Interaction between Labor Market Conditions and Educational Investment

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Abstract

We consider a two-period model in which an informed worker may invest in education in the first period, in which case she will not participate in the labor market in that period, but her diploma will allow her to signal a greater productivity in the second period. In this setting, we analyze the effects of a change in the labor market conditions on the worker´s educational decision. In a selective educational system wherein the rate at which the cost of education decreases with the worker´s productivity is sufficiently high, we found that the worker´s incentives to invest in education become stronger when the worker is more patient, when the expected future wage is greater, or when the cost of education is lower. Interestingly, the opposite occurs in a nonselective educational setting wherein the rate at which the cost of education decreases with the worker´s productivity is sufficiently low. Those results are robust to the worker´s risk preferences and to the specification of the prior distribution function of worker´s productivities.

Keywords: Education, risk preferences, selective educational system, separating equilibrium, signaling.

JEL Classification: C72, C79, D82, D83, I21, J01.

1 Introduction

In this article, we introduce a theoretical model in order to analyze the effects of changes in the labor market conditions on workers´ incentives to invest in education. Specifically, we consider a two-period signaling game in which a worker has private information on her productivity and decides whether she invests in education or not in the first period. If the worker does not invest in education, she will participate in the labor market in both periods, but if she invests in education, she will only work in the second period because she abandons the labor market during the investment period. When people decide whether they invest in education or not, they do not know the wages they will receive after obtaining their diploma because the labor market conditions in the future are uncertain. In order to introduce this real-world feature into the model, we assume that the monetary value of the worker´s productivity will depend on the price at which the product can be sold in the market and the worker will have to choose her level of investment in the first period without knowing the price of the product in the second period. Additionally, the positive correlation between the prices of the product in both periods will allow workers to use the current labor market conditions in order to predict those conditions in the future.

In this setting, we focus on a separating equilibrium in which only a worker with an ability greater than a certain threshold invests in education and this equilibrium may arise under two types of assumptions. First, when the cost of education decreases sufficiently with the worker´s ability, this equilibrium is obtained because the worker uses the educational investment as a signal of her productivity. Second, when the rate at which the cost of education decreases with the worker´s ability is sufficiently low, the separating equilibrium is also obtained, but now the reason why the worker with a low ability does not invest in education is the high opportunity cost of that investment.

In a selective educational system in which the cost of education decreases significantly with the worker´s ability, we obtain that the worker´s incentives to invest in education will become stronger when the worker is more patient, when the expected future price is greater or when the cost of education goes down. Interestingly, when the derivative of the cost of education with respect to the worker´s ability is sufficiently low, which will be referred to as the non-selective educational system, an increase in the worker´s level of patience, a rise in the expected future price and a lower cost of education will erode the incentives to invest in education. Finally, we found that these results are robust to the specification of the prior distribution of worker´s abilities and to the worker´s risk preferences.

In the selective educational system considered, an exogenous rise in the price of the product in period zero will lead to an increase in wages among non-educated workers who participate in the labor market in that period. When the positive correlation between the prices in periods zero and one is sufficiently low, those greater wages in period zero will cause a greater opportunity cost of schooling and the worker´s incentives to invest in education will be weaker. These results might explain previous empirical literature. For example, Hillman and Orians (2013) show that college enrollment is countercyclical, which suggests that the lower the wages, the greater the number of students who enroll in college. Similarly, Petrongolo and San Segundo (2002), Dellas and Sakellaris (2003), Giannelli and Monfardini (2003) and Tumino and Taylor (2015) show evidence suggesting that greater unemployment rates among youngsters cause a lower opportunity cost of education and a greater probability that they stay at school or university. However, when the positive correlation between the prices in periods zero and one is sufficiently high in our model, an exogenous increase in the price of the product in period zero will give rise to greater wages in that period, which will lead to much greater prices and wages in period one. As a result, the expected return to education will go up and consequently, the worker´s incentives to invest in education will be stronger. These results explain some evidence about the relationship between unemployment rates and educational investment. For instance, Petrongolo and San Segundo (2002), showed that higher unemployment rates among adults predict lower returns to education, which causes a lower probability of being enrolled into the schooling system. Similarly, Gilpin et al. (2015) found that enrollment and degree completion in for-profit colleges is positively related to employment growth and wages in related occupations.

Although a meta-regression analysis conducted by Havranek, Irsova and Zeylanova (2018) concluded that researchers have shown positive effects of a rise in tuition fees on the demand for education less often than they should, empirical literature of the effect of an exogenous reduction in the cost of college on the enrollment and degree completion is mixed. For example, McPherson and Schapiro (1991) obtained a positive relationship between the net cost of education and enrollment among middle and upper-income students and a negative relationship among low-income individuals. Furthermore, when McPherson and Schapiro (1991) estimated their regressions for private and public universities separately, their results showed that an increase in the cost of education at private institutions reduced the enrollment rate among low and medium-income students, but does not affect the enrollment rate significantly among high-income students. At public institutions, which are much less costly than private colleges, an increase in the net cost of education did not affect the enrollment rate significantly among low-income students and increased the enrollment rate among medium and high-income students. Likewise, Bruckmeier and Wigger (2014) found a negative effect of the introduction of academic fees on the enrollment rate in some German states, but they found a positive effect in those states in which students were eligible to non-interest-bearing loans, in those in which a high percentage of students do not pay tuition fees due to fee exemptions, and in some rich states where the real price of tuition fees was low. Similarly, Castleman and Long (2016) obtained a positive causal impact of grants that reduce the cost of education on college access and graduation, while Cohodes and Goodman (2014) showed that a tuition grant program that reduces the cost of college reduces the spending on higher education and the probability of degree completion. Interestingly, Cohodes and Goodman (2014) found that direct relationship between the cost of education and educational achievement in colleges in which the total cost of education, including fees, room, board and books, is much lower than in other colleges. Additionally, those students who received the grant in that study had a high income and got better grades at school than those who did not receive the grant. Using all that empirical literature, we conclude that there is a negative impact of an increase in the cost of education on enrollment and graduation rates in costly educational institutions and among low-income students, whereas the opposite occurs in those educational settings where students bear a lower cost of education and/or they belong to high-income classes. Our theoretical results suggest some explanation for that evidence. In particular, when there is an exogenous reduction in the cost of education caused by a reduction in the level of academic skills needed to get the diploma, we obtain that the worker´s incentives to invest in education become stronger in the selective educational setting, but the opposite occurs in the nonselective setting.

This article is organized as follows. In the next section, we briefly describe the contribution of our model to previous literature. In the third section, we present the model. In the fourth, we obtain our main results with a risk-neutral worker and under the assumption of a uniform prior distribution of worker´s abilities, whereas we analyze the robustness of those results to different assumptions about the worker´s risk preferences and to a different specification of the prior distribution of abilities in the fifth and sixth sections, respectively. Finally, we summarize the main conclusions in the seventh section and the proofs of all lemmas and propositions are included in the appendix.

2 Literature review

Since the seminal paper written by Spence (1973), theoretical models on the use of education as a signal of workers´ productivity in the labor market have proliferated. For example, Spence (1974) compares the results of the model when employers compete for hiring workers, when the employer is a monopsonist, in the efficient case, or with the maximization of different social welfare functions. Likewise, Spence (1976) analyzes what happens when employers offer two types of jobs and compete with respect to the signaling prerequisites for jobs as well as with salaries. Furthermore, Forges (1990) studies the effects of adding costless communication to an educational signaling model, Bedard (2001) adds a set of workers with constrained access to university, Frazis (2002) considers a model in which workers are unsure of their ability and Feltovich, Harbaugh and To (2002) introduce a model in which employers receive a noisy signal of workers´ productivity. Similarly, Spence (2002) also extends his seminal work by assuming that education increases the worker´s productivity, by introducing some companies which may learn the worker´s productivity at a cost, or by allowing the cost of education to increase with the worker´s productivity. Moreover, Gallice (2009) introduces a signaling model with two independent cohorts of workers who play the game only once, and some of those workers´ utility depend on the difference between their educational choice and the average education chosen by all the individuals belonging to the same cohort. Finally, Daley and Green (2014) develop a signaling model in which firms observe the worker´s level of education and their grade, which is a random variable, Perri (2019) considers a model in which workers may be employed in two sectors: a secondary sector where a worker´s productivity is the same regardless of her ability and a primary sector where that productivity is positively correlated with ability, and Jungbauer and Waldman (2023) study what happens when the employer cannot observe the educational level chosen by the worker, but that worker may send a true or false message about that education level to the employer, and the cost of that message depends on the worker´s level of honesty, which is private information.

