THEORY OF CONNECTIVITY AND APPORTIONMENT OF REPRESENTATIVE ACTIVITY CHAINS IN THE PROBLEM OF DECISSION-MAKING CONCERNING EARTHQUAKE POSSIBILITY

CRIADO F[1], GACHECHILADZE. T[2], JORJIASHVILI. N[2], MELADZE H.[2], SÁNCHEZ J.M.[3], SIRBILADZE G.[2], AND TSERTSVADZE. G[2]

ABSTRACT. In this paper a short model to expedite the study of a subjective estimate by means of a qualitative fuzzy technique has been developed, and recommendations for decision-making regarding the three principal elements of earthquakes (time, place and magnitude) has been formulated. Likewise, the problem related to the investigation of the possibility of using Atkins connectivity theory to deal with this subject has been studied.

Such a theory gives suitable information for decision-making concerning earthquake possibility in the shape of representative activity chains (precursors), which can not be obtained by other methods.

The most important and general result of this paper is a better understanding of the interaction of geophysical processes. Naturally, the development and application of indiscreet mathematical methods in earthquake prediction require further investigation.

Keywords: Fuzzy discrimination, connectivity analysis, earthquake, decision-making.

1. INTRODUCTION

To date, many earthquake analytical methods have been used, generally dependent on the characteristics of the analyzed precursors. These methods include, for instance, the analysis of the correlations between seismic patterns (Nikolaev 2001*a*), spectral analysis (Kiladze, Kachakidze, Kachakidze, Kukhianidze & Ramishvili 2001*b*), etc.; analysis generally based on a very limited picture of potential earthquake precursors (Kiladze, Kachakidze, Kachakidze, Kukhianidze & Ramishvili 2001*a*, Kiladze, Kachakidze, Kachakidze, Kukhianidze & Ramishvili 2001*b*), and which on many occasions take into account a sole factor (Kachachidze 2000, Nikolaev 2001*b*, Kiladze, Kukhianidze, Kachakidze, Ramishvili & Kachakhidze 2001, Kiladze, Kukhianidze, Kachakidze & Ramishvili 2000, Kachakhidze, Kachakhidze, Kiladze, Khukhianidze & Ramishvili 2001, Kachakhidze, Kachakhidze, Kiladze, Khukhianidze, Khvedelidze & Ramishvili 2001).

Over the last years, certain progress has been made in earthquake prediction. Several long-, intermediate- and short-term precursors of a different physical nature were discovered. Nearly all precursors are indiscreet on the background of strong noise and have several common features, as well as expressed regional variations. Therefore, earthquake prediction is essentially a problem of determining the parameters of a forthcoming catastrophe, i.e., its place, time and magnitude.

Two main problems must be solved to make further progress in the field of earthquake prediction. The first of which entails widening the experimental base by making new observations and collecting additional data. To start with, this would involve researching into global and regional prognostic parameters – image fields – of earthquake parameters

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[1] Department of Statistics and Operational Research, Campus de Teatinos, Faculty of Sciences, Malaga University, Spain. Phone/Fax: +34 952 13 29 70. Email: f_criado@uma.es.

[2] Department of Applied Mathematics and Computer Science. Tbilisi State University, Georgia.

[3] Doctoral student. Department of Statistics and Operational. Malaga University, Spain.

by means of astronomical observations. Other data of a geophysical nature to be collected include information on earthquake precursors (anomalies), which can be divided naturally into groups such as those related to the electric field, atmospheric pressure and so on.

The approach employed here is determined by the following factors:

1. the authors vision of the modern state of the problem of earthquake prediction;
2. the type of precursors characteristic of the region and available for measurement in real conditions;
3. the nature of the data and its analysis;
4. a suitable choice of data processing methods.

During the initial period dedicated to collecting and processing all available data it was believed that they were of a combined possibility-probability nature. A detailed analysis of this information was carried out and its possibility-probability nature was examined.

Based on these findings, the so-called fuzzy discrimination analysis (Norris, Pilsworth & Baldwin 1987) , along with the supplementary connectivity analysis (Atkin 1974, Criado, Gachechiladze, Meladze & Tsertsvadze 1998), was chosen as the most effective method for processing primary data.

The main advantage of the analytical method proposed in this paper is the opportunity it provides to study the combined data comprehensively.

2. DESCRIPTION AND STUDY OF THE PROBLEM

2.1. **Organization of information in a numerical-tabular database.** As in the discrimination analysis primary data, that is to say, the earthquake precursors, are presented in the form of frequency matrix (Norris et al. 1987).

$$\widehat{F} = \begin{array}{c} \\ \left(\begin{array}{cccc} M_1 & M_2 & ... & M_n \\ f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{array} \right) \end{array} \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array}, \qquad (1)$$

where M_k $(k = 1, \ldots, n)$ are values of earthquake power, A_l $(l = 1, \ldots, m)$ are certain activities (earthquake precursors and also constant characteristics of seismic regional data for forecasting “triggering” effects, etc.)(Criado, Gachechiladze, Meladze, Sánchez & Tsertsvadze 2001), f_{ij} $(i = 1, \ldots, m; j = 1, \ldots, n)$ is the relative frequency of A_i activity when the power of the earthquake is in the interval M_j. An initial analysis of this information is necessary for determining the precursor's image (horizontal entries of the frequency matrix).

For the vertical entries some subdivisions of intervals

$$[< 3],\ [3, 5],\ [> 5], \qquad (2)$$

which corresponds to weak, moderate and strong earthquakes respectively, will be proposed further on.

These intervals are consider to be values of fuzzy variables the linguistic description of which is

$$\begin{aligned} \text{“earthquake”} =& \text{weak earthquake}([< 3]) \text{ also} \\ & \text{moderate earthquake}([3, 5]) \text{ also} \\ & \text{strong earthquake}([> 5]) \end{aligned}$$

The so-called anomalies (precursors) in the behavior of some two-dimensional time functions (t_1, t_2) have been considered as activities (for example, anomalies in the behavior of the electrical field strain, vertical current, pressure gradient, and so on). The first component t_1 represents separate moments of measurement of function values during a ten-day period before an earthquake, with an interval of an hour. Component t_2 represents the year when the earthquake occurred. Such a structure corresponds to available data. Naturally, other types of data determine other structures.

A more detailed description of how to determine the "anomaly image" by fuzzy set-theoretic methods is cited in (Criado, Gachechiladze, Meladze, Tsertsvadze & Sirbiladze n.d.).

