Sobel, 2005).[4] In this literature, agents' utility depends on the utility of others, whereas in our case, agents' utility depends on the votes of others and not their utility.

It is common to consider agents' heterogeneity in models with social interaction. In particular, conformist agents are those that are influenced by others and independent agents are those that state their unbiased opinions irrespective of the consequences of doing so. Bernheim (1994) identifies in his model the role of independent agents as those whose preferences are extreme enough to refuse to conform. Herrera and Martinelli (2006), in a model of participation in elections, characterize conformists as those agents influenced by leaders, who also influence the election outcome (see also Glaeser et al., 1996). Rivas and Rodríguez-Alvarez (2014) consider leaders, independent agents, and

[3]We acknowledge that it could be very natural to think that when agents have different groups to conform to, they prefer to conform to those whose messages coincide with their own opinions. Our results hold under this assumption.

[4]See Sobel (2005) for a study of interdependent preferences applied to the ultimatum game case in which agents' preferences also depend on the consumption of others.

conformist agents in a model that includes information revelation prior to the voting stage, and Buechel et al. (2015) study a model of opinion formation in different discussion rounds in a social network framework consisting of leaders, conformist agents, counter-conformist agents, and honest agents.

Finally, there are other studies investigating voting procedures involving different thresholds. Maggi and Morelli (2006) present a model with a self-enforcing voting system. They conclude that unanimity is the optimal system if there is no external enforcement and majority rule is the ex-ante efficient rule. In addition, Buchanan and Tullock (1962) support the use of unanimity. The idea of considering other thresholds apart from the simple majority and unanimity rule also appears in Feddersen and Pesendorfer (1998), who suggest combining a super-majority rule with a larger jury in cases in which they want to reduce the probability of convicting the innocent.

5.3 The model and basic result

Let $N = \{1, ..., n\}$ be any finite set of agents. This group of agents has to decide whether to accept or reject a proposal. Let capital letters $C, S \subset N$ denote subsets of agents. We refer to the **true opinion** of an agent $i \in N$ as $t_i \in \{0, 1\}$, where $t_i = 0$ stands for agent i rejecting the proposal and $t_i = 1$ for accepting the proposal. Let $t = (t_1, ..., t_n) \in \{0, 1\}^n$ be a profile of true opinions.

To take a decision on the proposal, we ask agents to vote in favor of or against such a proposal. Therefore, agents are asked to announce a message in this sense.

A profile of messages is denoted by $m \in M$ where M is the set of messages. For any agent $i \in N$ and any profile of messages $m \in M$, let m_i denote the **message** of agent i and $m_{-i} \in M_{-i} = \times_{j \in N \setminus \{i\}} M_j$ the messages of all agents except i.

In most of the study, we assume that agents vote simultaneously.[5] Then, $M_i = \{0, 1\}$ is the set of messages for agent $i \in N$, where $m_i = 0$ means that agent i votes against

[5]Section 5 analyzes a sequential version of the game.

the proposal and $m_i = 1$ that agent i votes in favor of the proposal. Profiles of messages are denoted in two possible ways. When they are part of a given profile of messages, we use notation $m_i \in M_i$ to denote that this is the message of agent i in the profile. In other cases, we want to have a name for a given message, and we use superscripts, that is, $m_i^{t_i}$ stands for agent $i \in N$ voting according to her true opinion $t_i \in \{0, 1\}$. In addition, we write m_S^1 when $m_i = 1$ for all $i \in S$ and $S \subseteq N$, $m^1 = (m_1^1, ..., m_n^1)$ and $m^0 = (m_1^0, ..., m_n^0)$.

The description of the preferences differs from the standard case in that they depend not only on the decision taken but also on the profile of messages. Let $\{0, 1\} \times M$ be the **set of alternatives**, and $(x; m) \in \{0, 1\} \times M$ be an **alternative**, where $x \in \{0, 1\}$ stands for the decision taken. Let $\widetilde{\mathcal{R}}_i$ be the set of all possible preference relations for agent $i \in N$ defined on $\{0, 1\} \times M$ satisfying reflexivity, transitivity, and completeness. Let $\succeq_i \in \widetilde{\mathcal{R}}_i$ be a preference relation for agent $i \in N$, and $\succeq = (\succeq_1, ..., \succeq_n) \in \times_{i \in N} \widetilde{\mathcal{R}}_i$ be an admissible preference profile. Since we want to have a name for a given preference relation, we use superscripts, that is, $\succeq_i^{t_i}$ is the preference relation when agent i has opinion $t_i \in \{0, 1\}$. We also write $\succeq_C^1$ when $\succeq_i = \succeq_i^1$ for all $i \in C$ and $C \subseteq N$, $\succeq^1 = (\succeq_1^1, ..., \succeq_n^1)$, and $\succeq^0 = (\succeq_1^0, ..., \succeq_n^0)$.

Next, we define a social choice function.

Definition 7 *A* ***social choice function*** f *is a mapping from the set of admissible preference profiles to the set of alternatives,* $f : \widetilde{\mathcal{R}}^n \to \{0, 1\} \times M$.

We introduce two properties regarding agents' preference relations. We call the first property **selfishness**. An agent's preference relation satisfies selfishness if, when comparing two different pairs of alternatives, she prefers that alternative in which the decision matches her opinion.[6]

Definition 8 *Agent i's preference relation* $\succeq_i \in \widetilde{\mathcal{R}}_i$ *is* ***selfish*** *if for any true opinion,*

[6] Note that our property of selfishness is related to the notion of selfish preferences used in Sobel (2005), which refers to the preferences that do not directly depend on the messages of others.

$t_i \in \{0,1\}$ *and for any* $(x;m),(x';m') \in \{0,1\} \times M$ *such that* $x = t_i$ *and* $x' \neq t_i$, *we have* $(x;m) \succ_i (x';m')$.

The second property is referred to as **h−conformity**. Let $h = \{1, ..., n-1\}$. We interpret h as the minimal number of agents that any conformist voter wants to coincide with in situations in which her vote does not determine the decision. An agent's preference relation satisfies *h−conformity* if, when comparing two different pairs of alternatives with identical decision, she prefers the alternative in which the number of agents with the same message as hers is greater than or equal to h.[7]

Before presenting this concept, we define when two agents conform.

Definition 9 *For any* $m \in \times_{i \in N} M_i$, *and any* $i, j \in N$, *we say that agent* i ***conforms*** *to agent* j *if and only if* $m_i = m_j$.

Definition 10 *For any* $h \in \{1, ..., n-1\}$, *agent* i*'s preferences satisfy* **h−*conformity*** *if for any* $t_i \in \{0,1\}$ *and for any* $x \in \{0,1\}$:
$(x;m) \succ_i (x;m')$ *if and only if agent* i *conforms to at least* h *other agents given* m, *and conforms to at most* $h-1$ *other agents given* m'.

In addition, we define when an agent is conformist.

Definition 11 *For any* $h \in \{1, ..., n-1\}$, *an agent* $i \in N$ *is* ***conformist*** *if any admissible preference relation for* i *satisfies selfishness and* $h-$*conformity.*

We now illustrate that the properties of selfishness and $h-$conformity completely determine the set of admissible preferences of the agents. We also want to stress, as shown

[7] Moreno and Ramos-Sosa (2015) opens the door to more general versions of conformity. Agents may pay attention to some specific subsets of agents. Those subsets of agents could be either groups of experts, or a leader, or, as Kenneth Shepsle proposed to the authors, an agent may want to conform to some agents and not to others. Note that in these cases, the identity of the agents plays an important role.

in Example 1 that, when comparing alternatives, agents look first at the decision and, only after, at the vote of other agents. Therefore, agents' preferences are lexicographic.[8]

Example 12 *Let* $N = \{1, 2, 3\}$ *and* $h = 1$*. The set of admissible preference relations for agent* 1 *satisfying selfishness and* $h-$*conformity is given in the following two tables:*

Table 5.1: Preferences of agent 1 *when* $\succeq_1^0$.
$\succeq_1^0$
$\{(0;0,0,0),(0;0,0,1),(0;1,1,0),(0;1,1,1),(0;0,1,0),(0;1,0,1)\}$
$\{(0;1,0,0),(0;0,1,1)\}$
$\{(1;0,0,0),(1;0,0,1),(1;1,1,0),(1;1,1,1),(1;0,1,0),(1;1,0,1)\}$
$\{(1;1,0,0),(1;0,1,1)\}$

Table 5.2: Preferences of agent 1 *when* $\succeq_1^1$.
$\succeq_1^1$
$\{(1;0,0,0),(1;0,0,1),(1;1,1,0),(1;1,1,1),(1;0,1,0),(1;1,0,1)\}$
$\{(1;1,0,0),(1;0,1,1)\}$
$\{(0;0,0,0),(0;0,0,1),(0;1,1,0),(0;1,1,1),(0;0,1,0),(0;1,0,1)\}$
$\{(0;1,0,0),(0;0,1,1)\}$

Each alternative $(x; m_1, m_2, m_3)$ *consists of two components. The first component,* x*, corresponds to the decision taken. The second component consist of the messages of each of the agents. For instance, in alternative* $(1; 0, 1, 0)$ *the first component,* 1*, is the decision taken after agent* 1 *announces* $m_1 = 0$*, agent 2 announces* $m_2 = 1$ *and agent 3 announces* $m_3 = 0$*.*

Table 1 refers to the case in which agent 1 *is against the proposal. Agent* 1 *has four indifference classes, each row representing each class. The first indifference class includes the*

[8] An anonymous referee proposes to us another version of conformity in which agents want to conform to as many agents as possible. In Section 4, we show how this alternative version of conformity relates to that proposed in this study.

alternatives in which the decision coincides with agent 1's true opinion and she conforms to at least one other agent. The second indifference class includes the alternatives in which the decision coincides with agent 1's true opinion but she does not conform to anyone. Similarly, in the last two rows, the decision obtained differs from the true opinion but in the third row, agent 1 conforms to some other agent and in the last row, she does not.
Table 2 is interpreted in the same way as Table 1.

Example 1 illustrates that $h-$conformity and the true opinion of an agent completely determine her preference relation. For each h, there are only two admissible preference relations, each of which corresponds to the two possible true opinions of an agent.

For any $i \in N$, and any $h_i \in \{1, ..., n-1\}$, $\mathcal{R}_i^{h_i}$ denotes the set of admissible preference relations satisfying selfishness and $h-$conformity. When a preference relation is part of a given profile of preferences, we use notation $\succeq_i \in \mathcal{R}_i^{h_i}$ to denote that this is the preference relation of agent i in the profile.

Given h_i, asking whether an agent is in favor of or against the proposal is equivalent to asking for her preference relation. That is, when agent i sends message $m_i = 1$ ($m_i = 0$), we interpret this as if agent i says that her preference relation is $\succeq_i^1$ ($\succeq_i^0$).

Definition 13 *Given h_i and t_i, we say that agent i* ***votes truthfully*** *when $m_i = t_i$ for any $i \in N$.*

Therefore, when we refer to voting truthfully or truthful voting, we mean that agents announce their true preference relations, that is, their true opinions.

Now, we define the class of social choice function that we are interested in. Given $h_i \in \{1, ..., n-1\}$, for any preference profile $\succeq_i \in \mathcal{R}_i^{h_i}$, the proposal is accepted if a given number of agents q, where $q \in \{1, 2, .., n\}$, is in favor of the proposal, and it is rejected otherwise. We refer to q as the **threshold** needed for the proposal to be accepted. Next, we introduce the definitions of $q-$**threshold rule** and $q-$**truthful social choice function**.

Definition 14 *Given any* $q \in \{1, ..., n\}$*, a* $q-$***threshold rule*** $g^q(m)$ *is such that, for any* $m \in \times_{i\in N} M_i$ *chooses* $x = 1$ *if the number of agents such that* $m_i = 1$ *is greater than or equal to* q *and* $x = 0$ *otherwise.*

Definition 15 *Given* $(h_1, ..., h_n)$*, a* $q-$***truthful social choice function*** f^q*, is such that for any* $\succeq \in \times_{i\in N} \mathcal{R}_i^{h_i}$ *chooses an alternative* $(x; m)$*, where:*
i) $x = 1$ *if the number of agents such that* $t_i = 1$ *is greater than or equal to* q *and* $x = 0$ *otherwise, and*
ii) $m_i = t_i$ *for any* $i \in N$*.*[9]

Note that the $q-$truthful social choice function do not have full range. In Example 2, we illustrate it for the $1-$ and $2-$truthful social choice functions.

Example 16 *Let* $N = \{1, 2, 3\}$ *and* $(h_1, ..., h_n)$ *be given.*
The $1-$*truthful social choice function does not select alternative* $(0; 1, 0, 0)$ *for any preference profile whereas* $(1; 1, 0, 0)$ *is selected for preference profile* $(\succeq_1^1, \succeq_2^0, \succeq_3^0)$*.*
The $2-$*truthful social choice function does not select alternative* $(1; 1, 0, 0)$ *for any preference profile whereas* $(0; 1, 0, 0)$ *is selected for preference profile* $(\succeq_1^1, \succeq_2^0, \succeq_3^0)$*.*

Before defining the equilibrium concept used throughout this section, we make the following assumption.

Assumption A: Any agent knows her preference relation and the preference relations of the rest of the agents.[10]

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[9] *For every preference profile, the alternative selected by any* $q-$*truthful social choice function specifies the decision and vote of each agent. Alternatively, any* $q-$*truthful social choice function could specify the decision and number of agents voting for each option. The former could be related to an open ballot scenario whereas the latter is more related to a secret ballot scenario in which the decision is made public once all agents have voted. Our results remain valid under this alternative definition of a* $q-$*truthful social choice function.*

[10] A more realistic scenario would be one in which the preference relations of the agents are unknown. We assume complete information as a first step before analysing the implications of incomplete information in future research.

Definition 17 *For any* $(h_1, ..., h_n)$ *and any* $q \in \{1, ..., n\}$, $m \in M$ *is a* ***weakly undominated Nash equilibrium*** *of* (M, g^q) *at* $\succeq \in \times_{i \in N} \mathcal{R}_i^{h_i}$ *if for all* $i \in N$,
(1) m_i *is not weakly dominated and*
(2) for all $m'_i \in M_i$, $(g^q(m); m_i, m_{-i}) \succeq_i (g^q(m'_i, m_{-i}); (m'_i, m_{-i}))$.

Given $(h_1, ..., h_n)$ and $\succeq \in \times_{i \in N} \mathcal{R}_i^{h_i}$, $UN((M, g^q), \succeq, h)$ is the set of weakly undominated Nash equilibria of (M, g^q) at $\succeq$.

Proposition 1 shows that for any $(h_1, ..., h_n)$ and any $q-$threshold rule, there are undominated Nash equilibria yielding a decision different from that selected by the $q-$truthful social choice function for some preference profiles. In what follows, we refer to this by saying that the $q-$truthful social choice function is not **implemented** for a preference profile.

Proposition 1. *For any* $(h_1, ..., h_n)$ *and any* $q-$*threshold rule, there are* $\succeq \in \times_{i \in N} R_i^{h_i}$ *for which some undominated Nash equilibria do not implement the* $q-$truthful social choice function *for* $\succeq$.

Proof. Let $(h_1, ..., h_n)$ be such that $h_i \in \{1, ..., n-1\}$ for any $i \in N$. In order to prove the result, we present the following three cases that apply to the 1, n, and $q-$truthful social choice function, respectively, where $q \in \{2, ..., n-1\}$.
Case 1. 1$-$truthful social choice function. Let $\succeq \in \times_{i \in N} \mathcal{R}_i^{h_i}$ be such that $\succeq = \succeq^0$. We show that m^1 is a weakly undominated Nash equilibrium yielding $(1; m^1)$ where the 1$-$truthful social choice function is not implemented for $\succeq^0$.
Note that $g^1(m^1) = 1$ and $g^1(m_i^0, m_{-i}^1) = 1$ for any $i \in N$. By $h-$conformity, $(1; m^1) \succ_i^0 (1; m_i^0, m_{-i}^1)$ for any $i \in N$. Since $m^1 \in UN((M, g^1), \succeq, h)$, the 1$-$truthful social choice function is not implemented for $\succeq^0$. [11]
Case 2. n$-$truthful social choice function. Let $\succeq \in \times_{i \in N} \mathcal{R}_i^{h_i}$ be such that $\succeq = \succeq^1$. We show that m^0 is a weakly undominated Nash equilibrium yielding $(0; m^0)$ where the

[11] Note that for any profile different from $\succeq^0$, any weakly undominated Nash equilibrium implements the 1$-$truthful social choice function.

$n-$truthful social choice function is not implemented for $\succeq^1$.

Note that $g^n(m^0) = 0$ and $g^n(m_i^1, m_{-i}^0) = 0$ for any $i \in N$. By $h-$conformity, $(0; m^0) \succ_i^1 (0; m_i^1, m_{-i}^0)$ for any $i \in N$. Since $m^0 \in UN((M, g^n), \succeq, h)$, the $n-$truthful social choice function is not implemented for $\succeq^1$. [12]

Case 3. q−truthful social choice function where $\mathbf{q} \in \{\mathbf{2}, ..., \mathbf{n-1}\}$**.** For any preference profile, all agents voting in favor or against the proposal is a weakly undominated Nash equilibrium.

Let any $\succeq \in \times_{i \in N} \mathcal{R}_i^{h_i}$ and any $q \in \{2, ..., n-1\}$. Note that $g^q(m^0) = 0$ and $g^q(m_i^1, m_{-i}^0) = 0$ for any $i \in N$. By $h-$conformity, $(0; m^0) \succ_i^{t_i} (0; m_i^1, m_{-i}^0)$ for any $i \in N$ and $m^0 \in UN((M, g^q), \succeq, h)$ for any $\succeq$. Note also that $g^q(m^1) = 1$ and $g^q(m_i^0, m_{-i}^1) = 1$ for any $i \in N$. By $h-$conformity, $(1; m^1) \succ_i^{t_i} (1; m_i^0, m_{-i}^1)$ for any $i \in N$ and $m^1 \in UN((M, g^q), \succeq, h)$ for any $\succeq$. Therefore, $m^1, m^0 \in UN((M, g^q), \succeq, h)$ for any $q \in \{2, ..., n-1\}$ and the $q-$truthful social choice function is not implemented for any $\succeq$. ■

Proposition 1 offers a negative result. It shows that asking voters about their opinions does not implement any given $q-$truthful social choice function. In particular, for some $q-$truthful social choice functions, there are preference profiles for which the $q-$truthful social choice function is not implemented. We refer to a preference profile as **problematic**[13] for a $q-$truthful social choice function if it is not implemented. Alternatively, a $q-$truthful social choice function is problematic for a preference profile if it is not implemented.

With this idea in mind, we could perform two exercises. We could compare either preference profiles or $q-$truthful social choice functions.

A possible criterion that could be used to compare preference profiles is the following.

Definition 18 *A preference profile $\succeq$ is more problematic than $\succeq'$, if the set of $q-$truthful*

[12] Note that for any profile different from $\succeq^1$, any weakly undominated Nash equilibrium implements the $n-$threshold rule.

[13] We thank the anonymous referee who advised us to refer to these preference profiles as problematic and to adopt a criterion stating when a preference profile is more problematic than another.

social choice functions that cannot be implemented is greater in the sense of set inclusion for $\succeq$ than for $\succeq'$.

Applying this definition, we can present the following Corollary.

Corollary 19 *Preference profile $\succeq^0$ is problematic for any $q-$truthful social choice function such that $q < n$ and $\succeq^1$ is problematic for any $q-$truthful social choice function such that $q > 1$. Therefore, preference profiles $\succeq^0$ and $\succeq^1$ are not comparable among them. Any other preference profile different from $\succeq^1$ and $\succeq^0$ is problematic for any $q-$truthful social choice function such that $q \in \{2, ..., n-1\}$ but not for $q \in \{1, n\}$. Then, all of them are equally problematic. Finally, preference profiles $\succeq^0$ and $\succeq^1$ are the most problematic.*

A similar criterion could be used to compare $q-$truthful social choice functions.

Definition 20 *A $q-$truthful social choice function is more problematic than q' if the set of preference profiles in which the $q-$truthful social choice function cannot be implemented is greater in the sense of set inclusion than that for the $q'-$truthful social choice function.*

Applying this definition, we can present the following Corollary.

Corollary 21 *The $1-$truthful social choice function is problematic only for preference profile $\succeq^0$ and the $n-$truthful social choice function is problematic only for preference profile $\succeq^1$. Therefore, the $1-$ and $n-$truthful social choice functions are not comparable among them. Any $q-$truthful social choice function such that $q \in \{2, ..., n-1\}$ is problematic for any preference profile. Then, all these $q-$truthful social choice functions are equally problematic among them. Thus, the $1-$threshold and $n-$truthful social choice functions are the least problematic*

Table 3 offers a summary of which preference profiles and $q-$truthful social choice functions are problematic. A "✓" indicates which preference profile is problematic for a $q-$truthful social choice function and vice versa, as it is stated in Corollaries 1 and 2.

Table 5.3: Problematic preference profiles and $q-$truthful social choice functions

	$q-$thresholds				
Preference profiles $\succeq$	$q=1$	$q=2$	...	$q=n-1$	$q=n$
$\succeq^0$	$\times$	$\times$	$\times$	$\times$	$\checkmark$
$(\succeq_i^1, \succeq_{N\setminus\{i\}}^0)$	$\checkmark$	$\times$	$\times$	$\times$	$\checkmark$
$(\succeq_i^1, \succeq_j^1, \succeq_{N\setminus\{i,j\}}^0)$	$\checkmark$	$\times$	$\times$	$\times$	$\checkmark$
$(\succeq_{N\setminus\{i,j\}}^1, \succeq_i^0, \succeq_j^0)$	$\checkmark$	$\times$	$\times$	$\times$	$\checkmark$
$(\succeq_{N\setminus\{i\}}^1, \succeq_i^0)$	$\checkmark$	$\times$	$\times$	$\times$	$\checkmark$
$\succeq^1$	$\checkmark$	$\times$	$\times$	$\times$	$\times$

Finally, two remarks are in order. The first is about the truthful behavior of the agents in equilibrium. There are $q-$threshold rules in which the truth is a dominated strategy for the agents. Given $(h_1, ..., h_n)$, for some $q-$threshold rules, voting truthfully is not a weakly undominated Nash equilibrium. We illustrate this with the following example.

Example 22 *Let,* $N = \{1, 2, 3, 4, 5\}$ *and* $h_i = 2$ *for any* $i \in N$. *Let* $\succeq = (\succeq_1^1, \succeq_2^1, \succeq_3^1, \succeq_4^0, \succeq_5^0)$. *Take the* $3-$*threshold rule. In message* $m^t = (m_1^1, m_2^1, m_3^1, m_4^0, m_5^0)$, *all agents vote truthfully. Note that* $g^3(m^t) = 1$, $g^3(m_4^1, m_{N\setminus\{4\}}^t) = 1$ *and* $(1; m_4^1, m_{N\setminus\{4\}}^t) \succ_4 (1; m^t)$. *Therefore, all agents voting truthfully is not a weakly undominated Nash equilibrium of* (M, g^3).