In those models, workers choose a level of education and receive the returns to their educational investment in the same period and consequently, they do not take into account the opportunity cost generated by the wages lost during the educational period. For this reason, previous literature on educational signaling cannot explain the interaction between the labor market outcomes and workers´ incentives to use their level of education as a signaling device.

Although a few researchers have studied some interactions between the labor market conditions and workers´ incentives to signal through education, they have focused on the relationship between the use of education as a signal and current employers´ decisions about retaining or promoting their employee to a better paid job when retention or promotion may signal workers´ productivity in the labor market (Waldman, 1984, 1990, 2016). These models also assume that workers receive their wages in the same period regardless of the level of education they choose and consequently, they ignore the opportunity cost of education during the investment period.

Our model shares some similarities with that introduced by Kurlat and Scheuer (2021), in which they assume that firms can directly observe imperfect information on workers´ types and the quality of that information is heterogeneous across firms. Due to the introduction of those assumptions, they found that signaling decreases if the cost is higher, if the demand for workers increases, or if firms´ expertise improves. In our model, we also analyze the effects of an exogenous change in the cost of signaling (cost of education). Furthermore, an exogenous change in the price of the product in period zero in our model is equivalent to a change in the demand for workers in their model. Their results about the effects of an exogenous change in the cost of the signal on signaling are similar to those obtained in our model when the cost of education decreases sufficiently with the worker´s ability. However, the introduction of the opportunity cost of education allows us to obtain just the opposite relationship between signaling and that parameter when the relation between the cost of education and the worker´s ability is weaker. Besides, due to the introduction of correlation between the prices in both periods, our results suggest that a change in the demand for labor today may affect the educational investment differently depending on that correlation.

Additionally, in their model, education is a risky decision because equally productive and educated workers may receive different wages. The reason for this risk is that firms imperfectly evaluate workers´ productivities. In our model, the workers´ educational decision is risky because the conditions in the labor market after obtaining the diploma are uncertain. Lastly, unlike Kurlat and Scheuer´s model, we also analyze what happens when the worker is not risk neutral and when the prior distribution of worker´s abilities is not uniform.

3 Model

A worker has private information on her productivity, $t \in T = [t_0, t_n]$, where $0 < t_0 < t_n$. We assume that the prior distribution of this productivity is represented by a uniform distribution function and this is common knowledge. After observing her type, the worker chooses one of two possible levels of education: $e \in M = \{e_0, e_1\}$.

If the worker chooses e_0, it means that she does not study at the university and is employed in a company in periods 0 and 1. For simplicity, the cost of this level of education is assumed to be equal to zero. As companies compete a la Bertrand for hiring the worker, they will pay a wage equal to $P_0 E(t|e_0)$ to an uneducated worker in period 0, where $E(t|e_0)$ is the expected physical productivity among those workers who chose e_0 and P_0 is the price at which each unit of product can be sold in the market in period 0. In period 1, the uneducated worker will continue working in the labor market, in which case she receives a wage equal to the monetary value of her expected productivity: $P_1 E(t|e_0)$, where P_1 represents the price of the product in period 1. In period zero, when the worker decides whether she invests in education or not, she does not know the price of the product in period one and we assume that it follows a simple process: $P_1 = \rho P_0 + \varepsilon$, where $\rho > 0$ represents the positive correlation between the prices in both periods and ε is a random variable whose density function is $f(\cdot)$ and it represents fluctuations of the price around its long-run tendency. This density function is continuous and differentiable and is common knowledge. Moreover, the prior distribution of worker´s types and the distribution of ε are statistically independent. We assume that the support of ε is a closed interval: $\varepsilon \in [\overline{\varepsilon} - \Delta, \overline{\varepsilon} + \Delta]$. In order to avoid negative prices in period 1, we assume that $\Delta < \rho P_0 + \overline{\varepsilon}$.

Finally, when the worker chooses e_1, she will incur a cost of that level of education in period 1: $c(t, e_1)$. This cost increases with the level of education that the worker has to achieve in order to get the diploma, that is, $\frac{\partial c(t,e_1)}{\partial e_1} > 0$. As usual in models of educational signaling, we assume that the cost of education decreases with the worker´s type, that is, $\frac{\partial c(t,e_1)}{\partial t} < 0$. In this case, the worker will not work until completing the high level of education in period 1 and the competitive company will offer her a wage equal to the monetary value of her productivity, which is $P_1 E(t|e_1)$. As the price in period 1 is not known in period 0, the educational investment is risky.

Following standard notation, the discount factor will be represented by δ.

In this setting, we will obtain the perfect Bayesian equilibrium of the game, which satisfies the following conditions:

i. Each worker´s type will choose the level of education, e, that maximizes the discounted sum of utility levels: $u(t, e) = U[P_0 E(t|e_0) 1(e = e_0)] + \delta U[P_1 E(t|e) - c(t, e)]$, where $U(\cdot)$ is the worker´s utility function and $1(e = e_0)$ represents an indicator function, which is equal to one when the worker chooses the low level of education and zero otherwise.
ii. Given the level of education chosen by the worker, in each period the company will pay her a wage which is equal to the expected monetary value of productivity among those worker´s types with that level of education in equilibrium: $P_t E(t|e)$.

iii. The company´s beliefs must be consistent with the Bayesian rule in the equilibrium path. For example, when those worker´s types lower than t^* choose e_0 and those types greater than or equal to t^* choose e_1 in a separating equilibrium, the probability assigned by the company to type t after observing an educational level of e is given by $\mu(t|e) = \begin{cases} \frac{1}{t^*-t_0} \; if \; e = e_0 \\ \frac{1}{t_n-t^*} \; if \; e = e_1 \end{cases}$.

As shown by Mailath (1988), when the set of possible productivities of a worker forms a continuum, workers´ behavior in a separating equilibrium is completely determined because there is a unique separating equilibrium. In this article, we analyze the effects of a change in the labor market conditions on the separating equilibrium obtained.

4 Risk-neutral worker

In this section, we analyze the workers´ incentives to invest in education when they are risk-neutral.

The worker´s utility function, $U: \mathbb{R} \to \mathbb{R}$, is differentiable and strictly increasing, that is $U'(x) > 0 \; \forall x \in \mathbb{R}$. In order to simplify the model, the worker´s utility from no money is normalized to be equal to zero, that is, $U(0) = 0$. Additionally, due to risk-neutrality[1], $U''(x) = 0 \; \forall x \in \mathbb{R}$.

In this model, the level of education may reveal some information on the worker´s productivity in the labor market for two reasons. First, the educational investment has an opportunity cost, which is equal to the wage lost during the investment period, and that wage lost is the wage received by non-educated types in equilibrium, which increases with the expected productivity of those non-educated types. As a result, when the expected type

[1] In the next section, we will analyse the robustness of our results to the specification of the worker´s risk-preferences.

among non-educated workers is sufficiently high, only the highest worker´s types invest in education because their cost is lower than the additional wage received by educated workers. Second, as the cost of education decreases with the worker´s type, high-ability workers may invest in education in order to signal their higher productivity in the labor market.

In this article, we analyze what happens in two types of educational systems. First, we consider a scenario in which the rate at which the cost of education decreases with the worker´s productivity is sufficiently low. This setting is likely to arise in those contexts with undemanding universal educational systems in which even low ability students may complete a high level of education by putting a bit more effort than high ability students. This setting may also appear in educational institutions dominated by high-income students because rich low ability students might find it easy to spend more money on support tuition and other additional help in order to complete high levels of education. Second, we consider a scenario in which the rate at which the cost of education decreases with the worker´s productivity is sufficiently high. This set-up is more likely to appear in those contexts with selective educational systems in which only high ability students may complete the highest level of education.

<u>Nonselective educational system</u>. In this setting, we assume that the lowest type of worker will prefer the high to the low level of education when the employer identifies her type after observing the low level and pays the ex-ante expected wage after observing the high. This assumption can be written as:

Assumption 1. $U(t_0 P_0) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_0(\rho P_0 + \varepsilon)] f(\varepsilon) d\varepsilon < \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon) - c(t_0, e_1)\right] f(\varepsilon) d\varepsilon.$

This assumption is satisfied when the cost of education among the lowest types of worker is not too high. Similarly, we assume that the highest type of worker prefers the low to the high level of education when the wage among non-educated workers is the monetary value of the ex-ante productivity and the wage among educated workers is equal to the value of the productivity of the highest type, that is:

Assumption 2. $U\left(\frac{t_0+t_n}{2}P_0\right)+\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U\left[\frac{t_0+t_n}{2}(\rho P_0+\varepsilon)\right]f(\varepsilon)d\varepsilon>\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U[t_n(\rho P_0+\varepsilon)-c(t_n,e_1)]f(\varepsilon)d\varepsilon.$

This assumption holds when the cost of education among the highest-ability workers is sufficiently high.

