Entry Deterrence When the Potential Entrant is Your Competitor in a Different Market

Miguel Ángel Ropero

Department of Applied Economics, Faculty of Economics, University of Malaga, El Ejido, 6, 29071, Malaga, Spain

E-mail address: maropero@uma.es

Telephone number: +34 952 13 11 84

Abstract

In this article, we present a two-period model in which one firm operates in two markets: a monopoly and a duopoly. Assuming that this firm has private information on the cross-price elasticity of demand between the products sold in both markets, it limits its quantity supplied in the monopoly market in order to make its rival in the other market believe that entry into the monopolized market is unprofitable. As a result of this strategy, the average prices observed in both markets increase. This result suggests that the detrimental effects of entry deterrence on consumers´ welfare are stronger than those predicted by previous literature.

Keywords: Entry deterrence, pooling equilibrium, separating equilibrium, signalling game, undefeated equilibrium.

JEL Classification: D43, D82, L13.

1. Introduction

1.1. Motivation

When a firm considers expanding its business into a new market, the incumbent in that market will usually be a firm that is already competing with the potential entrant in a different market. This type of potential competition may arise when the patent for a product expires. For example, imagine that a company produces a patented board game A and at the same time, it competes with a rival in the market for a different game B, which is not patented. In this context, the cross-price elasticity of demand between both games is likely to be positive because consumers would substitute a game with the other when the price of the former increases. We can expect that the seller of both games will have more information on that cross-price elasticity of demand than the company selling only game B. When the patent of game A expires, its producer will have incentives to take advantage of its private information in order to prevent its rival in market B from entering market A.

Likewise, suppose that an airline is a monopolist on a route A connecting two cities, but it competes with another airline on a different route B, which connects one of the airports of route A with a third city. In this scenario, after selling flights on routes A and B, the monopolist is likely to have more information on passengers´ perception of the degree of substitutability between those flights than the airline which only operates on route B. Once again, the company flying on route A might try to convince its rival on route B that entry into the monopolized market is unprofitable. Hereafter, the monopolist in market A will be referred to as the "incumbent".

In this type of situations, we investigate whether the incumbent should increase or decrease its quantity sold in the monopoly market to deter entry. Additionally, we study the effects of entry deterrence on the prices observed in the threatened market and in the market where the incumbent is already competing with the potential entrant.

In order to accomplish this, this article analyses a two-period model in which a firm is a monopolist in market A and competes *a la* Cournot with another firm in market B.

Furthermore, the monopolist has private information of the cross-price elasticity of demand between the products sold in markets A and B. At the beginning of period 2, the potential entrant decides whether to enter market A or not after observing the prices and quantities sold in period 1. When the products are weak substitutes, entry into market A is profitable, but we find that the informed monopolist chooses the quantities in period 1 it would have chosen if the products were strong substitutes. Therefore, the incumbent constrains its quantities supplied in order to make the potential entrant believe that entry into market A is not profitable. As a result, not only will the price in the threatened monopoly market increase, but also the price in the duopoly market. In our model, the strategy used by the incumbent to deter entry will only be successful when the prices are sufficiently noisy. Otherwise, the potential entrant may learn that entry is profitable at the beginning of period 2 by simply observing the prices in period 1.

1.2. Related Literature and Contribution

Previous theoretical literature has addressed the monopolist´s use of its price set in the market to convince a potential competitor that it is unprofitable to enter the market. The monopolist´s price which deters the potential entrant from entering the market is known as the limit price. This branch of research typically assumes that the potential entrant is uncertain about the unit cost of production and depending on the specific assumptions about that uncertainty, some authors suggest that the monopolist chooses a lower price than the myopic optimal price to deter the entry of another competitor (Milgrom and Roberts, 1982; Toxvaerd, 2017), whereas other authors find the contrary (Harrington, 1986, 1987; Salonen, 1994; Jun and Park, 2010; Granero and Ordóñez de Haro, 2015)[1].

[1] Bagwell and Ramey (1991) propose a model of limit pricing in which they found that the pre-entry prices are not distorted.

In contrast to these articles, we assume that the potential entrant knows the cost of production because it is already competing with the incumbent in a different duopoly market. In our model, asymmetric information arises because the potential entrant is unaware of the cross-price elasticity of demand between the products sold in the duopoly and the monopoly markets before entering the latter. Moreover, the drop in the profit made by the potential entrant in the duopoly caused by the entry into the monopoly will be part of the cost of entry and it will depend on the incumbent´s reaction to entry. Thus, unlike previous research, the cost of entry will be endogenous in our model.

Furthermore, unlike the literature described above, our model suggests that empirical researchers on limit pricing should estimate not only the effects of the threat of entry on the price set in the threatened market, but also in other markets where the incumbent and the potential entrant are already competing with each other. In fact, empirical research on limit pricing tend to identify the threat of entry into a market with the presence of a competitor in a nearby market, but the authors do not estimate the effects of limit pricing in the threatened market on the price in other markets. For example, Goolsbee and Syverson (2008) and Gedge, Roberts, and Sweeting (2013) measured the threat of entry to a route between airports A and B using the presence of another airline in airports A and B separately. Similarly, Seamans (2013) used the presence of a company serving an adjacent area to identify the threat of entry in the cable TV industry. In all these cases, the potential entrant is already competing with the incumbent in nearby markets. As the incumbent is the only company selling its product in the threatened and the nearby markets, it may have some private information on the cross-price elasticity of demand between the products sold in both markets. In this context, our model predicts an increase in the price observed in both markets as a result of entry deterrence and a higher probability of entry deterrence when the prices observed are noisier.

Our results also bear some resemblance to those obtained by McGahan (1993) and Comino (2006). Under demand uncertainty, McGahan shows that a first mover in a market may not need to overinvest in capacity in order to discourage a second mover from a large-scale investment in the second period. In the same vein, Comino found that a first mover refrains from entering a profitable market in one period in order to make a second mover postpone its entry decision. In contrast to those models in which the second mover enters the market despite the strategy used by the first mover, the incumbent in our model uses its decisions in the first period in order to completely deter entry in the second period.

This article is also related to the literature on multimarket contact, in which firms also compete against one another in many markets, which usually leads firms to anticompetitive outcomes[2]. For example, Schmalensee (1978) and Judd (1985) developed a theoretical model in which a firm decides whether to enter or not different markets before a second mover. In this setting, Schmalensee (1978) found that the first mover introduces several similar products to leave no profitable niche for any additional entrant, whereas Judd (1985) only considers two possible markets and obtains that the first mover enters one of the markets, whereas the second enters the other, but he shows that the first mover cannot prevent entry by operating in all markets. Similarly, Bernheim and Whinston (1990) and Ciliberto and Williams (2014) show that multimarket contact may be a facilitator of tacit collusion and Feinberg (2013) found that exporters meeting rivals in other markets are reluctant to enter the home market of their rivals to avoid retaliation.

In all those models of multimarket contact, a firm uses certain strategies in one market to persuade its rival to be less aggressive in a different market, but there is no signal jamming

[2] Feinberg (1984) shows that firms constrain their quantities in a market for fear of retaliation in a different market where both firms compete with each other.

because the information is symmetric. However, in our model, the incumbent firm has private information on some market parameter and manipulates some market outcomes in order to interfere its rival´s updating process and prevent entry.