Thus for a fixed value of t_1 frequency matrix (1) is similar to the one shown in Table 1.

Table 1. Tabular representation of precursor frequencies

Earthquake strength		Weak earthquake(1)	Moderate earthquake(2)		Strong earthquake(3)		
Power anomaly		[< 3]	[3 − 4]	[4 − 5]	[5 − 6]	[6 − 7]	[> 7]
Anomaly of electrical field	$[\Delta\xi_{\min}, \Delta\xi^1]$	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
	$[\Delta\xi^1, \Delta\xi^2]$	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$[\Delta\xi^r, \Delta\xi^{\max}]$	$f_{(r+1)1}$	$f_{(r+1)2}$	$f_{(r+1)3}$	$f_{(r+1)4}$	$f_{(r+1)5}$	$f_{(r+1)6}$
Anomaly of pressure changes	$[\Delta\eta_{\min}, \Delta\eta^1]$	$f_{(r+2)1}$	$f_{(r+2)2}$	$f_{(r+2)3}$	$f_{(r+2)4}$	$f_{(r+2)5}$	$f_{(r+2)6}$
	$[\Delta\eta^1, \Delta\eta^2]$	$f_{(r+3)1}$	$f_{(r+3)2}$	$f_{(r+3)3}$	$f_{(r+3)4}$	$f_{(r+3)5}$	$f_{(r+3)6}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$[\Delta\eta^l, \Delta\eta^{\max}]$	$f_{(r+l+2)1}$	$f_{(r+l+2)2}$	$f_{(r+l+2)3}$	$f_{(r+l+2)4}$	$f_{(r+l+2)5}$	$f_{(r+l+2)6}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Group of constant factors	A_1	$f_{(k+1)1}$	$f_{(k+1)2}$	$f_{(k+1)3}$	$f_{(k+1)4}$	$f_{(k+1)5}$	$f_{(k+1)6}$
	A_2	$f_{(k+2)1}$	$f_{(k+2)2}$	$f_{(k+2)3}$	$f_{(k+2)4}$	$f_{(k+2)5}$	$f_{(k+2)6}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	A_s	$f_{(k+s)1}$	$f_{(k+s)2}$	$f_{(k+s)3}$	$f_{(k+s)4}$	$f_{(k+s)5}$	$f_{(k+s)6}$

In this table the precursors (anomalies) are naturally divided into groups: electrical field, atmospheric pressure, and so on. The whole range of anomaly changes is divided into intervals, each and every one of which is considered an activity.

Along with the aforementioned values indicating anomalies in the electric field, vertical current, pressure, gradient of pressure, magnetic field, the slow motion characteristics of the Earth's crust, and changes in the distribution of resilient wave velocity, data obtained from observing mineral waters, changes in the concentration of various chemical elements contained in subterranean waters and very noticeable precursors like the abnormal behavior of animals (especially fish) are also considered. Lightning, changes in the level of water in drill holes and "constant" characteristics such as seismologic zoning exactly forecasting "triggering" effects, can also be taken into account.

2.2. **Incidence matrix.** Instead of a frequency matrix $\widehat{F}$ of the discrimination analysis, let the "sub-matrix" corresponding to some interval of power and a certain geofield, normally atmospheric electrical field, be considered.

For example

$$\widehat{F}^{(k)} = \begin{array}{cccccc} M^{(k)}_{s_1} & M^{(k)}_{s_2} & M^{(k)}_{s_3} & M^{(k)}_{s_4} & M^{(k)}_{s_5} & \\ \left(\begin{array}{c} f_{s_1t_1} \\ f_{s_1t_2} \\ f_{s_1t_3} \\ f_{s_1t_4} \end{array}\right. & \begin{array}{c} f_{s_2t_1} \\ f_{s_2t_2} \\ f_{s_2t_3} \\ f_{s_2t_4} \end{array} & \begin{array}{c} f_{s_3t_1} \\ f_{s_3t_2} \\ f_{s_3t_3} \\ f_{s_3t_4} \end{array} & \begin{array}{c} f_{s_4t_1} \\ f_{s_4t_2} \\ f_{s_4t_3} \\ f_{s_4t_4} \end{array} & \left.\begin{array}{c} f_{s_5t_1} \\ f_{s_5t_2} \\ f_{s_5t_3} \\ f_{s_5t_4} \end{array}\right) & \begin{array}{c} A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array} \end{array}, \qquad (k = 1, 2, 3) \tag{3}$$

and replaced by a “linguistic” variant, where matrix elements are values of a linguistic variable. Here, the classical (non-fuzzy) case when this variable takes only two values: 1 (the given type of earthquake is accompanied by certain activity), and 0 (earthquake is not accompanied by this activity), is considered.

As an example, earthquakes occurring in Georgia from 1992-1995 were considered. Using Kaufman's experton theory the most typical intervals of activities were defined (Criado et al. 2001), this data constituting the horizontal entries of matrix (3). Vertical entries correspond to intervals of earthquake power grouped in three different matrices: weak ($k = 1$), moderate ($k = 2$) and strong ($k = 3$) earthquakes, and horizontal entries correspond to measures of the electrical field behavior 10 days before each earthquake.

For frequency sub-matrices one has:

$$F^{(2)} = \begin{pmatrix} 0.69 & 0.41 & 0.25 & 0.31 & 0.25 \\ 0.52 & 0.57 & 0.81 & 0.75 & 0.41 \\ 0.65 & 0.56 & 0.31 & 0.17 & 0.71 \\ 0.35 & 0.56 & 0.56 & 0.74 & 0.57 \end{pmatrix}$$

where

$$M^{(2)}_{s_1} = [4.30, 4.38], \quad M^{(2)}_{s_2} = [4.38, 4.46], \quad M^{(2)}_{s_3} = [4.46, 4.53],$$
$$M^{(2)}_{s_4} = [4.53, 4.61], \quad M^{(2)}_{s_5} = [4.61, 4.70].$$

$$F^{(3)} = \begin{pmatrix} 0.73 & 0.40 & 0.61 & 0.37 & 0.64 \\ 0.55 & 0.56 & 0.58 & 0.26 & 0.18 \\ 0.62 & 0.21 & 0.19 & 0.12 & 0.75 \\ 0.81 & 0.57 & 0.61 & 0.63 & 0.12 \end{pmatrix}$$

where

$$M^{(3)}_{s_1} = [5.1, 5.48], \quad M^{(3)}_{s_2} = [5.48, 5.86], \quad M^{(3)}_{s_3} = [5.86, 6.24],$$
$$M^{(3)}_{s_4} = [6.24, 6.62], \quad M^{(3)}_{s_5} = [6.62, 7.00].$$

$F^{(1)}$ is not presented because the reliability of data is very small. For this one the corresponding incidence matrix is directly evaluated by experts.

