

Outage Capacity of Rate-Adaptive Relaying for FSO Links with Nonzero Boresight Pointing Errors

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Abstract—An outage capacity analysis of a 3-way free-space optical (FSO) setup based on the optical path selection, either source-destination (S-D) or source-relay-destination (S-R-D), with a greater value of channel gain along with the use of repetition coding (RC) is analyzed over atmospheric turbulence channels with nonzero boresight pointing errors. The developed expression, which is validated through Monte Carlo simulation, is used to carefully analyze the performance and design of rate-adaptive relaying FSO systems from an information theory point of view. It is highlighted that a greater outage diversity is achieved by the obtained results without investing in extra hardware at the expense of an information rate-reduction.

Index Terms—FSO, outage capacity, atmospheric turbulence.

I. INTRODUCTION

FREE-space optical (FSO) communication systems have demonstrated to be an emerging broadband solution to efficiently solve the spectrum congestion problems in the existing radio-frequency (RF) networks [1]. Recently, several reported works have investigated the adoption of cooperative strategies in the context of terrestrial FSO links in order to create spatial diversity without investing in extra hardware with the main goal of improving the performance and achievable rates [2]–[6] (and references therein). One important issue in FSO communication systems that has not been taken into account in greater detail is to evaluate the effect of nonzero boresight on the performance of cooperative FSO systems for potential applications. Only a few works have been reported [4]–[6], but all of them present one thing in common: they did not analyze carefully its impact on performance. In this sense, we make an extra effort to exploit the derived solutions to study its impact as well as potential solutions to avoid a greater deterioration. Basically, we study the performance and design of rate-adaptive relaying FSO systems from an information theory point of view in the presence of nonzero boresight pointing errors.

In this letter, a novel closed-form asymptotic expression for the outage capacity is obtained for a 3-way cooperative FSO system over gamma-gamma (GG) atmospheric turbulence

channels with nonzero boresight pointing errors modeled by the Beckmann distribution. Thus, this closed-form solution is derived by exploiting the simplicity of the approximation of the Beckmann distribution presented in [11]. Unlike our previous work in [2], [3] and the other reported works [4]–[6], the motivation and purpose of this letter is to analyze the outage capacity of a cooperative FSO system in the presence of nonzero boresight pointing errors. This new cooperative protocol is able to achieve a robust and high outage diversity order at the expense of an information rate-reduction. The key idea is to take full advantage of rate-adaptive relaying to increase the outage performance. To do that, this novel relaying scheme is based on the optical path selection, either source-destination (S-D) or source-relay-destination (S-R-D), with a greater value of channel gain along with the use of repetition coding (RC) in all FSO links. In other words, the new feature is to exploit the potential time-diversity order available in the turbulent channel under nonzero boresight pointing error effects. The employment of time-diversity in cooperative FSO systems was firstly proposed in [3], where two separate channels over the same transmit and receive path are only implemented in the source-relay (S-R) link. An outage diversity analysis is also carried out as a function of the relay placement when not only different atmospheric turbulence conditions are considered, but also when different severity of nonzero boresight pointing errors is taken into consideration. It is concluded that relay-aided FSO systems based on rate-adaptive combined with selection-based technique are promising solutions to improve the outage capacity performance.

II. PROPOSED COOPERATIVE STRATEGY

We adopt a cooperative FSO system based on 3 separate intensity modulation and direct detection (IM/DD) FSO links, where On-Off keying (OOK) modulation is assumed. The proposed cooperative FSO protocol, which is based on the optical path selection with a greater value of channel gain together with time-diversity order of 2, is called as the PS-RC cooperative protocol. When the channel gain of the S-D link (h_{SD}) is greater than the channel gain of the S-R link (h_{SR}), i.e. $h_{SD} > h_{SR}$, the FSO communication is only based on the direct transmission to the destination node, obviating the S-R-D path or detect-and-forward (DF) based dual-hop FSO cooperative transmission. On the contrary, when $h_{SD} < h_{SR}$, the cooperative transmission is established successfully. The data received from the source node at the destination node as well as at the relay node are stored in a buffer for further

