On one contact problem of plane elasticity for a doubly connected domain: application to a hexagon

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Abstract. The paper addresses a problem of plane elasticity theory for a doubly connected body whose external boundary is a regular hexagon boundary and the internal boundary is the required full-strength hole including the origin of coordinates. Hexagon's two vertices are laid at the axis Oy and the middle points of its two opposite sides are laid at the axis Ox. This full-strength hole is cycle symmetric. It is assumed that to every link of the broken line of the outer boundary of the given body are applied absolutely smooth rigid stamps with rectilinear bases which are under action of the force P, that applies to their middle points. There is no friction between the surface of given elastic body and stamps. The unknown full-strength contour is free from outer actions. Using the methods of complex analysis, the analytical image of Kolosov-Muskhelishvili's complex potentials (characterizing an elastic equilibrium of the body), and unknown parts of its boundary are determined under the condition that the tangential normal moment arising at it takes a constant value. Such holes are called full-strength holes Numerical analysis are also performed and the corresponding graphs are constructed.

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1. Introduction

Applications of elastic plates weakened with full-strength holes are of great interest in several mechanical constructions (building practice, mechanical engineering, shipbuilding, aircraft construction, etc).

Boundary-value problems of the plane theory of elasticity and plate bending for infinite plates weakened by unknown full-strength holes with normal stresses acting on their boundaries and forces applied at infinity were analyzed in [1, 2, 8, 12, 17].

Boundary-value problems for a finite doubly-connected domain with a part of its boundary being unknown full-strength and the other part being a polygonal line are solved in [6, 7].

The axis-symmetric and cycle symmetric problems of the plane theory of elasticity and plate bending with partially unknown boundaries are studied in [3, 4, 5, 13, 14, 15, 16].

The most effective methods for studying these problems are the methods of the theory of analytical functions of a complex variable.

In this article, the cycle symmetric problem of plane elasticity theory for a regular hexagon weakened with full-strength hole is considered. Formulas of Kolovos-Muskhelishvili are used for investigating this problem. The solution is written in quadratures and the unknown full-strength hole of the plate is constructed

2. Problem Formulation and Solution Technique

Let an isotropic elastic body on the plane $z = x+iy$ occupy a doubly connected domain S, whose external boundary is a regular hexagon boundary and the internal boundary is the required full-strength hole.

The required full-strength hole includes the origin of coordinates. Hexagon's two vertices are laid at the axis Oy, and middle points of its two opposite sides are laid at the axis Ox (Fig. 1). Let to every link

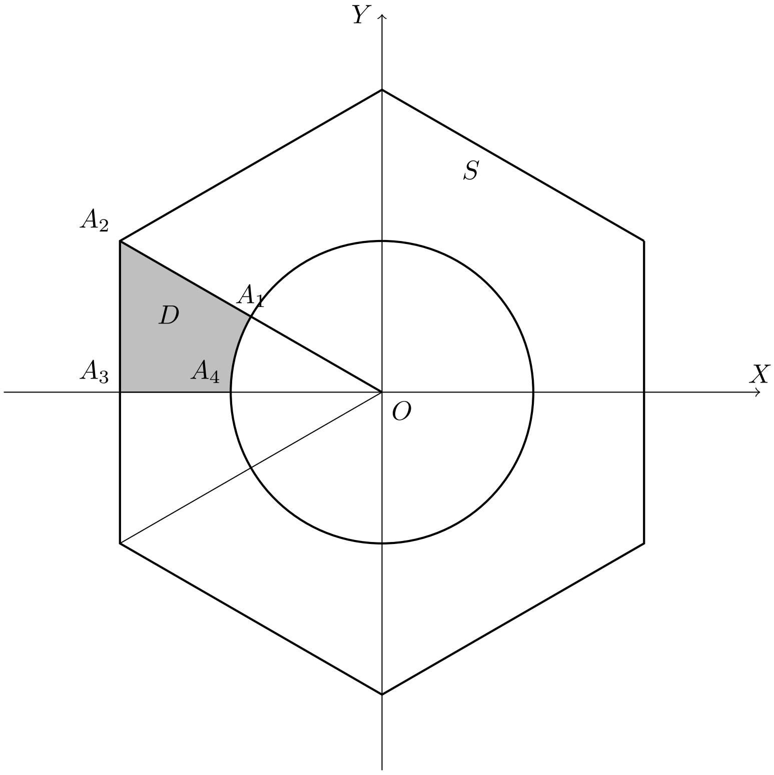


Figure 1. Graph of the posed problem

of the broken line of the outer boundary of the given body be applied absolutely smooth rigid stamps with rectilinear bases which are under action of the force P that applies to their middle points. There is no friction between the given elastic body and stamps. The unknown full-strength contour is free from outer actions.

Under the above assumptions, the normal displacement of every link of external boundary of broken line $\nu_n = \nu = \text{const}$. Tangential stresses $\tau_{ns} = 0$ along the entire boundary of the domain S.

