Institutional flexibility, political alternation, and middle-of-the-road policies*

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Abstract

This paper presents a novel explanation for political alternation in democracies, rooted in the benefit for the median voter of keeping policy from drifting too far to either extreme. Central to this argument is the idea that policy change is gradual and that this gradualism depends on the institutional flexibility/rigidity of the country. Built on this idea, we propose a model of dynamic elections and show that institutional rigidities cause alternation. We also show that, though institutional rigidities prevent governments from implementing extreme policies, they incentivize parties to polarize as much as they can. However, more flexible institutions can foster moderation. Last, we analyze the resilience of equilibrium policies to players' impatience and discuss extensions of our model, including office-motivated parties, the cost of alternation, alternation every two terms, and asymmetric policies.

Keywords: Gradual policy implementation; endogenous status quo; political alternation; polarization; moderation; efficiency; robustness

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1 Introduction

Although casual observation shows that left-wing and right-wing political parties take turns in office, there is relatively little work on the determinants of political alternation.[1] Empirical studies suggest that the electorate's natural disaffection with the elected government, particularly when the economy is not doing well, is the reason for party fluctuation (e.g., Bernhard and Leblang (2008), Walter (2009), Chwieroth and Walter (2017)). The idea of disaffection is also central to Roemer (1995)'s theory, inspired by Hirschman (1982), who considers an electorate whose preferences are continuously shifting away from the policy implemented by the elected government. Corruption scandals, proven incompetence, and simply disillusion with implemented policies are arguments that gravitate around this theory.

Though we believe that disappointment, in a general sense, is an important variable that explains part of the observed variation in political power, we also think that alternation may be a mechanism for the voters to assure that implemented policies do not move too far from the median voter's preferred policy. In fact, by replacing the ruling party every certain number of elections, voters guarantee that the incumbent has not enough time to propose, pass, and implement major policy changes. In this sense, alternation may also be a mechanism to keep policies at moderate positions.

Central to this argument is the idea that policy implementation takes time. The argument is that a country's institutions, both formal and informal, may preclude elected governments from setting rapid adjustments in policies and force them to implement new policies progressively, that is, through a series of gradual changes. Interestingly, casual observation supports this argument. Think, for example, about the ambitious health care reform in the United States, known as Obamacare, a process that started on March 23, 2010, when Barack Obama signed the Affordable Care Act, which included a timeline of progressive reforms that, before interrupted by Donald Trump, were planned to unfold over a decade (obamacarefacts.com).[2] Another example can be found in the plastics strategy of the European Union (EU) and the recently approved Single-Use Plastics Directive, which includes measures to reduce the use of single-use plastic items in the EU, new collection targets for plastic bottles, and so forth, with a calendar of implementations from 2021 to 2025 and 2030 (*The Guardian*, March 27, 2019). Other examples are the Hartz employment reforms in Germany, a package of reforms that include the restructuring of the German Federal Employment Agency, stricter sanctions, and the introduction of a low-wage sector, which were enacted in four stages (Hartz I–IV) between 2003 and 2005; the G-20 Initiative on Rationalizing and Phasing out Inefficient Fossil Fuel Subsidies, an agreement reached at the 2009 Pittsburgh Summit, establishing a compromise "to phase out and rationalize over the medium term inefficient fossil fuel subsidies" (*Reuters*, September 25, 2009); and the reform of the Spanish education system, with a calendar of implementations

[1] Since World War II, alternation is a regularity in Western democracies. Countries such as Denmark, Ireland, and Norway count more than ten alternations in office, and Australia, Canada, France, the United Kingdom, and the United States count just fewer than ten.

[2] See https://obamacarefacts.com/health-care-reform-timeline/ for a detailed calendar of the reforms scheduled for each year, from 2010 to 2022.

from 2014 to 2017 (*La Vanguardia*, December 3, 2013).[3]

This paper builds on this idea to explain political alternation and to propose a relation between institutional flexibility and policy extremism. To the best of our knowledge, this is new in the literature. In fact, a standard assumption in the political economy literature is that, once a party wins the election, its pursued policy is immediately implemented. The literature on agenda control (Romer and Rosenthal (1978)), divided government (Alesina and Rosenthal (1995, 1996, 2000)), legislative bargaining (Austen-Smith and Banks (1988), Baron and Diermeier (2001)), and within-party conflict (Roemer (2001)) makes the more realistic assumption that the final policy is a compromise between two or more political actors, usually the executive and the legislature. In any case, all these papers consider that, once a final policy is determined, it is fully operative. In contrast to this literature, in this paper, we consider that the policy proposed by the elected government is gradually introduced into society. Interestingly, we show that this novel ingredient provides a new explanation for parties' alternation in power.

To formalize this idea, we propose a model of dynamic elections between two political parties and a representative median voter. Political parties are policy motivated and have policy preferences at the extremes of the ideological interval, whereas the representative median voter prefers a moderate policy. Elections run at discrete time. Every election, the median voter selects one of the two policy-motivated parties and the elected party proposes a policy to implement during the term. We refer to the announced or stated policy as the *pursued policy*. The key assumption of the model is that pursued policies are gradually introduced into society; that is, during the term, the policy transits continuously from the inherited status quo policy to the pursued policy. It has a direct implication: the utility that the parties and the voter receive from the implemented policy changes daily, since policies are in continuous transition.

We formalize this idea of gradual policy implementation by means of a parameter, namely, a *country's institutional flexibility*, which determines both the speed of implementation of a new policy in society and the extent to which a government can change an existing policy. Alternatively, this parameter may be interpreted as the degree of the country's "policy inertia."[4] Note that the value of this parameter, together with the status quo policy in a given term and the pursued policy for that term, determines whether the time between two elections is enough for the pursued policy to be fully operative at the end of the term or

[3]Other examples include the gradual elimination of China's one-child policy and the progressive appointment or renovation of justices to the Supreme Court and other judicial branches in countries such as the United States and Spain.

[4]One possible interpretation for the assumption of gradual policy implementation is that there are policy issues that a government can only affect progressively, since they do not allow for drastic and sudden changes. For instance, labor reforms. This was the case with the Hartz employment reforms in Germany, and it is currently the case in Spain, with discussion about the new labor reform. In the case of the Hartz employment reforms, the process went through four stages, with Hartz I and II (enacted January 1, 2003) creating "staff services agencies" and new types of employment, such as minijobs; Hartz III (which came into effect January 1, 2004) restructuring and reforming job centers; and Hartz IV (enacted January 1, 2005) reducing social subsidies for unemployment beneficiaries. Arguments such as the division of power between the executive and the legislative, social resistance to changes and reforms, and the lobbying activities of interest groups may help explain the gradual implementation of policies in this case.

not. In this sense, changing the institutional flexibility parameter allows us to affect the range of policies that can indeed be implemented, which affects the parties' equilibrium behavior. Note also that, with the gradual implementation of policies, there is a linkage between terms, since the voter's optimal choice in a given term depends on the status quo policy, which further depends on the previous status quo, the previous pursued policy, and the institutional flexibility parameter.

Our results show that, for any player's discount factor, there is an equilibrium in which parties propose partisan policies, that is, policies that correspond to the parties' lines, and the median voter votes for a different party each election. We refer to this equilibrium as the *partisan-alternating equilibrium*, since policies are continuously swinging back and forth from term to term, from one party's partisan policy to the other. In this equilibrium, a different political party wins office each term. Hence, our model produces parties' alternation in power. We next analyze whether the partisan-alternating equilibrium is efficient. We obtain that any symmetric alternating outcome in which policies do not suffer from such sharp swings Pareto-dominates the partisan-alternating equilibrium. Moreover, we show that the milder the swings in policy, the more efficient the alternation is. Accordingly, in the rest of the paper we study which more efficient alternating outcomes can be sustained as equilibria of the game. In particular, we focus our attention on symmetric and stationary alternating strategies.[5]

Within this class of strategies, we show that, when a country's institutions are very rigid, in which case the implementation of new policies takes a long time, in (the unique) equilibrium policies are as polarized as the speed of implementation allows, although rigidity bounds polarization. In this sense, our results suggest that institutional rigidities introduce a trade-off to the parties: they limit the capacity of governments to implement extreme policies, but also incentivize governments to push policies as far as legal, political, and social restrictions allow.[6] In contrast, when institutions are sufficiently flexible, there are other (more efficient) equilibria, among which it is the Pareto-superior one, in which parties propose the median voter's preferred policy. The reason is that a country's institutional flexibility determines a party's ability to retaliate, that is, the effectiveness of punishment. Roughly speaking, when institutions are rigid, the threat to revert to the extreme partisan-alternating equilibrium is not very effective, since institutional rigidities will limit the capacity of the future governing party to choose extreme policies itself. However, when institutions are flexible, such a threat is more effective. Note that the result when institutions are sufficiently flexible follows the (Nash reversion) folk theorem logic (Friedman (1971)),[7] which states that for a high enough discount factor, the partisan strategy can be used as a trigger to enforce more moderate behavior in equilibrium.

[5] In Section 6.4 we elaborate on the possibility of asymmetric strategy profiles and argue that, for any possible asymmetric equilibrium, there exists a symmetric one that is more robust with respect to the discount factor.

[6] In line with the second part of the argument, the literature on divided government finds that the greater the strength of the legislative power (hence the weaker the executive power or, in terms of our model, the more rigid institutions are), the greater the parties' polarization. See Alesina and Rosenthal (1995, 1996, 2000), and Ghosh (2002).

[7] In such a case, our dynamic game with endogenous status quo approaches an (infinitely) repeated game. This is so since with more flexibility the status quo policy becomes less and less relevant to determine the next period payoffs.

Last, we investigate which of the more efficient equilibria are more resilient to impatience. This is an interesting exercise that allows us to select among the multiplicity of equilibria that we obtain for the case in which institutions are sufficiently flexible. We refer to this kind of equilibrium as a *robust-interior equilibrium*, which can be interpreted as second best in terms of stability that Pareto-dominates the more stable but less efficient partisan-alternating equilibrium (which is an equilibrium for any player's discount factor). Indeed, as pointed out by Blonski et al. (2011), a standard criterion of equilibrium selection that has often been used in applications of repeated Prisoner's dilemma games is the minimum discount factor for which cooperation is sustainable in equilibrium (i.e., the critical discount factor, as in Bruttel (2009)). We rely on this criterion to select among the multiplicity of equilibria in our scenario.[8]

We obtain that a robust-interior equilibrium exists only when the country's institutions are neither too flexible nor too rigid. We also obtain that the equilibrium policies in the robust-interior equilibria stand midway between the median voter's preferred policy and the parties' preferred policies, that is, they are *middle-of-the-road policies*.

This paper has two central messages. The first message is that the gradual implementation of policies provides a new explanation for parties' alternation in holding power. This result speaks directly to the literature on alternation. Empirical research has extensively documented the benefits of alternation (Horowitz et al. (2009), Milanovic et al. (2010), Besley et al. (2016)), an idea that is also central to some theoretical papers. For example, Lagunoff (2001) proposes a dynamic game in which greater political turnover leads to stronger support of civil liberties, and Acemoglu et al. (2011) show that political cooperation increases with party alternation, facilitating the sustainability of better policies. In contrast to this idea, it has also been argued that alternation may lead to time-inconsistent and inefficient policies that undermine economic growth (Alesina (1987), Persson and Svensson (1989), Alesina and Tabellini (1990), Battaglini and Coate (2008)). None of these papers, however, explains alternation, but, rather, they take it as given and analyze its effects. In this sense, the papers by Kramer (1977), Wittman (1977), Bendor et al. (2006), Forand (2014), and Nunnari and Zápal (2017), who propose models in which alternation arises as an equilibrium outcome, are closer to our work. These papers, however, consider models in which the source for alternation is the fact that only the challenger can experiment with new platforms, whereas the incumbent is constrained to propose the previously implemented policy. We contribute to this literature by considering a new ingredient, namely, institutional rigidities that imply gradual policy implementation, which provides a new logic to explain alternation.

The second message of our work is that it is institutional rigidity rather than institutional flexibility that may drive policy polarization. This result speaks to the literature on institutions and their effects on policy outcomes. This debate has received much attention in the last decades (e.g., Cox (1990), Myerson (1993*a*,*b*), Alesina and Rosenthal (1995, 1996, 2000), Persson and Tabellini (2000, 2005), Besley and Case

[8] See also the recent work by Dal Bó and Fréchette (2018), who survey experimental papers on repeated Prisoner's dilemma games and find that the cooperation rate is increasing in the difference between the effective discount factor and the critical discount factor.

(2003), Besley (2006), and more recently Pettersson-Lidbom (2008), Besley et al. (2010)). Much of this debate has focused on the effects of constitutions (electoral rules, term limits, district size, separation of powers, etc.) on policies. Related to this, the literature on bureaucratic inefficiency explores the connection between bureaucracy and policies (Huber and McCarty (2004), Gratton et al. (2021)), explaining the overproduction of laws, which may result in institutional rigidities, as an equilibrium outcome. We contribute to this literature by showing that a country's institutional flexibility may have effects on policies and, moreover, on policy polarization.

Our work is also related to the endogenous status quo literature. The relevance of the status quo policy in models of party competition was first highlighted by Grofman (1985), who nevertheless considered a static game with an exogenous status quo. Nowadays, the literature on legislative bargaining is possibly the most active in the study of endogenous status quo policies and its effects. Baron (1996) and Zápal (2020) show that an endogenous status quo has a moderating effect on policies, whereas Baron et al. (2011) and Dziuda and Loeper (2016, 2018) find a polarizing effect.[9] The literature has also identified other channels through which the dynamic linkage between periods may arise. Callander and Hummel (2014) focus on the idea of preemptive experimentation and rely on an information channel, whereas Callander and Raiha (2017) study how policy choices shape voters' future preferences. We contribute to this literature by identifying a new channel for the linkage between periods: a country's institutions and, more precisely, its institutional rigidities.