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detection. A block-fading later, the source node repeats the bit sequence transmitted, not being necessary to perform another selection when the source node is repeating the data. Both destination and relay are able to detect the received information from the source node in both block-fading, establishing the RC protocol. Note that channel side information (CSI) is known not only at the receiver but also at the transmitter. Without loss of generality, possible synchronization errors on outage capacity are beyond the main objective of this letter.

Knowing that the coherence time is on the order of milliseconds, we consider the time variations according to the theoretical block-fading model due to the frozen-atmosphere characteristics of scintillation, where the channel fade remains constant during a block and changes to a new independent value from one block to next. This temporal correlation can be overcome by means of long interleavers [7]. However, as in [8], we assume that the interleaver depth cannot be infinite and, hence, we can potentially benefit from a degree of time diversity order of 2. As proved experimentally in [9], a perfect interleaving can be done by simply transmitting the same information delayed at least the expected fade duration.

III. FSO CHANNEL MODEL

The received electrical signal for each terrestrial FSO link of this cooperative system is given by $y_m = h_m R x + z_m$, where R is the detector responsivity, x is the transmitted optical signal, h_m is the gain of the channel between the transmitter and the receiver, and z_m is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2 = N_0/2$. In the following section, the fading coefficient h_m for the links S-D, S-R and R-D is indicated by h_{SD} , h_{SR} and h_{RD} , respectively. Here, we assume that all fading coefficients are statistically independent. The transmitted optical signal is either 0 or $2P_t$ where P_t is the average optical power. The received electrical SNR can be defined as $\gamma(h_m) = 2P_t^2 R^2 h_m^2 / \sigma_n^2 = 4\gamma_0 h_m^2$, where $\gamma_0 = P_t^2 R^2 / N_0$ represents the received electrical SNR in absence of turbulence and pointing errors.

The channel gain is modeled as $h = L_m \cdot h_a \cdot h_p$, where L_m is the atmospheric attenuation, h_a is the atmospheric turbulence, and h_p are the pointing errors. The atmospheric attenuation, L_m , is computed by the exponential Beers-Lambert law as $L_m = e^{-\Phi d_m}$, where d_m is the FSO link distance in meters, and Φ is the atmospheric attenuation coefficient. To consider a wide range of turbulence conditions (moderate to strong), the GG turbulence model proposed in [10] of parameters α and β is assumed. Pointing errors at the receiver are modeled assuming a model of misalignment where the radial displacement r at the receiver follows the Beckmann distribution. Based on the analysis carried out in [11], the radial displacement r at the receiver can be simplified, and the Beckmann distribution can be accurately approximated by a modified Rayleigh distribution in order to study the effect of nonzero boresight on cooperative FSO systems. The attenuation due to pointing errors is approximated assuming a Gaussian spatial intensity profile of beam waist radius [12, eqn. (9)] as $h_p(r; z) \approx A_0 \exp(-2r^2/\omega_{z_{eq}}^2)$, where $A_0 = [\text{erf}(v)]^2$ is the fraction of the collected power at

$r = 0$, $v = \sqrt{\pi}a/\sqrt{2}\omega_z$, $\omega_{z_{eq}}^2 = \omega_z^2 \sqrt{\pi} \text{erf}(v)/2v \exp(-v^2)$ is the equivalent beam width, and a is the radius of a circular detection aperture.

