Selling Price for a Retailer Using a Comprehensive Methodology with Risk Management

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***Abstract*— Retail electricity markets are critical in assuring that final customers receive the full benefits of competition in wholesale electricity markets. *Retailers* act as a service interface between the *energy* market and customers. In this paper, a comprehensive methodology to solve the problem that retailers face in an electricity energy market, including risk management, is proposed. The methodology consist of two main stages, at the first stage, consumers are characterized and classified using clustering techniques, and at the second stage an optimization problem for each group of consumers is solved to get the optimal energy offer. The optimization problem considers uncertainty of price from the wholesale market through a scenario tree, and the Conditional Value at Risk (CVaR) as risk measure. The problem results into a quadratic optimization problem, and it is solved using the GAMS software.**

***Index Terms*-- Cluster, Hopfield-K-Means, optimal selling price, retailer, risk, scenario tree, tariff.**

I. Introduction

IN the last few years important changes have affected the Electricity Market [1]. One of the most important consequences has been the change to a competitive market [2], in which, consumers have several options to get the electricity, like for example buying it from the wholesale market or from a retailer [2].

In this new framework, retailers have to compete for clients (customers) and, considering that minimum quality of the service is fixed by the law, the main parameter to compete is the offered tariff. Therefore, retailers have to face the problem to offer the optimal tariff to get the maximum profit, and, at the same time, to manage the risk associated with different parameters in the trading, like the energy price from the wholesale market.

Retailers are interested in offering tariffs attractive to customers, therefore, in order to carry out an optimal strategy, they need to know the customers features as accurate as possible [3], in particular the voltage level they require and their load profile. Because of the set of clients is heterogeneous, it is usual to cluster them in groups as homogeneous as possible, typical (*DT*) and atypical (*DA*) clients [4], using clustering techniques [3] and quality indices [5] to evaluate the whole set of clusters. This is the method followed here to get optimal clusterings according with the objective to compute optimal tariffs.

Regarding the uncertainty that retailers have to cope with, the two main parameters are, the total amount of energy demanded by their clients, and the price they have to pay for that energy in the wholesale market. The profit of each retailer depends strongly on the fit between the previous factors, with uncertainty, and the tariffs it offers to the clients. Different approaches for an optimal fit have been studied, the typical approaches include the flat tariff (flat rate) [6-8] for the whole set of consumers, and different tariffs for each group of consumers [9, 10].

The main contribution of this paper is a comprehensive methodology for retailers to compute their optimal selling price with risk management. The methodology involves two main stages, at the first stage consumers are clustered, the optimal number of clusters is determined using quality indices, and at the second stage, the optimal tariff to offer to each group of clients (cluster) is computed through an optimization problem, that includes the uncertainty in price using a scenario tree, and the Conditional Value at Risk (*CVaR*) as risk measure. A flow diagram of the proposed methodology is depicted in Fig. 1.

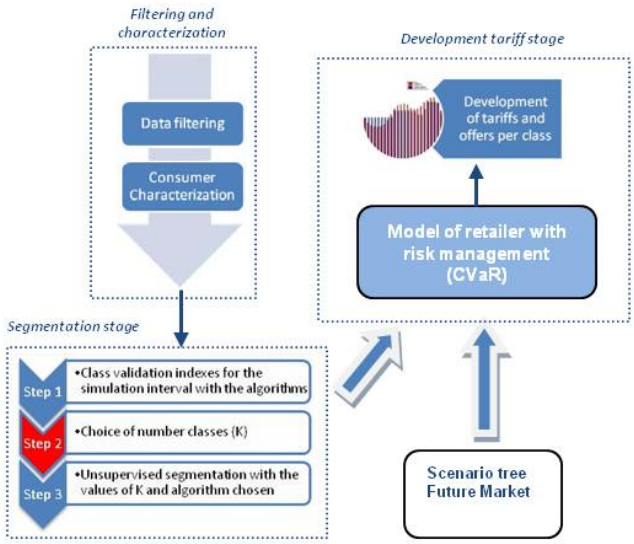

Fig. 1 Comprehensive methodology in two stages to determine tariffs for a retailer

This paper is organized as follows. In Section I, the comprehensive methodology to compute the optimal tariffs is described. The clustering stage is developed in Section II, the optimization problem to compute the tariffs, including the generation of the scenario tree, is described in Section III. An application to a study case is presented in Section IV. And conclusions are given in Section V.