Last, our paper is related to the literature on repeated elections games. The seminal work by Alesina (1988) first showed that the classical result of convergence to the median voter (Downs (1957), Wittman (1977, 1983)) is time inconsistent and therefore may not be sustained as an equilibrium if elections repeat in time, platforms are non-binding, and voters are rational and forward looking. We share with Alesina (1988) the result that repeated elections can sustain convergence if parties are patient enough, but not otherwise. Finally, our work is also related to that of Duggan (2000) and Van Weelden (2013), who consider models of repeated elections in which parties and voters do not communicate before the election and voters use a retrospective voting rule to decide their vote.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the one-election (or static) game, and in Section 4 we move to the infinite-horizon dynamic elections game, focusing on the study of the partisan-alternating equilibrium and its properties. In Section 5 we propose a class of alternating equilibria that Pareto-dominates the partisan-alternating equilibrium and analyze the robustness properties for different values of the institutional flexibility parameter. Section 6 discusses variations of the model, including i) the asymmetric distribution of voters and/or parties, ii) inefficiencies associated with a change in government, iii) alternation every two terms, iv) asymmetric (alternating) strategy profiles, v) office-motivated parties, and vi) heterogeneous discount factors. Finally,

[9] For a comprehensive survey, see Eraslan et al. (2020). Other papers with endogenous status quo and gradualism are those of Dixit et al. (2000), Bai and Lagunoff (2011), Bowen and Zahran (2012), Bowen et al. (2014), and Acemoglu et al. (2018).

Section 7 concludes the paper. All the proofs are presented in the Appendix.

2 The model

We consider a dynamic model of elections with endogenous status quo and complete information. In each election, two policy-motivated parties compete in a one-dimensional policy space $[-1, 1]$ for the vote of a representative median voter. The two political parties, L and R, are policy motivated and have preferred policies $\bar{x}^L = -1$ and $\bar{x}^R = 1$, respectively. The representative median voter, M, has preferred policy $\bar{x}^M = 0$.

Each $t \in \mathcal{N} \equiv \{1, 2, ...\}$ represents a term, which is the (usually four-year) period between two elections. At the beginning of each term t is an election in which the median voter selects the party to govern during the term. We denote by $v_t \in \{L, R\}$ the choice of the median voter in term t. The elected government in term t, v_t, inherits an *(endogenous) status quo* policy x_{t-1} from the previous term and announces the *pursued policy* p_t it would seek to implement during the present term t.[10] For simplicity, we assume $p_t \in [-1, 0]$ if $v_t = L$, and $p_t \in [0, 1]$ if $v_t = R$; that is, parties cannot propose pursued policies too different from their party's lines.[11]

Our key departure from the literature is that policies vary continuously in time; that is, the implemented policy during term t transitions continuously from x_{t-1} to p_t. The argument is that societies and institutions cannot change drastically from one day to the next. Hence, even if an incoming government would like its pursued policy to be fully operative immediately after the election, social and institutional variables may impede it and force the government to introduce new policies through a gradual process. Let $r > 0$ be the *institutional flexibility/rigidity* of a country. Then, for a given r, x_{t-1}, and p_t, the policy implemented at time $\tau \in [0, 1]$ of term t, where $\tau = 0, 1$ represents the beginning and end of the term, is

$$\varkappa_\tau (x_{t-1}, p_t) = \begin{cases} \min \{p_t, x_{t-1} + r\tau\} & \text{if } p_t \geq x_{t-1}, \\ \max \{p_t, x_{t-1} - r\tau\} & \text{if } p_t < x_{t-1}. \end{cases} \tag{1}$$

We draw attention to two ideas. First, parameter r sets the speed at which a new policy can travel from the inherited status quo x_{t-1}. Thus, the lower r is, the slower the pace of implementation of a new

[10] The model that we consider does not allow parties and voters to communicate before the election. This structure is reminiscent of the work of Duggan (2000) and Van Weelden (2013) and is used for simplicity. In fact, note that as long as platforms are non-binding, this is analogous to considering a more complex model in which political parties announce their platforms before the election.

[11] As will become clear later in the text, this assumption restricts the set of pursued policies of a party, but not the set of policies that the party may implement during a term. To see this, note that, since policies transit continuously from the status quo to the pursued policy, it may well occur that the governing party implements over a period of time a policy that does not belong to its preferred ideological spectrum. In this sense, this assumption is not as restrictive in our case as it may be in previous papers making a similar assumption (e.g., Alesina and Rosenthal (1989), James M. Snyder and Ting (2002)). Last, note also that, in this paper, we consider ideological political parties. Under this scenario, the assumption that parties can only propose policies that belong to their ideological spectrums seems very natural. For further elaboration of this assumption, see the discussion after Proposition 2.

policy, and the higher r is, the quicker the pace. In this sense, low values of r describe countries with rigid institutions (alternatively, countries with strong policy inertia, weak governments, or high social resistance to political reforms), and high values of r describe countries with flexible institutions.

Second, parameter r also describes the extent to which an incoming government can change an existing policy, since r bounds the distance from the status quo x_{t-1} that a new policy can travel. To see this, let us denote by x_t the *final implemented policy* at (the end of) term t. Then, from (1), we observe that this policy is either $x_t = \min\{p_t, x_{t-1} + r\}$ if $p_t \geq x_{t-1}$, or $x_t = \max\{p_t, x_{t-1} - r\}$ if $p_t < x_{t-1}$. Let $\bar{r} = \bar{x}^R - \bar{x}^L$ be the length of the policy interval $[-1, +1]$, i.e., $\bar{r} = 2$. Now, since $x_t \in [-1, +1]$, if $r \geq \bar{r}$, the final implemented policy x_t is never bounded by r and it always coincides with the pursued policy, i.e., $x_t = p_t$. However, if $r < \bar{r}$, both policies might not coincide. Hence, with the gradual implementation of policies, the focus and interest of the analysis moves from stated to implemented policies, since it is the final implemented policy at (the end of) term t that becomes the status quo at $t + 1$, which further affects how far the implemented policy at $t + 1$ can move away from x_t.

For a given term t, we assume that the utility to player $i \in \{M, L, R\}$ at time τ in term t depends on the distance between the policy implemented at time τ and the player's preferred policy $\bar{x}^i$. Since the policy transitions continuously during the term and the player receives utility from the full policy path, the utility to player $i \in \{M, L, R\}$ in term t is

$$u_i\left(x_{t-1}, p_t\right) = \int_{\tau=0}^{\tau=1} -\left(\bar{x}^i - \varkappa_\tau\left(x_{t-1}, p_t\right)\right)^2 d\tau, \tag{2}$$

where $\varkappa_\tau(x_{t-1}, p_t)$ is given by (1). Note that we assume that all the players have utility functions that are concave in the distance between the policy implemented and the player's preferred policy and that the two parties have preferences over the policy implemented both in and out of office. In this sense, we consider policy-motivated parties (Wittman (1977, 1983)).

We analyze the infinite-horizon version of the game. For any $t \in \mathcal{N}$, a history at term t, denoted by h^t, consists of the list of previously elected parties and their pursued policies:

$$h^t = \left((v_0, p_0), (v_1, p_1), ..., (v_{t-1}, p_{t-1})\right),$$

with $v_0 \in \{L, R\}$ randomly drawn with uniform probability. To make the game fully symmetric, we parametrize the initial policy by $x \in [0, 1]$ such that, if $v_0 = L$, then $x_0 = -x$ and, if $v_0 = R$, then $x_0 = x$. We denote by H^t the set of all possible histories at term t with $\mathcal{H} = \bigcup_{t \geq 1} H^t$.

Restricting attention to pure strategies, a strategy for the median voter in the dynamic game is $s_M : \mathcal{H} \longrightarrow \{L, R\}$. Regarding parties, a strategy for party L and R is $s_L : \mathcal{H} \longrightarrow [-1, 0]$ and $s_R : \mathcal{H} \longrightarrow [0, 1]$, respectively.[12] Let $s = (s_M, s_L, s_R)$ denote a (pure) strategy profile and let S be the set of all (pure) strategy profiles.

[12] For the sake of simplifying the definition of the parties' strategies, we assume that at every term t, the two parties propose a policy to implement, even if a party has not been elected. This has no effect on the results.

Now, for any $s \in S$, player i's payoff in the dynamic game is the discounted utility for the sequence of elections:

$$U_i(s) = \sum_{t=1}^{\infty} \delta^{t-1} u_i(x_{t-1}, p_t), \tag{3}$$

where $\delta \in (0,1)$ is the discount factor.[13] The equilibrium concept is subgame perfect equilibrium. We denote by $S^* \subset S$ the set of equilibrium strategy profiles.

3 The static game

Prior to the analysis of the dynamic game, let us first solve for the equilibria of the one-election game. We will refer to this game as the static game, in opposition to the dynamic elections game. The aim of this section is to understand what the equilibrium would look like if, after $t-1$ terms and a status quo x_{t-1}, there were a single election at t with the game ending after period t.

Note that, because r is finite, the static game is path dependent, that is, in each term t, the players' payoffs depend not only on the actions taken at t, but also on the status quo policy x_{t-1}.[14] Note also that x_{t-1} is the final implemented policy in term $t-1$; hence x_{t-1} depends itself on the status quo policy at $t-1$, x_{t-2}, and the actions taken by the players in term $t-1$, and so on. Despite this recursive dependence, given a status quo x_{t-1}, the equilibrium of the one-election game in term t is unique in terms of outcomes, as we show next.

Proposition 1. *Let x_{t-1} be the inherited status quo in term t. The equilibria of the static game in term t are those satisfying conditions (i) and (ii) below:*

(i) The median voter chooses:

$$v_t = \begin{cases} L \text{ if } x_{t-1} > 0, \\ R \text{ if } x_{t-1} < 0. \end{cases}$$

(ii) If elected,

- *party R chooses $p_t \in [\max\{\min\{x_{t-1}+r, 1\}, 0\}, 1]$,*
- *party L chooses $p_t \in [-1, \min\{\max\{x_{t-1}-r, -1\}, 0\}]$.*

Proof. See the Appendix.

There are two important comments on the result of Proposition 1. First, for a given x_{t-1} and r, if the equilibrium of the game is not unique, then all the equilibria are outcome equivalent. To see this, note that, as already stated, it is the implemented rather than the pursued policy that matters. This is

[13]See Section 6 for a brief discussion on the possibility of heterogeneous discount factors.

[14]When $r \to \infty$, the static game is not path dependent. This is the standard approach in the literature on political economy.

because it is the final implemented policy in term t, x_t, that determines the equilibrium outcome. Thus, for any stated policy p_t, the final implemented policy is either $x_t = \max\{\min\{x_{t-1} + r, 1\}, 0\}$ if $v_t = R$ or $x_t = \min\{\max\{x_{t-1} - r, -1\}, 0\}$ if $v_t = L$. Related to this idea, note that a particular and special equilibrium is that in which the parties propose their partisan policies and the voter votes for the party whose preferred policy is *farthest* from the status quo policy x_{t-1}. For obvious reasons, we will refer to this equilibrium of the static game as the *partisan equilibrium*. The intuition for this result is provided below.

Second, note that the equilibria described in Proposition 1 and, in particular, the prediction of the partisan equilibrium is sharply in contrast to the idea of policy convergence and the median voter theorem. Our framework does not produce convergence to the median because, in our model, the elected government is not tied by any binding platform. This argument is in line with the work by Alesina (1988), who shows that, when parties are policy motivated, platforms are non-binding, and voters are forward looking, convergence to the median is never an equilibrium in the static game. The logic for the result of Alesina (1988) also explains the result in our case. What differs between Alesina's work and ours is that, in the former, the voter has no incentive to vote for the party whose preferred policy is farthest from the status quo policy. This incentive, which will be key in explaining the alternation result that will appear in the dynamic game, depends crucially on r being finite. In fact, note that, if r were, instead, infinite, as for Alesina (1988), the median voter would never have a strict incentive to elect a new government, but would simply vote for the party with the closest party line to her own. If $-\bar{x}^L = \bar{x}^R$, as is the case in our model, the voter would select any party indistinctively.

4 The partisan alternating equilibrium

Next, we move to the analysis of the dynamic game. We start defining a strategy profile consisting of the repetition, from term to term, of the players' behavior in the partisan equilibrium of the static game. We will refer to this strategy as the *partisan-alternating strategy profile.*

Definition 1. ***(Partisan-alternating strategy profile)*** *A profile is a partisan-alternating strategy profile $s^{PA} \in S$ if in each term t:*

1. *The median voter chooses:*
$$v_t = \begin{cases} L \text{ if } x_{t-1} > 0, \\ R \text{ if } x_{t-1} < 0, \\ v \in \{L, R\} \backslash v_{t-1} \text{ otherwise.} \end{cases} \tag{4}$$

2. *Party $i \in \{L, R\}$ proposes its preferred policy $\bar{x}^i$; that is, $p_t = -1$ if $v_t = L$ and $p_t = 1$ if $v_t = R$.*

Note that, if $r \geq \bar{r}$, s^{PA} implies $x_t \in \{-1, 1\}$ for any term t, with $x_t \neq x_{t-1}$. That is, the final implemented policy swings back and forth from -1 to 1, and vice versa, from term to term. In the case

$r < \bar{r}$, the voter's strategy in term t prescribes starting voting for the party whose party line is farthest from the status quo x_{t-1} and to continue doing so until the policy goes across the median voter's preferred policy. Note that, in this case, there may be periods during which the policy remains on one side of the ideological spectrum and the power belongs to the other side. Though apparently contradictory, anecdotal evidence suggests that this is possible. Note, nevertheless, that for this to be the case, the initial policy needs to be extreme enough compared to the institutional flexibility of the country. Indeed, having reached such an extreme policy suggests either a previous exogenous shock or irrational behavior on the voters' side. Such an extreme situation may occur, for instance, after a dictatorship, when a democracy is installed or restored in a country.[15] Finally, from the term the policy goes across the median voter's preferred policy onward, s^{PA} describes alternation of parties in office, with policies swinging back and forth with distance $r = |x_t - x_{t-1}|$.