For example, the following matrix is obtained from $F^{(2)}$:

$$^{+}\widehat{R}_2 = \begin{array}{cccccc} M^{(2)}_{s_1} & M^{(2)}_{s_2} & M^{(2)}_{s_3} & M^{(2)}_{s_4} & M^{(2)}_{s_5} & \\ \left(\begin{array}{c} 1 \\ 1 \\ 1 \\ 0 \end{array}\right. & \begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \end{array} & \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} & \begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ 1 \\ 1 \end{array}\right) & \begin{array}{c} A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array} \end{array} \tag{4}$$

According to Atkin's terminology this matrix is called an incidence matrix; it determines some relation (called “connectivity”) on the Cartesian product $\{M^{(j)}_{s_i}\} \times \{A_{t_k}\}$. Thus, the vertical entries of this matrix correspond to certain cases of earthquakes, and horizontal ones to relevant activities.

By applying the connectivity theory to such an incidence matrix one can obtain representative chains of activities for a given earthquake.

The incidence matrices for weak and strong earthquakes are as follows:

$$
\begin{array}{c}
{}^{+}\widehat{R}_1 = \left(\begin{array}{ccccc|c}
M_{s_1}^{(1)} & M_{s_2}^{(1)} & M_{s_3}^{(1)} & M_{s_4}^{(1)} & M_{s_5}^{(1)} & \\
1 & 1 & 0 & 1 & 1 & A_{t_1} \\
0 & 0 & 1 & 1 & 0 & A_{t_2} \\
1 & 1 & 1 & 0 & 1 & A_{t_3} \\
1 & 1 & 0 & 1 & 0 & A_{t_4}
\end{array}\right) \\
{}^{+}\widehat{R}_3 = \left(\begin{array}{ccccc|c}
M_{s_1}^{(3)} & M_{s_2}^{(3)} & M_{s_3}^{(3)} & M_{s_4}^{(3)} & M_{s_5}^{(3)} & \\
1 & 0 & 1 & 0 & 1 & A_{t_1} \\
1 & 1 & 1 & 0 & 0 & A_{t_2} \\
1 & 0 & 0 & 0 & 1 & A_{t_3} \\
1 & 1 & 1 & 1 & 0 & A_{t_4}
\end{array}\right)
\end{array}
\qquad (5)
$$

The consideration of these incidence matrices does not remove uncertainty but as one can see it makes it possible to obtain the distribution of representative chains by connectivity levels and consequently to finding a generalized solution in the form of possibility distributions on the set of possible types of earthquakes. Atkin's method allows one to calculate the quantity of connectivity corresponding to any pair of columns or rows of the incidence matrix. This notion of connectivity is derived by viewing the rows or columns of the incidence matrix as polyhedron in a multi-dimensional space. The connectivity between two polyhedron is given by the number of shared faces. Two points are equivalent to a single face, three points to two faces, and so on. Thus, earthquakes $M_{s_1}^{(1)}$ and $M_{s_4}^{(1)}$ share a single face via activities A_{t_1} and A_{t_4}. Activities A_{t_2} and A_{t_4} share two faces via earthquakes $M_{s_2}^{(2)}, M_{s_3}^{(2)}$ and $M_{s_4}^{(2)}$.

2.3. **Connectivity level matrices and representative chains.** The connectivity between earthquakes power, ${}^{\pm}\widehat{C}_M^{(k)}$, and activities, ${}^{\pm}\widehat{C}_A^{(k)}$, is given by

$$
\begin{aligned}
{}^{+}\widehat{C}_M^{(k)} &= {}^{+}\widehat{R}_k^{T\,+}\widehat{R}_k - \widehat{\Omega}_M, \qquad & {}^{+}\widehat{C}_A^{(k)} &= {}^{+}\widehat{R}_k{}^{+}\widehat{R}_k^{T} - \widehat{\Omega}_A, \\
{}^{-}\widehat{C}_M^{(k)} &= {}^{-}\widehat{R}_k^{T\,-}\widehat{R}_k - \widehat{\Omega}_M, \qquad & {}^{-}\widehat{C}_A^{(k)} &= {}^{-}\widehat{R}_k{}^{-}\widehat{R}_k^{T} - \widehat{\Omega}_A,
\end{aligned}
\qquad (6)
$$

where ${}^{+}\widehat{R}_k^{T}$ is a transposition of ${}^{+}\widehat{R}_k$ matrix, ${}^{-}\widehat{R}_k = 1 -^{+} \widehat{R}_k$, $\widehat{\Omega}_M$ and $\widehat{\Omega}_A$ are simply all unity element matrices with and their dimensionality coincides correspondingly with $\widehat{R}_k^{T}\widehat{R}_k$ and $\widehat{R}_k\widehat{R}_k^{T}$ dimensions. Thus

$$
{}^{+}\widehat{C}_M^{(1)} = \begin{pmatrix} 3 & 3 & 1 & 2 & 2 \\ 3 & 3 & 1 & 2 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 2 & 2 & 1 & 3 & 1 \\ 2 & 2 & 1 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} =
$$

$$
= \begin{array}{c} \begin{array}{ccccc} M_{s_1}^{(1)} & M_{s_2}^{(1)} & M_{s_3}^{(1)} & M_{s_4}^{(1)} & M_{s_5}^{(1)} \end{array} \\ \begin{pmatrix} 2 & 2 & 0 & 1 & 1 \\ 2 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ M_{s_1}^{(1)} \\ M_{s_2}^{(1)} \\ M_{s_3}^{(1)} \\ M_{s_4}^{(1)} \\ M_{s_5}^{(1)} \end{array} \tag{7}
$$

$$
{}^{+}\widehat{C}_A^{(1)} = \begin{pmatrix} 4 & 1 & 3 & 3 \\ 1 & 2 & 1 & 1 \\ 3 & 1 & 4 & 2 \\ 3 & 1 & 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} =
$$

$$
= \begin{array}{c} \begin{array}{cccc} A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} \end{array} \\ \begin{pmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 3 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix} \end{array} \begin{array}{c} \\ A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array} \tag{8}
$$