Both assumptions 1 and 2 imply that the cost of the high level of education does not decrease with the worker´s type too much. This idea is reinforced by introducing the next assumption:

Assumption 3. $\left|\frac{\partial c(t,e_1)}{\partial t}\right|<\frac{P_0}{2\delta}\ \forall t\in[t_0,t_n].$

In this scenario, there may be a pooling equilibrium in which all worker´s types choose the low level of education and another pooling equilibrium in which all types choose the high level and those equilibria may survive standard refinements. However, we are interested in a separating equilibrium in which only those worker´s types greater than a certain threshold invest in education. This equilibrium will arise as long as employers have some incentives to separate low from high ability workers[2]. We will provide a simple example below.

<u>Selective educational system</u>. In this scenario, we assume that the lowest type of worker will prefer the low to the high level of education when the employer identifies her type after observing the former and pays the ex-ante expected wage after observing the latter. This assumption can be written as:

[2] For example, only high productivity workers may generate positive synergies over time within the companies where they work and those synergies may not be anticipated by workers at the time of hiring.

Assumption 1´. $U(t_0P_0)+\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U[t_0(\rho P_0+\varepsilon)]f(\varepsilon)d\varepsilon>\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U\left[\frac{t_0+t_n}{2}(\rho P_0+\varepsilon)-c(t_0,e_1)\right]f(\varepsilon)d\varepsilon.$

This assumption is satisfied when $c(t_0,e_1)$ is sufficiently high. Likewise, we assume that the highest type will prefer the high to the low level of education when the employer identifies her type after observing the high level of education and pays the monetary value of the ex-ante expected productivity after observing the low level, that is:

Assumption 2´. $U\left(\frac{t_0+t_n}{2}P_0\right)+\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U\left(\frac{t_0+t_n}{2}(\rho P_0+\varepsilon)\right)f(\varepsilon)d\varepsilon<\delta\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}U[t_n(\rho P_0+\varepsilon)-c(t_n,e_1)]f(\varepsilon)d\varepsilon.$

This assumption holds providing that $c(t_n,e_1)$ is sufficiently low.

Now assumptions 1´ and 2´ imply that the cost of education decreases sufficiently with the worker´s type, and this idea is reinforced by the following assumption:

Assumption 3´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right|>\frac{P_0}{2\delta}\ \forall t\in[t_0,t_n].$

In this scenario, although there may be pooling equilibria as well, they may be discarded by standard refinements, such as D1.

A Simple Example of the Nonselective Educational System. As the nonselective scenario is not standard and will lead to striking results, I will use an example to illustrate this context. Imagine that $T=[0,10]$ and the prior distribution of worker´s types is uniform with that support. Additionally, $\delta=1$, $P_0=1$ and the distribution of ε is uniform with support $[-0.5,0.5]$, that is, $\bar{\varepsilon}=0$ and $\Delta=0.5$. Finally, $\rho=1.1$ and $c(t,e_1)=2-\frac{t}{10}$ and $U(x)=x$. This example satisfies assumptions 1-3 and there is a separating equilibrium in which those types lower than $t^*=8.75$ choose e_0 and those greater than t^* choose e_1.

There might be other pooling equilibria as well, but as long as the employer has incentives to separate low from high productivity workers, this separating equilibrium will prevail. In this example, the employers only need to calculate the threshold of the separating equilibrium, $t^* = 8.75$ and they should offer uneducated workers a wage equal to the monetary value of the expected physical productivity of types lower than t^* and offer educated workers a wage equal to the monetary value of the expected physical productivity of types greater than t^*. In particular, the wage paid to an uneducated worker would be 4.375 ($\frac{0+8.75}{2}$) in period zero and 4.8125 (4.375×1.1) would be the expected wage paid to an uneducated worker in period one. Similarly, the expected wage paid to an educated worker in period one must be equal to 10.3125 ($1.1 \times \frac{8.75+10}{2}$). With those wage offers, only those worker´s types greater than 8.75 will have incentives to invest in education and companies will be able to separate low from high ability workers.

In both scenarios, a separating equilibrium will arise. In that equilibrium, there will be a worker´s type, t^*, who will be indifferent between both levels of education, those types lower than t^* will choose e_0 and those types greater than t^* will choose e_1. Under the assumptions of both scenarios, this is the unique separating equilibrium with a threshold. In that equilibrium, the indifferent worker´s type, t^*, will satisfy the following equation:

$$U\left(\frac{t_0+t^*}{2}P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon)d\varepsilon = \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon)d\varepsilon \qquad (1)$$

The left-hand side of this equation shows the expected utility obtained by those workers´ types who choose to work in period 0, which is equal to the sum of the discounted levels of utility obtained by those types of worker, whereas the right-hand side shows the expected utility obtained by those worker´s types who complete the high level of education in period

1, which is equal to the discounted expected utility obtained in that period from the wage received minus the cost of education.

As a limit case, we will analyze what happens when the cost of education does not change with the worker´s type, that is, let us assume that $c(t, e_1) = K\ \forall t \in T$. The equilibrium obtained in this setting allows us to isolate the effects of the introduction of the opportunity cost of education on the equilibrium obtained. Interestingly, the next lemma shows that this opportunity cost provides a mechanism in order to separate high from low-ability workers.

LEMMA 1. Under assumptions 1 and 2, when the cost of education does not change with t, there exists an equilibrium in which those types of worker lower than t^* *choose* e_0 *and those types greater than* t^* *choose* e_1, *where* $t^* \in T$.

As a result of the introduction of the opportunity cost in our model, this lemma shows that the employer may use the investment in education in order to distinguish low from high-productivity workers. Without the opportunity cost of education, it would be impossible to obtain this type of separating equilibrium when the cost of education is the same for all worker´s types. In particular, if the additional wage received by educated workers were greater than the cost of education, all worker´s types would choose the high level of education. Otherwise, no worker´s type would invest in education. Hence, separation would not arise. In equilibrium, lemma 1 shows that the cost of education, including the opportunity cost, is equal to the additional wage received by educated workers and for this reason, each type of worker is indifferent between investing and not investing in education.

In the example shown above, if the cost of the high level of education were $c(t, e_1) = 2\ \forall t \in T$, there would be a threshold separating equilibrium in which only those worker´s types greater than $t^* = 7$ would invest in education. In this equilibrium, non-educated workers would receive a wage of 3.5 in period zero and an expected wage of 3.85 in period one, whereas educated workers would receive an expected wage of 7.35 in period 1. A similar threshold separating equilibrium would arise in our example as long as the cost of education is constant and lies between 0.5 and 5.5. This outcome is not "stable" in the sense that all educated and non-educated types of worker are completely indifferent between investing and not investing in education, and this equilibrium would not be supported if a slight mass of types changed their choice. However, the equilibrium would be "stable" as long as the cost of education slightly decreases with the worker´s type. Therefore, lemmas 1 and 2 should only be considered as limit cases that analyze what happens when the type-dependency of the educational cost vanishes[3].

LEMMA 2. Under assumptions 1 and 2, when the cost of education does not change with the workers´ type, t^* *increases with* δ *and decreases with* K.

In the separating equilibrium considered, the cost of education of the indifferent type of worker, including the opportunity cost, must be equal to the profit from the educational investment. For this reason, when the discounted profit from investing in education goes up as a result of a rise in the discount factor (δ) or a drop in the cost of education (K), the opportunity cost of education for the indifferent type of worker must also increase. As the opportunity cost of education is the wage received by non-educated workers, it will only increase when the indifferent type goes up so that the average productivity of non-educated workers goes up.

After the analysis of the benchmark with constant cost of education, we perform some comparative statics analysis in our general model. Under the assumptions of the nonselective and selective educational scenarios, it is straightforward to see that there is a separating equilibrium like that described in lemma 1. In that equilibrium, the next proposition shows

[3] In this limit case, there may be other equilibria with an arbitrary partition of workers´ types. We are grateful to an anonymous referee for this comment.

that the effect of a change in the parameters of the model on the worker´s incentives to invest in education will depend on the scenario considered.

PROPOSITION 1. In the nonselective educational scenario (under assumptions 1, 2 and 3), t^* *increases with* δ *and with* ρ *and decreases with* e_1*. In the selective educational scenario (under assumptions 1´, 2´ and 3´),* t^* *decreases with* δ *and with* ρ *and increases with* e_1.