This article is organized as follows. The next section describes the model. In order to show what happens in the second period when the potential entrant enters the market, section 3 obtains the equilibrium of a one-period game with complete information and two firms competing with each other *a la* Cournot in two duopoly markets. Section 4 analyses what happens in the second period when the potential entrant stays out of the market, in which case we need to solve a one-period game in which two firms compete in a market, but one of those firms is a monopolist in a different market and is the only firm that knows the degree of substitutability between the products sold in both markets. Section 5 uses the equilibria of Sections 3 and 4 in order to obtain the consequences of entry deterrence in our model. Finally, Section 6 summarizes the main conclusions and the Appendix includes the proofs of our propositions.

2. The Model

In this section, we present the model. Firm 1 is a monopoly in market A, in which the demand curve is[3]:

$$p_t^A = \alpha - \beta Q_t^A - \rho Q_t^B + \xi_t \quad (1)$$

where p_t^j is the price in period t in market j, Q_t^j is the total quantity sold in period t in market j, $j \epsilon \{A, B\}$ and $t \epsilon \{1,2\}$, whereas α, β, and ρ are parameters which are stable over

[3] We borrow the specification of the demand curves from Singh and Vives (1984), but we assume that the intercept and the slope of the demand curves are the same in both markets. As shown in the online appendix, the results are robust to different values of those parameters and to a discount factor different from 1.

time with $\alpha, \beta, \rho > 0$ and $\beta > \rho$. Finally, ξ_t is a specific demand shock in market A in period *t* with a uniform distribution function, $\xi_t \sim U(-\bar{\xi}_A, \bar{\xi}_A)$.

Firm 1 also competes with firm 2 in market B, where both firms offer homogeneous goods. The demand curve in market B is:

$$p_t^B = \alpha - \beta Q_t^B - \rho Q_t^A + \varepsilon_t \tag{2}$$

where ε_t is the specific demand shock in market B with a uniform distribution function, $\varepsilon_t \sim U(-\bar{\varepsilon}_B, \bar{\varepsilon}_B)$. ξ_t and ε_t are statistically independent. In this set-up, ρ measures the degree of substitutability between the products sold in markets A and B. The greater ρ, the greater the cross-price elasticity of demand between both types of products in absolute value. The products sold in markets A and B are substitutes because $\rho > 0$.

The timing of the information is as follows. In period 0, nature chooses ρ which can take only two values[4]: $\rho \in \{\rho_L, \rho_H\}$, where $0 < \rho_L < \rho_H < \beta$. θ is the prior probability of ρ being equal to ρ_L and is common knowledge. In period 1, firm 1 is the only firm which observes the value of ρ before choosing its quantities supplied[5]. In period 2, each firm will observe the prices and quantities supplied in both markets in period 1 before making its decisions and firm 2 will decide whether to enter market A or not. If it enters the market, it will incur a cost, K, which may represent a lump-sum tax or a fixed cost incurred when

[4] We obtain the same type of equilibria with complements, $\rho < 0$. The results are available from the author upon request.

[5] The potential entrant knows α and β because it has experience selling a similar product in market B.

a company offers a new product. If the potential entrant enters market A, it will learn the true[6] value of ρ before making its quantity-decisions.

Firms compete *a la* Cournot in both periods and the constant unit cost of production, c, is the same for products A and B and for both firms. For the sake of simplicity, we assume that the discount factor is equal to 1 for both firms and this is common knowledge.

In this model, as firm 2 uses the realizations of the prices and quantities in period 1 in order to learn the true value of ρ, firm 1 has incentives to distort its quantity chosen in period 1 in order to interfere firm 2´s learning process and deter entry into market A. Under the above assumptions, this type of interference would be impossible when $\overline{\xi}_A$ and $\overline{\varepsilon}_B$ are too low because firm 2 would always learn that the products are weak substitutes $(\rho = \rho_L)$ when the prices observed in both markets in period 1 are high and they are strong substitutes $(\rho = \rho_H)$ when the prices are low. We assume that $\overline{\xi}_A > \frac{(\rho_H-\rho_L)(\alpha-c)(4\beta+\rho_H)}{6\beta(\beta+\rho_H)}$ and $\overline{\varepsilon}_B > \frac{(\rho_H-\rho_L)(\alpha-c)}{2(\beta+\rho_H)}$ to guarantee that firm 1 is able to jam the signal provided by the prices in period 1 and entry deterrence is possible.

The concept of equilibrium considered is a sequential equilibrium as defined by Kreps and Wilson (1982). Thus, the equilibrium must satisfy two conditions. First, each firm´s strategy is a best response to its rival´s strategy. Second, the uninformed firm´s beliefs in period 2 have to be consistent with Bayes´ rule in equilibrium. As those beliefs are

[6] In order to explain why the potential entrant observes ρ just after entering market A, let us assume that K includes the price paid for market research estimating the cross-price elasticity of demand. The consultancy which conducts that research sold this piece of information to firm 1 at the beginning of period 0 and in order to preserve the value of that information, the consultancy committed to selling the information on ρ to only those firms which are ready to compete in both markets.

unrestricted outside the equilibrium, there can be multiple sequential equilibria and we choose the unique equilibrium that survives the refinement proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993), which is known as undefeated equilibrium.

In this setting, an informed firm´s strategy specifies a quantity supplied in both markets and in both periods for each value of ρ. This strategy can be represented by the function:

$$s: \{\rho_L, \rho_H\} \times [0, +\infty)^4 \rightarrow [0, +\infty)^8$$

where $s(\rho_j, Q_1^A, Q_1^B, p_1^A, p_1^B) = \left(q_{i1}^j(\rho_j), q_{i2}^j(\rho_j, Q_1^A, Q_1^B, p_1^A, p_1^B)\right)$ and $q_{it}^j(\rho_j)$ is the quantity supplied by firm i in market j in period t when the true value of the unknown parameter is ρ_j. As the potential entrant updates its information on ρ at the beginning of period 2 by observing the prices and quantities in period 1, it will condition its decisions in period 2 on the previous market outcomes. Then, firm 1 anticipates this learning process and also conditions its quantities supplied in period 2, $q_{i2}^j(\rho_j, Q_1^A, Q_1^B, p_1^A, p_1^B)$, on the prices and quantities in period 1.

Similarly, firm 2´s strategy specifies its quantity supplied in market B in period 1, its entry decision and its quantities supplied in markets A and B in period 2 depending on its information in each period. This strategy can be represented by the function:

$$t: [0,1] \rightarrow [0, +\infty)^5$$

Where $t(\hat{\theta}) = \left(q_{21}^B, e_{22}^A(\hat{\theta}), q_{22}^{B0}(\hat{\theta}), q_{22}^{A1}(\hat{\theta}), q_{22}^{B1}(\hat{\theta})\right)$. Let us explain the notation used. In period 1, firm 2 only chooses its quantity supplied in market B given its prior information, q_{21}^B. In period 2, this firm will observe the prices and quantities in period 1 and will assign a posterior probability to $\rho = \rho_L$, which is represented by $\hat{\theta}$. There are three possible scenarios. First, when the prices in period 1 are so high that firm 2 learns that the products are weak substitutes ($\rho = \rho_L$), then $\hat{\theta} = 1$ and firm 2 enters market A. Second, when the prices in period 1 are so low that firm 2 learns that the products are strong substitutes ($\rho = \rho_H$), then $\hat{\theta} = 0$ and firm 2 does not enter market A. Finally, when the prices in period 1 take intermediate values, the potential entrant´s capacity to learn the unknown parameter will depend on the incumbent´s quantities chosen in the first period. Specifically, if the incumbent chooses the same quantities regardless of the value of ρ, firm 2 will receive no new information at the beginning of period 2, in which case $\hat{\theta} = \theta$ and firm 2 will not enter market A. If firm 1´s quantities chosen in period 1 are different for each value of ρ, these decisions will convey information on that parameter to firm 2, which will enter the market when $\rho = \rho_L$ ($\hat{\theta} = 1$) and stay out when $\rho = \rho_H$ ($\hat{\theta} = 0$). Thus, $\hat{\theta}$ summarizes all the new information gathered by firm 2 in period 1 and its entry decision, $e_{22}^{A}(\hat{\theta}) \in \{0,1\}$, depends on that parameter.