The second remark deals with Pareto dominance. We find that there is weakly undominated Nash equilibrium driving to alternatives that are Pareto dominated. In the case of any $q-$threshold rule different from n, when $\succeq = \succeq^0$, all agents voting in favor of the proposal is a weakly undominated Nash equilibrium yielding alternative $(1; m^1)$. However, alternative $(0; m^0)$ is strictly preferred to $(1; m^1)$ for any agent. For the $n-$threshold rule, when $\succeq = \succeq^1$, all agents voting against the proposal is a weakly undominated Nash equilibrium yielding alternative $(0; m^0)$. However, alternative $(1; m^1)$ is strictly preferred to $(0; m^0)$ for any agent.

5.4 Replacing conformist agents

In the previous section, we observe that all $q-$truthful social choice function are problematic for at least one preference profile when all agents are conformists. By contrast, when agents' preferences depend only on the decision, voting truthfully is the unique weakly undominated Nash equilibrium. We refer to these agents as independent agents.

Definition 23 *An agent $i \in N$ is **independent** if any admissible preference relation $\succeq_i$ satisfies selfishness and $(x; m) \sim_i (x; m')$ for all m,$m' \in M$.*

Let I be the set of independent agents and $N \backslash I$ the set of conformist agents. Henceforth, we write I to emphasize the partition between the set of independent and conformist agents. Thus, $I = \emptyset$ accounts for the case in which all agents are conformists, which is the case studied in the previous section, and $I = N$ for the case in which all agents are independents.

The composition of the set of agents taking the decision is important when implementing a given $q-$truthful social choice function. Assuming that the partition between conformist and independent agents can be chosen, the relevant question is to study which partition is the most appropriate to implement a given $q-$truthful social choice function by their associated $q-$threshold rule in the presence of conformism. This scenario represents a common situation in the corporate world, in which it is usual to contact external advisors when taking decisions.[14] These external advisors are asked to produce unbiased reports that are usually made available to the rest of the agents. In order to account for this situation, we make the following assumption.

[14]We assume that these external advisors have incentives only to be sincere and to represent the interests of their employers, and receive no gain by deviating from that behavior.

Assumption B: All agents know the number of independent agents and their messages.[15]

For any $q-$threshold rule and any $(h_1, ..., h_n)$, our objective is to calculate the number of independent agents guaranteeing that the $q-$truthful social choice function is implemented for any preference profile. We restrict our exercise to the symmetric case in order to obtain a closed formula.[16]

Assumption C: For any $i, j \in N \backslash I$, we consider that $h_i = h_j$ for any $h \in \{1, ..., n-1\}$.

Next, we provide the intuition behind our main result. First, we fix the number of agents n. Let O be the set of all linear orderings of the agents. Let $o' = (o_1, ..., o_n) \in O$ be an ordering in which o_k is the $k-th$ agent in ordering o'. Take a $q-$threshold rule and some h. Take one preference profile that is problematic for such $q-$truthful social choice function, say $\succeq \in \times_{i \in N} \mathcal{R}_i^h$. Check whether m^0 and m^1 are weakly undominated Nash equilibrium and take that equilibrium in which the $q-$truthful social choice function is not implemented, say $\overline{m}$. Take an ordering $o \in O$ of the agents. We now describe the procedure used to obtain our main result:

Step 1. Replace the first conformist agent in this ordering, o_1, with an independent agent with the same true opinion. Take $(m_{o_1}^{t_{o_1}}, \overline{m}_{N \backslash \{o_1\}})$. If it is a weakly undominated Nash equilibrium not implementing the $q-$truthful social choice function, go to Step 2. If not, stop and register that the number of independent agents is 1.

...

Step k. Replace the $k-th$ conformist agent in this ordering, o_k, with an independent agent with the same true opinion. Take $(m_{o_1}^{t_{o_1}}, ..., m_{o_k}^{t_{o_k}}, \overline{m}_{N \backslash \{o_1, ..., o_k\}})$. If it is a weakly undominated Nash equilibrium not implementing the $q-$truthful social choice function,

[15] We acknowledge that in a more general model, conformist agents may not observe the message of independent agents before they send their messages. For instance, conformist agents may know the number of independent agents but do not know their identities. This is an interesting scenario that we leave for further research.

[16] Admittedly, an exact number of independent agents could also be obtained for any given $(h_1, ..., h_n)$ but we consider that the symmetric case is a focal point.

go to Step $k+1$. If not, stop and register that the number of independent agents is k.

This procedure stops either in Step $n-1$ or before. If Step $n-1$ is reached, then it is clear that $(m_{o_1}^{t_{o_1}}, ..., m_{o_{n-1}}^{t_{o_{n-1}}}, \overline{m}_{o_n})$ is a weakly undominated Nash equilibrium implementing the $q-$truthful social choice function. We register the number of independent agents for ordering o. We repeat the same procedure for any ordering $o' \in O$. From all these pairs of orderings and numbers of independent agents, we select the maximal and associate that number with preference profile $\succeq$. We interpret this number as a measure of how problematic this preference profile is for the $q-$truthful social choice function. Following the exercise presented after Proposition 1, we now propose comparing preference profiles based on the number of independent agents associated with each of them. A preference profile $\succeq$ is more problematic than $\succeq'$ if the number of independent agents needed to implement the $q-$truthful social choice function is greater for $\succeq$ than for $\succeq'$. Therefore, we can identify the number of independent agents associated with the most problematic preference profile to the $q-$truthful social choice function for any given n and h.

We repeat the same procedure for any $h \in \{1, ..., n-1\}$, any $q-$truthful social choice function, and any n. In addition, we can propose a criterion to compare any pair of $q-$truthful social choice function. We say that a $q-$truthful social choice function is more problematic than q' if the number of independent agents to implement the $q-$truthful social choice function is greater than that for the $q'-$truthful social choice function.

We use Example 4 below to show the application of the procedure described above for some n and h.

Example 24 *Let* $N = \{1, 2, 3, 4, 5\}$ *and* $o = (o_1, ..., o_5)$. *Take the* $3-$*threshold rule and* $h = 3$. *Take preference profile* $(\succeq_1^1, \succeq_{N\setminus\{1\}}^0)$. *By Proposition 1,* m^0 *and* m^1 *are weakly undominated Nash equilibrium. For* $(\succeq_1^1, \succeq_{N\setminus\{1\}}^0)$, *the* $3-$*truthful social choice function selects* $(0; 1, 0, 0, 0, 0)$ *and the decision associated with* m^1 *is* 1. *Therefore,* $\overline{m} = m^1$. *Take ordering* $o = (1, 2, 3, 4, 5)$ *of the agents. We now apply the procedure.*

Step 1. *Replace agent* 1 *with an independent agent whose true opinion is to accept the* *proposal. Take* $(m_1^1, \overline{m}_{N\setminus\{1\}}) = (1,1,1,1,1)$. *Trivially, this is a weakly undominated Nash equilibrium not implementing the* $3-$*truthful social choice function. Therefore, we proceed to Step 2.*

Step 2. *Replace agent* 2 *with an independent agent whose true opinion is to reject the proposal. Take* $(m_1^1, m_2^0, \overline{m}_{N\setminus\{1,2\}}) = (1,0,1,1,1)$. *Since this is a weakly undominated Nash equilibrium not implementing the* $3-$*truthful social choice function, we proceed to Step 3.*

Step 3. *Replace agent* 3 *with an independent agent whose true opinion is to reject the proposal. Take* $(m_1^1, m_2^0, m_3^0, \overline{m}_{N\setminus\{1,2,3\}}) = (1,0,0,1,1)$. *By selfishness and since either agent* 4 *or agent* 5 *voting* 0 *change the decision from* 1 *to* 0*,* $(m_1^1, m_2^0, m_3^0, \overline{m}_{N\setminus\{1,2,3\}})$ *is not a weakly undominated Nash equilibrium. We stop and register that the number of independent agents is* 3*. Then, for* $o = (1,2,3,4,5)$*, the number registered is* 3*.*

We repeat the same procedure for ordering $o' = (2,3,4,5,1)$*.*

Step 1. *Replace agent* 2 *with an independent agent whose true opinion is to reject the proposal. Take* $(m_2^0, \overline{m}_{N\setminus\{2\}}) = (0,1,1,1,1)$. *Trivially, this is a weakly undominated Nash equilibrium not implementing the* $3-$*truthful social choice function. Therefore, we proceed to Step 2.*

Step 2. *Replace agent* 3 *with an independent agent whose true opinion is to reject the proposal. Take* $(m_2^0, m_3^0, \overline{m}_{N\setminus\{2,3\}}) = (0,0,1,1,1)$. *By selfishness and since either agent* 4 *or agent* 5 *voting* 0 *change the decision from* 1 *to* 0 $(m_2^0, m_3^0, \overline{m}_{N\setminus\{2,3\}})$ *is not a weakly undominated Nash equilibrium. We stop and register that the number of independent agents is* 2*. Then, for* $o' = (2,3,4,5,1)$*, the number registered is* 2*.*

In any ordering in which agent 1 occupies either the first or second position, the procedure is as in o *and the number of independent agents is* 3*. In any other ordering, the procedure is as in* o' *and the number of independent agents is* 2*. From all these pairs of orderings and numbers of independent agents, the maximal number of independent agents is* 3 *and we associate* 3 *with preference profile* $(\succeq_1^1, \succeq_{N\setminus\{1\}}^0)$*.*

We repeat the same procedure for every problematic preference profile for the $3-$*truthful* *social choice function. Table 4 offers the number of independent agents for each $q-$truthful social choice function and the preference profiles that are problematic for that rule. For example, the case of the $3-$truthful social choice function and preference profile $(\succeq_i^1, \succeq_{N\setminus\{i\}}^0)$ yields that the number of independent agents is 3, which corresponds to the number in the third cell of the second row.*

The first row in Table 4 corresponds to the number of independent agents associated with each $q-$truthful social choice function for preference profile $\succeq^0$. For the $q-$truthful social choice function such that $q = \{1, 2, 3\}$, this number is 2, as the $4-$truthful social choice function is 1 and this preference profile is not problematic for the $5-$truthful social choice function. For this preference profile, the most problematic $q-$truthful social choice functions are $q = \{1, 2, 3\}$, then the $4-$truthful social choice function, and finally, the $5-$truthful social choice function. The second and subsequent rows correspond to different preference profiles and, as can be observed in the table, the $q-$truthful social choice functions that are more problematic depend on the preference profiles shown in each row. The last row corresponds to the number of independent agents associated with the most problematic preference profile for each $q-$truthful social choice function.

Table 5.4: Independent agents when $n = 5$ and $h = 3$.

	$q-$thresholds				
Preference profiles $\succeq$	$q = 1$	$q = 2$	$q = 3$	$q = 4$	$q = 5$
$\succeq^0$	2	2	2	1	-
$(\succeq_i^1, \succeq_{N\setminus\{i\}}^0)$	-	3	3	2	-
$(\succeq_i^1, \succeq_j^1, \succeq_{N\setminus\{i,j\}}^0)$	-	4	4	3	-
$(\succeq_{N\setminus\{i,j\}}^1, \succeq_i^0, \succeq_j^0)$	-	3	4	4	-
$(\succeq_{N\setminus\{i\}}^1, \succeq_i^0)$	-	2	3	3	-
$\succeq^1$	-	1	2	2	2
$\#I$	2	4	4	4	2

The following theorem offers a formula that provides the number of independent agents as a function of n, q and h.

Theorem 25 *Let any n, $h \in \{1, ..., n-1\}$ and $q-$threshold rule. Every undominated Nash equilibria yields the decision selected by the $q-$truthful social choice function for any preference profile if*

$$\#I = \begin{cases} \min\{h, n-h\} & \textit{when } q \in \{1, n\}, \\ \min\{n-1, \max\{q-1, n-q\} + \min\{h, n-h\}\} & \textit{when } q \in \{2, ..., n-1\}. \end{cases}$$

Proof. See Appendix A. ■

We identify the most problematic $q-$truthful social choice function using the formula provided by the above theorem. It turns out that unanimous thresholds (either to accept or to reject the proposal) are the least problematic ones. Interestingly, as we remark below, after unanimity, the least problematic $q-$truthful social choice function is the majority.

Remark 26 *For any number of agents, n, and any $h = \{1, .., n-1\}$, there is a complete ranking of the $q-$truthful social choice function in terms of the number of independent agents. Moreover, unanimous thresholds are the least problematic and for any $q, q' \in \{2, .., n-1\}$, q is weakly less problematic than q' if and only if $\left|q - \frac{n+1}{2}\right| \leq \left|q' - \frac{n+1}{2}\right|$.*

To complete this section, we compare our way of modeling conformism with another reasonable and intuitive way of doing so. We can assume that agents' preferences satisfy selfishness but when it comes to conformity, they want to conform to as many agents as possible.[17] We now show that this property over the preferences of the agents is

[17] We thank an anonymous referee for proposing this alternative way of modeling conformity.

equivalent to considering that agents want to conform to at least the majority of agents, that is, $h = \frac{n}{2}$ when n is even and $h = \frac{n+1}{2}$ when n is odd.

Lemma 27 *Let any n and $q-$threshold rule. Suppose that the preferences of any conformist agent satisfy selfishness and that such an agent wants to conform to as many agents as possible. Every undominated Nash equilibria yields the decision selected by the $q-$truthful social choice function for any preference profile if*

$$\#I = \begin{cases} h & \text{if } q \in \{1, n\}, \\ n-1 & \text{if } q \in \{2, ..., n-1\}. \end{cases}$$

Proof. See Appendix A. ■

Interestingly, this alternative way of modeling conformity is very demanding in the sense that almost all agents must be independent to guarantee that any non-unanimous $q-$truthful social choice function is implemented. Still, the least problematic $q-$truthful social choice function are unanimity either to accept or to reject the proposal.

5.5 Sequential Voting

In this section, we study situations in which agents take turns when voting. As in Section 3, we maintain Assumption A but relax Assumptions B and C. Therefore, all agents are conformists and we do not impose uniformity of h. Suppose there is a fixed order of agents, indicating the sequence in which agents vote and, when voting, each agent knows what preceding agents have voted for. In what follows, *wlog*, we suppose that $o = \{1, 2, ..., n\}$, that is, agent 1 plays in the first stage, agent 2 plays in the second stage, and so on. Note that for any other ordering of the agents, say $o' \in O$, we can rename the agents and call agent 1 the first agent in o', agent 2 the second agent in o', etc.

We now introduce some additional notation. Let y be a node and let Y be a finite set of nodes that form a tree, with $Z \subset Y$ being the terminal nodes. For each $y \in Y$, let $i(y)$ be the agent playing in node y and let $\hat{h}(y)$ denote the set of nodes that are possible given what player $i(y)$ knows. Thus, if $y' \in \hat{h}(y)$, then $i(y') = i(y)$ and $\hat{h}(y') = \hat{h}(y)$. Let $\mathcal{H}_i$ be the set of information sets at which player i moves, that is, the set of information set at stage i of the game:

$$\mathcal{H}_i = \{S \subset Y : S = \hat{h}(y) \text{ for some } y \in Y \text{ with } i(y) = i\}.$$

A message for agent i is an action for each node y such that $i(y) = i$, and the message space for agent i is $M_i = \{0,1\}^{2^{i-1}}$.

We refer to $G^{\{1,..,n\},q} = ((M, g^q), \succeq)$ as a game for agents 1 to n, threshold q, and $\succeq \in \mathcal{R}^{h_i}$.

We introduce the following function $\alpha : \times_{i\in N} M_i \rightarrow \{0,1\}^n$, mapping each profile of messages, $m \in \times_{i\in N} M_i$, to the profile of actions, $\alpha(m) = (\alpha_1(m), ..., \alpha_n(m))$, where α_i is the action in m_i corresponding to the node y in which agent $i(y)$ plays given $h(y)$. In this context, we now define when two agents conform, the adaptation of the $q-$truthful social choice function and the $q-$threshold rule for the sequential game.

Definition 28 *For any $m \in \times_{i\in N} M_j$, and any $i, j \in N$, we say that agent i **conforms** to agent j if and only if $\alpha_i(m) = \alpha_j(m)$.*

Definition 29 *Given $(h_1, ..., h_n)$, a $q-$**truthful social choice function,** f^q, is such that for any $\succeq \in \times_{i\in N} \mathcal{R}_i^{h_i}$ chooses an alternative $(x; \alpha(m))$ where:*
i) $x = 1$ if the number of agents such that $t_i = 1$ is greater than or equal to q and $x = 0$ otherwise, and
ii) $\alpha_i(m) = t_i$ for any $i \in N$.

Definition 30 *Given any $q \in \{1, ..., n\}$, a $q-$**threshold rule** $g^q(m)$ is such that, for* *any $m \in \times_{i\in N} M_i$ chooses $x = 1$ if the number of agents such that $\alpha_i(m) = 1$ is greater than or equal to q and $x = 0$ otherwise.*

The equilibrium concept that we use is subgame perfect Nash equilibrium. Given game $G^{\{1,..,n\},q} = ((M, g^q), \succeq)$, $SPN(G^{\{1,..,n\},q})$ is the set of *subgame perfect Nash equilibria* of (M, g^q) at $\succeq$.

In Proposition 2, we make extensive use of whether an agent is pivotal. This remains valid at any stage of the game $G^{\{1,..,n\},q}$.

Definition 31 *Given any $q \in \{1, ..., n\}$, agent $i \in N$ is **pivotal** relative to $m_{-i} \in M_{-i}$ if there are $m_i, m_i' \in M_i$ such that $g^q(\alpha(m_i, m_{-i})) \neq g^q(\alpha(m_i', m_{-i}))$.*

Proposition 2 shows that any given $q-$truthful social choice function is implemented in any subgame perfect Nash Equilibrium of the game $G^{\{1,..,n\},q}$. The proof of Proposition 2 is divided in three Lemmas. Lemmas 2 and 3 offer direct proof for the $1-$ and the $n-$truthful social choice function, respectively. In the proof of these lemmas, we first analyze all preference profiles for which the decision of the $q-$truthful social choice function is to reject the proposal and then those for which the proposal is accepted. Then, we solve the game from stage n, in which agent n votes, to stage 1. In each stage, we analyze the equilibrium strategies for each of the voters, who vote taking into account $\hat{h}(y)$. In equilibrium, each agent votes according to her true opinion when she is pivotal relative to what the rest of agents previously voted for, and according to h when she is not pivotal. This completes the proof.

Finally, the proof for the $q-$truthful social choice function where $q \in \{2, .., n-1\}$ is presented in Lemma 4. We introduce some additional notation. We denote as $G^{\{1,...,k\},q} = \left(\times_{i=1}^{k} M_i, g^q, \{\succeq_i\}_{i=1}^{k}\right)$ a game consisting of agents 1 to k and threshold q, where $\succeq_i \in \mathcal{R}_i^{h_i}$ for any $i = \{1, ..., k\}$. The strategy of proof is to show that given the equilibrium strategy of the last agent in the sequence, the reduced game is either $G^{\{1,...,n-1\},q}$ if $\succeq_n^0$ or $G^{\{1,...,n-1\},q-1}$ if $\succeq_n^1$. First, we describe the equilibrium strategies of this last agent in two claims, A and B. Claim A applies when $\succeq_n^0$ and Claim B when $\succeq_n^1$. Note that the new last agent in the reduced game is agent $n-1$. Then, we show that applying this line of reasoning iteratively, we eventually end up in a game such that either $G^{\{1,...,k\},q}$ where $q=1$ or $G^{\{1,...,k\},q}$ where $q=k$. Once we obtain one of these two reduced games, the proof is completed applying either Lemma 2 or Lemma 3.

Proposition 2. *Let any* n, any $(h_1,...,h_n)$, *and any* $q-$*threshold rule.* Every *subgame perfect Nash Equilibrium yields the decision selected by the* $q-$truthful social choice function for any $\succeq \in \times_{i\in N}\mathcal{R}_i^{h_i}$.

Proof. See Appendix A. ■

Note that any $q-$truthful social choice function is implemented for any $(h_1,...,h_n)$. This implies that, under sequential voting, the presence of conformism does not affect the implementation of $q-$truthful social choice function. When agents play strategies that are a subgame perfect Nash equilibrium, it is as if all players were pivotal, because they anticipate the pivotality of those following them. Since conformity is lexicographically inferior to the agents' true opinions, the latter dominate the decisions of the players that are relevant for the equilibrium path.

In both versions of the game, simultaneous and sequential, conformity is present. Under simultaneous voting (Section 3), those equilibria not implementing a given $q-$truthful social choice function are generated by the presence of conformity. When we replace some conformist agents with independent agents (Section 4), we assume that conformist agents vote already knowing the number of independent agents and their messages. Therefore, we impose a sequence in which first, all independent agents vote simultaneously and subsequently, all conformist agents vote simultaneously. In this case, given that the replacement of conformist agents with independent agents stops when the procedure makes a conformist agent pivotal, conformity applies only when the decision is already set and it is always possible to implement the $q-$truthful social choice function. Under sequential voting, conformity does not affect the decision since agents vote selfishly in any situation in which they have the opportunity to do so.

Finally, the fact that in the sequential version of the game, any subgame perfect Nash equilibrium implements the $q-$truthful social choice function can be linked to the literature about contribution to a public good. In the literature, there are inefficient equilibria in the simultaneous version of the contribution game, but any equilibrium in the sequential version of the game is efficient (see Coats et al., 2009 for experimental evidence).

5.6 Concluding Remarks

The first message from this study is that the presence of conformity may alter decisions taken by agents. We show that no $q-$truthful social choice function is implementable in a simultaneous binary voting game if all agents are conformist.

The second message is that when designing committees in charge of taking decisions, there are several aspects to be considered: the number of agents in the committee in charge of the decision, the voting rule to use, the combination of independent and conformist agents in the committee, and the timing when agents announce their votes.

If the decisions are to be taken simultaneously, unanimous $q-$truthful social choice functions are the least problematic, followed by the majority $q-$truthful social choice function. Independent agents must be asked to reveal their opinions first and then, with this information in their hands, the rest of the agents vote and the decision is obtained. In the corporate world, a great majority of companies normally include in their committees external agents (what we call independent agents) who contribute with external and unbiased opinions to decision-making processes. We ignore what motivates the companies to do so, although we do know that this process helps to obtain decisions that correspond to the true opinions of agents.

Finally, when the committee takes decisions sequentially, the presence of conformism does not affect these decisions.

5.7 Appendix

This appendix includes the proofs of Theorem 1 and Lemmas 1, 2, 3, and 4. Before proceeding with the proofs, we need to introduce some notation. Let lower-case letters c, s denote the cardinality of subsets of agents $C, S \subset N$.

Proof of Theorem 1. We distinguish two cases, depending on the $q-$threshold rule. Case 1 presents the proof for the $1-$ and the $n-$threshold rule and Case 2 presents the proof for the remaining thresholds, $q \in \{2, ..., n-1\}$.

