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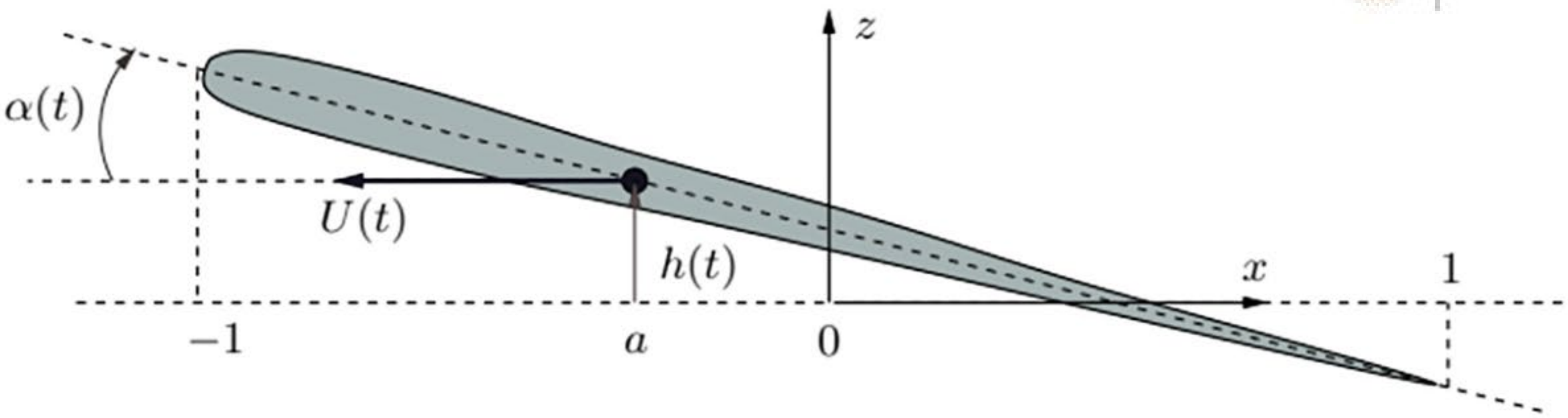

Unsteady propulsion of a two-dimensional flapping thin airfoil in a periodic stream

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- General formulation
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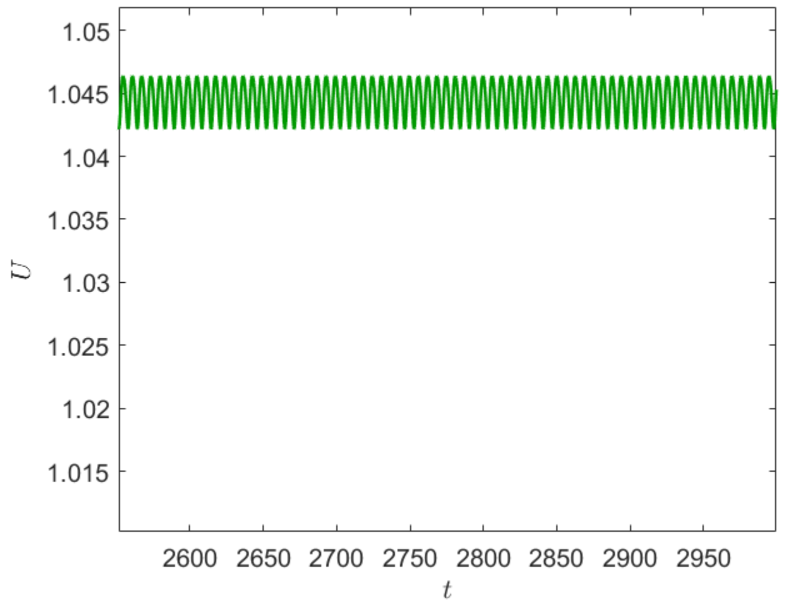
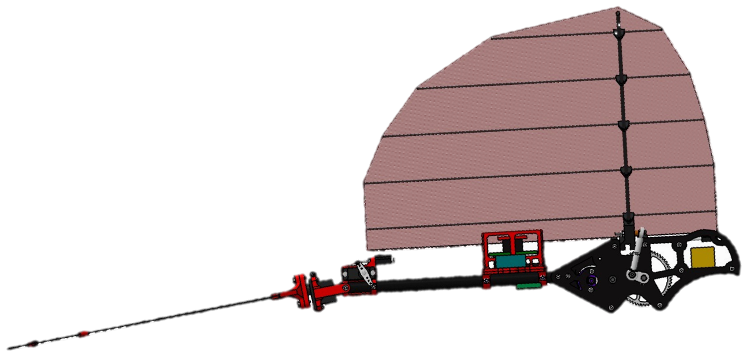
Problem definition: introduction