II. Clustering of Electricity Customers

In this section, the clustering stage, first stage, of the proposed method is described. The objective of this stage is to classify the electricity customers in an optimal number of clusters. Retailers will offer a tariff to each cluster of customers.

The data used in the clustering algorithms are previously preprocessed as follows:

1.- The raw data consist of the hourly demand for a set of consumers during a whole year.

2.- From the raw data, a set *Q*, of load patterns is computed. Each element in the set *Q* is an average load pattern (*CCM*) for a particular consumer *i*, and a particular configuration, for instance, a working day in a particular season, or the daily average over the whole year, or the daily average over one week, depending on the type of tariff (flat, seasonal, weekly) the retailer is interested to optimize.

3.- Filtering on the set *Q*, using the Kurtosis coefficient and the Mahalanobis' norm [11], with the aim of differentiate a subset of typical data (*DT*), and a subset of atypical data (*DA*).

4.- Identification of the main components for each element in the sets *DA* and *DT* (the elements in this sets are vectors). The aim is to consider only the relevant information, and to reduce the length of the corresponding vectors, getting the sets *DA** and *DT**. For this task many techniques have been described in the literature, the harmonic analysis of level three is used here [12].

5.- Normalization of the vector values in *DT** and *DA** to fit them in the interval [0, 1]. The resulting sets are X_{DT*} and X_{DA*}.

After the preprocessing, the resulting sets X_{DT*} and X_{DA*} are used as input for the clustering algorithm. The algorithm consists of two stages, with the segmentation at the first stage, and a testing through a validation index at the second stage, as it is described below. The segmentation is performed for different values of the number of clusters *K* in a certain range, then the validation index is computed for each result to select the optimal number of clusters in the tested range.

A. First stage: Segmentation

In the segmentation step, the data sets X_{DT*} and X_{DA*} are partitioned into a previously fixed number of clusters, *K*, with the aim to use these clusters in the optimization problem for the retailer. Many segmentation algorithms have been proposed in the literature [3, 4, 12], but the particular algorithm that is used here, is the so called Hopfield-K-Means (*HK*) algorithm, [12]. The *HK* algorithm is an hybrid algorithm with two main stages, at the first stage data are processed using a Hopfield neural network [13], and after that, using the K-Means algorithm [14] at the second stage. A flow diagram of the *HK* algorithm is depicted in Fig. 2.

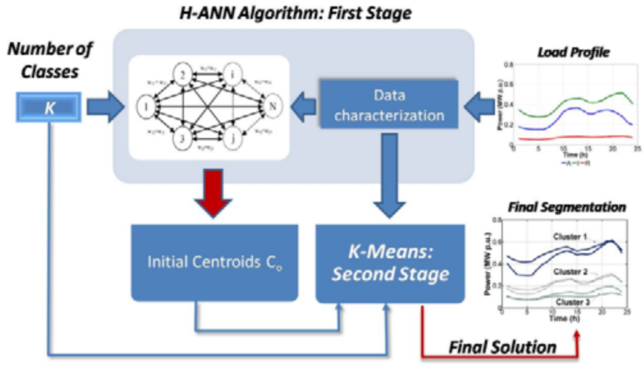

Fig. 2 Flow diagram of the Hopfield-K-Means algorithm

For the sake of the simplicity, let be *X* representing X_{DT*} or X_{DA*}. The centroid of the set *X*, named $C^{(X)}$, is computed according to (1).

$$
\begin{aligned}
&X(i,j) = x_j^i \qquad i = 1,\cdots,|X| \quad ; \quad j = 1,\cdots,v \\
&X = \left\{ x^{(i)}, i = 1,\cdots,|X| \right\} \\
&C^{(X)} = \frac{1}{|X|} \sum_{x \in X} x^{(i)}
\end{aligned}
\tag{1}
$$

where $x^{(i)}$ stands for the preprocessed data of client *i* in the set *X*, x_j^i is the component *j* of the vector $x^{(i)}$, $|Q|$ is the cardinality of the set *Q*, ($|Q| = |X|$), and v is the length of each vector $x^{(i)}$.

The set of clusters is a partition of the set *X*, a cluster *k* in the set *X* is named $X^{(k)}$. The results at each stage of the segmentation algorithm are:

1.- First stage: the Hopfield neural network is applied to the set *X* to get the partition $X_0^{(k)}$, and the centroids, $C_0^{(k)}$, of the clusters in this partition are computed.