We are now in position to study whether the partisan-alternating strategy profile constitutes an equilibrium and whether it is efficient. With respect to the first idea, point *(i)* of Proposition 2 below, asserts that the partisan-alternating strategy profile constitutes an equilibrium of the dynamic game for any discount factor $\delta \in (0, 1)$. Points *(ii)* and *(iii)* of this proposition state (limit) results for the case of very impatient players. To this situation, Proposition 2 states that the partisan-alternating strategy profile is either the only equilibrium (it is if $r \geq \bar{r}$) or, if it is not unique, then all the equilibrium strategy profiles are outcome equivalent to s^{PA} (it is if $r < \bar{r}$).

Proposition 2. *(i)* $s^{PA} \in S^*$ *for all* $\delta \in (0, 1)$*. (ii) If* $r \geq \bar{r}$, $\lim_{\delta \to 0} S^* = \{s^{PA}\}$*. (iii) If* $r < \bar{r}$*, then each* $s \in \lim_{\delta \to 0} S^*$ *is outcome equivalent to* s^{PA}*.*

Proof. See the Appendix.

The partisan-alternating equilibrium illustrates very well the logic behind the alternation result. The idea is simple: given the behavior of the parties, to alternate, i.e., voting each election for a different party, is a way for the voter to keep policies at moderate positions (at least for some period every term). Note that the alternation result depends crucially on r being finite and, more precisely, on the gradual implementation of policies that a finite r implies. In fact, if r were infinite, the voter would have no strict incentive to alternate, since the voter could never benefit from the more moderate positions that a policy takes on when traveling from an inherited status quo to the pursued policy.

[15] As an example, we can consider the case of Spain and, more precisely, the period after the death in 1975 of Francisco Franco, who imposed an extreme right-wing dictatorship in Spain for almost 40 years. After a short transition period led by a consensus "centrist" party (CDS, which later disappeared), in 1982 the left-wing Spanish Socialist Workers' Party (PSOE) won the first of four consecutive elections. Even today, these four consecutive elections constitute the longest period of governance by the same ruling party in Spain. During this period, 1982–1996, many leftist policies were progressively introduced in the country, moving status quo policies from far right to moderate and left-moderate policies. Since 1996, the Spanish political arena has been characterized by alternation in power between the right-wing party (Popular Party, PP) and left-wing party (PSOE).

Note that our assumption that $p_t \in [-1, 0]$ if $v_t = L$ and $p_t \in [0, 1]$ if $v_t = R$ simplifies our construction of the partisan-alternating strategy profile. Without it, we could think of deviations in which the elected party could propose policies arbitrarily close to zero on the opposite side of the party's preferred policy, so as to be reelected for a second term (in which, potentially, the party could move the policy toward its preferred policy).[16] These considerations would imply the construction of more complex (alternating) strategies for the median voter. We discuss such a possibility in Section 6, where we consider other possibilities of alternating equilibria, such as equilibria in which alternation occurs every two terms.

Finally, points *(ii) and (iii)* of Proposition 2 provide limit results for the uniqueness of the partisan-alternating equilibrium outcome when δ approaches zero. Regarding this case, in the next section we will show that, when we focus on a relevant class of strategies, i.e., symmetric and stationary alternating strategies, if r is low enough ($r < r_2$), there exists a threshold for δ such that only the extreme partisan-alternating equilibrium exists below it (see Proposition 4, case *(iii.b)*).

Next, we analyze whether the partisan-alternating equilibrium is efficient, that is, whether it maximizes individual players' payoffs. Recall that, by our assumption of symmetry, initial conditions are either $(v_0, x_0) = (R, x)$ or $(v_0, x_0) = (L, -x)$, each with probability one-half. Now, for each term t, let us define $|x_t^{PA}|$ as the absolute value of the final implemented policy in term t in the path induced by strategy s^{PA} and let $\hat{t} = \min\{t \in \mathcal{N} : -x + t \cdot r \geq 0\}$ be the first term in which alternation occurs. The result is as follows.

Proposition 3. *The partisan-alternating strategy profile s^{PA} is Pareto-dominated by any (symmetric) strategy profile in which, for all $t \geq 1$, v_t is given by (4) and $|p_t| = a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$. Additionally, the lower a is, the more efficient the strategy.*

Proof. See the Appendix.

Proposition 3 shows that the partisan-alternating equilibrium is inefficient, since it is Pareto-dominated by any symmetric strategy profile in which policies a and $-a$ alternate over time, with $a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$. To see the reason for this result, first note that, for the median voter, $\bar{x}^M = 0$; hence, the more extreme the a-profile, the lower the voter's utility. The result for the parties relies on two ideas. On the one hand, the concavity of players' utility function and the fact that, ceteris paribus r, the disutility to the non-elected party (while out of office) grows at an increasing rate with the extremism of the government's pursued policy. On the other hand, the symmetry of the initial conditions, which makes that any initial gain to a party for being in office in the first term is offset by the reverse situation, in which it is out of office (both situations being equally likely).

[16] Another possibility would be to consider deviations in which parties aim to be reelected forever, by permanently proposing policies close to zero on the opposite side of the party's preferred policy. Since the policy space is continuous, such a policy would need to be arbitrarily close to zero and, hence, would lead to the preferred policy of the median voter always being implemented. Given that all players are ideological (and there are no rents for holding office), this equilibrium outcome is well approximated by the analysis of the alternating 0-profile explored in Section 5. See Corollary 2 below.

These arguments also explain why the $a = 0$ profile Pareto-dominates any other symmetric a-profile. Despite this desirable property, note that the strategy profile described in Proposition 3 never constitutes an equilibrium for any $a \in [0, \min\{\left|x_{\hat{t}}^{PA}\right|, \left|x_{\hat{t}+1}^{PA}\right|\})$. The reason is this strategy profile does not penalize deviations. To account for this problem, in the next section we introduce a family of strategy profiles that can be Pareto-ranked and that penalizes deviations with reversion to the inefficient but stable strategy profile s^{PA}. We will show that such Pareto-improving strategy profiles may constitute an equilibrium.[17]

5 Middle-of-the-road alternating equilibria

Following the standard approach in the literature of dynamic and repeated games, in this section we will concentrate on equilibria in which parties use stationary strategies (e.g., Duggan (2000), Banks and Duggan (2008), Bernhardt et al. (2009, 2011), Van Weelden (2013)). We will further assume that the strategies are symmetric, of the class $|p_t| = a$.[18] Without loss of generality, we will consider $a \in [0, \hat{x}]$, with $\hat{x} = \min\{\frac{r}{2}, 1\}$ being the most extreme policy that can be implemented in any term for a given r. As for the median voter, we will follow Van Weelden (2013) and consider that the voter's strategy may depend both on the voter's payoff and the identity of the government in the previous term. A government's reelection can thus be contingent on its policy choice.[19] We will refer to an element of this class of (symmetric and stationary) strategy profiles as an *alternating a-profile*, with $a \in [0, \hat{x}]$, and denote it by $\widetilde{s}^a$.[20] We will further denote by $\widetilde{S} \equiv \{\widetilde{s}^a\}_{a \in [0,\hat{x}]} \subset S$ the set of (symmetric and stationary) alternating profiles parametrized by a.

Definition 2. ***(Alternating*** a***-profile)*** *For each $a \in [0, \hat{x}]$, a strategy profile is an alternating a-profile $\widetilde{s}^a \in S$ if:*

1. *In each term t, the median voter chooses v_t according to (4).*

2. *Political parties propose:*

   i) At $t = 1$, $|p_1| = \begin{cases} a \text{ if } v_1 \text{ is given by (4),} \\ 1 \text{ otherwise.} \end{cases}$

   ii) For any $t > 1$, $|p_t| = \begin{cases} a \text{ if, for all } t' \leq t,\ v_{t'} \text{ is given by (4) and } |p_{t-1}| = a, \\ 1 \text{ otherwise.} \end{cases}$

Additionally, with some abuse of terminology and in order to make the stationary path induced by the strategy profile $\widetilde{s}^a$ already present in the initial conditions, we will assume $x = a$, i.e., the initial conditions are either $(v_0, x_0) = (R, a)$ or $(v_0, x_0) = (L, -a)$, each with probability one-half.

[17] This family includes the partisan-alternating strategy profile as an extreme case.

[18] In Section 6.4, we discuss the effects of relaxing the symmetry assumption.

[19] As Van Weelden (2013) indicates (p. 1630), "*This still incorporates elements of stationarity, however, as she (the voter) gives each candidate a clean slate and evaluates him based only on her utility in the last period.*"

[20] Note that, in the case $r \geq \bar{r}$, $\hat{x} = 1$. In the case $r < \bar{r}$, we could extend the definition of $\widetilde{s}^a$ to all $a \in [0, 1]$, but it is straightforward to see that, for each $a \in (\hat{x}, 1]$, $\widetilde{s}^a$ is equivalent to $\widetilde{s}^{\hat{x}}$.

Two comments are worth mentioning here. First, the alternating a-profile prescribes the voter to vote for a different party each election and prescribes the parties to propose policy $|p_t| = a$ in term t if and only if no player has previously deviated. In case of a deviation, alternating a-profile prescribes the parties to propose their party lines and the voter to continue alternating forever.[21] Second, the alternating a-profile imposes initial conditions in accordance with the strategy profile. This formulation allows us to abstract from the particularities that different initial conditions may introduce in the game and to focus on the interesting part of the analysis, which is to understand under which conditions more efficient outcomes can be sustained as equilibria of the game.

Note that all the elements of $\widetilde{S}$ can be Pareto-ranked according to Proposition 3. The ranking establishes that any a-profile, with $a < 1$, Pareto-dominates the partisan-alternating profile s^{PA} and that the smaller the value of a, that is, the weaker the swings in policy from one term to the other, the more efficient the outcome. This idea is formalized next.

Corollary 1. *For all $a', a'' \in [0, \hat{x}]$ such that $a' < a''$, $\widetilde{s}^{a'}$ Pareto-dominates $\widetilde{s}^{a''}$.*

Proof. The result follows from Proposition 3.

With the efficiency result in mind, we next move to the equilibrium analysis. Let $\widetilde{S}^* = S^* \cap \widetilde{S}$ be the set of (symmetric and stationary) *alternating equilibria*, hereafter simply referred to as equilibria. The first result is as follows.

Proposition 4. *There exist r_1 and r_2, with $0 < r_1 < \bar{r} < r_2$, such that:*

(i) The alternating $\hat{x}$-profile is always an equilibrium, i.e., $\widetilde{s}^{\hat{x}} \in \widetilde{S}^*$.

(ii) If $r \leq r_1$, that is, institutional flexibility is low, the alternating $\hat{x}$-profile is the only equilibrium, i.e., $\widetilde{S}^* = \{\widetilde{s}^{\hat{x}}\}$.

(iii) If $r > r_1$, that is, institutional flexibility is high enough, there exists a continuum of alternating a-profiles that constitute equilibria for sufficiently patient players, with the a-profile below a certain cutoff. In particular, there exists the function $\bar{a}(r)$ and, for all $a \in [0, \bar{a}(r))$, there exists $\bar{\delta}(a, r) \in (0, 1)$ such that, for all $\delta \geq \bar{\delta}(a, r)$, the alternating a-profile is an equilibrium strategy, i.e., $\widetilde{s}^a \in \widetilde{S}^*$. Otherwise, it is not, i.e., $\widetilde{s}^a \notin \widetilde{S}^*$.

The cutoff functions $\bar{a}(r)$ and $\bar{\delta}(a, r)$ have the following properties:

[21] The punishment pattern that we consider consists of the natural idea of playing the partisan-alternating equilibrium, which is the "repetition" of the partisan equilibrium of the static game every term t. Since Abreu (1983), it has been well known that, in order to maintain cooperation in repeated games, players can use worse punishments than the reversion to the Nash equilibrium of the stage game. In this paper, we abstract from these more general but artificial punishment patterns, which typically include punishment phases in which players (including the deviating player) need to cooperate for a pre-specified number of periods to punish the deviating player.

(a) $\bar{a}(r)$ *is continuous in* r*. Moreover, if* $r < \bar{r}$*, then* $\bar{a}(r)$ *is strictly increasing, with* $\bar{a}(r) \in (0, r-1]$ *and* $\lim_{r \to r_1^+} \bar{a}(r) = 0$*. If* $r \geq \bar{r}$*, then* $\bar{a}(r) = 1$*.*

(b) $\bar{\delta}(a, r)$ *is continuous and strictly decreasing in* r*, with* $\bar{\delta}(a, r) > 0$ *and* $\lim_{r \to \infty} \bar{\delta}(a, r) = \frac{1-a}{3+a}$*. Moreover, if* $r < r_2$*, then* $\lim_{a \to \bar{a}(r)^-} \bar{\delta}(a, r) > 0$*. If* $r \geq r_2$*, then* $\lim_{a \to \bar{a}(r)^-} \bar{\delta}(a, r) = 0$*.*[22]

Proof. See the Appendix.

