

Some observations on short fatigue cracks under biaxial fatigue

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ABSTRACT.

This work presents a new methodology for evaluating crack initiation under biaxial conditions. The methodology consists of evaluating a number of crack parameters automatically with digital processing of high-magnification images of the crack. Five different strain conditions were evaluated on a low carbon ferritic-pearlitic steel specimen with tubular shape. A hole of 150 μm diameter was drilled to enforce the crack to initiate at a particular spot. Different combinations of axial and torsional strains were analysed during the initiation stage of the crack. Fatigue crack propagation curves clearly showed oscillations due to microstructure. It was also observed that these oscillations decreased as the torsional component of the strain was increased. Driving force was also evaluated in crack opening and sliding direction through COD and CSD. The results demonstrate that COD is sensitive to microstructural barriers and CSD is sensitive to crack deviations caused by the microstructure.

KEYWORDS

Biaxial fatigue; proportional loading; fatigue crack growth; crack initiation;

1. INTRODUCTION

Understanding the behaviour of engineering components under multiaxial load is crucial for a number of industries, including aerospace, ship building, automobile and power generation industries. Different models have been proposed in an attempt to characterise the behaviour under such conditions [1]. Stress-based models can be used to predict the fatigue life if the

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plastic strains are small [2]. Strain-based models are typically employed in the low-cycle fatigue regime where significant plasticity may occur [3]. Within strain-based models, critical plane approaches aim at predicting not only the fatigue life but also the crack direction of the crack [4, 5]. Energy models are based on evaluating the energy accumulated per cycle on the material which is used as a damage parameter [6, 7].

The fatigue life can be divided into two stages; initiation and propagation. The initiation stage includes crack nucleation and micro-crack growth up to a length of around 1 mm. Depending on the loading regime, a large fraction of the total life can be consumed in the initiation stage of the crack. Nevertheless, predictions made with most multiaxial models tend to be inaccurate in the initiation stage because they are based on continuum mechanics. Continuum mechanics assumptions such as homogeneity and isotropy do not hold true for initiation stage during which the crack size magnitude is similar to the characteristic microstructure dimensions. The material microstructure and the surface morphology are critical to the crack growth during the initiation stage [8]. During the initiation, fatigue crack growth is different to the propagation stage and oscillations are commonly observed on the growth curves. These are caused by grain and phase boundaries and other micro-structural features that continuum mechanics models do not take into account [9]. If strain amplitude does not exceed a certain threshold, small cracks may become arrested, thus being beneficial for the component. Non propagating cracks arise when the applied stress is insufficient to produce cyclic slip in grains or phases ahead of the crack-tip.

The models and tools available for long cracks are more advanced from the point of view of their applicability to real structural components with more industrial significance. For example, long propagation studies include interesting applications such as evaluation of crack-tip fields [10] on nuclear plant steam header [11]; evaluation of the stress intensity factor from displacement fields [12] in aerospace panels [13]; mixed-mode analysis [14] on a railway wheel [15]; or influence of surface treatments such as shot peening [16] on the fatigue life of offshore structures [17]. Conversely, small crack studies are less likely to be applied directly to real industrial cases. Since small cracks require more expensive and complex equipment to make observations [18], it is more difficult to bridge the gap between laboratory and industrial conditions. Consequently, studies dealing with short cracks have a greater statistical component [19]. The large developments in digital photography together with new imaging algorithms in recent times, will make it easier for small crack studies to move towards more industrial environments. The current work aims to study the fatigue

crack growth behaviour of small cracks under different biaxial conditions with a new methodology. The methodology combines powerful imaging systems with advanced digital image processing to characterise small crack growth under a range of biaxial conditions.

2. MATERIAL AND SPECIMEN

A low carbon steel (St-52-3N) was examined in this investigation. This material is widely used in structural applications in construction, manufacturing, ship building and offshore industries. The elastic modulus is 206 GPa and the yield stress is 369 MPa. The composition of the steel is shown in Table 1. Fig. 1 shows a micrograph of the St-52-3N steel. Ferrite and pearlite bands can be seen in Fig. 1 as vertical bands in white and black respectively.

Table 1. Chemical composition in weight % of St-52-3N steel. The balance is Fe.

Figure 1. Optical micrograph of St-52-3N steel used in all tests.

Tubular hollow specimens were used in this work to apply different combinations of torsion and tensile loads. A schematic of the geometry is shown in Fig. 2. All experiments were conducted with MTS 809 servo-hydraulic loading rig, allowing different biaxial loads to be applied on the specimens. Biaxial extensometer Epsilon 3550 was used to measure axial and angular strains. All the crack measurements were made with a black and white 1.2 megapixel digital camera coupled to a long distance microscope. Co-axial illumination was used to acquire all images. Two additional LED 3 W lamps were also used to improve light distribution on the surface and remove reflections. Fig. 3 shows the setup employed in the experiment with all the different elements used.