2.- Second stage: the K-Means algorithm is applied to the partition $X_0^{(k)}$ to get an improved partition with the same number of clusters, $X_f^{(k)}$, with centroids $C_f^{(k)}$, and this partition is the result of the *HK* algorithm.

B. Second stage: Validation index

The aim of this stage is to get the optimal number of cluster to be used in the segmentation stage, in which the number of clusters was a exogenous parameter, results for the number of clusters in a range are computed. The optimality, in this case, is defined according with the retailers' objective of computing optimal selling prices for each group of consumers. The number of clusters in the tested range that maximizes the value of the validation index is chosen as optimal.

The criteria adopted here to make the clusters are to maximize the compactness of each cluster and the distance among them. From the broad range of validation indices found in the literature [3-5,12], the Calinski index (*CH*) [15] has been chosen because it matches exactly the adopted criteria.

The Calinski index relates the dispersion among clusters, S_B and the dispersion inside the clusters, S_W, as is stated in (2).

$$CH = \frac{S_B}{S_W} \cdot \frac{|Q| - K}{K - 1} \tag{2}$$

The terms S_B and S_W stands for:

a) S_B is the average dispersion among clusters

$$S_B = \sum_{k=1}^{K} \left| X^{(k)} \right| \cdot (C^{(k)} - C^{(X)}) \cdot (C^{(k)} - C^{(X)})^t \tag{3}$$

b) S_W is the average dispersion inside the clusters.

$$S_W = \sum_{k=1}^{K} \sum_{i=1}^{\left| X^{(k)} \right|} (x^{(i)} - C^{(k)}) \cdot (x^{(i)} - C^{(k)})^t \tag{4}$$

III. Optimization problem for the selling price

In this section the optimization problem for computing the optimal selling price to offer to each cluster of consumers is described. A simplified flow diagram is depicted in Fig. 1. The key points of the optimization problem are:

1.- The objective is to maximize the net profit taking into account the associated risk.

2.- The risk comes from the uncertainty in the buying price from the wholesale market. This parameter is modelled through a scenario tree.

3.- The Conditional Value at Risk (*CVaR*) [16] is used as risk measure.

4.- Information about customers behaviour is considered through the clusters (Section II).

5.- Selling price and consumption are related by a linear inverse demand function [17].

These features of the optimization problem are described in detail in what follows.

A. Inverse demand function

The inverse demand function has as objective to deal with the uncertainty in the amount of energy demanded. It defines a explicit relation between demand and selling price. The model of linear inverse demand function used here, it is that defined in [18], where the line is defined by a point and the slope. An illustrative graphic of this function is showed in Fig. 3, and the mathematical expression is (5).

$$E_{ij} = E_{0ij} \left(1 + \frac{\beta_{ij} (\rho_{ij} - \rho_{0ij})}{\rho_{0ij}} \right) \tag{5}$$

where E_{ij} is the energy demanded of customer *i* at period *j*, ρ_{ij} is the selling price for customer *i* at period *j*, β_{ij} is the elasticity of demand (slope of the line), and $\left(E_{0ij}, \rho_{0ij} \right)$ is a point on the line that corresponds with a reference configuration, that it is assumed to be known.

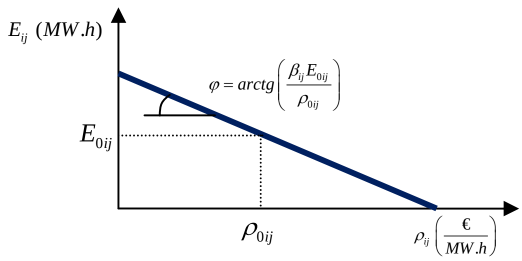

Fig. 3 Representation of the inverse demand function for a consumer *i* in a period *j*

The reference price, ρ_{0ij} is assumed to be a bit greater than the associated cost, as stated in (6).

$$\rho_{0ij} = Ko_{ij} \left(\alpha_{ij} + \gamma_{ij} \right) \tag{6}$$

where subscript i stands for a consumer, subscript *j* stands for a period, Ko_{ij} is a constant scalar greater than 1 [10], α_{ij} is the energy acquisition cost in the wholesale market, quantity corresponding to client *i* in period *j*, and γ_{ij} is the cost to access to the network [2].

