Research for Curricular Innovation in Action in the Mathematics Classroom

By José Luis Galán-García, José Luis González-Marí, Yolanda Padilla-Domínguez and Pedro Rodríguez-Cielos

Departamento de Matemática Aplicada. Escuela de Ingenierías Industriales. University of Málaga, Spain

jlgalan@uma.es, gmari@uma.es, ypadilla@ctima.uma.es, prodriguez@uma.es

Received: 27 September 2018 Revised: 1 March 2019 **DOI: 10.1564/tme_v27.3.02**

It is difficult for research in Mathematics Education to have any real impact on teaching practice in the classroom. Conscious of this, we have developed a research model for curricular innovation, involving the teachers responsible for its implementation, improving their training, and providing scientific information whilst maintaining the integrity of the phenomena analyzed "where they occur and exactly as they occur". The model is based on an analysis of the principal existing trends, and is tested in the development of Mathematical subjects at distinct educational levels, chief among them university courses in Engineering, where mixed research methods are used to analyse the effect that carrying out commands in a Computer Algebra System (CAS) has on the performance and attitude of students taking various Mathematics courses. Each teacher leads the study in his/her group and collaborates with the rest of the groups as an observer and assistant. Teaching protocols are employed in experimental and control groups; observation protocols, testing objectives, and individual questionnaires and interviews are also employed. The complete analysis provides data that confirms the fitness of the specific innovations applied (with a notable improvement of more than one point over 10 with α = 0, 05), which enables a direct and solidly-based change to the programmes and the development of their corresponding curriculums.

1. INTRODUCTION

Overall, we are in agreement with Schoenfeld (2000, pp. 641-642) who affirms "*Research in Mathematics Education has two main purposes, one pure and one applied: a) Pure (Basic Science) to understand the nature of mathematical thinking, teaching and learning; b) Applied (Engineering) to use such understanding to improve mathematics instruction.*" However, for a long time now, in the main, studies have been focused on discovering "what works" (Harel, G., 2006, p.58) in laboratory conditions[1] without considering that "*to respond to questions that concern the complexity of thoughts and human actions, especially those relevant to learning and teaching of mathematics, requires the adoption, or even, the invention of other research methodologies. For example, the well-known "teaching experiment" (Steffe & Thompson, 2000) focuses on questions of the development of mathematical knowledge in authentic situations[2] in the classroom*" (Lester, F., 2005, Harel, G. 2006, p.58).

There are a myriad of reasons why "*didactic research fails to reach the classroom*" (Becerra et.al., 2012, p.63) including; 1) the results are, in general, not directly applicable because they are isolated and have been analysed outside a real context, or in highly artificial conditions, 2) the teachers themselves are not involved and are either unfamiliar with the advances made or doubt their utility and therefore do not change their teaching style, 3) real conditions and the inertia of the usual processes complicate or prevent significant changes from being made, 4) research results are not sufficiently robust or validated, 5) theories and didactic research which attempt to offer practical solutions have largely been discredited, and 6) innovation as an alternative tool to improve the reality of the classroom provides only localised information with little scientific basis, and with difficulty, can the same results be reproduced in conditions different from the original setting.

In light of this situation, there are trends which attempt to take steps towards unifying the two basic approaches in a third way, which joins pure and applied research and curricular innovation in developing an "ordinary" didactic process, without ignoring the aforementioned basic approaches for, at least, some proposals. This is precisely the case of the trend, known as Research for Innovation (Bartolini, 1998; Arzarello, 1999) or more recently the trend known as Design Research (Kelly et al., 2008), which attempts to overcome the present difficulties by:

- Adapting to the conditions and real phenomena as soon as they happen;
- Respecting the "natural" processes, without causing a negative effect on ordinary development;
- Maximising operability with the minimum distortion, admitting only those changes that are strictly necessary and permitted under current legislation, in a holistic, global approach;
- Incorporating operational limitations and difficulties which enable phenomena to be studied to their full extent, without fragmentation, separation of factors or simplification.

The problem that they present is therefore of a methodological nature, but also includes the research fundamentals of Mathematics Education, that are operability and reach, the conditions of development and their innovation potential.

[1] Partially controlled conditions which create a reality different from what is hoped to be simulated.

[2] Didactic processes usually found in normal mathematics classrooms (Author's note).

The main purpose of this paper is the development of a new research model for curricular innovation, involving the teachers responsible for its implementation, improving their training, and providing scientific information whilst maintaining the integrity of the phenomena analyzed "where they occur and exactly as they occur".

2. THEORETICAL BACKGROUND AND RATIONALE

Research, through innovation in the mathematics classroom necessitates a general reflection on curricular design and development, an analysis of research methods and the different existing trends and models. We therefore detail below the principal points of reference on which our proposed model are based.

2.1 Curricular research and innovation in the mathematics classroom

2.1.1 Curricular research in Mathematics Education

The majority of the studies on Mathematics Education are closely linked to interests related to curricular design or development. At both levels of reflection there is very little information on the latter, the reason why:

> *"…it is necessary to promote, define and clarify, building on Mathematics Education, the field of study of curricular development and encourage the establishing of stable groups of researchers in this field who can work together in coordination and competitively."* (Rico, 1997, p. 267)

To define the notion of a Mathematics curriculum we take a generic approach which refers to the set of activities targeting training planning (Tyler, 1986; Stenhouse,1984), within a comprehensive, systemic model in accordance with the component scheme, and levels of specificity and dimensions as proposed by Rico (1998, p. 31 onwards). He is the author from whom we take the following statements as the principles for the model we are proposing here:

> *"… the curriculum is an open and dynamic process. As soon as materials are developed, they should be tested, the results evaluated, to identify errors and propose improvements, which would mean new materials with which to start the cycle anew. In this way, it would be possible to obtain a constant improvement of the curriculum."* (Rico, 1998, p.87)

2.1.2 Curricular Innovation in Mathematics

Innovation, curricular change and projects of innovation in mathematics all represent and have always represented a concern in the area of Mathematics Education especially in relation to sweeping curriculum reform and specific or extensive "curricular changes" with a narrow scope and carried out by only one group of people, (Coriat, 1997a, p. 200).