For similar reasons, firm 2´s quantities supplied in period 2 will also depend on $\hat{\theta}$: If the potential entrant does not enter market A, it will only choose its quantity sold in market B, $q_{22}^{B0}(\hat{\theta})$, but if it enters the market, it will learn the true value of the cross-price elasticity of demand and choose the quantities in both markets depending on whether $\rho = \rho_L$, *i.e.*, $q_{22}^{A1}(1), q_{22}^{B1}(1)$, or $\rho = \rho_H$, *i.e.*, $q_{22}^{A1}(0), q_{22}^{B1}(0)$.

3. Static Cournot Competition in Two Duopoly Markets with Complete Information

Hereafter, our main purpose is to determine the conditions under which a pooling equilibrium with entry deterrence will arise. In this equilibrium, the incumbent chooses the same quantities in both markets in period 1 irrespective of the true value of the unknown parameter. As a result of this strategy, the potential entrant will stay out of market A in period 2 when it does not learn anything new about ρ from the prices in period 1 and the prior probability of strong substitutes ($\rho = \rho_H$) is sufficiently high.

Despite the incumbent´s attempt to hide its private information, the potential entrant will learn the cross-price elasticity of demand when the prices take extreme values. Therefore, we will show that there will always be a non-negative probability that the incumbent fails to deter entry under the assumptions of the model.

In order to use backward-induction, in this section, we show the market outcomes in period 2 when firm 2 enters market A and learns ρ. In this case, both firms compete with perfect information. Then, each firm *i* will choose its quantities sold to maximize its total profit given the quantities chosen by firm *j*. That is:

$$\max_{q_{i2}^A, q_{i2}^B} E(\pi_{i2}) = \left[\alpha - \beta\left(q_{i2}^A + q_{j2}^A\right) - \rho\left(q_{i2}^B + q_{j2}^B\right) - c\right] q_{i2}^A$$
$$+ \left[\alpha - \beta\left(q_{i2}^B + q_{j2}^B\right) - \rho\left(q_{i2}^A + q_{j2}^A\right) - c\right] q_{i2}^B$$

Where $i, j \in \{1,2\}$. If we use the first-order conditions of these problems[7], we obtain the optimum quantities sold in these duopoly markets with perfect information:

$$q_{i2}^{Pj} = \frac{(\alpha - c)}{3(\beta + \rho)} \qquad (3)$$

Where q_{i2}^{Pj} denotes the optimum quantity chosen by firm *i* in market *j* in period 2 when the potential entrant enters market A and both firms compete with perfect information. *P* in the superscript stands for perfect information. As shown in equation (3), when the cross-price elasticity of demand between the products sold in both markets (ρ) increases, each firm has incentives to reduce its quantity supplied in a market to avoid cannibalizing its own profit in the other market.

Using the demand equations (1) and (2), we obtain the expected prices in both markets:

[7] The second order conditions are satisfied because $\beta > \rho$.

$$E(p_2^{PA}) = E(p_2^{PB}) = \frac{\alpha+2c}{3} \quad (4)$$

Finally, the expected total profit obtained by each firm in both markets is:

$$E\left(\pi_{i2}^A + \pi_{i2}^B\right) = \frac{2(\alpha-c)^2}{9(\beta+\rho)} \quad (5)$$

4. Static Cournot Competition in a Duopoly Market and a Monopoly with Asymmetric Information

At the beginning of period 2, firm 2 decides whether to enter market A or not. When the products sold in both markets are weak substitutes ($\rho = \rho_L$), but the incumbent chooses its quantities supplied in period 1 as if $\rho = \rho_H$, those quantities do not reveal the true value of the unknown parameter. Under our assumptions of the distributions of ξ_t and ε_t, when the prices observed in period 1 take intermediate values, firm 2 does not deduce anything new from the prices and quantities in period 1 and the posterior beliefs are equal to the prior beliefs. Then, firm 2 prefers to stay out of market A in that situation. This section determines the market outcomes in the second period in this scenario in which the incumbent deters entry.

When $\rho = \rho_k$, firm 1 chooses its quantities in order to maximize its expected profit:

$$\max_{q_{12}^A(\rho_k),q_{12}^B(\rho_k)} E(\pi_{12}|\rho = \rho_k)$$

$$= [\alpha - \beta q_{12}^A(\rho_k) - \rho_k(q_{12}^B(\rho_k) + q_{22}^B) - c]q_{12}^A(\rho_k)$$

$$+ [\alpha - \beta(q_{12}^B(\rho_k) + q_{22}^B) - \rho_k q_{12}^A(\rho_k) - c]q_{12}^B(\rho_k)$$

Where $k \in \{L, H\}$. Similarly, firm 2 chooses its quantity in market B to maximize its expected profit without knowing the true value of ρ:

$$\max_{q_{22}^B} E(\pi_{22}) = \theta[\alpha - \beta(q_{12}^B(\rho_L) + q_{22}^B) - \rho_L q_{12}^A(\rho_L) - c]q_{22}^B$$

$$+ (1-\theta)[\alpha - \beta(q_{12}^B(\rho_H) + q_{22}^B) - \rho_H q_{12}^A(\rho_H) - c]q_{22}^B$$

Thus, using the first-order conditions[8], we obtain the optimum quantities supplied:

$$q_{12}^{IA}(\rho_k) = \frac{(\alpha-c)}{2(\beta+\rho_k)} \tag{6}$$

$$q_{12}^{IB}(\rho_k) = \frac{(\alpha-c)(2\beta-\rho_k)}{6\beta(\beta+\rho_k)} \tag{7}$$

$$q_{22}^{IB} = \frac{(\alpha-c)}{3\beta} \tag{8}$$

Where $q_{12}^{Ij}(\rho_k)$ denotes the optimum quantity chosen by firm 1 in market *j* when $\rho = \rho_k$. Likewise, q_{22}^{IB} is the quantity chosen by firm 2 in market B. As the model described in this section is a game in which one player has private information on ρ, the *I*-superscript in these equations stands for imperfect information.