Analogously,

$$
{}^{+}\widehat{C}_M^{(2)} = \begin{array}{c} \begin{array}{ccccc} M_{s_1}^{(2)} & M_{s_2}^{(2)} & M_{s_3}^{(2)} & M_{s_4}^{(2)} & M_{s_5}^{(2)} \end{array} \\ \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ M_{s_1}^{(2)} \\ M_{s_2}^{(2)} \\ M_{s_3}^{(2)} \\ M_{s_4}^{(2)} \\ M_{s_5}^{(2)} \end{array}, \quad {}^{+}\widehat{C}_A^{(2)} = \begin{array}{c} \begin{array}{cccc} A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 3 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ -1 & 2 & 1 & 3 \end{pmatrix} \end{array} \begin{array}{c} \\ A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array}, \tag{9}
$$

$$
{}^{+}\widehat{C}_M^{(3)} = \begin{array}{c} \begin{array}{ccccc} M_{s_1}^{(3)} & M_{s_2}^{(3)} & M_{s_3}^{(3)} & M_{s_4}^{(3)} & M_{s_5}^{(3)} \end{array} \\ \begin{pmatrix} 3 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 & -1 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & -1 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ M_{s_1}^{(3)} \\ M_{s_2}^{(3)} \\ M_{s_3}^{(3)} \\ M_{s_4}^{(3)} \\ M_{s_5}^{(3)} \end{array}, \quad {}^{+}\widehat{C}_A^{(3)} = \begin{array}{c} \begin{array}{cccc} A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} \end{array} \\ \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 3 \end{pmatrix} \end{array} \begin{array}{c} \\ A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array} \tag{10}
$$

From the above -1 indicates that earthquakes $M_{s_2}^{(3)}$ and $M_{s_5}^{(3)}$, $M_{s_4}^{(3)}$ and $M_{s_5}^{(3)}$ are totally disconnected. A key feature of this classification of earthquakes and activities is that evidence both for and against each hypothesis is considered. This was the motivation for the positive and negative aspects of connectivity. Essentially, positive connectivity examines the connectivity of unity elements in the incidence matrix, i.e. the presence of some given activities appearing in earthquakes precursors, whilst negative connectivity examines zeros, i.e. the absence of such activities. Formulae (7, 8,9,10) are for positive connectivity. The corresponding expressions for negative connectivity are as follows:

$$
{}^{-}\widehat{C}_M^{(1)} = \begin{array}{cccccc}
M_{s_1}^{(1)} & M_{s_2}^{(1)} & M_{s_3}^{(1)} & M_{s_4}^{(1)} & M_{s_5}^{(1)} & \\
0 & 0 & -1 & -1 & 0 & M_{s_1}^{(1)} \\
0 & 0 & -1 & -1 & 0 & M_{s_2}^{(1)} \\
-1 & -1 & 1 & -1 & 0 & M_{s_3}^{(1)} \\
-1 & -1 & -1 & 0 & -1 & M_{s_4}^{(1)} \\
0 & 0 & 0 & -1 & 1 & M_{s_5}^{(1)}
\end{array}, \quad
{}^{-}\widehat{C}_A^{(1)} = \begin{array}{ccccc}
A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} & \\
0 & -1 & -1 & 0 & A_{t_1} \\
-1 & 2 & -1 & 0 & A_{t_2} \\
-1 & -1 & 0 & -1 & A_{t_3} \\
0 & 0 & -1 & 1 & A_{t_4}
\end{array} \tag{11}
$$

$$
{}^{-}\widehat{C}_M^{(2)} = \begin{array}{cccccc}
M_{s_1}^{(2)} & M_{s_2}^{(2)} & M_{s_3}^{(2)} & M_{s_4}^{(2)} & M_{s_5}^{(2)} & \\
0 & -1 & -1 & -1 & -1 & M_{s_1}^{(2)} \\
-1 & 0 & 0 & 0 & 0 & M_{s_2}^{(2)} \\
-1 & 0 & 1 & 1 & 0 & M_{s_3}^{(2)} \\
-1 & 0 & 1 & 1 & 0 & M_{s_4}^{(2)} \\
-1 & 0 & 0 & 0 & 1 & M_{s_5}^{(2)}
\end{array}, \quad
{}^{-}\widehat{C}_A^{(2)} = \begin{array}{ccccc}
A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} & \\
3 & 0 & 1 & -1 & A_{t_1} \\
0 & 0 & -1 & -1 & A_{t_2} \\
1 & -1 & 1 & -1 & A_{t_3} \\
-1 & -1 & -1 & 0 & A_{t_4}
\end{array} \tag{12}
$$

$$
{}^{-}\widehat{C}_M^{(3)} = \begin{array}{cccccc}
M_{s_1}^{(3)} & M_{s_2}^{(3)} & M_{s_3}^{(3)} & M_{s_4}^{(3)} & M_{s_5}^{(3)} & \\
-1 & -1 & -1 & -1 & -1 & M_{s_1}^{(3)} \\
-1 & 1 & 0 & 1 & -1 & M_{s_2}^{(3)} \\
-1 & 0 & 0 & 0 & -1 & M_{s_3}^{(3)} \\
-1 & 1 & 0 & 2 & 0 & M_{s_4}^{(3)} \\
-1 & -1 & -1 & 0 & 1 & M_{s_5}^{(3)}
\end{array}, \quad
{}^{-}\widehat{C}_A^{(3)} = \begin{array}{ccccc}
A_{t_1} & A_{t_2} & A_{t_3} & A_{t_4} & \\
1 & 0 & 1 & -1 & A_{t_1} \\
0 & 1 & 0 & 0 & A_{t_2} \\
1 & 0 & 2 & -1 & A_{t_3} \\
-1 & 0 & -1 & 0 & A_{t_4}
\end{array} \tag{13}
$$

Atkin's theory allows one to establish the connectivity not only between two polyhedron but also between chains of polyhedron connected to at least some given number of faces, i.e. chains of certain earthquakes power or activities. These representative chains can be determined directly from matrices ${}^{\pm}\widehat{C}_M^{(k)}$ and ${}^{\pm}\widehat{C}_A^{(k)}$. The results for this example are cited in tables 2-4.

q indicates the level of connectivity (number of faces). Brackets enclose those polyhedron connected at that level. Thus, the earthquake in the first group $\{M_{s_4}^{(1)}\}$ is self-connected at level 2; at that level $M_{s_1}^{(1)}$ and $M_{s_2}^{(1)}$ are connected. At level 2 $\{M_{s_4}^{(1)}\}$ and $\{M_{s_1}^{(1)}, M_{s_2}^{(1)}\}$ are isolated, becoming connected at level 1, along with $\{M_{s_5}^{(1)}\}$.