In general terms, only two categories of studies are recognised: those which follow a development model or a theoretical trend, and those which are based on a strategy of curricular innovation. Within the first group can be found the models; Behavioural, Modern Mathematics, Structuralism (Structure of disciplinary knowledge), Training (development of intellectual structures) and Integrated Learning (scientific content and structure together with intellectual training). The strategies for curricular innovation (Rico, 1997, p. 228) can also be of two types: the model of research-development-dissemination, and the model "focus on personality", which attempts to involve the teacher, either directly (full participation) or indirectly (as the final user of a product). We have taken an approach of direct participation with an integrated teaching model as the basis for our research model. The strategy centres on the personality of the teacher as the strategy for the integrated teaching model.

2.1.3 Research and curricular innovation

Innovation research aims to add a rigorousness, assurance and credibility to innovation in order to ensure the validity and scope of the effects of the changes made. Its aim therefore is to provide the means by which innovation can become an effective tool in effective solidly-based reforms that do not remain as just good intentions by reduced groups of teachers. The action research emerges as a wide-ranging model, complemented by the participation and coordination between specialists in research in Mathematics Education.

2.1.4 Research and curricular innovation and teaching training

In "ICME 3"[3] can be found for the first time, certain approaches that integrate and inter-relate research and curricular innovation with teacher training. This is illustrated by Stenhouse (op.cit):

> *"… the work developed in the fields of curriculum and teaching largely depends on perfecting ways of cooperative research between teachers with the simultaneous input from professional researchers, who support the work of the teachers."*

This confirms that there is a need for a model based on the figure of the teacher as a researcher[4], in research-based teaching and in research, based on a mixed methodology which focuses on schemes of action research. This arrangement therefore brings together research, innovation, and teaching in a coherent whole (Rico,1997) the results of which will have an immediate effect on teacher training, which in turn will affect common practices in the classroom, textbooks and institutional reform.

[3] ICME-3 (1976) Herman Athen and Heinz, Kunle, (eds.) Proceedings of the Third International Congress on Mathematical Education. Karlsruhe (Germany).

[4] In close collaboration with professional researchers (Author's Note).

2.2 Research methods in Mathematics Education

The study of real phenomena in the mathematics classroom not only implies an active participation of the teachers themselves, collaboration or an innovative intention, but also demands the use of diverse research models which can be divided into three main categories.

2.2.1 Non-empirical methods

In the initial phases of the studies it may be necessary to draw on the following well known procedures: Epistemological and phenomenological analysis (Freudhental, 1983; Puig, 1997), analyses of texts and resources in Mathematics (Coriat, 1997b), didactic analysis of mathematical content and teaching (González, 1998, 1999c); Gallardo and González, 2012).

2.2.2 Empirical methods

There are situations which require the use of qualitative methods (Guba and Lincoln, 1985; Taylor and Bogdan, 1986), action research procedures (Goyette and Lessard Hébert, 1988), interviews and surveys (Goetz and Lecompte, 1988; Cohen and Manion, 1990; Ortiz, 1998), or quantitative methods (Bisquerra, 1989).

2.2.3 Mixed methods

In general terms, we agree with the idea that in Mathematics Education "The small scale qualitative studies or large scale quantitative studies are essential to build useful and solidly-based knowledge to put into teaching practice…(But)[5]. Neither of them alone is sufficient, they are both necessary" (Hierbert and Grouws, 2007, p.398).

Until 2005, 70% of all studies in Mathematics Education used qualitative or quantitative methods (Hart, Smith, Swars and Smith, 2009, pp. 26-41). However, recently, there has been a tendency towards specific configurations of these two methods, depending on the characteristics of the studies, which has given rise to a new line of research. This is clearly marked by the use of Mixed Methods in Mathematics Education (Hiebert and Grouws, 2007; Castro and Godino, 2011), which constitute a "set or analysis of quantitative and qualitative data gathered at the same time or sequentially through established criteria and which integrate one or more stages in the research process" (Creswell, 2003, p.711).

The model which we propose uses mixed methods of research with a similar focus as those studies carried out in the field of engineer training (Borrego et al., 2009; Case and Light, 2011). The principles have a distinctly pragmatic basis (Johnson and Onwuegbuzie, 2004) and come from a concerted focus on Mathematics Education as "a discipline applied where research should improve education; the integration (of methods) is necessary not only to discover whether certain experiments in education improve learning and understanding but also to find out how these results are reached and why we can expect them to be repeatable elsewhere" (Hart et al., op.cit., p.39).

2.3 Trends and research models for curricular innovation in Mathematics Education: Critical analysis and consequences for the model

Research in Mathematics Education which proposes curricular innovation is carried out within frameworks made up of different stages and trends which we now analyse and which have contributed to the configuration of our proposed model.

2.3.1 Traditional trends

Bartolini (1998), Arzarello (1999) and Bartolini and others (1999), examining the deliberations of Grouws (1992), identify three traditional trends in research in Mathematics Education which are still valid for different approaches:

A. **Research based on a conceptual organisation of the discipline (also known as the scholastic philosophical tradition** (Grouws, op.cit). This trend focuses on improving teaching in generic situations based on a logical organisation of the discipline, with greater emphasis being placed on the result rather than the process itself, with the intention of influencing teaching programmes and texts. There is also a focus on the dissemination of initial training and the goodwill of professionals.
B. **Research on innovation in the classroom (pedagogical tradition).** This tradition promotes the production of paradigmatic examples in order to improve teaching in specific situations through action research, an intention to influence the curriculum, dissemination through permanent training and the leading by example and goodwill of teachers. The principal drawbacks to this trend are: transposing it to other classrooms, and the methodology research, the questioning of which has led to the third and final trend
C. **Research that observes and models the teaching-learning experience in the laboratory conditions (empirical scientific tradition).** The idea of this trend is to improve the understanding of the processes of teaching-learning in mathematics so as to have a solidly-based plan of intervention to put it into practice (majority of research by PME[6]).