A closed-form expression for the probability density function (PDF) of the composite GG fading channels and generalized misalignment errors was obtained in terms of the Meijer's G-function in [11, eqn. (16)]. In order to provide more insights and obtain simpler closed-form expressions for the outage capacity, the PDF of h_m is approximated by a single polynomial term as $f_{h_m}(h) \approx a_m h^{b_m-1}$ [2]. Therefore, different expressions for $f_{h_m}(h)$, depending on the relation between φ^2 and β when plane wave propagation is assumed [13], can be obtained as in [11, eqn. (18)] as follows

$$f_{h_m}(h) \doteq \begin{cases} \frac{\varphi^2 (\alpha\beta)^\beta \Gamma(\alpha-\beta) h^{\beta-1}}{(A_{\text{mod}} L_m)^\beta \Gamma(\alpha) \Gamma(\beta) (\varphi^2 - \beta)}, & \varphi^2 > \beta \\ \frac{\varphi^2 \Gamma(\alpha - \varphi^2) \Gamma(\beta - \varphi^2) h^{\varphi^2-1}}{(\alpha\beta)^{-\varphi^2} (A_{\text{mod}} L_m)^{\varphi^2} \Gamma(\alpha) \Gamma(\beta)}, & \varphi^2 < \beta \end{cases} \quad (1)$$

where A_{mod} was obtained in [11, eqn. (15)], $\varphi = \omega_{z_{eq}}/2\sigma_{\text{mod}}$, and σ_{mod} is the parameter that considers a nonzero boresight error in addition to the random jitter variances in each FSO link [11, eqn. (9)]. When the same jitter variance is considered for horizontal and elevation axes, σ_{mod}^2 reduces to $\sigma_{\text{mod}}^2 = ((3/2)\sigma_s^4 s^2 + \sigma_s^6)^{1/3}$, where s is the boresight error.

IV. OUTAGE CAPACITY ANALYSIS

In this section, we analyze the outage capacity performance at high SNR of the proposed cooperative protocol, showing that the average outage behaves asymptotically as $(O_c \gamma)^{-O_d}$, where O_d and O_c denote outage diversity order and coding gain, respectively.

Outage capacity applies to slowly-varying channels such as the FSO channel, where the instantaneous SNR is constant over a large number of transmission. Here, we assume that perfect knowledge of the CSI is available at the receiver of every node of this cooperative FSO system as well as the source node is able to send data at an code rate of R_0 bits/channel use. Thus, the outage capacity is defined as the probability of the instantaneous capacity of a channel state h for each of the involved FSO links, $C(h_m)$, is not sufficient to support data rate R_0 , and it is expressed as in [14]

$$P_{\text{out}} := \Pr[C(\gamma(h_m)) < R_0] = \Pr[4\gamma_0 h_m^2 < C^{-1}(R_0)] \\ = \int_0^{\sqrt{\frac{C^{-1}(R_0)}{4\gamma_0}}} f_{h_m}(h_m) dh_m = F_{h_m} \left(\sqrt{\frac{C^{-1}(R_0)}{4\gamma_0}} \right), \quad (2)$$

where $C^{-1}(R_0) = 2^{R_0} - 1$.

The received electrical SNR corresponding to the PS-RC cooperative protocol when the source node selects direct transmission with RC of order 2 is given by

$$\gamma_{\text{SD}} = \frac{2P_t^2 R^2}{2\sigma_n^2} (h_{SD1} + h_{SD2})^2 = 2\gamma_0 h_{SDT}^2. \quad (3)$$

Note that we define h_{SDT} as the sum of random variables (RVs) $h_{SD1} + h_{SD2}$ under the assumption that $h_{SD1} > h_{SR1}$ as determined by the cumulative density function (CDF) of the RV h_{SR1} as $F_{h_{SR1}}(h_{SD1})$. The CDF of the RV h_m can be expressed by a single polynomial term as

