Tesis Doctoral

DIRECT DEMOCRACY VERSUS REPRESENTATIVE DEMOCRACY: A THEORETICAL APPROACH

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A mis padres y a mi hermana.

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Doctoral, os la dedico.

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Introduction

For centuries, the suitability of the representative democracy as government system has not been questioned at all. Although there have been two theories of representation that have traditionally been distinguished, that is, the theory of likeness and the theory of substitution, it is the latter that has been used to defend the best aptitude of politicians against the citizens when making decisions. The theory of representation by likeness traces its origins to Cratylus, the dialogue written by Plato in 360 B.C. According to this theory, a representative should resemble what she represents. Thus, the existence of a similarity between politicians and citizens becomes necessary when talking about representation of the second by the first is pursued. As for the theory of representation by substitution, it was first proposed by Edmund Burke in his work A Philosophical Enquiry into the Origin of Our Ideas of the Sublime and Beautiful, first published in 1757. According to this theory, representation has to be understood as making present (again) what is (now) absent. In other words, it is a substitute or replacement for something that is absent. It is noteworthy that, unlike the previous case, this theory does not mention any criteria (likeness or something similar) that must be satisfied for a politician to be the representative of citizens. Implicitely, differences of varied nature between representatives and represented are allowed. It has been taking advantage of this possibility that many authors in both Economics and Political Science have relied on the assumption that politicians have better information than ordinary citizens about what should be done at any given time in order to support systems of political representation. Another advantage of representative democracy manifested in the literature is the reduction of the risk of the so-called "tyranny of the majority", recognized by authors such as Alexis de Tocqueville and John Stuart Mill, according to which a democratic majority would impose its will

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on a minority. Warning of the dangers of direct democracy, James Madison wrote in his work Federalist 10 published in 1787: *"It is of great importance... to guard one part of the society against the injustice of the other part. Different interests necessarily exist in different classes of citizens. If a majority be united by a common interest, the rights of the minority will be insecure."*

Life has been easy for representative democracy. Until now. The book Referendums: A Comparative Study of Practice and Theory, edited by David Butler and Austin Ranney and published in 1978 by the American Enterprise Institute (AEI) is considered one of the first attempts to collect the world experience with the instrument of the referendum. This work tries to justify the growing dissatisfaction of the electorate with the actions of the elected representatives through the increasing use of popular referendums in various nations around the world. The editors are based on referendum studies held in Australia, France, Ireland, Switzerland, Scandinavian countries, and the United States, among others. Some years later and as a result of this work, Ranney published his work The Referendum Device, whose review was included in American Political Science Review, one of the journals with the highest impact index within the category of Political Sciences. This text again analyzes the desire for referendums in democratic regimes. Referendums Around the World: The Growing Use of Direct Democracy (1994) and Referendums Around the World: The Continued Growth of Direct Democracy (2014) constitute later reissues of the work of Butler and Ranney. These texts continue to contribute to explain how referendums were affecting the way of doing politics in those countries where the instrument was most widespread.

The disaffection on the part of citizens with the system of political representation has been increasingly growing over the last years. Such increase has caused that, in recent times, certain sectors of society have ceased to see direct democracy as a mere complement to representative democracy and, instead, have begun to consider it as a potential substitute. In fact, we are witnessing a rising trend in popularity levels of government systems in which citizens are allowed to directly decide without the need for the traditional intermediation of politicians. The main purpose of this PhD dissertation is to

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approach the study of this phenomenon by comparing the systems of direct and representative democracy from a theoretical point of view. Faced with the undeniable success and popular acceptance of instruments of direct democracy such as referendums and popular consultations when making decisions, a very natural first question is when voters prefer a system of direct democracy or a system of representative democracy. In Chapter 1 we propose a model to investigate under which conditions voters prefer either one or the other system. In direct democracies voters choose a policy among two alternatives, under uncertainty about which policy better fits the realized state of the world; in representative democracies voters select a candidate who, once elected, chooses a policy having observed which is the realized state of the world. Voters and politicians' payoffs depend on a common component which is positive only if the policy fits the state of the world, and on a private ideological bias towards one of the policies. In direct democracies voters are uncertain about the future state of the world, while in representative democracies they are uncertain about the degree of ideological bias of the candidates, even if they know towards which policy each candidate is biased. We show that representative democracy is preferred if *(i)* the majority of voters are pragmatic (the common component prevails), and *(ii)* society is ideologically polarized, meaning that the majority of voters are ideological (the private component prevails), but the median voter is pragmatic. Direct democracy is the preferred instrument for collective choices in societies in which the majority of voters and the median voter are ideological, implying that the majority of voters have the same ideological bias, as, for instance, it occurs when the populist rhetoric of people against the elite succeeds. Moreover, we find that the demand of direct democracy is increasing in the polarization of the electorate, as long as no ideological group of voters becomes a majority.

As might be expected, the aforementioned success of direct democracy is recently resulting in the emergence of social movements and political parties asking for the direct participation of citizens in the decision making process, which would allow people to decide on each and every one of the issues that comes up for discussion. These groups not only firmly defend the system of direct democracy but also show a strong opposition to the system of representative democracy. Their main criticism is the possibility that

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politicians are self-seeking and end up behaving against people's interest. In this sense, they highlight the inability of representative democracy to implement what the majority of people desires for each specific issue. While the suitability of such an outcome could be discussed, we do not enter into this debate. We adopt instead a more positive approach to the matter. We wonder whether indeed there is no room for representative democracy to satisfy these groups' aspirations, being fully necessary for such purpose to abandon the traditional system of political representation and replace it by a system of direct democracy. In Chapter 2 we study under which conditions the two aforementioned procedures to make decisions may be equivalent in terms of the developed policies for each of the considered issues. We propose a model in which there is a finite number of issues on which a binary decision has to be made. In direct democracies voters directly vote for their more preferred decision on each issue, so majority voting issue-by-issue is the outcome in these systems. In representative democracies, we introduce a two-party system competition where the political platforms of these parties are known. Candidates representing each of the two parties will have these political platforms as the most preferred set of decisions for the bundle of issues. However, preferences of parties' candidates are not known, may those varying from policy-motivated to office-motivated. This assumption captures the essence of the criticism that systems of political representation are receiving related to the risk of having politicians who only pursue their own interests. We find necessary and sufficient conditions such that, irrespective from candidates' preferences, decisions made on each issue in representative democracies coincide with the decisions that would have been chosen in direct democracies by majority voting issue-by-issue. These conditions involve restrictions over pairs of preference profile of voters and political platforms of candidates. While we will formally state them in Chapter 2, we offer here an intuition of these conditions. First, there must be a degree of homogeneity among voters' preferences about the suitability of the implementation of decisions yielded by majority voting issue-by-issue such that this set of decisions is not defeated in pairwise comparisons by any other set of decisions for the bundle of issues. This would allow that, any potentially profitable deviation that might initially exist for any candidate would no longer be beneficial. Second, either there must be at least one candidate whose party political platform is in line with decisions made by majority voting on each issue, or there must

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be full divergence of political platforms of parties.[1] This double condition would allow, respectively, either the existence of a candidate who trivially has no incentives to deviate from what the majority of people desires for each issue, or the nonexistence of room for a mutually beneficial agreement for both candidates which may violate the interests of the majority.

Criticizing the opposite system by arguing the possible non-representativeness of the majority's preferences in its outcome is not something exclusive to those in favor of direct democracy.[2] Supporters of representative democracy question supporters of direct democracy when the latter claim that direct democracy is the right system to achieve such representativeness purpose. The former highlight the risk involved by the low turnout when a referendum is held. It might be the case that some people decide not to pronounce on a certain topic just because they are not interested on it. Participation thus would become a problem: since not enough people voted to make the decision relevant, it will be biased, not representing therefore the preferences of the majority. It is with this argument that they continue to defend the figure of a representative in charge of making decisions. However, supporters of direct democracy, far from giving up, rebut that argument. According to them, the damage suffered by society when decisions are made by self-seeking politicians could be even larger if the intensity of voters' preferences is taken into account: the politician might be choosing something that not only goes against society's preference, but something in fact "really bad" for a majority of voters. A solution that seeks to reconcile the views of both sides is proposed in Chapter 3, where we assess its performance according to a widely used principle for evaluating voting rules. We assume that decisions are made by a committee through scoring rules. The composition of such committee perfectly represents the distribution of voters' preferences over the set of alternatives. Committee members cannot abstain, removing thus the risk of underrepresentation warned by supporters of representative democracy, and they sincerely vote

[1] Of course, both conditions are compatible and may hold at the same time.

[2] As mentioned before, supporters of direct democracy criticize representative democracy as a result of the possible non-representativeness of the will of the majority due to the risk of decisions being made by self-seeking politicians.

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according to their preferences, being thus the outcome of the voting fully representative of the preferences of the electorate, which meets the demands of supporters of direct democracy. More specifically, we consider voting problems with an odd number of committee members and single-peaked preferences. We study whether there are scoring rules that are Condorcet consistent, that is, scoring rules that select, if exists, the alternative that wins to any other alternative in pairwise comparisons. With only three alternatives, there are scoring rules that yield the Condorcet winner only for committees of three and five agents. With four or more alternatives, only committees of three agents work. In all these scoring rules, the best and worst alternatives are assigned a score of 1 and 0, respectively, and any middle alternative a score between 0 and $\frac{1}{2}$. For five or more alternatives, the score of any middle alternative must be the same, and we call this family semiplurality scoring rules.

After Chapter 3, a section containing the main conclusions obtained from the development of the research objectives in Chapters 1, 2, and 3 is offered. Finally, a summary of this dissertation in Spanish and a section with the referenced bibliography end this essay.

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Chapter 1

Demand Of Direct Democracy

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1.1. Introduction

"We believe that major decisions can and should be legitimised by free and fair referendum - not just at the national level, but also at the regional and local level... We stand for direct democracy for the people. Let the peoples have their say!".

Europe of Freedom and Direct Democracy, a group in the European Parliament, `http://www.efddgroup.eu/about-us/direct-democracy`, April 6, 2019.

For some years now, many social movements asking for a more direct participation of citizens in the political process are emerging in several countries around the world. The idea that decisions should be made by the citizens themselves rather than being delegated to some elected representative is gaining supporters in recent times.

The purpose of this paper is to analyze under which conditions voters prefer representative democracy to direct democracy. In order to achieve our research goal we propose

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a model in which there are two possible policies, l (left) and r (right), to choose from. One of these two policies is *economically efficient*, that is, it is the one that better fits the realized state of the world. In direct democracy voters choose a policy among the two alternatives, while in representative democracy voters select a candidate who, once elected, chooses a policy having observed which is the realized state of the world. Voters and politicians' payoffs depend on a common component which is positive when the policy fits the state of the world, and on a private ideological bias towards one of the policies. We say that a voter (politician) is pragmatic when the common component prevails, while she is ideological when the private component predominates. In direct democracy voters are uncertain about the economically efficient policy, while in representative democracy they are uncertain about whether candidates are pragmatic or ideological, even if they know towards which policy each candidate is biased.

We find the conditions under which the majority of voters prefer a system of representative democracy. In this system, the economically efficient policy is always implemented with higher or equal ex-ante probability than in direct democracy. Consequently, pragmatic voters always prefer a system of representative democracy. The first case in which a society prefers a representative system is therefore when the majority of voters are pragmatic. Since the majority of voters are interested in the implementation of the economically efficient policy and politicians have better information about it, voters are willing to delegate the decision making process to representatives. The second case in which a system of representative democracy is preferred is when the majority of voters are ideological but the median voter is pragmatic. This occurs when the electorate is ideologically polarized between left and right. The intuition is the following. In this situation, the policy chosen in a direct democracy is the one that, ex-ante, is economically efficient with higher probability because the median voter is pragmatic. Assume, without loss of generality, that such policy is l. Thus, voters who are ideologically biased towards policy r along with pragmatic voters constitute a majority that continues to prefer representative democracy. Overall, ideological voters biased towards the policy that is not chosen in direct democracy prefer that the decision is made by an expert to try to avoid that the outcome of the vote is opposite to the one that they wish: representative democracy thus

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constitutes a moderating element between both ideologically opposed sides.

On the other hand, societies in which the majority of voters and the median voter are ideological, implying that the majority of voters have the same ideological bias, are inclined to a system of direct democracy. The reason is that the members of this majority group know what is their preferred policy, not needing the superior information available from the experts. These voters have common aspirations that will remain invariant regardless of what that information is. Thus, for this type of electorate a representative system does not suppose any type of advantage against the direct democracy; arguably, quite the reverse. Such majority of ideological voters are not willing to allow politicians to make decisions because there exists the risk that they end up choosing something totally different to their ambitions. This risk follows from the fact that with some positive probability the elected candidate is pragmatic and the policy different from the ideological bias of these voters is economically efficient.

It is noteworthy that this configuration of the electorate is nothing but a society in which there is a majority group of homogeneous voters in terms of their ideals and with interests opposed to the minority group constituted by the rest of the citizens in that society. In this sense, it is easy to directly connect our results with the populist rhetoric, which has traditionally defended the mobilization of a majority (the people) against a minority (the elite). Understanding this electorate as a populist society, we would be offering a theoretical explanation for the growing interest in the use of instruments of direct democracy shown by societies in which populist movements succeed.

Related Literature

Several papers express the benefits of using the instruments of direct democracy. According to Besley and Coate (2008), policy outcomes on specific issues may differ substantially from what the majority desires when citizens have only one vote to cast for candidates who have to decide on a bundle of issues. They show that citizens' initiatives and referendums prevent such problems from occurring. Matsusaka (2005) states that

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allowing citizens to participate in lawmaking leads to the prevalence of the median voter's preferences along different dimensions and therefore reduces the discretionary performance of the government. Empirical evidence on how direct democracy prevents politicians from increasing spending to favor special interest groups is offered by Santerre (1989) and Sanz (2015).

For its part, another strand in the literature highlights the inability of voters to make decisions due to lack of information. The seminal works of Madison (1787) and Siéyès (1789) stand up for the establishment of a representative democracy in which politicians with an informational advantage decide. Having politicians better informed than voters is a generally used assumption in the literature. The superior information available for the politicians may be of diverse nature. Roemer (1994) and Cukierman and Tommasi (1998) consider models where candidates are better informed than the electorate about how different policies map into outcomes. Schultz (1996, 2002), Martinelli (2001), and Jensen (2009) assume that politicians are better informed than voters about which is the right policy to adopt depending on which is the realization of the state of the world.

In contrast, there is literature that raises the possibility that politicians do not use this additional information for the benefit of citizens. Kartik and Preston McAfee (2007) propose a model in which only a fraction of candidates are committed to implement the policy that they consider to be the most appropriate to maximize the overall welfare of society, even when such policy may not be the most popular among voters. They denote this type of representatives as candidates "with character". However, candidates "without character" only seek to pander voters by carrying out the most popular policy, even knowing that such policy is not the one that maximizes the social welfare. In this vein, Morelli and Van Weelden (2013) introduce a parameter that measures the informational advantage of politicians over voters about the true state of the world and relate it to the probability of pandering by politicians. They find that the less informational advantage politicians have, the greater their incentives to pander. This implies that an increase on the information available to the voters can make all voters worse off since politicians are now more likely to pander.

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Papers cited above dealing with either direct or representative democracy focus exclusively on the analysis of one of these two systems, but without establishing a comparison between them as we will do. Kessler (2005) proposes a model to study the advantages of each of these regimes. Her findings are in line with the already pointed out benefits of both systems: policies chosen through direct democracy are closer to the preferences of the median voter, while policies chosen through representative democracy fit better to the existing state of the world given the superior information of politicians. As far as we know, Maskin and Tirole (2004) is the only paper in the literature that develops a theoretical analysis investigating the conditions for the suitability of each of these regimes. Assuming that the goal is the maximization of the social welfare they study whether, for that purpose, decisions should be made by the public directly, politicians subject to reelection, or independent judges. In this sense, they develop a normative analysis about the appropriateness of one or the other system. On the contrary, our purpose is to carry out a predictive analysis: we aim to determine which system will be demanded by a majority of voters.

In this sense, our paper also contributes to the literature about populism. There is empirical evidence that shows that individuals who are dissatisfied with the system of political representation are in favor of the use of instruments of direct democracy (Dalton 2004; Pauwels 2014; Hawkins and Rovira Kaltwasser 2017). This disaffection with representative democracy systems is one of the fundamental basis of the rhetoric of populism that repudiates the elite in power acting against the will of the people, demanding consequently for the return of decision-making power to the ordinary citizens (Fieschi and Heywood 2004). Voters with populist attitudes have been recently identified as those sharing preferences for both anti-elitism and people-centrism (Rooduijn 2014; Rooduijn et al. 2014). Mohrenberg et al. (2017) shows empirical evidence that those citizens with populist attitudes support direct democracy more than other citizens. Guiso et al. (2017) offers empirical evidence that the crisis of systemic economic security (mainly motivated by threats of globalization and migration) that Western countries are experiencing in the last decade undermines citizens' confidence in traditional political parties and institutions.

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Instead, citizens would be prone to vote for a party with a populist rhetoric that calls for the return of power to the people, which can be understood as a demand for direct democracy. To the best of our knowledge, our paper is the first to offer a theoretical justification about the reasons why a demand for direct democracy among citizens may exist. Furthermore, following well-established arguments in the literature, we are able to equate those societies that promote more direct citizen participation in the decision-making process with societies with populist attitudes, thus providing a theoretical framework for the empirical evidence about the successful of direct democracy among populist societies already existing in the literature.

The remainder of the paper is organized as follows. In Section 2 we set out the model. In Section 3 we derive equilibria under direct and representative democracy determining so the outcome under each system. In Section 4 we carry out an analysis of voters' expected utility in order to determine which of these systems is preferred by the majority of the electorate. Finally, we discuss and conclude in Section 5. Appendix A offers some explanatory notes. Appendix B provides the proofs of the results.

1.2. The Model

We study two voting systems: direct democracy and representative democracy. In direct democracy, voters directly cast their ballots for an alternative. In representative democracy, voters choose the representative, who will then choose the implemented policy.