2.3.2 Research for innovation

The authors propose a fourth trend which combines research with innovation so as to:

- Produce paradigmatic models to improve teaching in mathematics (model A);
- Study the conditions and factors that affect specific actions (models B and C);

[5] Author's Note.

[6] The International Group for the Psychology of Mathematics Education (PME).

- Produce innovative theoretical constructs to guide the actions of the teachers;
- Produce innovative didactic methods (specifically produced by this model).

The model has the following seven characteristics (González, 1999a; Galán, 2003; Padilla, 2003):

1. **Active and direct intervention of the teachers as "co-researchers"**: The teachers act as researchers in their own groups "developing studies of action research of a cooperative nature". The main limitations lie in the teacher training and the compatibility of research tasks and their day to day teaching obligations, which can be offset by collaboration with experts and situating the research within official guidelines and real conditions in the development.
2. **A "natural" and systemic consideration of the phenomena**: The studies try to explore the phenomena as they occur and where they occur, without reducing them, isolating their parts, excessively focusing on creating laboratory conditions or distorting their natural timing (Bartolini, 1998). We take this point of view, although without excluding other trends, approaches or complementary research.
3. **Concern for replicating the conditions of a real teaching-learning process, cutting down or eliminating entirely the distance between theory and practice.** One of the goals of this trend is to reach an understanding of the didactic processes so as to exert an influence over them more efficiently. The principal obstacle to this is the powerful socialising effect that a school has which favours the existence of routine practices which are difficult to modify if methodological controls and precautions are not taken (González, 1999d).
4. **Long-term studies and reconciling various tasks.** This type of research requires an extended period of time in the classroom with attention split between teaching tasks and research, which is difficult to achieve, but not impossible with cooperation and collaboration between institutions and the people involved.
5. **Particular attention paid to the moment in which the phenomena happen.** Normal educative processes happen over a long period of time and can therefore be analysed longitudinally. However, it is also necessary to carry out cross-cutting studies in which a small part of an isolated or unique process can be modified in order to analyse the result.
6. **A direct and rapid effect on the practices and training of the teachers involved.**
7. **Possible unwanted effects on the curricular development.** The model allows for the introduction of profound changes in the normal didactic process (Bartolini, 1998), which could have a negative effect on curricular development. Conversely, we believe it is necessary to control these unwanted effects and reconcile the changes introduced with the rest of the normal, didactic development.

2.3.3 Collaborative Action Research in Mathematics Education

Closely linked with the trend "Research for Innovation" is the approach that focuses on the development of qualitative research projects, designed and implemented by teachers and university experts. This is "a process of systematic study in which teachers, review their own teaching and their students' learning process through descriptive reports, discussion and exchange of ideas with other teachers as well as a critical reflection with the intention of improving educational practices in the classroom" (Raymond, 1994, p. 3). However, despite its advantages, which we incorporate into our own model, we believe it is necessary to employ mixed research methods, appropriately combined and adapted to the necessities of the study.

2.3.4 Local Theoretical Models

Although the goal of this approach is to "… analyse, in a localized and specific way, the distinct aspects or components of the phenomena of Mathematics Education…" (Filloy, Puig and Rojano, 2008, p. 328), we are interested in those approaches that recognise "…the necessity to model the study on local didactic phenomena in order to subject them to experimentation and thereby develop learning situations…in the classroom." (Fernández & Puig, 2002, p. 30).

Similarly, we share the local and practical character of the teaching and learning processes in mathematics: "Our research has always addressed teaching and learning processes, especially in the domain of mathematics." (Puig, , 2008, p. 98); "…This means to elaborate local theoretical models in order to understand the processes in play when specific mathematical content is taught, in the education system, to specific students, with the intention that these models are adequate for the phenomena being studied." (Fernández & Puig, 2002, p. 31), even though the authors recognise that "although the validity range of the models is not intended to go beyond the observed phenomena, the description of the model attempts to be in depth, complex and thorough…" (op.cit. p. 31).

The trend considers that the educational phenomena seen in mathematics allow different components or "models" to be used in studying all facets separately: Formal Competence Model, Cognition, or activity, Model, Teaching Model, and Communication Model. However, we think they are ideal to study isolated questions but not to tackle global problems.

2.3.5 Design Research, Teaching Experiments and REUBE trend

The trend "Design based research in education" arose from the proposal to implement innovative processes based on proofs and the adoption of scientifically based educational measures (Kelly et al., 2008) in order to improve the quality of teaching practices.

Cooperative work allows teachers to receive the information necessary to adapt and implement innovative processes or to develop a local teaching theory about an aspect or given topic (Gravemeijer & Van Eerde, 2009). The studies are developed around the idea of introducing new subject matter, new learning tools or new ways of organising the context (Confrey, 2006; Molina, M., Castro, E., Molina, J. L., 2011).

Within this trend can be found the "Teaching Experiment" (Steffe & Thompson, 2000) and the Renewal of University Education based on Proofs (Renovación de la Enseñanza Universitaria basada en Evidencias (REUBE)) (Becerra et al., 2012), which attempt to implement innovations in the classroom guided by the results of research on the discipline of teaching.

3. THE MODEL: RESEARCH FOR CURRICULAR INNOVATION IN ACTION IN THE MATHEMATICS CLASSROOM

Although all the aforementioned analysed trends are useful and share the same end goals, we believe that it is possible to have a model which is better adapted to real conditions and provides immediate scientific solutions to real educational problems in the mathematics classroom. This model can also provide certain guarantees that the process can be repeated elsewhere and ensures a greater validity, commonality and reliability of the knowledge discovered (Rodríguez, 1994, chap. 3, pp. 119-142). In what follows we examine the key aspects of the proposal.