Note that, even if the numerical values of r_1 and r_2 are not made explicit in the statement of Proposition 4, they can be computed explicitly (see the proof of the proposition). We start discussing points *(i)–(iii.a)* of Proposition 4, which are illustrated in Figure 1; point *(iii.b)* will be discussed after Corollary 2. Figure 1 depicts the amplitude of the alternating equilibria as a function of the country's institutional flexibility. The vertical axis represents a, which is a measure of policy polarization. Note that the higher a is, the greater the extremism of the equilibrium-implemented policies. The horizontal axis represents the parameter r, which captures the country's institutional flexibility. Recall that the higher r is, the greater the country's institutional flexibility. We observe that the alternating $\hat{x}$-profile, with $\hat{x} = \min\{\frac{r}{2}, 1\}$, is always an equilibrium. This is point *(i)* of Proposition 4. It is important to note that, since the $\hat{x}$-profile is equivalent to the partisan-alternating profile s^{PA} (see footnote 20), and the latter is an equilibrium for any $\delta \in (0, 1)$, then $\widetilde{S}^* \neq \emptyset$. Points *(ii)* and *(iii)* of the proposition characterize the alternating a-equilibria as a function of the institutional flexibility of the country. Point *(ii)* refers to the case $r \leq r_1$. Here, the result is that the only alternating a-equilibrium prescribes parties to propose, every term t, the most extreme policy compatible with r, that is, $|p_t| = \hat{x}$.[23] Then, in this case, the equilibrium is unique (within the class of alternating a-profiles). Point *(iii)* refers to the case $r > r_1$ where, for r high enough, the (Nash reversion) folk theorem logic (Friedman (1971)) applies (see footnote 7). Here we obtain that new alternating a-profiles can be sustained as equilibria. In particular, we obtain that the set of alternating a-equilibria is $\widetilde{S}^* = \{\widetilde{s}^a\}_{a \in [0, \bar{a}(r))} \bigcup \{\widetilde{s}^{\hat{x}}\}$, provided that players are sufficiently patient, i.e., $\delta \geq \bar{\delta}(a, r)$, with thresholds $\bar{a}(r)$ and $\bar{\delta}(a, r)$ satisfying the properties described in points *(iii.a)* and *(iii.b)* of the proposition, respectively. A multiplicity of equilibria therefore exists in this case.

The results of points *(i)–(iii.a)* of Proposition 4 have interesting implications. On the one hand, they show that rigid institutions limit the capacity of parties to implement partisan policies, but induce them to push policies as far as legal, political, and/or social restrictions allow. On the other hand, they also show that countries with flexible institutions *can* feature alternation with lower levels of political polarization (including the median voter's preferred policy) and hence more efficient outcomes. For intuition on this result, note that any incoming government considering the possibility of deviating needs to ponder two effects. The first is that, by deviating and taking the policy closer to its preferred policy, it will enjoy higher utility when in office and will create a more favorable status quo that may affect future policies.

[22]Note that for any given a in the domain $[0, \bar{a})$ (for a given value of r), such an a still remains in the domain when we increase r, since $\bar{a}(r)$ is an increasing function.

[23]Note that $r_1 < \bar{r}$. Hence, the case $r < r_1$ illustrates a situation in which a term is not enough to move the policy from one extreme of the ideological space to the other.

The second effect is that, after a deviation, the next governing party will retaliate and propose its preferred policy, something that the present government dislikes. Which of the two effects dominates determines whether a given strategy profile constitutes an equilibrium or not.

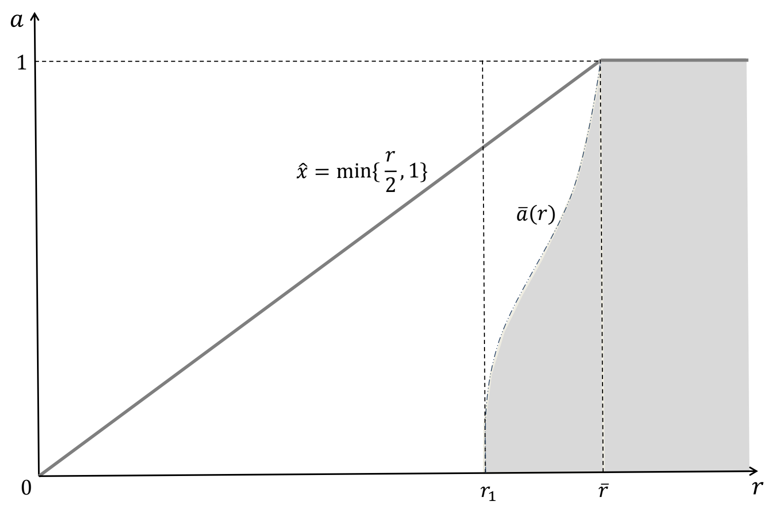

Figure 1: We represent $\hat{x}$ and the function $\bar{a}(r)$, which defines the amplitude of the alternating equilibria, as a function of the country's institutional flexibility r, for $\delta > \bar{\delta}(a, r)$. The equilibria are indicated by the thick gray lines and the shadowed area.

Having in mind the fact that parties' utilities are concave in distance, the crucial idea is that the country's institutional flexibility determines a party's ability to retaliate, that is, the effectiveness of punishment. Roughly speaking, when institutions are rigid, the threat to revert to the extreme partisan-alternating equilibrium is less effective, since institutional rigidities will limit the capacity of the next governing party to choose extreme policies when in office. However, when institutions are more flexible (higher r), such a threat is more effective, since the disutility to a party from a more distant policy grows at an increasing rate. This explains why the smaller r is, the greater the incentives of the governing party to use all the slack that r allows to bring the policy as close as possible to its preferred policy. The argument is that, by so doing, the governing party creates a more favorable status quo that will effectively limit the other party's ability to shift backward in retaliation. It is worth noting that these results depend crucially on r being finite, and, more precisely, on the fact that a finite r bounds policy changes.[24] Last, we note that the intuition behind these results has the flavor of the "Nuclear Peace" theory, according to which the threat of using strong weapons against an enemy (e.g., nuclear weapons) prevents the enemy's use of those same weapons, which induces stability and peace (Schelling (1966), Waltz (1981)).

[24]If r were infinite, the voter would be indifferent between extremes. This means that, after a deviation, the voter could implement different punishment strategies, such as finite-length punishments to any party that deviates from a moderate position. Note, however, that with infinite r, alternation may no longer hold.

Next, we present a corollary of Proposition 4 that states the conditions under which the Pareto-superior profile $a = 0$ is an equilibrium. The result states that, in our model, the median voter's preferred policy is an equilibrium only when the country has sufficiently flexible institutions and players are sufficiently patient.

Corollary 2. *The strategy profile $\widetilde{s}^0$ is an equilibrium if and only if $r > r_1$ and $\delta \geq \bar{\bar{\delta}}(0, r) \in (0, 1)$.*

Proof. The result follows from Proposition 4.

We now discuss point *(iii.b)* of Proposition 4, which characterizes $\bar{\bar{\delta}}(a, r)$, that is, the lower bound of δ that sustains an alternating a-equilibria. It shows that this lower bound is decreasing in r; i.e., the higher r is, the larger the range of values of δ for which such an equilibrium exists. Furthermore, point *(iii.b)* states that, for any $a < 1$, the smallest value of $\bar{\bar{\delta}}(a, r)$ (i.e., when r goes to infinity) is decreasing in a and converges to 0 as a approaches 1. In summary, Proposition 4 implies that, to know whether a particular non-partisan alternating a-profile is feasible, we need to check whether all three following conditions hold: $r > r_1$, $a < \bar{a}(r)$, and $\delta \geq \bar{\bar{\delta}}(a, r)$. If any of these conditions is not met, then the alternating a-profile is not an equilibrium. However, in the latter case, Proposition 4 also suggests that, by increasing the institutional flexibility parameter, such an alternating a-profile may eventually become an equilibrium, since $\bar{a}(r)$ is increasing in r and $\bar{\bar{\delta}}(a, r)$ is decreasing in r.

At this point, it is worth noting that despite $\bar{a}(r)$ is increasing in r, it does not imply that those alternating a-profiles involving more moderate policies are easier to sustain as equilibria. This will also depend on how $\bar{\bar{\delta}}(a, r)$ varies with a (indeed, as discussed above, its lower bound is decreasing in a). The analysis of which of the non-partisan alternating a-profiles is more easily sustained as an equilibrium, namely, for a larger set of discount factors, will be addressed by Proposition 5. For preliminary insight, in Figure 2 we represent $\bar{\bar{\delta}}(a, r)$ as a function of a for three different values of r, $r \in \{2.5, 2.9, 3.3\}$, satisfying $\bar{r} < 2.5 < 2.9 < r_2 < 3.3$. Given point *(iii)* of Proposition 4, we know that $\bar{a}(r) = 1$ for all $r \in \{2.5, 2.9, 3.3\}$, with $\lim_{a \to 1} \bar{\bar{\delta}}(a, 2.5) > \lim_{a \to 1} \bar{\bar{\delta}}(a, 2.9) > \lim_{a \to 1} \bar{\bar{\delta}}(a, 3.3) = 0$.

Figure 2 shows that function $\bar{\bar{\delta}}(a, r)$ when $r = 2.5$ presents quite a different pattern from the other two cases, $r = 2.9$ and $r = 3.3$, for which it is strictly decreasing in a (in line with the lower bound of $\bar{\bar{\delta}}(a, r)$ for $r \to \infty$, discussed above). This result suggests that, provided r is large enough, the more extreme a policy is, the easier (in terms of δ) it is to sustain the corresponding alternating a-profile as an equilibrium. However, for the lower value $r = 2.5$, we observe that $\bar{\bar{\delta}}(a, 2.5)$ is non-monotonic in a, first decreasing and then increasing in a, attaining its minimum at an interior value $\hat{a} \in (0, 1)$. Such a value $\hat{a}$ represents the policy that, apart from the extreme one, $\hat{x}$, can be sustained as an equilibrium for the lowest value of δ. This idea constitutes the basis of what will be defined as a *robust-interior equilibrium* in Definition 3 below. Note that, in the robust-interior equilibrium, policies will stand midway between the median voter's preferred policy and the party's preferred policy; that is, they will be *middle-of-the-road policies.*

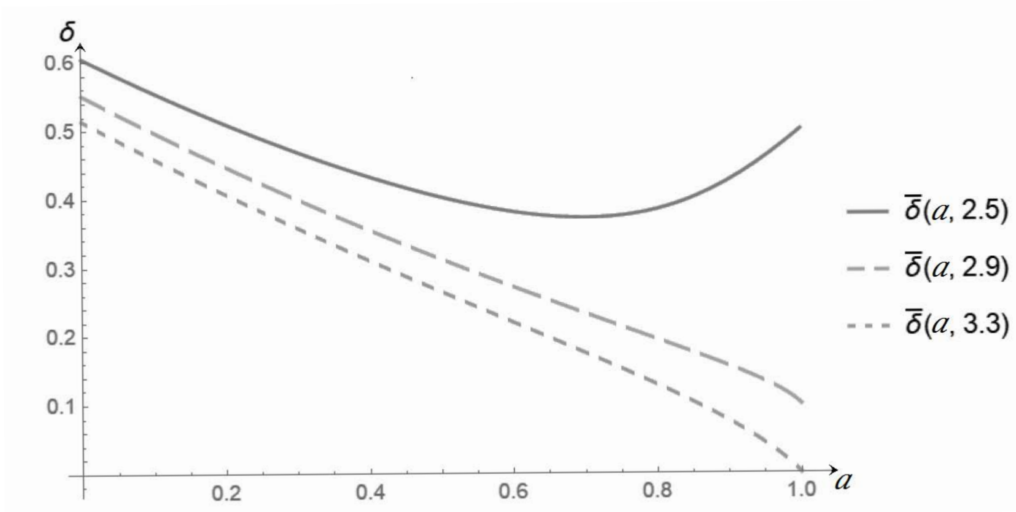

Figure 2: We represent $\bar{\delta}(a, r)$, which describes the lower bound of δ that sustains an alternating a-equilibrium, as a function of a, for three different values of r, $r \in \{2.5, 2.9, 3.3\}$.