Finally, when $\rho = \rho_k$, the expected prices in markets A and B will be:

$$E(p_2^{IA}) = \frac{3(\alpha+c)\beta-(\alpha-c)\rho_k}{6\beta} \tag{9}$$

$$E(p_2^{IB}) = \frac{\alpha+2c}{3} \tag{10}$$

Now, we obtain each firm´s expected profit when $\rho = \rho_k$:

$$E(\pi_{12}^I | \rho = \rho_k) = \frac{(\alpha-c)^2(13\beta-5\rho_k)}{36\beta(\beta+\rho_k)} \tag{11}$$

[8] Once again, firm 1´s second order conditions are satisfied because $\beta > \rho$. Additionally, we obtain that $\frac{\partial^2 E(\pi_{22})}{\partial q_{22}^{B2}} = -2\beta < 0$. Thus, the quantities obtained maximize firms´ profits.

$$E(\pi_{22}^{I}|\rho = \rho_k) = \frac{(\alpha - c)^2}{9\beta} \quad (12)$$

In this setting, a rise in the cross price elasticity of demand between the products sold in both markets causes two opposing effects on the expected prices. First, the prices decrease because competition between a company in one market and its rival in the other becomes fiercer (competition effect). Second, as the incumbent in market A competes with itself in market B, it decreases its quantity sold in each market when ρ increases (see equations (6) and (7)) in order to avoid cannibalizing its own profit in the other market and thanks to that drop in the quantities, the expected prices increase (cannibalization effect). In market A, the competition effect outweighs the cannibalization effect and the expected price decreases with ρ, but the price in market B does not change with ρ because both effects counteract each other. Therefore, the incumbent reduces the drop in its profit in market B when the products are stronger substitutes ($\rho = \rho_H$) by limiting its quantities supplied in both markets, but this behaviour also benefits the potential entrant in market B because its profit does not decrease with ρ (see equation 12). As shown in the previous section, if firm 2 enters market A, its profit in market B will decrease with ρ and the potential entrant needs to take it into account in order to decide whether it enters or not.

5. Entry deterrence

This section analyses the conditions under which the incumbent deters entry and the effects of entry deterrence. To start with, let us describe the intuition of our main results.

When the products sold in both markets are weak substitutes ($\rho = \rho_L$), firm 1 offers a lower quantity in market A in order to make its rival believe that the products are close substitutes ($\rho = \rho_H$) and that it is not profitable to enter. As a result of this lower quantity supplied, the price in the threatened market is higher than without the threat of entry as in the models proposed by Harrington (1986, 1987), Salonen (1994), Jun and Park (2010) and Granero and Ordoñez de Haro (2015). Unlike those models, the entrant does not stay out of market A for fear of a high cost, but to avoid a high drop in its profit in market B if the products sold in markets A and B were close substitutes. Interestingly, a higher price in market A also leads to a higher price in market B.

The next proposition shows the conditions under which entry deterrence arises:

PROPOSITION 1. There exist a value of $\overline{\xi}_A$: $\overline{\xi}_m$, *a value of* $\overline{\varepsilon}_B$: $\overline{\varepsilon}_m$, *two values of K:* $\underline{K}$ *and* $\overline{K}$, *and one value of* θ: $\bar{\theta} \in (0,1)$ *such that firm 1 deters entry into market A when* $\overline{\xi}_A > \overline{\xi}_m$, $\overline{\varepsilon}_B > \overline{\varepsilon}_m$, $K \in (\underline{K}, \overline{K})$ *and* $\theta < \bar{\theta}$ *in the unique undefeated pooling equilibrium of the model with the following strategies and beliefs:*

$$s\left(\rho, Q_1^A, Q_1^B, p_1^A, p_1^B\right) = \begin{cases} \left(q_{11}^{Ij}(\rho_H), q_{12}^{Ij}(\rho_L)\right) \ if \ \rho = \rho_L, \ p_1^A(\rho_L, -\overline{\xi}_A) < p_1^A < p_1^A(\rho_H, \overline{\xi}_A) \ and \ p_1^B(\rho_L, -\overline{\varepsilon}_B) < p_1^B < p_1^B(\rho_H, \overline{\varepsilon}_B) \\ \left(q_{11}^{Ij}(\rho_H), q_{12}^{Ij}(\rho_H)\right) \ if \ \rho = \rho_H, \ p_1^A(\rho_L, -\overline{\xi}_A) < p_1^A < p_1^A(\rho_H, \overline{\xi}_A) \ and \ p_1^B(\rho_L, -\overline{\varepsilon}_B) < p_1^B < p_1^B(\rho_H, \overline{\varepsilon}_B) \\ \left(q_{11}^{Ij}(\rho_H), q_{12}^{Pj}(\rho_k)\right) \qquad otherwise \end{cases} \tag{13}$$

$$t(\hat{\theta}) = \begin{cases} \left(q_{21}^{IB}, 0, q_{22}^{PB}, q_{22}^{PA}, q_{22}^{PB}\right) \ if \ \hat{\theta} = 0 \\ \left(q_{21}^{IB}, 0, q_{22}^{IB}, q_{22}^{PA}, q_{22}^{PB}\right) \ if \ \hat{\theta} = \theta \\ \left(q_{21}^{IB}, 1, q_{22}^{PB}, q_{22}^{PA}, q_{22}^{PB}\right) \ if \ \hat{\theta} = 1 \end{cases} \tag{14}$$

$$\hat{\theta} = \begin{cases} 0 \qquad if \ q_{11} = q_{11}^{Ij}(\rho_H) \ and \ \left(p_1^A < p_1^A(\rho_L, -\overline{\xi}_A) \ or \ p_1^B < p_1^B(\rho_L, -\overline{\varepsilon}_B)\right) \\ \theta \ if \ q_{11} = q_{11}^{Ij}(\rho_H), p_1^A(\rho_L, -\overline{\xi}_A) < p_1^A < p_1^A(\rho_H, \overline{\xi}_A) \ and \ p_1^B(\rho_L, -\overline{\varepsilon}_B) < p_1^B < p_1^B(\rho_H, \overline{\varepsilon}_B) \\ 1 \qquad if \ q_{11} = q_{11}^{Ij}(\rho_H) \ and \ \left(p_1^A > p_1^A(\rho_H, \overline{\xi}_A) \ or \ p_1^B > p_1^B(\rho_H, \overline{\varepsilon}_B)\right) \\ 1 \qquad otherwise \end{cases} \tag{15}$$

Where $q_{1t}^{Ij}(\rho_k)$ is the quantity supplied by firm 1 in market *j* in period *t* when $\rho = \rho_k$ and firm 2 does not know the true value of ρ. Likewise, q_{2t}^{IB} is the quantity supplied by firm 2 in market B in period *t* when it does not learn anything about ρ and stays out of market A. All these quantities are chosen as in the one-shot game described in section 4 and for this reason, the superscript *I* stands for imperfect information. Furthermore, $q_{12}^{Pj}(\rho_k)$ is the incumbent´s quantity supplied in market *j* in period 2 when firm 2 learns that $\rho = \rho_k$ from the prices in period 1. Specifically, as firm 2 would only enter market A after learning that $\rho = \rho_L$, $q_{12}^{Pj}(\rho_H)$ is firm 1´s quantity chosen in a one-period game with a monopoly and a duopoly similar to that described in section 4, but assuming that firm 2 knows that $\rho = \rho_H$, and $q_{12}^{Pj}(\rho_L)$ is the quantity chosen by the incumbent in market *j* in period 2 when it competes with its rival in both markets with perfect information as described in Section 3 assuming that $\rho = \rho_L$. Moreover, regardless of whether firm 2 enters market A or not, q_{2t}^{Pj} denotes the quantity chosen by the potential entrant in market *j* in period *t* when it knows the true value of ρ. Then, the superscript P stands for perfect information in all those quantities. Finally, $p_1^j(\rho_k, x)$ is the price in period 1 in market *j* when $\rho = \rho_k$ and the demand shock in market *j* is *x*.