3.1 Description

We refer to this approach as "Research for curricular innovation in action in the Mathematics classroom". This is a mixed methodological approach to didactic-curricular research through innovation in mainstream mathematics classrooms. This is applied in a classroom as the usual teaching process unfolds, with the participation of the teachers giving the class, and the minimum number of changes necessary to test the hypotheses being considered. Below we clarify the definition above in more detail:

Research

Although the studies can be seen as an enriching experience by the participants and the results could be seen as providing a "localised" improvement they should not be considered as mere experiences or simply as innovation efforts. In fact, the intention of the model is to protect and guide genuine research, in other words, regulated studies with a scientific aim, based on methodical and disciplined inquiry and focusing on the discovery and dissemination of solidly-based knowledge, validated and generalized as far as possible, repeatable and susceptible to critical examination by experts.

Innovation

The research developed, more than merely expanding the field of scientific knowledge in this area, must be explicitly targeted to provide information which enables an "immediate" implementation of genuine improvements in common teaching practices under similar conditions.

Curriculum

Innovation has to concentrate on the content of the official mathematics curriculum and the actual circumstances in which it is developed. To this end, the effects of certain curricular changes must be analysed always supposing that there exists prima facie evidence of their viability and positive effect on certain aspects of curricular development, without altering other conditions.

Action in the Mathematics classroom

Without underestimating the complementary role of interdisciplinary approaches, or dismissing other types of studies, the area of interest of the model centres on the processes of teaching and learning of mathematics in schools, where they happen and as they happen, which means that:

- the research is carried out as the mathematics class unfolds, in normal conditions and with a minimum number of changes allowed;
- the students and teachers take an active role and the teachers themselves act as researchers;
- the work is completed with the advice and participation of observers and at least one researcher in Mathematics Education;

The model and the studies that are detailed in the work of González (2004), Galán (2003), Padilla, (2003), Rodríguez (2004) and Larrubia (2006), comply with the conditions and characteristics that are detailed below:

3.2 Characteristics

Purpose

The purpose of the study is to reach a deeper understanding of the real processes at work in teaching-learning in mathematics without manipulating or distorting the nature or characteristics of the processes. It is also important to ascertain the teaching potential in curricular development and whether there is a real possibility to change or innovate existing curriculums and to discover how far reaching are the effects of different factors, evaluating the reach and real results of the changes "allowed" whilst at the same time identifying the limitations and difficulties in optimizing the process.

A study carried out in real conditions, has to, on the one hand, consider the phenomena as a whole and in all their complexity and also view them as singular, unrepeatable events; and on the other hand, take small steps rather than attempt extensive research studies which could become too unwieldly and too ambitious. In addition, special relationships are formed between the teachers, the research and the teacher training at the heart of the profession.

Conditions and Commitment

The following conditions and commitment serve as guidelines in both the inquiry process and permitted actions:

1. A scrupulous adherence to the reality of education, preserving the integrity and authenticity of the phenomena, which means:
    a) The research has to be carried out in normal, genuine conditions, thereby preventing the introduction of changes which indicate the presence of laboratory conditions.
    b) A reduced manoeuvrability. The study must be subject to limits that impose the reality of the classroom, the centre of learning, etc. (personal boundaries and limits on materials, time, curriculums, etc.); limitations and difficulties which are intended to control the variables, unwanted effects, the existence of rival hypothesis, outside influences and threats to validity.
2. Zero prejudice or innovative intention in the teaching process and its results
3. Total compatibility with the ordinary design with respect to the guidelines and guidance of the Educational Administration.
4. Durability and strength (permanent and sustainable changes)

Academic and curricular framework:

The framework is delimited by curricular design, national guidelines and their effect on teaching practice. The elements that make up the framework are the objectives, content, methodology guidelines, evaluation, resources, activities and other aspects regarding national curricular design.

Types of problems and studies

These are related to the effect that viable, legally permitted changes to the curriculum have on the development of ordinary teaching and the results of such changes.

The studies can be in isolation or made up of various coordinated studies, overtime and focused on replication and subsequent confirmation, in various equivalent groups, of the correctness of the curricular innovation under analysis.

Didactic Processes

The studies are carried out in real didactic processes, the curricular design and development and their impact. The standard version of the model has two didactic processes:

- Experimental, which centres on the changes which delimit the research problem posed and is reflected in an experimental teaching protocol;
- Control, which is detailed in a controlled teaching protocol which, in general, coincides with the ordinary didactic process without alteration.

Teaching Protocols

These should be clear, repeatable and include all aspects of a complete curricular design, in such a way as to permit an outside observer to say beyond doubt, that they have been followed faithfully or that there have been some deviations. The same is true for the protocol of the control process, which should include observable indicators of use to capitalise on the process and the existence of competitiveness in the experimental process.

Observation Processes

Attention should be paid to, among other aspects: suitability of the processes in the established protocols; the impartial behaviour of the teacher; deviations greater than expected; student behaviour; the characteristics and differences/similarities of the processes; the alternative, competitive nature of the control process.

Action areas and conditions

The research can be carried out in any classroom where mathematics is taught, which meets the conditions set out. It can be at a given level in a single centre of learning or in two or more centres and two or more levels and groups.

Differences to the usual process

- *Time frame*

The studies should be carried out throughout a normal academic course, which is not to say that they should be completed in a single course or that they should dominate the time dedicated to the course itself. The time frame will depend on the type of problem to be solved, the breadth of the problem and the real availability, and the approaches to be applied.