Prior to the definition and analysis of the robust-interior equilibrium, we briefly comment on the non-monotonicity result of $\bar{\delta}(a, 2.5)$ and, more precisely, on its source. To this aim, it is helpful to consider how changes in a affect each of the three main terms determining whether a deviation from an alternating a-profile is profitable or not. To illustrate these payoffs, consider, for instance, party L. The three terms are (i) the (equilibrium) payoff of first moving policy from a to $-a$ and subsequently from $-a$ to a (the latter discounted by δ), (ii) the (deviation) payoff of moving policy from a to -1, and (iii) the (punishment) payoff of first moving policy from -1 to 1 and subsequently from 1 to -1 (the latter discounted by δ). We can observe that the third term does not depend on the value of a, and it can be shown that, via the second term, an increase in a always decreases $\bar{\delta}$. Hence, the source for the non-monotonicity effect of a on $\bar{\delta}$ necessarily arises from the first term.[25]

Next, we elaborate on the idea of the robust-interior equilibrium, which aims to investigate which of

[25] To analyze how an increase in a affects $\bar{\delta}$ via the first term, that is, the equilibrium payoff, we need to consider how a change in a affects the equilibrium payoff and how the equilibrium payoff affects $\bar{\delta}$. It can be shown that $\bar{\delta}$ is decreasing in the equilibrium payoff. Regarding the effect of a on the equilibrium payoff, there are two forces: on the one hand, the effect through the concavity of the payoff function, by which the equilibrium payoff is decreasing in a, and, on the other hand, the effect through the "discounting", namely, the fact that the payoff from the policy traveling from a to $-a$ receives greater weight than the payoff from traveling back from $-a$ to a. We observe that, provided r is not very large, the effect of a on the discounting is non-monotonic and, in particular, it is positive for low values of a and negative for high values of a. This drives the non-monotonicity result. Intuitively, the idea is that when a is low, the policy travels for a short period of time every term, as pursued policies are quickly reached. In this case, an increase in a raises the equilibrium payoff of the elected party, which makes it easier to sustain equilibria. In contrast to this, when a is high, it takes longer for the pursued policy to be implemented. In this case, an increase in a raises even more this time lag, making the elected party worse off, which makes it more difficult to sustain equilibria.

the (non-extreme and more efficient) alternating a-equilibria are more resilient to impatience. In other words, our next goal is to study the alternating equilibria that are more robust to variations in the discount factor, in the sense of still remaining an equilibrium when we consider more impatient players. In doing so, it is important to remember that the inefficient partisan-alternating profile s^{PA} constitutes an equilibrium for any $\delta \in (0,1)$ and that, when $r \leq r_1$, this is the only equilibrium (within the class of alternating a-profiles), whereas when $r > r_1$, there are multiple equilibria of this class. In this sense, the analysis that follows will help us select among the multiplicity of equilibria that exist when $r > r_1$.

To this aim, we next focus on the (non-extreme) alternating a-profiles that constitute an alternating equilibrium. Within this class of strategies, we define a *robust-interior equilibrium strategy* as the alternating a-profile that is more robust to variations in δ, in the sense of being the last alternating a-profile to remain an (interior) equilibrium when we increase the degree of players' impatience. In this sense, we rely on a standard criterion that has often been used for equilibrium selection in applications of repeated Prisoner's dilemma games (see Blonski et al. (2011)). We denote a robust-interior equilibrium strategy by $\widetilde{s}^{\hat{a}}$, where $\hat{a}$ is the policy. Since the partisan-alternating profile s^{PA} is always an equilibrium for any $\delta \in (0,1)$, we can interpret an $\hat{a}$-profile as a "second best" that Pareto-dominates the s^{PA} profile. Last, note that because we consider interior equilibria, in the analysis that follows we will restrict attention to $\hat{a}$-profiles satisfying $\hat{a} < \hat{x}$.

Definition 3. ***(Robust-interior equilibrium)*** *A strategy $\widetilde{s}^{\hat{a}} \in \widetilde{S}$, with $\hat{a} \in [0, \hat{x})$, is a robust-interior equilibrium if there exists $\hat{\delta} \in (0,1)$ such that:*

1. *For all $\delta \geq \hat{\delta}$, $\widetilde{s}^{\hat{a}} \in \widetilde{S}^*$.*

2. *For all $\delta < \hat{\delta}$, $\widetilde{S}^* = \{\widetilde{s}^{\hat{x}}\}$.*

Next, we characterize the robust-interior equilibria of the game, using thresholds r_1 and r_2 introduced in Proposition 4. Since, by Proposition 4, if $r \leq r_1$ only the partisan equilibrium exists, we will focus on the case $r > r_1$.

Proposition 5. *Let $r > r_1$. There exists $\hat{r} \in (\bar{r}, r_2)$ such that:*

(i) If $r \geq \hat{r}$, there is no robust-interior equilibrium.

(ii) If $r < \hat{r}$, there exists a unique robust-interior equilibrium. The next two points characterize this equilibrium:

  (a) Let $\hat{a}(r) \in [0,1)$ be the policy such that $\widetilde{s}^{\hat{a}(r)} \in \widetilde{S}$ is a robust-interior equilibrium. Then $\hat{a}(r)$ is increasing in r, with $\lim_{r \to r_1^+} \hat{a}(r) = 0$ and $\lim_{r \to \hat{r}^-} \hat{a}(r) = 1$.

  (b) Let $\hat{\delta}(r) = \bar{\delta}(\hat{a}(r), r)$, with $\bar{\delta}(\cdot)$ as defined in Proposition 4. Then $\lim_{r \to r_1^+} \hat{\delta}(r) = 1$ and, for all $r < \hat{r}$, $\hat{\delta}(r)$ is strictly decreasing in r.

Proof. See the Appendix.

Note that the numerical value of $\hat{r}$ can be computed explicitly (see the proof of the proposition). Next, we discuss the results of Proposition 5, which are illustrated in Figure 3. The top panel of Figure 3 represents the amplitude of both the robust-interior equilibrium (thin black curve) and the partisan-alternating equilibrium (thick gray line), as a function of the country's institutional flexibility. The bottom panel represents the pairs (r, δ) for which the robust-interior equilibrium exists (shadowed area).

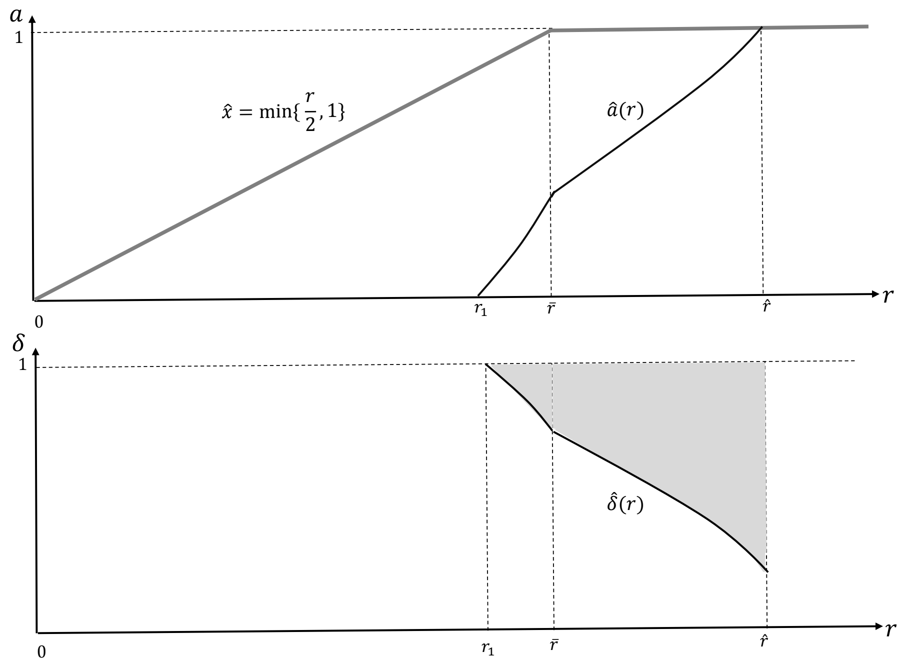

Figure 3: The top panel represents the function $\hat{a}(r)$ (thin black curve), which defines the amplitude of the robust-interior equilibrium, and $\hat{x}$ (thick gray line), which defines the amplitude of the alternating $\hat{x}$-equilibrium. The bottom panel shadows the pairs (r, δ) for which the corresponding robust-interior equilibria (represented above) exist. In both graphs, the horizontal axis represents the country's institutional flexibility r.

The top panel of Figure 3 shows that a robust interior equilibrium exists only for $r \in (r_1, \hat{r})$, an equilibrium that is furthermore unique.[26] These are points *(i)* and *(ii)* of Proposition 5. We also observe that, within the range $r \in (r_1, \hat{r})$, the greater the institutional flexibility r of a country is, the more extreme (and thus inefficient) the robust-interior equilibrium. In the lower limit, that is, when $r \to r_1$, we obtain that the robust-interior equilibrium of the game converges to the Pareto-efficient outcome (the $\widetilde{s}^0$ profile). The bottom panel of Figure 3 illustrates point *(ii.b)* of Proposition 5, elaborating on the lower bound of δ

[26] In line with this result, we note that the case $r = 2.5$ of Figure 2 corresponds to $r < \hat{r}$; hence, there is a robust interior equilibrium. In contrast, the cases $r = 2.9$ and $r = 3.3$ of Figure 2 correspond to $r > \hat{r}$; hence, there is no robust-interior equilibrium in these cases.

that sustains the robust-interior equilibrium, i.e., $\hat{\delta}(r) = \bar{\delta}(\hat{a}(r), r)$. Here we learn that, within the range $r \in (r_1, \hat{r})$, $\hat{\delta}(r)$ is decreasing in r. Hence, the greater the institutional flexibility of the country, the wider the range of values of δ for which the corresponding robust-interior equilibrium exists.

The results of Proposition 5 have interesting implications. First, they suggest that, when r is large enough, the more extreme the policy is, the easier it is to be sustained as an equilibrium (in terms of δ). Note that this result is in line with previous discussion on Figure 2. It also shows that, when r is intermediate, i.e., $r \in (r_1, \hat{r})$, this is no longer the case, since the non-partisan policies that are now easier to sustain as equilibria (in terms of δ) are interior policies. We refer to these robust interior policies as middle-of-the-road policies, since they stand midway between the median voter's preferred policy and the party's preferred one. In this sense, the result suggests that only when institutions are neither very flexible nor very rigid can robust policies be middle of the road. The next corollary elaborates on this idea. In particular, it uses Definition 3 and continuity arguments to show that, for values of the discount factor slightly above $\hat{\delta}$, an interior set of (non-partisan) middle-of-the-road policies can be sustained as alternating equilibria, that is, those policies lying in a neighborhood of the robust-interior equilibrium policy.

Corollary 3. *Let $r \in (r_1, \hat{r})$. If δ is low enough but above $\hat{\delta}(r)$, there is a continuum of (alternating) equilibria with middle-of-the-road policies lying in an interval $[a_1, a_2)$, with $0 \leq a_1 < a_2 < \bar{a}(r)$.*

Proof. The result follows from Proposition 5.

Last, the results of Proposition 5 suggest that the greater the institutional flexibility of the country, the larger the range of values of δ for which the corresponding robust-interior equilibrium exists. A reinterpretation of this result states that the greater the players' impatience, the higher the value of r required to sustain a robust-interior equilibrium which will, furthermore, convey more polarized policies. This result suggests that enjoying alternation with middle-of-the-road policies requires greater institutional flexibility the more impatient the society is.

6 Discussion

This section discusses relevant variations and assumptions of the model: i) asymmetric distribution of voters and/or parties, ii) inefficiencies associated with a change in government, iii) alternation every two terms, iv) asymmetric equilibria, v) office-motivated parties, and vi) heterogeneous discount factors.

It directly follows that the results of the paper are robust to the last two considerations. Regarding office-motivated parties, we may incorporate a rent for holding office into the utility function of the parties, given by expressions (2) and (3). Note that, as long as the parties are policy-motivated, as described by equation (2), the introduction of a rent for holding office will not affect the outcome of the game. To see it, simply note that, since the strategy of the median voter is always given by equation (4), which prescribes alternation and is invariant to deviations, incorporating a rent for holding office will affect a party's payoff in the same way both in the equilibrium and out of the equilibrium path. Regarding heterogeneous discount factors, if we consider this possibility, the binding condition to sustain an alternating a-profile as an equilibrium comes from the party with the lowest one. Thus, our main analysis still applies, just requiring that the minimum discount factor of the two parties satisfies the equilibrium conditions.

In the following, we discuss points i)-iv). All the extensions below consider the punishment pattern that we use in the previous section (after a deviation, parties propose their partisan policies forever and the voter alternates every term), and all the extensions, except for the last two, focus on (stationary and symmetric) alternating a-profiles.

6.1 Asymmetric distribution of voters and/or parties

The model considered so far assumes that the parties' preferred policies are equidistant to the median voter's preferred policy. In this section, we break this symmetry. In particular, without loss of generality, we consider players with the following preferred policies: $|\bar{x}^L| \in (0, 1)$, $\bar{x}^R = 1$, and $\bar{x}^M = 0$. Note that, in this case, $|\bar{x}^L - \bar{x}^M| < |\bar{x}^R - \bar{x}^M|$; that is, the median voter is closer to the left-wing party than to the right-wing party.[27]

Suppose, first, that r is small. Moreover, suppose $\frac{r}{2} \leq |\bar{x}^L|$. Here, the results in the main body of the paper hold, so we can state that $\widetilde{S}^* = \{\widetilde{s}^{\hat{x}}\}$. The reason is simple: because r is very small, any a-profile that does not exploit all the slack that r allows cannot be an equilibrium, since party R always gains by deviating (this could also be the case for party L). Additionally, the strategy $\widetilde{s}^{\hat{x}}$ is an equilibrium, because players maximize their payoffs, given the strategies of the other players. Now, suppose $r > 2|\bar{x}^L|$. We observe that the profile $\widetilde{s}^a$, with $a = |\bar{x}^L|$, in which party L proposes its preferred policy (and party R proposes the symmetric policy), is never an equilibrium. To see why, note that party R gains again by deviating, since this allows it to bring the policy closer to its preferred policy without being penalized for it (when R is out of office, the policy proposed by L will always be $a = |\bar{x}^L|$).