Let N be a unitary mass of voters and $X = \{l, r\}$ be the set of policies. Voters' utility has two components: an ideological component, which represents the ideological bias of each voter towards one of the policies, and a monetary component. The ideological component is private while the monetary one is common to every voter and depends on the realization of a random variable $\theta \in X$ which realizes after voters have voted. The monetary utility of a policy $x \in X$ is $v > 0$ if $\theta = x$ and zero otherwise. We then refer to the policy that maximizes the monetary utility of each voter as the policy which

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is **economically efficient**. Representatives, but not voters, observe the realization of θ. All voters have the same beliefs about which policy is economically efficient: l with probability p and r with probability $1-p$, where $p \in (0,1)$.[1] Formally, the utility of a voter $j \in N$ is given by:

$$u_j(x,\theta) = z_j(x) + \mathbb{1}\{\theta = x\}v \tag{1.1}$$

where $z_j(x)$ is the *ideological component* and $\mathbb{1}\{\theta = x\}v$ is the *monetary component* in which $\mathbb{1}\{\theta = x\}$ is an indicator function that takes unit value if the implemented policy is economically efficient and zero otherwise. We impose no restrictions on $z_j(\cdot)$, thereby accommodating many situations.

Let $b_j = z_j(r) - z_j(l)$ for every $j \in N$, so that $b_j < 0$ and $b_j > 0$ reflect ideological bias for l and r respectively. A voter j has no ideological bias if $b_j = 0$. Voters' biases are distributed according to $F(\cdot)$ with a positive density function $f(\cdot)$ and median m. Let $b_m = z_m(r) - z_m(l)$ denote the median voter's bias.

Voters are either *ideological* or *pragmatic*. A voter $j \in N$ is **ideological** if $|b_j| > v$, while she is **pragmatic** if $|b_j| \leq v$. Voter j is ideological if her bias is higher than the utility derived from the economically efficient policy and pragmatic otherwise. An ideological voter is concerned about the implementation of the policy towards which she is biased, regardless of the economically efficient policy. In contrast, a pragmatic voter is interested in the implementation of the economically efficient policy, regardless of her ideological bias. This happens when the difference of utility between l and r is compensated by v.[2]

Let Λ be the set of voters with $b_j < -v$, *i.e.*, ideological voters biased towards l, and $\lambda > 0$ be the proportion of these voters. Let P be the set of voters with $b_j > v$, *i.e.*, ideological voters biased towards r, and $\rho > 0$ be the proportion of these voters. Let Π be

[1] These probabilities are exogenous and common knowledge. Note that cases in which representative democracy does not suppose a comparative advantage against direct democracy in terms of available information, *i.e.*, $p = 0$ and $p = 1$, are out of our range of interest since there would be no room for delegation, so direct democracy would be trivially preferred.

[2] See *Appendix A* for a graphical explanation of both ideological biases and types of voters.

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the set of voters with $-v \leq b_j \leq v$, *i.e.*, pragmatic voters, and $\pi > 0$ be the proportion of these voters.[3] Obviously, $\lambda + \rho + \pi = 1$. We refer to $(\Lambda, \Pi, \mathrm{P})$ as an electorate.

By abuse of language, we distinguish two types of electorate: *ideological electorate* and *pragmatic electorate*. The electorate is **ideological** if the majority of voters are ideological, while the electorate is **pragmatic** if the majority of voters are pragmatic.

Definition 1. *The electorate is* ***ideological*** *if* $\lambda + \rho > \frac{1}{2}$. *Otherwise, it is pragmatic.*

An ideological electorate is **polarized** if no ideological group constitutes a majority by itself.

Definition 2. *The electorate is* ***polarized*** *if* $\lambda + \rho > \frac{1}{2}$, $\lambda < \frac{1}{2}$, *and* $\rho < \frac{1}{2}$.

If the electorate is polarized, then it is also ideological but not necessarily the opposite.

In a representative democracy there are two candidates: let $C = \{L, R\}$ denote the set of candidates and $c \in C$ a generic candidate. Both candidates are experts who observe the realization of the random variable θ.[4] The candidates have the same form of utility functions as the voters.[5] Candidate c's utility is:

$$u_c(x, \theta) = z_c(x) + \mathbb{1}\{\theta = x\}v \tag{1.2}$$

Let $b_c = z_c(r) - z_c(l)$ for both $c \in C$, which is interpreted as in the case of voters. We assume that $b_L < 0$ and $b_R > 0$, so that candidate L is biased for l and candidate R is biased for r. Candidates' ideological bias is common knowledge.

Like voters, candidates can be *ideological* or *pragmatic*. A candidate $c \in C$ is **ideological** if $|b_c| > v$, while she is **pragmatic** if $|b_c| \leq v$. An ideological candidate always

[3] Note that $\lambda, \rho, \pi > 0$ given that we assume a positive density function $f(\cdot)$.

[4] This is common knowledge.

[5] This assumption might be relaxed without affecting our results.

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prefers to implement the policy towards which she is biased regardless of the economically efficient policy, while a pragmatic candidate always prefers to implement the economically efficient policy. For each candidate $c \in C$, all voters believe that c is pragmatic with the same probability $\mu_c \in (0,1)$. These probabilities are exogenous and common knowledge. The probability of being pragmatic is not necessarily equal for both candidates.

Direct Democracy

In direct democracy voters choose a policy and the one which is voted by a majority is implemented, regardless of the realization of the random variable θ. Let $x^D \in X$ be the policy implemented in this voting system.

Let $\mathbb{E}u_j(x^D, \theta)$ denote the expected utility of a voter $j \in N$ in **direct democracy** when policy $x^D \in X$ is implemented. Equations (1.3) and (1.4) represent the cases $x^D = l$ and $x^D = r$ respectively.

$$\mathbb{E}u_j(l, \theta) = z_j(l) + pv \tag{1.3}$$

$$\mathbb{E}u_j(r, \theta) = z_j(r) + (1-p)v \tag{1.4}$$

Each voter $j \in N$ will cast her ballot for the policy (l or r) which maximizes her expected utility.

Representative Democracy

In representative democracy voters choose a candidate and the one which is voted by a majority is selected. Let $c^I \in C$ be the candidate selected in this voting system.[6] The winning candidate observes θ and implements the policy that maximizes her utility.

[6]For notational simplicity, we also refer to representative democracy as *indirect* democracy, so we use letter I in order to avoid confusion with the notation R used for candidates.

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Let $\mathbb{E}u_j(c^I, \theta)$ denote the expected utility of a voter $j \in N$ in **representative democracy** when candidate $c^I \in C$ is selected and such candidate implements the policy that maximizes her utility. Equations (1.5) and (1.6) represent the cases $c^I = L$ and $c^I = R$ respectively.

$$\mathbb{E}u_j(L, \theta) = (1-p)\mu_L\Big(v + b_j\Big) + pv + z_j(l) \tag{1.5}$$

$$\mathbb{E}u_j(R, \theta) = p\mu_R\Big(v - b_j\Big) + (1-p)v + z_j(r) \tag{1.6}$$

Each voter $j \in N$ votes for the candidate (L or R) which maximizes her expected utility.

1.3. Equilibria

We consider rational voters who vote for the alternative (a *policy* in direct democracy and a *candidate* in representative democracy) that they prefer. In case of being indifferent, we assume that they vote for the status quo alternative which, without loss of generality, we assume being policy l and candidate L respectively. In order to determine the outcome in each system, we identify an indifferent voter and the median voter whose preference determines the outcome. The indifferent voter in direct democracy, i^D, is the voter who is indifferent between policies l and r, *i.e.*, $\mathbb{E}u_{i^D}(l, \theta) = \mathbb{E}u_{i^D}(r, \theta)$. The indifferent voter in representative democracy, i^I, is the voter who is indifferent between candidates L and R, *i.e.*, $\mathbb{E}u_{i^I}(L, \theta) = \mathbb{E}u_{i^I}(R, \theta)$.

Lemmata 1 and 2 characterize the indifferent voter in both systems. Proofs of these Lemmata are in *Appendix B*.

Lemma 1. *In direct democracy, a voter $j \in N$ is indifferent between l and r if and only if $b_j = v(2p-1) \equiv b_{i^D}$. Every voter j such that $b_j < b_{i^D}$ ($b_j > b_{i^D}$) votes for policy l (r).*

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If $p > \frac{1}{2}$ ($p < \frac{1}{2}$), then $b_{i^D} > 0$ ($b_{i^D} < 0$): if policy l (r) is more likely to be economically efficient, then the indifferent voter in direct democracy must be ideological biased towards policy r (l). Clearly, if $p = \frac{1}{2}$, the indifferent voter has no ideological bias.

Lemma 2. *In representative democracy, a voter $j \in N$ is indifferent between L and R if and only if $b_j = \frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L} \equiv b_{i^I}$. Every voter j such that $b_j < b_{i^I}$ ($b_j > b_{i^I}$) votes for candidate L (R).*

The indifferent voter in representative democracy depends on the probability of each policy to be economically efficient (as in the case of direct democracy) and the voters' beliefs about candidates, namely whether they are pragmatic or ideological (μ_L, μ_R). A comparative statics analysis reveals that if candidate L is more likely to be pragmatic ($\mu_L > \mu_R$) and policy l is more likely to be economically efficient ($p > \frac{1}{2}$), then a pragmatic voter votes for candidate L since the ex-ante probability of having the economically efficient policy implemented is higher. Analogous interpretation applies for the case in which $\mu_L < \mu_R$ and $p < \frac{1}{2}$. Suppose that, as before, candidate L is more likely to be pragmatic, but now policy r is more likely to be economically efficient ($p < \frac{1}{2}$). Then, a pragmatic voter votes for candidate R only when the probability of her ideological bias (*i.e.*, policy r) being economically efficient is high enough to offset the risk entailed when voting a candidate who is ex-ante more likely to be ideological.

Understanding the behavior of a pragmatic voter becomes relevant given the fact that the indifferent voters in direct and representative democracy are always pragmatic. Lemma 3 states this result. Its proof is in *Appendix B*.

Lemma 3. *For every $p \in (0,1)$, $v \geq 0$, and $\mu_c \in (0,1)$ where $c \in C$, both the indifferent voters in direct and representative democracy are pragmatic.*

A voter who is indifferent between candidates in a representative system may strictly

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prefer a policy in direct democracy. As we have shown in Lemma 1, a comparison between b_j and b_{i^D} reveals the policy voted by j in direct democracy. By studying the relationship between b_{i^I} and b_{i^D} we can know the policy voted by i^I. Lemma 4 shows that the outcomes of this analysis depend on the voters' beliefs about the pragmatism of candidates. The proof of this Lemma is in *Appendix B*.

Lemma 4. *The indifferent voter in representative democracy i^I votes for policy l (r) in direct democracy if $\mu_L \leq \mu_R$ ($\mu_L > \mu_R$), for every $p \in (0,1)$.*

By Lemma 3 we know that i^I is pragmatic. Consider the case in which candidate R is more likely to be pragmatic ($\mu_L < \mu_R$). By Lemma 4, we know that i^I votes for policy l in direct democracy (*i.e.*, $b_{i^I} < b_{i^D}$). Figure 1.1 shows this situation. Here, a pragmatic voter who votes for policy l in direct democracy may end up voting for candidate R in representative democracy even though the ideological bias of such candidate is a policy different from the one that she prefers in direct democracy (see the dotted area in Figure 1.1). The reason is that selecting a candidate who, ex-ante, is more likely to implement the economically efficient policy generates an increase on the expected utility of pragmatic voters and such increase could be high enough to compensate for the risk that such candidate would eventually be ideological. Analogous interpretation applies to the case $\mu_L > \mu_R$. When $\mu_L = \mu_R$, both candidates are perceived by voters as equally likely to be pragmatic. Their ideological biases are therefore the only differentiating factor between candidates, so saying that a voter is indifferent between L and R is equivalent to say that such voter is indifferent between l and r.[7]

[7]We remind that in case of indifference between policies a voter votes for the status quo alternative in direct democracy, *i.e.*, policy l.

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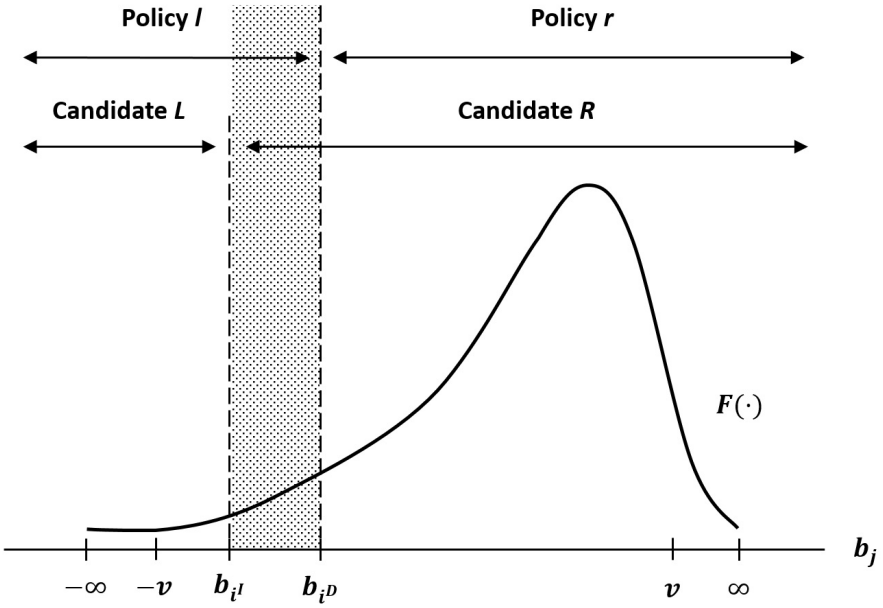

Figure 1.1 – Case $\mu_L < \mu_R$ and $p \in (0, 1)$

It is well-known that in a majoritarian voting system, the most preferred alternative by the median voter is selected. Lemma 5 states, for each possible configuration of parameters b_m, b_{iD}, and b_{iI}, the policy and the candidate that are selected in one and another system respectively. The proof for this result directly follows from Lemma 4.

Lemma 5. *The outcomes in direct and representative democracy are:*

(i) if $b_m \in \Big(-\infty, \min\{b_{iD}, b_{iI}\}\Big]$, *then* $x^D = l$ *and* $c^I = L$,

(ii) if $b_m \in \Big(\max\{b_{iD}, b_{iI}\}, \infty\Big)$, *then* $x^D = r$ *and* $c^I = R$, *and*

(iii) if $b_m \in \Big(\min\{b_{iD}, b_{iI}\}, \max\{b_{iD}, b_{iI}\}\Big]$, *then:*

(iii.i) $x^D = l$ *and* $c^I = R$ *if* $b_{iI} < b_{iD}$, *while*

(iii.ii) $x^D = r$ *and* $c^I = L$ *if* $b_{iI} > b_{iD}$.

By Lemma 5 we have that if $b_{iD} = b_{iI}$, then the selected candidate in representative democracy is the one whose ideological bias coincides with the policy that would be chosen

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in direct democracy. By Lemma 4 we know that this happens when voters believe that both candidates are equally likely to be pragmatic. However, this does not necessarily happen when $b_{i^D} \neq b_{i^I}$. In that case, the selected candidate in representative democracy could be ideologically biased towards the policy that is not chosen in direct democracy. Assume, without loss of generality, that $b_{i^I} < b_{i^D}$ (case illustrated in Figure 1.1). Suppose that $b_m \in (b_{i^I}, b_{i^D}]$. Note that the median voter is pragmatic since $|b_m| \leq v$.[8] In direct democracy, policy l is chosen, while candidate R is selected in representative democracy. It is not risky to claim that this situation represents a society in which a majority of voters show an interest by the implementation of the economically efficient policy: even though R's ideological bias is not the policy l, the median voter decides to vote for her since she is more likely to be pragmatic.

1.4. Electorate's Preference on Systems

Will voters incline towards direct democracy or representative democracy? The purpose of this section is to determine which is the system preferred by the majority of voters given the uncertainty about both the economically efficient policy and the pragmatism of each candidate. The answer hinges on the median voter. We compare the median voter's expected utility under direct democracy (*i.e.*, the utility before a *policy* is selected) with her expected utility under representative democracy (*i.e.*, the utility before a *candidate* is selected). We say that representative democracy is preferred to direct democracy by the majority of voters if $\mathbb{E}u_m(x^D, \theta) \leq \mathbb{E}u_m(c^I, \theta)$. Proposition 1 shows that the preferred voting system depends on both the type of the electorate and the type of the median voter of the distribution of voters' biases. The proof of this proposition is in *Appendix B*.

Proposition 1. *If the electorate is pragmatic, then representative democracy is the system preferred by the majority of voters. If the electorate is ideological, then:*

(i) if the median voter is pragmatic, representative democracy is the system preferred

[8] By Lemma 3 we have that $|b_{i^D}|, |b_{i^I}| \leq v$, so if $b_m \in (b_{i^I}, b_{i^D}]$, then $|b_m| \leq v$ trivially holds.

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by the majority of voters, while

(ii) if the median voter is ideological, direct democracy is the system preferred by the majority of voters.

Corollary 1 directly follows from Proposition 1.

Corollary 1. *Representative democracy is the system preferred by the majority of voters if and only if the median voter is pragmatic.*

From Proposition 1 we can distinguish three different situations.

When the electorate is pragmatic, a majority of voters are pragmatic and they prefer that the decision on which policy to implement be made by an expert. In fact, in representative democracy the economically efficient policy is always implemented with higher or equal ex-ante probability than in direct democracy. Note that here the median voter is necessarily pragmatic.

When the electorate is ideological and the median voter is pragmatic, a majority of voters are ideological but neither ideological voters biased towards l nor ideological voters biased towards r constitute a majority by themselves. This is what we call a **polarized electorate**. The fact that a society in which the majority of voters are not concerned about the economically efficient policy ends up preferring to delegate decisions on a candidate may seem, at first glance, counterintuitive. The polarization of the electorate between l and r is playing a key role in this. Since the median voter is pragmatic, in a direct democracy the policy that ex-ante is economically efficient with higher probability is chosen. Note that, due to the ideological polarization of the electorate, there is a group of voters who are dissatisfied with the outcome of direct democracy, that is, those ideological voters who are biased towards the policy that is not chosen in such system. Therefore, these voters prefer to delegate decisions to experts who may select their preferred policy with positive probability. Thus, in this situation there exists a majority coalition of voters

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supporting the representative democracy composed by the group of pragmatic voters and a group of ideological voters. For its part, the group of ideological voters who are biased towards the policy that is chosen in direct democracy clearly prefer that system. Such proportion of voters who are satisfied with direct democracy could be interpreted as a measure of the demand of direct democracy in the electorate. Proposition 2 shows how a shift in the distribution of voters' biases can increase the demand of direct democracy.