Case 1. Let $I \neq \emptyset$ and the $1-$threshold rule (the $n-$threshold rule is symmetric). If $\#I = \min\{h, n-h\}$, the $1-$truthful social choice function is implemented in equilibrium when $\succeq^0$. In doing so, we show that m_i^1 is weakly dominated by m_i^0 for any $i \in N \backslash I$. By assumption, any $i \in N \backslash I$ knows that there are $\#I$ voting for 0. We distinguish two cases:

Case 1.1. [Case 1.2.] $n - h > h$ $[n - h \leq h]$. Let $\#I = h$ $[\#I = n - h]$. Take any agent $i \in N \backslash I$. For any $m = (m_S^1, m_{N\backslash S}^0) \in UN((M, g^1), \succeq^0, h)$, $I \subseteq N \backslash S$. Since $n - s \geq \#I$ and by $h-$conformity, $(1; m_i^0, m_{S\backslash\{i\}}^1, m_{N\backslash S}^0) \succeq_i^0 (1; m_S^1, m_{N\backslash S}^0)$. By selfishness, $(0; m_i^0, m_{-i}^0) \succ_i^0 (1; m_i^1, m_{-i}^0)$ and m_i^1 is weakly dominated by m_i^0 for any $i \in N \backslash I$. Therefore, the result follows. Finally, if $\#I < h$ $[\#I < n - h \leq h]$ there are $m \in UN((M, q), \succeq^0, h)$ such that the $1-$truthful social choice function is not implemented. Let $m = (m_S^1, m_{N\backslash S}^0)$ where $S = N \backslash I$. For any $i \in S$, by $h-$conformity and since $\#I + 1 < h$, $(1; m_S^1, m_{N\backslash S}^0) \succ_i^0 (1; m_i^0, m_{S\backslash\{i\}}^1, m_{N\backslash S}^0)$.

Case 2. Let $I \neq \emptyset$ and any $q-$threshold rule such that $q \in \{2, ..., n-1\}$. We show that if $\#I = \min\{n-1, \max\{q-1, n-q\} + \min\{h, n-h\}\}$, the $q-$truthful social choice function is implemented in equilibrium for any $q \in \{2, .., n-1\}$ and any $\succeq$. In doing so, we show that for any $\succeq_i$, $m_i \neq t_i$ is weakly dominated by $m_i^{t_i}$ for any $i \in N \backslash I$. By assumption, any $i \in N \backslash I$ knows there are $\#I$ independent agents voting for their true opinions. For clarification, we divide the proof into three cases depending on the threshold.

Case 2.1. $q \in \{2, ..., \lfloor \frac{n+1}{2} \rfloor - 1\}$ (**Case 2.2.** when $q \in \{\lceil \frac{n+1}{2} \rceil + 1, ..., n-1\}$ is symmetric). Then, $q - 1 < n - q$. We distinguish two cases:

Case 2.1.1. $q-1 \leq \min\{h, n-h\}$**.** Then, $q \leq h+1$. Let $\#I = n-1$ and $\succeq = (\succeq_C^1, \succeq_{N\setminus C}^0) \in \times_{i\in N}\mathcal{R}_i^h$. Let agent i be the only conformist agent. If $c < q-1$, since $c < q-1 \leq h$, by $h-$conformity, $(0; m_i^0, m_{-i}) \succeq_i^{t_i} (0; m_i^1, m_{-i})$. If $c = q-1$ and $\succeq_i^1$ or $c = q$ and $\succeq_i^0$, since $c = q-1 \leq h$, by $h-$conformity, $(0; m_i^0, m_{-i}) \succeq_i^{t_i} (0; m_i^1, m_{-i})$. If $c = q-1$ and $\succeq_i^0$, by selfishness $(0; m_i^0, m_C^1, m_{N\setminus C\cup\{i\}}^0) \succ_i^0 (1; m_i^1, m_C^1, m_{N\setminus C\cup\{i\}}^0)$. If $c = q$ and $\succeq_i^1$, by selfishness $(1; m_i^1, m_{C\setminus\{i\}}^1, m_{N\setminus C}^0) \succ_i^1 (0; m_i^0, m_{C\setminus\{i\}}^1, m_{N\setminus C}^0)$. If $c > q$, when $c < h+1$, by $h-$conformity, $(1; m_i^0, m_{-i}) \succeq_i^{t_i} (1; m_i^1, m_{-i})$; when $c \geq h+1$, by $h-$conformity, $(1; m_i^1, m_{-i}) \sim_i^{t_i} (1; m_i^0, m_{-i})$. Finally, suppose that $\#I < n-1$. Let $\#I = n-2$ and suppose, *wlog,* that $1, 2 \in N\setminus I$. In addition, let $\succeq = (\succeq_C^1, \succeq_{N\setminus C}^0) \in \times_{i\in N}\mathcal{R}_i^h$ be such that $c = q$, and $\succeq_1^1, \succeq_2^1$. For $m = (m_1^0, m_2^0, m_{-\{1,2\}})$ such that for any $k \in I$, $m_k^{t_k}$ and $i, j \in \{1, 2\}$, $(1; m_i^0, m_j^0, m_{-\{i,j\}}) \succeq_i^1 (1; m_i^1, m_j^0, m_{-\{i,j\}})$.

Case 2.1.2. $q-1 > \min\{h, n-h\}$**.** We distinguish two subcases:

Subcase 2.1.2.a. [Subcase 2.1.2.b.] $h < n-h$ $[h \geq n-h]$**.** Then, $h \leq \frac{n}{2}-1$ $[h > \frac{n}{2}-1]$ and $q > h+1$ $[q < h+1]$. Let $\#I = n-q+h$ $[\#I = n-q+n-h]$ and $\succeq = (\succeq_C^1, \succeq_{N\setminus C}^0) \in \times_{i\in N}\mathcal{R}_i^h$. If $c < q$, at most $q-1$ independent agents vote for 1 and at least h independent agents vote for 0. Then, either by selfishness or by $h-$conformity, it is a weakly dominant strategy to vote for 0 for any $i \in N\setminus I$ such that $\succeq_i^0$. If $c \geq q$, at least h $[n-h]$ independent agents vote for 1. Either by selfishness or by $h-$conformity, it is a weakly dominant strategy to vote for 1 for any $i \in N\setminus I$ such that $\succeq_i^1$. Finally, suppose that $\#I < n-q+h$ $[\#I < n-q+n-h]$. Let $\#I = n-1-(q-h)$ $[\#I = n-q+n-h-1]$, and suppose $\succeq = (\succeq_C^1, \succeq_{N\setminus C}^0)$ is such that for $h-1$ $[n-h-1]$ independent agents $\succeq_i^1$, for $n-q$ independent agents $\succeq_j^0$ and all $k \in N\setminus I$ are such that $\succeq_k^1$. Note that the decision implemented by the $q-$truthful social choice function is 1. Let m be such that all independent agents tell the truth, and all conformist agents report $m_k = 0$. Since $h \leq \frac{n}{2}-1$ $[h > \frac{n}{2}-1]$, by $h-$conformity, $(0; m_k^0, m_{-k}) \succ_k^1 (0; m_k^1, m_{-k})$.

Case 2.3. n is even and $q \in \{\frac{n}{2}, \frac{n}{2}+1\}$.[18] Then, $n-q > q-1$. We distinguish two cases:

Case 2.3.1. $q-1 \leq \min\{h, n-h\}$**.** Then, $q \leq h+1$. Let $\#I = n-1$ and

[18] The proof when n is odd and $q = \frac{n+1}{2}$ is included in Moreno and Ramos-Sosa (2015).

$\succeq = (\succeq^1_C, \succeq^0_{N\backslash C}) \in \times_{i\in N}\mathcal{R}^h_i$. Let agent i be the only conformist agent. If $c \leq q$, the proof is analogous to Case 2.1.1. If $c > q$, when $q < c < h+1$, at least q independent agents vote for 1 and at most h agents vote for 1. Since $q \leq h+1 \leq q+1$,by $h-$conformity, $(1; m^0_i, m_{-i}) \succeq^{t_i}_i (1; m^1_i, m_{-i})$. If $q \leq h+1 \leq c$, it could be either $q \leq h+1 < c$ or $q < h+1 \leq c$. If $q \leq h+1 < c$, at least $h+1$ independent agents vote for 1. If $q < h+1 \leq c$, at least h independent agents vote for 1. Then, by $h-$conformity $(1; m^1_i, m_{-i}) \succeq^{t_i}_i (1; m^0_i, m_{-i})$. Finally, suppose that $\#I < n-1$. Let $\#I = n-2$ and suppose, *wlog,* that $1, 2 \in N\backslash I$. In addition, let $\succeq = (\succeq^1_C, \succeq^0_{N\backslash C})$ be such that $c = q-1$, and $\succeq^1_1, \succeq^1_2$. For $m = (m^0_1, m^0_2, m_{-\{1,2\}})$ such that for any $k \in I$, $m^{t_k}_k$, and $i, j \in \{1, 2\}$, since $q - 1 < n - q$ we have $(0; m^0_i, m^0_j, m_{-\{i,j\}}) \succeq^1_i (0; m^1_i, m^0_j, m_{-\{i,j\}})$.

Case 2.3.2. $q - 1 > \min\{h, n-h\}$**.** We distinguish two subcases:

Subcase 2.3.2.a. $h < n-h$**.** Then, $h \leq \frac{n}{2} - 1$ and $q > h+1$. Let $\#I = n-(q-h)$ and $\succeq = (\succeq^1_C, \succeq^0_{N\backslash C}) \in \times_{i\in N}\mathcal{R}^h_i$. If $c < q$, for $i \in N\backslash I$ such that $\succeq^1_i$, they vote for 1 when the independent agents voting for 1 are enough to conform to at least h. Otherwise, by $h-$conformity, they vote for 0. For the particular case in which $q-1$ independent agents vote for 1 and h independent agents vote for 0, since $q > h+1$ all $i \in N\backslash I$ such that $\succeq^0_i$ vote for 0 and all $i \in N\backslash I$ such that $\succeq^1_i$ vote for 1 as $q > h+1$. The rest of the proof is analogous to Subcase 2.1.2.a.

Subcase 2.3.2.b. $h \geq n-h$**.** Then, $h > \frac{n}{2} - 1$. Let $\#I = n - q + n - h$, and $\succeq = (\succeq^1_C, \succeq^0_{N\backslash C}) \in \times_{i\in N}\mathcal{R}^h_i$. If $c \geq q$, at least $n-h$ independent agents vote for 1, and there are strictly less than $n-q$ independent agents voting for 0. Given that $n-q > q-1$ and $q < h$, either by selfishness or by $h-$conformity, it is a weakly dominant strategy to vote for 0 for any $i \in N\backslash I$ such that $\succeq^0_i$. The rest of the proof is analogous to Subcase 2.1.2.b. ■

Proof of Lemma 1. We distinguish two cases, depending on the $q-$threshold rule. Case 1 presents the proof for the $1-$ and the $n-$threshold rule and Case 2 presents the proof for the remaining thresholds, $q \in \{2, ..., n-1\}$.

Case 1. Let the $1-$threshold rule (the $n-$threshold rule is symmetric). Then, the

$1-$truthful social choice function is not implemented when $\succeq^0$. Let $\#I = h$ and $h = \frac{n}{2}$ ($h = \frac{n+1}{2}$ if n is odd). By assumption, any conformist agent knows there are h independent agents voting for 0. Take any agent $i \in N \backslash I$. Since $\#I + i > \frac{n}{2} - i$, by $h-$conformity of agent $i \in N \backslash I$, $(0; m_i^0, m_{S \backslash \{i\}}^1, m_{N \backslash S}^0) \succeq_i^0 (1; m_S^1, m_{N \backslash S}^0)$. Finally, if $\#I = h - 1$, by $h-$conformity and since $\#I + i \leq h$, $(1; m_S^1, m_{N \backslash S}^0) \succ_i^0 (0; m_i^0, m_{S \backslash \{i\}}^1, m_{N \backslash S}^0)$.
Case 2. Let the $q-$threshold rule where $q \in \{2, ..., n-1\}$. We show that if $\#I = n - 1$, the $q-$truthful social choice function is implemented for any $q \in \{2, .., n-1\}$ and any $\succeq = (\succeq_C^1, \succeq_{N \backslash C}^0) \in \times_{i \in N} \mathcal{R}_i^h$. By assumption, the conformist agent knows there are $n - 1$ independent agents voting for their true opinions. Let agent i be the unique conformist agent. If $\succeq_i^0$, by selfishness $(0; m_i^0, m_C^1, m_{N \backslash C \cup \{i\}}^0) \succ_i^0 (1; m_i^1, m_C^1, m_{N \backslash C \cup \{i\}}^0)$ and if $\succeq_i^1$, by selfishness $(1; m_i^1, m_{C \backslash \{i\}}^1, m_{N \backslash C}^0) \succ_i^1 (0; m_i^0, m_{C \backslash \{i\}}^1, m_{N \backslash C}^0)$. Therefore, in both cases, the $q-$truthful social choice function is implemented in equilibrium.
Finally, suppose that $\#I < n - 1$. Let $\#I = n - 2$ and suppose *wlog* that $1, 2 \in N \backslash I$. For clarification, we divide the proof into four cases depending on the threshold. Take $q < \frac{n}{2}$. Let $\succeq = (\succeq_C^1, \succeq_{N \backslash C}^0)$ be such that $c = q$, and $\succeq_1^1, \succeq_2^1$. For $m = (m_1^0, m_2^0, m_{-\{1,2\}})$ such that for any $k \in I$, $m_k^{t_k}$, and $i, j \in \{1, 2\}$, $(0; m_i^0, m_j^0, m_{-\{i,j\}}) \succeq_i^1 (0; m_i^1, m_j^0, m_{-\{i,j\}})$. Take $q = \frac{n}{2}$ and n is even. Let $\succeq = (\succeq_C^1, \succeq_{N \backslash C}^0)$ be such that $c = q - 1$, and $\succeq_1^0, \succeq_2^0$. For $m = (m_1^1, m_2^1, m_{-\{1,2\}})$ such that for any $k \in I$, $m_k^{t_k}$, and $i, j \in \{1, 2\}$, $(1; m_i^1, m_j^1, m_{-\{i,j\}}) \succeq_i^0 (1; m_i^1, m_j^0, m_{-\{i,j\}})$. Take $q = \frac{n+1}{2}$ and n is odd. Let $\succeq = (\succeq_C^1, \succeq_{N \backslash C}^0)$ be such that $c = q - 1$, and $\succeq_1^0, \succeq_2^0$. For $m = (m_1^1, m_2^1, m_{-\{1,2\}})$ such that for any $k \in I$, $m_k^{t_k}$, and $i, j \in \{1, 2\}$, $(1; m_i^1, m_j^1, m_{-\{i,j\}}) \succeq_i^0 (1; m_i^1, m_j^0, m_{-\{i,j\}})$. Take $q > \frac{n}{2}$. Let $\succeq = (\succeq_C^1, \succeq_{N \backslash C}^0)$ be such that $c = q - 1$, and $\succeq_1^0, \succeq_2^0$. For $m = (m_1^1, m_2^1, m_{-\{1,2\}})$ such that for any $k \in I$, $m_k^{t_k}$, and $i, j \in \{1, 2\}$, $(1; m_i^1, m_j^1, m_{-\{i,j\}}) \succeq_i^0 (1; m_i^0, m_j^1, m_{-\{i,j\}})$. ■

The proof of Proposition 2 makes use of three lemmas. Lemma 2 applies for the $1-$truthful social choice function, Lemma 3 for the $n-$truthful social choice function, and Lemma 4 for the remaining $q-$truthful social choice functions, $q \in \{2, ..., n-1\}$.

Lemma 2. *For any number of agents n, the* $1-$truthful social choice function *is implemented in any subgame perfect Nash Equilibrium on R^{h_i}.*

Proof. Let n and the $1-$threshold rule. Let $\succeq^0$. Then, $x = 0$. We have to show that $SPN(G^{\{1,..,n\},1}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},1})$, $g^1(m) = 0$. We solve the game starting at stage n where agent n votes (remember that $o = (1,..,n)$). Since $\succeq_n^0$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n) = 0$ if agent n is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{n-1})$, and by $h-$conformity, $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ would be 0 or 1 depending on h if agent n is not pivotal. Out of the path of play, agent n is indifferent. We proceed to stage $n-1$, that is, to agent $n-1$. Applying the same reasoning iteratively, we reach stage 1. Again, since $\succeq_1^0$, in equilibrium, m_1 is such that $(m_2,..,m_n)$ as described above, by selfishness $\alpha_1(m_1,..,m_n) = 0$, since agent 1 is pivotal relative to $(m_2,..,m_n)$. Note that $g^1(m_1,..,m_n) = 0$. Then, $m \in SPN(G^{\{1,..,n\},1})$ and $g^1(m) = 0$.
Let $\succeq \in \mathcal{R}^{h_i}$ be such that for some $\succeq_i$, $\succeq_i^1$. Then, $x = 1$. We have to show that $SPN(G^{\{1,..,n\},1}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},1})$, $g^1(m) = 1$. In doing so, we show the proof by contradiction, that is, for any $m \in SPN(G^{\{1,..,n\},1})$, $g^1(m) = 0$. We solve the game starting at stage n where agent n votes. Let *wlog*, agent n in order o for whom $\succeq_n^1$. Since $\succeq_n^1$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n) = 1$ if agent n is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{n-1})$, and by $h-$conformity, $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ would be 0 or 1, depending on h whether agent n is not pivotal. Out of the path of play, agent n is indifferent. Note that for any $\widetilde{m}_i \in M_i$ where $i < n$, $g^1(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n) = 1$. Therefore, any $i < n$ is not pivotal relative to any $(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ for any $\widetilde{m}_j \in M_j$ for $j \neq i$, $j < n$. In equilibrium, for any $i < n$, agent i announces m_i according to $h-$conformity at any node in which she plays. Then, $m \in SPN(G^{\{1,..,n\},1})$ and $g^1(m) = 1$, and we get a contradiction. ■

Lemma 3. *For any number of agents n, the* $n-$truthful social choice function *is implemented in any subgame perfect Nash equilibrium on R^{h_i}.*

Proof. The proof is symmetric to Lemma 2. ■

The proof of Lemma 4 makes extensive use of Lemmas 2 and 3.

Lemma 4. *For any number of agents n, the $q-$truthful social choice function such that $q \in \{2, ..., n-1\}$ is implemented in any subgame perfect Nash Equilibrium on R^{h_i}.*

Proof. Let n and any $q-$threshold rule such that $q \in \{2, ..., n-1\}$. We show that for any $G^{\{1,...,n\},q}$, if agents play their equilibrium strategies, the reduced game obtained can be either $G^{\{1,...,k\},q}$ where $q = k$ or $G^{\{1,...,k\},q}$ where $q = 1$, which would depend on the preferences of the agents. We solve the game starting at stage n where agent n votes (remember that $o = (1, ..., n)$). Let $\succeq \in \mathcal{R}^{h_i}$. We distinguish two claims, depending on $\succeq_n$:

Claim A. If $\succeq_n^0$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1, ..., \widetilde{m}_{n-1}, m_n) = 0$ if agent n is pivotal relative to $(\widetilde{m}_1, ..., \widetilde{m}_{n-1})$, and by $h-$conformity, $\alpha_n(\widetilde{m}_1, ..., \widetilde{m}_{n-1}, m_n)$ would be 0 or 1, depending on h if agent n is not pivotal. Out of the path of play, agent n is indifferent. Then, the reduced game obtained after agent n plays her equilibrium strategy is a game with agents 1 to $n-1$ and $q-$threshold, that is, $G^{\{1,...,n-1\},q}$.

Claim B. If $\succeq_n^1$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1, ..., \widetilde{m}_{n-1}, m_n) = 1$ if agent n is pivotal relative to $(\widetilde{m}_1, .., \widetilde{m}_{n-1})$, and by $h-$conformity, $\alpha_n(\widetilde{m}_1, ..., \widetilde{m}_{n-1}, m_n)$ would be 0 or 1, depending on h whether agent n is not pivotal. Out of the path of play, agent n is indifferent. Then, the reduced game is $G^{\{1,...,n-1\},q-1}$.

We now distinguish several cases, depending on the number of agents such that $\succeq_i^1$.

Case 1. $\succeq$ such that $\#\{i \in N : \succeq_i^1\} < q-1$. We distinguish several cases, depending on the preferences of agents 2 to n. If $\succeq_k^0$, for $k = \{q+1, ..., n\}$, Claim A applies at stages n to $q+1$ and the reduced game is $G^{\{1,...,q\},q}$. If $\succeq_q^0$ and $\succeq_j^1$ for exactly one agent $j \in \{q+1, ..., n\}$, Claim B applies at stage j, and Claim A otherwise. Then, the reduced game is $G^{\{1,...,q\},q-1}$. To solve stage q, we apply Claim A and the reduced game is $G^{\{1,...,q-1\},q-1}$. If $\succeq_{q-1}^0$, and $\succeq_j^1$, $\succeq_k^1$ for exactly two agents, $j, k \in \{q, ..., n\}$. Claim B applies when we reach stages j and k, and Claim A otherwise. Then, the reduced game is $G^{\{1,...,q-1\},q-2}$. To solve stage $q-1$, we apply Claim A and the reduced game is

$G^{\{1,...,q-2\},q-2}$. By a similar reasoning, we end up in a case in which $\succeq_3^0$ and $\succeq_j^1$ for exactly $q-2$ agents $j \in \{4,...,n\}$. Claim B applies when each of the $q-2$ agents play, and Claim A otherwise. Then, the reduced game is $G^{\{1,2,3\},2}$. To solve stage 3, we apply Claim A and the reduced game is $G^{\{1,2\},2}$. In all of the abovementioned cases, the $q-$threshold equals the number of agents remaining in the game. Applying Lemma 3**,** the result follows.

Case 2. $\succeq$ such that $q < \#\{i \in N :\succeq_i^1\}$. We distinguish several cases depending on the preferences of agents 2 to n. If $\succeq_k^1$ for $k = \{n+2-q,...,n\}$, Claim B applies at stages n to $n+2-q$. Then, the reduced game is $G^{\{1,...,n+1-q\},1}$. If $\succeq_{n+1-q}^1$ and $\succeq_j^0$ for exactly one agent $j \in \{n+2-q,...,n\}$. Claim A applies when we reach stage j, and Claim B otherwise. Then, the reduced game is $G^{\{1,...,n+1-q\},2}$. To solve stage $n+1-q$, we apply Claim B and the reduced game is $G^{\{1,...,n-q\},1}$. If $\succeq_{n-q}^1$ and $\succeq_j^0$, $\succeq_k^0$ for exactly two agents, $j,k \in \{n+1-q,...,n\}$. Claim A applies when we reach stages j and k, and Claim B otherwise. Then, the reduced game is $G^{\{1,...,n+1-q\},2}$. To solve stage $n-q$, we apply Claim B and the reduced game is $G^{\{1,...,n-q-1\},1}$. By a similar reasoning, we end up in a case in which $\succeq_3^1$ and $\succeq_j^0$ for exactly $n-q-1$ agents $j \in \{4,...,n\}$. Claim A applies when each of the $n-q-1$ agents play, and Claim B otherwise. Then, the reduced game is $G^{\{1,2,3\},2}$. To solve stage 3, we apply Claim B and the reduced game is $G^{\{1,2\},1}$. In all of the abovementioned cases, for the obtained reduced games, the threshold is 1. Applying Lemma 2, the result follows.

Case 3. $\succeq$ such that $q-1 \leq \#\{i \in N :\succeq_i^1\} \leq q$. We distinguish two subcases.

Subcase 3.1. $n-q < q-1$. We distinguish several cases depending on the preferences of agents 2 to n. If $\succeq_k^0$ for $k = \{q+1,...,n\}$, Claim A applies at stages n to $q+1$. Then, the reduced game is $G^{\{1,...,q\},q}$. If $\succeq_{r+1}^0$ and $\succeq_j^1$ for exactly $q-r$ agents, $q > r > 1$, $j \in \{r+2,...,n\}$. Claim B applies when each of the $q-r$ agents play, and Claim A otherwise, and the reduced game is $G^{\{1,...,r+1\},r}$. To solve stage $r+1$, we apply Claim A and the reduced game is $G^{\{1,...,r\},r}$. Lemma 3 applies. If $\succeq_2^0$ and $\succeq_j^1$ for exactly $q-1$ agents, $j \in \{3,...,n\}$. Claim B applies in the stages in which each of the above $q-1$ agents play, and Claim A otherwise, and the reduced game is $G^{\{1,2\},1}$. Lemma 2 applies.

