On the solution of a contact problem for a rhombus weakened with a full-strength hole

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Abstract

This paper addresses a problem of plane elasticity theory for a doubly connected body whose external boundary is a rhombus with its diagonals lying at the coordinate axes OX and OY. The internal boundary is the required full-strength hole and the symmetric axes are the rhombus diagonals.

Absolutely smooth stamps with rectilinear bases are applied to the linear parts of the boundary, and the middle points of these stamps are under the action of concentrated forces, so there are no friction forces between the stamps and the elastic body. The hole boundary is free from external load and the tangential stresses are zero along the entire boundary of the rhombus. Using the methods of complex analysis, the analytical image of Kolosov-Muskhelishvili's complex potentials (characterizing an elastic equilibrium of the body), and the equation of an unknown part of the boundary are determined under the condition that the tangential normal stress arising at it takes the constant value. Such holes are called full-strength holes. Numerical analysis are performed and the corresponding graphs are constructed.

Keywords: plate elasticity theory, complex variable theory, stress state.

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1 Introduction

Mixed and contact problems belong to the category of problems of mechanics of deformed rigid bodies which are highly important for application and are the most difficult ones from the mathematical standpoint. It is through the mixed contact that loads are transferred to deformed bodies and it is in the contact zone that there occur stress concentrations that cause body failure, the emergence and spread of cracks. These phenomena cannot be prevented in the conditions of modern technological processes of production of mechanisms and machine parts.

Boundary-value problems of the plane theory of elasticity and plate bending for infinite plates weakened by unknown full-strength holes with normal stresses acting on their boundaries and forces applied at infinity were analyzed in [1, 2, 3].

Boundary-value problems for a finite doubly-connected domain with a part of its boundary being unknown full-strength and the other part being a polygonal line are solved in [4].

The axis-symmetric and cycle symmetric problems of the plane theory of elasticity and plate bending with partially unknown boundaries are studied in [5, 6, 7, 8, 9, 10, 11]. The most effective methods for studying these problems are the methods of the theory of analytical functions of a complex variable.

In this article, the axially symmetric problem of plane elasticity theory for a rhombus weakened with a full-strength hole is considered. Formulas of Kolosov-Muskhelishvili are used for investigating this problem. The solution is written in quadratures and the unknown full-strength hole of the plate is constructed

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2 Problem Formulation and Solution Technique

Let an isotropic elastic body on the plane $z = x + iy$ occupy a double connected domain S, whose external boundary is a rhombus with its diagonals lying at the coordinate axes OX and OY. The internal boundary is the required full-strength hole and the symmetric axes are the rhombus diagonals (Fig. 1).

Let to every link of the broken line (outer boundary of the given body) be applied absolutely smooth rigid stamps with rectilinear bases which displace to the normal under the action of concentrated normally compressive forces P applied to the stamp midpoints of polygon's sides. There is no friction between the given elastic body and stamps and the unknown full-strength contour is free from outer actions.

Under the above assumptions, the tangential stresses are zero $\tau_{ns} = 0$ along the entire boundary of the domain S and the normal displacements of every link of the external boundary $\nu_n = \nu$.

Consider the following problem: Find the shape of the unknown hole and the stress state of the given body such that the tangential normal stress σ_s arising at it would take constant value $\sigma_s = K = \text{const}$.

Since the problem is axially symmetric, then to investigate it, it is sufficient to consider the curvilinear quadrangle $A_1A_2A_3A_4$ denoted by D. The normal displacements and the tangential stresses are equal to zero $\nu_n = \tau_{ns} = 0$ at each segment $[A_1, A_2]$, $[A_3, A_4]$.

Let us introduce the following notations $\Gamma_1 = A_1A_2$, $\Gamma_2 = A_2A_3$, $\Gamma_3 = A_3A_4$, $\gamma = A_4A_1$, $\Gamma = \cup_{j=1}^{3}\Gamma_j$, $P_1 = \int_{\Gamma_1} \sigma_n\, ds$, $P_2 = \int_{\Gamma_2} \sigma_n\, ds$, $P_3 = \int_{\Gamma_3} \sigma_n\, ds$, σ_n is the normal stress. $P_2 = \int_{\Gamma_2} \sigma_n\, ds = -P$. Since the D is in the equilibrium state, then we have:

$$P_1 = P_2 \cos\beta = -P\cos\beta, \qquad P_3 = P_2 \sin\beta = -P\sin\beta$$

where $\beta = \angle A_1A_2A_3$.