It is really easy to understand the intuition behind this proposition. Under the nonselective scenario, when the discount rate or the expected price of the product in period 1 goes up, that is, when δ or ρ goes up, the future profit from investing in education will increase and the indifferent type´s opportunity cost of education should go up in order to compensate the rise in the expected future profit. For this reason, the indifferent worker´s type must increase so that the average productivity of those worker´s types who do not invest in education is higher. For similar reasons, when the monetary cost of education rises for the indifferent type, that is, when e_1 goes up, the opportunity cost of education of that type must go down to compensate it and consequently, the indifferent worker´s type decreases. However, in the selective scenario, the role of education as a signaling device outweighs the effect of its opportunity cost. As a result, when δ or ρ goes up, the discounted profit from the educational investment increases and more low-ability workers invest in education (t^* goes down). Similarly, when the cost of education goes up (greater e_1), some workers with lower ability will not find it profitable to signal by investing in education (t^* goes up).

In our model, the worker makes her educational decision in period 0 and a change in wages in that period will affect her incentives to invest in education. As the wage paid in period 0 depends on the price of the product in that period, we analyze the effect of an exogenous change in the price in period 0 on the worker´s incentives to invest in education in our next proposition.

PROPOSITION 2. There exists a certain threshold, $K \in \mathbb{R}^{+}$*, such that* $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ *when* $\rho \lesseqgtr K$ *in the nonselective educational scenario (under assumptions 1, 2 and 3) and* $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ *when* $\rho \lesseqgtr K$ *in the selective educational scenario (under assumptions 1´, 2´ and 3´).*

In this model, the effect of an improvement in the labor market today on the worker´s incentives to study will depend on the effect of that improvement on the expected prospects of the future labor market. In particular, when a rise in the current wage barely affects the wage expected by the worker in the future ($\rho < K$), a greater wage today will mainly imply a greater opportunity cost of studying at the university. On the contrary, when a greater wage today increases significantly the expected wage in the future ($\rho > K$), a greater wage today will mainly imply a greater expected profit from the educational investment. Then, in the selective scenario, a rise in P_0 will lead to an increase in the opportunity cost of education and a lower proportion of educated types (greater t^*) when $\rho < K$ and to a greater expected profit from education and a greater proportion of educated types (lower t^*) when $\rho > K$. In the nonselective scenario, when $\rho < K$, the increase in the opportunity cost of education caused by a rise in P_0 has to be compensated with a reduction in that opportunity cost for the indifferent type, which gives rise to a lower proportion of educated types (lower t^*). Similarly, when $\rho > K$, the increase in the expected profit from education caused by a rise in P_0 has to be compensated with a greater opportunity cost of education for the indifferent type and for this reason, t^* increases.

5 Results with different risk preferences

In this section, we obtain the equilibrium of the model when the worker is not risk-neutral. Now, the worker´s utility function is $U(x)$, which is continuous and twice differentiable, and we assume that $U'(x) > 0 \; \forall x \geq 0$. Once again, as a normalization, we assume that $U(0) = 0$ and therefore $U(x) > 0 \; \forall x > 0$.

In this setting, we consider the usual measure of the worker´s risk-preferences:

$$\gamma_x = -\frac{U''(x)}{U'(x)} x \qquad (2)$$

In this section, we consider two situations. First, we study what happens when the worker is risk-averse, in which case $U''(x) < 0, \gamma_x > 0 \; \forall x > 0$. In this scenario, γ_x is the Arrow-Pratt measure of relative risk-aversion. Second, we obtain our results when the worker is risk-lover: $U''(x) > 0, \gamma_x < 0 \; \forall x > 0$. In this scenario, γ_x is a measure of relative risk-loving.

As in the previous section, we consider the nonselective and selective scenarios:

Nonselective Educational System. Once again, we consider assumptions 1 and 2 and substitute assumption 3 with the following condition:

Assumption 4. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \frac{P_0}{2\delta \overline{U}(t)} U'\left(\frac{t_0+t}{2} P_0\right) - \frac{1}{\overline{U}(t)} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{ U'\left[\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0 + \varepsilon)\right)\right\} f(\varepsilon) \, d\varepsilon \; \forall t \in [t_0, t_n]$.

Where $\overline{U}(t) = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left[\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - c(t,e_1)\right] f(\varepsilon) d\varepsilon > 0$.

Selective Educational System. In this set-up, we consider assumptions 1´ and 2´ and substitute assumption 3´ with the following condition:

Assumption 4´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \frac{P_0}{2\delta \overline{U}(t)} U'\left(\frac{t_0+t}{2} P_0\right) - \frac{1}{\overline{U}(t)} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{ U'\left[\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0 + \varepsilon)\right)\right\} f(\varepsilon) \, d\varepsilon \; \forall t \in [t_0, t_n]$.

When the worker is risk-neutral, $\overline{U}(t) = \kappa = U'(x) \; \forall x$ and assumptions 4 and 4´ are equivalent to assumptions 3 and 3´ respectively.

Assuming that the prior distribution function of the worker´s productivity is uniform, $t \sim U[t_0, t_n]$, and that it is independent of the distribution of ε, once again, the worker´s type, t^*, who is indifferent between studying and not studying in a separating equilibrium will be given by equation (1).

The effects of changes in δ, e_1 and P_0 on t^* are the same as those described in propositions 1 and 2 in the nonselective and selective scenarios.

Now, we analyze the effects of a parallel change in the measure of the worker´s risk-preferences, that is, we assume that $|\gamma_x^n| = |\gamma_x^o| + \alpha$, where γ_x^o and γ_x^n are the old and new measures of risk-preferences and $\alpha \in \mathbb{R}^+$ is the change in that measure, which is the same for all levels of consumption. Our next proposition shows the relationship between this type of change in the workers´ risk-preferences and their incentives to invest in education.

PROPOSITION 3. There exists $m \in \mathbb{R}^+$ *such that:*

i. *When either the worker is risk-averse and the assumptions of the nonselective scenario are satisfied or the worker is risk-lover and the assumptions of the selective scenario are satisfied,* $\frac{\partial t^*}{\partial \alpha} \gtreqless 0$ *if* $t_n - t_0 \lesseqgtr m$.

ii. *When either the worker is risk-averse and the assumptions of the selective scenario are satisfied or the worker is risk-lover and the assumptions of the nonselective scenario are satisfied,* $\frac{\partial t^*}{\partial \alpha} \lesseqgtr 0$ *if* $t_n - t_0 \lesseqgtr m$.

There are three ingredients of our model that will help us to understand the results included in proposition 3. First, the cost of the educational investment has two components that lead to the signaling role of education. On the one hand, the opportunity cost of education, that is, the wage lost during the investment period increases with t^* and when t^* is sufficiently high, low-productivity workers prefer not to incur that cost. This the main reason why high productivity workers invest in education in the nonselective scenario. On the other hand, the monetary cost of education incurred by the indifferent type decreases with t^* and as a result, high-productivity workers signal their low cost of education by achieving the high educational level. This is the main reason why high productivity workers invest in education in the selective scenario. In our model, the nonselective educational environment is strengthened when the cost of education increases with t^*, whereas the selective scenario is reinforced otherwise.

Second, workers have to decide whether they invest in education or not in period 0 with perfect knowledge of the price in that period, but they do not know the price of the product in period 1 and for this reason, the wage received in that period is uncertain. Under these assumptions, the level of income obtained by the worker in equilibrium in period 1 is equal to the wage minus the cost of education:

$$I_1(e_i) = E(t|e_i)(\rho P_0 + \varepsilon) - 1(e = e_1)c(t, e_1)\ \forall i \in \{0,1\} \quad (3)$$

Where $E(t|e_i)$ is the expected productivity of those worker´s types who chose e_i in period 0.

As shown in equation (3), the wage received by the worker in period 1 is the sum of a certain ($E(t|e_i)\rho P_0$) and an uncertain component ($E(t|e_i)\varepsilon$). In the separating equilibrium considered, the educated worker takes greater risks than the uneducated because $\left|\frac{t^*+t_n}{2}\varepsilon\right| \geq \left|\frac{t_0+t^*}{2}\varepsilon\right|\ \forall\varepsilon$. Furthermore, the additional risk assumed by the educated worker ($\left|\frac{t_n-t_0}{2}\varepsilon\right|$) is proportional to $t_n - t_o$.

Finally, the certain component of income received by an educated type in period 1 ($\frac{t^*+t_n}{2}\rho P_0 - c(t, e_1)$) must be greater than that obtained by an uneducated type ($\frac{t_0+t^*}{2}\rho P_0$) when the worker is risk averse. Otherwise, no worker´s type would give up the wage in period

0 in order to invest in education, which is risky. For this reason, the risk premium demanded by an educated worker will differ from that demanded by an uneducated worker when that employee is risk-averse. Differently, when the worker is risk-lover, the certain component of the income received by an educated type in period 1 may be lower than that received by an uneducated type because a risk lover would be willing to pay a price in order to take the additional risk associated with education. As a result, the price the educated and uneducated workers would be willing to pay in order to take additional risk will be different when the worker is risk lover.