- Animals with flapping wings or fins to propel have a cruising velocity which oscillates around a mean value
- Experimental studies: focused on dynamic stall
- Formulation of thrust of a heaving and pitching airfoil under an oscillating airstream
- Generalization of lift from Greenberg [1]





Problem definition: motion

- Non–dimensional formulation, scaled with $U_s,\ c/2$
- Oscillation of the airfoil:

$$h(t) = h_0 \Re\left[e^{ikt}\right], \qquad \alpha(t) = \alpha_s + \alpha_0 \Re\left[e^{i(kt+\phi)}\right]$$

- Oscillating airstream: $U(t) = 1 + \sigma \Re\left[e^{i(k_1 t+\phi_1)}\right]$
- Reduced frequencies: $k = \frac{\omega c}{2U_s}$ and $k_1 = \frac{\omega_1 c}{2U_s}$
- Vertical displacement: $z_s(x,t) = h(t) - (x-a)\alpha(t)$
- Vertical velocity: $v_0(x,t) = -U(t)\alpha(t) + \dot{h}(t) - (x-a)\dot{\alpha}(t)$

Formulation: vortical impulse theory

- Aerodynamic forces defined by the vortical impulse theory:

$$C_L = \frac{L}{\frac{1}{2}\rho U_s^2 c} = -\frac{d}{dt}\left[\int_{-1}^{1} x\varpi_s dx + \int_{1}^{\infty} x\varpi_e dx\right]$$

$$C_T = \frac{T}{\frac{1}{2}\rho U_s^2 c} = -\frac{d}{dt}\left[\int_{-1}^{1} z_s\varpi_s dx + \int_{1}^{\infty} z_e\varpi_e dx\right]$$

$$C_M = \frac{M}{\frac{1}{2}\rho U_s^2 c^2} = \frac{1}{4}\frac{d}{dt}\left[\int_{-1}^{1} (x-a)^2\varpi_s dx + \int_{1}^{\infty} (x-a)^2\varpi_e dx\right]$$

Formulation: vorticity

- Contribution from the vortex–sheet wave separated:

$$\varpi_s(x,t) = \varpi_0(x,t) + \varpi_1(x,t), \qquad \Gamma_0(t) = \int_{-1}^{1} \varpi_0(x,t)dx$$

- Wake convected downstream:

$$\varpi_e(\xi,t) = \varpi_e(X), \qquad z_e(\xi,t) = z_e(X), \qquad X = \xi - \int_{t_i}^{t} U(t)dt$$

- For any function $f(\xi)$ satisfying $f(1) = 0$ (valid for the present case where U depends on time):

$$\frac{d}{dt}\int_1^{\infty} f(\xi)\varpi_e(X)d\xi = U(t)\int_1^{\infty} \frac{df(\xi)}{d\xi}\varpi_e(X)d\xi$$

Formulation: aerodynamic forces

- Lift force: $C_L = C_{L0} + C_{L1} + C_{L2}$

$$C_{L0} = U\Gamma_0, C_{L1} = \pi\left(\dot{U}\alpha + U\dot{\alpha} - \ddot{h} - a\ddot{\alpha}\right), C_{L2} = U\int_1^\infty \frac{\varpi_e(\xi,t)}{\sqrt{\xi^2-1}}d\xi$$

- Moment: $C_M = C_{M0} + C_{M1} + C_{M2} + \frac{a}{2}C_L$

$$C_{M0} = \frac{\pi}{2}U\left(U\alpha - \dot{h} - a\dot{\alpha}\right), \qquad C_{M1} = \frac{\pi\ddot{\alpha}}{16}, \qquad C_{M2} = -\frac{1}{4}C_{L2}$$

- Thrust: $C_T = -\alpha C_L + C_{T1} + C_{T2}$

$$C_{T1} = \pi\dot{\alpha}\left(\dot{h} + a\dot{\alpha} - U\alpha\right), C_{T2} = \int_1^\infty \left[\dot{h} + a\dot{\alpha} - \alpha U + \dot{\alpha}\left(\sqrt{\xi^2-1} - \xi\right)\right]\varpi_e d\xi$$

Harmonic motion: Circulation

- Circulation

$$\Gamma_0 = 2\pi\Re\left[\alpha_s + \left(-ikh_0 + \alpha_0 e^{i\phi} - \left(a - \frac{1}{2}\right)ik\alpha_0 e^{i\phi}\right)e^{ikt} + \alpha_s\sigma e^{i(\phi_1+k_1 t)} + \frac{1}{2}\alpha_0\sigma\left(e^{i(\phi_2+k_2 t)} + e^{i(\phi_3+k_3 t)}\right)\right]$$