This proposition shows that firm 1 deters entry when the prices are sufficiently noisy, ($\overline{\xi}_A > \overline{\xi}_m$ and $\overline{\varepsilon}_B > \overline{\varepsilon}_m$), the cost of entry is neither too high nor too low ($\underline{K} < K < \overline{K}$) and the prior probability of strong substitutes is sufficiently high ($\theta < \bar{\theta}$). Let us explain the intuition of each condition. When entry is profitable, firm 1 deters entry by choosing the quantities it would have chosen in the scenario in which entry is unprofitable, but despite this strategy, the potential entrant may learn the true state of the world by simply observing the prices in period 1 and enter market A. For this reason, only when the prices are sufficiently noisy ($\overline{\xi}_A > \overline{\xi}_m$ and $\overline{\varepsilon}_B > \overline{\varepsilon}_m$), they will not always reveal the unknown parameter and firm 1 may deter entry. When the cost of entry is too high ($K > \overline{K}$), firm 2 will never find it profitable to enter market A and the incumbent does not need to deter entry. On the contrary, when the cost of entry is too low ($K < \underline{K}$), entry is always profitable, and firm 1 cannot prevent firm 2 from entering the market. Finally, when firm 1 chooses the deterring quantities and the prices in period 1 do not reveal the value of ρ, firm 2 remains uninformed at the beginning of period 2 and bases its decisions on the prior probabilities. Thus, the potential entrant stays out of market A when the prior probability of the worst possible scenario after entry is too high ($\theta < \bar{\theta}$).

Equations (13)-(15) describe the pooling equilibrium obtained. In particular, each row of equation (13) includes the quantities supplied by firm 1 in markets A and B in periods 1 and 2. In equilibrium, when entry is profitable ($\rho = \rho_L$), the incumbent offers its optimum quantities for strong substitutes ($\rho = \rho_H$) in order to signal its rival that entry is unprofitable. Despite this strategy, firm 2 will enter market A if it learns that the products are weak substitutes ($\rho = \rho_L$) from the prices realized in period 1, in which case both firms will compete in both markets in period 2 with perfect information. When the prices in period 1 do not reveal the true value of ρ, firm 1´s strategy is successful and it preserves its monopoly power in market A.

Equation (14) describes the strategy chosen by firm 2 depending on its posterior beliefs. The first component of each row shows the quantity supplied in market B in period 1, the second represents the probability assigned to entering market A and the third includes the quantity in market B in period 2 without entry. Finally, the last two components show the quantities in markets A and B after entry. In equilibrium, firm 2 competes with its rival in period 1 as in the one-shot game described in section 4. In period 2, the potential entrant stays out of market A when it learns that the products are strong substitutes ($\hat{\theta} = 0$) by observing the prices in period 1, but it enters the market when the prices in period 1 reveal that the products are weak substitutes ($\hat{\theta} = 1$). In both cases, both firms compete with each other with perfect information in period 2. Lastly, when the prices in period 1 do not reveal the cross-price elasticity of demand ($\hat{\theta} = \theta$), firm 2 stays out of market A. In this case, both firms compete with each other with imperfect information in period 2 because firm 2 remains uninformed.

Now, it only remains to describe the posterior beliefs formed by the potential entrant in equilibrium. At the beginning of period 2, firm 2´s information will depend on the quantities and prices observed in period 1. When the price in one of the markets is so low (high) that it reveals that the products are strong (weak) substitutes, firm 2 will attach probability one to this scenario as shown in the first and third rows of equation (15). This revelation of ρ may occur even if the incumbent tries to hide its private information by choosing the pooling equilibrium quantities. However, when the prices in period 1 take intermediate values, they do not reveal anything new about the cross-price elasticity of demand and firm 2 remains uninformed ($\hat{\theta} = \theta$), as shown by the second row of equation (15). In our equilibrium, when $\rho = \rho_H$, firm 1 chooses its myopic optimum quantities in period 1. For this reason, when firm 1´s quantities observed in period 1 are different from those prescribed by our equilibrium, firm 2 will believe that $\rho = \rho_L$ as shown by the last row of equation (15) because only in this scenario, firm 1 chooses short-run suboptimal quantities in period 1 in order to deter entry. Therefore, it seems sensible to assume that a deviation from the equilibrium in period 1 may only come from an incumbent that knows that $\rho = \rho_L$, given the fact that only this type of informed firm might make a greater profit in that period by adjusting its quantities.

Unlike previous literature about limit pricing, firms sell their products in two different markets and consequently, the incumbent´s decisions made to deter entry affect the prices in both markets, as the next proposition shows.

PROPOSITION 2. When $\overline{\xi}_A > \overline{\xi}_m$, $\overline{\varepsilon}_B > \overline{\varepsilon}_m$, $K \in \left(\underline{K}, \overline{K}\right)$ *and* $\theta < \bar{\theta}$, *the average prices in markets A and B in period 1 are greater with the threat of entry than without it.*

As shown by proposition 1, the threat of entry increases the price in market A when the products are highly differentiated because the incumbent firm restricts its quantity sold in that market in order to hide its private information. This constraint to the quantity produced in market A reduces the level of competition in market B, which also increases the average price in that market. Hence, not only will entry deterrence reduce consumers´ surplus in the threatened market, but also in other markets where the incumbent firm and the potential entrant are already competing with each other.

Interestingly, the incumbent firm causes the potential entrant a positive externality in period 1 because it benefits from a higher price in market B as a result of entry deterrence in market A.

To finish off this section, we analyse the relationship between the probability of entry in our model and the noise of the signals provided by the prices in period 1.

PROPOSITION 3. The probability of entry deterrence increases with $\overline{\xi}_A$ *and with* $\overline{\varepsilon}_B$.

In equilibrium, the incumbent firm attempts to hide its private information in period 1 in order to deter entry and this strategy will only be successful when the prices do not reveal the true value of the unknown parameter to the potential entrant. For this reason, the noisier the prices are, the more likely it will be that the incumbent deters entry.

6. Conclusions

In this paper, we study a two-period model in which a company is a monopolist in market A and competes with another rival in market B. As this monopolist is the only firm which sells its products in both markets, it has private information on the cross-price elasticity of demand between the products sold in markets A and B. In this setting, when those products are highly differentiated, the monopolist in market A limits its quantity supplied in order to hide its private information and deter entry in this scenario. As a result of this strategy, the threat of expansion into the new market will increase not only the average price in the threatened market, but also in the other market where the incumbent and the potential entrant are arch-rivals. These results suggest that entry deterrence may be more detrimental to consumers than predicted in previous literature.

The results obtained would be similar in a model in which the informed incumbent is a monopolist in markets A and B and the potential entrant decides whether to enter market A or not without knowing the cross-price elasticity of demand between the products sold in both markets. In this case, when the products become closer substitutes, the incumbent would have even stronger incentives to reduce its quantity supplied in one market in order to avoid cannibalizing its own monopoly profit in the other market. Thus, entry deterrence would lead the incumbent to reduce the quantity supplied by even more than in our model to signal a high cross-price elasticity of demand and consequently, the price of the products in both markets would increase by more than in our model. As a case in point, when the patent of a drug is about to expire, Ellison and Ellison (2011) pointed out that some pharmaceutical companies tend to patent another drug of the same therapeutical category in order to deal with generic entry. In that case, the monopolist tries to induce consumers of the product with the expiring patent to switch to the other drug that has a greater remaining patent life and one way to do this is to raise the price of the older product[9].