Figure 2. Geometry of the dog-bone shaped tubular hollow specimen used in the experiments. All dimensions are in mm.

Figure 3. Experimental setup employed in this work.

3. EXPERIMENTS

The initiation stage of the crack growth was studied under combined proportional tension-compression and torsion tests [20]. All experiments were conducted under strain control

mode with the help of the biaxial extensometer, with cyclic sinus signal with zero mean strain ($R = -1$). The aim of the work was to study a wide range of biaxial loads, with angles between shear and axial strains, φ , going from 0 to 90° in 15° increments (see Fig. 4). Imaging of the crack during initiation was made possible by drilling a 150 μm diameter hole on the outer surface of the specimen [21]. The long-distance microscope was then focused on the hole, so that the crack initiation could be captured. Fig. 5 shows how the holes look in samples S2 and S4. The hole serves as a stress raiser thus increasing the chances of the crack nucleating there. The size of the hole was chosen so that fatigue behaviour remains the same as that of the specimen without the hole, but large enough so that nucleation of the crack occurs at the hole. The hole's influence on the fatigue life is negligible, as shown previously [22]. The setup allowed detection of the crack a few microns away from the edge of the hole. On average, detection of the crack was possible at distances of 6 μm from the edge of the hole. In practice, the crack did not nucleate at the hole in around 50% of the specimens tested. Only the samples where the crack nucleated at the hole are reported here. When the shear component was increased beyond $\varphi = 60^\circ$, it was not possible to nucleate the crack around the hole. Instead, the crack appeared either on the inside of the specimen or elsewhere. Consequently, it was not possible to conduct experiments with an angle greater than $\varphi = 60^\circ$. Table 2 summarises the different tension-compression and torsion strain combination studied.

Figure 4. Definition of angle between axial and shear strain amplitudes.

Table 2. Summary of five samples employed in this work, showing the angle (φ) between axial strain amplitude (ϵ_a) and shear strain amplitude (γ_a), as well as the fatigue life (N_f) of each sample.

4. FATIGUE CRACK MEASUREMENTS

The setup described previously (Fig. 3) allowed acquisition of high-magnification images while the specimens were being strained. Magnification factor of 1.1 $\mu\text{m}/\text{pixel}$ was achieved with working distance of approximately 380 mm, resulting in a field of view of $2 \times 1.5 \text{ mm}^2$. In addition, the surface of the specimen was etched for 30 seconds with nitric acid diluted with ethanol to create a random pattern on the surface. This pattern can be used for obtaining

displacement and strain fields with Digital Image Correlation (DIC). However, no DIC results are shown here. Fig. 5 shows some examples of images acquired during the experiment.

Figure 5. Images acquired for samples a) S2 and b) S4 after 950 and 1300 cycles respectively. Schematics showing how c) Crack Opening Displacement (COD), d) Crack Shear Displacement (CSD) and e) cracking angle throughout the experiment (α) are measured experimentally. The diameter of the hole in all images is 150 μm .

Around 120 images similar to those shown in Fig. 5 were acquired every 25 cycles. The image of maximum strain was then selected (Fig. 6.a) and analysed with an image processing algorithm. The algorithm evaluated the darkest pixels (minimum value in the grey scale) of the image and assumed they were inside the crack. Subsequently 20 grey values were added to the minimum value to account for small intensity differences occurring within the crack. This information was employed to binarise the image, as has been done in previous works [23]. The binarised image was then analysed with an edge-finding routine to detect the edges which coincide with the crack edges (Fig. 6.b) [24, 25]. Finally, the detected crack was superimposed on the original image (Fig. 6.c). The algorithm allowed automatic evaluation of the crack length for all images acquired. The results of monitoring the crack length against the number of cycles during the early stages of the crack are shown in Fig. 7. The crack growth rates were computed from a-N curves by numerical differentiation following ASTM standard [26] and are shown in Fig. 8.

Crack edges detected by the algorithm were also used to extract the cracking angle at for each loading case (α), the Crack Opening Displacement (COD) and the Crack Sliding Displacement (CSD). The cracking angle was obtained as the angle between the line joining the two crack tips. The COD and CSD were obtained by measuring the distance between the crack flanks in the opening direction and the growing direction respectively. Measurements of COD and CSD were taken at the edges of the hole drilled on the surface. Fig. 5c to 5e summarise the way cracking angle, COD and CSD are measured. The results of COD and CSD against the life fraction for the different straining conditions are shown in Fig. 9a and 9b respectively. The detected angle (α) for all samples is summarised in Table 3.