Representative $\widehat{C}_M^{(k)}$-chains describe how well the set of earthquakes represent a single group in terms of their exhibited activities. This analysis may be useful to discover whether different earthquakes tend to indicate similar "syndromes". The corresponding representative $\widehat{C}_A^{(k)}$-chains describe how well particular subsets of activities represent earthquakes as a whole and hence could be used to identify patterns of activities which are strongly indicative of earthquakes. The connection of information is implicit in connectivity concept, which can also be due to physical causes.

2.4. **Connectivity measure.** Now let the connectivity measure be constructed. Let us introduce the necessary definitions:

TABLE 2. Representative chains of weak earthquakes

positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_M^{(1)}$-chains	connectivity level	representative $-\widehat{C}_M^{(1)}$-chains
$q=2$	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}\}, \{M_{s_4}^{(1)}\}$	$q=2$	
$q=1$	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}, M_{s_4}^{(1)}, M_{s_5}^{(1)}\}, \{M_{s_3}^{(1)}\}$	$q=1$	$\{M_{s_3}^{(1)}\}, \{M_{s_5}^{(1)}\}$
$q=0$	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}, M_{s_3}^{(1)}, M_{s_4}^{(1)}, M_{s_5}^{(1)}\}$	$q=0$	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}, M_{s_3}^{(1)}, M_{s_5}^{(1)}\}, \{M_{s_4}^{(1)}\}$
positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_A^{(1)}$-chains	connectivity level	representative $-\widehat{C}_A^{(1)}$-chains
$q=3$	$\{A_{t_1}\}, \{A_{t_3}\}$	$q=3$	
$q=2$	$\{A_{t_1}, A_{t_3}, A_{t_4}\}$	$q=2$	$\{A_{t_2}\}$
$q=1$	$\{A_{t_1}, A_{t_3}, A_{t_4}\}, \{A_{t_2}\}$	$q=1$	$\{A_{t_2}\}, \{A_{t_4}\}$
$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$	$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_4}\}, \{A_{t_3}\}$

TABLE 3. Representative chains of moderate earthquakes

positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_M^{(2)}$-chains	connectivity level	representative $-\widehat{C}_M^{(2)}$-chains
$q=2$	$\{M_{s_1}^{(2)}\}, \{M_{s_2}^{(2)}\}$	$q=2$	
$q=1$	$\{M_{s_1}^{(2)}, M_{s_2}^{(2)}, M_{s_3}^{(2)}, M_{s_4}^{(2)}, M_{s_5}^{(2)}\}$	$q=1$	$\{M_{s_3}^{(2)}, M_{s_4}^{(2)}\}, \{M_{s_5}^{(2)}\}$
$q=0$	$\{M_{s_1}^{(2)}, M_{s_2}^{(2)}, M_{s_3}^{(2)}, M_{s_4}^{(2)}, M_{s_5}^{(2)}\}$	$q=0$	$\{M_{s_1}^{(2)}\}, \{M_{s_2}^{(2)}, M_{s_3}^{(2)}, M_{s_4}^{(2)}, M_{s_5}^{(2)}\}$
positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_A^{(2)}$-chains	connectivity level	representative $-\widehat{C}_A^{(2)}$-chains
$q=3$	$\{A_{t_2}\}, \{A_{t_4}\}$	$q=3$	$\{A_{t_1}\}$
$q=2$	$\{A_{t_2}, A_{t_4}\}, \{A_{t_3}\}$	$q=2$	$\{A_{t_1}\}$
$q=1$	$\{A_{t_2}, A_{t_3}, A_{t_4}\}$	$q=1$	$\{A_{t_1}, A_{t_3}\}$
$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$	$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_3}\}, \{A_{t_4}\}$

TABLE 4. Representative chains of strong earthquakes

positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_M^{(3)}$-chains	connectivity level	representative $-\widehat{C}_M^{(3)}$-chains
$q=3$	$\{M_{s_1}^{(3)}\}$	$q=3$	
$q=2$	$\{M_{s_1}^{(3)}, M_{s_3}^{(3)}\}$	$q=2$	$\{M_{s_4}^{(3)}\}$
$q=1$	$\{M_{s_1}^{(3)}, M_{s_2}^{(3)}, M_{s_3}^{(3)}, M_{s_5}^{(3)}\}$	$q=1$	$\{M_{s_2}^{(3)}, M_{s_4}^{(3)}\}, \{M_{s_5}\}$
$q=0$	$\{M_{s_1}^{(3)}, M_{s_2}^{(3)}, M_{s_3}^{(3)}, M_{s_4}^{(3)}, M_{s_5}^{(3)}\}$	$q=0$	$\{M_{s_2}^{(3)}, M_{s_3}^{(3)}, M_{s_4}^{(3)}, M_{s_5}^{(3)}\}$
positive-connectivity		negative-connectivity	
connectivity level	representative $+\widehat{C}_A^{(3)}$-chains	connectivity level	representative $-\widehat{C}_A^{(3)}$-chains
$q=3$	$\{A_{t_4}\}$	$q=3$	
$q=2$	$\{A_{t_1}\}, \{A_{t_2}, A_{t_4}\}$	$q=2$	$\{A_{t_3}\}$
$q=1$	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$	$q=1$	$\{A_{t_1}, A_{t_3}\}, \{A_{t_2}\}$
$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$	$q=0$	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$

Definition 1. Let $\vec{a}, \vec{b} \in [0,1]^n$. The set of ordered pair

$$\{(a_i, b_i): \text{ at least } a_i \text{ or } b_i > 0\}$$

is called the support of $\{(a_i, b_i) : i = 1, \ldots, n\}$.

Definition 2. Let $\vec{a}, \vec{b} \in [0,1]^n$. $\vec{a}$ and $\vec{b}$ are said to be "equivalent" if the support is not null and all pairs (a_i, b_i) are such that $a_i = b_i$.