Finally, consider r to be high enough. Recall that, in the main body of the paper, there are multiple equilibria in this case, including the Pareto-efficient outcome $a = 0$. It is interesting to note that breaking symmetry has one important consequence: the efficient profile $a = 0$ may no longer be an equilibrium (nor the profiles $a \sim 0$). To see this, suppose $r = 2$. It is clear that, if $|\bar{x}^L|$ is small, then party R would win by deviating, since the punishment it would incur for doing so is not costly enough to prevent the deviation. This idea suggests that only if $|\bar{x}^L|$ is sufficiently high we could obtain an equilibrium in which the policy implemented is the median voter's preferred policy. The reason for this is simple: how effective the punishment is depends on r (as in the main body of the text) and on the parties' ideologies $|\bar{x}^L|$ and $|\bar{x}^R|$. When $|\bar{x}^L|$ is small, the punishment to party R is small as well; hence equilibria are difficult to sustain. Put differently, what matters for sustaining equilibrium is the distance from the median voter's

[27] We can break the symmetry of the main body of the paper in at least two ways: i) changing the location of one party (and keeping constant the location of the median voter) and ii) changing the location of the median voter (and maintaining the location of the parties). Both alternatives are similar in terms of the resulting setup and outcomes.

ideal point to the closest of the two partisan bliss points.

Summarizing, the results on this variation of the model suggest that breaking the symmetry (while using the punishment pattern of the text) would push equilibria toward partisan lines, making it more difficult for moderate (and more efficient) alternating profiles to survive as equilibria of the game.

6.2 Inefficiencies associated with a change in government

Suppose now that voters bear costs from the alternation. These costs illustrate any class of switching costs and/or inefficiencies that accompany any change in government. Let c denote this cost and let us assume it to be a fixed cost. Of course, the effect of this cost is to reduce the voter's gain from the alternation.

Suppose c is small enough to not offset the voter's gain from alternation. In this case, even with cost c, the voter will find it optimal to alternate; hence the equilibrium/a that we predict are the same as those in the case without c. Suppose now c is high enough to affect the voter's optimal choice, but not so high as to make alternation unprofitable for the voter. Here, consider first an alternating a-profile where policies are moderate, that is, a is small. In this case, it is reasonable to consider that the a-profile constitutes an equilibrium, even if the voter has to pay a cost c for alternating, since the payoff to the voter from the alternation is high when a is small and, if deviating (and voting for the same party forever, which we believe is the best deviation), the voter's payoff would be smaller. Consider now an alternating a-profile with more extremist policies, that is, a is high. Because the payoff to the voter from the alternation decreases in a, it is easy to see that a cost c will exist for which this alternating a-profile is not an equilibrium. This result suggests the existence of a threshold for the cost such that, for costs above the threshold, only alternating a-profiles with moderate policies can be an equilibrium.

Altogether, we find that the results in this section suggest that introducing a cost for alternation would probably eliminate equilibria in which alternation conveys drastic policy changes and would not affect equilibria with moderate alternation. We thus predict that, in the presence of switching costs and/or inefficiencies that imply a cost to voters, if there are equilibria with alternation, then alternation would probably convey moderate (and thus more efficient) policies.

6.3 Alternation every two terms

We now discuss whether our model can produce an equilibrium in which alternation occurs with a frequency different from one. Casual observation shows that alternation often occurs every two terms. For example, three of the last four ex-presidents of the United States –Barack Obama, George W. Bush, and Bill Clinton– were each given two mandates. In this section, we consider this case and discuss whether our model can shed any light on this empirical observation.

First, note that for alternation every two terms to be an equilibrium, a necessary condition is that the median voter finds it optimal to vote in such a way (to vote for a different party after only the second term). This requires the parties not to propose their partisan policies the two terms in a row that they are in office. If they do so, however, the median voter would find it optimal to alternate after the first term, since this strategy would guarantee a period with moderate policies that, otherwise, the voter will not enjoy. Hence, alternation every two terms requires the elected government to propose different policies in the first and second mandates.

Along this line, consider the following profile of strategies, with r being high enough (for simplicity): the median voter alternates every two terms and the elected government proposes the voter's preferred policy in the first term and the party's partisan policy in the second term; that is, $p_t = 0$ in a government's first term in office and $|p_{t+1}| = 1$, when $v_t = v_{t+1}$, in the government's second term in office. Additionally, let us consider that, if any of the three players deviates, from that period onward, the parties propose their partisan policies forever and the voter alternates every term; that is, we use the same punishment pattern as in previous sections. Note that, given such strategies, for a sufficiently high δ, no player would benefit from a unilateral deviation and, hence, the strategies above would constitute an equilibrium of the game.[28]

In a similar vein, we can consider a profile of strategies in which everything is as before, except for the policies proposed in the first and second government mandates. Suppose, here, that the order is reversed, that is, the elected government now proposes its partisan policy in the first term and the voter's preferred policy in the second term. We believe that a similar explanation as for the equilibrium above may be used to show that, for sufficiently patient players, the profile may also constitute an equilibrium of the game.

Interestingly, the last profile describes a situation that seems to fit very well with real-world observations. In fact, casual evidence suggests that governments sometimes start their mandates with partisan policies and that more moderate policies come to the agenda during second mandates. In reference to former US Presidents Ronald Reagan, Bill Clinton, and George W. Bush and their second-term policies, Professor Daniel W. Drezner argues, "Recent second-termers have not reverted to their ideological bliss point – if anything, it's been the reverse; they've tacked away from their starting point."[29]

6.4 Asymmetric equilibria

Previous results focus on the use of symmetric alternating strategy profiles (alternating a-profiles; see Definition 2), which consider that parties' pursued policies are equidistant to the median voter's preferred policy. Next, we discuss the possibility of parties using asymmetric (alternating) strategy profiles. In

[28]To see why the conjectured strategy would constitute an equilibrium in this case, first consider the voter. Note that the voter would never deviate in the first term, since this would imply an extreme policy instead of a moderate one. Regarding the second term, the voter would not benefit from a deviation either, because, after one, alternation would occur every term, instead of every two terms, and the voter prefers alternation every two terms if r is high enough. Next, let us consider the parties. It is straightforward to see that a party would never deviate in the second term, when it is supposed to implement its preferred policy. Regarding the first term, we observe that a party that is patient enough would not have an incentive to deviate either, since, if it deviates, alternation would occur every term and, because of concavity (which implies that a party incurs high disutility when the other party implements its preferred policy), the party would be worse off.

[29]See "The dirty little secret about second-term presidents" (*Foreign Policy*, March 29, 2012).

particular, we consider an alternating a_L-a_R *profile*, with $a_L \in [0,1]$ and $a_R \in [0,1]$, that is constructed by replacing a in Definition 2 by a_L when $v_t = L$, and by a_R when $v_t = R$. Without loss of generality, we consider the case $a_L < a_R$.

We can say that, if r is below a certain threshold, then only those alternating a_L-a_R profiles such that $a_L + a_R = r$ constitute an equilibrium, and they are equilibria for any discount factor. Additionally, when r is high enough, it is straightforward to see that the binding condition to sustain an alternating a_L-a_R profile as an equilibrium comes from party L. To see it, note that the utility for playing the a_L-a_R profile is lower for party L than for party R, since a_R is closer to $\bar{x}^R$ than a_L is to $\bar{x}^L$. This fact implies that a deviation, which thereafter reverts behavior to the partisan alternating strategies, is more harmful for party R than for party L.

It can also be shown that, whenever an asymmetric alternating a_L-a_R profile is an equilibrium, the symmetric alternating a-profile, with $a = (a_L + a_R)/2$, is an equilibrium too. Indeed, the latter (symmetric) profile can be sustained as an equilibrium for lower values of δ than the former (asymmetric) profile can. To see this, consider a high enough r and compare party L's incentives for deviation under the symmetric strategy profile with those under the asymmetric strategy profile (recall that party L is the critical player). In both cases, the incentives to deviate at any period t can be separated into two parts: (i) the *net gain* from deviating derived at t and (ii) the *net loss* from deviating experienced in all subsequent periods $t+1, t+2, \ldots$. First, note that the net gain enjoyed at t is higher in the asymmetric case than in the symmetric one. This is due to the greater distance from $-a_L$ to $\bar{x}^L$, compared to that from $-a$ to $\bar{x}^L$, providing party L with higher potential profits for deviating from the equilibrium in the asymmetric case.[30] Second, the net loss experienced in all subsequent (punishment) periods is clearly smaller in the asymmetric case, since party L receives lower payoffs in the equilibrium path of the asymmetric case than in that of the symmetric case, given the same punishment payoffs in both scenarios. In summary, the incentives of the critical party (L) to deviate are strictly higher in the asymmetric alternating a_L-a_R profile than in the symmetric alternating a-profile, with $a = (a_L + a_R)/2$. Hence, we can conclude that, if it exists, the robust-interior equilibrium will necessarily convey symmetric strategies.

7 Conclusions

Despite political alternation being a fundamental feature of established democracies, there is no clear answer yet as to why parties take turns in power. Political disaffection or disillusion with the policies implemented by elected governments is surely something to consider. Citizens' changing preferences due to changing circumstances, such as a pandemic, an economic crisis, or a war, or to new information, such as a corruption scandal, should not be forgotten either. Beyond these arguments, which we consider should never be neglected, our work contributes to this discussion by proposing a novel explanation for political

[30] This is the case because, along the equilibrium path, in each term t, policy travels the same distance under the two strategy profiles; i.e., $a_R + a_L = 2a$.

alternation in democracies, an argument rooted in the median voter's benefit of keeping policies from drifting too far to either extreme.

Our work speaks to (at least) two literatures, the aforementioned literature on the logic behind alternation and research on institutions and their effects on policymaking. Our contribution to the former is our proposal of a new ingredient, institutional rigidity, that helps explain why rational voters may find it optimal to vote for a different political party each election. Our contribution to the latter is the identification of a new linkage between institutional rigidities and policy polarization, adding to the debate on the pros (e.g., safeguards against an autocrat) and cons (slower adaptation to changing circumstances) of institutional rigidities.

The results in this paper have clear policy implications. They suggest that, although institutional rigidities act as a safeguard against the implementation of extreme policies, they induce parties to polarize. Hence, a country's optimal design of its institutions should account for both rigidity and flexibility in a very precise combination.

What this precise combination is and whether this is something we can measure in the real world are important and challenging questions that remain unexplored. As usual, moving from theory to empirical work is not easy, since some of the essential features of the model are difficult to observe and/or measure (e.g., agents' degree of impatience or the speed of implementation of new policies). Despite these difficulties, we consider that testing our predictions with real-world data would be a very interesting exercise.[31] Another interesting exercise would be to better understand the incumbent's advantage in setting the status quo policy, a privilege that is due, here, to the gradual implementation of policies, and how important this advantage may be in shaping future policies.

Finally, the model has certain limitations, such as we consider complete information, exogenous institutional rigidities, and no entry of new parties. We believe that the study of such variations would provide a more comprehensive understanding of the linkage between alternation, institutional rigidity, and policy polarization. We leave these questions open for future research.

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[31]We could consider the current debate in the United States about increased political polarization and the existence of a unified versus a divided government as a possible "empirical test" of our results. Casual evidence suggests some support for our results that both sufficiently rigid and too flexible institutions may lead to polarization. In particular, the evidence suggests that (i) periods when the US president enjoys great political strength coincide with periods of strong US political polarization, such as during the last decades of the nineteenth century, and (ii) periods when the US President has very little political strength also coincide with periods of strong US political polarization, such as nowadays. For a measure of US politics polarization, see Poole and Rosenthal (1991). For a measure of political strength, we may consider the number of chambers controlled by the executive.

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8 Appendix

Proof of Proposition 1

Given equations (1)-(2), each party maximizes its utility by choosing a pursued policy p_t such that, in case of being elected, the implemented policy at the end of the term, x_t, is as close as possible to the party's partisan line. Since $\bar{x}^R = 1$, all $p_t \in [\max\{\min\{x_{t-1} + r, 1\}, 0\}, 1]$ are outcome equivalent for party R, yielding $x_t = \max\{\min\{x_{t-1}+r, 1\}, 0\}$. Likewise, since $\bar{x}^L = -1$, all $p_t \in [-1, \min\{\max\{x_{t-1}-r, -1\}, 0\}]$ are outcome equivalent for party L, yielding $x_t = \min\{\max\{x_{t-1} - r, -1\}, 0\}$. This proves point *(ii)* of the proposition.

Since the median voter's preferred policy is $\bar{x}^M = 0$, given *(ii)*, $u_M(x_{t-1}, p_t)$, defined by (1)-(2), is maximized by choosing $v_t = L$ if $x_{t-1} > 0$, and $v_t = R$ if $x_{t-1} < 0$. In case $x_{t-1} < 0$, both choices yield the same utility. This proves point *(i)* of the proposition. QED.

Proof of Proposition 2

We have a dynamic game, in which the choices at term $t-1$ determine the status quo at term t. We first prove point *(i)* of the proposition. Given the parties' strategies prescribed by s^{PA}, i.e., each party always proposes its preferred policy if elected; in each $t \geq 1$, the voter has incentives to follow (4) for each $\delta \in (0, 1)$ and $x_{t-1} \in [-1, 1]$, since there are no profitable deviations for her. Moreover, given the voter's behavior in (4), and our assumption that $p_t \in [-1,0]$ if $v_t = L$, and $p_t \in [0,1]$ if $v_t = R$, each party maximizes its utility by always proposing its preferred policy. This proves point *(i)*.

We now turn to points *(ii)* and *(iii)*. If $\delta \to 0$, then at any term t players only assign positive weight in their utility to their current payoffs. Recall that $\bar{r} = \bar{x}^R - \bar{x}^L = 2$. Therefore, if $r \geq \bar{r}$, in each $t \geq 1$, if elected, a party has strict incentives to propose its preferred policy (which is feasible for any $x_{t-1} \in [0,1]$). Given this, in case $r \geq \bar{r}$ the voter also has strict incentives to alternate at every term. This proves *(ii)*. In case $r < \bar{r}$, then for any $t \geq 1$, if elected, a party may have a set of optimal policies to choose, only one of them being feasible, and all of them being payoff equivalent. For party L, this set is given by $[-1, \min\{\max\{x_{t-1} - r, -1\}, 0\}]$ and, for party R, it is given by $[\max\{\min\{x_{t-1} + r, 1\}, 0\}, 1]$. Then, given this behavior of the parties, the unique best response of the voter is given by (4). This proves *(iii)*. QED.