From the statement of the problem it follows that the boundary conditions have the form

$$\nu_n = \begin{cases} 0, & t \in \Gamma_3 \cup \Gamma_1 \\ \nu, & t \in \Gamma_2 \end{cases} \tag{2.1}$$

$$\tau_{ns} = 0, t \in \Gamma \cup \gamma, \tag{2.2}$$

$$\sigma_n = 0, \quad \sigma_s = K, \quad t \in \gamma \tag{2.3}$$

$$P_2 = -P, \quad P_1 = -P\cos\beta, P_3 = -P\sin\beta \tag{2.4}$$

Let the points A_1, A_2, A_3, A_4 be counted in a positive direction and its affixes be denoted by the same symbols. Assume also that A_1 is the origin of the broken line Γ.

On the basis of the well-known Kolosov-Muskhelishvili's formulas [12], the problem reduces to finding the functions ψ, φ which are holomorphic in the domain D with the following conditions

$$\operatorname{Re} e^{-i\alpha(t)} \left(\chi\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)}\right) = 2\mu\nu_n(t), \qquad t \in \Gamma, \tag{2.5}$$

$$\operatorname{Re} e^{-i\alpha(t)} \left(\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)}\right) = C(t), \qquad t \in \Gamma, \tag{2.6}$$

$$\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)} = 0, \qquad t \in \gamma, \tag{2.7}$$

$$\operatorname{Re}\varphi'(t) = \frac{\sigma_n + \sigma_s}{4} = \frac{K}{4}, \qquad t \in \gamma \tag{2.8}$$

where χ, μ are elasticity constants, $C(t)$ is a piecewise-constant function, $\alpha(t)$ is the angle formed between the external normal n to contour and the abscissa axis Ox.

$$\alpha(t) = \alpha_k, \quad t \in \Gamma_k, \quad k = 1, 2, 3, \quad \alpha_1 = -\frac{\pi}{2}, \quad \alpha_2 = \frac{\pi}{2} - \beta, \quad \alpha_3 = \pi, \tag{2.9}$$

$$C(t) = \operatorname{Re}\left(e^{-i\alpha(t)} i \left(\int_{A_1}^{t} \sigma_n(s_0) e^{i\alpha(s_0)}\, ds_0\right)\right) \tag{2.10}$$

Taking into account (2.4) and (2.9), the (2.10) has following form:

$$C(t)=\begin{cases}0, & t\in\Gamma_1\\ -P\cos\beta\sin\beta, & t\in\Gamma_2\\ 0, & t\in\Gamma_3\end{cases} \tag{2.11}$$

Let $t\in A_kA_{k+1}$, $k=1,2,3$, then $t-A_k=i|t-A_k|e^{i\alpha_k}$. Hence we obtain

$$\operatorname{Re} te^{-i\alpha(t)}=\operatorname{Re} e^{-i\alpha(t)}A(t) \tag{2.12}$$

where $A(t)$ is a piecewise-constant function, $A(t)=A_k$, $t\in A_kA_{k+1}$, $k=1,2,3$. Taking into account (2.9) one obtains:

$$\operatorname{Re} e^{-i\alpha(t)}A(t)=\begin{cases}d_1\cos\alpha_2, & t\in\Gamma_2\\ 0, & t\in\Gamma_1\cup\Gamma_3\end{cases} \tag{2.13}$$

where $d_1=|0A_2|$.

Let functions $\varphi'(z)$, $\psi(z)$ be continuously extendable everywhere on the boundary of D domain, except perhaps the vertices of broken line Γ; in the neighborhood of A_k vertices the following condition holds

$$|\varphi'(z)|<M|z-A_k|^{-\delta_k},\quad |\psi(z)|<M|z-A_k|^{-\delta_k} \tag{2.14}$$

where $0\le\delta_k<1$, $k=2,3$; $0\le\delta_k<1/2$, $k=1,4$.

