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DISTRIBUTED EXPLICIT KNOWLEDGE AND COLLECTIVE AWARENESS

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Abstract

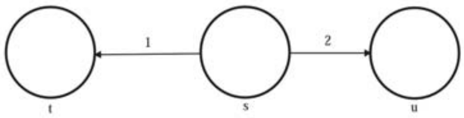
Our goal is to model communication in a group among agents with limited resources. Therefore we redefine the concept of distributed knowledge in order to distinguish between implicit and explicit knowledge. The most useful tool to do this is the concept of awareness, as a way of limiting and selecting what the agents really know. In this sense we propose a collective awareness that ensures full explicit communication and shows the dynamic aspects of the information exchange in a group. Depending on the definition of this collective awareness we are able to model the various ways in which the agents behave during communication.

1 Introduction

The standard definition of distributed knowledge (see [2]) intends to capture the knowledge flow between the agents in a group. The group is treated as another agent (the 'wise man') and acquires some knowledge that no individual agent on his own could posses. This new knowledge is derived from the information exchange between the agents in the group. Consider the standard epistemic language with the distributed knowledge operator D_G (where G is a set of agents). Let $\mathcal{M} = (S, R_1, \dots, R_n, V)$ be any Kripke model (more details below) and $\models$ be the usual satisfaction relation, defined on the model. Then we have that for any $s \in S$:

$$\mathcal{M}, s \models D_G\varphi \quad \text{iff} \quad \mathcal{M}, t \models \varphi, \text{for all } t \text{ such that } (s,t) \in \textstyle\bigcap_{i \in G} R_i \qquad (D1)$$

This definition gives rise to strange cases in which we can model a group knowledge that has been established without a justified information exchange between the agents in the group, we could call these type of situations 'mysterious knowledge'. Imagine a simple case: Agent 1 knows that the movie X is not shown at at 17:00h, agent 2 knows it, too; but both (as a group), after communication, know that the movie is shown at 17:00h (p). How is this possible? Consider the following model $\mathcal{M}_{\text{movie}} = (S, R_1, R_2, V)$, where:



- $S = \{s, t, u\}$; $R_1 = \{(s, s), (s, t)\}$; $R_2 = \{(s, s), (s, u)\}$; $V(q) = \emptyset$ for all atom q of the language.

We have that $\mathcal{M}, s \models D_G p$, i.e., the group G (where $G = \{1, 2\}$) knows p at s, according to the definition above, in $(D1)$ (since $\bigcap_{i \in G} R_i = \emptyset$). But p is no logical consequence of the combined knowledge of the agents at s, since this knowledge is consistent and $\neg p$ is part of it. This is a case of 'mysterious knowledge' where we cannot justify coherently how the group acquires their knowledge. In our example we use a frame with no special properties on the accessibility relations. In [5] the authors present another case of mysterious knowledge where the accessibility relations are equivalence relations.

To avoid this kind of situations, we can use a different distributed knowledge definition that considers the logical consequences of the knowledge of the group members, as in [3]. But, although this situation could be fixed, the information flow happens in an ideal context. The communication that is being modelled focuses on implicit knowledge and belief. It does not take into account agents with limited reasoning, real agents. Our goal is to approach the distributed knowledge of a group whose members have limited reasoning resources.

When a group of agents establishes communication they exchange different kinds of information: knowledge, beliefs, doubts, mistakes, etc. If we focus only on the knowledge, then it should be clear that this knowledge is always explicit. If the agents have limited reasoning resources then we need an appropriate tool that distinguishes between what is implicit or explicit. Fagin et al., in [2], refer to the awareness of the agent as a way of limiting their knowledge (see also [1]).

The information exchange between the agents in a group modifies each individual awareness. Therefore the explicit knowledge of the group, seen as the knowledge of the wise man, depends on the awareness of the group. We could then consider a 'collective awareness' that will be applied to the explicit knowledge of the group in the same way that the individual awareness is applied to the explicit knowledge of the agent. In other words, if we have $K_i^e\varphi \equiv K_i\varphi \wedge A_i\varphi$, where $K_i^e\varphi$ means 'agent i explicitly knows φ', $K_i\varphi$ means 'agent i implicitly knows φ' and $A_i\varphi$ means 'agent i is aware of φ'; we can establish in a natural way that for any group of agents G we have: $D_G^e\varphi \equiv D_G\varphi \wedge A_G\varphi$, where $D_G^e\varphi$ means 'φ is distributed explicit knowledge among the agents in G', $D_G\varphi$ means 'φ is distributed implicit knowledge among the agents in G' and $A_G\varphi$ means 'group G is aware of φ'.