In summary, the nonselective or selective scenario will be reinforced depending on whether the cost of education increases or decreases with the indifferent type, educated types take greater risk than uneducated types in equilibrium and both types of workers will value the same risk differently in the separating equilibrium considered.

Now, it is easy to understand the results shown in proposition 3. We start with the analysis of the risk-averse worker. We distinguish two cases. First, when $t_n - t_o$ is sufficiently high, an increase in the level of risk aversion will make the cost of the additional risk associated with the educational investment much greater and as a result, the cost of education will increase with t^* by more, or will decrease with t^* by less. Therefore, an increase in the level of risk aversion strengthens the nonselective educational scenario and weakens the selective scenario. For those reasons, when the degree of risk aversion goes up, more types invest in education in the nonselective scenario, that is, t^* decreases (see part i of proposition 3) and fewer types choose the high level of education in the selective scenario, that is, t^* goes up (see part ii of proposition 3). Second, when $t_n - t_o$ is sufficiently low, the expected productivities among educated and uneducated workers will be similar and the wage received will barely increase with education. In this case, the income received by educated workers is lower than that received by uneducated workers and consequently, the only reason why the highest worker´s types invest in education must be because the risk premium demanded by educated types is lower than that demanded by uneducated workers, which only happens when the risk premium demanded by the worker decreases with her level of consumption. When the indifferent type goes up in equilibrium, the expected income received by educated types in equilibrium goes up because their expected productivity is greater and their cost of education is lower. Hence, when the level of risk aversion goes up, the drop in the risk premium demanded by educated types increases with the indifferent type and this relationship weakens the nonselective scenario and reinforces the selective environment. For those reasons, when the worker´s risk aversion goes up, fewer types invest in education in the nonselective scenario as shown by part i of proposition 3 and more types invest in education in the selective scenario as shown by part ii.

To finish off the analysis of proposition 3, we describe the workers´ incentives when they are risk-lover. Once again, when $t_n - t_o$ is sufficiently high, the risk associated with the educational investment increases significantly with t^*, which reduces the cost of education incurred by the risk-loving worker. Consequently, when the level of risk-loving goes up, that drop in the cost of education with t^* increases. For this reason, the nonselective scenario is weakened and fewer types invest in education in that setting as shown by part ii of proposition 3, whereas the selective environment is reinforced and more types invest in education in that scenario as shown by part i. Lastly, when $t_n - t_o$ is sufficiently low, the risk associated to education is low. In addition to that, uneducated and educated workers receive similar wages in period 1, but the net income received by educated types will be lower than that received by uneducated types because the former have to pay the educational cost. Therefore, the only reason why the highest types incur the cost of education must be that their measure of risk loving is greater, which happens when the level of risk-loving goes down with the level of consumption. Therefore, when the indifferent type goes up in equilibrium, educated types receive a greater income as a result of a greater expected productivity and a lower educational cost and their level of risk-loving will go down, which causes an increase in the cost of education. When the level of risk loving is greater, that additional cost of the risky educational investment is even greater. Consequently, the nonselective scenario is reinforced and the selective setting is weakened when $t_n - t_o$ is sufficiently low and the effects of a change in α on t^* are reversed as shown by proposition 3.

In this section, we have shown the relationship between the workers´ preferences for risk and their incentives to invest in education in our model. Regardless of whether the worker is risk-averse or risk-lover, the effect of a change in other parameters on the separating equilibrium obtained are the same as those described by propositions 1 and 2. As shown by propositions 1, 2 and 3, the comparative statics considered crucially depends on whether the assumptions of the nonselective or selective scenario are satisfied. For this reason, we add two lemmas in order to determine some parametric conditions under which those settings will arise.

LEMMA 3. If the worker is risk-averse, there exist $m, \overline{\gamma}_1, \overline{\gamma}_2, \Delta_1, \Delta_2 \in \mathbb{R}^+$ *such that the assumptions of the nonselective scenario will be satisfied as long as* $t_n - t_0 > m, \gamma_c > \overline{\gamma}_1 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1 \; \forall t \in [t_0, t_n]$*, whereas the assumptions of the selective scenario will hold as long as* $t_n - t_0 < m, \gamma_c > \overline{\gamma}_2 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_0, t_n]$.

LEMMA 4. If the worker is risk-lover, there exist $m, \overline{\gamma}_3, \overline{\gamma}_4, \Delta_3, \Delta_4 \in \mathbb{R}^+$ *such that the assumptions of the nonselective scenario will be satisfied providing that* $t_n - t_0 < m, \gamma_c > \overline{\gamma}_3 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3 \; \forall t \in [t_0, t_n]$*, whereas the assumptions of the selective scenario will hold as long as* $t_n - t_0 > m, \gamma_c > \overline{\gamma}_4 \; \forall c > 0$ *and* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4 \; \forall t \in [t_0, t_n]$.

Like in the previous section with risk-neutrality, those lemmas show that the nonselective (selective) scenario is more likely to appear when rate at which the cost of education decreases with the worker´s ability is sufficiently low (high). As shown in our analysis of proposition 3, when the worker is risk-averse, the risk associated with the educational investment strengthens the nonselective (selective) environment when $t_n - t_0$ is sufficiently high (low). For this reason, when the worker´s measure of risk aversion is sufficiently high, lemma 3 shows that the nonselective (selective) scenario is more likely to appear if $t_n - t_0$ is sufficiently high (low). Finally, when the worker is risk-lover, our analysis of proposition 3 shows that the risk associated with education strengthens the selective (nonselective) setting when $t_n - t_0$ is sufficiently high (low). As a result, when the worker is sufficiently risk-lover, lemma 4 shows that the selective (nonselective) scenario is more likely to arise if $t_n - t_0$ is sufficiently high (low).

Our results suggest that the effect of changes in the labor market conditions on workers´ incentives to invest in education will depend on the rate at which the educational cost decreases with students´ ability, on students´ risk preferences and on employers´ uncertainty about workers´ productivity in the labor market. Specifically, lemmas 3 and 4 will guide future empirical researchers in order to determine the conditions under which a specific change may affect educational attainment in one way or another.

6 Robustness to the specification of the prior distribution of worker´s types

In previous sections, we assumed that the worker´s type was drawn from a uniform distribution. In this section, we analyze the effect of specifying other types of distribution on the results obtained. In order to see the extent to which our results can be generalized, we will use the general relationship between the sender´s incentives to signal and the prior distribution of sender´s types obtained by Adriani and Sonderegger (2019) and Jewit (2004).

Now, let $g: T \rightarrow [0,1]$ denote the prior density function from which the sender´s type is drawn at the beginning of the signaling game we are describing. In our model, the signal can only take two values and the sender´s incentives to choose the high value is given by the following expression in the separating equilibrium considered:

$$\phi(t^*) = E(t|t > t^*) - E(t|t < t^*) \quad (4)$$

Where t^* is the worker´s type who is indifferent between the low and high values of the signal. In this setting, $\phi(t^*) > 0$ measures the worker´s incentive to invest in education.

We summarize the results obtained by Adriani and Sonderegger (2019) and Jewitt (2004) in the next lemma.

LEMMA 5. $\phi'(t^*)$ *will depend on the shape of* $g(t)$ *as follows:*

I. *If* g *is an everywhere increasing (decreasing) function, then* ϕ *will be decreasing (increasing) everywhere.*

II. *If* g *is strictly increasing and then decreasing (unimodal), then there exists* $t_m \in [t_0, t_n]$ *such that* ϕ *is strictly decreasing when* $t < t_m$ *and strictly increasing when* $t > t_m$*. Moreover, if* g *is symmetric, then* t_m *coincides with the mode of* g.

III. *If* g *is strictly decreasing and then increasing, then there exists* $t_M \in [t_0, t_n]$ *such that* ϕ *is strictly increasing when* $t < t_M$ *and strictly decreasing when* $t > t_M$*. Moreover, if* g *is symmetric, then* t_M *coincides with the anty-mode of* g.

The proof of this lemma can be found in Adriani and Sonderegger[4] (2019).

In this setting, when the worker is risk-neutral, the type who is indifferent between investing or not investing in education, t^*, will be given by:

[4] See their lemmas 1 and 2.

$$U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] = \delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \quad (5)$$

Once again, we consider both scenarios previously described.