- *Methodology*

The study requires the methodology to be:

A. *Mixed*, which should include, depending on the circumstances: Non-empirical methods: Didactic analysis, Analysis of mathematical content, Historical Analysis, Phenomenological analysis, Epistemological Analysis, Analysis of text books and mathematical books, Task analysis, etc.
   Qualitative and quantitative empirical methods: Concerning the intervention itself (Ethnographic techniques and immersion in natural groups; experimental techniques (experimental group-control group); techniques of action research); With respect to gathering information (surveys, interviews, questionnaires, objective testing; ethnographic methods: participant observation, external observation, triangulation, epistemological reflection, analysis of logs; consultation with experts, opinion of teachers, etc.; Regarding data analysis: Descriptive methods, comparison of parameters, variance analysis.
B. *Complex*: as befits the complexity of the problem and the circumstances.
C. *Adaptable*: the study should be adaptable to the problem posed and the real conditions of development of the teaching process, the changes made, the personal

circumstances of the participants, official guidelines or physical circumstances.

D. *Compatible*: it should be compatible with the design and development of the curriculum and the conditions of the study[7].

- *Variables*

Dependent variables: Performance, scoring of a standard objective test before and after the process is applied, etc.; independent and concurrent: time dedicated to exercises and problem solving, initial past performance, scoring, etc.

- *Phases of the study*

Theoretical phase, preparation and basis: Background, didactic analysis, preliminary, exploratory, theoretical and empirical studies: Equivalence testing of groups and delimitation of any differences found; verification of normality and equality of means and variances, randomness, composition and origin of the sample groups, etc.; elaboration of: Protocols of process and observation; control and logging instruments (audio, video, note books, log pages, etc.); design of empirical studies;

Experimental phase: Development of experimental and control approaches, observation, controls and tests. Assurance of: equivalence in the two approaches, where this does not affect study changes; guarantees of impartiality, elegance of procedures, "threats to validity"; logs and fidelity controls of the protocols; elimination and control of extraneous variables and unwanted effects;

Analysis phase: the results and conclusions drawn.

4. RESEARCH AND INNOVATION IN MATHEMATICAL COURSES IN UNIVERSITY STUDIES IN ENGINEERING

The model described has been used to study some aspects of the usual design and development of the curriculum for solving quadratic equations in the 3rd course of the ESO (Educación Secundaria Obligatoria - Compulsory Secondary Education-) (Larrubia, 2006) and in mathematical courses of engineering studies, an area of phenomena that has seldom been studied in research regulated by Mathematics Teaching[8].

In relation to engineering degrees, there have been three studies (Galán, 2003; Padilla, 2003; Rodríguez, 2004), all based on changes to conventional teaching practices in the following subjects, commonly found in the curriculum of the "Telecommunication Engineering" degree:

1. Line Integrals in the subject "Vector Analysis and Differential Equations" (first year).
2. Multiple Integrals in the subject "Vector Analysis and Differential Equations" (first year).
3. Derivation and Integration of Complex Variable Functions in the subject "Advanced Mathematics" (second year).

In all three cases an analysis is carried out considering the effects that a new mixed teaching methodology, has on the following factors: learning, attitude, performance of students, and the teaching process and its comprising elements. This also encompasses those programmes that include DERIVE (known as experimental) which are compared to traditional or more common teaching methodologies (known as control). Simultaneously, they take advantage of the experimental development to reflect on the validity and reliability of the data, the innovative potential, the relevance and usefulness of the theoretical and methodological framework, and the future development of this line of research.

In accordance with the established methodology and in order to evaluate the model, the empirical studies are divided into two different parts, although interrelated and applied simultaneously:

- Quantitative study, aiming to compare the effects of the two didactic approaches
- Qualitative study, aiming to gather specific and independent information from the quantitative study so as to expand on the data provided by said study.

The experimental study fits a quasi-experimental design or a dual pre-test/post-test design. In the qualitative study the following are used: instruments and techniques of participant observation, triangulation, interviews and surveys to obtain evidence on speculations, the research problem, the model used, the method used in relation to performance, validity of teaching process and to what extent the teaching approach is innovative.

The practical development is adjusted in each study in the following stages (See Figure 1):

Stage 1: Pre-test	• Random assignation of the students to the control and experimental subgroups. • Random assignation of the teaching processes to the two student subgroups. • Prior level test (pre-test): the test consists of solving various exercises and questions on the themes, previously developed in each of the study's subjects.
Stage 2: Teaching	• Phases both teaching processes have in common. Development of those aspects, the two processes have in common. • Differential phase of both teaching processes. Development of differential aspects in each of the processes. • Observations. In each study, two observers wrote detailed reports on what happened and decided whether this fitted or not with the expected methodological development.

[7] It is not possible, for example, to carry out successive cycles of research in action in the same subject or to dedicate many hours to research tasks in detriment to the curricular design.

[8] It is important to highlight the interest of the Department of Applied Mathematics at the University of Málaga in this research to improve the didactic process and improve the quality of training of future engineers.

	• DERIVE files. At the end of the experimental process, the students recorded all that has happened throughout the process in DERIVE files (See Figures 2. and 3.)
Stage 3: Post-test	• Evaluation test (post-test). A commonly used written test was applied to both groups at the end of the teaching processes in order to evaluate the performance levels reached.
Stage 4: Surveys and Interviews	• Surveys. Once the teaching processes had finished, the students took a survey so as to obtain complementary information on the development of the experience. • Interviews. The students were interviewed individually to gather information on attitudes, motivation, level of interest, suitability of teaching method applied, negative effects of the experimental process and to check data consistency.

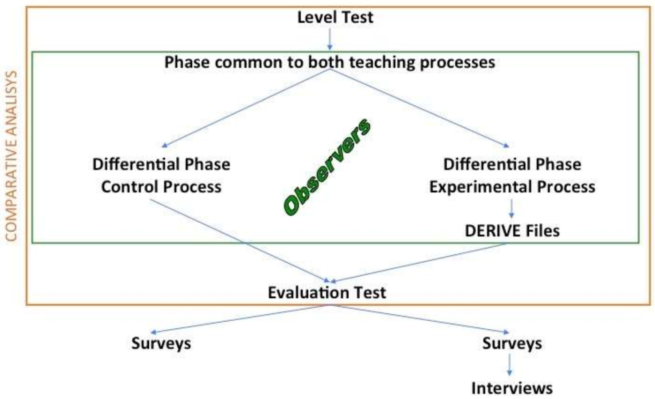

Figure 1 Diagram of studies carried out

The instruments discussed, with the exception of the DERIVE files and interviews, were applied to both groups, experimental and control in each of the three studies.