Our aim is to present the minimum conditions under which we can define the collective awareness, that is, under which expressions such as 'group G is aware of

φ' make sense. We want to analyze the results of combining this notion with two different senses of D_G in order to describe the different concepts of explicit group knowledge: ($D1$) previously defined and ($D2$) presented later.

Group knowledge attempts to reflect how the knowledge is gained after an information exchange between the members of the group. This can be thought of as a new agent representing the group (the wise man) who possesses the information resulting from the exchange. Since the information being exchanged is explicit knowledge, the awareness of the wise man needs to be the result of the interaction of the individual awarenesses. In a natural way this knowledge exchange needs to have an impact on the collective awareness of the group.

On the formal account we can reduce the collective awareness to the intersection of all the information of the individual awarenesses (*pure awareness intersection*). Nevertheless, being less strict about this matter we can suppose that the collective awareness will contain *at least* this intersection (*awareness intersection*) (AI). This less strict version enables us to reflect the dynamic aspects of communication, the fact that after the information exchange the individual awarenesses acquire the new shared information. The way in which we represent this notion is by allowing the collective awareness of the wise man to include the additional information that results from communication and that does not belong to, or cannot be a result of, the awareness intersection.

On the other hand, the new information of the collective awareness, that does not belong to any individual, has its origin in the communication itself. Then we can state the *principle of limited collective awareness* (PLCA), according to which the content of the collective awareness can only be generated by the interaction of the information of the individual awarenesses.

In general, the agents can communicate everything they know or not, depending on the context. Thus, there will be some information that they will not communicate to the others, but which nevertheless belongs to their individual awareness. We can consider both these type of models and, furthermore, distinguish between those cases where all the information they communicate is knowledge (*full rational communication*) and where not necessarily all of it is knowledge (*partial rational communication*).

2 Epistemic logic with distributed knowledge and collective awareness

Consider a countable set of propositional letters $\mathcal{P}$ and a finite set of agents $\mathcal{A}g = \{1, \ldots, n\}$, the language $\mathcal{L}^{DA^c}$ of epistemic logic with distributed knowledge and collective awareness is given by the following definition:

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \rightarrow \varphi \mid K_i\varphi \mid K_i^e\varphi \mid D_G\varphi \mid D_G^e\varphi \mid A_i\varphi \mid A_G\varphi$

(where $p \in \mathcal{P}$, $i \in G \subseteq \mathcal{A}g$)

A frame is a tuple $\mathcal{F} = (S, R_1, \ldots, R_n, \mathcal{A}_1, \ldots, \mathcal{A}_n, \mathcal{A}_G)$, where:

1. S is a non-empty set of sates (also called 'worlds').
2. $R_i \subseteq S \times S$ for all $1 \leq i \leq n$. Each R_i is an accessibility relation for agent i.
3. $\mathcal{A}_i : S \longrightarrow 2^{\mathcal{L}^{DA^c}}$ for all $1 \leq i \leq n$.
4. $\mathcal{A}_G : S \longrightarrow 2^{\mathcal{L}^{DA^c}}$, where $\mathcal{A}_G$ satisfies: for any $s \in S$,

   (AI): $\bigcap_{i \in G} \mathcal{A}_i(s) \subseteq \mathcal{A}_G(s)$,

   (PLCA): $\mathcal{A}_G(s) \subseteq For(ATOM(\bigcup_{i \in G} \mathcal{A}_i(s)))$.

   where

   $For(ATOM(\bigcup_{i \in G} \mathcal{A}_i(s)))$ is the set of formulas generated by the atoms that appear in the formulas of $\bigcup_{i \in G} \mathcal{A}_i(s)$ (*the awareness set in s*).

Note that we did not specify any special properties for the individual awareness on item 3. It is wellknown [2] that $\mathcal{A}_i$ can have different properties (closed under subformulas, generated by a subset of primitive propositions, etc.). Analogous considerations can be expressed regarding collective awareness and depending on the properties of the individual awarenesses. Note also that the two general conditions of 'awareness intersection' and 'limited collective awareness', included on item 4, are the minimum intuitive requirements we impose on the collective awareness frames.

(AI) says that $\mathcal{A}_G(s)$ contains at least the intersection of individual awarenesses (before communication) and it can be expanded with new information (after communication). This new information represents the modifications of the individual awarenesses after communication. This mechanism works similarly to the D_G operator. This operator collects what the agents know after communication, but the model can only reflect, as a picture, what the agents know before communication. In this regard we are dealing with static models.