Nonselective Educational System. We assume that the following conditions will be satisfied:

Assumption 5. $U(t_0 P_0) + \delta U\big[t_0\big(\rho P_0 + E(\varepsilon)\big)\big] < \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big) - c(t_0, e_1)\big]$.

Assumption 6. $U[E(t)P_0] + \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big)\big] > \delta U\big[t_n\big(\rho P_0 + E(\varepsilon)\big) - c(t_n, e_1)\big]$.

Assumption 7. $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| < \frac{P_0}{\delta}\frac{\partial E(t|t < t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big)\ \forall t^* \in [t_0, t_n]$.

Selective Educational System. We assume that the following conditions will be satisfied:

Assumption 8. $U(t_0 P_0) + \delta U\big[t_0\big(\rho P_0 + E(\varepsilon)\big)\big] > \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big) - c(t_0, e_1)\big]$.

Assumption 9. $U[E(t)P_0] + \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big)\big] < \delta U\big[t_n\big(\rho P_0 + E(\varepsilon)\big) - c(t_n, e_1)\big]$.

Assumption 10. $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \frac{P_0}{\delta}\frac{\partial E(t|t < t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big)\ \forall t^* \in [t_0, t_n]$.

As in the previous section, we also assume that the worker´s utility from the minimum level of consumption in period 1 is positive, that is, $U(t_0 P_0) > 0$.

Now, we can determine the effect of a change in the parameters of the model on the worker´s incentives to invest in education.

PROPOSITION 4. For any prior distribution of sender´s types, t^* *increases with* δ *and decreases with* e_1 *in the nonselective scenario and* t^* *decreases with* δ *and increases with* e_1 *in the selective setting. Furthermore, there exists a certain threshold,* $K \in \mathbb{R}^+$*, such that* t^* *decreases (increases) with* P_0 *when* $\rho < K$ $(\rho > K)$ *in the nonselective set-up, but* t^* *increases (decreases) with* P_0 *when* $\rho < K$ $(\rho > K)$ *in the selective scenario.*

These results are the same as those included in propositions 1 and 2, which means that the relationship between the worker´s incentives to invest in education and the labor market outcomes is robust to the specification of the prior distribution of sender´s types when the worker is risk-neutral.

When the prior distribution of worker´s types is concentrated on the highest types or when $g(\cdot)$ is increasing ($\phi(\cdot)$ decreases everywhere as shown in lemma 5), then the nonselective scenario is more likely to appear. On the contrary, when the prior distribution of worker´s types is concentrated on the lowest types or when $g(\cdot)$ is decreasing ($\phi(\cdot)$ increases everywhere as shown in lemma 5), then the selective scenario is more likely to arise. Therefore, some sufficient conditions for each scenario are shown in our next lemma.

LEMMA 6. There exists $c_1 \in \mathbb{R}^+$ *such that the nonselective scenario will arise providing that* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1 \; \forall t \in [t_0, t_n]$ *and* $g(\cdot)$ *is increasing everywhere. Likewise, there exists* $c_2 \in \mathbb{R}^+$ *such that the selective scenario will arise as long as* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2 \; \forall t \in [t_0, t_n]$ *and* $g(\cdot)$ *is decreasing everywhere.*

This lemma shows that the nonselective scenario will be more likely to arise in those labor markets where most workers are highly productive and the educational system is universal. On the contrary, the selective environment will arise as long as most workers are less productive and the educational system is more selective.

7 Conclusions

In this article, we analyze the effects of a change in the labor market conditions on people´s incentives to invest in education. In order to meet this goal, we extend Spence´s model of signaling by introducing the opportunity cost of education caused by the wage lost during the investment period. Specifically, we consider a two-period game in which a worker with private information on her productivity in the labor market has to decide whether she invests in education or not in the first period. Unlike the classical model, we assume that the worker cannot participate in the labor market during the investment period, but she may use a high level of education as a signal of her higher productivity in the second period, in which case she will receive a greater wage.

In this setting, the wage lost during the investment period is a key component of the cost of education in our model. Specifically, an equilibrium arises in which only those workers with the highest ability invest in education. In this equilibrium, the wage received by uneducated workers is sufficiently high because sufficiently high types of worker invest in education. As a result, the wage lost by educated workers during the investment period is so high that low ability workers do not have incentives to choose the high level of education. Consequently, the employer will be able to identify those workers with the highest productivity by using the educational credentials. In contrast to the classical model, we found that this separating equilibrium may arise even when the monetary cost of education does not change with the worker´s ability, that is when the single-crossing condition is not satisfied.

On the contrary, when the single-crossing condition is satisfied and the cost of education decreases sufficiently with the worker´s ability, the classical signaling equilibrium will also arise and high-ability workers will use a high level of education as a signal of their productivity. As expected, when the worker is more patient, when prospects of the labor market conditions in the future improve, or when the cost of education goes down, the incentives to invest in education become stronger in this scenario. Interestingly, the opposite occurs when the cost of education does not decrease sufficiently with the student´s ability.

We also identify some sufficient conditions under which the nonselective or the selective scenario will arise. The rate at which the cost of education decreases with the student´s ability plays a key role and the nonselective (selective) setting is more likely to arise when that rate is sufficiently low (high). Apart from that condition, our results suggest that the nonselective environment is more likely to be obtained when workers are sufficiently risk-averse and employers´ uncertainty on the workers´ productivity is sufficiently high, or when workers are sufficiently risk-lover and employers´ uncertainty on workers´ productivity is sufficiently low. Furthermore, a prior distribution of worker´s productivities sufficiently concentrated on the highest types will also lead to the nonselective scenario even if workers are risk neutral. Under those conditions, employers will be able to use education as a mechanism in order to separate low from high ability workers even if low ability students can easily achieve a high educational level. Similarly, the selective scenario is more likely to appear when workers are sufficiently risk-averse and employers´ uncertainty on workers´ productivity is sufficiently low, or when workers´ level of risk-loving is sufficiently high and employers´ uncertainty on workers´ productivity is sufficiently high. In addition to that, a labor market dominated by low-productivity workers will also lead to the selective setting. In that setting, high-productivity workers will have strong incentives to use a high level of education as a signal of their greater productivity.

Finally, we found that the relationship between the labor market conditions and workers´ incentives to invest in education is robust to the specification of workers´ risk preferences and to the prior distribution of workers´ types considered.

Appendix

Proof of Lemma 1. To start with, we define a function representing the worker´s additional profit from choosing e_0 when those types lower than $t \in T$ choose e_0 and those greater than t choose e_1:

$$F(t,\delta,K) = U\left(\frac{t_0+t}{2}P_0\right) + \delta\left[\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - K\right) f(\varepsilon)d\varepsilon\right] \quad \text{(A.1)}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \to \mathbb{R}$. Under assumptions 1 and 2, δ and K must take values such that $F(t_0, \delta, K) < 0$ and $F(t_n, \delta, K) > 0$. Since $\frac{\partial F}{\partial t} = \frac{P_0}{2} U'\left(\frac{t_0+t}{2} P_0\right) > 0 \; \forall t \in [t_0, t_n]$, there exists a unique $t^* \in [t_0, t_n]$ such that $F(t^*) = 0$, which is equation (1) defining the indifferent worker´s type in a separating equilibrium. QED.

Proof of Lemma 2. Given the function defined in (A.1), the indifference equation shown in (1) is equivalent to $F(t^*, \delta, K) = 0$. Then, we obtain the following derivatives:

$$\frac{\partial F(t^*,\delta,K)}{\partial \delta} = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - K\right) f(\varepsilon) d\varepsilon \tag{A.2}$$

Since $U(0) = 0$, $U'(\cdot) > 0$ and $\frac{t_0+t}{2} P_0 > 0 \; \forall t \in [t_0, t_n]$, then $U\left(\frac{t_0+t^*}{2} P_0\right) > 0$. As a result, $\frac{\partial F(t^*,\delta,K)}{\partial \delta} < 0$ so that the indifference equation (1) is satisfied.

Similarly, it is straightforward to see that $\frac{\partial F(t^*,\delta,K)}{\partial K} > 0$. Finally, since $\frac{\partial F(t^*,\delta,K)}{\partial t^*} = \frac{P_0}{2} U'\left(\frac{t_0+t^*}{2} P_0\right) > 0$, we use the implicit function theorem in order to obtain the desired results:

$$\frac{\partial t^*}{\partial \delta} = -\frac{\frac{\partial F(t^*,\delta,K)}{\partial \delta}}{\frac{\partial F(t^*,\delta,K)}{\partial t^*}} > 0 \tag{A.3}$$

$$\frac{\partial t^*}{\partial K} = -\frac{\frac{\partial F(t^*,\delta,K)}{\partial K}}{\frac{\partial F(t^*,\delta,K)}{\partial t^*}} < 0 \tag{A.4}$$

This completes the proof of lemma 2. QED.