Consider the problem: Find the stress state of the body and the shape of the unknown hole such that the tangential normal stress σ_s arising at it would take the constant value $\sigma_s = K = \text{const}$.

Since the problem is cycle axially symmetric, then to investigate the state problem, it is sufficient to consider the curvilinear quadrangle $A_1A_2A_3A_4$ which will be denoted by D. The normal displacements and the tangential stresses are equal to zero $v_n = \tau_{ns} = 0$ at each segment $[A_1, A_2], [A_3, A_4]$.

Let introduce the following notations $\Gamma_1 = A_1A_2$, $\Gamma_2 = A_2A_3$, $\Gamma_3 = A_3A_4$, $\gamma = A_4A_1$, $\Gamma = \sum_{j=1}^{3} \Gamma_j$. A_3 is the middle point of hexagon side. Since D is in the equilibrium state, then we have:

$$\int_{\Gamma_1} \sigma_n \, ds = -P, \qquad \int_{\Gamma_2} \sigma_n \, ds = \frac{-P}{2}, \qquad \int_{\Gamma_3} \sigma_n \, ds = -\frac{\sqrt{3}}{2} P,$$

where σ_n is the normal stress.

The boundary conditions are

$$\nu_n = \begin{cases} 0, & t \in \Gamma_3 \cup \Gamma_1, \\ \nu, & t \in \Gamma_3, \end{cases} \tag{1}$$

$$\begin{gathered} \tau_{ns} = 0, \qquad t \in \Gamma \cup \gamma, \\ \sigma_n = 0, \qquad \sigma_s = K, \qquad t \in \gamma \end{gathered} \tag{2}$$

$$\int_{\Gamma_1} \sigma_n \, ds = -P, \qquad \int_{\Gamma_2} \sigma_n \, ds = \frac{-P}{2}, \qquad \int_{\Gamma_3} \sigma_n \, ds = -\frac{\sqrt{3}}{2} P \tag{3}$$

On the basis of the well-known Kolosov-Muskelishvili's formulas [10], the problem reduces to finding the functions ψ, φ which are holomophic in the domain D with the following conditions

$$\Re e^{-i\alpha(t)}(\chi\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)}) = 2\mu\nu_n(t), \qquad t \in \Gamma, \tag{4}$$

$$\Re e^{-i\alpha(t)}(\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)}) = C(t), \qquad t \in \Gamma, \tag{5}$$

$$\varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)} = 0, \qquad t \in \gamma, \tag{6}$$

$$\Re\varphi'(t) = \frac{\sigma_n + \sigma_s}{4} = \frac{K}{4}, \qquad t \in \gamma, \tag{7}$$

where χ, μ are elasticity constants, $C(t)$ is a piecewise constant function, $\alpha(t)$ is the angle formed between the external normal n to contour and the abscissa axis Ox.

$$\alpha(t) = \alpha_k, \qquad t \in \Gamma_k, \qquad k = 1, 2, 3, \qquad \alpha_1 = \frac{\pi}{3}, \qquad \alpha_2 = \pi, \qquad \alpha_3 = \frac{3\pi}{2}, \tag{8}$$

$$C(t) = \Re\left(e^{-i\alpha(t)} i \left(\int_{A_1}^{t} \sigma_n(s_0) e^{i\alpha(s_0)}\, ds_0\right)\right) \tag{9}$$

Taking into account (3) and (8), the (9) has following form:

$$C(t) = \begin{cases} -\dfrac{\sqrt{3}P}{2} & t \in \Gamma_2 \\ 0 & t \in \Gamma_1 \cup \Gamma_3 \end{cases} \tag{10}$$

Let, $t \in A_k A_{k+1}$, $k = 1, 2$ and $t \in A_4 A_1$, then $t - A_k = i|t - A_k|e^{i\alpha_k}$. Hence we obtain

$$\Re t e^{-i\alpha(t)} = \Re e^{-i\alpha(t)} A(t) \tag{11}$$

where $A(t)$ is a piecewise-constant function, $A(t) = A_k$, $t \in A_k A_{k+1}$, $k = 1, 2$ and $t \in A_4 A_1$.

Taking into account (8) one obtains:

$$\Re e^{-i\alpha(t)} A(t) = \begin{cases} \dfrac{\sqrt{3}}{2} a & t \in \Gamma_2 \\ 0 & t \in \Gamma_1 \cup \Gamma_3 \end{cases}$$

where a is the length of the regular hexagon side.