Subcase 3.2. $n-q>q-1$. We distinguish several cases, depending on the preferences of agents 2 to n. If $\succeq_k^1$ for $k=\{n+2-q,...,n\}$, Claim A applies at stages n to $n+2-q$ and the reduced game is $G^{\{1,...,n+1-q\},1}$. If $\succeq_{r+1}^1$ and $\succeq_j^0$ for exactly $n-q-r$ agents, $n-q>r>0$, $j\in\{r+2,...,n\}$. Claim A applies when each of the $n-q-r$ agents play, and Claim B otherwise, and the reduced game is $G^{\{1,...,r+1\},2}$. To solve stage $r+1$, we apply Claim B and the reduced game is $G^{\{1,...,r\},1}$. Lemma 2 applies. If $\succeq_3^1$ and $\succeq_j^0$ for exactly $n-q-r$ agents, $j\in\{3,...,n\}$. Claim A applies when each of the $n-q-r$ agents play, and Claim B otherwise, and the reduced game is $G^{\{1,2\},1}$. Lemma 2 applies.
Subcase 3.3. $n-q=q-1$. We distinguish several cases, depending on the preferences of agents 2 to n. If $\succeq_{r+1}^0$ and $\succeq_j^1$ for exactly $q-r$ agents, $q>r>1$, $j\in\{r+2,...,n\}$. Claim B applies when each of the $q-r$ agents play, and Claim A otherwise, and the reduced game is $G^{\{1,...,r+1\},r}$. To solve stage $r+1$, we apply Claim A and the reduced game is $G^{\{1,...,r\},r}$. Lemma 3 applies. If $\succeq_{r+1}^1$ and $\succeq_j^0$ for exactly $n-q-r$ agents, $n-q>r>0$, $j\in\{r+2,...,n\}$. Claim A applies when each of the $n-q-r$ agents play, and Claim B otherwise, and the reduced game is $G^{\{1,...,r+1\},2}$. To solve stage $r+1$, we apply Claim B and the reduced game is $G^{\{1,...,r\},1}$. Lemma 2 applies. ■

Proof of Proposition 2. This follows from Lemmas 2, 3, and 4.

Chapter 6

Conformity, Information and Truthful voting

Coauthored with Bernardo Moreno and Ismael Rodríguez-Lara (Middlesex University). Submitted.

6.1 Introduction

People frequently face binary decisions that require their opinion: board members choosing whether to accept or reject a proposal (e.g., projects and budgets), senators and congressmen voting on whether to pass a bill, or citizens voting in a referendum. In these cases, agents might have a clear and strong opinion on what the best decision is. If they do not pay attention to anything else, one can expect them to vote for the option that best suits their opinions, that is, to vote truthfully. However, agents may decide to misrepresent their opinion and conform to other agents by voting for the alternative option, especially if they know that their vote will not influence the adopted decision.[1] In this paper, we argue that the desire to conform to other agents in a voting game might

[1] Conformity can be defined as the tendency of agents to align attitudes, beliefs and behaviors with those of some other agents (Myers, 2012).

be affected by the voting rule, which in turn determines the pivotality of agents. We also discuss the effects of information in a conformity setting. Specifically, we investigate whether agents will be more likely to vote truthfully when they are informed that other agents will vote truthfully.

The literature on conformity voting (summarized in Section 6.2) frequently assumes that agents may want to vote for the winning option. Arguably, decisions are not always adopted using majority rules, therefore voting for the winning option is not always equivalent to voting with the majority group. One of the goals of the current paper is to re-examine the idea of conformity to account for decisions that require more than majority to be adopted. The fact that many institutions employ different rules calls for modeling the issue of conformity under different voting rules. In some States of the United States, for example, juries require unanimity for finding a defendant guilty. In the Council of the European Union, unanimity is also used for EU membership, while a supermajority is required when the Council votes on a proposal by the Commission or the High Representative of the Union for Foreign Affairs and Security Policy.

Our goal in this paper is to study, both theoretically and empirically, how conformity and information affect agents' tendency to vote truthfully under different voting rules. On a theoretical level, we build a model of complete information in which a group of five agents face a binary decision, with one of the options requiring a certain degree of support (majority, supermajority or unanimity) to be passed. Agents prefer one of the two possible options, A and B. There is a group of three agents who prefer option B and a group of two agents preferring option A. In what follows, we refer to agents preferring option A (B) as type-A agents (type-B agents). We distinguish three scenarios. In the first scenario, agents only care about the adopted decision. Therefore, voting truthfully is the unique undominated Nash equilibrium. In the second scenario, all agents are conformists. They care about whether their preferred option is chosen and whether they vote the same as the other agents. Conformist agents' preferences are lexicographical in the sense that they always prefer their option to be elected and would conform if they are not pivotal. In this case, agents voting truthfully is an undominated Nash equilibrium and there are also several equilibria in which some group of agents does not vote truthfully. In the third scenario, we introduce heterogeneous types, and it is common information that two type-B agents will vote for their preferred option. In this setting, we show that truthful voting remains being an undominated Nash equilibrium and the set of equilibria in which agents do not vote truthfully shrinks, compared with the case of conformity.

To empirically investigate these predictions, we conduct a laboratory voting game. Following, among others, Gerber et al. (1998), Morton and Williams (1999), Battaglini et al. (2010) or Bassi et al. (2011), we employ monetary incentives to induce subjects to have preferences over alternatives that correspond to our theory. In our experiment, we consider two different types of subjects. Each type receives the highest possible payoff if the adopted decision coincides with their type. In our baseline treatment, subjects also receive a small additional payoff, regardless of how they vote. In our treatment with conformity, the additional payoff is received only if the subjects' decision coincides with the decision of any other agent in their group. In a third treatment, a group of two type-B agents is forced to vote truthfully, and this is known by all the agents before they vote.[2]

Consistent with our theoretical predictions, we find that agents are more likely to vote truthfully in the baseline treatment, compared with the treatment in which conformity is induced. We also provide evidence on the effects of the different voting rules (majority, supermajority and unanimity) on the likelihood of each type of agent voting truthfully. Our findings indeed reveal an interplay between conformity and the pivotality of agents. We show that the voting rule does not have any effect on the likelihood of voting truthfully when there is no conformity, as predicted by our theoretical model. In the presence of conformity, the effect of the voting rule seems to depend on the type of the agent. We find that all type-B agents vote truthfully when a majority is required, while type-A agents

[2] Note that each of the treatments corresponds to a theoretical scenario. Note also that we deliberately abstract from the issue of whether subjects conform, focusing our attention on the effects of inducing conformity in a voting setting.

are more likely to conform. Under supermajority, option B is selected only if at least one type-A agent votes for it. We find that type-A (type-B) agents vote more (less) truthfully under supermajority, compared with majority rule. With unanimity, type-B agents are in the majority group but anticipate the pivotality of type-A agents and vote less truthfully as a result. Finally, our data support the effects of being informed about the vote of some agents as truthful voting becomes more pervasive. The effects of conformity and information translate into efficiency losses and gains, respectively. Specifically, we find that conformity (information) decreases (increases) the average total payoffs and the likelihood of obtaining the maximum possible total payoff.

The next section offers a review of the extant literature on conformity. In Section 6.3, we present our theoretical model and the results for this particular setup. In Section 6.4, we present the experimental design. We summarize the testable hypotheses in Section 6.5. The results are discussed in Section 6.6. Section 6.7 concludes. Additional material such as the experimental instructions and the non-parametric analysis of our data are relegated to the Appendix.

6.2 Literature Review

The term conformity is often used in the literature on social influence to indicate agreement with the majority position, brought about either by a desire to 'fit in' or be liked (normative conformity) or because of a desire to be correct (informational conformity).[3] In the current paper, we consider a voting game in which agents have a preferred option and obtain the maximum possible payoff if this is the adopted decision; therefore, we focus on normative conformity. There is an ample literature studying *information cascades*, which, according to Anderson and Holt (1997), occur "when initial decisions coincide in a way that it is optimal for each of the subsequent individuals to ignore his or her private signals and follow the established pattern." Readers interested in this type of

[3]See Myers (2012) or Goeree and Yariv (2015) for further discussion on the two types of conformity and Sherif (1937), Asch (1955) or Milgram (1965, 1974) for seminal contributions in the field.

conformity can consult, among other contributions, Banerjee (1992), Bikhchandani et al. (1992), Dekel and Piccione (2000) or Barberà and Nicolò (2016) for theoretical models, Coleman (2004), Morton et al. (2012), and Morton et al. (2015) for empirical studies and Hung and Plott (2001), Morton and Williams (1999, 2000) and Goeree and Yariv (2015) for experimental evidence.

In the experimental literature on conformity voting, Hung and Plott (2001) consider a setting in which agents decide sequentially after receiving an informative signal concerning the correct decision. In one of the treatments, the authors focus on informational influence by assuming that agents gain utility if they announce the correct decision. Regarding the effects of normative influence, Hung and Plott (2001) consider a treatment in which agents receive a larger payoff if their individual decision coincides with the group decision, which is determined by majority rule. In another treatment, the payoff for voting for the group's decision is larger than the payoff obtained from voting for the correct option. In our setting, we consider the case of different voting rules. Furthermore, agents in our model vote simultaneously and receive the largest payoff only if the group decision coincides with their preferred option. Our agents' preferences are in line with Dutta and Sen (2012), Gerber et al. (1998), Bassi et al. (2011) and Morton et al., (2012), in the sense that agents have preferences defined not only on the outcomes but also on the messages of the rest of the agents. Although agents in our model have incentives to conform to someone else, we do not assume that subjects want to vote for the same option as the majority. This, in turn, also differentiates our paper from bandwagon behavior, which refers to the desire to vote for the predicted winner in an election (Morton et al., 2015; Morton and Ou, 2015). In this literature, agents' preferences are frequently modeled by assuming that agents experience a benefit or cost depending on whether they vote with the majority (Luzzati, 1999; Hung and Plott, 2001; Morton et al., 2012; Battaglini et al. 2010; Bassi et al. 2011; Michaeli and Spiro, 2015). We relax this assumption in at least two ways. First, we assume that agents are willing to conform only if they cannot change the outcome of the election. Second, we assume that agents may not consider the vote of the majority as a reference point but simply care about voting for the same option as any other member of the group. Our experiment is then suited to study the interplay between truthful voting and conformity in a setting in which (pivotal) agents may prefer to vote with the minority if their votes determine the elected choice.

Our model is highly related to that of Bassi et al. (2011). In this model, agents are told their types, to create an identity for them. As in our model, some agents are part of the majority group and others of the minority group. The main difference lies in the fact that Bassi et al. (2011) provide incentives for agents to deviate from their assigned types to be on the winning side of an election (bandwagoning voting), while we only penalize agents who vote alone. Höchtl et al. (2012) is another paper related to ours. They consider two types of agents (rich and poor) who vote for the level of redistribution in a majority setting. The authors find that the composition of the group (whether rich or poor are in the majority group) is decisive in obtaining redistributive results, thereby suggesting the effects of pivotality in a majority setting. We extend these findings by investigating the effects of different voting rules when we induce conformity among agents such that the adopted decision does not necessarily coincide with that supported by the majority group.

Our paper provides a set of important experimental findings regarding the relevance of voting rules for truthful voting. Overall, there is no wide consensus indicating the appropriate voting rule, which depends on the issue at hand and the features of the voting procedure. The theoretical work of Feddersen and Pesendorfer (1998) pleads for the use of the majority rule instead of the unanimity rule in a Condorcet-winner setting (see also Guarnaschelli et al. 2000). Assuming strategy-proof rules in the voting stage of the game, Barberà and Nicolò (2016) study under which of such voting rules informed agents are more prone to disclose truthful information to uninformed agents, finding that majority rule is better inducing information disclosure. The majority rule is also preferred in Rivas and Rodriguez-Alvarez (2014) in a model of deliberation in which some agents want to conform to leaders, while Moreno and Ramos-Sosa (2015) suggest that the decision in equilibrium is more likely to differ from that obtained by truthful voting using the majority rule compared with the unanimity rule. Our results contribute to this literature by providing experimental evidence on the effects of different voting rules on behavior in a setting with conformity. This relates our paper to the experimental work of Anderson et al. (2015), who find no difference between the use of majority and unanimity rules in a Condorcet-winner setting.

Finally, there are other studies in the literature that examine the effects of having agents who vote for their preferred opinion on the outcome. The social choice literature finds that their presence benefits the implementation of social choice correspondences that satisfy no veto power (Dutta and Sen, 2012). There is also a benefit in models of information aggregation (Buechel et al., 2015) or information disclosure (Barberà and Nicolò, 2016). In a conformity setting, Rivas and Rodríguez-Alvarez (2014) and Moreno and Ramos-Sosa (2015) show that conformist agents become more truthful when it is common information that other agents are voting truthfully. This, in turn, helps to implement the socially desired option, which is the one that matches the state of nature (Rivas and Rodríguez-Alvarez, 2014) or would be selected if agents voted truthfully (Moreno and Ramos-Sosa, 2015).

6.3 Model

Consider five agents $N = \{1, 2, 3, 4, 5\}$ who have to vote between option A and option B. Agents have their preferences defined over two possible options. Agents $1, 2$ and 3 are *type-B agents* because they prefer option B to option A. Agents 4 and 5 are *type-A agents* because they prefer option A to option B. The list of types $t = (t_1, t_2, t_3, t_4, t_5) = (B, B, B, A, A)$ is common information, where $t_i \in \{A, B\}$ stands for the type of agent $i \in N$.

Agents vote simultaneously for one of the two options (abstention is not allowed). Let $M_i = \{A, B\}$ be the set of messages for agent $i \in N$, where $m_i = A$ ($m_i = B$) stands for agent i voting for option A (option B).[4] Let $M = \times_{i \in N} M_i$ be the set of messages and $m_{-i} \in M_{-i} = \times_{j \in N \backslash \{i\}} M_j$ be the messages of all agents except i. The set of all agents except i is denoted by $N_{-i} = N \backslash \{i\}$. We denote the profile of messages by $m \in M$.

The voting rule is such that for option B to be adopted, at least q agents have to vote for it. We refer to $q \in \{3, 4, 5\}$ as the *voting rule*. A voting rule $q = 3$ ($q = 4$) [$q = 5$] implies that option B requires simple majority (supermajority) [unanimity] to be adopted. We define $q(m)$ as a mapping from M onto $\{A, B\}$, where $q(m) = B$ if q or more agents vote for option B. Otherwise, $q(m) = A$.

Agents' preferences are defined over alternatives that consist of the adopted decision and the profile of messages announced by the agents. Agents' preferences can then be written as follows:

$$u(t_i, q, m) = v(t_i, q(m)) + c(m)$$

where

$$\begin{aligned} v(t_i, q(m)) &= \begin{cases} v & \text{if } q(m) = t_i \\ 0 & \text{if } q(m) \neq t_i \end{cases} \\ c(m) &= \begin{cases} c & \text{if } m_i = m_j, \text{ for any } j \in N_{-i} \\ 0 & \text{if } m_i \neq m_j, \text{ for any } j \in N_{-i} \end{cases} . \end{aligned}$$

The first term $v(t_i, q(m))$ specifies what agents receive if the adopted decision coincides with their type. Thus, if option B receives the required support, agents $1, 2$ and 3 receive a payoff of v whereas agents 4 and 5 receive a payoff of 0. Agents may also receive an additional payoff $c(m) = c$ if their message coincides with that of any other agent $j \in N_{-i}$. If $m_i = m_j$, we say that agent i conforms to agent $j \in N_{-i}$. Otherwise, $c(m) = 0$. We say that there is conformity if $c(m)$ is as described above.

[4] In our proofs, we also use m_i^A (m_i^B) to denote that agent i votes for option A (B).

We assume that $v > c > 0$ to capture the idea that agents do not place their desire to follow the rest of the agents before obtaining their preferred option. That is, agent i will conform only if her message is irrelevant to the adopted decision; i.e., preferences to conform are lexicographical.[5]

The aim of this paper is to investigate the effects of introducing conformity into the voting behavior of the agents. In particular, we want to study the extent to which truthful behavior is affected. We hereafter refer to agent $i \in N$ voting truthfully if she votes for her own type, that is, $m_i = t_i$. We also want to study voting behavior when it is common information that some agents will always vote for their type.

Next, we define the equilibrium concept that we use throughout our paper.

Definition. For any $q \in \{3, 4, 5\}$, $m \in M$ is a *weakly undominated Nash equilibrium* (WUNE) if for any $i \in N$,
(1) m_i is not weakly dominated and
(2) for any $m'_i \in M_i$, $u_i(t_i, q, m_i, m_{-i}) \geq u_i(t_i, q, m'_i, m_{-i})$.

In our baseline scenario, we assume that all agents receive $c(m) = c$ regardless of the profile of messages $m \in M$. Then, there is no conformity.

Proposition 1. *In the baseline scenario, all agents voting truthfully is the unique WUNE, that is,*
i) type-A agents vote for A for any $q = \{3, 4, 5\}$ *and*
ii) type-B agents vote for B for any $q = \{3, 4, 5\}$.

Proof. If there is no conformity, $u_i(t_i, q(m)) = v(t_i, q(m)) + c$. For any $m_{-i} \in M_{-i} = \times_{j \in N \setminus \{i\}} M_j$ such that the vote of agent $i \in N$ does not determine the adopted decision, $u_i(t_i, q, m_i, m_{-i}) = u_i(t_i, q, m'_i, m_{-i})$ where $m_i = t_i$ and $m'_i \neq t_i$. For any $m_{-i} \in M_{-i} = \times_{j \in N \setminus \{i\}} M_j$ such that the vote of agent $i \in N$ determines the adopted decision, $v + c = u_i(t_i, q, m_i, m_{-i}) > u_i(t_i, q, m'_i, m_{-i}) = c$ where $m_i = t_i$ and $m'_i \neq t_i$. ■

[5] Admittedly, this is not the unique way of defining conformity. We could have opted for requiring an agent to conform to some subset of N_{-i}. See Moreno and Ramos-Sosa (2015) for a discussion.

Proposition 1 shows that all agents voting truthfully is the unique WUNE for any voting rule $q \in \{3, 4, 5\}$.

In our conformity scenario, we assume that $c(m) = c$ when $m_i = m_j$ for some $j \in N_{-i}$. Then, agents' payoffs depend on the adopted decision and how their messages relate to other agents' messages.

Proposition 2. *In the conformity scenario, there are weakly undominated Nash equilibria in which:*
i) type-A agents vote for A and for B for any $q \in \{3, 4\}$ and
ii) type-B agents vote for A and for B for any $q = \{3, 4, 5\}$.

Proof. We show that there are WUNE in which agents do not vote truthfully. Take $q = 3$ or $q = 4$ and consider the cases in which all agents vote unanimously for one of the options. In these cases, agents get $v + c$ or c, depending on whether their preferred option is being elected or not. Any agent switching her message and voting alone would be worst off, because the adopted decision will not change and $c(m) = 0$ (i.e., the agent that switches gets v or 0, depending on whether her preferred option is being elected). When $q = 5$, these arguments apply for any type-B agent as well. However, all agents voting for B is not an equilibrium profile as any type-A agent (say agent 5) can deviate and vote truthfully to change the elected decision, getting $v > c$. Indeed, $u_5(A, q, m_5^A, m_{-5}) > u_5(A, q, m_5^B, m_{-5})$ for any $m_{-5} \in M_{-5}$, where $u_5(A, q, m_5^A, m_{-5}) = v$ if $m_j = B$ for any $j \in N \backslash \{5\}$, and $u_5(A, q, m_5^A, m_{-5}) = v + c$ otherwise. ■

Proposition 2 shows that in the conformity scenario, voting truthfully remains as an equilibrium strategy for all the agents. Moreover, there are also other equilibria in which agents vote for an option that does not coincide with their type.[6] This is true for all agents and voting rules, except for type-A agents when $q = 5$. Type-A agents vote truthfully in that case because they are pivotal and their decision determines the

[6]Note that the arguments in Proposition 1 can be applied to show that voting truthfully is also a WUNE in this setting.

adopted decision. Interestingly, for $q = 3$, all agents voting for option B (option A) is an equilibrium. As no agent is pivotal, type-A (type-B) agents voting truthfully would be penalized. Hence, type-A (type-B) agents are conforming to type-B (type-A) agents.

Finally, we study voting behavior when it is common information that some agents will always vote for their type and conformity is still induced in the preferences of the agents. This will be our informational scenario in which agents 1 and 2 vote for their preferred option.

Proposition 3. *In the informational scenario, there are weakly undominated Nash equilibria in which:*
i) type-A agents vote for A and B for any $q \in \{3, 4\}$ *and*
ii) type-B agents vote for B for any $q \in \{3, 4, 5\}$.

Proof. We show that there are WUNE in which type-A agents do not vote truthfully.[7] Note that all agents voting unanimously for A and only one agent voting for B is dismissed as agents 1 and 2 are forced to vote for B.
Take $q = 3$ or $q = 4$ and consider the case in which all agents vote unanimously for B. In this case, this is a WUNE since any type-A agent (say agent 4) voting truthfully is worst off as $u_4(A, q, m_4^B, m_{-4}^B) = c > 0 = u_4(A, q, m_4^A, m_{-4}^B)$. Note that, in this case, type-A agents vote for B. The remaining type-B agent (agent 3) in this case votes truthfully as $u_3(B, q, m_3^B, m_{-3}^B) = v + c > v = u_3(B, q, m_3^A, m_{-3}^B)$. Besides, when $q = 3$, the profile of messages where only two agents vote for B is not an equilibrium due to agent 3 voting truthfully can make the decision be B when voting for B. When $q = 4$, agent 3 also votes truthfully for any $m_{-3} \in M_{-3} = \times_{j \in N \setminus \{3\}} M_j$ since $u_3(B, q, m_3^B, m_{-3}) \geq u_3(B, q, m_3^A, m_{-3})$. Therefore, two agents voting for B not being 1 and 2 are profiles of strategies that are not possible and agent 3 always votes truthfully.
Take $q = 5$. Consider the case in which all agents vote unanimously for B. In this case, this is not a WUNE since any type-A agent (say agent 4) voting truthfully makes the decision

[7] Following Proposition 1, voting truthfully is a WUNE for each possible $q \in \{3, 4, 5\}$.

be A. That is, she is strictly better off as $u_4(A, q, m_4^A, m_{-4}^B) = v > c = u_4(A, q, m_4^B, m_{-4}^B)$. Besides, agent 3 voting for A is not a WUNE since $u_3(B, q, m_3^B, m_{-3}) \geq u_3(B, q, m_3^A, m_{-3})$ for any $m_{-3} \in M_{-3} = \times_{j \in N \setminus \{3\}} M_j$. Therefore, for $q = 5$, the unique WUNE is all agents voting truthfully. ■

Compared with the conformity scenario, in the informational scenario the equilibrium strategies reduce to only two: all agents voting truthfully and all agents unanimously voting for B (except for $q = 5$).[8] When we examine the possible effects of the voting rule in the informational scenario, we then note that type-A agents might vote for any option when $q = 3$ or $q = 4$, but voting truthfully is a dominant strategy when $q = 5$. As for type-B agents, voting truthfully is a weakly dominant strategy for any possible voting rule.[9]

In the next section, we present an experiment that is designed to test our theoretical predictions.