Combining (2.5) and (2.6) and then differentiating with respect to the arc abscissa s, and taking into account that $\alpha(t)$, $C(t)$, $\nu_n(t)$ are piecewise constant functions, one obtains

$$\operatorname{Im}\varphi'(t)=0,\qquad t\in\Gamma. \tag{2.15}$$

Equalities (2.8) and (2.15) are the Keldysh-Sedov problem for domain D:

$$\begin{aligned}\operatorname{Re}\left(\varphi'(t)-\frac{K}{4}\right)&=0, & t\in A_4A_1,\\ \operatorname{Im}\left(\varphi'(t)-\frac{K}{4}\right)&=0, & t\in\Gamma.\end{aligned} \tag{2.16}$$

Problem (2.16) has a unique solution [13] (see more details in [14])

$$\varphi'(z)=\frac{K}{4} \tag{2.17}$$

Hence one obtains

$$\varphi(z)=\frac{K}{4}z \tag{2.18}$$

where the constant was neglected.

Substituting the value $\varphi(t)$, $C(t)$ defined by formulas (2.18) and (2.11) into the boundary conditions (2.6)-(2.7), and taking into account (2.13) one gets the following problem

$$\operatorname{Re}\left[e^{-i\alpha(t)}\left(\frac{K}{2}t+\overline{\psi(t)}\right)\right]=C(t)=\begin{cases}0, & t\in\Gamma_1\\ -P\cos\beta\sin\beta, & t\in\Gamma_2\ ,\\ 0, & t\in\Gamma_3\end{cases} \tag{2.19}$$

$$\operatorname{Re} te^{-i\alpha(t)}=\operatorname{Re} e^{-i\alpha(t)}A(t),\qquad t\in\Gamma, \tag{2.20}$$

$$\frac{K}{2}t+\overline{\psi(t)}=0,\qquad t\in\gamma \tag{2.21}$$

Let the function $z=\omega(\zeta)$, $\zeta=\xi+i\eta$ maps the semicircle $|\zeta|<1$, $\operatorname{Im}\zeta>0$ conformally onto the domain D. It is assumed the vertices A_k of rhombus line correspond to the points a_k of the semicircle $|\zeta|=1$, $\operatorname{Im}\zeta>0$, $a_k=\omega^{-1}(A_k)$, $k=1,2,3,4$. It is assumed, that $a_1=1$, $a_4=-1$, $a_3=i$.

Here we can fix three points and the remaining ones are to be defined. Then the diameter $-1 \le \xi \le 1$ is mapped onto arc A_4A_1 and the semi-circumference γ_0: $|\gamma_0| = 1$, $\operatorname{Im}\zeta > 0$ is mapped onto the broken line Γ. The point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \pi/2$ is to be defined. Hence, by virtue of (2.19), (2.20), (2.21) for functions $\psi_0(\zeta) = \psi(\omega(\zeta))$ and $\omega(\zeta)$ one obtains:

$$\operatorname{Re} e^{-i\alpha(\sigma)}\overline{\psi_0(\sigma)} = -\frac{K}{2}\operatorname{Re} e^{-i\alpha(\sigma)}A(\sigma) + C(\sigma), \qquad \sigma \in \gamma_0, \tag{2.22}$$

$$\operatorname{Re} e^{-i\alpha(\sigma)}\omega(\sigma) = \operatorname{Re} e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0, \tag{2.23}$$

$$\frac{K}{2}\omega(\sigma) + \overline{\psi_0(\sigma)} = 0, \qquad \sigma \in (-1, 1), \tag{2.24}$$

$$\psi_0(\zeta) = \psi(\omega(\zeta)). \tag{2.25}$$

For simplicity, the piecewise constant functions $\alpha(\omega(\sigma))$, $A(\omega(\sigma))$, $C(\omega(\sigma))$ will again be denoted by $\alpha(\sigma)$, $A(\sigma)$, $C(\sigma)$.