- Wake vorticity, depends on

$$X = \xi - \int_{t_i}^{t} U(t)dt = \xi - t - \frac{\sigma}{k_1}\sin(k_1 t + \phi_1)$$

- Following Greenberg [1] we assume $\dfrac{\sigma}{k_1} = \dfrac{2U_\sigma}{\omega_1 c} \equiv k_\sigma^{-1} \ll 1$

- Wake vorticity

$$\varpi_e(\xi, t) = -2\pi\alpha_s\delta(\xi - \infty) + ge^{ik(t-\xi)} + ge^{ik_1(t-\xi)} + ge^{ik_2(t-\xi)} + ge^{ik_3(t-\xi)}$$

Harmonic motion: forces

- Lift:

$$C_L(t) = \pi\big(\dot{U}\alpha + U\dot{\alpha} - \ddot{h} - a\ddot{\alpha}\big) + 2\pi\alpha_s + U\Re\big[G_0\mathcal{C}(k)e^{ikt} + G_{01}\mathcal{C}(k_1)e^{ik_1t} + G_{02}\mathcal{C}(k_2)e^{ik_2t} + G_0\mathcal{C}(k_3)e^{ik_3t}\big]$$

- Moment

$$C_M(t) = \frac{\pi}{2}\left[a\dot{U}\alpha + \left(a - \frac{1}{2}\right)U\dot{\alpha} - \left(\frac{1}{8} + a^2\right)\ddot{\alpha} - a\ddot{h}\right] + \pi\left(a + \frac{1}{2}\right)U\alpha_s$$
$$+\left(a + \frac{1}{2}\right)\frac{U}{2}\Re\big[G_0\mathcal{C}(k)e^{ikt} + G_{01}\mathcal{C}(k_1)e^{ik_1t} + G_{02}\mathcal{C}(k_2)e^{ik_2t} + G_0\mathcal{C}(k_3)e^{ik_3t}\big]$$

- Thrust

$$C_T = -\alpha C_L + \pi\dot{\alpha}\big(\dot{h} + a\dot{\alpha} - U\alpha\big) - \big(\dot{h} + a\dot{\alpha} - U\alpha\big)2\pi\alpha_s$$
$$-\big(\dot{h} + a\dot{\alpha} - U\alpha\big)\Re\left[\frac{2i}{\pi}G_0\mathcal{C}_1(k)e^{ikt} + \frac{2i}{\pi}G_{01}\mathcal{C}_1(k_1)e^{ik_1t} + \frac{2i}{\pi}G_{02}\mathcal{C}_1(k_2)e^{ik_2t} + \frac{2i}{\pi}G_{03}\mathcal{C}_1(k_3)e^{ik_3t}\right]$$
$$-\dot{\alpha}\Re\left[G_0\left(-\frac{2}{\pi k}(1 + ik)\mathcal{C}_1(k) - \frac{i}{k}\mathcal{C}(k)\right)e^{ikt} + G_{01}\left(-\frac{2}{\pi k_1}(1 + ik_1)\mathcal{C}_1(k_1) - \frac{i}{k_1}\mathcal{C}(k_1)\right)e^{ik_1t}\right.$$

Harmonic motion: average coefficients

- Lift:

$$\bar{C}_L = \alpha_s \left(2\pi + \frac{\sigma^2 \mathcal{F}(k_1)}{2}\right)$$

- Moment:

$$\bar{C}_M = \frac{\alpha_s}{2}\left(a + \frac{1}{2}\right)\left(2\pi + \frac{\sigma^2 \mathcal{F}(k_1)}{2}\right)$$

- Thrust

$$\begin{aligned}\bar{C}_T = &-2(kh_0)^2\mathcal{G}_1(k) + kh_0\alpha_0[-2\mathcal{F}_1(k)\cos(\phi) + k\mathcal{G}_1(k)\cos(\phi)\,(3-4a) + 2\mathcal{G}_1(k)\sin(\phi) - k\mathcal{F}_1(k)\sin(\phi)] \\ &+\alpha_0^2\left[2k\mathcal{F}_1(k)(1-a) + 2k^2\mathcal{G}_1(k)(1-a)\left(a - \frac{1}{2}\right)\right] - (\sigma\alpha_s)^2[\pi\mathcal{F}(k_1) + 2\mathcal{G}_1(k_1)] \\ &- (\sigma\alpha_0)^2[\pi\mathcal{F}(k_2) + 2\mathcal{G}_1(k_2) + \pi\mathcal{F}(k_3) + 2\mathcal{G}_1(k_3)]\end{aligned}$$