Definition 3. Let $\vec{a}, \vec{b} \in [0,1]^n$. $\vec{a}$ and $\vec{b}$ are said to be "absolute disparate" if it have null support or in all pairs in the support of $\{(a_i, b_i) : i = 1, \ldots, n\}$, exactly one of component a_i or b_i is zero.

Definition 4. Let $\vec{a}, \vec{b} \in [0,1]^n$. The connectivity measure of $\vec{a}$ and $\vec{b}$ is defined as follows:

- If $\vec{a}$ and $\vec{b}$ are "absolutely disparate", the connectivity measure is zero.
- If $\vec{a}$ and $\vec{b}$ are "equivalent" the connectivity measure is the unity.
- Otherwise, the connectivity measure is given by the proportion of pairs (a_i, b_i) in the support such that $a_i = b_i = 1$.

Notice that the connectivity measure can be identified with a scalar product of vectors $\vec{a}$ and $\vec{b}$ and that all aforesaid can be applied to both fuzzy and non-fuzzy vectors, but for the latter one can obtain the unique expression

$$c(\vec{a}, \vec{b}) = \left(\sum_{i=1}^{n} (a_i \wedge b_i) \right) / \left(\sum_{i=1}^{n} (a_i \vee b_i) \right), \tag{14}$$

whilst for the former the expression of the scalar product is no longer unique.

For example, let

$$\vec{a} = (1,1,0,0,1), \qquad \vec{b} = (1,0,0,1,1),$$
$$\{(a_i, b_i), i = 1, \ldots, 5\} = \{(1,1), (1,0), (0,0), (0,1), (1,1)\}.$$

The support of this set is according to Definition 1

$$\{(a_i, b_i), \text{ at least } a_i \text{ or } b_i > 0\} = \{(1,1), (1,0), (0,1), (1,1)\}.$$

In this case vectors $\vec{a}$ and $\vec{b}$ are, according to Definitions 2 and 3, neither equivalent nor absolutely disparate, and according to Definition 4 the connectivity measure can be obtained as follows:

$$c(\vec{a}, \vec{b}) = \begin{pmatrix} \text{proportion of pairs } (a_i, b_i) \text{ in the support} \\ \text{such that } (a_i = b_i = 1) \end{pmatrix} =$$
$$= \frac{\sum_{i=1,\ldots,5} (a_i \wedge b_i)}{\sum_{i=1,\ldots,5} (a_i \vee b_i)} = \frac{2}{4} = \frac{1}{2} \qquad \text{see (14)}$$

Let the generalization of Definition 4 for chains of components > 2 be considered.

Definition 5. Let $\vec{a}^{(1)}, \ldots, \vec{a}^{(n)} \in [0,1]^n$. The connectivity measure of $\{\vec{a}^{(i)}\}_{i=1,\ldots,n}$ is defined in the following way:

$$c(\{\vec{a}^{(i)}\}_{i \in I}) = \min_{i,j \in I, i \neq j} c(\vec{a}^{(i)}, \vec{a}^{(j)}), \qquad I = 1, \ldots, n \tag{15}$$

TABLE 5. Connectivity measures of representative chains (weak earthquakes)

positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_M^{(1)}$-chains	connectivity measure	representative $-\widehat{C}_M^{(1)}$-chains
1	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}\}$		no representative chains
0.25	$\{M_{s_1}^{(1)}, M_{s_2}^{(1)}, M_{s_4}^{(1)}, M_{s_5}^{(1)}\}$		
positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_A^{(1)}$-chains	connectivity measure	representative $-\widehat{C}_A^{(1)}$-chains
0.4	$\{A_{t_1}, A_{t_3}, A_{t_4}\}$		no representative chains

TABLE 6. Connectivity measures of representative chains (moderate earthquakes)

positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_M^{(2)}$-chains	connectivity measure	representative $-\widehat{C}_M^{(2)}$-chains
0.25	$\{M_{s_1}^{(2)}, M_{s_2}^{(2)}, M_{s_3}^{(2)}, M_{s_4}^{(2)}, M_{s_5}^{(2)}\}$	1.0	$\{M_{s_3}^{(2)}, M_{s_4}^{(2)}\}$
positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_A^{(2)}$-chains	connectivity measure	representative $-\widehat{C}_A^{(2)}$-chains
0.6	$\{A_{t_2}, A_{t_4}\}$		
0.4	$\{A_{t_2}, A_{t_3}, A_{t_4}\}$	0.5	$\{A_{t_1}, A_{t_3}\}$

TABLE 7. Connectivity measures of representative chains (strong earthquakes)

positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_M^{(3)}$-chains	connectivity measure	representative $-\widehat{C}_M^{(3)}$-chains
0.75	$\{M_{s_1}^{(3)}, M_{s_3}^{(3)}\}$	0.67	$\{M_{s_2}^{(3)}, M_{s_4}^{(3)}\}$
positive-connectivity		negative-connectivity	
connectivity measure	representative $+\widehat{C}_A^{(3)}$-chains	connectivity measure	representative $-\widehat{C}_A^{(3)}$-chains
0.75	$\{A_{t_2}, A_{t_4}\}$	0.67	$\{A_{t_1}, A_{t_3}\}$
0.20	$\{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$		

The following is a simple rule of apportionment of representative chains: in Tables 2-4 such chains are fairly large and highly connected (excluding chains with one component). The level of connectivity according to Definition 5 is attached to each selected chain. From this one, tables 5-7 are obtained.

Tables 5-7 establish the distribution of connectivity measures by representative chains. The authors were especially interested in uncertainty distribution by representative chains of activities. For any certain image of activities possibility distribution by earthquake powers has the following form:

$$\delta(M^{(j)}) = \frac{1}{2}\left(\chi_{\text{Large}}\left(\frac{\sum_i c_i P(Q_i \mid \{A^{(j)}\})}{\sum_i c_i}\right) + \chi_{\text{small}}\left(\frac{\sum_k d_k P(R_k \mid \{A^{(j)}\})}{\sum_k d_k}\right)\right), \quad (16)$$

where $M^{(j)}$ indicates the group to which the earthquake belongs. c_i is the measure of connectivity corresponding to chain Q_i. $P(Q_i \mid \{A^{(j)}\})$ denotes the proportion of activities in Q_i which are presented in $\{A^{(j)}\}$, d_k is the connectivity measure of negative connectivity of the representative chain R_k, $P(R_k \mid \{A^{(j)}\})$ is the corresponding proportion in the case of negative connectivity. "Large" and "Small" are fuzzy subsets of interval $[0, 1]$ ("Large"={fuzzy subset "number near to 1" of interval $[0, 1]$}, "Small"={fuzzy subset "number near to 0" of interval $[0, 1]$}) and χ_{Large} and χ_{Small} are their respective membership functions, χ_{Large} increasing monotonic and χ_{Small} decreasing monotonic.