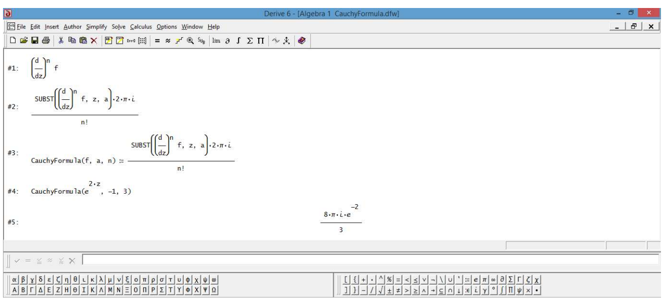

Figure 2 Command programming task carried out by students

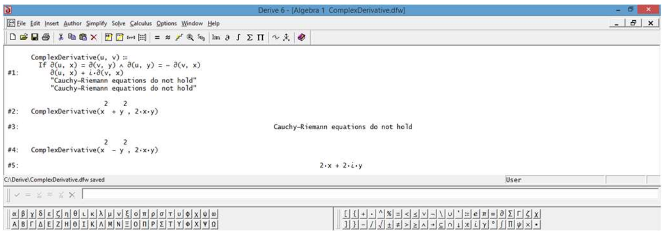

Figure 3 Command programming task carried out by students

5. RESULTS AND CONCLUSIONS

The specific software considered is not relevant for the process. Any CAS which allows programming can be considered. In the three studies, the CAS considered was DERIVE since the authors are expert in the use of this CAS in both, Mathematics Educations (Aguilera-Venegas, G. et. al., 2015; Aguilera-Venegas, G. et. al., 2017) and Engineering applications (Aguilera, G. et. al. 2010; Aguilera, G. et. al. 2014; Galán-García, J.L, et. al. 2014; Galán-García, J.L. et. al. 2018).

The process followed has yielded the following results and conclusions (commons in the three studies):

- It is possible to implement, in accordance with the principles and conditions of the established theoretical framework, a mixed teaching method, comprising board lessons and I.T laboratory classes based on the DERIVE programs.
- The performance of the students significantly improved in all cases.
- The mixed teaching method: a) helps in learning concepts and procedures; b) improves the attitude of the students in terms of attention, motivation, interest and participation; c) is compatible with the usual design and curricular development; d) improvement of the mastery and proficiency of the students in the subjects' content, measured through objective tests, designed with the usual criteria applied; e) favours the leading role of the students themselves, compared to technological means and increases their degree of participation and levels of independence in the work carried out in the class.
- In all three studies the following conditions were met: With respect to the reality of the classroom, zero prejudice, innovative intent, compatibility of the design with regard to the directives and guidelines of the department, durability and robustness of the innovations implemented, assessing the phenomena as a whole and in all their complexity, active participation of teachers, both in their own, natural groups and in collaboration with each other.
- In regard to the purpose of this line of research: The reality of the situation was analysed without any extraneous manipulations; viable and compatible changes were made to the curriculum; tasks and novel uses for them were introduced so as to optimise the process and improve the results; knowledge based on curricular phenomena was obtained and disseminated; the involvement of the teachers was successfully achieved in

the research into teaching practice itself as the core to generating innovation and positioned professional knowledge.

It is evident that ideal conditions for developing research as it should be, are not to be found in normal mathematics classrooms. The education system, the participants, the centres and the classrooms are not yet prepared for research into the actual practice as it is unfolding. Having said which, we believe that the classroom is in fact the ideal place with the ideal conditions for carrying out "applied" research on Mathematics Education. Without doubt, this type of setting has added difficulties due to the limitations of the actual process, so there needs to be changes made which help and enhance this type of research. However, we cannot underestimate the value of isolated studies on performance, difficulties, problem studies, behaviour in class or text books, carried out under laboratory conditions and focusing on only one part of the phenomena observed. They are worthy of consideration and should be taken into account in subsequent studies.

When we talk about the limitations and difficulties observed in the studies we are referring to:

- Real conditions, the requirements demanded by the official guidelines and having to design curriculums for complex core subjects, leave very little margin to carry out the tests and complicate the implementation of complete processes of action research. However, working in groups has allowed studies to be carried out which could be considered as cycles of an overall process of this type from a methodological point of view and taking into account the most common aspects.
- The resulting extension of each of the three studies was due to the restrictions imposed by the theoretical framework of the research. Despite this, the strength of the common process of inquiry has permitted us to examine, with some confidence, new subjects and curricular changes.

Lastly, in relation to the continuation and future prospects of these studies and this line of research, we consider it important to highlight the following aspects:

- The studies should be repeated with new groups of students, new subject areas and with the participation of other teachers of these subjects
- Studies should be repeated in other degree courses with a comparative analysis of the results.
- A more in depth look at the qualitative aspects of curricular development is necessary.
- A deeper look at the reasons for the changes detected: learning, characteristics, types, evolution, etc.; influence of the programme and the tasks themselves have on performance; analysis of other possible factors.
- Research into other curricular changes with respect to: evaluation, methodology, tasks, applications, use of new technologies from other angles (Internet, etc.).