B. Scenario tree for prices

The retailers have to make offers to the clients, with contracts that usually include several months. And the buying price for the daily wholesale market, over the interval, is unknown at the moment of signing the contract. That information is of vital importance for computing the optimal selling price, a common approach to manage this uncertainty consist of using forecasting models [6, 8] or scenario trees [19].

The scenario tree approach has been adopted here. The scenarios have been computed using historical data from the daily wholesale market. Each scenario contains the price for all the periods in the considered interval of contract.

The algorithm for computing the scenarios is an adaptation of the previously defined algorithm for clustering clients. The algorithm involves two main stages:

1.- At the first stage, the segmentation algorithm *HK* and the Calinski validation index (*CH*), defined in Section II, are applied to the historical data to get a first partition of the price profiles, with a optimal number of subsets according to the Calinski index.

2.- At the second stage, the subsets in the previous partitionare partitioned again, but this time the number of subsets in each new partition is previously fixed (the same value for all the subsets), and from each one of the resulting subsets a scenario is computed as the average of the price profiles in the subset.

For the partition at the second stage, a histogram is represented for each subset from the first stage. The values represented in the histogram correspond to the norm (7) computed over each price profile, the price profiles have been previously processed using harmonic analysis of level three (*FFT3*) [12]. Similar price profiles has a similar value of norm (7), so the number of columns in the histogram is the number of subsets at this second stage. A flow diagram of the whole algorithm is depicted in Fig. 4.

$$d(p^{(i)}) = \sqrt{\sum_{j\in J}\sum_{l\in L}\frac{(p_j^{(i)} - p_l^{(i)})^2}{2}} \qquad J = L = \{1,2,5,8\} \qquad (7)$$

where $p_j^{(i)}$ is the component *j* of the price profile *i*, that results from the application of the harmonic analysis of level three (*FFT3*) to the initial data.

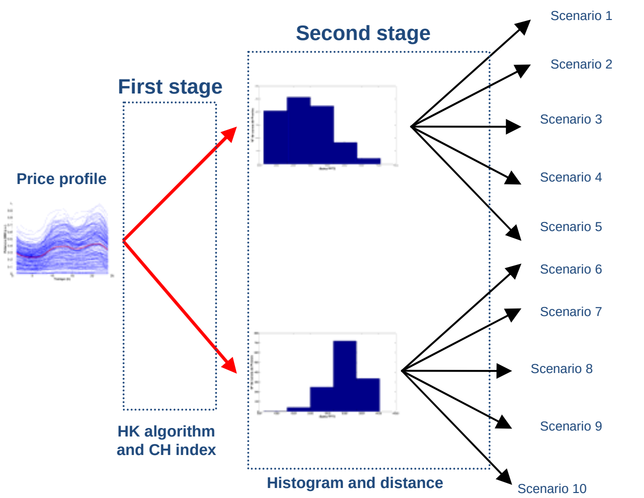

Fig. 4 Flow diagram for the scenario tree creation

To validate the resulting scenarios two type of tests have been performed:

a) Average and variance of prices on the set of scenarios is compared with average and variance of prices on the set of historical data, and they must be very close.

b) A different number of partitions at the second stage that leads to trees with different number of branches, are tested in the optimization problem. The number of branches were progressively increased until the changes in the result from the optimization problem were small enough.

C. *Mathematical formulation*

In this section, the optimization-based model is described. It includes the objective function, risk management constraints and operational and market constraints.

C.1 *Objective function*

In this article, it is assumed that the retailer can buy energy either in the daily or future energy markets or both. Moreover, the retailer also manages the consumers feed-in tariffs [2].