Consider the new unknown function $W(\zeta)$ defined by

$$W(\zeta) = \begin{cases} \frac{K}{2}\omega(\zeta), & |\zeta| < 1, \quad \operatorname{Im}\zeta > 0 \\ -\overline{\psi_0(\bar\zeta)}, & |\zeta| < 1, \quad \operatorname{Im}\zeta < 0 \end{cases} \tag{2.26}$$

By virtue of (2.24) it easy to verify that $W(\zeta)$ is a holomorphic function into the circle $|\zeta| < 1$.

From (2.26) one obtains

$$W^+(\xi) = \frac{K}{2}\omega(\xi), \qquad W^-(\xi) = -\overline{\psi_0(\xi)}, \qquad -1 < \xi < 1, \tag{2.27}$$

$$W^+(\sigma) = \frac{K}{2}\omega(\sigma), \quad \sigma \in \gamma_0, \qquad W^-(\sigma) = -\overline{\psi_0(\bar\sigma)}, \quad \sigma \in \gamma_0^*. \tag{2.28}$$

The signs $(+)$ and $(-)$ refer to the upper and to the lower edges, respectively. γ_0^* is the reflection of γ_0 with respect to X-axis. By virtue of (2.18) and (2.20) one obtains

$$W^+(\xi) - W^-(\xi) = 0, \qquad -1 < \xi < 1.$$

that means that $W(\zeta)$ is a holomorphic function into the circle $|\zeta| < 1$.

By virtue of (2.22) - (2.23) the function $W(\zeta)$ defined by (2.26) satisfies the boundary conditions:

$$\operatorname{Re} e^{-i\alpha(\sigma)}W(\sigma) = \frac{K}{2}\operatorname{Re} e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0, \tag{2.29}$$

$$\operatorname{Re} e^{-i\alpha(\sigma)}W(\sigma) = -C(\sigma) + \frac{K}{2}\operatorname{Re} e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0^*, \tag{2.30}$$

Conditions (2.29) and (2.30) can be rewritten in the following form

$$\operatorname{Re} e^{-i\alpha(\sigma)}W(\sigma) = f(\sigma), \qquad \sigma \in \gamma' \tag{2.31}$$

where $\gamma' = \gamma_0 \cup \gamma_0^*$, $\alpha(\sigma) = \alpha(\bar\sigma)$, $\sigma \in \gamma_0^*$, $\alpha_1 = -\pi/2$, $\alpha_2 = \pi/2 - \beta$, $\alpha_3 = \pi$, $\beta = \angle A_1A_2A_3$.

If $\sigma \in \gamma_0$, then

$$f(\sigma) = \frac{K}{2}Ree^{-i\alpha(\sigma)}A(\sigma) = \begin{cases} \frac{K}{2}d_1\cos\alpha_2, & \sigma \in (a_2, a_3) \\ 0, & \sigma \in (a_1, a_2) \cup (a_3, a_4) \end{cases}, \qquad d_1 = |0A_2| \tag{2.32}$$

If $\sigma \in \gamma_0^*$, then

$$f(\sigma) = \frac{K}{2}\operatorname{Re} e^{-i\alpha(\sigma)}A(\sigma) - C(\sigma) = \begin{cases} 0, & \sigma \in (\bar a_2, a_1^*), \\ P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2, & \sigma \in (\bar a_3, \bar a_2) \\ 0, & \sigma \in (a_4, \bar a_3) \end{cases} \qquad a_1^* = e^{2\pi i} \tag{2.33}$$

Thus, the problem in question has been reduced to the Riemann-Hilbert problem with piecewise-constant coefficients. The solution of this problem was obtained in [15] (by reducing it to a linear-conjugation problem). Here the problem is reduced to the Dirichlet problem for a circle and its solution is presented by Schwarz formula, which is computationally convenient.