Proposition 2. *Let* $(\Lambda, \Pi, \mathrm{P})$ *and* $(\Lambda', \Pi', \mathrm{P}')$ *be two polarized electorates such that* $\Lambda \subseteq \Lambda'$, $\mathrm{P} \subseteq \mathrm{P}'$ *with at least one strict inclusion, and* $\min\{\lambda', \rho'\} \geq \max\{\lambda, \rho\}$. *Then, the demand of direct democracy from* $(\Lambda, \Pi, \mathrm{P})$ *to* $(\Lambda', \Pi', \mathrm{P}')$ *increases.*

Proof of Proposition 2 is straightforward and is omitted. From $(\Lambda, \Pi, \mathrm{P})$ to $(\Lambda', \Pi', \mathrm{P}')$, a fraction of initially pragmatic voters (*i.e.*, voters in Π) becomes ideological. Since the cardinality of both groups of ideological voters in $(\Lambda', \Pi', \mathrm{P}')$ is at least as high as the cardinality of the largest ideological group in $(\Lambda, \Pi, \mathrm{P})$ and is necessarily one of these two groups that will demand direct democracy, the demand of direct democracy increases. It is noticeable that a shift in the distribution of voters' biases as the one proposed in Proposition 2 is consistent with the definition of polarization increase offered by Esteban and Ray (1994). Corollary 2 directly follows from Proposition 2 and this observation.

Corollary 2. *The demand of direct democracy is increasing in electorate's polarization.*

From Corollary 2 follows that when the electorate is polarized and therefore representative democracy is still the system preferred by the majority of voters, an increase in the polarization of the electorate increases the demand of direct democracy.

Finally, when the electorate is ideological and the median voter is ideological, a majority of voters are ideological and furthermore one of the ideological groups constitutes a majority by itself. Since at least 50% of voters are interested in the implementation of an specific policy regardless of which policy is the economically efficient one, direct

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democracy is the preferred system. It allows voters to implement whatever policy they desire, thus preventing a candidate from ending up implementing a different policy. This is where our results connect with literature about **populist movements**. Traditionally, populist movements have been understood as the mobilization of a majority (the people) against a minority (the elite). The majority group composed by ideological voters all biased towards the same policy would constitute the majority that, according to a theory of populism, shares common aspirations and aims at achieving them at all costs (Guiso et al. 2017). For its part, the rest of voters on that society (that is, ideological voters biased towards the opposite policy and pragmatic voters) would constitute the minority that does not present the same ambitions as the majority group, motivating thus the confrontation between both groups and capturing therefore the essence of the populism (Akkerman et al. 2014).

1.5. Conclusion

When a decision has to be made, voters may prefer to directly decide or delegate to informed representatives. We have proposed a model to study when voters would show a predisposition towards one or another option. We find that the preference of a society to be governed by a system of direct or representative democracy depends on both the type of the electorate and the type of the median voter. As long as the electorate is pragmatic, representative democracy is the preferred system. The informational advantage of representatives is enough for an electorate concerned about the implementation of the economically efficient policy to have incentives to delegate. When the electorate is ideological, though, we have to look at the type of the median voter in order to determine the preferred system. If the electorate is ideological and the median voter is pragmatic, then representative democracy is the preferred system. Since no ideological group has a strict majority in the society, a system of political representation is preferred even when the majority of voters are ideological. Although there is a majority of voters who do not care about what is economically efficient, there is no consensus among all of them on what action should be carried out. Instead, there exists a clear division of the electorate

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into two groups defending opposing policies. This situation in which neither of these two groups constitutes a majority by itself is what we have identified as a polarized electorate. As a consequence of this polarization, whatever the outcome in direct democracy is, there is always a group of ideological voters who oppose such a policy. Trying to avoid that something opposed to their own ideological bias is chosen, the group of ideological voters biased towards the policy that would not be chosen in direct democracy prefers to delegate their vote to representatives. These ideological voters, along with the existing pragmatic voters in the society, constitute a majority coalition in favour of the representative democracy. Moreover, the demand of direct democracy is increasing in the polarization of the electorate, as long as no ideological group becomes a majority. On the contrary, when an ideological group becomes a majority, and therefore both electorate and median voter are ideological, direct democracy is the preferred system. This is a society in which there is a majority of voters who do not care about what is economically efficient and agree on what decision should be made. Such an electorate prefers a system that guarantees the implementation of the policy that they desire, rather than running the risk of allowing a representative to choose. This situation is what we have associated with societies in which populist movements have been successful. In this way, our model predicts that an increase in populism will increase the demand for direct democracy.

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1.6. Appendix A

Figure 1.2 shows how a voter j who is ideologically biased towards a certain policy could be either pragmatic or ideological depending on whether the magnitude of such bias is greater or less than the monetary utility in absolute terms.

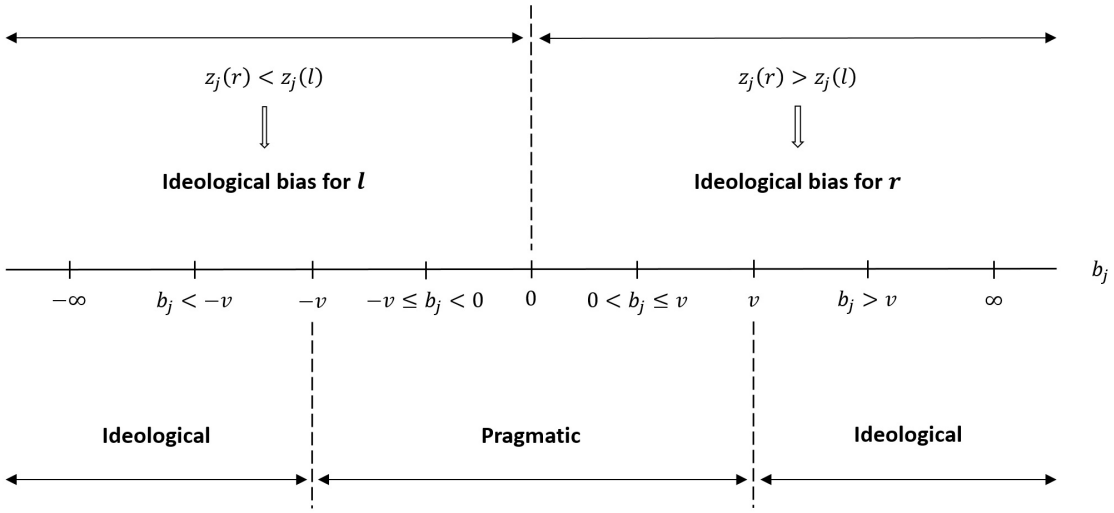

Figure 1.2 – Ideologically biased pragmatic or ideological voter

Consider a voter j such that $b_j < -v$. Since $b_j < 0$ (*i.e.*, $z_j(r) < z_j(l)$), we know that voter j is ideologically biased towards policy l. In addition, when $b_j < -v$, the difference between the ideological component evaluated in l and the ideological component evaluated in r is so large that it cannot be compensated by the monetary component of the utility function. Thus, voter j is concerned about the implementation of policy l, regardless what the economically efficient policy is. Note that, by symmetry, a voter j such that $b_j > v$ will be concerned about the implementation of policy r above all things. For this reason, in both cases we say that voter j is *ideological*. Consider now a voter j such that $-v \leq b_j < 0$. In this case, voter j is still ideologically biased towards policy l. However, here the utility derived from the monetary component when the economically efficient policy is implemented is larger than the difference between the ideological component evaluated in l and the ideological component evaluated in r. This implies that, although voter j is still ideologically biased towards policy l, she ends up preferring policy r when it is the economically efficient policy. By symmetry, a voter j such that $0 < b_j \leq v$ will prefer policy l when it is economically efficient, even though she is ideologically biased

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towards policy r (*i.e.*, $b_j > 0$). Consequently, we refer to voter j as *pragmatic* voter in both cases.

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1.7. Appendix B

PROOF OF LEMMA 1. Let i^D be such that:

$$\mathbb{E}u_{i^D}(l,\theta) = \mathbb{E}u_{i^D}(r,\theta) \tag{1.7}$$

From Equation (1.3) we have that the expected utility of i^D when policy l is implemented is:

$$\mathbb{E}u_{i^D}(l,\theta) = z_{i^D}(l) + pv \tag{1.8}$$

and, from Equation (1.4), the expected utility of i^D when policy r is implemented is:

$$\mathbb{E}u_{i^D}(r,\theta) = z_{i^D}(r) + (1-p)v \tag{1.9}$$

By substituting (1.8) and (1.9) in (1.7) such condition can be rewritten as:

$$\begin{aligned} z_{i^D}(l) + pv &= z_{i^D}(r) + (1-p)v \\ \Leftrightarrow z_{i^D}(r) - z_{i^D}(l) &= v(2p-1) \\ \Leftrightarrow b_{i^D} &= v(2p-1) \end{aligned} \tag{1.10}$$

Consider now a voter $j \in N$ such that $b_j < b_{i^D}$. Note that this condition is equivalent to:

$$\begin{aligned} b_j &< v(2p-1) \\ \Leftrightarrow z_j(r) - z_j(l) &< v(2p-1) \\ \Leftrightarrow \mathbb{E}u_j(l,\theta) &> \mathbb{E}u_j(r,\theta) \end{aligned} \tag{1.11}$$

which implies that the expected utility of j when policy l is implemented is higher than her expected utility when policy r is implemented. Therefore, voter j votes for policy l. By symmetry, if $b_j > b_{i^D}$, then voter j votes for policy r.

■

PROOF OF LEMMA 2. Let i^I be such that:

$$\mathbb{E}u_{i^I}(L,\theta) = \mathbb{E}u_{i^I}(R,\theta) \tag{1.12}$$

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From Equation (1.5) we have that the expected utility of i^I when the candidate L is elected is:

$$\mathbb{E}u_{i^I}(L,\theta) = (1-p)\mu_L\Big(v + b_{i^I}\Big) + pv + z_{i^I}(l) \tag{1.13}$$

and, from Equation (1.6), the expected utility of i^I when the candidate R is elected is:

$$\mathbb{E}u_{i^I}(R,\theta) = p\mu_R\Big(v - b_{i^I}\Big) + (1-p)v + z_{i^I}(r) \tag{1.14}$$

By substituting (1.13) and (1.14) in (1.12) such condition can be rewritten as:

$$\begin{aligned}(1-p)\mu_L\Big(v + b_{i^I}\Big) + pv + z_{i^I}(l) &= p\mu_R\Big(v - b_{i^I}\Big) + (1-p)v + z_{i^I}(r)\\ \Leftrightarrow z_{i^I}(r) - z_{i^I}(l) &= \frac{v\Big(2p - 1 - p\mu_R + (1-p)\mu_L\Big)}{1 - p\mu_R - (1-p)\mu_L}\\ \Leftrightarrow b_{i^I} &= \frac{v\Big(2p - 1 - p\mu_R + (1-p)\mu_L\Big)}{1 - p\mu_R - (1-p)\mu_L}\end{aligned} \tag{1.15}$$

Consider now a voter $j \in N$ such that $b_j < b_{i^I}$. Note that this condition is equivalent to:

$$\begin{aligned}b_j &< \frac{v\Big(2p - 1 - p\mu_R + (1-p)\mu_L\Big)}{1 - p\mu_R - (1-p)\mu_L}\\ \Leftrightarrow z_j(r) - z_j(l) &< \frac{v\Big(2p - 1 - p\mu_R + (1-p)\mu_L\Big)}{1 - p\mu_R - (1-p)\mu_L}\\ \Leftrightarrow \mathbb{E}u_{i^I}(L,\theta) &> \mathbb{E}u_{i^I}(R,\theta)\end{aligned} \tag{1.16}$$

which implies that the expected utility of j when candidate L is elected is higher than her expected utility when candidate R is elected. Therefore, voter j votes for candidate L. By symmetry, if $b_j > b_{i^I}$, then voter j votes for candidate R.

■

PROOF OF LEMMA 3. We divide this proof into two parts:

- First, we prove that $|b_{i^D}| \leq v$, or equivalently, $|v(2p-1)| \leq v$. Note that this is equivalent to prove that conditions (1) and (2) hold:

  (1) $v(2p-1) \leq v$, which holds since $p \leq 1$ is always the case.

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(2) $v(2p-1) \geq -v$, which holds since $p \geq 0$ is always the case.

Hence, we have that $|v(2p-1)| \leq v$ for all $p \in (0,1)$ and $v \geq 0$.

- Second, we prove that $|b_{i^I}| \leq v$, or equivalently, $\left|\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}\right| \leq v$. Note that this is equivalent to prove that conditions (3) and (4) hold:

(3) $\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L} \leq v$

(4) $\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L} \geq -v$

We distinguish three cases depending on the values of μ_L and μ_R. We show that conditions (3) and (4) hold for each of these cases:

a. Suppose that $\mu_L = \mu_R$. Then, conditions (3) and (4) are equivalent to conditions (1) and (2) respectively. Therefore, they hold for all $p \in (0,1)$ and $v \geq 0$.

b. Suppose that $\mu_L < \mu_R$. Then, condition (3) requires that $\mu_L \leq 1$, and condition (4) requires that $\mu_R \leq 1$, which are always the case. Therefore, they hold for all $p \in (0,1)$ and $v \geq 0$.

c. Suppose that $\mu_L > \mu_R$. By symmetry to case *b.*, conditions (3) and (4) also hold for all $p \in (0,1)$ and $v \geq 0$.

Hence, we have that $\left|\frac{v\left(2p-1-p\mu_R+(1-p)\mu_L\right)}{1-p\mu_R-(1-p)\mu_L}\right| \leq v$ for all $p \in (0,1)$, $v \geq 0$ and every $\mu_c \in (0,1)$ where $c \in C$.

■

PROOF OF LEMMA 4. By Lemma 1 we know that the relationship between b_{i^I} and b_{i^D} reveals the policy voted by i^I. Let $p \in (0,1)$. First, notice that $b_{i^I} = b_{i^D} \equiv v(2p-1)$ (*i.e.*, i^I is indifferent between l and r in direct democracy) when $\mu_L = \mu_R$. Now

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assume, without loss of generality, that $b_{i^I} < b_{i^D}$ (*i.e.*, i^I votes for l in direct democracy). By Lemmas 1 and 2, this condition can be rewritten as:

$$\frac{v\Big(2p-1-p\mu_R+(1-p)\mu_L\Big)}{1-p\mu_R-(1-p)\mu_L} < v(2p-1) \tag{1.17}$$

which holds when:

$$\mu_L < \mu_R \tag{1.18}$$

By symmetry, $b_{i^I} > b_{i^D}$ (*i.e.*, i^I votes for r in direct democracy) when $\mu_L > \mu_R$.

■

PROOF OF PROPOSITION 1. We divide this proof into two parts.

- First, we study which is the system preferred by the majority of voters. We distinguish three cases, which correspond to the cases identified in Lemma 5. For each of these cases, we compare the expected utility of the median voter in direct and representative democracy.

Case 1. Suppose that $b_m \in \Big(-\infty, \min\{b_{i^D}, b_{i^I}\}\Big]$. Then representative democracy is preferred to direct democracy if:

$$\begin{aligned} &\mathbb{E}u_m(l,\theta) \leq \mathbb{E}u_m(L,\theta) \\ \Leftrightarrow\ & z_m(l)+pv \leq (1-p)\mu_L\Big(v+b_m\Big)+pv+z_m(l) \\ \Leftrightarrow\ & b_m \geq -v \end{aligned} \tag{1.19}$$

Case 2. Suppose that $b_m \in \Big(\max\{b_{i^D}, b_{i^I}\},\ \infty\Big)$. Then representative democracy is preferred to direct democracy if:

$$\begin{aligned} &\mathbb{E}u_m(r,\theta) \leq \mathbb{E}u_m(R,\theta) \\ \Leftrightarrow\ & z_m(r)+(1-p)v \leq p\mu_R\Big(v-b_m\Big)+(1-p)v+z_m(r) \\ \Leftrightarrow\ & b_m \leq v \end{aligned} \tag{1.20}$$

Case 3. Suppose that $b_m \in \Big(\min\{b_{i^D}, b_{i^I}\},\ \max\{b_{i^D}, b_{i^I}\}\Big]$ and:

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(3.1) $b_{i^I} < b_{i^D}$. Then representative democracy is preferred to direct democracy if:

$$\begin{aligned} &\mathbb{E}u_m(l,\theta) \leq \mathbb{E}u_m(R,\theta) \\ \Leftrightarrow\ & z_m(l) + pv \leq p\mu_R\Big(v - b_m\Big) + (1-p)v + z_m(r) \\ \Leftrightarrow\ & b_m \geq \frac{v\Big(-1 + p(2-\mu_R)\Big)}{1-p\mu_R} \end{aligned} \tag{1.21}$$

which always holds in the considered interval $(b_{i^I}, b_{i^D}]$ since $\frac{v\left(-1+p(2-\mu_R)\right)}{1-p\mu_R} \leq b_{i^I}$ for all $p \in (0,1)$, $v \geq 0$, and $\mu_c \in (0,1)$ where $c \in C$. Therefore, we have that representative democracy is preferred to direct democracy throughout the interval $(b_{i^I}, b_{i^D}]$.

(3.2) $b_{i^I} > b_{i^D}$. Then representative democracy is preferred to direct democracy if:

$$\begin{aligned} &\mathbb{E}u_m(r,\theta) \leq \mathbb{E}u_m(L,\theta) \\ \Leftrightarrow\ & z_m(r) + (1-p)v \leq (1-p)\mu_L\Big(v + b_m\Big) + pv + z_m(l) \\ \Leftrightarrow\ & b_m \leq \frac{v\Big(-1 + 2p + (1-p)\mu_L\Big)}{1-(1-p)\mu_L} \end{aligned} \tag{1.22}$$

which always holds in the considered interval $(b_{i^D}, b_{i^I}]$ since $b_{i^I} \leq \frac{v\left(-1+2p+(1-p)\mu_L\right)}{1-(1-p)\mu_L}$ for all $p \in (0,1)$, $v \geq 0$, and $\mu_c \in (0,1)$ where $c \in C$. Therefore, we have that representative democracy is preferred to direct democracy throughout the interval $(b_{i^D}, b_{i^I}]$.

Note that, by Lemma 3, $|b_{i^D}| \leq v$ and $|b_{i^I}| \leq v$. Thus, from *Cases 1*, *2*, and *3* it is derived that representative democracy is the system preferred by the majority of voters as long as $|b_m| \leq v$. Equivalently, representative democracy is the system preferred by the majority of voters if the median voter is pragmatic, while direct democracy is the system preferred by the majority of voters if the median voter is ideological.