Then, retailers incomes, *I*, can be written as

$$I = N\sum_{w=1}^{|\Omega|}\pi_w \sum_{i,k=1}^{K}\sum_{j=1}^{p} n_k E_{ijw}\rho_{ij} \qquad (8)$$

where

$$E_{ijw} = E_{0ij}\left(1 + \frac{\beta_{ij}\left(\rho_{ij} - \rho_{0ijw}\right)}{\rho_{0ijw}}\right) \qquad (9)$$

and

$$\rho_{0ijw} = Ko_{ij}\left(\frac{\alpha_{ijw}^{SPOT} + \alpha_{ij}^{FORWARD}}{2} + \gamma_{ij}\right) \qquad (10)$$

The notation used for equations (8-10) is the following: *N* is the number of period in the interval of offer, for instance, one year; n_k is the consumer number of group *k*; *K* is the group numbers in the segmentation phase, *p* is the number of periods of the tariff offer i.e. for a daily tariff p=24. π_w is the probability of scenario *w*; Ω is scenario set and $|\Omega|$ is the cardinal of Ω. E_{ijw} is the energy demand (MW.h) of customer *i* at period *j* and scenario *w*, ρ_{ij} is the energy selling price (€/MW.h) for customer *i* at period *j*; ρ_{oijw} is the reatailer's initial offer for customer *i* at period *j* at scenario *w;* α_{ijw}^{SPOT} is the buying price (€/MW.h) in the daily energy market for customer *i* at period *j* and scenario *w*; $\alpha_{ij}^{FORWARD}$ is the buying price (€/MW.h) for the future energy market for customer *i* at period *j*; γ_{ij} is the energy price (€/MW.h) of the feed-in tariff €/MW.h for customer *i* at period *j*.

The retailer's expenses, *E*, are:

$$E = N\sum_{w=1}^{|\Omega|}\pi_w \sum_{i,k=1}^{K}\sum_{j=1}^{p} n_k\left(E_{ijw}\gamma_{ij} + E_{ijw}^{SPOT}\alpha_{ijw}^{SPOT} + E_{ij}^{FORWARD}\alpha_{ij}^{FORWARD}\right) + \sum_{i=1}^{n}\sum_{j=1}^{p} P_{ij}\sigma_{ij} \qquad (11)$$

where E_{ijw}^{SPOT} is the daily market energy demand (MW.h) of customer *i* at period *j* and scenario *w*; $E_{ij}^{FORWARD}$ is the forward market energy demand MW.h of customer *i* at period *j*; P_{ij} is the contracted power MW of customer *i* at period *j* in the feed-in tariff and σ_{ij} is the price (€/MW) of the feed-in tariff of customer *i* at period *j*.

Then, the retailer's benefits for a given scenario *w*, $B_p(w)$, can be written as:

$$\begin{aligned} B_p(w) = N\sum_{i,k=1}^{K}\sum_{j=1}^{p} n_k \Big[E_{ijw}\left(\rho_{ij}-\gamma_{ij}\right) - E_{ijw}^{SPOT}\alpha_{ijw}^{SPOT} \\ - E_{ij}^{FORWARD}\alpha_{ij}^{FORWARD} \Big] - \sum_{i=1}^{n}\sum_{j=1}^{p} P_{ij}\sigma_{ij} \end{aligned} \tag{12}$$

C.2 Risk management

The *CVaR* is used in the optimization problem for risk management. There is a value of the profit for each scenario, therefore a profit distribution, and the *CVaR* is computed on that distribution. The *CVaR* is incorporated to the optimization problem according to [16], including the additional term (13) in the objective function and two additional constraints (17). The resulting objective function in the optimization problem has the expression (14).

$$CVaR = \varsigma - \frac{1}{1-\theta}\sum_{w=1}^{|\Omega|}\pi_w Q_w \tag{13}$$

$$B_T(\rho) = \lambda\sum_{w=1}^{|\Omega|}\pi_w B_p(w) + (1-\lambda)CVaR \tag{14}$$

where θ and Q_w are auxiliary variables, θ is the level of confidence, θ in [0,1) and represents the accumulated probability of the scenarios with smaller profits, and λ is a weighting factor that represents the risk aversion of the retailer, λ in [0,1]. A value of $\lambda = 0$ corresponds to a retailer with a maximum risk aversion, and a value of $\lambda = 1$ a retailer that does not take into account the risk.

C.3 Model constrainst

In the proposed model, the following constraints are considered. First, the selling price is bounded by the following equation:

$$\rho_{ij} \geq Km_{ij}\sum_{w=1}^{|\Omega|}\pi_w\left(\frac{\alpha_{ijw}^{SPOT}+\alpha_{ij}^{FORWARD}}{2}+\gamma_{ij}\right) \tag{15}$$

where Km_{ij} is a constant to control the lower bound. Similarly, and following [10], the selling price is also bounded by:

$$\begin{aligned} &\frac{E_{ijw}\rho_{ij}}{E_{iw}^{TOTAL}} \leq Ks_{ij}\left(\frac{\alpha_{ijw}^{SPOT}+\alpha_{ij}^{FORWARD}}{2}+\gamma_{ij}\right) \\ &i = 1,2,\ldots,K \\ &j = 1,2,\ldots,p \\ &w = 1,2,\ldots,\Omega \end{aligned} \tag{16}$$

where Ks_{ij} is a constant to control the upper bound and E_{iw}^{TOTAL} is the total energy demand of customer i for period j at scenario *w*.