Function $e^{2i\alpha(\sigma)}$ is given by

$$e^{2i\alpha(\sigma)} = \frac{X(\sigma)}{\overline{X(\sigma)}}, |\sigma| = 1, \tag{2.34}$$

Where $X(\zeta)$ is presented, as

$$X(\zeta) = \frac{(\zeta - a_2)^{\frac{\alpha_1-\alpha_2}{\pi}+1}(\zeta - a_3)^{\frac{\alpha_2-\alpha_3}{\pi}+1}(\zeta - \bar{a}_3)^{\frac{\alpha_3-\alpha_2}{\pi}}(\zeta - \bar{a}_2)^{\frac{\alpha_2-\alpha_1}{\pi}}}{\zeta}\sqrt{\bar{a}_2\bar{a}_3} \tag{2.35}$$

By virtue of conditions (2.34), the condition (2.31) will have the form

$$\frac{W(\sigma)}{X(\sigma)} + \frac{\overline{W(\sigma)}}{\overline{X(\sigma)}} = \frac{2fe^{i\alpha(\sigma)}}{X(\sigma)}. \tag{2.36}$$

The condition (2.36) presents the boundary condition of Dirichlet problem, whose solution is presented by Schwartz formula

$$\frac{W(\zeta)}{X(\zeta)} = \frac{1}{2\pi i}\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}(\sigma+\zeta)\,d\sigma}{X(\sigma)\sigma(\sigma-\zeta)} \tag{2.37}$$

Since the function $X(\zeta)$ has a simple pole at the point $\zeta = 0$, then the function $W(\zeta)/X(\zeta)$ has the first order zero at this point $\zeta = 0$ and from (2.37) one holds:

$$\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}\,d\sigma}{X(\sigma)\sigma} = 0. \tag{2.38}$$

i.e.

$$\frac{K}{2}d_1\cos\alpha_2 e^{\alpha_2 i}\int_{a_2}^{a_3}\frac{d\sigma}{X(\sigma)\sigma} + \left(P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2\right)e^{\alpha_2 i}\int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma} = 0 \tag{2.39}$$

Thus we obtain equation with respect to two unknown parameters a_2, K.

We could choose some value of K and then determine the parameter a_2. In this case, the problem becomes more complicated. For the purpose of computations, it is more convenient to calculate K for each fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \pi/2$ and for the given P.

Taking into account (2.38), the formula (2.37) will have the form

$$W(\zeta) = \frac{\zeta X(\zeta)}{\pi i}\int_{\gamma'}\frac{f(\sigma)e^{i\alpha(\sigma)}\,d\sigma}{X(\sigma)\sigma(\sigma-\zeta)} \tag{2.40}$$

i.e.

$$W(\zeta) = \frac{\zeta X(\zeta)}{\pi i}\left(\frac{K}{2}d_1\cos\alpha_2 e^{\alpha_2 i}\int_{a_2}^{a_3}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\zeta)}\right.$$
$$\left. + \left(P\cos\beta\sin\beta + \frac{K}{2}d_1\cos\alpha_2\right)e^{\alpha_2 i}\int_{\bar{a}_3}^{\bar{a}_2}\frac{d\sigma}{X(\sigma)\sigma(\sigma-\zeta)}\right) \tag{2.41}$$

By virtue of (2.26), equation of the contour $z = \omega(\xi)$ is presented by

$$\omega(\xi) = \frac{2W(\xi)}{K}, \qquad -1 < \xi < 1. \tag{2.42}$$

From relationship (2.26), the functions ω and ψ are defined. Thus equation of the contour and stress state of body is defined.

Remark 1. *Let us introduce the following notations*

$$A = \int_{a_2}^{a_3} \frac{d\sigma}{X(\sigma)\sigma}, \qquad B = \int_{\bar{a}_3}^{\bar{a}_2} \frac{d\sigma}{X(\sigma)\sigma}$$

From equality (2.39) one obtains

$$K = -\frac{2PB\, ctg\, \beta}{d_1(A+B)} \tag{2.43}$$

The solution of problem exists if

$$A + B \neq 0 \tag{2.44}$$

3 Construction of the of rhombus full-strength hole

Let us consider some concrete cases. Let the length $|0A_2| = 1.5$, the value of angle $\beta = \angle A_1A_2A_3$ be changed for $0 < \beta < \pi/2$. The full-strength contours are defined for the different rhombuses which are obtained by changing the parameter β, $0 < \beta < \pi/2$.