$F_{h_m}(h) \doteq (\alpha_m/b_m)h^{b_m}$, as deduced from Eq. (1). This is due to the fact that all fading coefficients are statistically independent. In this way, we obtain the outage probability of the non-cooperative transmission, $P_{\text{out}}^{\text{SD}}$, assuming CSI at the receiver and transmitter as follows

$$P_{\text{out}}^{\text{SD}}(R_0) := \Pr \left[h_{SDT}^2 \leq \frac{C^{-1}(R_0)}{2\gamma_0} \right] \\ = \int_0^{\sqrt{\frac{C^{-1}(R_0)}{2\gamma_0}}} f_{h_{SDT}}(h)dh = F_{h_{SDT}} \left(\sqrt{\frac{C^{-1}(R_0)}{2\gamma_0}} \right). \quad (4)$$

Knowing that h_{SD_1} and h_{SD_2} are statistically independent and that the resulting PDF of their sum h_{SDT} can be determined by using the moment-generating function (MGF) of their corresponding PDFs obtained via single-sided Laplace and its inverse transforms, an approximate expression for the PDF, $f_{h_{SDT}}(h)$, can easily be obtained as

$$f_{h_{SDT}}(h) \doteq \frac{a_{SD}^2 \Gamma(b_{SD}) \Gamma(b_{SD} + b_{SR})}{(a_{SR})^{-1} b_{SR} \Gamma(2b_{SD} + b_{SR})} h^{2b_{SD} + b_{SR} - 1}. \quad (5)$$

Note that $b_m = \min(\beta_m, \varphi_m^2)$ as deduced from Eq. (1). From the above expression, the closed-form asymptotic solution for the outage capacity corresponding to the direct transmission is obtained by substituting Eq. (5) into Eq. (4).

Next, we analyze the cooperative transmission, i.e., when the source node selects transmission via S-R-D path. In this case, a DF based dual-hop FSO cooperative system with RC is employed. In this dual-hop, the outage capacity performance is dominated by the worst hop. Thus, the received electrical SNR of this dual-hop FSO system with RC is given by $\gamma_{\text{SRD}} = \min(\gamma_{\text{SR}}, \gamma_{\text{RD}})$, where γ_{SR} and γ_{RD} are the received electrical SNRs at relay node and destination node, respectively. Hence, we can express γ_{SRD} as

$$\gamma_{\text{SRD}} = 2\gamma_0 \min \left((h_{SR_1} + h_{SR_2})^2, (h_{RD_1} + h_{RD_2})^2 \right) \\ = 2\gamma_0 \min(h_{SR_T}^2, h_{RD_T}^2) = 2\gamma_0 h_{SRD}^2. \quad (9)$$

It should be noted that h_{SR_T} represents the sum of RVs $h_{SR_1} + h_{SR_2}$ under the assumption that $h_{SR_1} > h_{SD_1}$ as determined by the term $F_{h_{SD_1}}(h_{SR_1})$. In this way, the CDF corresponding to h_{SRD} is easily derived from [15] as

$$F_{h_{SRD}}(h) = F_{SR_T}(h) + F_{RD_T}(h) - F_{SR_T}(h)F_{RD_T}(h). \quad (10)$$

Both $F_{SR_T}(h)$ and $F_{RD_T}(h)$ are obtained via MGF as in Eq. (5). Therefore, $F_{SR_T}(h)$ is expressed in closed-form as

$$F_{SR_T}(h) \doteq \frac{a_{SD} a_{SR}^2 \Gamma(b_{SR}) \Gamma(b_{SD} + b_{SR})}{b_{SD} (b_{SD} + 2b_{SR}) \Gamma(b_{SD} + 2b_{SR})} \\ \times h^{\frac{b_{SD} + 2b_{SR}}{2}}. \quad (11)$$

At the same time, $F_{RD_T}(h)$ is also expressed as

$$F_{RD_T}(h) \doteq \frac{a_{RD}^2 \gamma_0 (b_{RD})^2}{2b_{RD} \Gamma(2b_{RD})} h^{b_{RD}}. \quad (12)$$