Our findings can be easily extended to a model with more than two periods if we assume that the demand shocks in markets A and B are statistically independent over time and they are uniformly distributed. However, the robustness of the results to serially correlated demand shocks in a model with infinite periods or to demand shocks with a different distribution is an open question for future research. Furthermore, we assume that other parameters of the model, such as the intersections between the demand curves and the vertical axis, or the price elasticities of demand in both markets, are uncorrelated with the cross-price elasticity of demand. Under weaker assumptions, the potential entrant might use its information on previous prices and sales and the correlation between the known and the unknown parameters in order to update its information. Finally, there may be more than one incumbent firm and more than one potential entrant. Thus, there are different ways of extending the model in the future in order to better understand entry deterrence when the incumbent firms are already competing with the potential entrants in a different market.

Appendix

Proof of proposition 1. To prove that equations (13)-(15) contain a sequential equilibrium, we follow three steps. Firstly, we prove that the strategies chosen by both firms form a Nash equilibrium. Secondly, we prove that the posterior beliefs are consistent with Bayes´ rule in equilibrium. Third, we also analyse the type of restrictions on the posterior beliefs outside the equilibrium path such that this equilibrium is unique.

[9] See footnote 31 in Ellison and Ellison (2011).

First Step: Nash equilibrium. Quantity Decisions.

First scenario ($\rho = \rho_H$). Period 1. When $\rho = \rho_H$, in period 1 both firms choose their optimum quantities in a one-shot game with incomplete information, as shown in Section 4. That Section proves that these quantities form a Bayesian Nash equilibrium of this one-shot game.

Period 2. In period 2, we have two possible scenarios. First, if firm 2 does not learn the true value of ρ by observing the prices in period 1, it does not enter market A and we obtain the same quantities in period 2 as those obtained in period 1. Second, if firm 2 can infer the true value of ρ from the prices in period 1, it will not enter market A and we obtain a Nash equilibrium of a one-period game in a monopoly and a duopoly market with perfect information[10]. Therefore, the decisions made by both firms in periods 1 and 2 form a Nash equilibrium when $\rho = \rho_H$ because each firm´s decision in each period is a best response to its rival´s decision.

Second scenario ($\rho = \rho_L$). Period 1. When $\rho = \rho_L$, once again, firm 2 chooses its optimum quantity in period 1 given its prior beliefs as described in Section 4. Now, we need to prove that firm 1 also chooses optimum quantities in period 1 given its rival´s strategy. According to the concept of sequential equilibrium, firm 2´s beliefs are unrestricted outside the equilibrium. For example, the equilibrium considered is consistent with firm 2 believing that $\rho = \rho_L$ and entering market A when firm 1´s choices in period 1 are outside the equilibrium. Given those beliefs, it suffices to prove that in period 1 firm 1 is better off by choosing the quantities shown in equations (6) and (7)

[10] In this case, the one-period model coincides with that shown in section 4, except that firm 2 knows the true value of ρ. The optimum decisions and profits are available from the authors upon request.

when $\rho_k = \rho_H$ and deterring entry into market A, than by choosing its optimum quantities for a one-shot game (shown in equations (6) and (7) when $\rho_k = \rho_H$) and facing a new competitor in market A in period 2. This will be our next objective.

As shown in Section 4, when $\rho = \rho_L$, if firm 1 chooses its optimum quantities for a one-shot game in period 1, its expected profit is given by equation (11). This profit will be denoted as $E(\pi_{11}|\rho = \rho_L, No\ deterrence)$. In this case, firm 1 does not deter entry and both firms compete in both markets with complete information in period 2. Thus, Section 3 shows that firm 1´s expected profit in period 2 is given by equation (5) when $\rho = \rho_L$. This profit will be referred to as $E(\pi_{12}|\rho = \rho_L, No\ deterrence)$.

When $\rho = \rho_L$, but firm 1 attempts to deter entry in period 1 by choosing the quantities shown in equations (6) and (7) after substituting ρ_K with ρ_H, its total expected profit is:

$$E(\pi_{11}|\rho = \rho_L, Deterrence) = \frac{(\alpha-c)^2(13\beta^2+26\beta\rho_H-18\beta\rho_L-5\rho_H^2)}{36\beta(\beta+\rho_H)^2} \quad \text{(A.1)}$$

At the beginning of period 2, there are two possible scenarios depending on whether the prices observed in period 1 reveal the true value of ρ or not. First, let us consider the scenario in which prices are informative. Specifically, firm 2 learns ρ by simply observing the price in market A in period 1 providing that, at least, one of the following conditions is satisfied:

$$p_1^A(\rho_H, \xi_1) = \alpha - \beta Q_1^A - \rho_H Q_1^B + \xi_1 < \alpha - \beta Q_1^A - \rho_L Q_1^B - \overline{\xi}_A = p_1^A(\rho_L, -\overline{\xi}_A) \quad \text{(A.2)}$$

$$p_1^A(\rho_H, \overline{\xi}_A) = \alpha - \beta Q_1^A - \rho_H Q_1^B + \overline{\xi}_A < \alpha - \beta Q_1^A - \rho_L Q_1^B + \xi_1 = p_1^A(\rho_L, \xi_1) \quad \text{(A.3)}$$

Where $p_1^A(x, y)$ is the realization of the price in market A in period 1 when $\rho = x$ and $\xi_1 = y$. Putting inequalities (A.2) and (A.3) together, we conclude that firm 2 learns ρ by observing the price in market A provided that $\xi_1 < -\overline{\xi}_A + (\rho_H - \rho_L)Q_1^B$ or $\xi_1 > \overline{\xi}_A -$

$(\rho_H - \rho_L)Q_1^B$. Using the same reasoning, we obtain that firm 2 learns ρ by observing the price in market B in period 1 provided that $\varepsilon_1 < -\overline{\varepsilon}_B + (\rho_H - \rho_L)Q_1^A$ or $\varepsilon_1 > \overline{\varepsilon}_B - (\rho_H - \rho_L)Q_1^A$. Therefore, the probability that firm 2 will learn ρ from the prices in markets A and B in period 1 is:

$$\Pr(Learning) = \Pr\{\xi_1 \in [-\overline{\xi}_A, -\overline{\xi}_A + (\rho_H - \rho_L)Q_1^B] \cup \xi_1 \in [\overline{\xi}_A - (\rho_H - \rho_L)Q_1^B, \overline{\xi}_A] \cup \varepsilon_1 \in [-\overline{\varepsilon}_B, -\overline{\varepsilon}_B + (\rho_H - \rho_L)Q_1^A] \cup \varepsilon_1 \in [\overline{\varepsilon}_B - (\rho_H - \rho_L)Q_1^A, \overline{\varepsilon}_B]\} \quad (A.4)$$

Then, the probability that firm 2 does not learn ρ by observing the prices in period 1 is:

$$q = \Pr(No\ Learning) = Pr\{[-\overline{\xi}_A + (\rho_H - \rho_L)Q_1^B < \xi_1 < \overline{\xi}_A - (\rho_H - \rho_L)Q_1^B] \cap [-\overline{\varepsilon}_B + (\rho_H - \rho_L)Q_1^A < \varepsilon_1 < \overline{\varepsilon}_B - (\rho_H - \rho_L)Q_1^A]\} \quad (A.5)$$