Particular cases

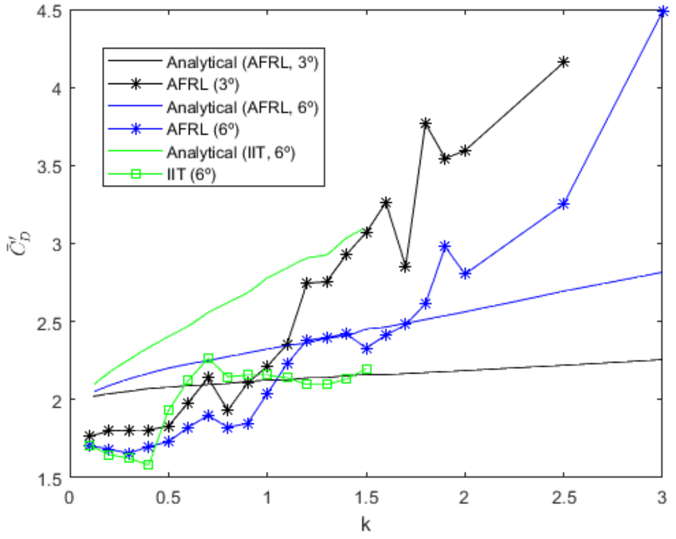
- Case $k = k_1$: $k_3 = 0$, new constant term in the circulation Γ_0
- Case $k = 2k_1$: new constant terms for all force coefficients
- Case $2k = k_1$: new constant terms for the thrust coefficient
- Case $k = 0$:

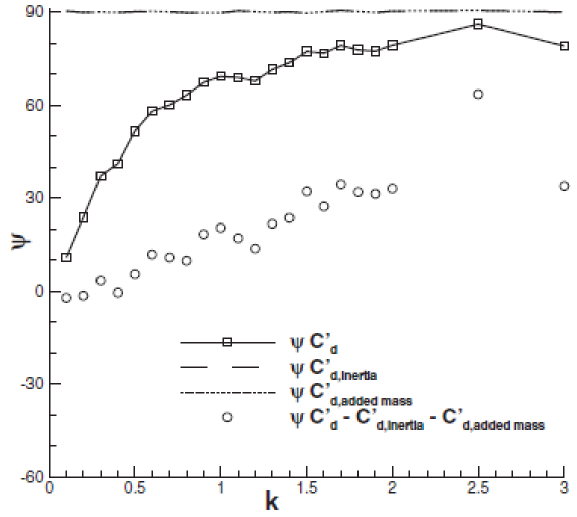
$$C_L = 2\pi\alpha_s + \pi\sigma^2\alpha_s F(k) + \pi\sigma\alpha_s\left(2\left(1 + F(k_1)\right)\cos(k_1 t) - \left(k + 2G(k_1)\right)\sin(k_1 t)\right)$$
$$+\pi\sigma^2\alpha_s(F(k_1)\cos(2k_1 t) - G(k_1)\sin(2k_1 t))$$

$$C_T = -\sigma^2\alpha_s^2\left(2G_1(k_1) + \pi F(k_1)\right)$$
$$+\sigma\alpha_s^2\left(\left(-4G_1(k_1) - 2\pi F(k_1)\right)\cos(k_1 t) + \left(\pi k + 2\pi G(k_1) - 4F_1(k_1)\right)\sin(k_1 t)\right)$$
$$+\sigma^2\alpha_s^2\left(\left(-2G_1(k_1) - \pi F(k_1)\right)\cos(2k_1 t) + \left(\pi G(k_1) - 2F_1(k_1)\right)\sin(2k_1 t)\right)$$

Validation approach

- Comparison with experimental results from the literature
- Problem: considering friction drag
- Different cases:
  - $k = 0$ (Granlund, [4])
  - Pitching
  - Heaving
- Comparison with numerical studies





Summary

- Description of the forces obtained
- Generalization of Greenberg [1]
- Obtention of thrust of an airfoil in a pulsating flow
- Small effects on average values
- Considerable effects on the oscillations
- Validation with experimental and numerical results

Cites

[1] J. M. Greenberg. Airfoil in sinusoidal motion in a pulsating stream. Technical Report TR 1326, NACA, 1947.

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[3] R. Fernandez-Feria. Linearized propulsion theory of flapping airfoils revisited. Phys. Rev. Fluids, 1:084502, 2016.

[4] K. Granlund, B. Monnier, M. Ol, and D. Williams. Airfoil longitudinal gust response in separated vs. attached flows. *Physics of Fluids*, 26(2):027103, 2014.

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