To construct the required rhombus' full-strength hole, at first, the arc $\gamma = A_4A_1$ of the required full-strength contour is constructed. Having defined K by formula (2.43) for every fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \pi/2$, $a_1 = 1$, $a_4 = -1$, $a_3 = i$ and given P:

$$K = -\frac{2PB \operatorname{ctg} \beta}{d_1(A+B)}$$

the equation of the contour $z = \omega(\xi)$, is presented by formula (2.42):

$$\omega(\xi) = \frac{2W(\xi)}{K}, \qquad -1 < \xi < 1,$$

where the $W(\xi)$ is defined as (2.41):

$$W(\xi) = \frac{\xi X(\xi)}{\pi i} \left(\frac{K}{2} d_1 \cos \alpha_2 e^{\alpha_2 i} \int_{a_2}^{a_3} \frac{d\sigma}{X(\sigma)\sigma(\sigma - \xi)} + \left(P \cos\beta \sin\beta + \frac{K}{2} d_1 \cos\alpha_2 \right) e^{\alpha_2 i} \int_{\bar{a}_3}^{\bar{a}_2} \frac{d\sigma}{X(\sigma)\sigma(\sigma - \xi)} \right), \qquad -1 < \xi < 1.$$

for each pair of parameters (θ_2, K), $(a_2 = e^{i\theta_2})$ and the function $X(\zeta)$ is presented as (2.35):

$$X(\zeta) = \frac{(\zeta - a_2)^{\frac{\alpha_1 - \alpha_2}{\pi}+1} (\zeta - a_3)^{\frac{\alpha_2 - \alpha_3}{\pi}+1} (\zeta - \bar{a}_3)^{\frac{\alpha_3 - \alpha_2}{\pi}} (\zeta - \bar{a}_2)^{\frac{\alpha_2 - \alpha_1}{\pi}}}{\zeta} \sqrt{\bar{a}_2 \bar{a}_3}$$

where $\alpha_1 = -\pi/2$, $\alpha_2 = \pi/2 - \beta$, $\alpha_3 = \pi$.

Thus, the part A_4A_1 of the required full-strength contour is constructed by $z = \omega(\xi)$. Since the problem is axially symmetric, the other parts of this graphic are obtained by its axis symmetric mapping with respect to coordinate axes Ox and Oy.

Here as an illustration, some graphics of full-strength contours are presented for the following parameters (Figure 2).

For illustration we present some numerical calculations and plot for concrete case in Figure 3 where

$$x = -1.5, -1.4, \ldots, 1.5$$

$$g_0(x) = \begin{cases} f_2(x) & \text{if } -1.5 < x \le 0 \\ f_1(x) & \text{if } 0 \le x < 1.5 \\ 0 & \text{otherwise} \end{cases} \qquad \omega_1(\xi) = \frac{2W_1(\xi)}{K}$$

$$g_1(x) = \begin{cases} f_4(x) & \text{if } -1.5 < x \le 0 \\ f_5(x) & \text{if } 0 \le x < 1.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\xi_4 = -0.9, -0.8, \ldots, 0.9$$

$$W(\xi_4) = \omega_1(\xi_4) \qquad h(\xi_4) = \mathrm{Re}\,(W(\xi_4)) - v \cdot \mathrm{Im}\,(W(\xi_4))$$

$$u(\xi_4) = -\omega_1(\xi_4) \qquad g(\xi_4) = -\mathrm{Re}\,(W(\xi_4)) + v \cdot \mathrm{Im}\,(W(\xi_4))$$

Numerical results can be found in Table 1.

4 Conclusion

The shape of the contour of the required hole and the stress state of the given body are determined, provided that the tangential normal stress σ_s arising at contour of required hole takes the constant value. Full-strength contours are found by means of complex analysis. The considered problem with partially unknown boundaries is reduced to the known boundary value problem of the theory of analytic functions by means of the developed method. The solutions are presented in quadratures and full-strength contours are constructed.

The plates weakened by a hole with full-strength contours have the most strength and the least weight (in comparison with the other holes). It is proved in [2] that the weight of a plate weakened by hole with full-strength contour is less than 40% of that of a plate weakened by circular hole with the same strength.