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- Second, we study the preference for one or another system depending on the type of the electorate.

  i. *Pragmatic electorate.* By definition, the electorate is pragmatic if the majority of voters are pragmatic. This is equivalent to say that $\pi \geq 0.5$. Consequently, $\lambda + \rho \leq 0.5$. Given that $\lambda, \rho > 0$, the previous condition implies that $\lambda, \rho < 0.5$. Therefore, the median voter of the distribution will necessarily belong to the proportion of voters denoted by π. In other words, if the electorate is pragmatic, then the median voter is necessarily pragmatic. From *Cases 1*, *2*, and *3* above we know that representative democracy is the system preferred by the majority of voters if the median voter is pragmatic. Thus, we conclude that if the electorate is pragmatic, then representative democracy is the system preferred by the majority of voters.

  ii. *Ideological electorate.* By definition, the electorate is ideological if the majority of voters are ideological. This is equivalent to say that $\lambda + \rho > 0.5$. Consequently, $\pi < 0.5$. In this case, the median voter is not necessarily either pragmatic or ideological. The type of the median voter depends on the specific configuration of parameters λ and ρ as follows:

     - Assume that $\lambda + \rho > 0.5$. Therefore, the electorate is ideological. If either $\lambda \geq 0.5$ or $\rho \geq 0.5$, then the median voter belongs to the proportion of voters denoted by either λ or ρ respectively. In other words, the median voter is ideological. From *Cases 1*, *2*, and *3* above we know that direct democracy is the system preferred by the majority of voters if the median voter is ideological. Thus, we conclude that if the electorate is ideological and the median voter is ideological, then direct democracy is the system preferred by the majority of voters.
     - Assume that $\lambda + \rho > 0.5$. Therefore, the electorate is ideological. If $\lambda < 0.5$ and $\rho < 0.5$, then the median voter belongs to the proportion of voters denoted by π. In other words, the median voter is pragmatic. From *Cases*

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1, *2*, and *3* above we know that representative democracy is the system preferred by the majority of voters if the median voter is pragmatic. Thus, we conclude that if the electorate is ideological and the median voter is pragmatic, then representative democracy is the system preferred by the majority of voters.

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Chapter 2

Implementing Direct Democracy Via Representation

2.1. Introduction

"Ordinary citizens will become protagonists, abandoning the current system of delegate democracy in the hands of politicians... We are not a party, we are not a caste, one man, one vote, one man, one vote".

Five Star Movement, an Italian Political Movement, `https://www.theguardian.com/world/2019/feb/13/digital-political-parties-democratic-m5s`, February 13, 2019.

A significant phenomenon in current society is the emergence of social movements with an anti-representative democracy rhetoric. These groups criticize systems based on power delegation due mainly to the risk of politicians being self-seeking and ending up behaving against people's interest. They claim the need for direct participation of citizens in the decision making process to guarantee the representativeness of society's preferences in the final outcome. As a result of the recent success of this discourse among the population, the emergence of political parties in favor of direct democracy has become a generalized fact in Europe, having those arised in several countries. *Freedom and Direct Democracy* in Czech Republic, *Alternative for Germany* in Germany, *Five Star Movement* in Italy,

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Podemos in Spain, *Direktdemokraterna* in Sweden, and *Something New* in United Kingdom are just some examples of parties demanding the right of people to decide directly by using direct democracy instruments such as referendums and popular consultations. These organizations defend that citizens should be able to decide on each and every one of the issues that comes up for discussion. When decisions about several issues have to be made, these political parties demand that, for each of these issues, it should be carried out what the majority of people desires for such specific issue.

Claims of these corporations about the need to shift towards a system of direct democracy in order to achieve such outcome could be interpreted as an omen about the inability of representative democracy to implement what the majority of people desires for each specific issue. We develop a positive analysis of this matter. The purpose of this paper is to study whether there is some room for representative democracy to implement the majority preferences on each issue or, on the contrary, it is totally necessary for such end to abandon the traditional system of political representation and replace it by a system of direct democracy. As far as we know, Coffman (2016) is the only paper in the literature that develops a theoretical analysis investigating the conditions under which representative democracy implements the choices made by people in direct democracy. The author considers the existence of a single issue for which there are a finite number of alternatives. She focuses on the case where the decision made in direct democracy leads to a strict ordering of these alternatives and looks for conditions under which the candidate with this ordering is elected. On the contrary, we consider that there are a finite number of issues for each of which a binary choice has to be made. We aim to find conditions under which decisions for each issue that people would have made, one at a time, by independent referendums are implemented by the elected candidate when she decides for the bundle of issues. To this extent, our paper delimits a problem that, until now, had been attributed to any representative democracy. Besley and Coate (2008) show that, as a consequence of the bundling of issues that is inherent when electing a representative, policies that diverge far from what the majority desire may be implemented on specific issues by the elected politician. Assuming that the goal is the prevalence of the median voter's preferences, the authors argue that this error of representative democracy would

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create a role for citizens' initiatives. In this sense, we identify societies in which such bundling of issues does not result in representative democracies implementing socially undesirable outcomes, thus not being necessary to resort to the direct participation of citizens.

Formally, we investigate whether there are necessary and sufficient conditions such that decisions made on each issue in a representative democracy coincide with the decisions that would have been chosen in a direct democracy by majority voting issue-by-issue. We propose a model in which there are a finite number of issues on which a binary decision has to be made. For convenience, we assume that there are no complementarities among issues, so the order in which decisions on different issues are made has no effect on the choice made for each issue. In direct democracy, voters directly vote for their more preferred decision on each issue, so majority voting issue-by-issue is the outcome in these systems. In representative democracy, we introduce a two-party system competition where the political platforms of these parties are known. Candidates representing each of the two parties will have these political platforms as the most preferred set of decisions for the bundle of issues. Candidates also have preference for being in office. Unlike for preferences over issues, preference for being in office is not known, may thus candidates' preferences vary from policy-motivated to office-motivated. This assumption captures the essence of the criticism that systems of political representation are receiving from groups in favor of direct democracy related to the risk of having politicians who only pursue their own interests.

We find conditions that guarantee that, irrespective from candidates' preferences, representative democracy coincides with direct democracy. These necessary and sufficient conditions involve restrictions over pairs of preference profile of voters and political platforms of candidates. First, there must be no set of decisions for the bundle of issues such that is preferred by a majority of voters to the set of decisions yielded by majority voting issue-by-issue. In other words, the outcome of majority voting issue-by-issue has to be a Condorcet winner relative to the voters' preference profile on the bundle of issues. This would allow that, any potentially profitable deviation that might initially exist for

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any candidate would no longer be beneficial. Second, either there must be at least one candidate such that her most preferred set of decisions for the bundle of issues coincides with the outcome of majority voting on each issue, or candidates' preferences are such that there is no issue for which both candidates share their more preferred decision.[1] This double condition would allow, respectively, either the existence of a candidate who trivially has no incentives to deviate from what the majority of people desires for each issue, or the nonexistence of room for a mutually beneficial agreement for both candidates which may violate the interests of the majority.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the results along with their corresponding proofs, as well as an interpretation of them. Section 4 offers a discussion about the divergence of results in direct and representative democracy when our conditions are not met. Finally, Section 5 concludes.

2.2. The Model

There are q issues and for each of them a binary decision has to be made. Let $K = \{1, ..., q\}$ be the set of issues and $k \in K$ an arbitrary issue. Let $x = \left(x^1, ..., x^q\right)$ be a vector of decisions on q issues where $x^k \in \{-1, 1\}$ denotes the decision for issue k. We call a vector x an alternative and $A \equiv \{-1, 1\}^q$ the set of alternatives.

Let $N = \{1, ..., n\}$ be an odd finite set of voters. Each voter $i \in N$ has strict separable preferences defined over the set of alternatives, which means that for each issue k her most preferred decision remains invariant regardless of the decisions for all the other issues. This implies that for each issue k a voter i has either -1 or 1 as her most preferred decision for such specific issue.[2] For voter $i \in N$, let $\mathcal{P}_i$ be the set of all strict separable preference relations defined on A, with typical element P_i. Let

[1] Of course, both conditions are compatible and may hold at the same time.

[2] See Le Breton and Sen (1999) for a deep understanding on how every strict separable preference relation over the set of alternatives induces a unique strict preference relation over each issue.

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$P_N = (P_1, ..., P_n)$ be a voters' preference profile. Given a preference profile of voters, let $x^k_{maj}(P_N) \in \{-1, 1\}$ be the decision preferred by a majority of voters for issue k and $x_{maj}(P_N) = \left(x^1_{maj}(P_N), ..., x^q_{maj}(P_N)\right) \in A$ the alternative for which the decision on each issue k is made by majority voting. We say that an alternative is a Condorcet winner relative to the voters' preference profile when no other alternative defeats it in pairwise comparisons. Formally:

Definition 1. *An* ***alternative $y \in A$*** *is a* ***Condorcet winner at P_N*** *if there is no $z \in A$ such that $\#\{i \in N \,/\, z\, P_i\, y\} > \frac{n}{2}$.*

Let L and R be two candidates, each representing a different political party. We denote the set of candidates by $C = \{L, R\}$. We introduce the electoral game played by these candidates. Knowing the preference profile of voters, each candidate announces an alternative, which is the one that she implements in case of winning the election. Let m_L and m_R be the announced alternatives by candidates L and R respectively where $m_L, m_R \in A$. Once alternatives have been announced, each voter sincerely votes for the candidate who announces a more preferred alternative. When candidates announce different alternatives, having an odd number of voters with strict preferences over alternatives implies that one of the two candidates necessarily wins the election. However, when the announced alternatives are the same, both candidates would be equally likely to win the election. We refer to this situation as a "tie" between candidates.[3] We use w_L or w_R to denote that candidate L or candidate R wins the election and *tie* for the coinciding announcements case. Let $O = \{w_L, tie, w_R\}$ be the set of office-outcomes. Candidates have preferences over the set of alternatives and also for being in office. We define an electoral-outcome as a pair of office-outcome and alternative. Let $\mathcal{E} \equiv O \times A$ be the set of electoral-outcomes, with typical element $(o; x)$ where $o \in O$ and $x \in A$. Candidates have separable preferences defined over the set $\mathcal{E}$. Additionally, preference for being in office implies that given any alternative x, each candidate prefers to win rather than to tie and to tie rather than to lose.[4] For candidate $c \in C$, let $\mathcal{P}_c$ be the set of all these preference

[3]For each voter, the decision on which candidate to vote for is comparable to a coin-flipping game.

[4]The term *to lose* refers to the case in which the opponent candidate wins.

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relations defined on $\mathcal{E}$, with typical element P_c. Let $P_C = (P_L, P_R)$ be a candidates' preference profile. For convenience, we refer to the most preferred alternative on A of each candidate as her "top". Given a preference profile of candidates, let τ_L and τ_R be the tops of candidates L and R respectively where $\tau_L, \tau_R \in A$. For each candidate $c \in C$, let $\tau_c = \left(\tau_c^1, ..., \tau_c^q\right)$ where $\tau_c^k \in \{-1, 1\}$ is the decision preferred by candidate c for issue $k \in K$.

Definition 2. *Given a pair of tops (τ_L, τ_R), we say that a preference profile of candidates* ***P_C is consistent with (τ_L, τ_R)*** *if $\tau_L(P_L) = \tau_L$ and $\tau_R(P_R) = \tau_R$.*

Note that different preference profiles of candidates can generate the same pair of tops (τ_L, τ_R).[5] We highlight the situation in which, issue by issue, the most preferred decision of one of the candidates is the opposite to the most preferred decision of the other candidate. We refer to this condition as maximal top-differentiation between candidates.

Definition 3. *We say that candidates are* ***maximally top-differentiated*** *when, for each issue, the most preferred decision of candidate L is the opposite to the most preferred decision of candidate R. Formally, $\tau_L = -\tau_R$.*

While preferences of voters are known, about candidates only their respective tops are known. Thus, a society in our context is defined by a preference profile of voters and a pair of tops of candidates. Let the triplet (P_N, τ_L, τ_R) define a society. Our aim is to study when in a representative democracy where candidates play the electoral game, the decision made for each issue coincides with the decisions that would have been chosen in a direct democracy where voters directly vote over each single issue. We focus on Nash

[5] Let $P_C = (P_L, P_R)$ and $P'_C = (P'_L, P'_R)$ be two preference profiles of candidates such that $P_L = P'_L$, which clearly implies that $\tau_L(P_L) = \tau_L(P'_L)$, while $P_R \neq P'_R$, with *(i)* $\tau_R(P_R) = \tau_R(P'_R)$; *(ii)* for each $y, z \in A$, $(w_R; y)\ P_R\ (tie; z)$, and *(iii)* for each $y, z \in A$ such that $(w_R; y)\ P'_R\ (w_R; z)$, we have that $(w_L; y)\ P'_R\ (w_R; z)$. Even when at both P_R and P'_R candidate R has the same top, at P_R she is office-motivated since she gives priority to be in office regardless of the policy to be carried out, while at P'_R she is policy-motivated since she is willing to refuse to be in office as long as a certain policy was implemented.

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Equilibrium concept of the electoral game.

Definition 4. *Given a society (P_N, τ_L, τ_R) and a preference profile of candidates P_C consistent with (τ_L, τ_R), we say that announcements* ***m_L and m_R constitute a Nash Equilibrium of the electoral game*** *if no candidate $c \in C$ has incentives to deviate and announce some $m'_c \in A \setminus \{m_c\}$.*

2.3. On When Representative Democracy Equals Direct Democracy: The Results

For every possible society, our purpose is to find under which conditions $x_{maj}(P_N)$ is the unique Nash Equilibrium outcome of the electoral game for each preference profile of candidates which is consistent with (τ_L, τ_R).

Lemma 1 states that, if there is a Nash Equilibrium of the electoral game, then both candidates are announcing the same alternative. Intuitively, it is easy to see why a situation in which different alternatives are announced cannot be sustained as equilibrium. If this would be the case, we know that the candidate announcing a more preferred alternative by a majority of voters wins the election and carries out her announced alternative. By preference for being in office, the losing candidate has incentives to deviate and announce the same alternative as the winning candidate since, given the implementation of such alternative, this candidate prefers to tie rather than to lose.

Lemma 1. *If announcements m_L and m_R constitute a Nash Equilibrium of the electoral game, then both candidates announce the same alternative.*

Proof. Suppose not, *i.e.*, let $m_L = y$ and $m_R = z$ where $y \neq z$ and $y, z \in A$ such that m_L and m_R are a Nash Equilibrium of the electoral game. Suppose, without loss of generality, that $(w_L; y)$ is the obtained electoral-outcome after voting by voters. Consider

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candidate R. Let $m'_R \in A$ such that $m'_R = y$. Note that, if candidate R deviates and announces $m'_R = y$, then $(tie; y)$ is the resulting electoral-outcome. By preference for being in office, $(tie; y)\ P_R\ (w_L; y)$, so announcing $m'_R = y$ is a profitable deviation for candidate R when $m_L = y$. Therefore, m_L and m_R are not a Nash Equilibrium of the electoral game.

■

We now identify a necessary condition for having $x_{maj}(P_N)$ as the unique Nash Equilibrium outcome of the electoral game. Proposition 1 states that only if $x_{maj}(P_N)$ is a Condorcet winner, there is room for the achievement of such purpose.

Proposition 1. *Given any society (P_N, τ_L, τ_R), suppose that $x_{maj}(P_N)$ is the unique Nash Equilibrium outcome of the electoral game for every P_C consistent with (τ_L, τ_R). Then $x_{maj}(P_N)$ is a Condorcet winner at P_N.*

Proof. By contradiction, suppose that for some society $x_{maj}(P_N)$ is not a Condorcet winner at P_N. Then, there exists $y \in A$ such that $\#\{i \in N \,/\, y\, P_i\, x_{maj}(P_N)\} > \frac{n}{2}$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for at least one of the candidates, say L, $P_L \in \mathcal{P}_L$ and, for each $h, s \in A$, we have $(w_L; h)\ P_L\ (tie; s)$. By Lemma 1, we know that, $m_L = m_R = x_{maj}(P_N)$ is the only Nash Equilibrium of the electoral game. Let $m'_L \in A$ be such that $m'_L = y$. Since $\#\{i \in N \,/\, y\, P_i\, x_{maj}(P_N)\} > \frac{n}{2}$, $(w_L; y)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By assumption, $(w_L; y)\ P_L\ (tie; x_{maj}(P_N))$, so announcing $m'_L = y$ is a profitable deviation for candidate L when $m_R = x_{maj}(P_N)$, which contradicts that m_L and m_R) is a Nash Equilibrium of the electoral game.

■

Proposition 2 shows that having $x_{maj}(P_N)$ as a Nash Equilibrium outcome of the electoral game is guaranteed when the issue-by-issue majority voting outcome is a Condorcet winner.

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Proposition 2. *Let (P_N, τ_L, τ_R) be a society such that $x_{maj}(P_N)$ is a Condorcet winner at P_N. Then, $x_{maj}(P_N)$ is a Nash Equilibrium outcome of the electoral game for every P_C consistent with (τ_L, τ_R).*

Proof. Let $m_L = m_R = x_{maj}(P_N)$. Note that $(tie; x_{maj}(P_N))$ is the resulting electoral-outcome for these announcements. Since $x_{maj}(P_N)$ is a Condorcet winner at P_N, for each $m'_L \in A \setminus \{x_{maj}(P_N)\}$, $(w_R; x_{maj}(P_N))$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office, $(tie; x_{maj}(P_N))$ P_L $(w_R; x_{maj}(P_N))$, so announcing m'_L is not a profitable deviation for candidate L when $m_R = x_{maj}(P_N)$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a Nash Equilibrium of the electoral game.
■

Theorem 1 states necessary and sufficient conditions for the issue-by-issue majority voting outcome, that is, $x_{maj}(P_N)$, to be the unique Nash Equilibrium outcome of the electoral game.

Theorem 1. *Given any society (P_N, τ_L, τ_R), $x_{maj}(P_N)$ is the unique Nash Equilibrium outcome of the electoral game for every P_C consistent with (τ_L, τ_R) if and only if $x_{maj}(P_N)$ is a Condorcet winner at P_N and:*

(i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or

(ii) candidates are maximally top-differentiated.

Proof. We prove this theorem by showing the following two claims.

CLAIM 1 (Necessity). *If $x_{maj}(P_N)$ is the unique Nash Equilibrium outcome of the electoral game for every P_C consistent with (τ_L, τ_R) then $x_{maj}(P_N)$ is a Condorcet winner at P_N and (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or (ii) candidates are maximally top-differentiated.*

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That $x_{maj}(P_N)$ is a Condorcet winner at P_N follows from Proposition 1. By contradiction, suppose that neither (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, nor (ii) candidates are maximally top-differentiated. Then, (1) $\tau_L^r = -x_{maj}^r(P_N)$ for some $r \in K$, (2) $\tau_R^s = -x_{maj}^s(P_N)$ for some $s \in K$, and (3) $\tau_L^t = \tau_R^t$ for some $t \in K$. Next, we show that there exists some $y \in A \setminus \{x_{maj}(P_N)\}$ such that $m_L = m_R = y$ is a Nash Equilibrium of the electoral game for some P_C consistent with (τ_L, τ_R). We distinguish two cases:

- Case 1. $\tau_L^t = \tau_R^t = -x_{maj}^t(P_N)$ *for some* $t \in K$.