The values of Km_{ij} and Ks_{ij} are set according to the retailer commercial strategy.

The constraints associated with the risk management are:

$$\begin{aligned} &Q_w \geq \varsigma - B_p(w) \qquad \forall\, w\in\Omega \\ &Q_w \geq 0 \qquad\qquad\quad \forall\, w\in\Omega \end{aligned} \tag{17}$$

Finally, the energy balance equations for customers are:

$$E_{ijw} = E_{ijw}^{SPOT} + E_{ij}^{FORWARD} \tag{18}$$

$$E_{iw}^{TOTAL} = \sum_{j=1}^{p} E_{ijw} \tag{19}$$

C.4 Optimization problem

The formulation of the optimization problem is:

$$\begin{aligned} &\max \quad B_T(\rho) = \lambda\sum_{w=1}^{|\Omega|}\pi_w B_p(w) + (1-\lambda)CVaR \\ &\textit{Sujeto a}: \\ &\rho_{ij} \geq Km_{ij}\sum_{w=1}^{|\Omega|}\pi_w\left(\frac{\alpha_{ijw}^{SPOT}+\alpha_{ij}^{FORWARD}}{2}+\gamma_{ij}\right) \\ &\frac{E_{ijw}\rho_{ij}}{E_{iw}^{TOTAL}} \leq Ks_{ij}\left(\frac{\alpha_{ijw}^{SPOT}+\alpha_{ij}^{FORWARD}}{2}+\gamma_{ij}\right) \\ &E_{ijw} = E_{ijw}^{SPOT} + E_{ij}^{FORWARD} \\ &E_{iw}^{TOTAL} = \sum_{j=1}^{p} E_{ijw} \\ &Q_w \geq 0 \qquad w\in\Omega \\ &Q_w \geq \varsigma - B_p(w) \qquad w\in\Omega \end{aligned} \tag{20}$$

In this optimization problem it is calculated selling prices (ρ_{ij}) that maximizes the objective function. The expected benefit of the retailer (*Bc*) will be given by:

$$B_c = \sum_{w=1}^{|\Omega|}\pi_w B_p(w) \tag{21}$$

IV. Experimental Results

The case study used consisted of information regarding 230 medium voltage (20 kV) lines, of which 19 feed administrative customers (*A*), 175 feed industrial customers (*I*) and 36 feed residential customers (*R*). The information was provided by a distribution utility. The *CCM* of each customer has been obtained by the average daily load curves of the period provided, each having 24 values.

We apply the filtering procedure and characterized data described in Section III obtaining a typical data set (*DT*) with

219 customers and other customer-specific (*DA*) with 11 clients.

A. *Segmentation Stage Results.*

The *HK* algorithm is applied to the *DT** and *DA** data sets. The Calinski index is used to obtain optimal clusters. Theses results can be observed in Fig. 5 and 6.

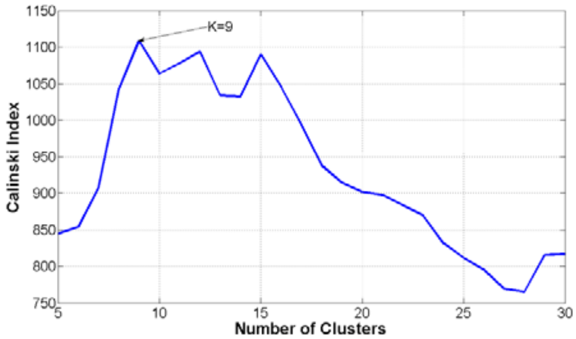

Fig. 5 Calinski index for the data set *DT**

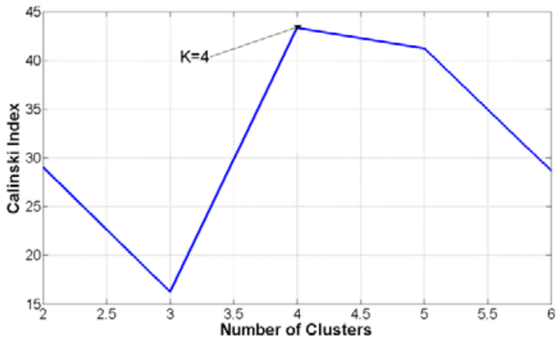

Fig. 6 Calinski index for the data set *DA**

The number of customers is 9 for typical customers (*DT*) and 4 for specific clients (*DA*). An independent tariff will be elaborated for each of these groups.