Figure 6. Image of sample S1 at maximum strain after applying 800 cycles. a) Raw image, b) image of the crack detected with the algorithm and c) detected crack superimposed on original image. The diameter of the hole is 150 μm .

Figure 7. Crack length versus number of cycles for samples a) S1, b) S2, c) S3, d) S4 and e) S5.

Figure 8. Fatigue crack propagation rates (da/dN) versus crack length for samples a) S1, b) S2, c) S3, d) S4 and e) S5.

Figure 9. Evolution of a) COD and b) CSD during the experiments.

Table 3. Summary of SWT angles, detected angles throughout the experiment (α) and difference between experimental angle and SWT.

5. ANALYSIS OF RESULTS AND DISCUSSION

It is observed in Table 2 and Fig. 7 that fatigue lives increase from 875 to 2425 cycles as the angle φ increases from 0° to 60° . That is, for the strain ranges studied, increasing the shear component produces longer fatigue lives. Fig. 7 also shows that the growth of the cracks emanating to the right hand and left hand of the hole are similar for samples S1, S2 and S5. Differences between right and left cracks are observed for samples S3 and S4. The smallest crack length that the algorithm was able to measure was between 6 and 24 μm for samples S1 and S4 respectively. These lengths were measured from the edge of the hole to the crack tip. The final crack length was computed by adding the length measured from the edge of the hole to the distance between the hole and the crack tip. The resolution for detecting the crack in its early stages depended on the illumination and the contrast of the acquired images. Crack morphology also influenced the resolution for crack detections, similarly to previous works [24].

Fig. 8 shows oscillations in the growth rate. These oscillations are due to microstructure and are often observed during crack initiation. The barrier effect is produced by the pearlite bands which are the phases with greatest strength [22]. Since the propagation rate curves are plotted on logarithmic scale, it is not easy to compare the magnitude of oscillations for the

different angles. Thus the average oscillation for all angles was computed. The extent of each oscillation was obtained by subtracting each local minimum to the subsequent local maximum. The average extent of the oscillations for all angles studied is shown in Fig. 9. These values are an estimate of how large the acceleration and retardation transients are. Large oscillations are indicative of great accelerations and/or great retardations. It can be seen in Fig. 9 that acceleration and retardation transients increase as the angle φ decreases. That is, increasing the axial component of the strain produces stronger accelerations and retardations.

Figure 10. Average oscillation on da/dN curve for the five different angles, φ , studied.

The current tests do not seem to follow the trend depicted by McDowell model (Fig. 11) [27, 28] developed for uniaxial loading. The model predicts higher level of oscillations as the load is increased. However, the data shown in Fig. 8 seems to follow a different trend, with higher stress level being related to larger oscillations. It is possible that increasing the torsional component, reduces the oscillations typically observed in small cracks and the effect of the torsional component is more pronounced than the mean stress effect. Further experiments should be conducted in order to evaluate separately the mean stress effect and the torsional component effect. Fig. 8 also shows that growth rates depend weakly on crack length, as it is normally observed in the short crack regime [29]. This can be seen in the overall small slope of the curves.

Figure 11. Typical evolution of crack growth rate as a function of crack length and stress amplitude. Adapted from [27].

The greater damaging effect of lower angle φ is evident from Fig. 7 since lower angles φ yield shorter fatigue lives. Accordingly, the combination of strains applied at lower angles φ (i.e. with higher tensile component) produces an overall higher level of stresses. This behaviour can also be studied with the Smith-Watson-Topper (SWT) damage model [30]. The damage under multiaxial loading can be evaluated with the SWT damage parameter:

$$SWT = s_{n,\max} \frac{De_n}{2} \quad (1)$$

where $De_n/2$ is the maximum normal strain amplitude and $s_{n,max}$ is the maximum normal stress acting on the plane subjected to a maximum $De_n/2$. This criterion was developed for materials failing due to crack growth in mode I where fatigue growth is controlled from early stages by crack growth through planes perpendicular to the direction of maximum strain and maximum normal stress. Fig. 12 shows the evolution of SWT versus fatigue life for all samples. It is clear from Fig. 12 that the combination of strains with smaller torsional component are less damaging since they have a larger SWT damage parameter.