REFERENCES

Aguilera, G. et. al. (2010). Automated generation of contrapuntal musical compositions using probabilistic logic in Deirve. *Mathematics and Computer in Simulation* 80 (6) 1200–1211. DOI: http://dx.doi.org/10.1016/j.matcom.2009.04.012

Aguilera, G. et. al. (2014). An accelerated-time simulation of car traffic on a motorway using a CAS. *Mathematics and Computer in Simulation* 104 21–30. DOI: http://dx.doi.org/10.1016/j.matcom.2012.03.010

Aguilera-Venegas, G. et. al. (2015). Teaching semantic tableaux method for propositional classical logic with a CAS. *International Journal for Technology in Mathematics Education 22*(2) 85–92.

Aguilera-Venegas, G. et. al. (2017). Parametrization of curves and line integrals with a CAS. *International Journal for Technology in Mathematics Education 24*(4), 179–190.

Arzarello, F. (1999). Linee di tendenza della ricerca per l'innovazione in Italia. *Actas Jornadas Hispano-Luso-Italianas de Educación Matemática*. Escola de Verao. Santarem. Portugal.

Bartolini (1998). Linee di tendenza della ricerca per l'innovazione in Italia: Analisi di un caso pa radigmatico. *Actas Jornadas Hispano-Luso-Italianas de Educación Matemática*. Escola de Verao. Santarem. Portugal.

Bartolini, M.; Boni, M.; Ferri, F.; Garuti, R. (1999). Early Approach to Theoretical Thinking: Gears in Primary School. *Educational Studies in Mathematics* 39, 67–87.

Becerra et. al. (2012). Renovación de la Enseñanza Universitaria Basada en Evidencias. Una metodología de acción flexible. *Perfiles Educativos* v. XXXIV núm. 135, pp. 62-77.

Bisquerra, R. (1989).- Métodos de investigación educativa. Barcelona: CEAC.

Case, J.; Light, G. (2011). Emerging methodologies in Engineering Education Research. Journal of Engineering Education, v. 100, nº 1, pp. 186-210.

Castro, W. F.; Godino, J. D. (2011). Métodos mixtos de investigación en las contribuciones a los Simposios de ls SEIEM (1997-2010). En: M. Marín et al. (eds.): *Investigación en Educación Matemática SEIEM XV*. Pp. 99-117. Ciudad Real.

Cohen y Manion (1990). *Métodos de investigación educativa*. La Muralla. Madrid.

Confrey, J. (2006). The evolution of design studies as methodology. En R.K. Sawyer (Ed.), *The Cambridge Handbook of the Learning Sciences* (pp. 135-152). New York, NY: Cambridge University Press.

Coriat, M. (1997a). Materiales, recursos y actividades: un panorama. En: Rico, L. (coord.) *La Educación Matemática en la Enseñanza Secundaria*. Vol. 12. ICE Universidad Barcelona.

Creswell, J. W. (2003). *Research Dessign: Qualitative, quantitative and mixes approaches*, Thousand Oaks, CA: Sage.

Fernández-Lajusticia, A.; Puig, L. (2002). Una actividad matemática organizada en el marco de los modelos teóricos locales: Razón y proporción en la Escuela Primaria. En Murillo y otros (Eds.). *Actas del VI Simposio de la Sociedad Española de Investigación en Educación Matemática (SEIEM)*, pp. 29-46. Universidad de la Rioja. Logroño.

Filloy, E.; Puig, L. y Rojano, T. (2008). El estudio teórico local del desarrollo de competencias algebraicas. *Enseñanza de las Ciencias*, *25*(3), pp. 327-342

Freudhental, H. (1983). *Didactical phenomenology of Mathematical Structures*. Dordrecht Reidel.

Galán, J. L. (2003). Integrales Múltiples con Derive. Un estudio de innovación curricular en primer curso de Ingeniería Técnica de Telecomunicaciones. PhD Thesis. University of Málaga, Málaga, Spain. I.S.B.N.: 84-688-8421-9.

Galán-García, J.L. et. al. (2014). An accelerated-time simulation for traffic flow in a smart city. *Journal of Computational and Applied Mathematics* 270, 557–563. DOI: http:// dx.doi.org/10.1016/j.cam.2013.11.020

Galán-García, J.L. et. al. (2018). Improving CAS capabilities: New rules for computing improper integrals. *Applied Mathematics & Computation* 316, 525–540. DOI: http://dx.doi.org/10.1016/j.amc.2016.12.024

Gallardo, J.; González, J. L., (2012). El Análisis Didáctico como método para el tratamiento de los antecedentes bibliográficos en la investigación en Educación Matemática. Capítulo en: Lupiañez, J. L. (ed.). *El Análisis Didáctico y la investigación en Educación Matemática*. En prensa.

Goetz y Lecompte, (1988). *Etnografía y diseño cualitativo en investigación educativa*. Madrid: Morata.

González, J. L. (1999a). Comentario a la Ponencia "Linee di tendenza della ricerca per l'innovazione in Italia: Analisi di un caso paradigmatico". *Actas Jornadas Hispano-Luso-Italianas de Educación Matemática*. Escola da Verao. Santarem. Portugal.

González, J. L. (1999b). Contribución al panel "Qualidade da Investigaçao": Relevancia de la investigación para la calidad de la enseñanza. *Actas Jornadas Hispano-Luso-Italianas de Educación Matemática*. Escola da Verao. Santarem. Portugal.

González, J. L. (1999c). Didactical Analysis: A non-empirical qualitative method for research in Mathematics Education. En: Schwank, I. (edit.). *Proceedings of European Research in Mathematics Education I (ERME I)*. II: Group 7. Osnabrüeck, Alemania.

González, J. L. (1999d). Estructura y organización curricular. En: *Proyecto Docente*. Documento inédito en poder del autor. Departamento de Didáctica de la Matemática, de las Ciencias Sociales y de las Ciencias Experimentales. Universidad de Málaga.

González, J. L. (2000). Aproximación a un marco teórico y metodológico específico para la investigación en Educación Matemática. En: *Actas III Simposio de la Sociedad Española de Investigación en Educación Matemática (SEIEM)*, páginas 213–225. Valladolid.

Goyette y Lessard Hébert, (1988). *La investigación-acción*. Barcelona: Laertes.