Hence, finding an optimal shape is of great practical importance and the investigation of problems of plane elasticity with a partially unknown boundary are topical and of great practical and theoretical value.

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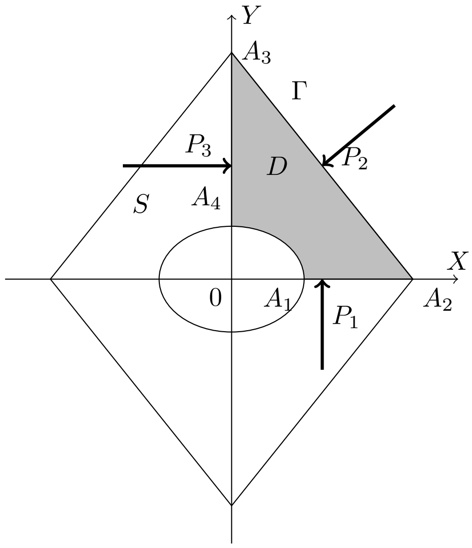

Figure 1: Graph of the posed problem

$u(\xi_4)$	$\omega_1(\xi_4)$	$h(\xi_4)$	$g(\xi_4)$
$-0.011 - 0.416i$	$0.011 + 0.416i$	$0.011 - 0.416i$	$-0.011 + 0.416i$
$-0.024 - 0.416i$	$0.024 + 0.416i$	$0.024 - 0.416i$	$-0.024 + 0.416i$
$-0.038 - 0.415i$	$0.038 + 0.415i$	$0.038 - 0.415i$	$-0.038 + 0.415i$
$-0.054 - 0.413i$	$0.054 + 0.413i$	$0.054 - 0.413i$	$-0.054 + 0.413i$
$-0.071 - 0.41i$	$0.071 + 0.41i$	$0.071 - 0.41i$	$-0.071 + 0.41i$
$-0.09 - 0.407i$	$0.09 + 0.407i$	$0.09 - 0.407i$	$-0.09 + 0.407i$
$-0.111 - 0.402i$	$0.111 + 0.402i$	$0.111 - 0.402i$	$-0.111 + 0.402i$
$-0.134 - 0.395i$	$0.134 + 0.395i$	$0.134 - 0.395i$	$-0.134 + 0.395i$
$-0.158 - 0.387i$	$0.158 + 0.387i$	$0.158 - 0.387i$	$-0.158 + 0.387i$
$-0.185 - 0.376i$	$0.185 + 0.376i$	$0.185 - 0.376i$	$-0.185 + 0.376i$
$-0.214 - 0.362i$	$0.214 + 0.362i$	$0.214 - 0.362i$	$-0.214 + 0.362i$
$-0.244 - 0.345i$	$0.244 + 0.345i$	$0.244 - 0.345i$	$-0.244 + 0.345i$
$-0.276 - 0.324i$	$0.276 + 0.324i$	$0.276 - 0.324i$	$-0.276 + 0.324i$
$-0.309 - 0.298i$	$0.309 + 0.298i$	$0.309 - 0.298i$	$-0.309 + 0.298i$
$-0.343 - 0.266i$	$0.343 + 0.266i$	$0.343 - 0.266i$	$-0.343 + 0.266i$
$-0.377 - 0.228i$	$0.377 + 0.228i$	$0.377 - 0.228i$	$-0.377 + 0.228i$