Proof of Proposition 3

Let $\hat{t} = \min\{t \in \mathcal{N} : -x + t \cdot r \geq 0\}$ be the first term in which alternation occurs. The utility to player $i \in \{M, L, R\}$ from strategy s^{PA} is:

$$\begin{aligned}
U_i(s^{PA}) \quad = \quad & \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)r) + \delta^{\hat{t}-1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r)\right. \\
& + \delta^{\hat{t}} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r) + \delta^{\hat{t}+1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r) \\
& \left. + \delta^{\hat{t}+2} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r) + \ldots\right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)r) + \delta^{\hat{t}-1} u_i(x-(\hat{t}-1)r, x-\hat{t}r) + \delta^{\hat{t}} u_i(x-\hat{t}r, x-(\hat{t}-1)r)\right. \\
& \left. + \delta^{\hat{t}+1} u_i(x-(\hat{t}-1)r, x-\hat{t}r) + \delta^{\hat{t}+2} u_i(x-\hat{t}r, x-(\hat{t}-1)r) + \ldots\right)
\end{aligned}$$

$$\begin{aligned}
= \quad & \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)r) + \frac{\delta^{\hat{t}-1} u_i(-x+(\hat{t}-1)r, -x+\hat{t}r) + \delta^{\hat{t}} u_i(-x+\hat{t}r, -x+(\hat{t}-1)r)}{1-\delta^2}\right) \\
& + \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)\,r) + \frac{\delta^{\hat{t}-1} u_i(x-\left(\hat{t}-1\right)r, x-\hat{t}r) + \delta^{\hat{t}} u_i(x-\hat{t}r, x-\left(\hat{t}-1\right)r)}{1-\delta^2}\right),
\end{aligned}$$

which, since $\left|-x+\hat{t}r\right| = \left|x_{\hat{t}}^{PA}\right|$ and $\left|-x+\left(\hat{t}-1\right)r\right| = \left|x_{\hat{t}-1}^{PA}\right|$, can be rewritten as:

$$\begin{aligned}
U_i(s^{PA}) \quad = \quad & \sum_{k=0}^{k=\hat{t}-2} \delta^k \frac{u_i(-x+kr, -x+(k+1)\,r) + u_i(x-kr, x-(k+1)\,r)}{2} + \\
& \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}}^{PA}\right|)}{2} + \\
& \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}}^{PA}\right|)}{2}.
\end{aligned}$$

Now consider a strategy, say s', in which for all $t \geq 1$, v_t is given by (4) and $|p_t| = a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$. It follows that the utility to player $i \in \{M, L, R\}$ from strategy s' is:

$$\begin{aligned} U_i(s') &= \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(-x+kr, -x+(k+1)\,r) + \delta^{\hat{t}-1} u_i(-x+\left(\hat{t}-1\right)r, a) + \tfrac{\delta^{\hat{t}} u_i(a,-a)+\delta^{\hat{t}+1} u_i(-a,a)}{1-\delta^2}\right) \\ &+ \frac{1}{2}\left(\sum_{k=0}^{k=\hat{t}-2} \delta^k u_i(x-kr, x-(k+1)\,r) + \delta^{\hat{t}-1} u_i(x-\left(\hat{t}-1\right)r, -a) + \tfrac{\delta^{\hat{t}} u_i(-a,a)+\delta^{\hat{t}+1} u_i(a,-a)}{1-\delta^2}\right), \end{aligned}$$

which, since $\left|-x+\hat{t}r\right| = \left|x_{\hat{t}}^{PA}\right|$ and $\left|-x+\left(\hat{t}-1\right)r\right| = \left|x_{\hat{t}-1}^{PA}\right|$, can be rewritten as:

$$\begin{aligned} U_i(s') &= \sum_{k=0}^{k=\hat{t}-2} \delta^k \frac{u_i(-x+kr, -x+(k+1)\,r) + u_i(x-kr, x-(k+1)\,r)}{2} + \\ & \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a)}{2} + \\ & \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a,-a) + u_i(-a,a)}{2}. \end{aligned}$$

Hence,

$$\begin{aligned} U_i(s') - U_i(s^{PA}) &= \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a) - \left(u_i(-\left|x_{\hat{t}-1}^{PA}\right|, |x_{\hat{t}}^{PA}|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}-1}^{PA}\right|)\right)}{2} + \\ & \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a,-a) + u_i(-a,a) - \left(u_i(-\left|x_{\hat{t}-1}^{PA}\right|, |x_{\hat{t}}^{PA}|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}-1}^{PA}\right|)\right)}{2}. \end{aligned}$$

Since $a \in [0, \min\{|x_{\hat{t}}^{PA}|, |x_{\hat{t}+1}^{PA}|\})$ and, by (4), the three players' utility functions are concave in the distance between the policy implemented and the players' preferred ideologies (with $\bar{x}^L = -1$, $\bar{x}^R = 1$ and $\bar{x}^M = 0$), we get that, for all $i \in \{M, L, R\}$,

$$\begin{aligned} u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a) &> \left(u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}-1}^{PA}\right|)\right), \\ u_i(a,-a) + u_i(-a,a) &> \left(u_i(-\left|x_{\hat{t}-1}^{PA}\right|, \left|x_{\hat{t}}^{PA}\right|) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -\left|x_{\hat{t}-1}^{PA}\right|)\right), \end{aligned}$$

which in turn implies that $U_i(s') - U_i(s^{PA}) > 0$.

An analogous reasoning shows that if we consider a strategy, say s'', in which for all $t \geq 1$, v_t is given by (4) and $|p_t| = a' \in [0, a)$, then, for all $i \in \{M, L, R\}$:

$$\begin{aligned} U_i(s'') - U_i(s') &= \delta^{\hat{t}-1} \frac{u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a') + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a') - \left(u_i(-\left|x_{\hat{t}-1}^{PA}\right|, a) + u_i(\left|x_{\hat{t}-1}^{PA}\right|, -a)\right)}{2} + \\ & \frac{\delta^{\hat{t}} + \delta^{\hat{t}+1}}{1-\delta^2} \frac{u_i(a',-a') + u_i(-a',a') - (u_i(a,-a) + u_i(-a,a))}{2}, \end{aligned}$$

and, by the same argument, $U_i(s'') - U_i(s') > 0$. This completes the proof. QED.

Proof of Proposition 4

Point *(i)* of the proposition follows directly from Proposition 2 (recall that $\hat{x} = \min\{\frac{r}{2}, 1\}$).

We then turn to analyze the additional equilibria in $\widetilde{S}^*$. To check if a strategy $\widetilde{s}^a$, with $a \in [0, \hat{x})$, is an equilibrium, given the symmetry of the path induced by $\widetilde{s}^a$ and without loss of generality, we can focus on any term t at which $x_{t-1} = -a$ and $v_{t-1} = L$, and check that party R does not have any incentive to deviate in the continuation game. To see this, note that M does not have any incentive to deviate from $\widetilde{s}^a$ at any t, since a deviation induces both parties (and the median voter himself) to turn to the repetition, forever, of the (inefficient) equilibrium of the static game, which cannot yield a higher utility to M than the continuation utility derived from $\widetilde{s}^a$. Then, we only need to check the incentives of party R to deviate at term t, from $\widetilde{s}^a$, to its preferred policy $p_t = 1$. Note that from $t+1$ onwards (i.e., after the deviation), we can safely assume that all players will keep playing according to $\widetilde{s}^a$, since it prescribes to turn to play according to the extreme alternating strategy profile, which as shown in Proposition 2, is an equilibrium for all $\delta \in (0, 1)$.

Hence consider some term t such that $x_{t-1} = -a$ and $v_{t-1} = L$. The utility induced by following the strategy $\widetilde{s}^a$ to R in the continuation game is:

$$\begin{aligned} U_R^F(a, r, \delta) &= u_R(-a, a) + \delta u_R(a, -a) + \delta^2 u_R(-a, a) + \delta^3 u_R(a, -a) + ... \\ &= \frac{u_R(-a, a)}{1-\delta^2} + \frac{\delta u_R(a, -a)}{1-\delta^2}. \end{aligned} \tag{5}$$

In contrast, if R deviates to its preferred policy $p_t = 1$ (and hereafter all the players follow the prescription of $\widetilde{s}^a$), then the utility to R in the continuation game is:

$$\begin{aligned} U_R^D(a, r, \delta) &= u_R(-a, x^D) + \delta u_R(x^D, x^C) + \delta^2 u_R(x^C, x^D) + \delta^3 u_R(x^D, x^C) + ... \\ &= u_R(-a, x^D) + \frac{\delta u_R(x^D, x^C)}{1-\delta^2} + \frac{\delta^2 u_R(x^C, x^D)}{1-\delta^2}, \end{aligned} \tag{6}$$

where, the "deviation policy", x^D, and the subsequent "continuation policy in the next term", x^C, are defined as:

$$\begin{aligned} x^D &= \min\{r-a, 1\}, \\ x^C &= \max\{x^D - r, -1\} = \max\{\min\{-a, 1-r\}, -1\}. \end{aligned}$$

Therefore,

$$\widetilde{s}^a \in S^* \iff U_R^F(a, r, \delta) \geq U_R^D(a, r, \delta). \tag{7}$$

The next lemma restricts the set $\widetilde{S}^*$.

Lemma 1: If $a < \frac{r}{2}$ and $r - a \leq 1$, $\widetilde{s}^a \notin \widetilde{S}^*$.

Proof of Lemma 1. Assume, for the sake of contradiction, that $a < \frac{r}{2}$, $r - a < 1$ and $\widetilde{s}^a \in \widetilde{S}^*$. Consider some term t such that $x_{t-1} = -a$ and $v_{t-1} = L$. Then, $U_R^F(a, r, \delta)$ and $U_R^D(a, r, \delta)$ are given by (5) and (6). Since $r - a \leq 1$, $x^D = r - a$ and $x^C = -a$, we get:

$$U_R^F(a, r, \delta) - U_R^D(a, r, \delta) = \frac{u_R(-a, a) - u_R(-a, r-a)}{1-\delta^2} + \frac{\delta(u_R(a, -a) - u_R(r-a, -a))}{1-\delta^2}. \tag{8}$$

Given (2), since $\bar{x}^R = 1$, we know that the function $u_R(x_{t-1}, p_t)$ is strictly increasing in both arguments. Since $a < \frac{r}{2}$, it follows that $r - a > a$ which, by (8), yields $U_R^F(a, r, \delta) - U_R^D(a, r, \delta) < 0$, a contradiction with $\widetilde{s}^a \in \widetilde{S}^*$. This proves Lemma 1.

Given the condition $r - a \leq 1$, since $a \leq 1$, Lemma 1 is only relevant for the case $r \in (0, \bar{r}]$. In particular, Lemma 1 implies that:

(I) If $r \leq 1$, for all $a \in [0, \frac{r}{2})$, $\widetilde{s}^a \notin \widetilde{S}^*$.

(II) If $r \in (1,2]$, for all $a \in [r-1, \frac{r}{2})$, $\widetilde{s}^a \notin \widetilde{S}^*$.

Point (I) already shows that, if $r \leq 1$, then $\widetilde{S}^* = \{\widetilde{s}^{\frac{r}{2}}\}$. Hence, in the following we consider the case $r > 1$. Moreover, by point (II) we can focus on strategies with:

$$a \in [0, \min\{r-1, 1\}). \tag{9}$$

Assuming $r > 1$ and $a \in [0, \min\{r-1,1\})$, we have that, in (6):

$$\begin{aligned} x^D &= 1, \\ x^C &= \max\{1-r, -1\}. \end{aligned}$$

Hence,

$$\begin{aligned}
u_R(-a,a) &= -\int_0^{\frac{2a}{r}} (1+a-r\tau)^2 d\tau - \int_{\frac{2a}{r}}^1 (1-a)^2 d\tau, && (10)\\
u_R(a,-a) &= -\int_0^{\frac{2a}{r}} (1-a+r\tau)^2 d\tau - \int_{\frac{2a}{r}}^1 (1+a)^2 d\tau, && (11)\\
u_R(-a,1) &= -\int_0^{\frac{1+a}{r}} (1+a-r\tau)^2 d\tau, && (12)\\
u_R(1, \max\{1-r,-1\}) &= -\int_0^{\min\{\frac{2}{r},1\}} (r\tau)^2 d\tau - \int_{\min\{\frac{2}{r},1\}}^1 2^2 d\tau, && (13)\\
u_R(\max\{1-r,-1\},1) &= -\int_0^{\min\{\frac{2}{r},1\}} \left(\min\left\{\frac{2}{r},1\right\} \cdot r - r\tau\right)^2 d\tau. && (14)
\end{aligned}$$

By replacing (10)-(14) in expressions (5) and (6), we obtain the function:

$$f(a,r,\delta) = (1-\delta^2)\left(U_R^F(a,r,\delta) - U_R^D(a,r,\delta)\right),$$

where $f(a,r,\delta)$ is a degree-3 polynomial in a:

$$f(a,r,\delta) = \sum_{j=0}^{j=3} \beta_j (r,\delta) \cdot a^j,$$

with coefficients:

$$\begin{aligned}
\beta_0(r,\delta) &= \frac{1-\delta^2 + r(9\delta - 3) - 12r\delta \min\{1, \frac{2}{r}\} + r^3\delta(1+\delta)\min\{1,\frac{2}{r}\}^3}{3r},\\
\beta_1(r,\delta) &= -\frac{(\delta-1)(1+2r+\delta)}{r},\\
\beta_2(r,\delta) &= -\frac{3+r+(r-4)\delta+\delta^2}{r},\\
\beta_3(r,\delta) &= -\frac{(\delta-5)(1+\delta)}{3r}.
\end{aligned}$$

Hence, for any $a \in [0, \min\{r-1,1\})$, given (7), to verify if $\widetilde{s}^a$ is an equilibrium we need to check if $f(a,r,\delta) \geq 0$.