  Suppose, without loss of generality, $\tau_L^t = \tau_R^t = 1$ and $x_{maj}^t(P_N) = -1$. Let $Y = \{Y \in A : y^t = -x_{maj}^t(P_N) = 1\}$. By construction, $x_{maj}(P_N) \notin Y$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for each $y \in Y$ and $z \in A \setminus Y$, we have $(w_R; y)\ P_L\ (w_L; z)$ and $(w_L; y)\ P_R\ (w_R; z)$. Let $h \in Y$ be such that for each $k \in K \setminus \{t\}$, $h^k = x_{maj}^k(P_N)$. Since $h \in Y$ we know that $h^t = -x_{maj}^t(P_N) = 1$, so clearly $h \neq x_{maj}(P_N)$. We now show that $m_L = m_R = h$ is a Nash Equilibrium of the electoral game. Note that $(tie; h)$ is the resulting electoral-outcome. Observe that candidate L has no incentives to deviate to some $z \in A \setminus Y$ since by assumption $(w_R; h)\ P_L\ (w_L; z)$ and by preference for being in office $(tie; h)\ P_L\ (w_L; z)$. Thus, candidate L would only consider deviations to alternatives that belong to the set Y. Let $m'_L \in Y$ such that $m'_L = g$ where $g \in Y \setminus \{h\}$. Since $g \neq h$, there exists at least $k \in K \setminus \{t\}$ such that $g^k \neq h^k$. By construction, $h^k = x_{maj}^k(P_N)$ while $g^k = -x_{maj}^k(P_N)$. Since $x_{maj}(P_N)$ is a Condorcet winner, by separability of voters' preferences we have $\#\{i \in N \,/\, h\ P_i\ g\} > \frac{n}{2}$. Hence, $(w_R; h)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office $(tie; h)\ P_L\ (w_R; h)$, so announcing $m'_L = g$ is not a profitable deviation for candidate L when $m_R = h$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a Nash Equilibrium of the electoral game.

- Case 2. *For every* $t \in K$ *with* $\tau_L^t = \tau_R^t$ *we have* $\tau_L^t = \tau_R^t = x_{maj}^t(P_N)$.

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Then, from points (1), (2), and (3), $\tau_L^r = -x_{maj}^r(P_N) = -\tau_R^r$ for some $r \in K$, and $\tau_R^s = -x_{maj}^s(P_N) = -\tau_L^s$ for some $s \in K$. Suppose, without loss of generality, $\tau_L^r = 1$, $x_{maj}^r(P_N) = \tau_R^r = -1$, $\tau_R^s = 1$, and $x_{maj}^s(P_N) = \tau_L^s = -1$. Let $Q = \{Q \in A : q^r = \tau_L^r = -x_{maj}^r(P_N) = 1 \text{ and } q^s = \tau_R^s = -x_{maj}^s(P_N) = 1\}$. By construction, $x_{maj}(P_N) \notin Q$. Let $P_C = (P_L, P_R)$ be a candidates' preference profile consistent with (τ_L, τ_R) such that, for each $q \in Q$ and $z \in A \setminus Q$, we have $(w_R; q)$ P_L $(w_L; z)$ and $(w_L; q)$ P_R $(w_R; z)$. Let $h \in Q$ be such that for each $k \in K \setminus \{r, s\}$, $h^k = x_{maj}^k(P_N)$. Since $h \in Q$ we know that $h^r = \tau_L^r = -x_{maj}^r(P_N) = 1$ and $h^s = \tau_R^s = -x_{maj}^s(P_N) = 1$, so clearly $h \neq x_{maj}(P_N)$. We now show that $m_L = m_R = h$ is a Nash Equilibrium of the electoral game. Note that $(tie; h)$ is the resulting electoral-outcome. Observe that candidate L has no incentives to deviate to some $z \in A \setminus Q$ since by assumption $(w_R; h)$ P_L $(w_L; z)$ and by preference for being in office $(tie; h)$ P_L $(w_L; z)$. Thus, candidate L would only consider deviations to alternatives that belong to the set Q. Let $m'_L \in Q$ such that $m'_L = g$ where $g \in Q \setminus \{h\}$. Since $g \neq h$, there exists at least $k \in K \setminus \{r, s\}$ such that $g^k \neq h^k$. By construction, $h^k = x_{maj}^k(P_N)$ while $g^k = -x_{maj}^k(P_N)$. Since $x_{maj}(P_N)$ is a Condorcet winner, by separability of voters' preferences we have $\#\{i \in N \,/\, h\, P_i\, g\} > \frac{n}{2}$. Hence, $(w_R; h)$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office $(tie; h)$ P_L $(w_R; h)$, so announcing $m'_L = g$ is not a profitable deviation for candidate L when $m_R = h$. The analysis for candidate R follows an analogous reasoning. Therefore, m_L and m_R are a Nash Equilibrium of the electoral game.

CLAIM 2 (Sufficiency). *If $x_{maj}(P_N)$ is a Condorcet winner at P_N and (i) $x_{maj}(P_N) = \tau_c$ for some $c \in C$, or (ii) candidates are maximally top-differentiated, then $x_{maj}(P_N)$ is the unique Nash Equilibrium outcome of the electoral game for every P_C consistent with (τ_L, τ_R).*

That $x_{maj}(P_N)$ is a Nash Equilibrium outcome of the electoral game for each preference profile of candidates consistent with (τ_L, τ_R) follows from Proposition 2. It remains

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to be shown that it is unique. By contradiction, suppose that there exists a Nash Equilibrium of the electoral game such that $x_{maj}(P_N)$ is not the resulting outcome. From Lemma 1 we have that there are m_L and m_R such that $m_L = m_R = y$ where $y \in A \setminus \{x_{maj}(P_N)\}$ which are a Nash Equilibrium of the electoral game with $(tie; y)$ as the resulting electoral-outcome. We distinguish two cases:

- Case 1. *$x_{maj}(P_N) = \tau_c$ for some $c \in C$.*

  Assume, without loss of generality, that $\tau_L = x_{maj}(P_N)$. Let $m'_L = x_{maj}(P_N)$. Since $x_{maj}(P_N)$ is a Condorcet winner, $(w_L; x_{maj}(P_N))$ is the resulting electoral-outcome when m'_L and m_R are the candidates' announcements. By preference for being in office and separability of candidates' preferences, $(w_L; x_{maj}(P_N))$ P_L $(tie; y)$, so announcing $m'_L = x_{maj}(P_N)$ is a profitable deviation for candidate L when $m_R = y$. Therefore, $m_L = y$ and $m_R = y$ are not a Nash Equilibrium of the electoral game.

- Case 2. *Candidates are maximally top-differentiated.*

  We distinguish two subcases:

  - Subcase 2.1. *For some $c \in C$, $\tau_c = x_{maj}(P_N)$.*

    We come back to Case 1.

  - Subcase 2.2. *For each $c \in C$, $\tau_c \neq x_{maj}(P_N)$.*

    We distinguish two subsubcases:

    - Subsubcase 2.2.1. *For some $c \in C$, $\tau_c = m_c = y$.*

      Assume, without loss of generality, that $\tau_L = m_L = y$. By maximal top-differentiation, $y = -\tau_R$. Let $m'_R = x_{maj}(P_N)$. Since $x_{maj}(P_N)$ is a Condorcet winner, $(w_R; x_{maj}(P_N))$ is the resulting electoral-outcome when m_L and m'_R are the candidates' announcements. By preference for being in office and separability of candidates' preferences, $(w_R; x_{maj}(P_N))$ P_R $(tie; -\tau_R)$, so announcing $m'_R = x_{maj}(P_N)$ is a profitable deviation for

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candidate R when $m_L = y$. Therefore, $m_L = y$ and $m_R = y$ are not a Nash Equilibrium of the electoral game.

- Subsubcase 2.2.2. *For each* $c \in C$, $\tau_c \neq m_c = y$.
  Since $y \neq x_{maj}(P_N)$ by construction, there exists at least $h \in K$ such that $y^h \neq x^h_{maj}(P_N)$. Assume, without loss of generality, that $y^h = 1$ and $x^h_{maj}(P_N) = -1$. Furthermore, by maximal top-differentiation, we know that $\tau^h_L = -\tau^h_R$. Assume, without loss of generality, that $\tau^h_L = y^h = 1$ and $\tau^h_R = x^h_{maj}(P_N) = -1$. Let $m'_R = \tilde{m}_R$ where for each $k \in K \setminus \{h\}$, $\tilde{m}^k_R = y^k$ and for $h \in K$, $\tilde{m}^h_R = -y^h = x^h_{maj}(P_N) = -1$. Since $x_{maj}(P_N)$ is a Condorcet winner, by separability of voters' preferences we have $\#\{i \in N \,/\, \tilde{m}_R \, P_i \, y\} > \frac{n}{2}$. Thus, $(w_R; \tilde{m}_R)$ is the resulting electoral-outcome when $m_L = y$ and $m'_R = \tilde{m}_R$ are the candidates' announcements. Note that $\tilde{m}^h_R = \tau^h_R$. By preference for being in office and separability of candidates' preferences, $(w_R; \tilde{m}_R) \; P_R \; (tie; y)$, so announcing $m'_R = \tilde{m}_R$ is a profitable deviation for candidate R when $m_L = y$. Therefore, $m_L = y$ and $m_R = y$ are not a Nash Equilibrium of the electoral game.

■

Theorem 1 identifies necessary and sufficient conditions for the two considered procedures to make decisions, that is, by direct vote of voters over each single issue or allowing that decisions are made in a electoral game, to be equivalent in terms of the decision made for each issue when tops of candidates is all that is known about their preferences. We explain now why conditions in Theorem 1 are necessary and sufficient for having the outcome of direct democracy as the unique Nash Equilibrium outcome of the electoral game in representative democracy.

We start by proving necessity. When $x_{maj}(P_N)$ is not a Condorcet winner, the existence of a Nash Equilibrium with $x_{maj}(P_N)$ as outcome is not guaranteed. Suppose that there is a candidate who is office-motivated.[6] Since $x_{maj}(P_N)$ is not a Condorcet winner,

[6]Office-motivation is understood as the preference of the electoral-outcome in which the candidate

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there is at least one alternative that defeats $x_{maj}(P_N)$ in pairwise comparison. Thus, the office-motivated candidate has incentives to deviate and announce some alternative which is preferred to $x_{maj}(P_N)$ by a majority of voters. This would allow her to win the election, which is the most important for such candidate regardless of the alternative to be implemented. For its part, when neither there is a candidate with $x_{maj}(P_N)$ as top nor candidates are maximally top-differentiated, the uniqueness of a Nash Equilibrium with $x_{maj}(P_N)$ as outcome is not guaranteed. Assume that the tops of both candidates are equal and different from $x_{maj}(P_N)$. Suppose that both candidates are policy-motivated.[7] Then, the top of these candidates, which is different from $x_{maj}(P_N)$, can be sustained as a Nash Equilibrium outcome of the electoral game.

We show now sufficiency of the conditions. Assume first that $x_{maj}(P_N)$ is a Condorcet winner and there is at least one candidate with $x_{maj}(P_N)$ as top. Existence of the equilibrium is guaranteed since both candidates announcing $x_{maj}(P_N)$ is a Nash Equilibrium of the electoral game. By Lemma 1, candidates announce the same alternative in equilibrium. Note that the tie between candidates and the implementation of $x_{maj}(P_N)$ is the electoral-outcome in this case. Since $x_{maj}(P_N)$ is a Condorcet winner, no candidate has incentives to deviate and announce a different alternative. In case of doing so, the deviant candidate loses the election while her opponent wins and carries out the alternative $x_{maj}(P_N)$. But, by preference for being in office, such candidate prefers to tie and implement $x_{maj}(P_N)$ rather than losing having still that alternative implemented, so no candidate has a profitable deviation. To show uniqueness of the equilibrium, suppose that both candidates are announcing a same alternative but different from $x_{maj}(P_N)$. Announcing $x_{maj}(P_N)$ is a profitable deviation for the candidate with such alternative as top: if she announces $x_{maj}(P_N)$, she wins the election and carries out her top, which is the best possible scenario for such candidate. Therefore, no alternative other than $x_{maj}(P_N)$

wins and her least preferred alternative is carried out over the electoral-outcome in which she ties and her top is implemented.

[7]Policy-motivation is understood as the preference of the electoral-outcome in which the candidate loses and her top is carried out over the electoral-outcome in which she wins and her second most preferred alternative is implemented.

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can be sustained as a Nash Equilibrium outcome of the electoral game. Assume now that $x_{maj}(P_N)$ is a Condorcet winner and candidates are maximally top-differentiated. Existence of the equilibrium is similar to the previous case. For the uniqueness of the equilibrium, suppose again that both candidates are announcing a same alternative but different from $x_{maj}(P_N)$. Consider one issue for which the decision announced by candidates is different from the decision included in $x_{maj}(P_N)$. By maximal top-differentiation, there is necessarily a candidate that, for such issue, has the decision contained by $x_{maj}(P_N)$ on it as her most preferred decision. Consider a variant of the alternative initially announced by the candidates, in which decisions announced for all the issues remain the same as at the beginning except for the issue at hand, which would now become the decision specified by $x_{maj}(P_N)$. Note that, if the above candidate announces this modified alternative wins the election and carries out such alternative. By separability of preferences and preference for being in office, this electoral-outcome is preferred by this candidate to the initial electoral-outcome in which she tied and an alternative more distant from her top was carried out. Thus, there is a candidate with a profitable deviation so no alternative other than $x_{maj}(P_N)$ can be sustained as a Nash Equilibrium outcome of the electoral game.

2.4. What If Our Conditions Are Not Met? A Discussion

Necessity and sufficiency of our conditions implies that, as long as at least one of them is not fulfilled, the equivalence between direct democracy and representative democracy in terms of outcomes ceases to exist. In this section, we describe the different outcomes that are obtained in both systems when any of the conditions is not met and offer a discussion about the suitability of each of these regimes when decisions derived from each of them do not coincide.

Several criticisms of decisions made in direct democracy have been identified in the literature. The Sen's Liberal Paradox (Amartya, 1970) states that respecting individual

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preferences of voters in each issue may lead to a Pareto dominated outcome. In our context, this paradox implies that there would be some set of decisions for the bundle of issues (*i.e.*, an alternative) that is preferred by every voter to the set of decisions yielded from majority voting issue-by-issue. A slightly less worrisome problem in terms of social efficiency but still relevant is the Ostrogorski's Paradox (Ostrogorski, 1970, Rae and Daudt, 1976). These authors find that choosing issue-by-issue by majority rule when decisions are dichotomous may result in a majority defeated overall outcome. There would be therefore an alternative that is preferred by a majority of voters to $x_{maj}(P_N)$. Related to Ostrogorski's Paradox is Anscombe's Paradox (Anscombe, 1976). It says that when decisions involving dichotomous choices are made by majority rule on each issue, a majority of voters could disagree in a majority of issues. Having the majority voting issue-by-issue output (*i.e.*, $x_{maj}(P_N)$) as a Condorcet winner guarantees that none of these paradoxes is occurring, so it could be said that the outcome of direct democracy is desirable in this case. Assume that $x_{maj}(P_N)$ is a Condorcet winner relative to the voters' preference profile P_N but neither there is a candidate with $x_{maj}(P_N)$ as top nor candidates are maximally top-differentiated. It can be argued that, under this framework, direct democracy is recommendable. Direct democracy is ensuring the representativeness of society's preferences in the final outcome currently demanded by numerous social movements and political parties, having furthermore such outcome no criticism from a social welfare perspective. Thus, a natural requirement to state that representative democracy works well is to also provide $x_{maj}(P_N)$ as a result. However, even when the majority voting issue-by-issue will be a Nash Equilibrium outcome of the electoral game, an alternative other than $x_{maj}(P_N)$ could be also sustained as an equilibrium of the game: by Proposition 2 we know that $x_{maj}(P_N)$ being Condorcet winner is a sufficient condition for the existence of a Nash Equilibrium with $x_{maj}(P_N)$ as outcome, while uniqueness of such equilibrium is not guaranteed due to not having neither a candidate with $x_{maj}(P_N)$ as top nor maximal top-differentiation of candidates. Note that this is a situation in which social requests for the establishment of direct democracies would make sense as a consequence of the threat posed by the existence of politicians exclusively moved by private interests contrary to the interests of the majority of the electorate. On the one hand, having no candidate with $x_{maj}(P_N)$ as top may be due to the increased activity of lobbies and special-interest

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groups, which prevent the existence of a candidate with the preferences of the median voter. For its part, the fact of not having maximally top-differentiated candidates could lead to a failure of the electoral competition. Electoral competition between traditional left-wing and right-wing political parties comes to motivate the convergence towards the preferences of the median voter. It is, therefore, a way to discipline self-seeking politicians to end up acting in the best interests of the majority. However, the current disappearance of the traditional distinction between left and right ideologies creates distrust of electoral competition among people, thus motivating a preference for direct democracies.

Assume now that either there is a candidate with $x_{maj}(P_N)$ as top or candidates are maximally top-differentiated but $x_{maj}(P_N)$ is not a Condorcet winner relative to the voters' preference profile P_N. The first thing we notice is that the outcome of direct democracy could be suffering from some of the paradoxes mentioned above since it is not a Condorcet winner, making therefore such an outcome socially undesirable. Thus, it seems natural to think that there would be room for a better performance of representative democracy in this context. However, unlike the previous case, it is not possible here to carry out an immediate comparison between the outcomes of both systems in order to suggest the suitability of one of them. We no longer have an alternative that can be clearly defended as desirable, as was previously the case with the Condorcet winner. Buechel (2014) finds that, on our preferences domain, if $x_{maj}(P_N)$ is not a Condorcet winner, then a Condorcet winner does not exist.[8] This could be understood as a situation in which voters are very heterogeneous on their preferences, in such a way that given any alternative, some majority coalition among voters preferring a different alternative can always arise. This creates some difficulties in obtaining an outcome in representative democracy. The existence of cycles on voters' preferences implies that a Nash Equilibrium outcome of the electoral game may not exist. The non-existence of an alternative in whose implementation a majority of voters agree joint with the difficulty of reaching an agreement between two politicians with totally different ideological positions would eventually result in an incessant existence of profitable deviations for each of them, thus

[8]Buechel (2014) finds that, on the domain of separable preferences, if there exists a Condorcet winner, then it coincides with the median alternative, which in our case is given by $x_{maj}(P_N)$.