Table 1: Example for numerical calculations

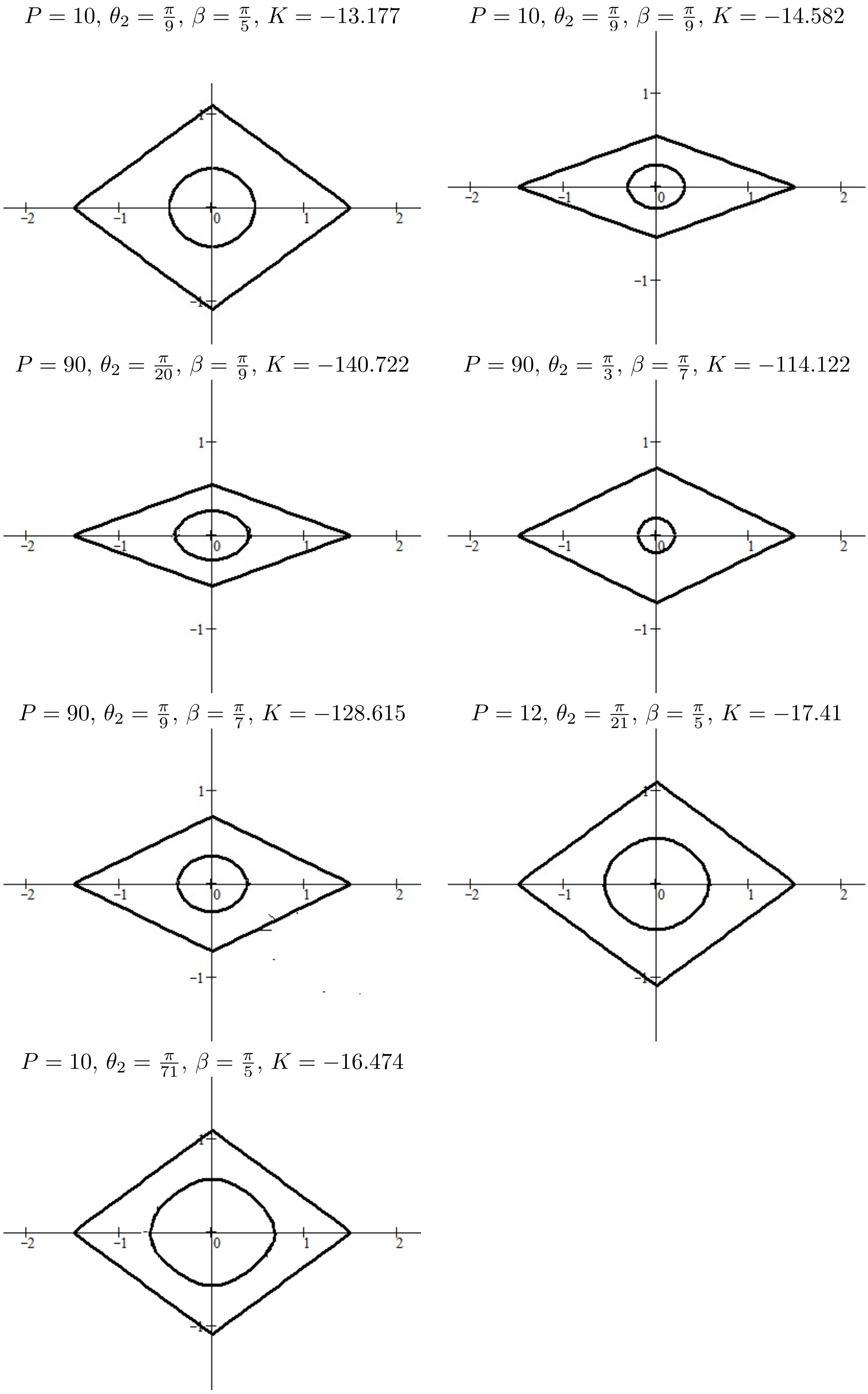

Figure 2: Graphics of full-strength contours for different parameters

$P = 10$, $\theta_2 = \frac{\pi}{9}$, $\beta = \frac{\pi}{5}$, $K = -13.177$

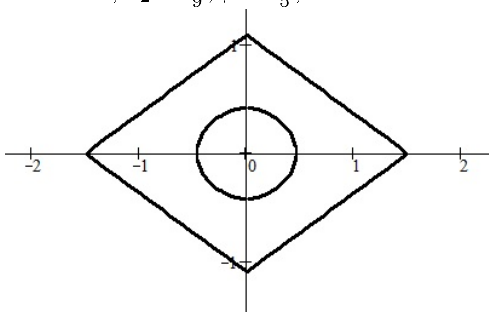

Figure 3: Example for numerical calculations