Therefore, the outage capacity of the considered DF based dual-hop FSO system is given by

$$P_{\text{out}}^{\text{SRD}}(R_0) := \Pr \left[h_{SRD}^2 \leq \frac{C^{-1}(R_0)}{2\gamma_0} \right] \\ = \int_0^{\sqrt{\frac{C^{-1}(R_0)}{2\gamma_0}}} f_{h_{SRD}}(h)dh = F_{h_{SRD}} \left(\sqrt{\frac{C^{-1}(R_0)}{2\gamma_0}} \right). \quad (13)$$

Finally, the asymptotic outage capacity corresponding to the PS-RC cooperative protocol at the destination node is given by $P_{\text{out}}(R_0) \doteq P_{\text{out}}^{\text{SD}}(R_0) + P_{\text{out}}^{\text{SRD}}(R_0)$, and it can be seen at the bottom of this page in Eq. (14). Clearly, the asymptotic outage capacity expression is dominated by $b_{\min} = \min(2b_{SD} + b_{SR}, b_{SD} + 2b_{SR}, 2b_{RD})$. More importantly, the outage diversity order gain O_d relative to the non-cooperative link S-D is given by $O_d = b_{\min}/b_{SD}$.

It should be noted that unlike other reported papers on performance analysis that usually obtain the sophisticated Meijer G-function, this function does not offer a significant insight on the achievable performance. For that reason, this asymptotic expression is simpler and easier to compute the performance of cooperative FSO systems with nonzero boresight pointing errors due to the fact that a typical outage performance target is set to 10^{-6} for most practical FSO links.

V. RESULTS AND DISCUSSION

Now, we present some numerical results in Figs. 1 and 2 for different relay locations, which is represented by (x_R, y_R) in a Cartesian coordinate system. Here, the parameters α_m and β_m are calculated from [10] and a value of $\lambda = 1550$ nm is assumed. Different weather conditions are adopted such as haze visibility of 4 km for moderate turbulence, and clear visibility of 16 km for strong turbulence.

In Fig. 1(a), the outage diversity order O_d is depicted as a function of the horizontal displacement of the relay node x_R when different relay locations are considered. Note that the condition $\varphi^2 > \beta$ is satisfied in this figure for each FSO link and, hence, the achievable outage diversity does not depend on nonzero boresight pointing errors, thus allowing a better performance. In comparison with similar cooperative protocols [2], [3], and other reported works [4]–[6], [12], it is concluded that the available outage diversity order is always greater than 1.75, achieving a value of ≈ 4 for specific relay locations by considering only one relay and without investing in extra hardware at the expense of an information rate-reduction. By other way, it has also been demonstrated that nonzero boresight displacements present a strong impact on outage diversity, limiting the achievable rate. More interestingly, a notable improvement in outage capacity performance is observed when comparing to the two transmitters case, i.e. $O_d = 2$, presenting

$$P_{\text{out}}(R_0) \doteq \frac{a_{SD}^2 a_{SR} \Gamma(b_{SD}) \Gamma(b_{SD} + b_{SR})}{(2b_{SD} + b_{SR}) b_{SR} \Gamma(2b_{SD} + b_{SR})} \left(\frac{C^{-1}(R_0)}{2\gamma_0} \right)^{\frac{2b_{SD} + b_{SR}}{2}} + \frac{a_{RD}^2 \gamma_0 (b_{RD})^2}{2b_{RD} \Gamma(2b_{RD})} \left(\frac{C^{-1}(R_0)}{2\gamma_0} \right)^{b_{RD}} \\ + \frac{a_{SD} a_{SR}^2 \Gamma(b_{SR}) \Gamma(b_{SD} + b_{SR})}{b_{SD} (b_{SD} + 2b_{SR}) \Gamma(b_{SD} + 2b_{SR})} \left(\frac{C^{-1}(R_0)}{2\gamma_0} \right)^{\frac{b_{SD} + 2b_{SR}}{2}}. \quad (14)$$