Proof of Proposition 1. First of all, we rewrite equation (1) as the following function of the indifferent worker´s type, t^*:

$$F(t^*, \delta, P_0, \rho, e_1) = U\left(\frac{t_0+t^*}{2} P_0\right) + \delta\left\{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \right.$$

$$\left.\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon) d\varepsilon\right\} = 0 \tag{A.5}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \times (0, +\infty) \times \mathbb{N} \to \mathbb{R}$ is the real valued function defined in equation (A.5). If we derive this function with respect to each variable, we obtain the following expressions:

$$\frac{\partial F}{\partial t^*} = \kappa\left[\frac{P_0}{2} + \delta c_1(t^*, e_1)\right] \tag{A.6}$$

$$\frac{\partial F}{\partial \delta} = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon) d\varepsilon \tag{A.7}$$

$$\frac{\partial F}{\partial P_0} = \kappa\left(\frac{t_0+t^*}{2} - \delta\frac{t_n-t_0}{2}\rho\right) \tag{A.8}$$

$$\frac{\partial F}{\partial \rho} = -\delta\kappa P_0 \frac{t_n-t_0}{2} \tag{A.9}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*, e_1) \tag{A.10}$$

Where $\kappa = U'(x)\ \forall x \in \mathbb{R}$, $c_1(t^*, e_1) = \frac{\partial c(t^*, e_1)}{\partial t^*}$ and $c_2(t^*, e_1) = \frac{\partial c(t^*, e_1)}{\partial e_1}$.

It is easy to see that $\frac{\partial F}{\partial t^*} > 0$ under assumption 3 and $\frac{\partial F}{\partial t^*} < 0$ under assumption 3´.

Additionally, since $U(0) = 0$, $U'(\cdot) > 0$ and $\frac{t_0+t^*}{2} P_0 > 0\ \forall t^* \in [t_0, t_n]$, then $U\left(\frac{t_0+t^*}{2} P_0\right) > 0$. As a result, $\frac{\partial F}{\partial \delta} < 0$ so that equation (A.5) is satisfied.

Finally, as shown by equations (A.9) and (A.10), $\frac{\partial F}{\partial \rho} < 0$ and $\frac{\partial F}{\partial e_1} > 0$. Thus, if we use the implicit function theorem, we obtain the desired results under assumption 3:

$$\frac{\partial t^*}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial t^*}} > 0 \tag{A.11}$$

$$\frac{\partial t^*}{\partial \rho} = -\frac{\frac{\partial F}{\partial \rho}}{\frac{\partial F}{\partial t^*}} > 0 \tag{A.12}$$

$$\frac{\partial t^*}{\partial e_1} = -\frac{\frac{\partial F}{\partial e_1}}{\frac{\partial F}{\partial t^*}} < 0 \tag{A.13}$$

Under assumption 3´, these inequalities are reversed because $\frac{\partial F}{\partial t^*} < 0$ and this completes the proof of proposition 1. QED.

Proof of Proposition 2. From equation (A.8), we see that $\frac{\partial F}{\partial P_0} > 0$ when $\rho = 0$ and $\frac{\partial F}{\partial P_0}$ decreases with ρ. Then, there will be a threshold, $K \in \mathbb{R}^+$, such that $\frac{\partial F}{\partial P_0} > 0$ when $\rho < K$, whereas $\frac{\partial F}{\partial P_0} < 0$ when $\rho > K$. Once again, we use the implicit function theorem in order to obtain the desired results. In particular, under assumptions 1, 2 and 3, we conclude that $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho > K$. Likewise, under assumptions 1´, 2´ and 3´, we obtain that $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho > K$. QED.

Proof of Proposition 3. First of all, we rewrite equation (1) of the indifferent worker´s type, t^*:

$$F(t^*, \delta, P_0, \rho, e_1) = U\left(\frac{t_0+t^*}{2} P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left\{U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon = 0 \tag{A.14}$$

Where $F: [t_0, t_n] \times [0,1] \times [0, +\infty) \times (0, +\infty) \times \mathbb{N} \rightarrow \mathbb{R}$.

Next, we obtain the following derivative:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left\{\frac{\rho P_0+\varepsilon}{2} U'\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - \left(\frac{\rho P_0+\varepsilon}{2} - \frac{\partial c(t^*,e_1)}{\partial t^*}\right) U'\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon \tag{A.15}$$

In order to find out the sign of $\frac{\partial F}{\partial t^*}$, we rewrite the derivative shown in (A.15) as:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) - \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{U'\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] - U'\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon)\, d\varepsilon \quad \text{(A.16)}$$

Using the mean value theorem (Lagrange theorem), we know that there exists $c_\varepsilon^* \in \mathbb{R}$ such that equation (A.16) can be expressed as[5]:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) - \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] U''(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon)\, d\varepsilon \quad \text{(A.17)}$$

Where c_ε^* may be different for each value of ε. Lastly, we substitute the second derivative of the utility function in the above expression with our measure of the worker´s relative risk-preference and obtain the following result:

$$\frac{\partial F}{\partial t^*} = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\partial c(t^*,e_1)}{\partial t^*} U'\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon)\, d\varepsilon \quad \text{(A.18)}$$

Where $\gamma_{c_\varepsilon^*} = -\frac{U''(c_\varepsilon^*)c_\varepsilon^*}{U'(c_\varepsilon^*)}$. Therefore, $\frac{\partial F}{\partial t^*}$ is the sum of three components:

i. $A = \frac{P_0}{2} U'\left(P_0 \frac{t_0+t^*}{2}\right) > 0$ because the worker´s utility function is strictly increasing.

ii. $B = \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left[\frac{t_n-t_0}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon.$

[5] If $\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon) < \frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)$, then $c_\varepsilon^* \in \left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon), \frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right)$. Otherwise, $c_\varepsilon^* \in \left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1), \frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right)$.

iii. $C = \delta \frac{\partial c(t^*, e_1)}{\partial t^*} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U' \left[\frac{t^* + t_n}{2} (\rho P_0 + \varepsilon) - c(t^*, e_1) \right] f(\varepsilon)\, d\varepsilon$. This component is a negative number because $\frac{\partial c(t^*, e_1)}{\partial t^*} < 0$ and the utility function is strictly increasing.

In the separating equilibrium considered, $F(t^*) = 0$, as shown by equation (A.14).

In order to determine the sign of B, we rewrite it in the following way:

$$B = \delta \left[\frac{t_n - t_0}{4} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} (\rho P_0 + \varepsilon)^2 \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon - \right.$$

$$\left. c(t^*, e_1) \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0 + \varepsilon}{2} \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon)\, d\varepsilon \right] \qquad \text{(A.19)}$$

We consider two cases:

<u>Case I</u>. (Risk Aversion). In this case, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) > 0\ \forall c_\varepsilon^*$ and $B \lesseqgtr 0$ providing that $t_n - t_0 \lesseqgtr m$, where:

$$m = \frac{c(t^*, e_1) \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \left(\frac{\rho P_0 + \varepsilon}{2} \right) \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon) d\varepsilon}{\frac{1}{4} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} (\rho P_0 + \varepsilon)^2 \frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) f(\varepsilon) d\varepsilon} > 0 \qquad \text{(A.20)}$$

In this context, $|B|$ increases when $\gamma_{c_\varepsilon^*}$ goes up for all values of c_ε^*. In the nonselective scenario, $\frac{\partial F}{\partial t^*} > 0$ and when $t_n - t_0 < m$, then $B < 0$ and a parallel rise in $\gamma_{c_\varepsilon^*}$ will cause a drop in $\frac{\partial F}{\partial t^*}$ for all values of t^*. Thus, the value of t^* at which $F(t^*) = 0$ will increase[6]. In the selective scenario, $\frac{\partial F}{\partial t^*} < 0$ and when $t_n - t_0 < m$, then $B < 0$ and a parallel rise in $\gamma_{c_\varepsilon^*}$ will cause a drop in $\frac{\partial F}{\partial t^*}$. Thus, the value of t^* at which $F(t^*) = 0$ will go down. Clearly,

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[6] As shown by equation (A.14), the first component of $F(t^*)$ is $U\left(\frac{t_0 + t^*}{2} P_0\right)$, which increases with $t^*$and that increase with t^* does not depend on γ, but the rate at which the remaining part of $F(t^*)$ changes with t^* depends on γ as shown by B. Therefore, $F(t^*)$ is equal to zero when t^* is equal to the value at which $U\left(\frac{t_0 + t^*}{2} P_0\right)$ is equal to the absolute value of the remaining part of $F(t^*)$.

when $t_n - t_0 > m$, all those effects of a parallel change in $\gamma_{c_\varepsilon^*}$ on t^* are reversed because $B > 0$.