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preventing the achievement of an equilibrium in the electoral competition.

2.5. Conclusion

There are more and more social movements and political parties claiming that the true respect of the will of a society lies in carrying out, for each single issue, what the majority desires. Given the risk of having self-seeking politicians in representative democracy, these groups defend the better suitability of direct democracy when respecting the interests of society. We have proposed a model to study when a system of representative democracy would be equivalent to a system of direct democracy in terms of the developed policies. We find necessary and sufficient conditions such that, decisions made on each issue in representative democracies coincide with the decisions that would have been chosen in direct democracies by majority voting issue-by-issue, regardless of whether candidates are either policy-motivated or office-motivated. First, the majority voting issue-by-issue output (*i.e.*, $x_{maj}(P_N)$) has to be a Condorcet winner relative to the voters' preference profile P_N. Second, either there is a candidate with $x_{maj}(P_N)$ as her most preferred alternative or *top*, or candidates are maximally top-differentiated, which means that there is no issue for which both candidates have the same more preferred decision. First condition can be interpreted as the existence of some degree of homogeneity among voters' preferences about the suitability of the implementation of decisions yielded by majority voting issue-by-issue. Second condition can be understood as the existence of either at least one candidate whose preferences are in line with preferences of society, or candidates with sufficiently different preferences. The concurrent fulfillment of both conditions allows that, on the one hand, any potentially profitable deviation from $x_{maj}(P_N)$ that might initially exist for any candidate would no longer be beneficial and, on the other hand, either there exists a candidate who trivially has no incentives to deviate from what the majority of people desires for each issue, or there is no room for a mutually beneficial agreement for both candidates which may violate the interests of the majority. Thus, social demands claiming the need of removing the traditional system of political representation and establishing a direct democracy in order to respect the will of the majority is only meaningful

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in societies in which these conditions are not being met, since implementation of direct democracy via representation would be guaranteed. Situations in which some of these conditions does not hold deserve special attention, due to the breakdown of the equivalence between the systems of direct and representative democracy. The case in which $x_{maj}(P_N)$ is a Condorcet winner but neither there is a candidate with $x_{maj}(P_N)$ as top nor candidates are maximally top-differentiated is especially interesting. Even when the implementation of the majority voting issue-by-issue output is socially desirable, representative democracy might be selecting something totally different. This event might be due to the increased activity of lobbies and special-interest groups, which prevent the existence of a candidate with the preferences of the median voter. Similarly, a failure of the electoral competition could be the cause. Electoral contest between traditional left-wing and right-wing political parties used to motivate the convergence towards the preferences of the median voter. When they disappeared, the electoral competition abandoned its disciplining role of self-seeking politicians that forced them to behave according to the will of the majority. Thus, direct democracy would be recommendable under this framework. For its part, when either there is a candidate with $x_{maj}(P_N)$ as top or candidates are maximally top-differentiated but $x_{maj}(P_N)$ is not a Condorcet winner, it is not possible to carry out a comparison between the outcomes of direct democracy and representative democracy in order to suggest the suitability of one of them. The non-existence of an alternative in whose implementation a majority of citizens agree hinders the existence of an equilibrium in the electoral competition: cycles on voters' preferences would imply that incessant profitable deviations for each of the candidates may exist, thus preventing the achievement of a stable situation between the two politicians in the electoral game.

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Chapter 3

Condorcet Consistent Scoring Rules And Single-Peakedness

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3.1. Introduction

When only two alternatives are at stake, majority voting is the best method, the one satisfying well-known properties (see May, 1952). The challenge is to extend majority voting among pairs to situations where there are more than two alternatives. Condorcet proposed choosing the alternative defeating every other alternative in pairwise comparisons (Condorcet, 1785). Borda proposed assigning points to each candidate according to preferences and choosing the alternative with the highest total score (Borda, 1781). These two proposals lead to the two most acknowledged families of voting rules, Condorcet winner rules and scoring (or positional) rules (Smith, 1973; Young, 1975). The Condorcet consistent criterion, requiring the election of the Condorcet winner when it exists, is a widely used principle for evaluating alternative voting rules. On an unrestricted domain of preferences, for any odd number of voters and any number of alternatives higher than

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two, each scoring rule is not Condorcet consistent for some preference profile (Fishburn, 1973). In this paper we consider the set of single-peaked preferences and we are interested in analyzing which scoring rules, if any, are Condorcet consistent.

Finding out what is the "optimal" size of a group of voters has been paid attention in the literature. In the context of the Condorcet's jury theorem, where a group of voters wants to reach the "correct" decision by majority voting, the number of voters that should be included in the group depends on the probability of voting for the correct decision of each voter (Condorcet, 1875; Grofman, Owen, and Feld, 1983).

While Section 2 presents the model, Section 3 presents the main results and their proofs. Other results are placed in the Appendix.

3.2. The Model

Let $A = \{a_1, a_2, ..., a_m\}$ be a finite set of $m \geq 3$ *alternatives* where $a_1 < a_2 < ... < a_m$, and $N = \{1, ..., n\}$ be an odd finite set of $n \geq 3$ *agents*. Each agent i's preferences are linear orders over alternatives, denoted by R_i, which are single-peaked on A. For any a_l, $a_k \in A$, $a_l R_i a_k$ denotes that a_l is strictly preferred to a_k by i at R_i. A *preference* R *is single-peaked* on A if *(i)* there exists an alternative $p(R)$, the peak of R, such that $p(R)Ra_l$ for any $a_l \in A$, and *(ii)* for every pair of alternatives $a_l, a_k \in A$ such that $p(R) < a_l < a_k$ or $a_k < a_l < p(R)$, $a_l R a_k$. Let $\mathcal{S}$ denote the set of all single-peaked preferences on A. Elements in $\mathcal{S}^n$ are called *preference profiles*, and are denoted by $R_N = (R_1, ..., R_n)$. A *voting rule* $r : \mathcal{S}^n \to 2^A \backslash \varnothing$ selects a non-empty set of alternatives for each preference profile. We define two types of voting rules: the Condorcet winner rule and the scoring rules.

We say that an alternative a_j defeats alternative a_k in majority comparison if a strict majority of agents prefers a_j over a_k. When n is odd and preferences are single-peaked, there always exists a unique alternative that defeats any other by majority comparisons, which is called the Condorcet winner. The *Condorcet winner rule CW* assigns to each

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preference profile the Condorcet winner, that is, the unique alternative that defeats every other alternative in pairwise majority comparisons.

Fix a vector $w = (w_0, ..., w_{m-1}) \in \mathbb{R}^m$ satisfying $w_0 \leq w_1 \leq ... \leq w_{m-2} \leq w_{m-1}$ and $w_0 < w_{m-1}$. Without loss of generality, we assume that $w_0 = 0$ and $w_{m-1} = 1$. Points are assigned to every alternative in such a way that if alternative x is in the j'th top position according to R_i, then x receives $s_w(x, R_i) = w_{m-j}$ points from agent i. Given a preference profile R_N and an alternative x, let $s_w(x, R_N) = \sum_{i=1}^{n} s_w(x, R_i)$ be the score assigned to x at R_N. The w-scoring rule S_w assigns to each preference profile the set of alternatives with the highest score.[1]

Given A and N, we say that a w-scoring rule is *Condorcet consistent* if for any $R_N \in \mathcal{S}^n$, $CW(R_N) \in S_w(R_N)$.

For each m, call *semiplurality rules* the family of w-scoring rules where $w_{m-2} = w_1 \in \left[0, \frac{1}{2}\right]$. Note that plurality is a member of this family for $w_1 = 0$. The Borda count for m alternatives is the w-scoring rule where $|w_l - w_{l+1}| = |w_{l+1} - w_{l+2}|$ for all $l = 0, ..., m-2$. The Borda count is a semiplurality rule only for $m = 3$ and $w_1 = \frac{1}{2}$.

3.3. Results and Discussion

For each size of the society and the set of alternatives, we characterize the subset of w-scoring rules that are Condorcet consistent. We focus on the cases of three or more alternatives because in the case of two alternatives each w-scoring rule trivially chooses the Condorcet winner. We obtain positive results for specific situations, but only with three or five agents. The characterization is obtained in Propositions 1, 2, and 3.

[1] Our definition of scoring rules is from Moulin (1988). See Young (1975) and also Bossert and Suzumura (2018) for the equivalent definition of positional scoring rules.

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Proposition 1. *Let $n = 3$. (i) For $m \neq 4$, a w-scoring rule is Condorcet consistent if and only if $w_{m-2} \in \left[0, \frac{1}{2}\right]$ and $w_{m-2} = w_1$. (ii) For $m = 4$, a w-scoring rule is Condorcet consistent if and only if $w_{m-2} \in \left[0, \frac{1}{2}\right]$.*

For $m \neq 4$, this is the family of semiplurality scoring rules with $w_{m-2} \in \left[0, \frac{1}{2}\right]$.

Proposition 2. *Let $n = 5$. (i) For $m = 3$, a w-scoring rule is Condorcet consistent if and only if $w_{m-2} = \frac{1}{3}$. (ii) For $m \geq 4$, no w-scoring rule is Condorcet consistent.*

Proposition 3. *For $n > 5$ and any $m \geq 3$, no w-scoring rule is Condorcet consistent.*

To prove the "only if" implication ($\Rightarrow$) of Propositions 1 and 2 it is useful to state the following two Lemmas, proved in the Appendix, that present the scoring rules violating Condorcet consistency for three and five agents.

Lemma 1. *Let $n = 3$. (a) For any $m \geq 3$, no w-scoring rule with $w_{m-2} \in \left(\frac{1}{2}, 1\right]$ is Condorcet consistent. (b) For any $m \geq 5$, no w-scoring rule with $w_{m-2} \in \left[0, \frac{1}{2}\right]$ and $w_{m-3} > w_1$, or with $w_{m-2} \in \left[0, \frac{1}{2}\right]$ and $w_{m-2} > w_{m-3} = w_1$ is Condorcet consistent.*

Lemma 2. *Let $n = 5$. (a) For $m = 3$, no w-scoring rule such that $w_{m-2} \in \left[0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, 1\right]$ is Condorcet consistent. (b) For $m \geq 4$, no w-scoring rule is Condorcet consistent.*

Proof of Proposition 1. The proof of the "only if" implication ($\Rightarrow$) of both Parts *(i)* and *(ii)* is straightforward by Parts (a) and (b) of Lemma 1. It remains to prove the "if" implication of both Parts *(i)* and *(ii)*. We provide a proof for Part *(ii)*.
(ii) Let $n = 3$ and $m = 4$. The w-scoring rule such that $w_{m-2} \in \left[0, \frac{1}{2}\right]$ is Condorcet consistent. We distinguish the following four cases:

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Case 1: $R_N \in \mathcal{S}^n$ such that $p(R_1) = p(R_2) = p(R_3) = p$. Clearly $CW(R_N) = \{p\} = S_w(R_N)$.

Case 2: $R_N \in \mathcal{S}^n$ such that $p(R_1) = p(R_2) = a_t < p(R_3) = a_{t+l}$ for some $t \in \{1, ..., m-1\}$ and $l \leqslant m - t$. Note that $CW(R_N) = a_t$. Also, observe on the one hand that $s_w(a_t, R_N) \geq 2 \geq 1+2w_{m-2}$ since $w_{m-2} \leq \frac{1}{2}$. On the other hand, $s_w(a_{t+l}, R_N) \leq 1+2w_{m-2}$ holds. Moreover, for all $x \notin \{a_t, a_{t+l}\}$, $s_w(x, R_N) \leq 3w_{m-2} \leq \frac{3}{2}$ since $w_{m-2} \leq \frac{1}{2}$.Thus, $s_w(x, R_N) \leq \frac{3}{2} < 2 \leq s_w(a_t, R_N)$. Thus, $CW(R_N) = a_t \in S_w(R_N)$.

Case 3: $R_N \in \mathcal{S}^n$ such that $p(R_1) = a_t < p(R_2) = p(R_3) = a_{t+l}$ for some $t \in \{1, ..., m-1\}$ and $l \leqslant m - t$. This case is symmetric to Case 2 exchanging the roles of a_t and a_{t+l}.

Case 4: $R_N \in \mathcal{S}^n$ such that $p(R_1) < p(R_2) < p(R_3)$. Note first that $CW(R_N) = p(R_2)$. When computing we obtain that $s_w(p(R_1), R_N) \leq 1 + w_{m-2} + w_1$, $s_w(p(R_3), R_N) \leq 1 + w_{m-2} + w_1$, and $1 + w_{m-2} + w_1 \leq s_w(p(R_2), R_N) \leq 1 + 2w_{m-2}$. Furthermore, for any alternative $x < p(R_1)$, $s_w(x, R_N) \leq s_w(p(R_1), R_N)$ and for any alternative $y > p(R_3)$, $s_w(y, R_N) \leq s_w(p(R_3), R_N)$. Thus, $CW(R_N) = p(R_2) \in S_w(R_N)$.

Wrapping up, S_w is Condorcet consistent. This shows Part *(ii)* that can be replicated for Part *(i)*. Thus, the proof ends. ■

To prove Proposition 2, we need some notation. For $m = 3$, there are only four single-peaked preferences, say R, R', R'', and R''' defined as follows: $a_1Ra_2Ra_3$, $a_2R'a_3R'a_1$, $a_2R''a_1R''a_3$, and $a_3R'''a_2R'''a_1$. For any $R_N \in \mathcal{S}^n$, let n_1 be the number of agents with preferences R, n_2 the number of agents with preferences R', n_3 the number of agents with preferences R'', and $n_4 \equiv n - (n_1 + n_2 + n_3)$ the number of agents with preferences R'''. Note that the triple (n_1,n_2,n_3) uniquely defines a profile.

Proof of Proposition 2. Part *(ii)* of Proposition 2 is proved since it coincides with Part (b) of Lemma 2. To prove Part *(i)* of Proposition 2, observe that Part (a) of Lemma 2 shows the "only if" implication ($\Rightarrow$) of Part *(i)*. It only remains to prove the "if" implication of Part *(i)*: We have to show that for $n = 5$ and $m = 3$, the w-scoring rule such that $w_{m-2} = \frac{1}{3}$ is Condorcet consistent.

Let $R_N \in \mathcal{S}^n$ and (n_1, n_2, n_3) be the triple uniquely associated to R_N. Since $w_{m-2} = \frac{1}{3}$,

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then $s_w(a_1, R_N) = n_1 + \frac{1}{3}n_3$, $s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3}$, and $s_w(a_3, R_N) = n_4 + \frac{1}{3}n_2$. Consider five cases.

Case 1: $n_1 = 5$. Observe that $CW(R_N) = a_1$, and $s_w(a_1, R_N) = 5 > s_w(a_2, R_N) = \frac{5}{3} > s_w(a_3, R_N) = 0$ which shows Condorcet consistency.

Case 2: $n_1 = 4$. Observe that $CW(R_N) = a_1$, $s_w(a_1, R_N) \geq 4$, $s_w(a_3, R_N) \leq 1$, and $s_w(a_2, R_N) \leq \frac{7}{3}$, which shows Condorcet consistency.

Case 3: $n_1 = 3$. Observe that $CW(R_N) = a_1$ and that $s_w(a_1, R_N) = 3 + \frac{1}{3}n_3 > s_w(a_3, R_N) = (2 - n_2 - n_3) + \frac{1}{3}n_2 = 2 - n_3 - \frac{2}{3}n_2$. To get Condorcet consistency, it remains to show that $s_w(a_1, R_N) \geq s_w(a_2, R_N)$. Since $n = 5$ and $n_1 = 3$, then $0 \leq n_2 + n_3 \leq 2$, and for any possible such tuple (n_2, n_3), we can check that $3 + \frac{1}{3}n_3 \geq \frac{2}{3}(n_2 + n_3) + \frac{5}{3}$. This ends the proof of Case 3.

Case 4: $n_1 = 2$. Note that $CW(R_N) \neq a_1$. Moreover, $CW(R_N) = a_3$ if and only if $n_4 = 3$. The Condorcet consistency of this subcase is proved by symmetry to Case 3 exchanging the roles of n_1 and a_1 by n_4 and a_3, respectively.

Note that $CW(R_N) = a_2$ if and only if $n_4 \leq 2$, equivalently, $1 \leq n_2 + n_3 \leq 3$.

Subcase 4.1: $n_4 = 0$, or equivalently, $n_2 + n_3 = 3$. Note that $s_w(a_3, R_N) = \frac{1}{3}n_2 \leq 1$. Moreover, $1 < 2 + \frac{1}{3}n_3 = s_w(a_1, R_N)$ and since $n_3 \leq 3$, $2 + \frac{1}{3}n_3 < s_w(a_2, R_N) = 2 + \frac{5}{3}$. Thus, we get Condorcet consistency.

Subcase 4.2: $n_4 = 1$, or equivalently, $n_2 + n_3 = 2$. Since $n_2 \leq 2$, note first that $s_w(a_3, R_N) = 1 + \frac{1}{3}n_2 < 1 + \frac{2}{3} \leq s_w(a_1, R_N) = 2 + \frac{1}{3}n_3$. Moreover, since $n_3 \leq 2$, $s_w(a_1, R_N) = 2 + \frac{1}{3}n_3 \leq 2 + \frac{1}{3}2 < 3 = \frac{2}{3}2 + \frac{5}{3} = s_w(a_2, R_N)$. Thus, we get Condorcet consistency.

Subcase 4.3: $n_4 = 2$, or equivalently, $n_2 + n_3 = 1$. Since $n_3 \leq 1$, $s_w(a_1, R_N) = 2 + \frac{1}{3}n_3 \leq \frac{7}{3} = s_w(a_2, R_N)$. Similarly, since $n_2 \leq 1$, $s_w(a_3, R_N) = 2 + \frac{1}{3}n_2 \leq \frac{7}{3} = s_w(a_2, R_N)$. Thus, we get Condorcet consistency.