6.4 Experimental Design

A total of 390 subjects were recruited to participate in our computerized sessions (Fischbacher, 2007). Subjects were Economics or Business students from the undergraduate population of the Universidad de Valencia, with no previous experience in similar experiments. Subjects were invited to participate in our experiment using the recruitment system of the laboratory (LINEEX).

[8] Therefore, all voting rules are strategy-proof because the decision adopted in the informational scenario coincides with that obtained when all agents vote truthfully.

[9] Although the theoretical result also holds when only one type-B agent votes truthfully, we force two agents to do so for experimental purposes. By forcing two agents to vote truthfully, the payoff for conformity $c > 0$ is always guaranteed. Moreno and Ramos-Sosa (2015) provide the number of forced agents (who they refer to as independent agents) within the voting group guaranteeing that the adopted decision always coincides with the decision obtained when all agents vote truthfully. Their results hold for any voting rule $q \in \{1, n\}$, any number of agents n and any list of agents' types t. In our setting, having two forced agents guarantees that the adopted decision will coincide with the decision under truthful voting for $q = 3$ and $q = 5$.

We ran a total of 13 sessions with 30 subjects each. At the beginning of each session, subjects were randomly assigned a type (Player A or Player B), which was held constant throughout the session. Subjects were told that they were in a group of 5 subjects. It was common information that each group consisted of 2 Players A and 3 Players B.

When subjects were informed of their types, they were asked to vote between option A and option B across three different rounds. In each of the rounds, option B required a different number of votes $q = \{3, 4, 5\}$ to be the adopted decision. It was common information that if option B did not receive at least q votes, then option A would be the adopted decision. In our experiment, q was announced to subjects at the beginning of each round. The order of q was randomly selected, and we balanced the number of observations across sequences of decisions; e.g., we had the same number of observations for sequences $3, 4, 5$ and $5, 3, 4$. As for the feedback, subjects voted in each q while receiving no information whatsoever regarding previous decisions in their groups. Thus, subjects voted without knowing what other subjects in their group had voted for or what decision had been adopted in previous rounds. This, in turn, implies that our method for eliciting the relevant behavior is free of historical contagion and learning effects.

At the end of the experiment, one round was randomly selected for payment. Payoffs for each subject depended on whether *i*) the adopted decision in that round coincided with their own type and *ii*) they voted for the same option as any other subject in their group.

Our experiment relies on a between-subjects design (i.e., subjects only participate in one of the three possible treatments). We summarize these treatments below:

- **Baseline** (BL, 120 subjects, 24 groups). Subjects received 75 ECUs if the adopted decision coincided with their own type. Subjects received an additional amount of 25 ECUs, regardless of the option for which they decided to vote.

- **Conformity** (CON, 120 subjects, 24 groups). Subjects received 75 ECUs if the adopted decision coincided with their own type. They received the additional payoff of 25 ECUs only if their vote coincided with that of any other subject in their group.

- **Informational** (INF, 150 subjects, 30 groups). Payoffs were as in the conformity treatment, but it was common information that 2 subjects in the role of Player B would be given no option but to vote for option B. The subjects forced to vote for B were the same throughout the session. It was common information that they had to vote for option B in each round, as this was the only option that appeared on their computer screen.[10]

Instructions in each treatment were read aloud by the session monitor, and subjects were allowed to ask any question in private before starting the treatment. We minimized the probability of subjects failing to understand how payoffs were generated using a pre-experimental quiz, in which subjects were asked to compute the payoffs of randomly generated examples.

Each session lasted for approximately 1 hour, and the subjects received approximately 7.5 Euros for participating (10 ECUs = 1 Euro). The experiment included an additional phase in which individual characteristics were elicited. Our questionnaire, together with a translated version of our original instructions, is presented in Appendix A. We note that the questionnaire includes gender, age, cognitive abilities, risk aversion, social preferences, trust, happiness, satisfaction and inequality. We will use these variables as controls in our econometric analysis in Section 6.5.

6.5 Testable Hypotheses

Our first treatment (BL) resembles our theoretical model for the case in which $c(m) = c$ for any $m \in M$; therefore, subjects are expected to vote truthfully in this treatment (Proposition 1). We induce conformity in our second treatment (CON) by paying subjects the additional amount of money only if they vote for the same option as any other member

[10] We decided to have two subjects instead of the computer in the role of forced agents to avoid any concern about social preferences; e.g., type-A agents can vote differently depending on whether they impose an externality on the computer or on another human subject. For the effects of social preferences on bandwagon voting, see Morton and Ou (2015) or Corazzini and Greiner (2007).

of their group. Given our theoretical results (Proposition 2), we predict that truthful voting will be less pervasive in this treatment. The first hypothesis, which we want to reject, is then as follows:

Hypothesis 1. *The presence of conformity will not affect the likelihood of voting truthfully, which will be the same in BL and CON.*

We expect that conformity will affect the likelihood of truthful voting for both types of agents, except when option B requires unanimity. Proposition 2 highlights that type-A agents vote truthfully in that case because they are pivotal; therefore, we do not expect any effect of conformity if $q = 5$. In that vein, we can expect that inducing conformity will have more effects on type-B agents when $q = 5$, compared with the case in which $q = 3$, thereby suggesting that the effects of conformity might be affected by the decision rule and how pivotal agents are.

In our third treatment (INF), it is common knowledge that two type-B agents are forced to vote for B. Our theoretical results display that voting truthfully is the unique equilibrium strategy for the other type-B voter (Proposition 3). As a result, we expect type-B agents to vote more truthfully in INF, compared with CON. The hypothesis that we want to reject is then as follows:

Hypothesis 2. *In a conformity setting, the presence of two type-B agents voting for their types will not affect the likelihood of truthful voting by the other type-B agent, which will be the same in CON and INF.*

Proposition 2 shows that type-A agents can vote for A or B when $q = \{3, 4\}$, whereas voting truthfully is the unique WUNE for them when $q = 5$. The same is true when forced agents are included; therefore, we do not expect any positive effect of knowing the presence of forced agents on the likelihood of type-A agents voting truthfully.

While we can examine the effects of conformity and the presence of forced agents on voting truthfully when there are different voting rules, we can also test for the effect of the voting rule in each particular setting. We have shown that voting truthfully is the unique WUNE in our BL treatment; therefore, the voting rule should not affect agents' behavior in this treatment. This argument is also valid for type-B agents in the INF treatment. In the CON treatment, however, type-A agents may vote for option A or B when $q = 3$ or $q = 4$, but they should vote truthfully when $q = 5$. The hypothesis that we want to reject is then as follows:

Hypothesis 3. *The voting rule does not have any effect on the agents' likelihood of voting truthfully in BL, CON and INF.*

Our prediction that type-A agents might vote more truthfully when $q = 5$, compared with $q = 3$ or $q = 4$, highlights the importance of the pivotality of agents on the likelihood of voting truthfully. Along these lines, type-B agents can anticipate that increasing the voting rule makes it more difficult for option B to be chosen in any possible setting. Type-B agents can then be less likely to vote truthfully when the voting rule becomes more stringent in CON.

6.6 Results

This section presents our experimental evidence. Section 6.6.1 focuses on the first two treatments (BL vs. CON) to show the effects of conformity on the likelihood of voting truthfully. In Section 6.6.2, we assess whether knowing that two agents are voting truthfully influences agents' voting behavior in a conformity setting (CON vs. INF). To provide some insights into efficiency, we evaluate our findings in terms of total surplus in Section 6.6.3.

6.6.1 On the effects of conformity

Figure 6.1 depicts the effect of conformity by plotting the average likelihood of voting truthfully in BL and CON, separately for each type of agent. Error bars reflect standard error of the mean. The results disaggregated by voting rule are presented in Table 6.1 below the figure.

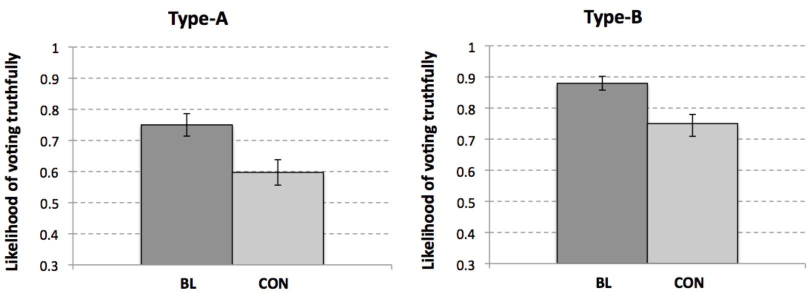

Figure 6-1: Effect of conformity on the likelihood of voting truthfully.

In line with our theoretical prediction, we observe that the presence of conformity decreases the average frequency of voting truthfully for type-A (from 0.75 to 0.60) and type-B agents (from 0.87 to 0.75). As is shown in Table 6.1, conformity decreases the likelihood of type-A agents voting truthfully for any possible voting rule, with the smallest effect when $q = 5$. For type-B agents, conformity decreases the likelihood of truth-telling for each possible voting rule, except when $q = 3$.[11]

	Type-A		**Type-B**	
	BL	CON	BL	CON
$q = 3$	0.71	0.46	0.90	0.94
$q = 4$	0.85	0.71	0.90	0.74
$q = 5$	0.69	0.62	0.83	0.57
N	48	48	72	72

Table 6.1: Frequency of truthful voting in BL and CON for each possible voting rule.

Table 6.1 allows us to also observe the effect of the voting rule on the likelihood of voting truthfully. When there is no conformity, Proposition 1 shows that there is a

[11] A non-parametric analysis suggests that these effects are statistically significant (see Appendix B).

unique WUNE in which all agents vote truthfully regardless of the voting rule. Our findings seem to support this prediction by suggesting no effect of the voting rule on the likelihood of voting truthfully in the BL treatment. The voting rule, however, seems to have a significant effect on the likelihood of voting truthfully in the CON treatment. In particular, agents are less likely to vote truthfully under those voting rules under which agents are less pivotal ($q = 3$ for type-A agents and $q = 5$ for type-B agents).[12]

One final aspect that is worth mentioning is that our model predicts that all agents will vote truthfully in the BL treatment. We find, however, that type-B agents are more likely to vote truthfully than type-A agents. This is an interesting finding in line with Bassi et al. (2011), who report that members in the minority group do not vote truthfully if decisions are made by majority rule.

In what follows, we perform an econometric analysis to study the behavior of type-A and type-B agents in greater detail. For each type of agent, we estimate a logit model for the likelihood of voting truthfully. The set of independent variables includes a dummy variable that takes value 1 if there is conformity (d_{CON}) and two dummy variables for the value of the voting rules ($d_{q4} = 1$ if $q = 4$ and $d_{q5} = 1$ if $q = 5$). Our specification also includes the interaction terms between the treatment and the voting rules ($d_{CON}d_{q4}$ and $d_{CON}d_{q5}$) to capture the (possibly different) effects of conformity on the likelihood of voting truthfully depending on the voting rule. We control for individual heterogeneity by including the responses to the questionnaire as independent variables.

Figure 6.2 depicts the average marginal effect (ME) of conformity, together with the 95% confidence intervals, for each possible voting rule.[13] The results for type-A agents (type-B agents) are presented in the left panel (right panel). The reported effects take into account Ai and Norton (2003) and Karaca−Mandic et al. (2012).[14]

[12] This is much in line with the comment made by Charles Plott to the authors in the sense that "the votes under majority-rule institutions are typically overwhelming majorities because the minority, when anticipating a loss on the vote, just go along with the majority."

[13] Appendix B contains the estimates of our logit regressions. As we discuss in Appendix B, our findings are robust if we instead consider a linear probability model.

[14] Ai and Norton (2003) and Karaca−Mandic et al. (2012) discuss the correct way of estimating

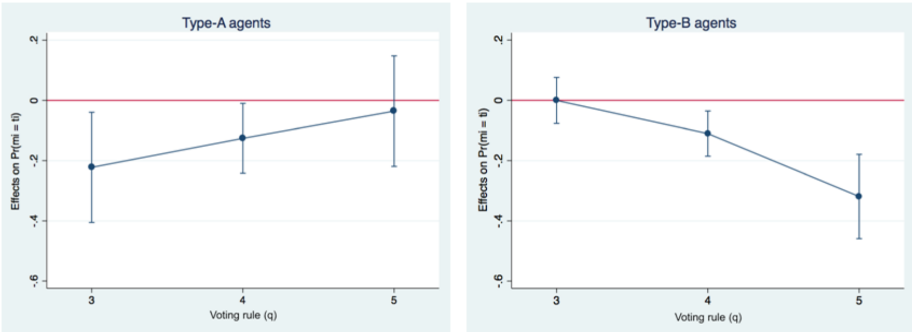

Figure 6-2: Marginal effects of conformity on the likelihood of voting truthfully after logit specification.

We observe that conformity decreases the likelihood of type-A agents voting truthfully when $q = 3$ (ME $= -0.222$, $p = 0.017$) and $q = 4$ (ME $= -0.126$, $p = 0.003$). Consistent with our theoretical results, we do not observe any effect of conformity on the likelihood of type-A agents voting truthfully when $q = 5$ (ME $= -0.036$, $p = 0.703$). For type-B agents, we cannot reject the null hypothesis of conformity having no effect on the likelihood of voting truthfully when $q = 3$ (ME $= -0.0003$, $p = 0.993$). This null hypothesis is rejected at any common significance level when $q = 4$ (ME $= -0.11$, $p = 0.004$) or $q = 5$ (ME $= -0.319$, $p < 0.001$).[15] We conclude that these findings provide evidence against our first hypothesis. The behavioral pattern observed in Figure 6-2 also suggests that inducing conformity is more likely to affect behavior when agents are not pivotal.

Observation 1. *Conformity decreases the likelihood of voting truthfully. The effects of conformity depend on how pivotal agents are.*

Next, we study the effects of the voting rule on the likelihood of agents voting truth-

marginal effects in nonlinear models that include interaction terms. Hereafter, all the MEs reported in our paper take their work into account.

[15] Our results indicate that conformity reduces the likelihood of type-A agents voting truthfully by approximately 13%. The estimates for type-B agents suggest a reduction in the likelihood of voting truthfully of 14%. Statistically, both effects are significant at the 1% level.

fully in BL and CON by means of a logit analysis, where the explanatory variables are the dummies for the voting rules (d_{q4} and d_{q5}). The estimated MEs are summarized in Table 6.2.

	Type-A		Type-B	
	BL	CON	BL	CON
($d_{q4} = 1$ if $q = 4$)	0.149	0.25***	0.000	-0.208***
	(0.09)	(0.10)	(0.05)	(0.06)
($d_{q5} = 1$ if $q = 5$)	-0.021	0.167	-0.069	-0.375***
	(0.10)	(0.10)	(0.06)	(0.06)
Controls	Yes	Yes	Yes	Yes
Wald χ^2-test	25.73**	19.55*	19.43	29.19***
Pseudo- R^2	0.09	0.08	0.08	0.17
Observations	144	144	216	216

Table 6.2: Marginal effects of the voting rule on the likelihood of voting truthfully in BL and CON.

We support our conjecture suggesting no effect of the voting rule on the likelihood of voting truthfully in the BL treatment. For the CON treatment, there is no-clear cut prediction regarding how subjects should vote under each voting rule. In that case, our findings suggest that type-A agents are more likely to vote truthfully as the voting rule becomes more stringent, whereas type-B agents appear to vote less truthfully as the voting rule becomes more stringent. This confirms the relationship between the likelihood of voting truthfully and the pivotality of agents.[16]

[16] Rivas and Rodriguez-Alvarez (2014) show that agents are more likely to reveal truthful information under the majority rule. In contrast, we find that the majority rule induces type-B (type-A) agents to vote more (less) truthfully, compared with the unanimity rule.

Observation 2. *i) The voting rule has no effect on the likelihood of voting truthfully when there is no conformity. ii) Under conformity, the voting rule has an effect on the likelihood of voting truthfully, with agents being more likely to vote truthfully when they are pivotal.*

Overall, these findings highlight the importance of conformity and the pivotality of agents on the likelihood of voting truthfully. Next, we investigate the influence on voting truthfully when agents know that there are two forced agents voting for their preferred option.

6.6.2 On the effects of information

We replicate our previous analysis and show the effect on voting when agents know that some agents are voting truthfully. Figure 6.3 and Table 6.3 report in aggregate levels the likelihood of voting truthfully in INF for each type of agent. For the sake of comparison, we include the results for CON. Importantly, two type-B agents were forced to vote for their preferred option in our INF treatment. The reported data for the INF treatment do not consider these agents; i.e., they focus only on the type-B agents who were not forced to vote truthfully but allowed to vote for either of the two options. Error bars reflect standard errors of the mean.

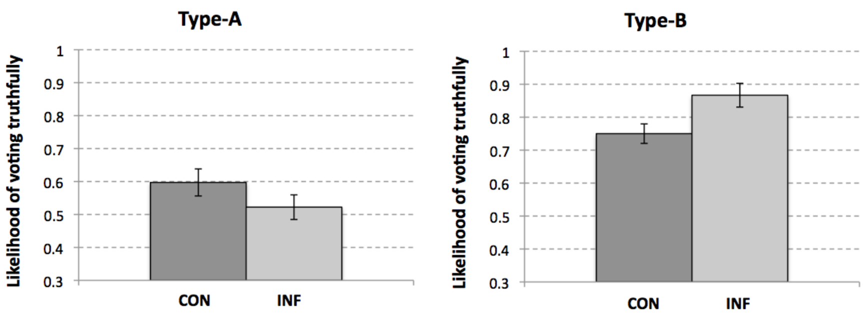

Figure 6-3: Effect of information on the likelihood of voting truthfully.

	Type-A		Type-B	
	CON	**INF**	**CON**	**INF**
$q = 3$	0.46	0.35	0.94	0.97
$q = 4$	0.71	0.55	0.74	0.97
$q = 5$	0.62	0.67	0.57	0.67
N	48	30	72	60

Table 6.3: Frequency of truthful voting in CON and INF for each possible voting rule.

We observe that the presence of forced agents increases the likelihood of type-B agents voting truthfully (from 0.75 to 0.86), which is in line with our theoretical predictions. The results seem to be consistent for each possible voting rule. For type-A agents, the presence of forced agents seems to decrease their likelihood of voting truthfully (from 0.60 to 0.52). This negative effect occurs for each possible voting rule except when $q = 5$, which is the only voting rule under which voting truthfully is the unique WUNE strategy for type-A agents in both treatments due to their pivotality.[17]

Next, we perform our econometric analysis. We estimate a logit specification for the probability of each type of agent voting truthfully. We control for individual heterogeneity and include a dummy variable that takes value 1 if there are forced agents (d_{INF}) and two dummies for the voting rules (d_{q4} and d_{q5}). We allow for different effects of the presence of forced agents depending on the voting rules by including the interaction terms. The logit estimates, together with the results of a linear probability model, are presented in Appendix B. The estimated MEs and the corresponding 95% confidence interval are summarized in Figure 6.4.

The MEs indicate that the presence of forced agents decreases the likelihood of type-A agents voting truthfully. The effect is significant when $q = 3$ (ME $= -0.164$, $p = 0.057$). However, we cannot reject the null hypothesis of no effect of the presence of forced agents

[17]See Appendix B for the results of the non-parametric analysis.

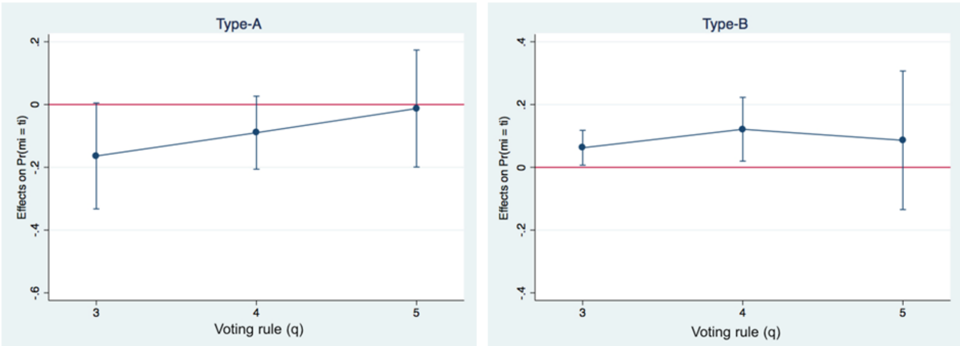

Figure 6-4: Marginal effects of information on the likelihood of voting truthfully after logit specification.

on the behavior of type-A agents when $q = 4$ and $q = 5$ (ME $= -0.090$, $p = 0.131$, and ME $= -0.013$, $p = 0.894$, respectively).

Consistent with our theoretical prediction, we observe a positive effect of the presence of forced agents on the likelihood of type-B agents voting truthfully when $q = 3$ (ME $=$ 0.062, $p = 0.028$) and $q = 4$ (ME $= 0.121$, $p = 0.052$). The effect is not significant when $q = 5$ (ME $= 0.086$, $p = 0.444$).

When we estimate the average effect of the presence of forced agents on the likelihood of type-A agents voting truthfully, we find that this is not significant at any common level (ME $= -0.089$, $p = 0.12$). The overall effect for type-B agents is, however, significant (ME $= 0.090$, $p = 0.043$).

Observation 3. *When agents know that two type-B agents are forced to vote truthfully, the likelihood of voting truthfully increases for the remaining type-B agents.*

Finally, we can examine the effect of the voting rule on the likelihood of voting truthfully in INF. Our results suggest that type-A agents are more likely to vote truthfully when $q = 4$ and $q = 5$ (ME $= 0.20$, $p = 0.029$, and ME $= 0.317$, $p = 0.001$, respectively). As a result, we conclude that the voting rule has a positive effect on the likelihood of type-A agents voting truthfully. For type-B agents, there is no effect of the voting rule when we compare the likelihood of voting truthfully in $q = 3$ and $q = 4$. Surprisingly, type-B agents are less likely to vote truthfully in the INF treatment when $q = 5$ (ME $= 0.310$, $p < 0.001$). This occurs despite voting truthfully being the unique WUNE strategy for type-B agents under all possible voting rules. Although we find that the presence of forced agents increases truthful behavior, our findings suggest that type-B agents are less likely to vote truthfully as we increase the stringency of the voting rule, probably as a response to type A's agents voting more truthfully when the voting rule becomes more stringent. These findings, in turn, support the idea that the pivotality of agents affects the likelihood of voting truthfully.

6.6.3 Maximizing total surplus

We consider the issue of efficiency losses in this section. If agents care about the welfare of the group (Coate and Conlin, 2004; Feddersen and Sandroni, 2006), they will vote to maximize total payoffs. In our setting, this occurs when option B is selected, regardless of whether all agents are voting for B. Our previous findings suggest that conformity decreases truthful voting, whereas receiving information fosters it. How do these results translate into efficiency gains or losses in terms of total surplus?

We use the behavioral data in each treatment to form all possible configurations of groups consisting of three type-B agents and two type-A agents.[18] We then compute the average expected payoff and the likelihood of receiving the maximum possible payoff in each treatment for each possible voting rule. These results are reported in Table 6.4.