(PLCA) says that after communication, the collective awareness cannot have more information than the information contained by the set of formulas generated by the atoms of the awareness set. For instance, if no agent in the group has notice about the trigeminus, it is impossible that the information 'the trigeminus is a nerve' can appear in the collective awareness after communication.

A model is a tuple $\mathcal{M} = \{\mathcal{F}, V\}$, where $\mathcal{F}$ is a frame and V is a valuation function $V : \mathcal{P} \longrightarrow 2^S$ such that V associates every $p \in \mathcal{P}$ with a subset of S, intuitively the states in which p is true. In addition, a satisfaction relation $\models$ between models and formulas in $\mathcal{L}^{DA^c}$ can be defined. We write $\mathcal{M}, s \models \varphi$ to mean that the formula φ is true at (satisfied in) state s in $\mathcal{M}$ and it can be inductively defined as follows:

$\mathcal{M}, s \models p$ iff $s \in V(p)$ (for each $p \in \mathcal{P}$)
$\mathcal{M}, s \models \neg\varphi$ iff $\mathcal{M}, s \not\models \varphi$
$\mathcal{M}, s \models \varphi \wedge \psi$ iff $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, s \models \psi$
$\mathcal{M}, s \models \varphi \rightarrow \psi$ iff $\mathcal{M}, s \not\models \varphi$ or $\mathcal{M}, s \models \psi$
$\mathcal{M}, s \models K_i\varphi$ iff for all t such that $(s, t) \in R_i$: $\mathcal{M}, s \models \varphi$
$\mathcal{M}, s \models A_i\varphi$ iff $\varphi \in \mathcal{A}_i(s)$
$\mathcal{M}, s \models K_i^e\varphi$ iff $\mathcal{M}, s \models K_i\varphi$ and $\mathcal{M}, s \models A_i\varphi$
$\mathcal{M}, s \models A_G\varphi$ iff $\varphi \in \mathcal{A}_G(s)$

We can extend the satisfaction relation with both the following alternatives for distributed knowledge, $(D1)$ introduced before and $(D2)$ below (see [3]):

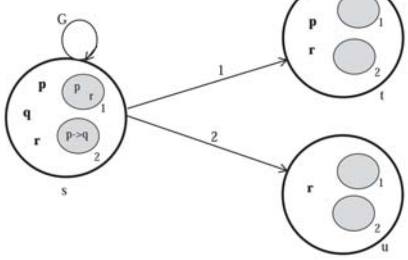
$\mathcal{M}, s \models D_G\varphi$ iff $\mathcal{M}, t \models \varphi$, for all t such that $(s, t) \in \bigcap_{i \in G} R_i$ (D1)
$\mathcal{M}, s \models D_G\varphi$ iff $\{\psi \in \mathcal{L}^{A^c} \mid \mathcal{M}, s \models K_i\psi$ for some $i \in G\} \Vdash \varphi$ (D2)

We also have:

$\mathcal{M}, s \models D_G^e\varphi$ iff $\mathcal{M}, s \models D_G\varphi$ and $\mathcal{M}, s \models A_G\varphi$

Note that in $(D2)$ we use $\mathcal{L}^{A^c}$, i.e., the language resulting from $\mathcal{L}^{DA^c}$ by dropping D_G and D_G^e, avoiding this way circularity in the definition, as pointed out in [3]. On the other hand, the symbol $\Vdash$ is a relation of *logical consequence* between a set of formulas and one formula. In general, $\Phi \Vdash \varphi$ means that for every model $\mathcal{M}$ and every state s in $\mathcal{M}$, if all formulas in Φ are satisfied in s, φ is also satisfied in s. In addition, the notions of *satisfiability* and *validity* are defined as usual.

Example 1. *In the following model* $\mathcal{M} = (S, R_1, R_2, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_G, V)$ *we take* s *as the actual state and* $G = \{1, 2\}$. *We define the model only attending to atoms* p, q, r*:*



- $S = \{s, t, u\}$.
- $R_1 = \{(s, s), (s, t)\}$; $R_2 = \{(s, s), (s, u)\}$.
- $\mathcal{A}_1(s) = \{p, r\}$; $\mathcal{A}_2(s) = \{p \rightarrow q\}$*;* $\mathcal{A}_1(t) = \mathcal{A}_2(t) = \mathcal{A}_1(u) = \mathcal{A}_2(u) = \emptyset$.
- $\mathcal{A}_G(s)$ *is defined below in different ways;* $\mathcal{A}_G(t) = \mathcal{A}_G(u) = \emptyset$.