Case 5: $n_1 = 1$. Note first that $CW(R_N) \neq a_1$. Moreover, by definition of CW the following two statements hold: (1) $CW(R_N) = a_3$ if and only if $n_4 \in \{3, 4\}$ and (2) $CW(R_N) = a_2$ if and only if $n_4 \leq 2$. We first show Condorcet consistency for situation in (1). Since $n_4 \in \{3, 4\}$, or equivalently $0 \leq n_2 + n_3 \leq 1$, $s_w(a_3, R_N) = n_4 + \frac{1}{3}n_2 \geq 3 > \frac{7}{3} \geq s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3}$ and also $s_w(a_3, R_N) = n_4 + \frac{1}{3}n_2 \geq 3 > \frac{4}{3} \geq s_w(a_1, R_N) = 1 + \frac{1}{3}n_3$. To show Condorcet consistency for situation in (2), that is,

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when $CW(R_N) = a_2$, note that $n_4 \leq 2$ if and only if $2 \leq n_2 + n_3 \leq 4$. Observe that $s_w(a_1, R_N) = n_1 + \frac{1}{3}n_3 = 1 + \frac{1}{3}n_3$ is always smaller than $s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3}$. Note that since $n_4 \leq 2$, $s_w(a_3, R_N) = n_4 + \frac{1}{3}n_2 \leq 2 + \frac{1}{3}n_2$ which is smaller or equal than 3 when $n_2 \leq 3$. Since $2 \leq n_2 + n_3$, $s_w(a_3, R_N) \leq 3 \leq s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3}$. For the case where $n_2 > 3$, that is, $n_2 = 4$, we have that $n_3 = n_4 = 0$. Clearly, $s_w(a_3, R_N) = n_4 + \frac{1}{3}n_2 = \frac{4}{3} < s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3} = \frac{13}{3}$. Thus, we prove Condorcet consistency.

Case 6: $n_1 = 0$. Note first that $CW(R_N) \neq a_1$ and observe that $s_w(a_1, R_N) = \frac{1}{3}n_3$ is always smaller than $s_w(a_2, R_N) = \frac{2}{3}(n_2+n_3)+\frac{5}{3}$. Moreover, on the one hand, $CW(R_N) = a_2$ if and only if $n_2 + n_3 > n_4$ and $CW(R_N) = a_3$ if and only if $n_2 + n_3 < n_4$, since being n odd and $n_1 = 0$, $n_2 + n_3 \neq n_4$. On the other hand, the scoring single winner is a_2, that is, $s_w(a_2, R_N) = \frac{2}{3}(n_2 + n_3) + \frac{5}{3} > s_w(a_3, R_N) = 5 - n_3 - \frac{2}{3}n_2$ if and only if $4n_2 + 5n_3 > 10$, which holds if and only if $n_2 + n_3 > n_4$. To check the first "if and only if" implication note that $\frac{2}{3}(n_2 + n_3) + \frac{5}{3} > 5 - n_3 - \frac{2}{3}n_2$ if and only if $\frac{4}{3}n_2 + \frac{5}{3}n_3 - \frac{10}{3} > 0$, which holds if and only if $4n_2 + 5n_3 > 10$. The latter "if and only if" implication is proved by checking that for each triple n_2, n_3 and n_4 with $n_2 + n_3 > n_4$ for the smallest value of $n_2 + n_3$ (that is, 3), the inequality $4n_2 + 5n_3 > 10$ holds. Observe that if the latter inequality holds for the smallest value of $n_2 + n_3$, it will also hold for all possible cases. Then, we also have that $CW(R_N) = a_3$ if and only if $n_2 + n_3 < n_4$. We get Condorcet consistency which ends the proof. ■

Proposition 3 states an impossibility result to obtain Condorcet consistent w-scoring rules for seven or more agents. Its proof is in the Appendix.

Summarizing, in this paper, we show that if the size of the group of voters is three, a sub-family of the scoring rules is Condorcet consistent for any number of alternatives. We also show that when there are three alternatives at stake, if the size of the group of voters is between three and five, then some scoring rules are Condorcet consistent.

Lepelley (1996) considers also single-peaked preferences but concentrates on different Condorcet criteria. Moreno and Puy (2005) analyze Condorcet consistency of scoring

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rules but in a setting where the agenda is not fixed. Bossert and Suzumura (2019) propose a generalization of positional voting rules allowing for non-additive criteria to be included. Skowron, Faliszewski, and Slinko (2019) characterize a class of multi-winner scoring rules satisfying axioms in the spirit of Young's characterization for single-winner scoring methods. In the context of experimental economics, Slater (1958) proposes to estimate the "optimal" group size as the size preferred by group members. The author considers groups of two to seven individuals from a single population who meet several times to discuss about some human relations problem and submit a group solution to it. Slater finds that five-person groups are optimal. Hackman and Vidmar (1970) extend Slater's analysis to the case in which groups members are drawn from two different populations and where three types of intellectual tasks to be carried out by those groups are distinguished. In line with Slater's conclusion, they find that agents in the experiment consider a group size between four and five members as optimal.

3.4. Appendix

We prove Proposition 3 and Lemmas 1 and 2 used for the proof of Propositions 1 and 2. We first introduce and prove six Lemmas.

Define four preferences used in the proofs of Lemmas 3 to 7: $R \in \mathcal{S}$ is such that $a_1 R a_2 R ... R a_{m-1} R a_m$, $R' \in \mathcal{S}$ is such that $a_2 R' a_3 R' ... R' a_m R' a_1$, $R'' \in \mathcal{S}$ is such that $a_m R'' a_{m-1} R'' ... R'' a_2 R'' a_1$, and $R''' \in \mathcal{S}$ is such that $a_3 R''' a_4 R''' a_5 R''' ... R''' a_m R''' a_2 R''' a_1$. To prove these lemmas, we propose preference profiles for which Condorcet consistency fails.

Lemma 3. *For any $n \geq 3$ and $m \geq 3$, no w-scoring rule with $w_{m-2} \in \left(\frac{1}{2}, 1\right]$ is Condorcet consistent.*

Proof. Let $R_N \in \mathcal{S}^n$ where $\#\{i \in N : R_i = R\} = \frac{n+1}{2}$ and $\#\{i \in N : R_i = R'\} = \frac{n-1}{2}$. Note that $CW(R_N) = a_1$ and $x \notin S_w(R_N)$ for $x \in A \backslash \{a_1, a_2\}$ since $s_w(a_2, R_N) >$

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$s_w(x, R_N)$. Moreover, $s_w(a_2, R_N) = \frac{n-1}{2} + \frac{n+1}{2}w_{m-2} > \frac{n-1}{2} + \frac{n+1}{4}$ and $\frac{n+1}{2} + \frac{n-3}{4} \geq \frac{n+1}{2} = s_w(a_1, R_N)$ since $n \geq 3$. Thus, $S_w(R_N) = \{a_2\}$. ■

Lemma 4. *For any $n \geq 5$ and $m \geq 4$, no w-scoring rule with $w_{m-2} < 1$ is Condorcet consistent.*

Proof. Let $R_N \in \mathcal{S}^n$ where $\#\{i \in N : R_i = R\} = \frac{n-1}{2}$, $\#\{i \in N : R_i = R'\} = 1$, and $\#\{i \in N : R_i = R'''\} = \frac{n-1}{2}$. Note that $CW(R_N) = a_2$, $s_w(a_2, R_N) = \frac{n-1}{2}w_{m-2} + 1 + \frac{n-1}{2}w_1$, and $s_w(a_3, R_N) = \frac{n-1}{2} + w_{m-2} + \frac{n-1}{2}w_{m-3}$. Since $\frac{n-1}{2} = 1 + \frac{n-3}{2}$ and $w_{m-2} < 1$, we obtain that $s_w(a_3, R_N) > \frac{n-1}{2}w_{m-2} + 1 + \frac{n-1}{2}w_{m-3}$. Moreover, $\frac{n-1}{2}w_{m-2} + 1 + \frac{n-1}{2}w_{m-3} \geq s_w(a_2, R_N)$ since $w_{m-3} \geq w_1$. Thus, $a_2 = CW(R_N) \notin S_w(R_N)$. ■

Lemma 5. *For any $n > 5$ and $m = 3$, no w-scoring rule with $w_{m-2} \in \left[0, \frac{1}{2}\right]$ is Condorcet consistent.*

Proof. Distinguish three cases: (1) $w_{m-2} < \frac{n-3}{2(n-2)}$, (2) $w_{m-2} > \frac{n-3}{2(n-2)}$, and (3) $w_{m-2} = \frac{n-3}{2(n-2)}$.

Case 1: let $R_N \in \mathcal{S}^n$ such that $\#\{i \in N : R_i = R\} = \frac{n-1}{2}$, $\#\{i \in N : R_i = R'\} = 1$, and $\#\{i \in N : R_i = R''\} = \frac{n-1}{2}$. Note that $CW(R_N) = a_2$. We can check that $s_w(a_2, R_N) = (n-1)w_{m-2} + 1$, $s_w(a_3, R_N) = w_{m-2} + \frac{n-1}{2}$, and thus, $s_w(a_3, R_N) > s_w(a_2, R_N)$ if and only if $w_{m-2} < \frac{n-3}{2(n-2)}$.

Case 2: let $R_N \in \mathcal{S}^n$ such that $\#\{i \in N : R_i = R\} = \frac{n+1}{2}$, $\#\{i \in N : R_i = R'\} = 2$, and $\#\{i \in N : R_i = R''\} = \frac{n-5}{2}$. Note that $CW(R_N) = a_1$. We can check that $s_w(a_2, R_N) = (n-2)w_{m-2} + 2$, $s_w(a_1, R_N) = \frac{n+1}{2}$, and thus, $s_w(a_2, R_N) > s_w(a_1, R_N)$ if and only if $w_{m-2} > \frac{n-3}{2(n-2)}$.

Case 3: let $R_N \in \mathcal{S}^n$ such that $\#\{i \in N : R_i = R\} = \frac{n+1}{2}$, $\#\{i \in N : R_i = R'\} = 3$, and $\#\{i \in N : R_i = R''\} = \frac{n-7}{2}$. Note that the profiles is well-defined since $n > 5$. Note that $CW(R_N) = a_1$. We can check that $s_w(a_2, R_N) = (n-3)w_{m-2} + 3$, $s_w(a_1, R_N) = \frac{n+1}{2}$, and thus, $s_w(a_3, R_N) > s_w(a_1, R_N)$ if and only if $w_{m-2} > \frac{n-5}{2(n-3)}$, which holds for all $n > 1$, since $w_{m-2} = \frac{n-3}{2(n-2)}$. ■

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Lemma 6. *For any $n = 5$ and $m = 3$, no w-scoring rule with $w_{m-2} \in \left[0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \frac{1}{2}\right]$ is Condorcet consistent.*

The profiles of preferences in Cases (1) and (2) in the proof of Lemma 5 work to prove Lemma 6.

Lemma 7. *For any $n = 3$ and $m \geq 5$, no w-scoring rule with $w_{m-2} \in \left(0, \frac{1}{2}\right]$ and $w_{m-3} > w_1$ is Condorcet consistent.*

Proof. Let $R_N \in \mathcal{S}^n$ such that $\#\{i \in N : R_i = R\} = 1$, $\#\{i \in N : R_i = R'\} = 1$, and $\#\{i \in N : R_i = R''\} = 1$. Note that $CW(R_N) = a_2$. We can check that $x \notin S_w(R_N)$ for $x \in A\backslash\{a_1, a_2, a_3\}$ since $s_w(a_2, R_N) > s_w(x, R_N)$. Moreover, since $s_w(a_2, R_N) = 1+w_{m-2}+w_1 < s_w(a_3, R_N) = 1+w_{m-2}+w_{m-3}$ by $w_{m-3} > w_1$, we get that $CW(R_N) = a_2 \notin S_w(R_N)$. ■

Lemma 8. *For any $n = 3$ and $m \geq 5$, no w-scoring rule with $w_{m-2} \in \left[0, \frac{1}{2}\right]$ and $w_{m-2} > w_{m-3} = w_1$ is Condorcet consistent.*

Proof. Take any $R_N \in \mathcal{S}^3$ such that the preferences of the three agents are $\widetilde{R}$, $\overline{R}$, and $\widehat{R} \in \mathcal{S}$, where $a_2\widetilde{R}a_1\widetilde{R}a_3\widetilde{R}...\widetilde{R}a_{m-1}\widetilde{R}a_m$, $a_3\overline{R}a_2\overline{R}a_4\overline{R}a_1\overline{R}a_5\overline{R}...\overline{R}a_m$, and $a_4\widehat{R}a_5\widehat{R}a_3\widehat{R}a_2\widehat{R}a_1\widehat{R}...\widehat{R}a_m$. Note that $CW(R_N) = a_3$. We can check that $x \notin S_w(R_N)$ for $x \in A\backslash\{a_2, a_3, a_4\}$ since $s_w(a_2, R_N) = 1 + w_{m-2} + w_1 > w_{m-2} + 2w_1 \geq s_w(x, R_N)$. Moreover, $s_w(a_2, R_N) = 1+w_{m-2}+w_1 > s_w(a_3, R_N) = s_w(a_4, R_N) = 1+2w_1$ since $w_{m-2} > w_1$. Then, we get that $CW(R_N) = a_3 \notin S_w(R_N) = \{a_2\}$. ■

The proof of Lemma 1 is straightforward by Lemmas 3, 7, and 8.

The proof of Lemma 2 is straightforward by Lemmas 3, 4, and 6.

The proof of Proposition 3 is straightforward by Lemmas 3, 4, and 5.

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General Conclusions

In this PhD dissertation we have focused on the study of two of the most widespread forms of government around the world: direct democracy and representative democracy. The recent success of direct democracy systems among the population to the detriment of the popularity of the traditional systems of political representation has led us to address this issue. From a theoretical perspective, we have answered a series of questions that explain why societies may have a preference for one or another system of government, as well as describe those situations in which the employment of one or another system (and the way in which it is applied) becomes a really relevant concern due to the social consequences that this could entail.

In Chapter 1 we have proposed a model to investigate under which conditions voters prefer either a system of direct democracy or a system of representative democracy. We find that the preference of a society to be governed by either one or the other system depends on both the type of the electorate and the type of the median voter. As long as the electorate is pragmatic, representative democracy is the preferred system. The informational advantage of representatives is enough for an electorate concerned about the implementation of the economically efficient policy to have incentives to delegate. When the electorate is ideological, though, we have to look at the type of the median voter in order to determine the preferred system. If the electorate is ideological and the median voter is pragmatic, then representative democracy is the preferred system. This situation is what we have identified as a polarized electorate: no ideological group constitutes a majority by itself and a clear division of the electorate into two groups defending opposing policies exists. Trying to avoid that something opposed to their own

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ideological bias is chosen, the group of ideological voters biased towards the policy that would not be chosen in direct democracy prefers to delegate their vote to representatives. These ideological voters, along with the existing pragmatic voters in the society, constitute a majority coalition in favour of the representative democracy. Moreover, the demand of direct democracy is increasing in the polarization of the electorate, as long as no ideological group becomes a majority. On the contrary, when an ideological group becomes a majority, and therefore both electorate and median voter are ideological, direct democracy is the preferred system. Such an electorate prefers a system that guarantees the implementation of the policy on which the majority agree, rather than running the risk of allowing a representative to choose. This situation is what we have associated with societies in which populist movements have been successful. In this way, our model predicts that an increase in populism will increase the demand for direct democracy.

In Chapter 2 we have proposed a model to study when a system of representative democracy would be equivalent to a system of direct democracy in terms of the developed policies. We find necessary and sufficient conditions such that, decisions made on each issue in representative democracies coincide with the decisions that would have been chosen in direct democracies by majority voting issue-by-issue, regardless of whether candidates are either policy-motivated or office-motivated. First, the majority voting issue-by-issue output has to be a Condorcet winner relative to the voters' preference profile. Second, either there is a candidate with such an output as her most preferred alternative or *top*, or candidates are maximally top-differentiated, which means that there is no issue for which both candidates have the same more preferred decision. First condition can be interpreted as the existence of some degree of homogeneity among voters' preferences about the suitability of the implementation of decisions yielded by majority voting issue-by-issue. Second condition can be understood as the existence of either at least one candidate whose preferences are in line with preferences of society, or candidates with sufficiently different preferences. The concurrent fulfillment of both conditions allows that, on the one hand, any potentially profitable deviation from the majority voting issue-by-issue output that might initially exist for any candidate would no longer be beneficial and, on the other hand, either there exists a candidate who trivially has no incentives to deviate from what

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the majority of people desires for each issue, or there is no room for a mutually beneficial agreement for both candidates which may violate the interests of the majority. Thus, social demands claiming the need of removing the traditional system of political representation and establishing a direct democracy in order to respect the will of the majority is only meaningful in societies in which these conditions are not being met.

In Chapter 3 we have proposed a solution that seeks to reconcile both sides of supporters of direct democracy and representative democracy, offering a response to the concerns raised to each of them when the system contrary to their liking is adopted. Supporters of representative democracy question the suitability of direct democracy to achieve the representativeness of the majority's preferences due to the risk involved by the low turnout when a referendum is held. If not enough people vote to make the decision relevant, it will be biased, not representing therefore the preferences of the majority. For their part, supporters of direct democracy criticize representative democracy as a result of the possible non-representativeness of the will of the majority due to the risk of decisions being made by self-seeking politicians. They go even further by claiming that the damage suffered by society when decisions are made by self-seeking politicians could be even larger if the intensity of voters' preferences is taken into account. The proposed solution is to have decisions made by a committee through scoring rules that are Condorcet consistent. The composition of such committee perfectly represents the distribution of voters' preferences over the set of alternatives. Committee members cannot abstain, removing thus the risk of underrepresentation warned by supporters of representative democracy, and they sincerely vote according to their preferences, being thus the outcome of the voting fully representative of the preferences of the electorate, which meets the demands of supporters of direct democracy. We find that if the size of the committee is three, a sub-family of the scoring rules is Condorcet consistent for any number of alternatives. We also find that when there are three alternatives at stake, if the size of the committee is between three and five, then some scoring rules are Condorcet consistent.

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Summary In Spanish - Resumen En Español

Durante siglos, la idoneidad de la democracia representativa como sistema de gobierno no ha sido cuestionada en absoluto. Si bien han sido dos las teorías de la representación que han sido tradicionalmente distinguidas, esto es, la teoría de la semejanza y la teoría de la sustitución, es esta última la que ha sido empleada para defender la mejor habilidad de los políticos frente a los ciudadanos a la hora de tomar decisiones. La teoría de la representación como semejanza remonta sus orígenes a la obra *Crátilo*, el diálogo escrito por Platón en el año 360 a.C. Según esta teoría, un representante debería semejarse a lo que representa. Así, se hace necesaria la existencia de una similitud entre políticos y ciudadanos para poder hablar de representación de los segundos por parte de los primeros. En cuanto a la teoría de la representación como sustitución, fue propuesta por primera vez por Edmund Burke en su obra *Indagación filosófica sobre el origen de nuestras ideas acerca de lo sublime y de lo bello*, publicada en 1757. De acuerdo a esta teoría, la representación ha de entenderse como un hacer presente (de nuevo) lo que está (ahora) ausente. Dicho de otro modo, se trata de un sustituto o reemplazo de algo que está ausente. Resulta llamativo que, a diferencia del caso anterior, esta teoría no menciona ningún criterio (semejanza o algo similar) que deba ser satisfecho para que un político pudiera ser el representante de los ciudadanos. Implícitamente, se permite que existan diferencias de diversa naturaleza entre representantes y representados. Beneficiándose de esta posibilidad, muchos autores, tanto en el ámbito de la Economía como en el de las Ciencias Políticas, se han fundamentado en el supuesto de que los políticos disponen de mejor información que los ciudadanos ordinarios acerca de lo que debe hacerse en cada

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momento para defender los sistemas de representación política. Otra ventaja de la democracia representativa manifestada en la literatura es la reducción del riesgo de la llamada "tiranía de la mayoría", reconocida por autores tales como Alexis de Tocqueville y John Stuart Mill, según la cual una mayoría democrática impondría su voluntad sobre una minoría. Advirtiendo de los peligros de la democracia directa, James Madison escribía en su obra *Federalista 10* publicada en 1787: "Resulta de gran importancia... para proteger a una parte de la sociedad frente a la injusticia de la otra parte. Intereses diferenciados necesariamente existen en diferentes clases de ciudadanos. Si una mayoría está unida por un interés común, los derechos de la minoría estarán en peligro."