[18] Given our data, we can form a total of 961,056 different groups in BL and CON. We can form at least 53,100 groups in INF (when we do not combine the behavior of different subjects in the role of forced type-B voters).

	BL	CON	INF
A. Average expected payoff			
$q = 3$	338.9	335.4	338.4
$q = 4$	290.6	285.4	314.0
$q = 5$	279.1	269.3	272.2
	302.9	296.7	308.2
B. Likelihood of getting the highest possible payoff			
$q = 3$	0.85	0.50	0.54
$q = 4$	0.21	0.03	0.20
$q = 5$	0.06	0.03	0.07
	0.37	0.19	0.27

Table 6.4: Efficiency and total surplus.

As we increase the stringency of the voting rule, we expect the likelihood of selecting option B to decrease. Table 6.4 shows that the average expected payoff and the likelihood of receiving the highest possible payoff decreases with the voting rule, in every possible treatment. When we compare across treatments, Table 6.4 shows that CON results in a smaller average expected payoff and a lower likelihood of receiving the highest possible payoff for any possible voting rule, compared with the BL treatment. Efficiency improves in INF, as the expected payoff and the likelihood of maximizing the total payoff increase for each possible voting rule.

Observation 4. *i) The likelihood of maximizing total payoffs is higher when there is no conformity. ii) With the presence of forced agents, the likelihood of maximizing total payoffs increases. iii) In every setup, the socially efficient outcome is less likely as the voting rule becomes more stringent.*

Our result that information improves efficiency in a conformity setting supports the theoretical works of Dutta and Sen (2012), Barberà and Nicolò (2016), Buechel et al.

(2015), Rivas and Rodríguez-Alvarez (2014), Battaglini et al. (2010) and Moreno and Ramos-Sosa (2015), who highlight the benefits of assuming that some agents always vote truthfully in a variety of settings.

6.7 Conclusion

This paper studies the effects of conformity and information in a binary-decision voting game in which agents are heterogeneous with respect to their preferred outcome. We show that conformity reduces the likelihood of truthful voting for all agents. When agents are informed that two type-B agents will vote truthfully, we expect the other type-B agent to also vote truthfully. Our experimental data support these predictions and highlight the importance of the voting rule in driving the results. In particular, we find that the willingness to vote truthfully and the effects of information depend upon the voting rule, thereby suggesting that there is an interplay between conformity voting and the pivotality of the agents.

Considerable attention has been devoted to the incentives of agents to vote truthfully in the literature on Social Choice. There are numerous studies characterizing social choice functions that satisfy strategy-proofness (i.e., truth-telling is a weakly dominant strategy in the direct mechanism). Arguably, there are social choice functions satisfying desirable properties that fail to be strategy-proof. If we insist on the desirability of those social choice functions, we should measure their manipulability. Although we lack a unanimously way of doing so theoretically, laboratory experiments provide an excellent means of empirically addressing this question. This paper is part of a more ambitious project in which we want to test several well-known social choice functions in their appropriate domains of definition. The current paper can be understood as a first step in this direction.

6.8 Appendix A

6.8.1 Translated Instructions (originally in Spanish)

Welcome to the experiment!

This is an experiment to study decision-making. Instructions are simple and if you follow them carefully you will be paid an amount of money at the end of the experiment. Your earnings in this experiment may depend on your decisions and the decisions of other participants. Your earnings will be given confidentially so neither the other participants in the lab nor the instructors will know your payoffs. Similarly, your identity will be anonymous throughout the whole experiment. Thus, nobody will know the decisions you have taken during the experiment.

Please, from no on, do not communicate with other participants during the experiment. If you have any questions please raise your hand. Out of this type of questions, any kind of communication among the participants of the experiment is forbidden and will be subject to immediate exclusion from the experiment.

What's the experiment about?

In the experiment, there are two types of participants: A and B. Before starting the experiment, the computer will randomly decide whether you are a type-A or type-B participant. The type assigned by the computer will remain throughout the experiment.

In total, this experiment consists of three rounds. Before starting the first round, we will match you with other participants in the lab to form a group of 5 participants. Your group will remain throughout the whole experiment and consists of 2 type-A participants and 3-type B participants.

[**BL and CON**] In each of the rounds, you can choose between voting for Option A or voting for Option B to determine which is the Chosen Option of that round. Before you have to vote, we will announce to you (and to all the members of your group) how many votes Option B needs to be chosen in that round.

[**INF**] In each of the rounds, the computer force two type-B participants to vote for

Option B. If you are a type-B participant and you are forced by the computer to vote for Option B you can only choose that option in the three rounds of the experiment. The rest of the members of the group can choose in each round whether to vote for *Option A* or *Option B* to determine which is the *Chosen Option* of that round. Before you have to vote, we will announce to you (and to all the members of your group) how many votes Option B needs to be chosen in that round.

Option B may need 3, 4 or 5 votes to be chosen, depending on the round.

- If Option B gets the number of votes needed, Option B will be the *Chosen Option* in that round.

- If Option B doesn't get the number of votes needed, Option A will be the *Chosen Option* in that round.

Baseline treatment

How can I earn money in this experiment?

At the end of the experiment, we will announce which has been the Chosen Option in each round and what were your earnings in each round.

Your earnings in each round will depend, in part, on the Chosen option. You will also receive an additional amount just for casting your ballot. The total earnings you can receive are explained in detail as follows:

Earnings for the chosen option: If the Chosen Option in your group coincides with your type you will receive 75 ECUS. That is, if you are a type-A participant and Option A is the Chosen Option for the members of your group, you will receive 75 ECUS in that round. In the same way, if you are a type-B participant and Option B is the Chosen Option for the members of your group, you will receive 75 ECUS in that round.

Earnings for casting your ballot: Independently on whether you choose Option A or Option B, you will receive 25 ECUS just for casting your ballot. This gain is received independently on your type of participant and the Chosen Option in that round.

To determine your earning in one round, we sum your earnings for the chosen option and your earnings for casting the ballot. At the end of the experiment, after informing you about your total earnings in each of the three rounds of the experiment, the computer will randomly choose one round and we will pay you out depending on the earnings obtained in that round.

Your will receive the equivalent in EURO according to the exchange rate 10 ECUS = 1 Euro.

Next, we present some examples in order to show you how payoffs are calculated.

(Examples)

Now, the computer will randomly choose whether you are a type-A or type-B participant in this experiment. Next, we present a screen with two more examples. We ask you to pay attention to these examples in order to understand correctly how earnings are calculated before starting the experiment as after that you will take a simple test to check that you have already understood everything. If after reading the examples or solving the test you have doubts about how earnings are calculated, please, raise your hand and ask the instructors. Knowing how earnings are calculated can help you to obtain more money during the experiment.

Besides, bear in mind that:

1. Your type (A or B) will be the same and in your group there always be 2 type-A participants and 3 type-B participants.

2. To determine which is the Chosen Option in one round, we check whether Option B has obtained a determined number of votes in that round (3 votes, 4 votes or 5 votes). This information may change from round to round and will be announced to all the members of your group before voting.

3. Your earnings will depend on whether the Chosen Option in one round coincides with your type of participant. Besides, you will receive an additional amount just

for casting the ballot (regardless of your vote).

Conformity and informational treatment:

How can I earn money in this experiment?

At the end of the experiment, we announce which has been the Chosen Option in each round and what were your earnings in each round.

Your earnings in each round will depend, on the one hand, on the Chosen Option and on the other, on the number of members of your group voting for the same option than you. The total earnings you can receive are explained in detail as follows:

Earnings for the chosen option: If the Chosen Option in your group coincides with your type you will receive 75 ECUS. That is, if you are a type-A participant and Option A is the Chosen Option for the members of your group, you will receive 75 ECUS in that round. In the same way, if you are a type-B participant and Option B is the Chosen Option for the members of your group, you will receive 75 ECUS in that round.

Earnings for coinciding: If the option you have voted for in one round has in total 2 or more votes (that is, if somebody else has vote for the same option than you), then you will receive 25 ECUS. You will receive these earnings regardless of your type of participant and the Chosen Option in the round.

To determine your earnings in one round, we sum your earnings for the Chosen Option

and your earnings for coinciding. At the end of the experiment, after informing you about your total earnings in each of the three rounds of the experiment, the computer will randomly choose one round and we will pay you out depending on the earnings obtained in that round.

Your will receive the equivalent in EURO according to the exchange rate 10 ECUS = 1 Euro.

Next, we present some examples in order to show you how payoffs are calculated.

(Examples)

Now, the computer will randomly choose whether you are a type-A or type-B participant in this experiment. Next, we present a screen with two more examples. We ask you to pay attention to these examples in order to understand correctly how earnings are calculated before starting the experiment as after that, you will take a simple test to check that you have already understood everything. If after reading the examples or solving the test you have doubts about how earnings are calculated, please, raise your hand and ask the instructors. Knowing how earnings are calculated can help you to obtain more money during the experiment.

Besides, bear in mind that:

1. Your type (A or B) will be the same and in your group there always be 2 type-A participants and 3 type-B participants.

2. To determine which is the Chosen Option in one round, we check whether Option B has obtained a determined number of votes in that round (3 votes, 4 votes or 5 votes). This information may change from round to round and will be announced to all the members of your group before voting.

3. Your earnings will depend on whether the Chosen Option in one round coincides with your type of participant. Besides, you will receive an additional amount if someone else has voted for the same option than you.

[INF only] 4. There are 2 type-B participants in your group that are forced to vote for Option B in the three rounds of the experiment.

6.8.2 Questionnaire

- **Age:** What is your age?. . . years.

- **Gender:** What is your gender? (00 male, 01 female).

- **Risk aversion:** We elicited risk attitudes using the investment decision in Gneezy and Potters (1997). Each participant hypothetically received 10 Euros and was asked to choose how much of it, x, she wanted to invest in a risky option and how much she wished to keep. The amount invested yielded a dividend equal to 2.5x with 1/2 probability, being lost otherwise. The money not invested in the risky option (10-x) was kept by the participant. In this situation, the expected value of investing is positive and increasing in the amount invested; therefore a risk-neutral (or risk-loving) participant should invest the 10 Euros, whereas a risk-averse participant will invest less. The amount not invested in the risky asset is a natural measure of risk aversion.

- **Trust:** We use the question in the GSS to elicit attitudinal trust. Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people? (Trust = 1 if the answer is "most people can be trusted").

- **Cognitive abilities:** We use the Cognitive Reflection Test (CRT) in Frederick (2005).

- **Social preferences:** We use the answers to the social value orientation (SVO) in Van Lange et al. (1997) to classify subjects as individualistic, prosocial, competitive, or others.

- **Satisfaction:** "How do you feel in this moment with your life?" (1-7-scaled answer from 1 (very satisfied) to 7 (Not at all satisfied).

- **Happiness:** "Taking everything into consideration, would you call yourself..." (01 not very happy, 02 quite happy, 03 very happy).

- **Inequality:** "Consider the following situation: Two secretaries with the same age do exactly the same work. However, one of them earns 20 euros per week more than the other. The one that is paid more is more efficient and faster, while working. Do you believe it is fair that one earns more than the other?" ($Ineq = 1$ if Yes).

6.9 Appendix B

6.9.1 Non-parametric analysis

Effect of conformity

Table 6.1 in the main text shows the effect of conformity by reporting the average likelihood of voting truthfully in BL and CON, for each type and each possible voting rule separately. If we consider the Chi-2 test (one-side alternative), we find that the presence of conformity decreases the average frequency of voting truthfully from 0.83 to 0.69 ($p < 0.001$). The negative effect of conformity on voting truthfully is significant for both type-A ($p < 0.003$) and type-B agents ($p < 0.001$).

When we can look at the effect of conformity for each possible voting rule in Table 6.1, we observe that conformity decreases the likelihood of type-A agents voting truthfully when the voting rule is $q = 3$ ($p = 0.006$), and when it is $q = 4$ ($p = 0.042$). As expected, conformity has no effect for type-1 voters when the voting rule is equal to $q = 5$ ($p = 0.52$). As for type-B agents, we expect that conformity decreases the likelihood of truth telling for each possible voting rule. Our findings suggest no significant effect when the voting rule is equal to $q = 3$ ($p = 0.34$), but when the voting rule equals to $q = 4$ ($p = 0.004$) or $q = 5$ ($p < 0.001$). Overall, all these findings are in line with our prediction.

Table 1 allows us to observe also the effect of the voting rule on the likelihood of voting truthfully. When there is no conformity, Proposition 1 shows that there is a unique WUNE in which all agents vote truthfully regardless of the value of the voting rule. Our findings seem to support this prediction by suggesting no effect of the voting rule on the likelihood of voting truthfully in the BL treatment. The voting rule, however, seems to have a significant effect in the likelihood of voting truthfully in the CON treatment. In principle, we expect for type-A agents to vote more truthfully in $q = 5$. When we do pairwise comparisons we find that the likelihood of truth telling increases in $q = 5$ with respect to $q = 3$, but the effect is weakly significant ($p = 0.10$). In fact, it seems that type-A agents are more likely to vote truthfully in the CON treatment when $q = 4$. As for type-B agents, the likelihood of voting truthfully seems to decrease in the value of the voting rule. We note that our theoretical result do not provide any prediction for this behavior.

Effect of information

The introduction of two type-B agents forced to vote truthfully does not affect the likelihood of voting truthfully for type-A agents ($p = 0.18$) but it does for type-B agents ($p = 0.012$).

When we look at the effects of forced agents for each possible voting rule in Table 1, we observe that knowing that two agents are forced to vote for B does not affect the likelihood of type-A agents voting truthfully when the voting rule is $q = 3$ ($p = 0.253$) or q = 5 ($p = 0.652$). The effect is, however, significant when the voting rule is $q = 4$ ($p = 0.046$). As for type-B agents, the non-parametric analysis rejects the null hypothesis that forced agents do not affect the likelihood of voting truthfully when the voting rule is equal to $q = 4$ ($p = 0.004$). This is, however, rejected when the voting rule equals $q = 4$ ($p = 0.64$) or $q = 5$ ($p = 0.36$).

We can also test the effect of the voting rule on the likelihood of voting truthfully in the INF treatment. Pairwise comparisons suggest a positive effect of the voting rule on the likelihood of type-A agents voting truthfully. This is because the likelihood of voting truthfully is smaller in $q = 3$ than in $q = 4$ ($p = 0.014$) or $q = 5$ ($p < 0.001$). If we compare $q = 4$ and $q = 5$ we find no effect of the voting rule $p = 0.190$. As for type B agents, the voting rule has no effect on the likelihood of voting truthfully when we compare $q = 3$ and $q = 4$ ($p = 1$), but type-B agents are less likely to vote truthfully when $q = 5$ compared with any of the other two voting rules ($p = 0.003$).

6.9.2 Econometric analysis

Effect of conformity

The first two columns of Table B.1 present the maximum likelihood estimates of our logit regression, for each of the two types of agents separately. We complement our analysis by reporting the results of a linear probability model. In this latter case, the estimates can be interpreted as the effects of each independent variable on the probability of voting truthfully. The set of independent variables include controls for gender, risk aversion, trust, or social value orientation of the subject (see Appendix A for further details on the questionnaire).

We see in the first column that conformity has a significant effect on the likelihood of type-A agents voting truthfully. The marginal effect reported in Figure 2 in the main text indicates that the likelihood of voting truthfully indeed decreases by 22.2 percent when $q = 3$. This is in line with the results of the linear probability model in column 3, which estimates a decrease of 22.8 percent. The Chi-2 test suggests that the effect of conformity is also significant when $q = 4$, but not when $q = 5$. Thus, we can observe that the estimates of conformity in the linear probability regression (-0.228) is roughly in the same magnitude (but different sign) than the estimate when we interact the treatment variable with the voting rule $q = 5$ (0.187).

Our analysis for type-B agents in columns 2 and 4 suggests that conformity has no effect on the likelihood of voting truthfully when $q = 3$, but in the rest of the cases. Thus, note that the dummy variable for the treatment condition is not significantly different from 0, but the interaction terms with the voting rules are significant. The estimates of the linear probability model indeed suggest that the negative effect of conformity when $q = 4$ is around 18.1 percent ($0.027 - 0.208$). It is roughly 27.9 percent when $q = 5$. Recall that the marginal effects after the logit regressions reported in the main text are -0.11 and -0.32 respectively.

Dependent variable: Probability of voting truthfully				
	Logit model		Linear Probability model	
	Type-A	Type-B	Type-A	Type-B
dq_4 (=1 if q = 4)	0.911	-1.10 e16	0.146	-2.62e-15
	(0.57)	(0.56)	(0.09)	(0.05)
dq_5 (=1 if q = 5)	-0.103	-0.629	-0.021	-0.069
	(0.50)	(0.56)	(0.10)	(0.06)
d_{CON} (=1 if conformity)	-0.990**	0.518	-0.228**	0.027
	(0.49)	(0.68)	(0.10)	(0.05)
$d_{CON}\, dq_4$	0.206	-1.852**	0.104	-0.208**
	(0.74)	(0.84)	(0.14)	(0.080)
$d_{CON}\, dq_5$	0.822	-2.003**	0.187	-0.306***
	(0.68)	(0.81)	(0.15)	(0.09)
Age	-0.039	0.049*	-0.009	0.006**
	(0.04)	(0.03)	(0.01)	(0.003)
Women	-0.669*	0.278	-0.131*	0.041
	(0.34)	(0.31)	(0.07)	(0.04)
Risk aversion	-0.026	0.121**	-0.005	0.015**
	(0.05)	(0.06)	(0.01)	(0.01)
Trust	-0.111	0.096	-0.025	0.011
	(0.32)	(0.32)	(0.06)	(0.04)
Inequality	0.503	0.014	0.104	0.006
	(0.33)	(0.34)	(0.07)	(0.04)
CRT	-0.077	0.708	-0.019	0.094
	(0.45)	(0.50)	(0.09)	(0.06)
Individualistic	-0.274	-0.294	-0.032	-0.031
	(0.50)	(0.50)	(0.09)	(0.07)
Prosocial	-0.346	0.025	-0.046	0.009
	(0.46)	(0.49)	(0.08)	(0.06)
Competitive	-0.084	-0.609	-0.001	-0.078
	(0.80)	(0.86)	(0.14)	(0.13)
Satisfaction	-0.003	-0.167	-0.001	-0.023
	(0.12)	(0.14)	(0.02)	(0.02)
Happiness	0.058	-0.337	0.010	-0.045
	(0.28)	(0.31)	(0.06)	(0.04)
Constant	2.130	1.546	0.946***	0.810***
	(1.35)	(1.43)	(0.28)	(0.18)
Wald-test	32.38***	42.41***	2.96***	2.93***
(Pseudo) R^2	0.09	0.13	0.11	0.131
Observations	288	432	288	432

Table B. 1. Econometric analysis for the effect of conformity[19]

[19]Notes: Robust standard errors are reported in parentheses. The log-pseudo likelihood in the logit regressions is -165.94 and -179.33 for type-A and type-B agents respectively. Significance *** p<0.01, ** p<0.05, * p<0.1.

Effect of knowing that others vote truthfully

The first two columns of Table B.2 present the maximum likelihood estimates of our logit regression, for each of the two types of agents separately. We complement our analysis by reporting the results of a linear probability model.

In line with our discussion in the main text, we find that information affects differently to type-A (who tend to vote less truthfully with the presence of forced voters) and type-B agents (who tend to vote more truthfully). When $q = 3$, we find that the effect for type-B agents is not significant according to the linear probability model, but it is when q = 4. This is the main difference between the logit model reported in the main text and the linear probability model. We also note that the reported estimates for the logit model in Table B.2 cannot be used to compute the marginal effects, according to Ai and Norton (2003). The correct marginal effects are reported in the main text.

Dependent variable: Probability of voting truthfully				
	Logit model		Linear Probability model	
	Type-A	Type-B	Type-A	Type-B
dq_4 (=1 if q = 4)	1.077**	-1.871***	0.250**	-0.208***
	(0.45)	(0.64)	(0.10)	(0.06)
dq_5 (=1 if q = 5)	0.693	-2.665***	0.167	-0.375***
	(0.44)	(0.62)	(0.11)	(0.07)
d_{INF} (=1 if information)	-0.522	0.353	-0.123	-0.006
	(0.41)	(1.13)	(0.10)	(0.05)
$d_{INF}\,dq_4$	-0.239	1.871	-0.050	0.208**
	(0.60)	(1.63)	(0.14)	(0.08)
$d_{INF}\,dq_5$	0.647	-0.178	0.150	0.065
	(0.63)	(1.31)	(0.15)	(0.12)
Age	-0.034	0.053	-0.008	0.005
	(0.03)	(0.03)	(0.01)	(0.003)
Women	-0.425	0.313	-0.097	0.046
	(0.29)	(0.35)	(0.07)	(0.05)
Risk aversion	0.026	0.164***	0.006	0.020***
	(0.06)	(0.06)	(0.01)	(0.01)
Trust	0.047	0.113	0.011	0.013
	(0.25)	(0.35)	(0.06)	(0.05)
Inequality	-0.127	-0.152	-0.028	-0.016
	(0.31)	(0.39)	(0.07)	(0.05)
CRT	-0.439	0.664	-0.099	0.083
	(0.40)	(0.548)	(0.09)	(0.07)
Type_i	-0.746	-0.366	-0.166	-0.043
	(0.63)	(0.69)	(0.14)	(0.09)
Type_p	-0.647	-0.242	-0.143	-0.022
	(0.59)	(0.62)	(0.13)	(0.08)
Type_c	-0.522	0.172	-0.119	0.043
	(0.81)	(1.02)	(0.172)	(0.13)
Satisfaction	0.050	-0.272*	0.0114	-0.036
	(0.11)	(0.15)	(0.03)	(0.02)
Happiness	0.118	-0.374	0.027	-0.047
	(0.26)	(0.40)	(0.06)	(0.05)
Constant	1.128	2.405	0.752**	0.904***
	(1.39)	(1.72)	(0.32)	(0.23)
Wald-test	20.34	46.51***	1.60*	4.86***
(Pseudo) R^2	0.06	0.20	0.08	0.18
Observations	324	303	324	303

Table B.2. Econometric analysis for the effect of information

Chapter 7

General Conclusions

This thesis studies theoretically the strategic behavior of two key components of democracy: political parties and voters.

Among the great variety of topics regarding political parties, this thesis focuses on two-party political competition when one of the parties is an office-seeking traditional party whilst the other has preferences for ambiguity.

The first scenario analyzed is the one in which the ambiguous party acts as an assembly party. Under this setting, results show that more moderate assemblies increase the options of winning the elections. Interestingly, we find that the location of the assembly induces the traditional party to locate at different platforms. When the assembly is centrist, the traditional party moves away from the median voter location. However, when the assembly is extremist, the traditional party locates at a moderate position. As a result, the assembly party generates divergence between the platforms of the parties, in contrast to the Downsian model. The current importance of direct democracy inside new political parties in how they elect their candidates and representatives is an interesting point to study in a future research related to assembly parties.

In the second scenario, the ambiguous party is represented by a party which consists of two internal factions: an ideological and an opportunistic. Under a sequential setting, we show that a party that gives equal weights to its two factions needs considerable valence advantage to win the elections, even with a first mover advantage. Interestingly, despite the multiplicity of equilibria, the Downsian equilibrium does not hold in this case and parties have incentives to offer divergent platforms. From the voters' viewpoint, parties with heterogeneous factions generate more pluralistic platforms. From the parties' viewpoint, a multi-faction party is worst-off when competing with a traditional party. An extension of this model must study this question simultaneously, considering different weights to each of the factions.