- $V(p) = \{s, t\}$; $V(q) = \{s\}$; $V(r) = S$.

Before communication we have:

- $\mathcal{M}, s \models K_1 p \qquad \mathcal{M}, s \models K_1 r \qquad \mathcal{M}, s \models K_2(p \rightarrow q)$

The agents can interchange information. And what happens after communication? We can contemplate several possibilities. In all of them we have the same result using (D1) or (D2):

1. • $\mathcal{A}_G(s) = \bigcap_{i \in G} \mathcal{A}_i(s) = \varnothing$ *(there is no communication at all)*
   The distributed implicit knowledge is infinite ($\mathcal{M}, s \models D_G p, \mathcal{M}, s \models D_G r, \ldots$), and the distributed explicit knowledge does not increase (it was empty and remains empty).
2. • $\mathcal{A}_G(s) = \{r\}$
   1 does not speak about all his knowledge. 2 does not speak at all. We will focus on the distributed explicit knowledge. As a consequence: $\mathcal{M}, s \models D^e_G r$ (only!).
3. • $\mathcal{A}_G(s) = \{p, q, r, p \rightarrow q\}$
   1 speaks about all his knowledge. 2 speaks about all his knowledge. In particular they can conclude q, since $\mathcal{M}, s \models D_G q$ and, as $q \in \mathcal{A}(s)$, we obtain $\mathcal{M}, s \models D^e_G q$.
4. • $\mathcal{A}_G(s) = \{p, p \rightarrow q\}$
   1 speaks about part of his knowledge. 2 speaks about all his knowledge. Although they communicate p and $p \rightarrow q$ they cannot conclude q despite all (they may lack Modus Ponens). Indeed, though $\mathcal{M}, s \models D_G q$ we have $\mathcal{M}, s \not\models D^e_G q$, since $q \notin \mathcal{A}(s)$.

3 Models with rational information flow

We say that there is 'rational information flow' in a group whenever the collective awareness of a group acquires knowledge from the individual agents after communication. We can define two versions of rational information flow: (*i*) all the acquired information needs to be knowledge (strong version), or (*ii*) at least part of the acquired information is knowledge (weak version). We can also specify two ways in which the information flows: either all explicit knowledge is acquired or only part of it.

We are interested in collecting classes of structures with rational communication flow and in defining the concept of explicit distributed knowledge in relation to this property. Note that the minimum conditions (AI) and (PLCA) do not commit themselves to neither of these versions. There is also no guarantee that those intersections cannot be empty. However, if there is a real knowledge exchange between the agents in the group those sets can never be empty. The rational communication flow models are of interest because they ensure fruitful knowledge exchange, where the agents have really learned new information.

In what follows we will use the following notation: We will call $KS_G(\mathcal{M}, s)$ and $KS^e_G(\mathcal{M}, s)$ respectively *implicit knowledge set* and *explicit knowledge set* of a group of agents G in a state s of a model $\mathcal{M}$, defined as follows:

$$KS_G(\mathcal{M}, s) = \{\psi \in \mathcal{L}^{DA^c} \mid \mathcal{M}, s \models K_i\psi \text{ for some } i \in G\}$$

$$KS^e_G(\mathcal{M}, s) = \{\psi \in \mathcal{L}^{DA^c} \mid \mathcal{M}, s \models K^e_i\psi \text{ for some } i \in G\}$$

Consider the following four possibilities for A_G that reflect different forms of communication:

$$\mathcal{A}_G(s) = \textstyle\bigcap_{i\in G} \mathcal{A}_i(s) \qquad (A1)$$

$$\mathcal{A}_G(s) \subseteq \{\psi \in \mathcal{L}^{DA^c} \mid KS^e_G(\mathcal{M}, s) \Vdash \psi\} \qquad (A2)$$

$$\{\psi \in \mathcal{L}^{DA^c} \mid KS^e_G(\mathcal{M}, s) \Vdash \psi\} \subseteq \mathcal{A}_G(s) \qquad (A3)$$

$$\{\psi \in \mathcal{L}^{DA^c} \mid KS^e_G(\mathcal{M}, s) \Vdash \psi\} \cap \mathcal{A}_G(s) \neq \emptyset \qquad (A4)$$

Combining these notions of $\mathcal{A}_G$ with $(D1)$ and $(D2)$ we are able to model many ways of knowledge transfer between the agents.