La vida ha sido fácil para la democracia representativa. Hasta ahora. El libro *Referendums: A Comparative Study of Practice and Theory*, editado por David Butler y Austin Ranney y publicado en 1978 por el Instituto Norteamericano de la Empresa (American Enterprise Institute - AEI) es considerado uno de los primeros intentos por recoger la experiencia mundial con el instrumento del referéndum. Dicha obra trata de justificar la creciente insatisfacción del electorado con la actuación de los representantes electos mediante el uso, cada vez mayor, de referéndums en varias naciones de todo el mundo. Los editores se basan en estudios de referéndums celebrados en Australia, Francia, Irlanda, Suiza, países escandinavos y Estados Unidos, entre otros. Algunos años más tarde y fruto de este trabajo, Ranney publicó su obra *The Referendum Device*, cuya reseña fue incluida en la revista American Political Science Review, una de las que cuentan con un mayor índice de impacto dentro de la categoría de Ciencias Políticas. Este texto analiza nuevamente el deseo de referéndums en los regímenes democráticos. *Referendums Around the World: The Growing Use of Direct Democracy* (1994) y *Referendums Around the World: The Continued Growth of Direct Democracy* (2014) constituyen posteriores reediciones de la obra de Butler y Ranney. Tales textos continúan contribuyendo a explicar cómo los referéndums estaban afectando a la forma de hacer política en aquellos países entre los que dicho instrumento estaba más extendido.

El descontento de los ciudadanos con el sistema de representación política ha venido creciendo cada vez más en los últimos años. Dicho aumento ha provocado que, reciente-

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mente, algunos sectores de la sociedad hayan dejado de ver la democracia directa como un mero complemento de la democracia representativa y, en su lugar, hayan comenzado a considerarla como un potencial sustituto. De hecho, venimos presenciando una tendencia creciente en los niveles de popularidad de los sistemas de gobierno en los que los ciudadanos pueden decidir directamente sin necesidad de la tradicional intermediación de los políticos. El principal objetivo de esta Tesis Doctoral es abordar el estudio de este fenómeno comparando los sistemas de democracia directa y democracia representativa desde un punto de vista teórico. Ante el innegable éxito y aceptación popular de los instrumentos de democracia directa tales como los referéndums y las consultas populares a la hora de tomar decisiones, una primera pregunta muy natural es cuándo los votantes prefieren un sistema de democracia directa o un sistema de democracia representativa. En el Capítulo 1 proponemos un modelo para investigar bajo qué condiciones los votantes prefieren uno u otro sistema. En las democracias directas los votantes eligen una política de entre dos alternativas, bajo incertidumbre acerca de qué política se ajusta mejor a la realización del estado del mundo; en las democracias representativas los votantes eligen a un candidato quien, una vez elegido, selecciona una política tras haber observado el estado del mundo realizado. Los pagos de los votantes y de los políticos dependen de un componente común que es positivo sólo si la política se ajusta al estado del mundo, y de un sesgo ideológico privado hacia una de las políticas. En las democracias directas los votantes tienen incertidumbre acerca del estado del mundo futuro, mientras que en las democracias representativas tienen incertidumbre acerca del grado del sesgo ideológico de los candidatos, aun cuando la política hacia la cual cada candidato está sesgado es conocida. Nosotros demostramos que la democracia representativa es preferida si *(i)* la mayoría de votantes son pragmáticos (prevalece el componente común), y *(ii)* la sociedad está ideológicamente polarizada, lo que significa que la mayoría de votantes son ideológicos (prevalece el componente privado), pero el votante mediano es pragmático. La democracia directa es el instrumento preferido para la toma colectiva de decisiones en sociedades en las que la mayoría de votantes y el votante mediano son ideológicos, lo que implica que la mayoría de votantes tienen el mismo sesgo ideológico, como sucede, por ejemplo, cuando la retórica populista de la gente contra la élite tiene éxito. Además, descubrimos que la demanda de democracia directa es creciente en la polarización del electorado, siempre y

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cuando ningún grupo ideológico de votantes llegue a ser una mayoría.

Como cabría esperar, el mencionado éxito de la democracia directa viene resultando desde hace algún tiempo en la emergencia de movimientos sociales y partidos políticos que exigen la participación directa de los ciudadanos en el proceso de toma de decisiones, lo cual permitiría a la gente decidir sobre todos y cada uno de los temas que se planteasen a debate. Estos grupos no sólo defienden firmemente el sistema de democracia directa si no que también muestran una fuerte oposición hacia el sistema de democracia representativa. Su principal crítica es la posibilidad de que los políticos sean egoístas y acaben comportándose en contra de los intereses de la gente. En este sentido, destacan la incapacidad de la democracia representativa para implementar lo que la mayoría de la gente desea para cada tema en concreto. Si bien la idoneidad de dicho resultado podría ser discutido, nosotros no entraremos en ese debate. En su lugar, adoptamos un enfoque más positivo del asunto. Nos preguntamos si, de hecho, no hay espacio para que la democracia representativa satisfaga las aspiraciones de estos grupos, siendo totalmente necesario para tal objetivo abandonar el tradicional sistema de representación política y reemplazarlo por un sistema de democracia directa. En el Capítulo 2 estudiamos bajo qué condiciones los dos procedimientos de toma de decisiones mencionados serían equivalentes en términos de las políticas llevadas a cabo para cada uno de los temas considerados. Proponemos un modelo en el que hay un número finito de temas sobre los que debe tomarse una decisión binaria. En las democracias directas los votantes votan directamente por su decisión más preferida para cada tema, de manera que la votación por mayoría tema a tema es el resultado obtenido en estos sistemas. En las democracias representativas, nosotros presentamos una competición bipartidista donde las plataformas políticas de ambos partidos son conocidas. Los candidatos que representan cada uno de estos partidos tendrán tales plataformas políticas como su conjunto de decisiones para el agregado de temas más preferido. Sin embargo, las preferencias de los candidatos de los partidos no son conocidas, pudiendo éstas variar desde la motivación por la política hasta la motivación por la oficina. Este supuesto captura la esencia de la crítica arrojada hacia los sistemas de representación política relativa al riesgo de tener políticos que únicamente persiguen sus propios intereses. Nosotros encontramos condiciones necesarias y suficientes tales que,

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independientemente de las preferencias de los candidatos, las decisiones tomadas para cada tema en las democracias representativas coinciden con las decisiones que habrían sido elegidas en las democracias directas mediante votación por mayoría tema a tema. Estas condiciones suponen restricciones sobre pares de perfil de preferencias de votantes y plataformas políticas de los candidatos. Si bien en el Capítulo 2 las formularemos formalmente, aquí ofrecemos una intuición de tales condiciones. En primer lugar, debe existir un cierto grado de homogeneidad entre las preferencias de los votantes acerca de la idoneidad de la implementación de las decisiones derivadas de la votación por mayoría en cada tema tal que este conjunto de decisiones no sea derrotado en comparaciones a pares por ningún otro conjunto de decisiones para el agregado de temas. Esto permitiría que, cualquier desviación potencialmente beneficiosa que pudiese existir inicialmente para alguno de los candidatos dejase de ser beneficiosa. En segundo lugar, o bien debe haber al menos un candidato cuyas preferencias estén alineadas con las decisiones tomadas mediante votación por mayoría en cada tema, o bien los candidatos deben tener preferencias lo suficientemente diferentes.[2] Esta doble condición posibilitaría, respectivamente, o bien la existencia de un candidato que trivialmente no tendrá incentivos a desviarse de lo que la mayoría de gente desea para cada tema, o la no existencia de lugar para un acuerdo mutuamente beneficioso para ambos candidatos que pudiese violar los intereses de la mayoría.

Criticar el sistema contrario argumentando la posible no representatividad de las preferencias de la mayoría en su resultado no es algo exclusivo de aquellos en favor de la democracia directa.[3] Los partidarios de la democracia representativa cuestionan a los partidarios de la democracia directa cuando estos últimos aseguran que la democracia directa es el sistema apropiado para lograr dicho objetivo de representatividad. Los primeros destacan el riesgo que supone la baja participación cuando se celebra un referéndum. Podría suceder que algunas personas decidiesen no pronunciarse acerca de un determinado tema simplemente porque no están interesados en él. La participación se convertiría por

[2]Por supuesto, ambas condiciones son compatibles y podrían cumplirse al mismo tiempo.

[3]Como se mencionó anteriormente, los partidarios de la democracia directa critican la democracia representativa a consecuencia de la posible no representatividad de la voluntad de la mayoría debido al riesgo que supone que las decisiones hayan sido tomadas por políticos egoístas.

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tanto en un problema: dado que no votaron las suficientes personas como para hacer la decisión relevante, esta estará sesgada, no representando por tanto las preferencias de la mayoría. Es este argumento con el que ellos siguen defendiendo la figura de un representante encargado de la toma de decisiones. Sin embargo, los partidarios de la democracia directa, lejos de rendirse, rebaten dicho argumento. Según ellos, el daño sufrido por la sociedad cuando las decisiones son tomadas por políticos egoístas podría ser incluso mayor si se tiene en cuenta la intensidad de las preferencias de los votantes: el político podría estar eligiendo algo que, no sólo fuese en contra de las preferencias de la sociedad, si no que de hecho fuese "realmente malo" para una mayoría de votantes. En el Capítulo 3 se propone una solución que trata de reconciliar las visiones de ambos bandos. Nosotros evaluamos su actuación de acuerdo a un principio ampliamente usado a la hora de evaluar reglas de votación. Asumimos que las decisiones son tomadas por un comité mediante reglas de puntuación. La composición de dicho comité perfectamente representa la distribución de las preferencias de los votantes sobre el conjunto de alternativas. Los miembros del comité no pueden abstenerse, eliminando así el riesgo de la subrrepresentación advertido por los partidarios de la democracia representativa, y votan sinceramente de acuerdo a sus preferencias, siendo por tanto el resultado de la votación totalmente representativo de las preferencias del electorado, lo cual satisface las exigencias de los partidarios de la democracia directa. Más concretamente, consideramos problemas de votación con un número impar de miembros del comité y preferencias unimodales. Estudiamos si existen reglas de puntuación que sean consistentes en el sentido de Condorcet, es decir, reglas de puntuación que seleccionan, si existe, la alternativa que vence a cualquier otra alternativa en comparaciones a pares. Con sólo tres alternativas, existen reglas de puntuación que eligen al ganador de Condorcet únicamente con comités de tres y cinco agentes. Con cuatro o más alternativas, sólo los comités de tres agentes funcionan. En todas estas reglas de puntuación, a la mejor y peor alternativas se les asigna una puntuación de 1 y 0 respectivamente, y a cualquier alternativa intermedia una puntuación entre 0 y $\frac{1}{2}$. Con cinco o más alternativas, la puntuación de cualquier alternativa intermedia debe ser la misma, y denotaremos esta familia como las reglas de puntuación semiplurales.

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Main Results - Principales Resultados

En esta Tesis Doctoral nos hemos centrado en el estudio de dos de las formas de gobierno más extendidas alrededor del mundo: la democracia directa y la democracia representativa. El reciente éxito de los sistemas de democracia directa entre la población en detrimento de la popularidad de los tradicionales sistemas de democracia representativa nos ha llevado a abordar este tema. Desde una perspectiva teórica, hemos respondido a una serie de cuestiones que explican porqué las sociedades podrían tener una preferencia por uno u otro sistema de gobierno, así como describir aquellas situaciones en las que el empleo de uno u otro sistema (y el modo en el que es aplicado) se convierte en una preocupación realmente relevante dadas las consecuencias sociales que ello podría implicar.

En el Capítulo 1 hemos propuesto un modelo para investigar bajo qué condiciones los votantes prefieren bien un sistema de democracia directa, o bien un sistema de democracia representativa. Descubrimos que la preferencia de una sociedad por ser gobernada por uno u otro sistema depende de ambos el tipo del electorado y el tipo del votante mediano. Siempre y cuando el electorado sea pragmático, la democracia representativa es el sistema preferido. La ventaja informativa de los representantes es suficiente como para que un electorado preocupado por la implementación de la política económicamente eficiente tenga incentivos a delegar. Por su parte, cuando el electorado es ideológico, debemos fijarnos en el tipo del votante mediano para determinar cuál es el sistema preferido. Si el electorado es ideológico y el votante mediano es pragmático, entonces la democracia representativa es el sistema preferido. Esta situación es la que hemos identificado como un electorado polarizado: ningún grupo ideológico constituye una mayoría por sí mismo y existe una clara división del electorado en dos grupos defendiendo políticas opuestas. Tratando de evitar que algo contrario a su propio sesgo ideológico sea elegido, el grupo de votantes ideológicos sesgados hacia la política que no sería elegida en la democracia directa prefiere delegar su voto en los representantes. Estos votantes ideológicos, junto con los votantes pragmáticos existentes en la sociedad, constituyen una coalición mayoritaria en favor de la democracia representativa. Además, la demanda de democracia directa es creciente en la polarización del electorado, siempre y cuando ningún grupo ideológico

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llegue a suponer una mayoría. Por el contrario, cuando un grupo ideológico pasa a ser una mayoría, y por tanto ambos el electorado y el votante mediano son ideológicos, la democracia directa es el sistema preferido. Tal electorado prefiere un sistema que garantice la implementación de la política en la cual la mayoría están de acuerdo, en lugar de correr el riesgo de permitir a un representante que decida. Esta situación es la que hemos asociado con las sociedades en las que los movimientos populistas han sido exitosos. De este modo, nuestro modelo predice que un aumento del populismo aumentará la demanda de democracia directa.

En el Capítulo 2 hemos propuesto un modelo para estudiar cuando un sistema de democracia representativa sería equivalente a un sistema de democracia directa en términos de las políticas desarrolladas. Encontramos condiciones necesarias y suficientes tales que, las decisiones tomadas para cada tema en democracias representativas coincide con las decisiones que habrían sido tomadas en democracias directas mediante votación mayoritaria tema a tema, independientemente de si los candidatos están bien motivados por la política o bien motivados por la oficina. En primer lugar, el resultado de la votación por mayoría tema a tema tiene que ser un ganador de Condorcet relativo al perfil de preferencia de los votantes. En segundo lugar, o bien hay un candidato con tal resultado como su alternativa más preferida o *top*, o bien los candidatos están máximamente top-diferenciados, lo cual significa que no hay ningún tema para el cual ambos candidatos tengan la misma decisión más preferida. La primera condición puede ser interpretada como la existencia de un cierto grado de homogeneidad entre las preferencias de los votantes acerca de la idoneidad de la implementación de las decisiones derivadas de la votación por mayoría en cada tema. La segunda condición puede ser entendida como la existencia de o bien al menos un candidato cuyas preferencias estén en línea con las preferencias de la sociedad, o bien candidatos con preferencias lo suficientemente diferentes. El cumplimiento simultáneo de ambas condiciones permite que, por un lado, cualquier desviación con respecto al resultado de la votación por mayoría tema a tema potencialmente beneficiosa que pudiese inicialmente existir para cualquiera de los candidatos dejaría de ser beneficiosa y, por otro lado, permite que o bien exista un candidato que, trivialmente, no tendría incentivos a desviarse de lo que la mayoría de la gente desea para cada tema, o bien que no haya sitio

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para un acuerdo mutuamente beneficioso para ambos candidatos que pudiese violar los intereses de la mayoría. De este modo, las demandas sociales reclamando la necesidad de eliminar el tradicional sistema de representación política y establecer una democracia directa con tal de respetar la voluntad de la mayoría tienen sentido únicamente en sociedades en las que estas condiciones no se estén cumpliendo.

En el Capítulo 3 hemos propuesto una solución que busca reconciliar a los bandos de partidarios de la democracia directa y de la democracia representativa, ofreciendo una respuesta a las preocupaciones surgidas a cada uno de ellos cuando el sistema contrario al de su agrado es adoptado. Los partidarios de la democracia representativa cuestionan la idoneidad de la democracia directa para lograr la representatividad de las preferencias de la mayoría, debido al riesgo que supone la baja participación cuando se celebra un referéndum. Si no vota la suficiente gente como para hacer que la decisión sea relevante, esta estará sesgada, no representando por tanto las preferencias de la mayoría. Por su parte, los partidarios de la democracia directa critican la democracia representativa a causa de la posible no representatividad de la voluntad de la mayoría debido al riesgo que supone el hecho de que las decisiones puedan ser tomadas por políticos egoístas. Ellos van incluso más allá al afirmar que el daño sufrido por la sociedad cuando las decisiones son tomadas por políticos egoístas podría ser incluso mayor si la intensidad de las preferencias de los votantes fuese tenida en cuenta. La solución propuesta consiste en que las decisiones sean tomadas por un comité mediante reglas de puntuación que sean consistentes en el sentido de Condorcet. La composición de tal comité representa perfectamente la distribución de las preferencias de los votantes sobre el conjunto de alternativas. Los miembros del comité no pueden abstenerse, eliminando así el riesgo de subrepresentación advertido por los partidarios de la democracia representativa, y votan sinceramente de acuerdo a sus preferencias, siendo por tanto el resultado de la votación totalmente representativo de las preferencias del electorado, lo cual satisface las demandas de los partidarios de la democracia directa. Descubrimos que si el tamaño del comité es tres, entonces una sub-familia de las reglas de puntuación es consistente en el sentido de Condorcet para cualquier número de alternativas. También encontramos que cuando hay tres alternativas en juego, si el tamaño del comité se encuentra entre tres y cinco, entonces

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hay algunas reglas de puntuación que son consistentes en el sentido de Condorcet.

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