If we use the uniform distributions and substitute the quantities in period 1 in (A.5) with our pooling equilibrium quantities, we obtain that probability in equilibrium:

$$q^* = \left[1 - (\rho_H - \rho_L)\frac{(\alpha - c)(4\beta + \rho_H)}{6\beta(\beta + \rho_H)\overline{\xi}_A}\right]\left[1 - (\rho_H - \rho_L)\frac{(\alpha - c)}{2(\beta + \rho_H)\overline{\varepsilon}_B}\right] \quad (A.6)$$

Under our assumptions, $\overline{\xi}_A > \frac{(\rho_H - \rho_L)(\alpha - c)(4\beta + \rho_H)}{6\beta(\beta + \rho_H)}$ and $\overline{\varepsilon}_B > \frac{(\rho_H - \rho_L)(\alpha - c)}{2(\beta + \rho_H)}$, we can guarantee that $q^* > 0$. From (A.6), we deduce that firm 2 will never learn ρ when $\overline{\xi}_A$ and $\overline{\varepsilon}_B$ tend to infinity.

When $\rho = \rho_L$, if firm 1 chooses the pooling equilibrium quantities in order to deter entry into market A, there will be two cases. First, firm 1 will preserve its monopoly power in market A with probability equal to q^* and as shown in Section 4, its expected profit in period 2 is shown by equation (11) when replacing ρ_k with ρ_L. We denote this profit made by firm 1 without new information as $E(\pi_{12}|\rho = \rho_L, Deterrence, No\ Learning)$. Second, firm 2 learns that $\rho = \rho_L$ in period 2 with probability equals to $1 - q^*$, in which case it enters market A with complete information and firm 1´s profit is shown by equation (5) when replacing ρ with ρ_L. This profit with compete information will be referred to as $E(\pi_{12}|\rho = \rho_L, Deterrence, Learning)$.

Therefore, firm 1 tries to deter entry whenever

$$E(\pi_{11}|\rho = \rho_L, Deterrence) + q^* E(\pi_{12}|\rho = \rho_L, Deterrence, No\ Learning) + (1 - q^*) E(\pi_{12}|\rho = \rho_L, Deterrence, Learning) > E(\pi_{11}|\rho = \rho_L, No\ deterrence) + E(\pi_{12}|\rho = \rho_L, No\ deterrence) \quad \text{(A.7)}$$

This inequality is equivalent to the next one:

$$q^* > \frac{18\beta(\rho_H - \rho_L)^2}{5(\beta - \rho_L)(\beta + \rho_H)^2} = m(\rho_H, \rho_L) \quad \text{(A.8)}$$

Therefore, firm 1 deters entry when $q^* > m(\rho_H, \rho_L)$. It is straightforward to prove that $m(\rho_H, \rho_L) < 1$, and using equation (A.6), it becomes clear that $q^* \to 1$ when $\overline{\xi}_A \to \infty$ and $\overline{\varepsilon}_B \to \infty$. Therefore, there will be a value of $\overline{\xi}_A$: $\overline{\xi}_m$ and a value of $\overline{\varepsilon}_B$: $\overline{\varepsilon}_m$ such that $q^* > m(\rho_H, \rho_L)$ when $\overline{\xi}_A > \overline{\xi}_m$ and $\overline{\varepsilon}_B > \overline{\varepsilon}_m$.

Period 2. When $\rho = \rho_L$, given the strategy chosen by firm 1, firm 2 will enter market A if the prices observed in period 1 reveal the true value of ρ and will not enter otherwise. In the former case, the quantities chosen in period 2 form a Nash equilibrium of a one-period game with perfect information as shown in Section 3, whereas in the latter case, the quantities chosen form a Bayesian-Nash equilibrium of a one-shot game with asymmetric information (see Section 4). So far we have proven that the quantities chosen by both firms in periods 1 and 2 form a Nash equilibrium.

Entry decision. To finish the first step of our proof, we show that firm 2´s choice is a best response to its rival´s strategy. At the beginning of period 2, firm 2 may face two different scenarios: It may or may not learn ρ by observing the prices in period 1. We study both cases separately.

Case 1. Learning. We need to prove that firm 2 does not enter market A when $\rho = \rho_H$, but it enters the market when $\rho = \rho_L$. If firm 2 stays out despite having perfect information, it is straightforward to obtain its equilibrium expected profit in period 2 in a game with a duopoly and a monopoly and perfect information:

$$E(\pi_{22}|No\ entry, Learning) = \frac{(\alpha-c)^2}{9\beta} \quad \text{(A.9)}$$

If firm 2 enters market A, its expected profit in period 2 is that shown in equation (5) minus the entry cost, K. After comparing these profits with and without entry for different values of ρ, it is easy to see that whenever $K_1 < K < K_2$, we can guarantee that firm 2 enters market A when $\rho = \rho_L$ and stays out when $\rho = \rho_H$, where $K_1 = \frac{(\alpha-c)^2(\beta-\rho_H)}{9\beta(\beta+\rho_H)}$ and $K_2 = \frac{(\alpha-c)^2(\beta-\rho_L)}{9\beta(\beta+\rho_L)}$. Under our assumptions, $K_1 < K_2$ and consequently, there will be some values of K that makes firm 2 enter market A only when $\rho = \rho_L$.

Case 2. No Learning. Now, we analyse firm 2's entry decision in period 2 when the prices observed in period 1 do not reveal information on ρ. In such a case, the profit made by firm 2 in period 2 when it does not enter market A is given by equation (12). When firm 2 enters market A in period 2, it learns ρ and competes with its rival with perfect information as shown in section 3, in which case firm 2´s profit is shown in equation (5). Using this equation, we obtain that firm 2´s ex-ante expected profit from entry will be:

$$E(\pi_{22}|Entry, Before\ Learning) = \frac{2(\alpha-c)^2(\beta+\hat{\rho})}{9(\beta+\rho_L)(\beta+\rho_H)} - K \quad \text{(A.10)}$$

Where $\hat{\rho} = \theta\rho_H + (1-\theta)\rho_L$. When this profit is lower than that obtained without entry (equation (12)), firm 2 will stay out of market A without new information. Hence, entry deterrence will occur whenever:

$$K > \frac{2(\alpha-c)^2(\beta+\hat{\rho})}{9(\beta+\rho_L)(\beta+\rho_H)} - \frac{(\alpha-c)^2}{9\beta} = K_3 \qquad \text{(A.11)}$$

Now, we need to put the two cases analysed together. First, when the prices in period 1 reveal the true value of ρ and $K_1 < K < K_2$, we have proven that firm 2 enters market A after learning that $\rho = \rho_L$ and stays out after deducing that $\rho = \rho_H$. Similarly, in the second case in which the prices in period 1 do not reveal the unknown parameter, firm 2 prefers to stay out of market A when $K > K_3$. As it is straightforward to prove that $K_1 \leq K_3 \leq K_2 \ \forall\theta \in [0,1]$ and K_3 increases with θ, this implies that there will be a value of θ, which is $\bar{\theta} \in (0,1)$, such that firm 2´s entry decision is optimal whenever $\theta < \bar{\theta}$ and $K_3 < K < K_2$. In proposition 1, $\underline{K} = K_3$ and $\overline{K} = K_2$.