The average load curves of each customer group are shown in Fig. 7.

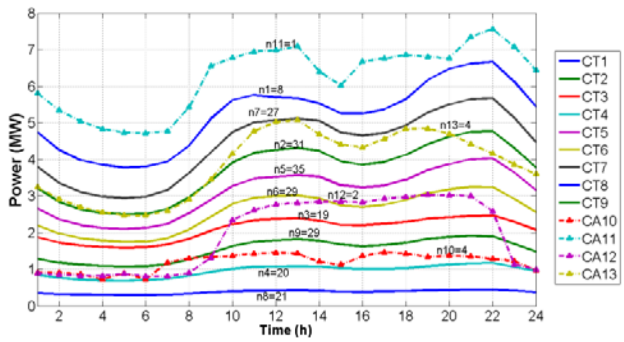

Fig. 7 Average load profiles for each cluster, and number of clients in each cluster

In Fig. 7, CT1 to CT9 customers corresponds to typical customers (*DT*) and CA10 to CA13 customers corresponds to the specific customers (*DA*).

B. *Results for the study case.*

At this stage we have considered the daily market prices provided by OMIE [21] for the years 2005 to 2010. The scenarios generated were 10 and they correspond to the price profile shown in Fig. 8.

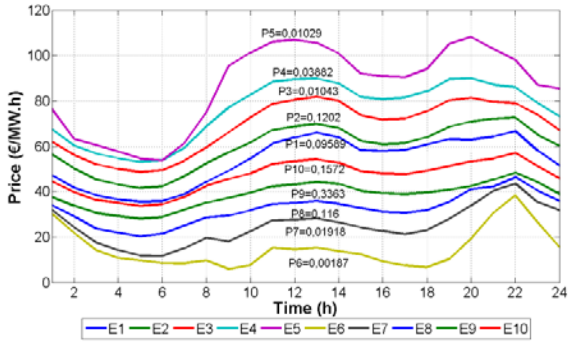

Fig. 8 Values and probabilities of the price profiles for the daily market

For the forward market, it is considered to bid for energy in the time periods from 10 to 12 h and from 18 to 20 h. The block of energy for each specific average customer is 0.5 MW. It is assumed a price of 50 € / MW.h.

For the feed-in tariffs, it has been considered the prices published in [20] assuming all customers belonging to type 6.2. It has also been considered daily and hourly offers for each typical and specific customer. Moreover, it has been considered N=365; $Ko_{ij} = 1{,}1$, $Km_{ij} = 0{,}1$, $\theta = 0{,}95$ and $Ks_{ij} = 2.4$. The results can be observed in Fig. 9 and 10.