We first consider the case $r \in (1, \bar{r}]$. In such a case condition (9) becomes $a \in [0, r-1)$ and the conditions such that $f(a,r,\delta) \geq 0$ are:

$$\begin{aligned}
r_1 < \quad & r \quad \leq \bar{r}, && (15)\\
0 \leq \quad & a \quad < \bar{a}_L(r), && (16)\\
\bar{\delta}_L(a,r) \leq \quad & \delta \quad < 1, && (17)
\end{aligned}$$

with $r_1 = \sqrt{3}$, where $\bar{a}_L(r)$ is the second highest real root of the following cubic polynomial in α:

$$h(\alpha; r) = 4\alpha^3 - 3r\alpha^2 + r(r^2 - 3),$$

and:

$$\bar{\delta}_L(a,r) = \frac{12a^2 + 4a^3 - 3r - 6ar - 3a^2r + r^3}{2(1 + 3a + 3a^2 + a^3 - r^3)} + \frac{1}{2}\sqrt{\frac{\sigma(a,r)}{(1 + 3a + 3a^2 + a^3 - r^3)^2}}, \tag{18}$$

with

$$\begin{aligned} \sigma(a,r) = {} & 4 + 24a + 12a^2 - 48a^3 + 108a^4 + 120a^5 + 36a^6 - 12r \\ & -12ar - 48a^2r - 144a^3r - 132a^4r - 36a^5r + 9r^2 \\ & +36ar^2 + 54a^2r^2 + 36a^3r^2 + 9a^4r^2 - 4r^3 - 12ar^3 \\ & +60a^2r^3 - 12a^3r^3 + 6r^4 - 36ar^4 + 6a^2r^4 + r^6, \end{aligned} \tag{19}$$

being $\bar{\delta}_L(a,r) \in (0,1)$ for all (a,r) satisfying (15)-(16). This proves point *(ii)*.

Since all the coefficients of $h(\alpha; r)$ are themselves polynomials in r (hence, continuous functions) and, for the whole interval $r_1 \leq r \leq \bar{r}$, the three roots (in α) of the polynomial $h(\alpha; r)$ are real,[32] it follows that $\bar{a}_L(r)$ is continuous in r in such an interval (see Uherka and Sergott (1977)). Moreover, algebraic calculations show that the derivative of $\bar{a}_L(r)$ is strictly positive in the interval $r_1 < r < \bar{r}$, and that:

$$\begin{aligned} \lim_{a \to r_1^+} \bar{a}_L(r) &= 0, \\ \text{and } \bar{a}_L(\bar{r}) &= 1. \end{aligned}$$

We now study the equilibrium conditions for the case $r > \bar{r}$. Note that, in such a case, condition (9) becomes $a \in [0,1)$ and the conditions such that $f(a,r,\delta) \geq 0$ are:

$$r > \bar{r}, \tag{20}$$

$$0 \leq a < 1, \tag{21}$$

$$\bar{\delta}_H(a,r) \leq \delta < 1, \tag{22}$$

where:

$$\bar{\delta}_H(a,r) = \frac{16 + 16a + 4a^2 - 9r - 3ar}{2(7 + 4a + a^2)} + \frac{\sqrt{3}}{2}\sqrt{\frac{\rho(a,r)}{(7 + 4a + a^2)^2}}, \tag{23}$$

with:

$$\begin{aligned} \rho(a,r) = {} & 76 + 128a + 152a^2 + 64a^3 + 12a^4 - 68r - 140ar \\ & -68a^2r - 12a^3r + 27r^2 + 18ar^2 + 3a^2r^2, \end{aligned} \tag{24}$$

being $\bar{\delta}_H(a,r) \in (0,1)$ for all (a,r) satisfying (20)-(21).

To prove point *(iii)* we define:

$$\bar{a}(r) = \begin{cases} \bar{a}_L(r) \text{ if } r_1 < r \leq \bar{r} \\ 1 \text{ if } r > \bar{r}, \end{cases} \tag{25}$$

$$\bar{\delta}(a,r) = \begin{cases} \bar{\delta}_L(a,r) \text{ if } r_1 < r \leq \bar{r} \\ \bar{\delta}_H(a,r) \text{ if } r > \bar{r}. \end{cases} \tag{26}$$

[32] Three distinct real roots in the interior of the interval ($\sqrt{3} < r < 2$) and two distinct real roots (one of them with multiplicity two) at the extremes ($r = \sqrt{3}$ and $r = 2$).

We now prove that function $\bar{\delta}(a,r)$ is continuous. In this respect, it can be checked that $\bar{\delta}_H(a,r)$ is defined for all $r > \bar{r}$ and $a \in [0,1)$ and that $\bar{\delta}_L(a,r)$ is defined for all $r \in (r_1, \bar{r}]$ and $a \in [0, \bar{a}_L(r))$. Hence, $\bar{\delta}(a,r)$ is well defined in its domain. Moreover, it can be checked that, for all $r > \bar{r}$ and $a \in [0,1)$, $\lim_{x \to r^+} \bar{\delta}_H(a,x) = \lim_{x \to r^-} \bar{\delta}_H(a,x)$. Likewise, for all $r \in (r_1, \bar{r}]$ and $a \in [0, \bar{a}_L(r))$, $\lim_{x \to r^+} \bar{\delta}_L(a,x) = \lim_{x \to r^-} \bar{\delta}_L(a,x)$. For the threshold case ($r = \bar{r}$), we get $\lim_{r \to \bar{r}^-} \bar{\delta}_L(a,x) = \lim_{r \to \bar{r}^+} \bar{\delta}_H(a,x)$. Hence, for all (a,r) in the domain of $\bar{\delta}(a,r)$, $\lim_{x \to r} \bar{\delta}(a,x)$ exists. Finally, it can be checked that, for all $r > \bar{r}$ and $a \in [0,1)$, $\lim_{x \to r} \bar{\delta}_H(a,x) = \bar{\delta}_H(a,r)$, and that for all $r \in (r_1, \bar{r}]$ and $a \in [0, \bar{a}_L(r))$, $\lim_{x \to r} \bar{\delta}_L(a,x) = \bar{\delta}_L(a,r)$. This proves continuity.

Algebraic calculations show that for all $r > \bar{r}$ and $a \in [0,1)$, $\bar{\delta}_H(a,r) > 0$ and $\frac{\partial \bar{\delta}_H(a,r)}{\partial r} < 0$, and that for all $r \in (r_1, \bar{r}]$ and $a \in [0, \bar{a}_L(r))$, $\bar{\delta}_L(a,r) > 0$ and $\frac{\partial \bar{\delta}_L(a,r)}{\partial r} < 0$. Hence, it follows that, for all $r > r_1$ and $a \in [0, \bar{a}(r))$,

I) $\bar{\delta}(a,r) > 0$ and

II) $\frac{\partial \bar{\delta}(a,r)}{\partial r} < 0$.

Computing the limit of the function as $r \to \infty$, we get:

$$\lim_{r \to \infty} \bar{\delta}(a,r) = \frac{1-a}{3+a}.$$

Regarding the (left) limit as $a \to \bar{a}(r)$, algebraic calculations show that:

$$\begin{aligned} \lim_{a \to \bar{a}(r)^-} \bar{\delta}(a,r) &> 0 \text{ for } r \in (r_1, r_2), \text{ and} \\ \lim_{a \to 1^-} \bar{\delta}(a,r) &= 0 \text{ for } r \geq r_2, \end{aligned}$$

with $r_2 = 3$. This completes the proof. QED.

Proof of Proposition 5

First consider the case $r_1 < r \leq \bar{r}$ and $a < \bar{a}(r)$, i.e., conditions (15) and (16) defined in the proof of Proposition 4 hold. Then, given (25) and (26), we get that:

$$\frac{\partial \bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow a \leq \hat{a}_L(r), \tag{27}$$

$$\frac{\partial \bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow \hat{a}_L(r) \leq a < \bar{a}(r), \tag{28}$$

where $\hat{a}_L(r)$ is the third highest real root of the following degree-7 polynomial in α:

$$k_L(\alpha; r) = \sum_{j=0}^{j=7} \lambda_j(r) \cdot \alpha^j,$$

with all the coefficients of $k_L(\alpha; r)$ being themselves polynomials in r (hence, continuous functions), given by the following expressions:

$$\begin{aligned}
\lambda_0(r) &= -6r^2+3r^3+2r^4+2r^5+6r^6-r^7-2r^8,\\
\lambda_1(r) &= 24r-2r^2+7r^3-10r^4-52r^5+16r^6+13r^7,\\
\lambda_2(r) &= -12r+16r^2+26r^3+106r^4-94r^5-14r^6,\\
\lambda_3(r) &= 32-172r-12r^2-6r^3+198r^4-40r^5,\\
\lambda_4(r) &= 144-208r-62r^2-101r^3+88r^4,\\
\lambda_5(r) &= 208-56r-50r^2-57r^3,\\
\lambda_6(r) &= 112+28r-12r^2,\\
\lambda_7(r) &= 16+12r.
\end{aligned}$$

Hence, given (27)-(28), if $r_1 < r \leq \bar{r}$, we conclude that within the interval $0 \leq a < \bar{a}(r)$, the threshold $\bar{\delta}(a,r)$ reaches its minimum at $a = \hat{a}_L(r)$.

Now consider the case $r > \bar{r}$. In such a case, by (25), $\bar{a}(r) = 1$. Then, assuming $r > \bar{r}$ and $a < 1$, given (26), we get that:

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow [r < \hat{r} \wedge a \leq \hat{a}_H(r)] \vee [r = \hat{r} \wedge a < \hat{a}_H(r)] \vee r > \hat{r}, \tag{29}$$

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow r \leq \hat{r} \wedge a \geq \hat{a}_H(r), \tag{30}$$

with $\hat{r} = 2\sqrt{2}$, where $\hat{a}_H(r)$ is the third highest real root of the following degree-4 polynomial in α:

$$k_H(\alpha; r) = \sum_{j=0}^{j=4} \eta_j(r) \cdot \alpha^j,$$

with all the coefficients of $k_H(\alpha; r)$ being themselves polynomials in r (hence, continuous functions), given by the following expressions:

$$\begin{aligned}
\eta_0(r) &= -48-60r+52r^2+15r^3,\\
\eta_1(r) &= 96-272r-44r^2+18r^3,\\
\eta_2(r) &= 352-32r-68r^2+3r^3,\\
\eta_3(r) &= 160+64r-12r^2,\\
\eta_4(r) &= 16+12r.
\end{aligned}$$

Since $\hat{a}_H(\hat{r}) = 1$ and we are considering the case $r > \bar{r}$ and $a < 1$, the expressions (29)-(30) can be rewritten as:

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \leq 0 \Longleftrightarrow [r < \hat{r} \wedge a \leq \hat{a}_H(r)] \vee r \geq \hat{r}, \tag{31}$$

$$\frac{\partial\bar{\delta}(a,r)}{\partial a} \geq 0 \Longleftrightarrow r < \hat{r} \wedge a \geq \hat{a}_H(r), \tag{32}$$

which, in turn, imply that:

a) If $r < \hat{r}$, the threshold $\bar{\delta}(a,r)$ reaches its minimum at $a = \hat{a}_H(r)$.

b) If $r \geq \hat{r}$, then $\frac{\partial \bar{\delta}(a,r)}{\partial a} < 0$ and, therefore, there is not a minimum of $\bar{\delta}(a,r)$ in the interval $a \in [0,1)$, so there is no robust-interior equilibrium. This proves point *(i)* of the proposition.

To prove point *(ii)* we define:

$$\hat{a}(r) = \begin{cases} \hat{a}_L(r) \text{ if } r_1 < r \leq \bar{r} \\ \hat{a}_H(r) \text{ if } \bar{r} < r < \hat{r} \end{cases} \tag{33}$$

and note that algebraic calculations show that:

I) If $r_1 < r \leq \bar{r}$, $\frac{\partial \hat{a}_L(r)}{\partial r} > 0$.

II) If $\bar{r} < r < \hat{r}$, $\frac{\partial \hat{a}_R(r)}{\partial r} > 0$.

III) $\hat{a}_L(r_1) = 0$, $\hat{a}_L(\bar{r}) = \hat{a}_H(\bar{r})$ and $\hat{a}_H(\hat{r}) = 1$.

Finally, by substituting (33) in (26), we obtain $\hat{\delta}(r) = \bar{\delta}(\hat{a}(r), r)$. Algebraic calculations show that:

IV) $\lim_{r \to r_1^+} \hat{\delta}(r) = 1$.

V) If $r_1 < r \leq \bar{r}$, $\frac{\partial \hat{\delta}(r)}{\partial r} < 0$.

VI) If $\bar{r} < r < \hat{r}$, $\frac{\partial \hat{\delta}(r)}{\partial r} < 0$.

VII) $\bar{\delta}_L(\hat{a}_L(\bar{r}), \bar{r}) = \bar{\delta}_H(\hat{a}_H(\bar{r}), \bar{r})$.

This completes the proof. QED.