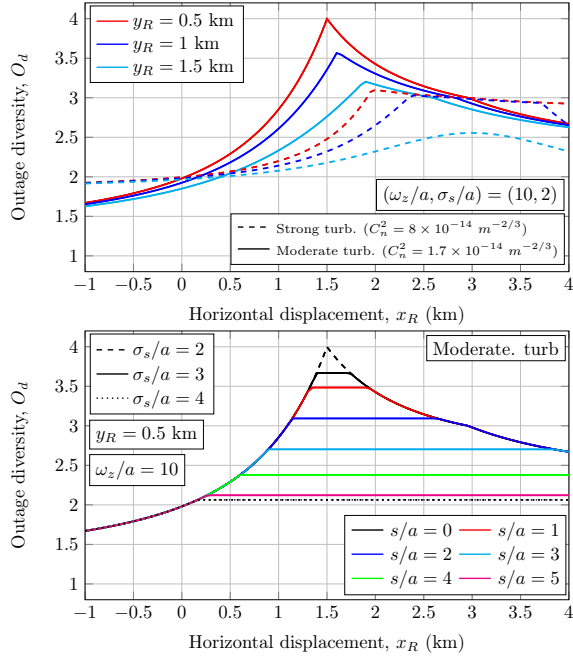


Fig. 1: Outage diversity order under different nonzero boresight pointing errors over moderate and strong turbulence.

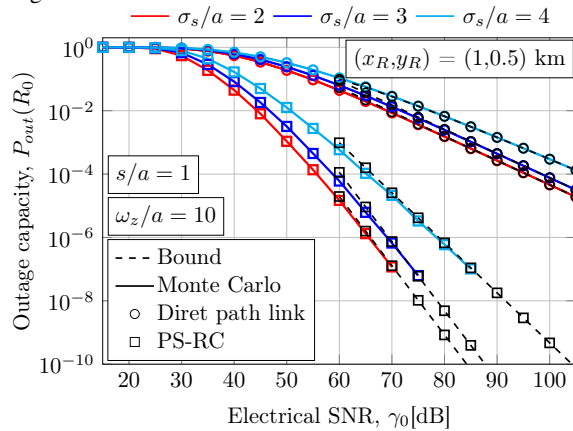


Fig. 2: Outage capacity for a code rate $R_0 = 0.5$ bits/channel use.

an outage diversity order gain quite superior to 2 when larger amounts of misalignment are not assumed. It can also be observed that the optimal relay placement depends on turbulence conditions and nonzero boresight pointing errors according to the outage diversity. In Fig. 1(b), the effect of nonzero boresight is studied from an outage diversity point of view. These results demonstrate that nonzero boresight errors present a larger performance degradation at high SNR.

In Fig. 2, the results corresponding to this asymptotic outage capacity analysis for moderate turbulence and different severity of pointing errors such as $\sigma_s/a = \{2, 3, 4\}$ are illustrated for a code rate $R_0 = 0.5$ (bits/channel use). Monte Carlo simulations are included as a reference, confirming the accuracy and usefulness of the derived results. Additionally, we consider the performance analysis for the direct path link to establish the baseline performance [3, eqn. (7)].

VI. CONCLUSION

As a concluding remark, we can say that a significant improvement in outage capacity is achieved by fully exploiting

the potential time-diversity available in S-D, S-R and R-D turbulent channel. In other words, combining rate-adaptive with cooperative communication results in significant improvements in terms of the achievable outage capacity when the effect of nonzero boresight errors takes place. Moreover, the impact of scintillation and nonzero boresight pointing errors on performance is mitigated by employing selective relaying. Nevertheless, optimization methods for the beam width can also be applied to reduce such damaging effect. Finally, it has been demonstrated that rate-adaptive relaying is an original method that can be applied to practical FSO networks to extend the coverage area and achieve spatial diversity.

From results proposed in this paper, we think that to do research on cooperative FSO systems based on rate-adaptive relaying that use the CSI information of all FSO links in the presence of nonzero boresight pointing errors may be an interesting topic for future work.