Second Step: Consistency. Let $\hat{\theta} = Pr\{\,\rho = \rho_L | Q_{11}^*\}$ denote the posterior probability assigned by firm 2 to the scenario in which $\rho = \rho_L$ after observing the equilibrium quantities chosen by firm 1 in period 1. In equilibrium, after firm 2 observes the quantities and prices in period 1, there may be two possible scenarios. First, when firm 2 learns ρ from the prices in period 1, $\hat{\theta} = 1$ after learning that $\rho = \rho_L$ and $\hat{\theta} = 0$ after learning that $\rho = \rho_H$. Second, when the prices in period 1 do not reveal the true value of ρ, $\hat{\theta} = \theta$, because firm 2 does not learn anything new about the unknown parameter and the posterior beliefs are prescribed by Bayes´ rule in equilibrium:

$$\hat{\theta} = Pr\{\,\rho = \rho_L | Q_{11}^*\} = \frac{Pr\{\rho=\rho_L\}Pr\{Q_{11}^* | \rho = \rho_L\}}{Pr\{Q_{11}^*\}} = \frac{\theta\cdot 1}{\theta\cdot 1 + (1-\theta)\cdot 1} = \theta \qquad \text{(A.12)}$$

Where $Pr\{\rho = \rho_L\}$ is the probability of a low value of ρ, $Pr\{Q_{11}^* | \rho = \rho_L\}$ is the probability that firm 1 chooses its equilibrium quantities in period 1 when $\rho = \rho_L$ and $Pr\{Q_{11}^*\}$ is the probability that firm 2 observes the equilibrium quantities chosen by firm 1. This probability is equal to 1 because firm 1 chooses the same quantities in both possible scenarios in our pooling equilibrium.

Third Step: Restrictions on the out-of-equilibrium beliefs. Finally, we analyse the type of restrictions on the off-the-equilibrium beliefs under which our equilibrium is unique. In this particular signalling game, the informed player sends a message with two components in period 1. These components are the quantities supplied in markets A and B, respectively. At the same time, the uninformed player chooses an action, which is its quantity supplied in market B. In period 2, the uninformed player observes the message sent by the informed one and chooses whether entering market A or not and its quantities supplied at the same time as its informed rival.

In this context, a pooling equilibrium consists of a set of strategies in which firm 1 sends the same message in period 1 irrespective of the true value of the unknown parameter, whereas a separating equilibrium consists of a set of strategies in which firm 1 sends a different message in each state of the world. As a result, the posterior beliefs will be equal to the prior beliefs in a pooling equilibrium, whereas the posterior beliefs will be degenerate in a separating. Using this terminology, our equilibrium is pooling. Under the conditions of proposition 1, we have just proven that there will not be a separating equilibrium in which each type of informed player offers its short-run optimum quantities in period 1, because in period 1 the informed firm would rather offer the quantities considered in proposition 1 when $\rho = \rho_L$. In other words, one type of informed player would prefer to mimic the other type.

Now we will use the restrictions on the off-the-equilibrium beliefs introduced by Mailath, Okuno-Fujiwara and Postlewaite (1993) to show that our proposed pooling equilibrium is the unique undefeated pooling equilibrium. Imagine that a certain message, m_1, is out of the equilibrium path, but it is sent by certain type of informed player, t_1, in a different equilibrium. If t_1 is better off in the second equilibrium than in the first, the uninformed player must believe that m_1 was sent by t_1 when this message is observed out of the initial equilibrium path. If the uninformed player´s beliefs in the initial equilibrium are not consistent with this restriction, Mailath, Okuno-Fujiwara and Postlewaite (1993) consider that the initial equilibrium is defeated by the other. Therefore, the undefeated equilibrium will be the equilibrium which is not defeated by any other equilibrium.

We will show that any other pooling equilibrium that differs from the equilibrium proposed in proposition 1 would be defeated by ours. Specifically, assume that there is another pooling equilibrium in which both types of informed player choose the same quantities in period 1, but these quantities differ from those in our proposed equilibrium. If this were the case, it would mean that the type of firm 1 that knows that $\rho = \rho_H$ chooses short-run suboptimal quantities in period 1 given its rival´s choice in that period. As the alternative equilibrium is also a pooling equilibrium, the posterior beliefs are equal to the prior beliefs. Under this condition, we have proven that this type of firm 1 chooses its short-run optimal quantities in period 2 in our pooling equilibrium given its rival´s information. Hence, it would be impossible for this type of informed player to make a greater profit in period 2, given its rival´s decision in an alternative pooling equilibrium, because it is already making its maximum possible profit in our proposed equilibrium. In conclusion, in any other pooling equilibria, this type of informed player would be worse off than in our proposed equilibrium, which means that any alternative pooling equilibria would be defeated by ours[11]. In fact, when $\rho = \rho_H$, if firm 2 observed the quantities chosen by the informed firm in our proposed equilibrium, which would be out of the equilibrium path in an alternative pooling equilibrium, the uninformed firm would believe that $\rho = \rho_H$ and it would not enter market A. Therefore, the posterior beliefs in the alternative pooling equilibrium, which have to be equal to the prior beliefs, would not be consistent with the previous restriction. This completes the proof of proposition 1.

Proof of proposition 2. When $\rho = \rho_L$, firm 1 chooses its quantities in period 1 as if $\rho = \rho_H$ (see equations (6) and (7)) and the expected prices in both markets are:

$$E(p_1^A|Entry\ deterrance) = \frac{\alpha(\beta-\rho_L)+2\alpha\beta+c\rho_L}{6\beta} + \frac{\alpha(\rho_H-\rho_L)+(\beta+\rho_L)c}{2(\beta+\rho_H)} \quad \text{(A.13)}$$

$$E(p_1^B|Entry\ deterrance) = \frac{\alpha+2c}{3} + \frac{(\rho_H-\rho_L)(\alpha-c)}{2(\beta+\rho_H)} \quad \text{(A.14)}$$

As shown in Section 4, if firm 1 does not deter entry in period 1, the expected prices of equations (9) and (10) can be written as:

$$E(p_1^A|No\ entry\ deterrance) = \frac{\alpha(\beta-\rho_L)+2\alpha\beta+c\rho_L}{6\beta} + \frac{c}{2} \quad \text{(A.15)}$$

$$E(p_1^B|No\ entry\ deterrance) = \frac{\alpha+2c}{3} \quad \text{(A.16)}$$

The first addend of the right-hand side of equations (A.13) and (A.15) coincide, but the second addend in (A.13) is greater than that in (A.15) because $\alpha > c$. Similarly, the second addend of the right-hand side of equation (A.14) is positive and the first addend

[11] Our pooling equilibrium also survives the intuitive criterion developed by Cho and Kreps (1987) because there are no out-of-equilibrium messages which are equilibrium dominated for any types of sender.

is equal to the price shown in (A.16). Thus, $E\left(p_1^j \middle| Entry\ deterrance\right) > E\left(p_1^j \middle| No\ entry\ deterrance\right)$.

Proof of proposition 3. When $\rho = \rho_L$, firm 1 tries to deter entry by choosing the pooling equilibrium quantities in period 1. After observing these quantities, firm 2 stays out of market A when the prices in period 1 does not reveal ρ. Thus, the probability of entry deterrence is equal to θq^* and it is immediate that q^* increases with $\overline{\xi}_A$ and with $\overline{\varepsilon}_B$ (see equation (A.6)).

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