Gravemeijer, K. & Van Eerde, D. (2009). Design Research as a means for building a knowledge base for teachers and teaching in Mathematics Education. *The Elementary School Journal,* v. 109 (5).

Grouws, D. (1992). *Handbook of Research on Mathematics Teaching and Learning*. Mac Millan Publishing Company, New York.

Guba, E. G.; Lincoln, (1985). *Naturalistic Inquiry*. Sage Publication Inc. California. USA.

Harel, G. (2006). Mathematics Education Research, its nature and its purpose. A discussion on Lester paper. *Zentralblatt für Didáktik der Mathematik (ZDM), 38*(1), pp. 58-62.

Hart, L.; Smith, S.; Swars, S.; Smith, M. (2009). An examination of research methods in Mathematics Education (1995-2005). *Journal of mixed methods research*, v. 3, nº 1, pp. 26-41.

Hiebert, H.; Grouws, D. (2007). The effects of classroom mathematics teaching on student`s learning. En: F. K. Lester (eds.). *Second Handbook of research on mathematics teaching and learning*, pp. 371-404. Charlotte N. C.: NCTM and IAP.

ICME-3 (1976). Hermann Athen and Heinz, Kunle, (eds.): *Proceedings of the Third International Congress on Mathematical Education. Karlsruhe* (Germany).

Johnson, R. B.; Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *19*(4), 2-9.

Kelly, A. E.; Lesh, R. A.; Baek, J. Y. (2008). *Handbook of Design Research Methods in Education. Innovations in Science, Technology, Engineering and Mathematics Learning and Teaching.* New York: Routledge.

Larrubia, J. J. (2006). *La resolución de ecuaciones de segundo grado con puzzle algebraico. Un estudio en 3º curso de Educación Secundaria Obligatoria*. PhD Thesis. University of Málaga, Málaga, Spain.

Lawton, (1986). *Curriculum studies and Educational Planning*. Hodder and Stoughton, London.

Molina, M.; Castro, E-; Molina, J. L. (2011). Un acercamiento a la investigación de diseño a través de los experimentos de enseñanza. *Enseñanza de las Ciencias*, *29*(1), pp. 75-88

Ortiz, A. (1998). Entrevistas semiestructuradas en la investigación en Educación Matemática. Newstead, K. and Olivier, A. (edit.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education (PME22).* Stellenbosch (South Africa).

Padilla, Y. (2003). *Integrales de Línea con Derive. Un estudio de innovación curricular en primer curso de Ingeniería Técnica de Telecomunicaciones*. PhD Thesis. University of Málaga, Málaga, Spain. I.S.B.N.: 84-688-8420-0.

Puig, L. (1997). Análisis Fenomenológico. En: Rico, L. (coord.) *La Educación Matemática en la Enseñanza Secundaria*. Vol. 12. ICE Universidad Barcelona.

Puig, L. (2008). Sentido y elaboración del componente de competencia de los modelos teóricos locales en la investigación de la enseñanza y aprendizaje de contenidos matemáticos específicos. *PNA*, *2*(3), pp. 87-107.

Raymond, A. (1994). Collaborative Action Research in Mathematics Education: A tale of two teacher-researcher. *PME XVI*: Baton Rouge LA. Nº 141.

Rico, L. (1997a). *Bases Teóricas del Currículo de Matemáticas en Educación Secundaria*. Madrid: Síntesis.

Rico, L. (1998). Concepto de Currículum desde la Educación Matemática. *Revista de Estudios del Currículum*, *1*(4):7–42.

Rodríguez, P. (2004). *Derivación e integración de funciones de variable compleja con DERIVE: Un estudio de innovación curricular en segundo curso de Ingeniería Técnica de Telecomunicación*. PhD Thesis. University of Málaga, Málaga, Spain. I.S.B.N.: 84-688-9473-7.

Romberg, T. (1991). Características problemáticas del currículum escolar de Matemáticas. *Revista de Educación*, (294), 323–406

Romberg, T. (1992). Perspectives on Scholarship and Research methods. En: Grouws, D. (edit.), *Handbook of Research on Mathematics Teaching and Learning*, pp. 49–64. Mac Millan, Nueva York.

Sierra, M. (1999). Comentario a la Ponencia “Linee di tendenza della ricerca per l'innovazione in Italia: Analisi di un caso paradigmatico”. *Actas Jornadas Hispano-Luso-Italianas de Educación Matemática*. Escola de Verao. Santarem. Portugal

Steffe, L. P.; Thompson, P. W.; von Glasersfeld (2000). Teaching experiment methodology: Underlying principles and essential elements. En: A. E. Kelly y R. A. Lesh (Eds.). *Handbook of research design in Mathematics and Science Education* (256-306). Mahwah: Erlbaum.

Stenhouse, L. (1984). *Investigación y desarrollo del currículo*. Morata. Madrid

Stenhouse, L. (1987). *La investigación como base de la enseñanza*. Morata. Madrid.

Taylor y Bogdan, (1986). *Introducción a los métodos cualitativos de investigación*. Paidós. Buenos Aires.

BIOGRAPHICAL NOTES

Dr. José Luis Galán-García, Dr. José Luis González-Marí and Dr. Pedro Rodríguez-Cielos are Associate Professors and Dra. Yolanda Padilla-Domínguez is an Assistant Professor of different mathematic subjects in Engineering at the University of Málaga, Spain. One of their research interests is the use of CAS in mathematics lectures in the different Engineering degrees they teach. Dr. José Luis Galán-García, Dra. Yolanda Padilla Domínguez and Dr. Pedro Rodríguez-Cielos obtained their Ph.D Degrees in Mathematics Education with Dr. José Luis González-Marí as supervisor. The authors have presented more than 100 talks in different International Conferences on the topic “CAS in Math subjects in Engineering” and several papers on different uses of CAS (both, in Education and in Engineering applications) in different relevant journals.