REFERENCES

- [1] M. A. Khalighi and M. Uysal, "Survey on free space optical communication: A communication theory perspective," *Communications Surveys & Tutorials, IEEE*, vol. 16, no. 4, pp. 2231–2258, 2014.
- [2] R. Boluda-Ruiz, A. Garcia-Zambrana, C. Castillo-Vazquez, and B. Castillo-Vazquez, "Adaptive selective relaying in cooperative free-space optical systems over atmospheric turbulence and misalignment fading channels," *Opt. Express*, vol. 22, no. 13, pp. 16629–16644, 2014.
- [3] C. Castillo-Vazquez, R. Boluda-Ruiz, B. Castillo-Vazquez, and A. Garcia-Zambrana, "Outage performance of DF relay-assisted FSO communications using time-diversity," *Photon. Technol. Lett., IEEE*, vol. 27, no. 11, pp. 1149–1152, 2015.
- [4] J.-Y. Wang, J.-B. Wang, M. Chen, Y. Tang, and Y. Zhang, "Outage analysis for relay-aided free-space optical communications over turbulence channels with nonzero boresight pointing errors," *Photonics Journal, IEEE*, vol. 6, no. 4, pp. 1–15, 2014.
- [5] P. Wang, T. Cao, L. Guo, X. Liu, H. Fu, R. Wang, and Y. Yang, "Multihop FSO over exponentiated Weibull fading channels with nonzero boresight pointing errors," *IEEE Photon. Technol. Lett.*, vol. 28, pp. 1747–1750, 2016.
- [6] G. Varotsos, H. Nistazakis, A. Stassinakis, G. Tombras, V. Christofilakis, and C. K. Volos, "Outage performance of mixed, parallel and serial DF relayed FSO links over weak turbulence channels with nonzero boresight pointing errors," in *Modern Circuits and Systems Technologies (MOCAST), IEEE 7th International Conference on*, pp. 1–4, 2018.
- [7] H. E. Nistazakis, E. A. Karagianni, A. D. Tsigopoulos, M. E. Fafalios, and G. S. Tombras, "Average capacity of optical wireless communication systems over atmospheric turbulence channels," *J. of Lightwave Technol., IEEE/OSA*, vol. 27, no. 8, pp. 974–979, 2009.
- [8] F. Xu, A. Khalighi, P. Caussé, and S. Bourennane, "Channel coding and time-diversity for optical wireless links," *Opt. Express*, vol. 17, no. 2, pp. 872–887, 2009.
- [9] C. H. Kwok, R. V. Penty, and I. H. White, "Link reliability improvement for optical wireless communication systems with temporal-domain diversity reception," *Photon. Technol. Lett., IEEE*, vol. 20, no. 9, pp. 700–702, 2008.
- [10] L. Andrews, R. Phillips, and C. Hopon, *Laser beam scintillation with applications*. SPIE press, vol. 99, 2001.
- [11] R. Boluda-Ruiz, A. Garcia-Zambrana, C. Castillo-Vázquez, and B. Castillo-Vázquez, "Novel approximation of misalignment fading modeled by Beckmann distribution on free-space optical links," *Opt. Express*, vol. 24, no. 20, pp. 22 635–22 649, Oct 2016.
- [12] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," *J. Lightwave Technol., IEEE/OSA*, vol. 25, no. 7, pp. 1702–1710, July 2007.
- [13] N. Wang and J. Cheng, "Moment-based estimation for the shape parameters of the gamma-gamma atmospheric turbulence model," *Opt. Express*, vol. 18, no. 12, pp. 12 824–12 831, 2010.
- [14] A. Goldsmith, *Wireless communications*. Cambridge university press, 2005.
- [15] A. Papoulis, "Probability, Random Variables, and Stochastic Processes," 3rd ed. Tata-Mcgraw-Hill, 1991.