<u>Case II</u>. (Risk Loving). In this case, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) < 0 \; \forall c_\varepsilon^*$ and $B \lesseqgtr 0$ providing that $t_n - t_0 \gtreqless m$.

Once again, $|B|$ increases when $\gamma_{c_\varepsilon^*}$ goes up for all values of c_ε^*. When $t_n - t_0 < m$, then $B > 0$ and an increase in $\gamma_{c_\varepsilon^*}$ for all values of c_ε^* will cause a rise in B. As a result, $\frac{\partial F}{\partial t^*}$ will increase in the nonselective scenario and the value of t^* at which $F(t^*) = 0$ will go down. In the selective scenario, $\frac{\partial F}{\partial t^*} < 0$ and an increase in $\gamma_{c_\varepsilon^*}$ for all values of c_ε^* will cause a rise in B and $\left|\frac{\partial F}{\partial t^*}\right|$ will go down, which implies that the value of t^* at which $F(t^*) = 0$ will go up. On the contrary, the sign of the effects of a change in the measure of the worker´s risk loving on t^* will be reversed in each scenario when $t_n - t_0 > m$ because $B < 0$.

This completes the proof of proposition 3. QED.

Proof of Lemma 3. As shown by assumptions 4 and 4´, the nonselective scenario arises when $\frac{\partial F}{\partial t^*} > 0$ and the selective scenario appears when $\frac{\partial F}{\partial t^*} < 0$. As shown in the proof of proposition 3, $\frac{\partial F}{\partial t^*} = A + B + C$, where $A > 0, C < 0$ and $|B|$ goes up when γ_c goes up for all levels of c.

When the worker is risk-averse, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) > 0$ and the sign of B depends on $t_n - t_0$. In particular, there exists $m \in \mathbb{R}^+$ such that $B \lesseqgtr 0$ if $t_n - t_0 \lesseqgtr m$. We distinguish two cases:

Case I. When $t_n - t_0 < m$, then $B < 0$. In this case, there exist $\overline{\gamma}_2, \Delta_2 \in \mathbb{R}^+$ such that $A < -(B + C)$ when $\gamma_c > \overline{\gamma}_2 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_o, t_n]$. Therefore, $\frac{\partial F}{\partial t^*} = A + B + C < 0$ when $t_n - t_0 < m, \gamma_c > \overline{\gamma}_2 \; \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_2 \; \forall t \in [t_o, t_n]$.

Case II. When $t_n - t_0 > m$, then $B > 0$. In this case, there exist $\overline{\gamma}_1, \Delta_1 \in \mathbb{R}^+$ such that $A + B > -C$ providing that $\gamma_c > \overline{\gamma}_1\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1\ \forall t \in [t_o, t_n]$. Hence, $\frac{\partial F}{\partial t^*} = A + B + C > 0$ when $t_n - t_0 > m, \gamma_c > \overline{\gamma}_1\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_1\ \forall t \in [t_o, t_n]$.

This completes the proof of lemma 3. QED.

Proof of Lemma 4. Once again, we use the expression of the derivative shown in the proof of proposition 3: $\frac{\partial F}{\partial t^*} = A + B + C$.

When the worker is risk-lover, $\frac{\gamma_{c_\varepsilon^*}}{c_\varepsilon^*} U'(c_\varepsilon^*) < 0$ and there exists $m \in \mathbb{R}^+$ such that $B \lesseqgtr 0$ if $t_n - t_0 \gtreqless m$. We distinguish two cases:

Case I. When $t_n - t_0 < m$, then $B > 0$. In this case, there exist $\overline{\gamma}_3, \Delta_3 \in \mathbb{R}^+$ such that $A + B > -C$ when $\gamma_c > \overline{\gamma}_3\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3\ \forall t \in [t_o, t_n]$. Therefore, $\frac{\partial F}{\partial t^*} = A + B + C > 0$ when $t_n - t_0 < m, \gamma_c > \overline{\gamma}_3\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \Delta_3\ \forall t \in [t_o, t_n]$.

Case II. When $t_n - t_0 > m$, then $B < 0$. In this case, there exist $\overline{\gamma}_4, \Delta_4 \in \mathbb{R}^+$ such that $A < -(B + C)$ providing that $\gamma_c > \overline{\gamma}_4\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4\ \forall t \in [t_o, t_n]$. Hence, $\frac{\partial F}{\partial t^*} = A + B + C < 0$ when $t_n - t_0 > m, \gamma_c > \overline{\gamma}_4\ \forall c > 0$ and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \Delta_4\ \forall t \in [t_o, t_n]$.

This completes the proof of lemma 4. QED.

Proof of Proposition 4. To start with, we rewrite the indifference condition included in equation (5) as:

$$F[t^*, \delta, P_0, E(P_1|P_0), e_1] = U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] -$$

$$\delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] = 0 \qquad \text{(A.21)}$$

If we derive this function with respect to the indifferent type, we obtain:

$$\frac{\partial F}{\partial t^*} = \kappa P_0 \frac{\partial E(t|t < t^*)}{\partial t^*} - \delta\kappa \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big) + \delta\kappa \frac{\partial c(t^*, e_1)}{\partial t^*} \quad \text{(A.22)}$$

Recall that $\kappa = \frac{\partial U(x)}{\partial x}$, which is constant because the worker is risk-neutral. Under assumption 7 (assumption 10), $\frac{\partial F}{\partial t^*} > 0$ ($\frac{\partial F}{\partial t^*} < 0$).

Similarly, we obtain the following derivatives:

$$\frac{\partial F}{\partial \delta} = U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] - U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \quad \text{(A.23)}$$

$$\frac{\partial F}{\partial P_0} = \kappa[E(t|t < t^*) - \delta\phi(t^*)\rho] \quad \text{(A.24)}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*, e_1) \quad \text{(A.25)}$$

Now, we determine the signs of these derivatives. Recall that the indifference condition included in equation (5) can be expressed as:

$$U(E(t|t < t^*)P_0) = \delta\big\{U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] -$$
$$U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big]\big\} \quad \text{(A.26)}$$

The left-hand side of this equation is positive because $U(E(t|t < t^*)P_0) > U(t_0 P_0) > 0$. Therefore, the right-hand side of the equation is also positive, which implies that $\frac{\partial F}{\partial \delta} < 0$. It is also clear that $\frac{\partial F}{\partial e_1} > 0$. By using the implicit function theorem, we conclude that $\frac{\partial t^*}{\partial \delta} > 0$, and $\frac{\partial t^*}{\partial e_1} < 0$ in the nonselective scenario, whereas $\frac{\partial t^*}{\partial \delta} < 0$ and $\frac{\partial t^*}{\partial e_1} > 0$ in the selective scenario. Finally, $\frac{\partial F}{\partial P_0}$ in equation (A.24) is positive when $\rho = 0$ and strictly decreases with ρ. Hence, there exists $K \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$, which implies that $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ when $\rho \lesseqgtr K$ in the nonselective scenario and $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$ in the selective scenario.

QED.

Proof of Lemma 6. We need to analyze the sign of $\frac{\partial F}{\partial t^*}$, which is shown in equation (A.22).

As shown by lemma 5, when f is an everywhere increasing function, then $\frac{\partial \phi(t^*)}{\partial t^*} < 0\ \forall t \in [t_0, t_n]$. As a result, equation (A.22) shows that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| = 0$. Since $\frac{\partial F}{\partial t^*}$ decreases with $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right|$, there exists $c_1 \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| < c_1$.

Hence, we have proven that $\frac{\partial F}{\partial t^*} > 0$ as long as g is an everywhere increasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1\ \forall t \in [t_0, t_n]$, which is the first part of lemma 6.

Similarly, lemma 5 shows that $\frac{\partial \phi(t^*)}{\partial t^*} > 0\ \forall t \in [t_0, t_n]$ when g is an everywhere decreasing function. From equation (A.22), we can guarantee that $\frac{\partial F}{\partial t^*} < 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \max_{t^* \in [t_0, t_n]} \left\{\frac{P_0}{\delta} \frac{\partial E(t|t < t^*)}{\partial t^*}\right\} = c_2$.

Therefore, we have proven that $\frac{\partial F}{\partial t^*} < 0$ as long as g is an everywhere decreasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2\ \forall t \in [t_0, t_n]$, which is the second part of lemma 6 and this completes the proof. QED.

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