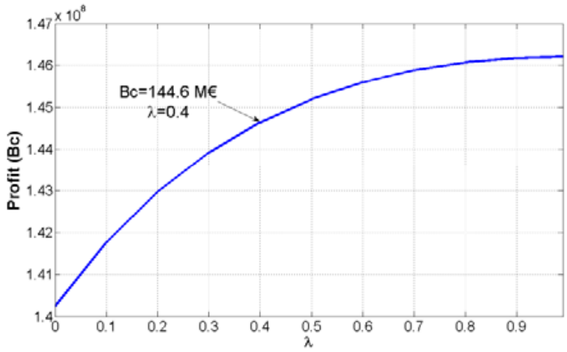

Fig. 9 Expected profit for Ks = 2.4 for the clients in the subset DT

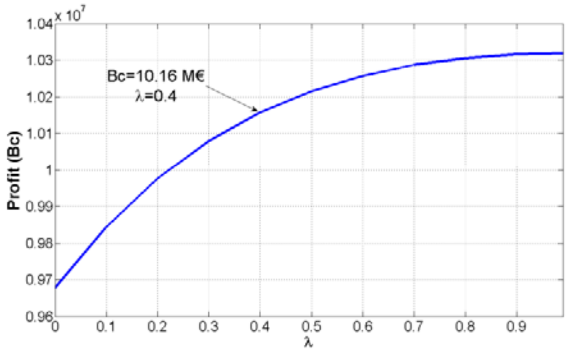

Fig. 10 Expected profit for Ks = 2.4 for the clients in the subset DA

These figures show the retailer's expected profits for each group of customers. It can be observed that the profit increases monotonically with the value of λ and from a value of $\lambda = 0.7$ the expected benefits remain practically constant, for both *DA* and *DT* customers. In this case, the simulations were obtained for $\lambda = 0.4$ for *DT* and *DA* customers. The retailer expected benefits for customers *DT* and *DA* are 144.6 M€ and 10.1 M€, respectively. These profits can be observed in Fig. 11.

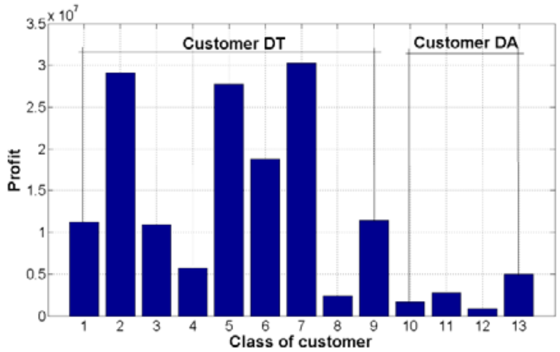

Fig. 11 Expected profit for each king of client.

The results of the tariffs offered for each client are shown in Fig. 12 and 13.

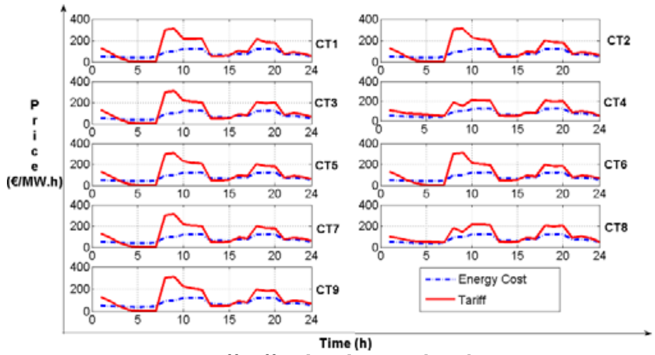

Fig. 12 Tariffs offered to clients in the subset DT

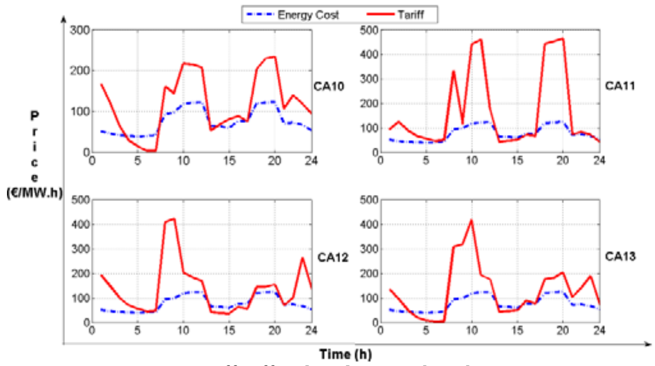

Fig. 13 Tariffs offered to clients in the subset DA

For *DT* customers, it can be observed that the offered tariffs are similar among them. There is a relation among load profiles depicted in Fig. 7. Moreover, for lower energy costs periods (0-8 h. y de 13-17 h.), offered prices are lower or equal to buying energy prices.

For *DA* customers, there is a significant deviation among offers; however, there is a clear trend of lower prices for valley periods and higher offer for peak periods. Note that CA11 is the customer with the highest energy demand and it gets the higher offers. The opposite happens with customer CA10.

V. Conclusions

A comprehensive methodology for the determination of retailer selling prices has been proposed. The offered selling prices for DT and DA customers follow load profiles showing higher prices for peak periods and lower prices in valley hours. These offers can be used as an indirect control demand tool. Another important feature of this methodology is that it can be obtained retailer selling prices for different risk adverse behavior and given selling price bounds.

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