Let, for example, $\{A^{(j)}\} = \{A_{t_1}, A_{t_2}, A_{t_3}, A_{t_4}\}$. Using tables 5-7 and (16) after some simple calculations one gets:

$$\begin{aligned}
\delta(M^{(1)}) &= \frac{1}{2}\left(\chi_{\text{Large}}(1) + \chi_{\text{Small}}(0)\right) = \frac{1}{2}(1+1) = 1,\\
\delta(M^{(2)}) &= \frac{1}{2}\left(\chi_{\text{Large}}(0.56667) + \chi_{\text{Small}}(1)\right) = \frac{1}{2}\left(\chi_{\text{Large}}(0.56667) + 0\right) = \frac{1}{2}\left(\chi_{\text{Large}}(0.56667)\right),\\
\delta(M^{(3)}) &= \frac{1}{2}\left(\chi_{\text{Large}}(0.55263) + \chi_{\text{Small}}(1)\right) = \frac{1}{2}\left(\chi_{\text{Large}}(0.55263) + 0\right) = \frac{1}{2}\left(\chi_{\text{Large}}(0.55263)\right)
\end{aligned}$$

3. CONCLUSION

To make a final "classic" decision using the connectivity analysis an additional principle is needed. For example, the decision can be taken according to the maximum of function $\delta(M^{(j)})$, or as the final decision one can accept the non-fuzzy subset nearest to fuzzy $\delta(M^{(j)})$. In general, the form of χ_{Large} and χ_{small} will be decisive in final decision-making.

It is easy to obtain that in the example developed in this paper, independently of the form of χ_{Large}, the best decision is $M^{(1)}$.

Connectivity theory permits one to establish the connectivity not only between two polyhedron but also between chains of polyhedron connected with at least some given number of faces, i.e., chains of certain earthquakes or precursors.

The most important feature of the connectivity analysis (and the discrimination analysis as well) is the possibility of multi-factor decision-making.

Using the connectivity analysis together with the discrimination analysis is convenient to improve the reliability of decision-making.

4. A NON-FUZZY CASE IMPLEMENTATION

With the object of provide a didactic example, we have implemented the calculations given in this paper in *octave* for the non-fuzzy case, a system for numeric calculus, available for a lot of platforms and under the GNU license.

4.1. **Basic matrix calculation.** With the aim of calculating ${}^{-}\hat{R}_k = 1 - {}^{+}\hat{R}_k$, it is defined the following function

```
function s = Rm(R)
  s = ones(rows(R), columns(R)) - R;
endfunction
```

The function `ones(r,c)` gives a matrix $r \times c$ filled with 1's.

For calculating ${}^{+}\hat{C}_M^{(k)} = {}^{+}\hat{R}_k^{\,T}\,{}^{+}\hat{R}_k - \hat{\Omega}_M$ and ${}^{-}\hat{C}_M^{(k)} = {}^{-}\hat{R}_k^{\,T}\,{}^{-}\hat{R}_k - \hat{\Omega}_M$

```
function s = CM(R)
  s = R' * R - ones(columns(R), columns(R));
endfunction
```

and for calculating ${}^{+}\hat{C}_A^{(k)} = {}^{+}\hat{R}_k\,{}^{+}\hat{R}_k^{\,T} - \hat{\Omega}_A$ and ${}^{-}\hat{C}_A^{(k)} = {}^{-}\hat{R}_k\,{}^{-}\hat{R}_k^{\,T} - \hat{\Omega}_A$

```
function s = CA(R)
  s = R * R' - ones(rows(R), rows(R));
endfunction
```

the argument being `R` and `Rm(R)`, respectively.

For example, if matrices `R1`, `R2` and `R3` has been previously defined as given in this paper, for calculating

- ${}^{-}\hat{R}_2$, use `Rm(R2)`,
- ${}^{-}\hat{C}_M^{(2)}$, use `CM(Rm(R2))`,
- ${}^{+}\hat{C}_M^{(3)}$, use `CM(R3)`,
- ${}^{+}\hat{C}_A^{(1)}$, use `CA(R1)`,
- ${}^{-}\hat{C}_A^{(2)}$, use `CA(Rm(R2))`.

4.2. **Connectivity groups related functions.** The next one is a utility function. Let us check that if an item `x` exists in given set `S`:

```
function s = is_in_set(x, S)
  s = (length(union(S, create_set([x]))) == length(S));
endfunction
```

The following function is used to calculate the set of elements that are directly connected at level `l` with element `i`, following the connectivity matrix `CM`:

```
function r = connect_with(i, CM, l)
  r = create_set([]);
  for j = 1:rows(CM)
    if CM(i,j) >= l
      r = union(r, create_set([j]));
    endif
  endfor
endfunction
```

In order to calculate the connectivity group to which element `i` belongs at level `l`, it is calculated the group of elements to which it is directly connected at level `l`, which will be then expanded iteratively with the corresponding group of all the elements directly connected at level `l` to some element of the group. This way, when it is not possible to expand the group anymore, it is obtained the set of all the elements that are accessible from node `i` following a chain of elements connected at level `l`

```
function r = connect_group_of(i, CM, l)
  changed = 1;
  s = connect_with(i, CM, l);
  while (changed)
    changed = 0;
    r = s;
    for i = 1:length(s)
      r = union(r, connect_with(s(i), CM, l));
      if length(r) != length(s)
        changed = 1;
      endif
    endfor
    s = r;
  endwhile
endfunction
```

In order to calculate the connectivity groups at level `l`, one tries to find the connectivity groups of all the elements at this level:

```
function r = connect_groups_at_level(CM, l)
  r.l = l;
  sets = list();
  notused = 1:rows(CM);

  while length(notused) > 0
    c = connect_group_of(notused(1), CM, l);
    if length(c) > 0
      sets = append(sets, c);
    endif
    notused = complement(union(c, create_set([notused(1)])), \
                         notused);
  endwhile
  r.sets = sets;
endfunction
```

The result of this function requires some explanation. It gives a structure with two fields: field `l` has the value of connectivity level and field `sets` contains a list of lists of connected elements at level `l`.