The results of the cracking angle during the experiment (α) are summarised in Table 3, along with the crack orientation as predicted by SWT model [30]. According to SWT, the crack will initiate at a plane subjected to maximum $De_n/2$. In order to evaluate the crack plane, the range of normal strain was evaluated from 0° to 180° in 5° steps. The results of the cracking angle measured experimentally and as predicted by SWT are shown in Table 3. The angle predicted by SWT increases as the torsional component is increased. The experimental angle also increase with the torsional component, but the trend is not as clear. This is certainly caused by the microstructure, which can influence greatly the initiation point and the initiation angle [31]. Overall SWT predictions underestimated the cracking direction.

COD can be used to characterise crack growth behaviour in the initiation stage [32]. In this work, COD information is complemented with CSD, following the ideas of Li [33]. The results showing the evolution of the COD and CSD during each experiment are shown in Fig. 9. It can be seen that the methodology employed does not allow measurement of COD and CSD below fraction lives of approximately 0.3. For life fractions below 0.3, the crack tip displacements are too small to be resolved with the current imaging system. This could be certainly improved with higher camera resolution. It can be seen that COD increases gradually throughout the experiment for all loading cases, as one would expect for increasing crack lengths (see Fig. 7). The lower damaging effect of increasing torsional component can also be seen in Fig. 9a because the highest COD is observed for 0° , followed by 15° , 30° , 45° and 60° , in agreement with SWT damage parameter (Fig. 12). It is also observed in Fig. 9a that curves do not increase steadily, but instead there exists some oscillation which in most cases take the form of plateaus, or periods during which COD does not increase. To evaluate whether these oscillations are due to random error or correspond to real behaviour of the crack, these were compared to the growth rates shown in Fig. 8. Fig. 13 shows the growth rate and the change in COD throughout the experiment ($dCOD/dN$) for sample S2 in the same plot. COD values were numerically differentiated in a similar way to a-N data to obtain

da/dN data (Fig. 8). Fig. 13 shows that da/dN and dCOD/dN curves follow similar trends, with the largest rises and drops occurring at approximately the same time. The correlation between both curves is not exact because they are both obtained by differentiation. The error inherent to crack length and COD measurements is largely amplified through the differentiation. Similar trends were also observed for the rest of the samples. Such results suggest that microstructure oscillations in the growth rate can also be detected by the COD measurements.

The evolution for CSD is slightly different to that of COD (see Fig. 13). Unlike in the case of COD, the highest CSD corresponds to $\varphi = 15^\circ$ followed by $\varphi = 0^\circ$. Large scattering is observed for the rest of loading conditions ($\varphi = 30^\circ, 45^\circ$ and 60°) probably due to CSD values being too small to be captured by the system. The non-negligible CSD values for $\varphi = 15^\circ$ and $\varphi = 0^\circ$ are probably related to the crack growing at an angle that is not perpendicular to the maximum principal strain direction. If the crack grows exactly in the direction that maximises the principal strain, only opening mode takes place. However, if the crack angle deviates from the direction maximising the principal strain, then a shear component in the driving force appears [34]. Table 3 summarises the angle that maximises the range of principal strain (named SWT angle), the measured angle and the difference between both angles for all loads studied. The largest angle differences occur for $\varphi = 15^\circ$ followed by $\varphi = 0^\circ$. A comparison of angle differences in Table 3 and Fig. 9b suggests that the deviation from the theoretical angle in the 15° and 0° cracks induces a sliding component on the driving force. These deviations from the theoretical angle are most likely caused by the microstructure.

Figure 12. SWT damage parameter versus fatigue life. The loading angles (φ) that identify each experiment are also included.

Figure 13. Crack growth rate and change in COD during the test for sample S2.

6. CONSLUSIONS

A methodology for characterising crack initiation under biaxial loading conditions has been described. It included automatic evaluation with digital processing algorithms and edge-finding routine of crack length, crack angle and two driving force parameters such as COD

and CSD. The setup allowed detection of the crack between 6 and 24 μm from the edge of a hole drilled so that the crack could be observed from its early stages. Different biaxial conditions were investigated in five tubular specimens of low carbon ferritic-pearlitic steel. Strains ranged from pure tension-compression axial strain to strain states with an increasing torsional component and a decreasing axial component. Typical oscillations with acceleration and retardation transients were observed throughout all the strain states studied. These oscillations were ascribed to phase boundaries existing in the material. The retardations seem to appear when the crack meets a pearlite band. In addition, the magnitude of the oscillations decreases as the torsional component increases. It was not possible to correlate the different oscillation levels with previous models describing crack growth at the initiation stage.

COD and CSD measurements have been shown to be useful in characterising the different combinations of loads. In addition, it has been shown that COD is sensitive to acceleration and retardations due to microstructure. It has also been shown that CSD is very small for all cases except for the cases when the crack grows at an angle substantially different to that predicted by SWT.

The new methodology can be easily adapted to study small crack behaviour in industrial environments.

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