In the third scenario, both parties have a preference for ambiguity and for winning the elections. Under a situation in which office-seeking incentives are very rewarding, parties sacrifice their taste for ambiguity and converge to the ideal policy of the median voter. In the opposite case in which ambiguity is very rewarding, an equilibrium fails to exist since parties have incentives to deviate to more ambiguous platforms even when this means losing the elections. We conclude therefore that with risk-neutral voters when the distribution of voters is known, parties cannot choose ambiguous platforms if they want to win the elections. A possible direction in which this model can be extended is to consider that parties have an ideology and deviating from this ideal policy of the parties comes at a positive cost.

From both the first and the second scenario, we can reach to the conclusion that the traditional party moves along the policy space as it needs to differentiate from the ambiguous party in order to attract voters from the center and from one of the sides of the median. This result applies regardless of the ambiguous party has enough valence advantage to win the elections, as it influences the strategies of the traditional party even when the ambiguous party is defeated. We also find that it is not sufficient to have a first mover advantage for an ambiguous party to win the elections. The only possibility it has relies on having enough valence advantage to defeat a traditional party. From the three scenarios analyzed, we also obtain that ambiguity makes insider voters display a kind of risk aversion, despite voters being risk-neutral. This risk aversion comes from the fact that a risk-neutral voter, when locating between the two policies of the ambiguous party, derives a greater utility from voting the traditional party than from voting the ambiguous party when the mean policy of the ambiguous party coincides with the unique location of the traditional party. Therefore, we can also conclude that ambiguity is a disadvantage for a party when competing with a traditional one even with risk-neutrality. A future research in which we can study how the valence advantage of one party influences voters' risk attitudes would be an interesting direction to extend this analysis.

Moving towards the second research area of this thesis, voters, the study focuses on how the presence of conformity may alter decisions taken by agents under a complete information setting. When all agents are conformist, no $q-$threshold rule can be implemented in a simultaneous binary voting game. A solution to deal with this problem is replacing conformist by independent agents, who only consider their opinion when voting. Under a simultaneous setting, it is possible to make the effect of conformity disappear with a sufficient number of conformist agents replaced by independents. In case the decisions are taken sequentially, conformity does not affect these decisions. This study is a first step to analyze an incomplete information version of this model where, instead of replacing conformist by independent agents, we can add to the vote group a certain number of independent agents, as boards of directors already do at the companies.

To finish, experimental data supports the fact that inducing conformity in the preferences of the agents by means of an extra payoff for coinciding with the vote of some other agent reduces the likelihood of voting truthfully. Besides, when agents vote knowing that there are independent agents, this leads them to vote more truthfully. Results also show the effect of the voting rule on agents' decisions, being less sincere the less pivotal they are. Once we know the effects of inducing conformity in the preferences of the agents, it would be interesting to know how the voting rule and thus, the pivotality of the agents makes agents to conform to the decision most preferred by the group of agents when conformity is not induced in the preferences of the agents. Not only the performance of the subjects when conducing the experiment but also the beliefs they have about the behavior of the rest of subjects gives enough evidence to reach to some conclusions regarding agents' reasoning in taking decisions.

After analyzing conformity both theoretically and experimentally, we can reach to some conclusions regarding the pivotality of agents, the voting rules and the truthful behavior of the agents. First, we find that conformity influences the final decision only when agents cannot be pivotal, which occurs under a simultaneous setting without independent agents. Second, after analyzing the number of independent agents needed to implement the truth for any voting rule, it turns out that unanimous rules are the ones needing less independents. After unanimity, simple majority appears to be better than any other supermajority rule. Third, experimental results suggest there is an interplay between the voting rule and truthful voting, since agents vote more truthfully the more favorable a voting rule is to obtain a particular outcome. That is, there is an effect of the voting rule on agents voting truthfully only when they can be pivotal. Future research may study why agents do not vote truthfully in binary voting games with complete information in which preferences for conformity are not induced. May a voter sacrifice her favorite option when she realizes that the society prefers the other option or is there a fear of voting alone or with the minority?

Chapter 8

Resumen en español

Esta sección se destina a resumir los principales aspectos de la tesis en español. En el primer apartado, se introduce el tema de investigación y en la siguiente sección, se presentan los objetivos que se persiguen dentro de la misma. A continuación, se resume cada una de las principales contribuciones que conforman la tesis. El apartado final se destina a presentar las conclusiones.

8.1 Introducción

La presente tesis estudia de manera teórica el comportamiento estratégico de dos partes fundamentales de la democracia: los partidos políticos y los votantes.

8.1.1 Partidos Políticos

Dentro del amplio abanico de temas de estudio referente a las estrategias de los partidos políticos, esta tesis se centra en la competencia política para el caso en el que dos partidos políticos compiten para ganar las elecciones. En particular, analiza casos de competencia política caracterizados por tener a uno de los partidos ejerciendo de partido tradicional y a otro partido que es percibido por los votantes como ambiguo. El partido tradicional viene representado por una única plataforma política y su objetivo se limita a ganar las elecciones. En cambio, el partido ambiguo representa varias plataformas políticas y los votantes son incapaces de relacionar una única plataforma política con ese partido. La literatura ya ha encontrado respuesta a cuáles serían las estrategias de equilibrio de dos partidos políticos tradicionales que compiten por ganar las elecciones, el equilibrio Downsiano, que muestra que ambos partidos se sitúan en la posición del votante mediano (Downs, 1957), es decir, aquella plataforma posicionada en el punto medio del espacio de políticas. Con esta tesis, se pretende conocer cuáles serían las consecuencias, en términos de la estrategia política utilizada por cada uno de los partidos en equilibrio, cuando uno de ellos es un partido tradicional y el otro partido se caracteriza por su ambigüedad. La razón para estudiar las competencia entre estos tipos de partidos políticos viene motivada por la creciente aparición de nuevos partidos políticos que intentan atraer a votantes descontentos con los partidos tradicionales, que intentan no ser relacionados con una única ideología política y que pueden ser definidos como partidos ambiguos, al no ofertar una única política concreta.

El hecho de que un partido político sea ambiguo hace que los votantes asocien varias estrategias políticas a ese partido. Esto implica que el partido político ambiguo va a ser evaluado por los votantes como una lotería en la que los votantes dan una probabilidad positiva determinada a cada una de las políticas que anuncia este partido político. Esta tesis presenta el estudio de las estrategias de competencia política entre un partido tradicional y un partido ambiguo en dos escenarios: cuando el partido ambiguo es un partido asambleario y cuando el partido ambiguo está compuesto de varias facciones internas. Por último, se estudia un caso más general en el que dos partidos políticos sin una ideología concreta tienen un doble objetivo: ser ambiguos y ganar las elecciones.

8.1.2 Votantes

En la vida real, se detectan situaciones en la que los individuos, a la hora de votar por una alternativa, ceden ante las presiones sociales y votan por alternativas menos preferidas para poder encajar mejor en el grupo, ser aceptado o para crear un vínculo con las personas con las que se coincide en el voto. A este comportamiento se le conoce en psicología social como *conformity*, que se define como "tipo de influencia social que conlleva un cambio en el comportamiento de un individuo para sentirse mejor dentro de un grupo".

Este comportamiento se ha estudiado en psicología social desde mediados del siglo XX. En 1950, el científico Solomon Asch fue pionero en confirmar esta actitud social mediante un experimento en un laboratorio. Más tarde, Deutsch y Gerard (1955) identifican dos tipos de influencia social: informacional y normativa. La influencia informacional ocurre cuando los individuos actualizan sus preferencias teniendo en cuenta la opinión del resto de individuos. En cambio, la influencia normativa describe el comportamiento que se considera en este estudio, en el que los individuos manifiestan una opinión condicionada por la opinión general que ya hay en el grupo.

En casos de votación, esta conducta puede determinar que la decisión elegida no sea la que represente las preferencias reales de los individuos y éstos, al imponerse a sí mismos la necesidad de coincidir con ciertos miembros del grupo, voten una opción distinta a la preferida. Esto puede representar un grave problema en escenarios tales como juicios, consejos de administración, comités de calidad, tribunales, entre otros comités de decisión.

Para estudiar este tipo de comportamiento y sus consecuencias, se presenta un modelo en el que los votantes tienen bien definidas sus preferencias de acuerdo a dos alternativas posibles: mantener la opción actual o cambiarla. A partir de ahí, se introduce un tipo de preferencias lexicográficas que hace que los agentes voten por su opción favorita si ven que ésta tiene opciones a ser elegida, o voten de acuerdo a lo que va a votar un grupo de referencia cuando su voto ya no afecta a la decisión final. El primer escenario a estudiar es meramente teórico. En él se aplica este tipo de preferencias para n agentes, cualquier regla de la mayoría (mayoría simple, supermayoría, unanimidad, etc.) y se considera que los votantes quieren coincidir con cualquier número de integrantes del grupo. En el segundo escenario, se analiza de manera experimental un perfil de preferencias concreto para 5 individuos.

8.2 Objetivos

El objetivo general de esta tesis es analizar, bajo un punto de vista teórico y utilizando las distintas herramientas proporcionadas por la teoría de juegos, el comportamiento estratégico de dos partes fundamentales de la democracia: los partidos políticos y los votantes.

Los objetivos específicos vienen condicionados por la finalidad de cada uno de los artículos científicos que conforman la tesis y que se desarrollan entre los capítulos 2 y 6.

- **Objetivo 1:** Modelo de competencia política donde un partido asambleario y un partido tradicional compiten para ganar las elecciones. Se demuestra cuáles son las estrategias políticas que llevarían a cabo dichos partidos políticos en equilibrio y se comprueba de qué manera la existencia de un partido asambleario determina la estrategia política que toma un partido tradicional.

- **Objetivo 2.** Modelo de competencia política donde un partido con dos facciones internas y un partido tradicional compiten para ganar las elecciones. Se estudian las estrategias políticas de ambos partidos ante la existencia de incertidumbre acerca de la ventaja que un partido tiene sobre otro cuando deciden sus estrategias de manera secuencial.

- **Objetivo 3.** Modelo de competencia política entre dos partidos políticos no ideológicos caracterizados por proponer estrategias ambiguas. Ante esta situación, se estudian cuáles serían las estrategias políticas que les permitirían cumplir con sus dos principales objetivos: representar al electorado y ganar las elecciones.

- **Objetivo 4.** Modelo de elección binaria donde los votantes tienen en cuenta dos características a la hora de votar: primero, cuál es su opción preferida y, segundo, qué va a votar el resto de votantes. Bajo esta situación, se identifican las consecuencias del fenómeno del "conformity" sobre la manera de votar de los individuos. Asimismo, se propone como solución reemplazar un número concreto de individuos por agentes independientes y no influenciables.

- **Objetivo 5.** Estudio mediante un experimento de un caso particular del modelo presentado en el Objetivo 4. Se valida de manera experimental la fórmula propuesta para solucionar el problema del conformity y se analizan los resultados obtenidos.

8.3 Contribución

En esta sección, se desarrolla brevemente el trabajo relacionado con cada uno de los objetivos específicos de esta tesis. Cada uno de los apartados de esta sección contiene una introducción al tema de estudio, se explica brevemente la metodología utilizada y los resultados a los que se llegan a través de la misma.

8.3.1 Equilibrio Downsiano ante un partido asambleario

Escrito junto con Socorro Puy. Publicado en el libro The Political Economy of Governance: Institutions, Political Performance and Elections. Editores: Schofield, N. y G. Caballero. Springer (2015). ISBN: 978-3-319-15550-0. DOI 10.1007/978-3-319-15551-7.

A lo largo del Capítulo 2, se estudia un escenario de competencia política entre un partido tradicional (Partido A) y en el que el partido ambiguo ejerce de partido asambleario (Partido B).

Se define como partido asambleario aquél que decide sus propuestas políticas mediante asambleas. En el marco teórico que se propone, el partido asambleario organiza dos asambleas, una pre-electoral y otra post-electoral. En la asamblea pre-electoral, se deciden qué propuestas va a representar el partido cuando se celebren las elecciones. Durante la asamblea pre-electoral, los asistentes, sean afiliados o no al partido, pueden proponer su política preferida como propuesta del partido. Las múltiples propuestas que pueden aflorar en la asamblea pre-electoral hacen que el partido asambleario sea evaluado por los votantes como una lotería, dándole una probabilidad positiva a cada una de las propuestas surgidas en la asamblea. En el caso de que el partido asambleario ganara las elecciones, se celebraría una asamblea post-electoral en el que las propuestas definidas en la asamblea pre-electoral puedan ser votadas por los asistentes a la asamblea para decidir qué política de todas las propuestas implementará el partido. Para ello, se toma la distancia euclídea entre la política preferida del votante y las propuestas finales del partido. Por razones de simplificación, y siguiendo el modelo del Ciudadano-Candidato de Osborne and Slivinski (1996), se considera que sólo dos propuestas salen elegidas en la asamblea pre-electoral. Esas dos propuestas están simétricamente localizadas alrededor del votante mediano x_M, es decir, una a la izquierda, $x_M - \varepsilon$, y otra a la derecha del votante mediano, $x_M + \varepsilon$, donde ε es la distancia máxima que hace impedir que haya una tercera propuesta en la asamblea (ver Proposición 2 de Osborne and Slivinski, 1996). Tras conocer las propuestas del partido asambleario, el partido tradicional selecciona aquella política que le permita ganar las elecciones. Aparte de las propuestas del partido asambleario y la política del partido tradicional, cada partido tiene asociado un parámetro que mide la percepción general que tienen los votantes sobre cada partido, a lo que la literatura en competencia política denomina *valence*. El valence recoge la percepción general que genera un partido político respecto al otro. Un valor positivo de valence para un partido concreto podría indicar si la sociedad considera a ese partido como más preparado para dirigir el país, si lo considera menos corrupto, más cercano o carismático, etc. Por ejemplo, si la sociedad tiene una percepción general acerca de que el partido tradicional es menos corrupto (algo que los votantes valoran de manera positiva), el valence favorecerá al partido tradicional respecto al partido ambiguo, de manera que los votantes obtendrán una utilidad extra, representada por el parámetro β, si votan al partido tradicional. En nuestro caso, se recurre a la diferencia entre los valence de ambos partidos, $\Delta\beta$, para definir el grado de protesta social en contra de los partidos tradicionales, cuyo valor indica si la sociedad en general prefiere más a un partido asambleario que a un partido tradicional (si el valor del parámetro valence es más alto para el partido asambleario (β_B) que para el partido tradicional (β_A), tenemos que $\Delta\beta = \beta_B - \beta_A > 0$). Se asume que los partidos desconocen el valor del grado de protesta social y si uno u otro partido tiene cierta ventaja respecto al otro.

La función de utilidad de los partidos políticos y de los votantes nos permite conocer las estrategias de equilibrio de ambos partidos ante unas elecciones de este tipo. Para ello, se define como equilibrio político al par compuesto por las propuestas acordadas en la asamblea pre-electoral y la política del partido tradicional que maximiza su probabilidad de ganar.

A la hora de resolver el juego electoral, se tienen en cuenta las siguientes etapas:

1. El partido asambleario (Partido B) organiza una asamblea pre-electoral donde todo el que desee puede hacer una propuesta política.

2. El partido tradicional (Partido A) decide su estrategia política x_A.

3. Se celebran las elecciones.

4. Si el partido tradicional gana, la plataforma x_A se implementa. Si el partido asambleario gana, se celebra una asamblea post-electoral en la que todo el que desee puede votar y en la que se decide cuál de las propuestas elegidas en la asamblea pre-electoral será implementada.

Se estudian dos escenarios, cuando todo el electorado participa en la asamblea y cuando la participación es parcial. En el caso de que todo el electorado participe en la asamblea, se obtiene el grado de presión social $\Delta\beta$ necesario para que el partido asambleario gane las elecciones (Proposición 1). Para grados de presión social inferiores, se identifican dos políticas de equilibrio que permitirían ganar al partido tradicional: una situándose en el punto medio entre la propuesta pre-electoral de la asamblea más de izquierdas ($x_M - \varepsilon$) y el mediano (x_M), es decir, $x_A = x_M - \frac{\varepsilon}{2}$; y en el punto medio entre el mediano y la propuesta pre-electoral de la asamblea más de derechas ($x_M + \varepsilon$), es decir, $x_A = x_M + \frac{\varepsilon}{2}$ (Proposición 2). En este segundo resultado, se obtiene que el partido tradicional tiene que diferenciarse de la política del votante mediano para poder atraer a una mayoría del electorado. Cuando el partido tradicional se sitúa a la izquierda del votante mediano, obtiene el apoyo de la mayoría que está a la izquierda del mediano, incluido éste. En cambio, si el partido tradicional se sitúa a la derecha del votante mediano, obtendrá el apoyo de la mayoría que está a la derecha del mediano.

En el escenario en el que hay participación parcial en la asamblea, se pueden distinguir dos situaciones: que la asamblea sea centrista o sea extremista. Se considera que una asamblea es centrista si el votante mediano del electorado asiste a la asamblea, es decir, está situado entre las dos propuestas de la asamblea. En ese caso, se obtiene cuál es el grado de protesta social necesario para que el partido asambleario gane las elecciones (Proposición 3). Además, se obtiene que, en equilibrio, el partido tradicional se situará en el punto medio entre el mediano del electorado y la propuesta de derechas de la asamblea si la asamblea es de centro-izquierda ($x_M^a + \varepsilon$), es decir, $x_A = \frac{x_M + x_M^a + \varepsilon}{2}$; y en el punto medio entre el mediano del electorado y la propuesta de izquierdas de la asamblea si la asamblea es de centro-derechas ($x_M^a - \varepsilon$), e decir $x_A = \frac{x_M + x_M^a - \varepsilon}{2}$. Se considera que una asamblea es extremista si el votante mediano no asiste a la asamblea (la posición del votante mediano está fuera de las propuestas de la asamblea). En este caso, el partido asambleario gana las elecciones si el grado de presión social es lo suficientemente grande como para superar la distancia entre las posiciones del votante mediano de la asamblea y del electorado (Proposición 4). Además, en los casos en los que la asamblea es extremista, el partido tradicional opta por ofrecer la posición del votante mediano del electorado ($x_A = x_M$). Es decir, la existencia de una asamblea extremista hace que el partido tradicional proponga políticas más moderadas, mientras que una asambleas centrista hace que el partido tradicional se diferencie de las propuestas de la asamblea, para así atraer votantes centristas y extremistas.

Este análisis permite concluir que la existencia de un partido asambleario que propone dos propuestas, una a cada lado de la posición del votante mediano, induce al partido tradicional a proponer políticas distintas a la posición del votante mediano, lo que contradice la convergencia obtenida por el modelo Downsiano. Además, la existencia de un partido político asambleario no sólo tiene efectos cuando éste gana las elecciones, sino que también tiene efectos indirectos cuando pierde, ya que afecta la estrategia política del partido tradicional, haciendo que éste sea más moderado o extremista dependiendo de lo extremista o moderada que sea la asamblea.

8.3.2 Competencia política entre partidos con facciones internas heterogéneas

Escrito junto con la Prof. Socorro Puy.

A lo largo del Capítulo 3, se estudia un escenario de competencia política en el que uno de los partidos es percibido como ambiguo al estar formado por varias facciones internas.

Se consideran dos partidos: el Partido L y el Partido R. El Partido L tiene dos facciones: una ideológica y otra oportunista. La facción ideológica es una estrategia política que está fijada y que se ocupa de mantener los principios ideológicos de los fundadores del partido. Esta facción puede interpretarse como la "marca" del partido o como la media de las distintas políticas que el partido ha implementado a lo largo de su existencia y se definirá como x_L^E. La facción oportunista es la facción estratégica dentro del partido y se encarga de maximizar las opciones que tiene el partido de ganar las elecciones. Esta facción propone aquella plataforma que maximice el número de situaciones en las que el partido ganaría las elecciones y se definirá como x_L^O. El Partido R tiene una única facción oportunista, cuyo objetivo es ofrecer aquella estrategia política dentro del espacio electoral $[0, 1]$ que maximice sus opciones de ganar las elecciones. A este nivel del análisis, se introduce la siguiente restricción: que la facción oportunista esté a la derecha de la política representada por la facción ideológica.

Dado que cada facción dentro del Partido L tiene el mismo peso dentro del partido, éste es evaluado por los votantes como una lotería, donde cada facción tiene un 50% de posibilidades de ser la política que implemente el partido en caso de ganar las elecciones. Como el Partido R sólo consta de una facción y es oportunista, el Partido R está asociado a una única política, x_R y se le denomina como partido tradicional. Cada partido tiene asociado un valence en el que todos los individuos están de acuerdo. Tal y como se hizo en el caso de estudio anterior, se recurre a la diferencia entre los valence de ambos partidos para definir qué partido tiene una ventaja en *valence* sobre el otro partido, $\Delta\beta$. Nuevamente, se asume que existe incertidumbre sobre qué partido tiene ventaja en valence sobre el otro.

En cuanto a los votantes, la utilidad que reciben por votar a cierto partido depende del valor del valence y de la distancia euclídea entre la política ideal del votante y las propuestas finales del partido. Votarán a aquel partido que derive una mayor utilidad y en caso de que sea la misma se abstendrán. En este caso, hay que tener en cuenta que el Partido L propone dos políticas y la utilidad del votante viene determinada por el valor del valence y la distancia euclídea entre la política ideal del votante y cada una de las políticas propuestas por el Partido L, ponderado por las posibilidades de que cada una de ellas sea implementada por el partido.

A la hora de resolver el juego electoral, se tienen en cuenta las siguientes etapas:

1. Dada la propuesta fijada por la facción ideológica del Partido L, x_L^E, la facción oportunista del Partido L propone aquella política que maximice el pago esperado bajo la incertidumbre generada por el valence advantage.

2. El partido R observa la política anunciada por el Partido L y anuncia la política que maximiza el pago esperado bajo la incertidumbre generada por el valence advantage, x_R.

3. Se celebran las elecciones y los votantes optan por uno de los dos partidos o se abstienen en caso de indiferencia.

Se resuelve el juego secuencial aplicando inducción hacia atrás, es decir, resolviendo la etapa 3 primero, luego la mejor respuesta del Partido R y, por último, la decisión óptima del Partido L, una vez que conoce la mejor respuesta del Partido R. Tras resolver el juego se obtienen las estrategias de equilibrio de ambos partidos, de donde se destaca que el Partido L elegirá cualquier estrategia a la derecha del votante mediano y por debajo de la distancia del doble del mediano y la política fija de la facción ideológica, es decir, $x_L^{O^*} \in [x_M, 2x_M - x_L^E]$. Por su parte, el Partido R tiene una única estrategia política y se situará en el punto medio entre el votante mediano y la política oportunista del Partido L, es decir, $x_R^* = \frac{x_L^{O^*}+x_M}{2}$. En el caso de que el Partido L tenga ventaja en valence sobre el Partido R, éste ganará las elecciones. En caso contrario, la victoria se la llevará el Partido R.

Del resultado se debe destacar la multiplicidad de equilibrios obtenidos y la ausencia del equilibrio Downsiano en los mismos, dándose equilibrios divergentes donde cada partido se sitúa a un lado distinto del votante mediano. Asimismo, el hecho de que un partido tradicional compita con un partido formado por varias facciones sólo crea desventajas para éste último, al ganar únicamente cuando tiene suficiente ventaja en valence.