If we assume $(A1)$, then two different things may happen: either there is no communication at all, or everything the agents communicate is already known by them. If we assume $(A2)$, $(A3)$ or $(A4)$ there can be information that the group explicitly knows without the need that any of their members do. The group acquires this knowledge after deriving it from the knowledge of their members. On the other hand, regarding the specific case of $(A2)$, the collective awareness is only 'rational'; that is, it only contains the logical consequences of the explicit knowledge of their members.

In the case of $(A3)$ and $(A4)$ the collective awareness has a 'rational core', $\{\psi \in \mathcal{L}^{DA^c} \mid KS^e_G(\mathcal{M}, s) \Vdash \psi\}$, standing for the information that can be derived from the explicit knowledge set. Regarding $(A3)$ the agents communicate all their knowledge. But the collective awareness is not necessarily reduced to its rational core. This strikes us more intuitive since the awareness can contain inconsistent information. Assuming $(A4)$, there can be members of the group that do not communicate all their knowledge. This has a direct impact on the explicit knowledge of the group which does not contain everything its members really know.

4 Classes of models and full explicit communication

The *Principle of Full Communication* establishes that whenever φ is considered group knowledge, it should be possible for the members of the group to establish φ through communication. It is argued by Van der Hoek et al., in [5], that group knowledge should comply with this principle. They formulate it as follows:

$$\mathcal{M}, s \models D_G\varphi \text{ implies } KS_G(\mathcal{M}, s) \Vdash \varphi$$

The authors use the language $\mathcal{L}^D$ (resulting from $\mathcal{L}^{DA^c}$ by dropping the explicit epistemic and awareness operators). A dissertation about the class of models that comply with this principle using (D1) can be found in [4]. Since we want to deal with real agents whose reasoning resources are limited, the knowledge that is being established through communication needs to be explicit. Hence we can establish the principle of *full explicit communication*:

$$\mathcal{M}, s \models D^e_G\varphi \text{ implies } KS^e_G(\mathcal{M}, s) \Vdash \varphi$$

Full explicit communication can be studied combining the definitions of distributed implicit knowledge, $(D1)$ or $(D2)$, and the conditions on the collective awareness, $(A1)$-$(A4)$ above. The result of this combination is given by the following propositions:

Proposition 1. *In the following classes of models distributed explicit knowledge does not comply with the principle of full explicit communication:*

1. *The class of models that satisfies* $(D1)$ *and either* $(A1)$ *or* $(A3)$ *or* $(A4)$.
2. *The class of models that satisfies* $(D2)$ *and either* $(A1)$ *or* $(A3)$ *or* $(A4)$.

Proof. We will prove in item 2 the case $(D2)$ and $(A4)$. Let $G = \{1, 2\}$ and consider the model $(S, R_1, R_2, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_G, V)$, where:
$S = \{s\}$; $R_1 = \{(s, s)\}$; $R_2 = \emptyset$; $\mathcal{A}_1(s) = \{p, q\}$; $\mathcal{A}_2(s) = \emptyset$; $V(p) = \{s\}$; $V(\varphi) = \emptyset$ for all atom φ in $\mathcal{P}$ distinct of p. Assume also that $\mathcal{A}_G(s) = \{p, q\}$ which satisfies (A4), because $KS^e_G(\mathcal{M}, s) = \{p\}$. Now, we have that $\mathcal{M}, s \models D_G q$ (since $\mathcal{M}, s \models K_2 q$) and as $q \in \mathcal{A}_G(s)$, we obtain $\mathcal{M}, s \models D^e_G q$, but $\{p\} = KS^e_G(\mathcal{M}, s) \nVdash q$. □

The following result is immediate:

Proposition 2. *In the following classes of models distributed knowledge complies with the principle of full explicit communication:*

1. *The class of models that satisfies* $(D1)$ *and* $(A2)$.
2. *The class of models that satisfies* $(D2)$ *and* $(A2)$.

5 Conclusions and future work

We have seen that collective awareness is an adequate concept for modeling communication with information exchange in a group of agents. This notion reflects, in a static way, the dynamics of communication allowing changes and integrating new information. But there are still many unexplored areas in this field, such as: (*i*) Exploring more classes of models that comply with the principle of full explicit communication and defining formal systems to deal with this concept syntactically. (*ii*) Redefining explicit distributed knowledge specifying the type of information that collective awareness can contain. (*iii*) Analyzing the concepts of distributed explicit knowledge and collective awareness from the perspective of Dynamic Epistemic Logic (DEL) and Public Announcement Logic (PAL).

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