In order to calculate the connectivity groups, one calculates the limits of the values of connectivity levels, and calculates all the groups of connectivity at each level between these limits:

```
function r = connect_groups(CM)
  mn = min(min(min(CM)),0);
  mx = max(max(CM));
  r = list();
  for i = mn:mx
    r = append(r, connect_groups_at_level(CM, i));
  endfor
endfunction
```

The return value is a list of structures like that described in the previous function, which conforms with the distribution of connectivity groups at the different levels.

4.3. **Connectivity measure related functions.** The next function calculates the connectivity measure of two elements. In order to calculate the connectivity level of a pair $\{A_i, A_j\}$, we pass as `R` the corresponding matrix `R1`, `R2` or `R3`. In order to calculate the connectivity level of a pair $\{M_i, M_j\}$, one has to pass the transposed matrix `R'`.

```
function r = con_pair(i,j, R)
  S = R([i,j], :);
  if sum(max(S)) > 0
    r = sum(min(S)) / sum(max(S));
  else
    r = 0;
  endif
endfunction
```

The connectivity measure of a list of nodes `l` is defined as the minimum of the connectivity measures of each pair of elements of `l`:

```
function r = mc(l, R)
  a = [];
  for i = 1:length(l)
    for j = (i+1):length(l)
```

```
      a(length(a) + 1) = con_pair(l(i),l(j),R);
    endfor
  endfor
  if length(a) > 0
    r = min(a);
  else
    r = 0;
  endif
endfunction
```

The connectivity measure distribution of the representative chains is calculated by collecting every representative chain at every level and dropping the duplicated entries. Parameter `r` is the result given by function `connect_groups`.

```
function a = connect_measures(r, R)
  a = list();
  for i = 1:length(r)
    if nth(r,i).l > 0
      for j = 1:length(nth(r,i).sets)
        if length(nth(nth(r,i).sets,j)) > 1
          l.set = nth(nth(r,i).sets, j);
          l.mc  = mc(l.set, R);
          a = append(a, l);
        endif
      endfor
    endif
  endfor
  i = 1;
  while  i < length(a)
    j = i + 1;
    while j <= length(a)
      if length(nth(a,i).set) == length(nth(a,j).set)
        if min(nth(a,i).set == nth(a,j).set) = 1
          a = splice(a,j,1);
        endif
      endif
      j++;
    endwhile
    i++;
  endwhile
endfunction
```

The return of this function is a list of structures that have two fields: a field `set`, which is the representative chain and a field `mc`, which represents the associated measure of connectivity.

4.4. **Connectivity measure distribution.** The next function calculates the proportion of elements of list `t1` belonging to list `t2`.

```
function p = Prop(t1, t2)
  s1 = create_set(t1);
  s2 = create_set(t2);
  p = length(intersection(s1,s2)) / length(s1);
endfunction
```

The connectivity distribution for a given activities chain `li` is calculated by means of the following function:

```
function r = connect_distr(CM, li)
  scp = list();
  sc  = list();
  for i = 1:length(CM);
    el = nth(CM,i);
    in.mc   = el.mc;
    in.prop = Prop(el.set, li);
    in.set  = el.set;
    scp = append(scp, in);
    sc  = append(sc, el.mc);
  endfor
  scm = 0;
  for i = 1:length(sc)
    scm += nth(sc,i);
  endfor
  scpm = 0;
  for i = 1:length(scp);
    scpm += (nth(scp, i).mc * nth(scp, i).prop);
  endfor
  if scm > 0
    r = scpm / scm;
  else
    r = 0
  endif
endfunction
```

Parameter `CM` is the result of the `connet_measures` function.

This function gives the parameter for χ_{Large} and χ_{Small} when they are evaluated for ${}^{+}C_M^{(k)}$ and ${}^{-}C_M^{(k)}$ respectively.

For example, to calculate the fuzzy distribution for a activities chain $\{A_1, A_3, A_4\}$:

$$\delta(M^{(1)}) = \frac{1}{2}\left(\chi_{\text{Large}}(\texttt{p1}) + \chi_{\text{Small}}(\texttt{p2})\right),$$

one obtains the parameters `p1` and `p2` as:

- `p1 = connect_distr(connect_measures(connect_groups(CA(R1)), R1), [1,3,4])`,
- `p2 = connect_distr(connect_measures(connect_groups(CA(Rm(R1))), Rm(R1)), [1,3,4])`

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Biographies

Francisco Criado Torralba: was born in 1945. He received the Ph. D. degree from Malaga University, Malaga, Spain, in 1979. Currently, he lectures on statistics and operational research at Malaga University. He has published a large number of papers on mathematical systems modeling, fuzzy systems, game theory and control theory, as well as having participated actively in several EU research programs.

Tamaz Gachechiladze: was born in 1929. He is an Assistant Professor of the Department of Stochastic Processes Theory, Tblisis State University, Georgia. His current research interests include mathematical cybernetics and informatics.

Nato Jorjiashvili: was born in 1977. Assistant of the Department of stochastic processes theory Tbilisi State University, Georgia. Current research interests: Mathematical cybernetics, Informatics.

Hamlet Meladze: was born in 1939. He is currently Chair of Computer Mathematical Providing and Information Technologies, and Full Professor in the Department of Applied Mathematics and Computer Sciences, Tbilisi State University, Georgia. His current research interests include mathematical cybernetics and informatics.

José María Sánchez Sáez: was born in 1970. He graduated with degree in Mathematics in the Málaga University. Presently he is working on his doctoral thesis in the Statistics and Operational Research Department. Current research interest include statistics and informatics.

Guram Tsertsvadze: was born in 1933. He is Chair of Mathematical Cybernetics and Informatics, and Full Professor in the Department of Applied Mathematics and Computer Sciences, Tbilisi State University, Georgia. His current research interests include mathematical cybernetics and informatics.

Sirbiladze Gia: was born in 1959. Ph.D. Assistant professor of the Department of stochastic processes theory Tbilisi State University, Georgia. Current research interests: Informatics, Fuzzy Mathematics, Fuzzy Statistics.

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