8.3.3 Partidos ambiguos sin capacidad de serlo

Escrito junto con la Prof. Socorro Puy. En revisión en revista JCR.

A lo largo del Capítulo 4, se estudia un escenario de competencia política en el que ambos partidos tienen cierta preferencia por ser ambiguos. En concreto, se estudia el caso en que los dos partidos tienen preferencias por ganar las elecciones y por ser ambiguos, es decir, están dispuestos a ofrecer un abanico de propuestas políticas para así recoger las demandas de la mayor parte del electorado posible. Se supone, además, que estos partidos no son ideológicos. Dada la importancia por la ambigüedad, la plataforma de los partidos pueden ser políticas únicas o puede ser una lotería, que consistiría en la distribución de probabilidades sobre algunas políticas del espacio de políticas $[0, 1]$. El caso de estudio se limita a loterías que puedan ser discretas o continuas y que sigan una distribución uniforme. Las loterías continuas vienen definidas por dos parámetros, el punto medio de la lotería, x y el nivel de ambigüedad ε que define el intervalo de políticas que pueden ser implementadas, es decir, $[x-\varepsilon, x+\varepsilon]$. Las loterías discretas están formadas por un número finito de políticas con la peculiaridad de que la distancia que hay entre cada política tiene que ser la misma. Las loterías discretas vienen definidas por tres parámetros, el número de políticas m, la política media x y el nivel de ambigüedad ε.

A la hora de votar, los votantes sólo tienen en cuenta su función de utilidad, que viene representada por la distancia entre las políticas propuestas por los partidos y la política ideal del votante. Cuanto más grande sea esa distancia, menos utilidad tendrá el votante. De esta manera, los votantes tienen preferencias single-peaked, caracterizadas por preferir aquellas políticas que más se acerquen a su política ideal.

El concepto de equilibrio que se utiliza para conocer las estrategias de los partidos y al ganador de las elecciones es el equilibrio de Nash en estrategias débilmente no dominadas. Es decir, la estrategia que elija el partido es aquella en la que el partido reciba más utilidad (maximice sus opciones de ganar las elecciones) dada la estrategia que haga el otro partido.

Lo primero que se examina es si existe alguna lotería que domine débilmente a todas las demás, para un mismo punto medio y para un mismo nivel de ambigüedad. Por ejemplo, si se compara una lotería con dos propuestas (cada propuesta corresponde a los extremos de la lotería), con una lotería de tres propuestas, se obtiene que la lotería de 3 propuestas domina débilmente a la de dos, ya que hay votantes dentro del rango de la lotería, a los que llamaremos *insiders*, que prefieren la lotería de 3 propuestas. Esta comparativa se puede realizar para cualquiera dos pares de loterías con distinto número de propuestas. Por último, se compara una lotería continua con una lotería con m políticas y comprobamos que la lotería continua domina débilmente a cualquier lotería discreta. Esto implica que, si un partido político se decantara por una lotería, ésta tendría que ser una lotería continua.

Teniendo en cuenta que las loterías discretas no son estrategias de equilibrio, se analizan situaciones de equilibrio donde las estrategias de los partidos se reducen a dos: la lotería continua y una única política. Se analiza qué características tienen que darse en una situación de equilibrio. En primer lugar, si existe un equilibrio, en él los partidos políticos tienen que empatar. Es decir, no se da ninguna situación de equilibrio en donde haya un partido que gane y otro que pierda. En segundo lugar, los partidos tienen que proponer un mismo nivel de ambigüedad. Además, los partidos tienen que proponer la misma política o tienen que proponer una política equidistante al votante mediano. En ambos casos, el partido que se desvíe a una estrategia más cercana al votante mediano es el que ganaría. Esto nos lleva a una única situación, en la que los dos partidos proponen la política del votante mediano.

Teniendo en cuenta lo anterior, se analizan distintos escenarios dependiendo del peso que tenga para los partidos políticos ganar las elecciones. Si los partidos políticos dan prioridad a ganar las elecciones (caso en el que la utilidad que obtienen por ganar las elecciones es mayor que la utilidad que obtendrían por proponer plataformas ambiguas), los partidos convergen al equilibrio Downsiano, es decir, ambos empatan proponiendo la política ideal del votante mediano. Si la utilidad por ser ambiguo es suficientemente grande, no existen equilibrios. Para encontrar un equilibrio donde ambos partidos propongan una política ambigua, es decir, la lotería continua, tenemos que irnos a un escenario donde no se obtiene ninguna utilidad por ganar las elecciones y en la que ambos partidos propondrán la lotería continua con el máximo grado de ambigüedad posible. Se puede concluir, por tanto, que el hecho de que se diseñe un escenario donde los partidos políticos le den cierta importancia a proponer políticas ambiguas no es suficiente para obtener un equilibrio distinto del equilibrio Downsiano.

8.3.4 Votación bajo "conformity"

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En el Capítulo 5 se analiza el comportamiento de los votantes bajo la influencia del *conformity.*

Un grupo de n individuos tiene que decidir si acepta o rechaza una propuesta. Cada uno de los individuos está a favor o en contra de la propuesta. Los individuos votan y si el número de individuos que vota a favor de la propuesta es igual o superior a cierta regla de la mayoría, la propuesta se acepta. Supongamos que las preferencias de estos individuos no sólo se caracterizan por tener en cuenta si su opción favorita es estar a favor o en contra de la propuesta. Además, cuando los individuos ven que su voto no va a afectar a la decisión final, los votantes se fijan en lo que votan el resto de miembros del grupo, dándole importancia a coincidir con el voto de alguno de ellos. Es decir, las preferencias de estos agentes son tales que, primero prefieren que su opción preferida sea la decisión final. Cuando ven que esto no va a ocurrir, prefieren votar lo mismo que ciertos individuos del grupo (aparece el fenómeno del conformity).

Teniendo en cuenta las preferencias de los individuos y bajo información completa, se examina cómo afectan a los resultados obtenidos, es decir, si se obtienen decisiones finales que no se obtendrían si todos votasen sólo teniendo en cuenta cuál es su opción preferida. El primer escenario que se presenta es aquel en el que los individuos votan de manera simultánea y en el que los individuos pueden querer coincidir en el voto con cualquier número de agentes. En este sentido, se obtienen equilibrios en los que la decisión final es distinta a la obtenida si todos votasen sólo de acuerdo a su opinión. Esto ocurre independientemente del número de individuos que forme el grupo, para cualquier regla de la mayoría que se considere para tomar la decisión y para cualquier número de individuos con los que los votantes quieran coincidir.

Cómo propuesta para solventar esta situación, se propone ir reemplazando a estos individuos cuyas preferencias están afectadas por el *conformity* por otros individuos independientes, cuyas preferencias se caracterizan por votar sólo teniendo en cuenta su opción preferida, sin importarles lo que vote el resto de agentes, lo que se considera en este estudio como votación sincera. Esta medida es bastante usada en comités de decisión en los que se recurren a consultores externos que aportan su opinión insesgada acerca de la decisión a tomar y que son contratados únicamente por su reputación como individuos independientes y no influenciados. Como supuesto, se asume que todos los votantes quieren coincidir en el voto con un mismo número de personas (una persona, la mayoría del grupo, etc.).

El segundo resultado de este estudio propone un mecanismo que indica el número de individuos a reemplazar por agentes independientes y que garantizaría la decisión que se obtiene cuando todos los agentes votan según su opción preferida para cualquier perfil de preferencias de los agentes. Este número de independientes depende del número total de agentes que forman parte del grupo, de la regla de la mayoría que se escoja para tomar la decisión y del número de individuos con el que los agentes quieren coincidir en el voto, que se asume que es el mismo para todos los individuos. Además, se analiza el caso particular en el que los individuos quieren votar lo mismo que la mayoría del grupo. En este caso, el mecanismo también funciona y te indica el número de agentes independientes que permite obtener la decisión que se elegiría si todos los votantes votaran de manera sincera.

Por último, se estudia el caso en que los agentes, en vez de tomar la decisión de manera simultánea, votan de manera secuencial. Se asume que todos los agentes tienen preferencias afectadas por el *conformity* y que cada uno de los agentes puede coincidir en el voto con el número de individuos que prefiera (se relaja el supuesto anterior en el que se asume que todos los agentes quieren coincidir con el mismo número de personas). En este caso, se obtiene que el fenómeno del *conformity* en las preferencias de los agentes no afecta a la decisión final, independientemente del número total de agentes que formen el grupo de votación y de la regla de la mayoría que se escoja. Este último resultado viene condicionado por el comportamiento pivotal de los agentes, que se define como la posibilidad que tiene un individuo de ser decisivo en el resultado de una elección a la hora de obtener uno u otro resultado. En un escenario secuencial, los agentes tienen más opciones de ser pivotal que en un escenario con votación simultánea, ya que anticipan el comportamiento pivotal de aquellos agentes que les siguen. Dado que el fenómeno del *conformity* es lexicográficamente inferior a la opción preferida por los individuos, éstos prefieren votar por su opción preferida. Esto no implica que no haya conformity, implica que ésta sólo se da una vez la decisión ya se ha tomado.

8.3.5 Conformity, información y votación sincera

Escrito junto con los Prof. Bernardo Moreno e Ismael Rodríguez-Lara (Middlesex University). En revisión en revista JCR.

En el Capítulo 6 se analiza el comportamiento de los votantes bajo la influencia del *conformity* a través de un experimento en un laboratorio, donde se inducen estas preferencias a los individuos.

Siguiendo el estudio anterior descrito en el Capítulo 5, los agentes tienen que tomar una decisión de manera simultánea habiendo dos posibles opciones, A o B. El grupo de agentes está compuesto por cinco agentes. Las preferencias de los agentes vienen determinadas de manera que tres de los agentes prefieren la opción B y los dos restantes prefieren la opción A. Primero, se estudia de manera teórica tres escenarios distintos en los que se considera información completa. En el primer escenario, los agentes sólo tienen en cuenta su opción preferida a la hora de votar. De acuerdo con el modelo teórico, el único equilibrio de Nash no dominado es aquel en el que todos los miembros del grupo votan por su opción preferida.

En el segundo escenario, se considera que todos los individuos tienen preferencias afectadas por el *conformity,* en el sentido que, los votantes quieren votar por su opción preferida salvo que no puedan influir en el resultado, caso en el que preferirán votar lo mismo que al menos otro agente. En este caso, aparte del equilibrio del primer escenario, se encuentran equilibrios de Nash en el que algunos agentes no votan de manera sincera.

En el tercer escenario, los agentes saben, antes de votar, que hay dos agentes cuya opción preferida es B, que están votando de manera sincera. En este caso, se mantiene el equilibrio de Nash no dominado en el que todos los agentes votan de manera sincera y se reducen los equilibrios en el que había individuos que no votaban por su opción preferida.

Para comprobar si los resultados teóricos presentados anteriormente para este caso pueden obtenerse empíricamente, se lleva a cabo un experimento en un laboratorio. Para inducir a los sujetos del experimento a tener las mismas preferencias que en la teoría, se recurre a darles incentivos monetarios. Se consideran dos tipos de agentes: A y B. Cada tipo recibe el máximo pago si la decisión final coincide con su tipo. En el primer escenario, los individuos reciben una cantidad extra independientemente de lo que voten. En el segundo escenario, donde se quiere inducir a los individuos preferencias afectadas por el *conformity*, los sujetos reciben un pago adicional si su decisión coincide con la decisión de algún agente dentro del grupo, independientemente de que la decisión final sea o no su opción favorita. En el tercer escenario, aparte de inducir a los individuos preferencias afectadas por el *conformity*, los votantes saben que dos agentes de tipo B son forzados a votar de manera sincera, es decir, a votar por B. Los pagos en este último escenario son similares a los del segundo escenario.

Para llevar a cabo el experimento, se reclutaron 390 sujetos, siendo todos estudiantes de Economía o Empresa de la Universidad de Valencia. En total se organizaron 13 sesiones con 30 sujetos cada una. Al comienzo de cada sesión, se le asignaba a cada sujeto un tipo, A o B, de manera aleatoria, que se mantenía a lo largo de toda la sesión. Todos los sujetos sabían que cada grupo estaba formado por 5 agentes, 3 de tipo B y 2 de tipo A. Tras haber sido informados de sus tipos, los sujetos tenían que votar por una u otra opción en 3 rondas distintas. En cada ronda, para aceptar la opción B, se les anunciaba el número de votos que se requiere para que la opción B sea la decisión final, habiendo tres posibilidades: mayoría simple (3 votos a favor de B), supermayoría (4 votos a favor de B) o unanimidad (5 votos a favor de B). Si la opción B no obtiene el apoyo necesario en la ronda, la decisión final es la opción A. El orden de las reglas de mayoría se seleccionaba de manera aleatoria de manera que todas las secuencias, si era 3, 4, 5; 4, 5, 3; etc.; tenían el mismo número de observaciones. En ningún momento, los sujetos recibían información acerca de cómo habían votado el resto de participantes en las rondas previas. Al final del experimento, una de las rondas era seleccionada de manera aleatoria para pagar a los individuos de acuerdo a cómo lo hubieran hecho en esa ronda. Los pagos a los sujetos dependían de si la decisión final en cada ronda coincidía con el tipo del agente y de si había votado lo mismo que otro sujeto dentro del grupo.

Las hipótesis que se quieren contrastar mediante el experimento son las siguientes:

Hipótesis 1: La presencia de *conformity* no afectará a la probabilidad de votar de manera sincera, siendo la probabilidad igual en el primer y segundo tratamiento.

Hipótesis 2: Con *conformity*, la presencia de dos agentes de tipo B forzados a votar de manera sincera no afectará a la probabilidad de los restantes tipo B a votar de manera sincera, siendo dicha probabilidad igual en el segundo y tercer tratamiento.

Hipótesis 3: La regla de la mayoría utilizada no afectará a la probabilidad de los agentes de votar de manera sincera en ninguno de los tratamientos.

Siguiendo los resultados teóricos, se encuentra que los agentes tienden a votar de manera sincera en el primer escenario en comparación con el escenario en el que se induce *conformity* (en términos medios, los tipos A pasan de 0.75 a 0.6 mientras que los tipo B pasan de 0.87 a 0.75). Asimismo, se encuentra una relación positiva entre la probabilidad de votar de manera sincera y la pivotalidad de los individuos. Para el segundo tratamiento se obtiene que los votantes de tipo A son más sinceros cuando se requiere unanimidad para obtener la opción B (si un votante de tipo A vota A, no se obtiene B) mientras que los votantes de tipo B son más sinceros con la regla de la mayoría (si un tipo B deja de votar por B, lo más seguro es que salga A). En el primer tratamiento sin inducir conformity, se obtiene que la regla de la mayoría no tiene ningún tipo de efecto en la probabilidad de votar de manera sincera de los individuos.

Los votantes también tienden a votar de manera más sincera cuando saben que hay dos agentes votando su opción preferida (en términos medios, los tipo A pasan a ser menos sinceros, de 0.60 a 0.52, excepto cuando se requiere unanimidad, mientras que los tipo B pasan a ser más sinceros, del 0.75 al 0.86 con respecto al segundo tratamiento). Además, se mantiene la relación que existe entre la probabilidad de ser sincero y ser pivotal, al encontrar que los tipo A son más sinceros conforme la regla de la mayoría requiere más votos a la opción B (pasar de mayoría simple a unanimidad), mientras que los tipo B son menos sinceros cuando se requiere unanimidad, probablemente como respuesta a que los tipo A van a ser pivotales y van a impedir que la opción B salga.

Para finalizar, se estudia si el hecho de inducir conformity afecta a obtener un resultado más o menos eficiente, analizando, para ello, el excedente total (la suma de los pagos obtenidos por todos los agentes en cada uno de los tratamientos y para cada regla de la mayoría). Se obtiene que sin conformity la probabilidad de maximizar los pagos totales es mayor y que esta probabilidad aumenta ante la presencia de los dos agentes tipo B forzados a votar por B. También se obtiene que, conforme la regla de la mayoría requiere más apoyo para la opción B, más se aleja de el resultado socialmente eficiente, que se maximiza cuando se obtiene la opción B.

8.4 Conclusiones generales

Esta tesis estudia teóricamente el comportamiento estratégico de dos partes fundamentales de la democracia: los partidos políticos y los votantes.

La parte destinada al estudio de los partidos políticos se centra en la competición política entre dos partidos políticos cuando uno de ellos representa a un partido tradicional que sólo quiere ganar las elecciones, mientras que el otro tiene ciertas preferencias por la ambigüedad.

En el primer escenario, el partido ambiguo actúa de partido asambleario. Se muestra que cuando la asamblea propone políticas más moderadas, se incrementan sus opciones de ganar las elecciones. Además, se obtiene que la plataforma donde se sitúe la asamblea hace que el partido tradicional se mueva por el espacio electoral, haciendo que se sitúe fuera de la posición del votante mediano cuando la asamblea sea centrista. Por tanto, se obtiene que el partido asambleario genera divergencia entre las plataformas políticas de los partidos, en contraste con el modelo Downsiano en el que los dos partidos tradicionales se sitúan en el votante mediano. La demanda por aplicar democracia directa dentro de los nuevos partidos políticos en su forma de elegir a los candidatos o las políticas a llevar a cabo son un tema interesante a desarrollar en un futuro estudio sobre los partidos asamblearios.

En el segundo escenario, el partido ambiguo viene representado por un partido que consiste en dos facciones internas: una ideológica y otra oportunista. Bajo un marco secuencial, se muestra que un partido con dos facciones necesita mucha ventaja en valence para ganar las elecciones, incluso en el caso de elegir la plataforma política ya conociendo qué va a hacer su oponente. A pesar de la multiplicidad de equilibrios, en este escenario tampoco se obtiene el equilibrio Downsiano ya que los partidos tienen incentivos a diferenciarse. Desde el punto de vista del votante, un partido con distintas facciones genera plataformas políticas más plurales. Sin embargo, desde el punto de vista de los partidos, un partido con varias facciones perjudica al partido en comparación con un partido tradicional. Una extensión de este modelo debe estudiar esta cuestión de manera simultánea, y considerando que las facciones puedan tener distinto grado de importancia dentro del partido.

En el tercer escenario analizado, ambos partidos tienen preferencia por la ambigüedad y por ganar las elecciones. En una situación en las que los incentivos por ganar las elecciones son muy altos, los partidos convergen a la política del votante mediano, sacrificando su preferencia por la ambigüedad. En caso de que los incentivos por ser ambiguos sean muy altos, no se obtiene equilibrio alguno, ya que los partidos tienen incentivos a desviarse a estrategias más ambiguas, aunque impliquen la pérdida de las elecciones. Por tanto, con votantes neutrales al riesgo y cuando la distribución de votantes es conocida, los partidos no pueden ganar las elecciones con una política ambigua. Una extensión de este modelo interesante de realizar es comprobar si se obtiene el mismo resultado cuando los partidos sí son ideológicos y desviarse de su ideología hacia una política más ambigua conllevase un coste.

Tras analizar el primer y el segundo escenario, se llega a la conclusión de que un partido tradicional se mueve por el espacio político cuando necesita diferenciarse de un partido ambiguo con el objetivo de atraer a votantes tanto del centro como de alguno de los lados del mediano. Este resultado se obtiene independientemente de si el partido asambleario tiene suficiente valence para ganar las elecciones, ya que influye sobre las estrategias del partido tradicional incluso cuando pierde las elecciones. De los tres escenarios analizados, se obtiene que la ambigüedad de un partido hace que ciertos votantes desarrollen un tipo de aversión al riesgo, aunque su utilidad sea definida como neutral. Esta aversión al riesgo viene del hecho de que un votante neutral al riesgo, cuando se sitúa en medio de las dos políticas del partido ambiguo, obtiene una mayor utilidad del partido tradicional que del asambleario cuando ambos ofrecen la misma política (en términos medios, para el partido asambleario). Por tanto, se puede concluir que la ambigüedad es un inconveniente para un partido que compite contra un partido tradicional incluso cuando los votantes son neutrales al riesgo. Una posible extensión de este estudio sería analizar cómo el valence que tiene un partido puede influir en las actitudes frente al riesgo de los votantes. ¿El hecho de que un partido sea generalmente considerado como populista hará más averso al riesgo a un votante, o hará más tolerantes al riesgo a los votantes cuando los partidos son más corruptos?

El segundo tema de estudio se centra en estudiar cómo la presencia de conformity puede alterar las decisiones de los votantes ante una situación de información completa. Cuando todos tienen preferencias afectadas por el conformity, se obtiene decisiones que no coincide con lo que se obtendría cuando todos los votantes votan de manera sincera. Para solventar este problema, se recurre a reemplazar a este tipo de agentes por votantes independientes, que sólo consideran su opinión cuando votan y siempre votan de manera sincera. Si la votación se desarrolla de manera simultánea, es posible hacer que el efecto del conformity desaparezca si se reemplaza a un número suficiente de agentes por independientes. En caso de que la votación se realice de manera secuencial, el conformity no afecta a la decisión obtenida. Este estudio es un primer paso para analizar una versión del modelo con información incompleta en el que, en vez de reemplazar agentes afectados por el conformity por agentes independientes, se añada al grupo de votación a agentes independientes, tal y como se hacen en los consejos de administración de las compañías hoy en día.

Para finalizar, los datos obtenidos en el experimento respaldan el hecho de que inducir conformity en las preferencias de los agentes por medio de un pago extra por coincidir con el voto de algún otro agente reduce la probabilidad de votar de manera sincera. Además, cuando los agentes saben que hay agentes independientes, esto les conlleva a votar de manera más sincera. Los resultados también muestran el efecto de la regla de votación sobre la manera de votar de los agentes, siendo menos sincero cuando menos pivotales son. Una vez se conoce los efectos del conformity en las preferencias de los agentes, sería interesante saber cómo la regla de votación y la pivotalidad de los agentes haga que los agentes voten por la decisión más preferida por el grupo cuando no se induce conformity en las preferencias de los agentes. No sólo la actuación de los sujetos en el experimento es importante, también sería muy útil entender cuáles son sus creencias sobre el comportamiento del resto de los sujetos durante la toma de decisiones de los agentes en este tipo de situaciones.

Tras analizar conformity tanto teórica como experimentalmente, se pueden llegar a varias conclusiones con respecto a la pivotalidad de los agentes, las reglas de votación y el comportamiento sincero de los agentes. Primero, se obtiene que el conformity influye el voto de los agentes cuando éstos no pueden ser pivotales, lo que ocurre cuando la votación es simultánea sin agentes independientes. Segundo, se puede llegar a la conclusión de que las reglas de votación unánimes son más efectivas para obtener la decisión que se obtendría cuando los agentes votan de manera sincera, ya que necesitan menos agentes independientes. Tras unanimidad, la mayoría simple necesitaría menos independientes que cualquier otra regla de votación. Tercero, el resultado experimental sugiere una interacción entre la regla de votación y la probabilidad de votar de manera sincera, ya que los agentes votan más sinceramente cuanto más favorable sea la regla de votación para la decisión que más les interese. Es decir, hay un efecto de la regla de votación en la votación sincera de los agentes sólo cuando éstos pueden ser pivotales. Una extensión de este estudio puede estar centrado en estudiar por qué los agentes no votan de manera sincera cuando hay información completa y las preferencias por conformity no están inducidas. ¿Puede un votante sacrificar su opción favorita cuando se da cuenta que la sociedad prefiere la otra opción, o deja de votar su opción favorita sólo por miedo a votar sólo o con una minoría?

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