The functions $\varphi(z)$, $\bar{z}\varphi'(z) + \psi(z)$ will be assumed to be continuous everywhere on the boundary of the domain D, while functions $\varphi'(z)$, $\psi(z)$ are continuously extendable on the boundary D, with the exclusion of the point A_2 in the neighborhood of which the following estimate is admitted:

$$|\varphi'(z)| < M|z - A_2|^{-\delta}, \qquad |\psi(z)| < M|z - A_2|^{-\delta}, \qquad 0 \le \delta < 1 \tag{12}$$

Summarizing (4) and (5) and differentiating with respect to arc abscissa s and taking into account that $\alpha(t)$, $C(t)$, $\nu_n(t)$ are the piecewise constant functions on Γ, we obtain

$$\Im\varphi'(t) = 0, \qquad t \in \Gamma. \tag{13}$$

Equalities (7) and (13) are the Keldysh-Sedov problem for domain S:

$$\begin{gathered} \Re\left(\varphi'(t) - \frac{K}{4}\right) = 0, \qquad t \in A_4 A_1, \\ \Im\left(\varphi'(t) - \frac{K}{4}\right) = 0, \qquad t \in \Gamma. \end{gathered} \tag{14}$$

Problem (14) has a unique solution [9]

$$\varphi'(z) = \frac{K}{4} \tag{15}$$

Hence we obtain

$$\varphi(z) = \frac{K}{4} z. \tag{16}$$

Here we neglect the constant summands.

Substituting the values $\varphi(t)$, $C(t)$ into the boundary conditions (5) - (6) and taking into account (12) one gets the following problem

$$\Re\left[e^{-i\alpha(t)}\left(\frac{K}{2}t+\overline{\psi(t)}\right)\right] = C(t) = \begin{cases} -\frac{\sqrt{3}P}{2} & t \in \Gamma_2 \\ 0 & t \in \Gamma_1 \cup \Gamma_3 \end{cases} \tag{17}$$

$$\Re t e^{-i\alpha(t)} = \Re e^{-i\alpha(t)}A(t), \qquad t \in \Gamma, \tag{18}$$

$$\left(\frac{K}{2}t+\overline{\psi(t)}\right) = 0, \qquad t \in \gamma \tag{19}$$

Let the function $z = \omega(\zeta)$, $\zeta = \xi + i\eta$ map semicircle $|\zeta| < 1$, $\Im\zeta > 0$ conformally onto the domain D. It is assumed the vertices A_k of the hexagon line correspond to the point a_k of the semicircle $|\zeta| = 1$, $\Im\zeta > 0$, $a_k = \omega^{-1}(A_k)$, $k = 1, 2, 3$. It is assumed that $a_1 = 1$, $a_4 = -1$, $a_3 = i$. Here we can fix three points and the remaining ones are to be defined. Then the diameter $-1 \leq \xi \leq 1$ is mapped onto arc A_4A_1 and the semi-circumference $\gamma_0 : |\gamma_0| = 1$, $\Im\zeta > 0$ is mapped onto the broken line Γ. The point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \frac{\pi}{2}$ is to be determined.

Because of (17) - (19) the boundary conditions become

$$\Re e^{-i\alpha(\sigma)}\overline{\psi(\sigma)} = -\frac{K}{2}\Re e^{-i\alpha(t)}A(\sigma) + C(\sigma), \qquad \sigma \in \gamma_0, \tag{20}$$

$$\Re e^{-i\alpha(\sigma)}\omega(\sigma) = \Re e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0, \tag{21}$$

$$\begin{gathered} \frac{K}{2}\omega(\sigma) + \overline{\psi_0(\sigma)} = 0, \qquad \sigma \in (-1, 1), \\ \psi_0(\zeta) = \psi(\omega(\zeta)). \end{gathered} \tag{22}$$

Since $\alpha(t)$, $A(t)$, $C(t)$ are the piecewise-constant functions, $\alpha(\omega(\sigma))$, $A(\omega(\sigma))$, $C(\omega(\sigma))$ are denoted by $\alpha(\sigma)$, $A(\sigma)$, $C(\sigma)$.

Let us introduce a piecewise-holomorphic function $W(\zeta)$ in the circle $|\zeta| < 1$.

$$W(\zeta) = \begin{cases} \frac{K}{2}\omega(\zeta) & |\zeta| < 1, \Im\zeta > 0, \\ -\overline{\psi_0(\bar{\zeta})} & |\zeta| < 1, \Im\zeta < 0 \end{cases} \tag{23}$$

Taking into account (23)

$$W^{+}(\xi) = \frac{K}{2}\omega(\xi), \qquad W^{-}(\xi) = -\overline{\psi_0(\xi)}, \qquad -1 < \xi < 1,) \tag{24}$$

$$W^{+}(\sigma) = \frac{K}{2}\omega(\sigma), \qquad \sigma \in \gamma_0, \qquad W^{-}(\sigma) = -\overline{\psi_0(\bar{\sigma})}, \qquad \sigma \in \gamma_0^{*} \tag{25}$$

where the signs $(+)$ and $(-)$ mark, as usually, the limit values the function takes at the upper and lower edges and γ_0^{*} is the mirror image of γ_0 about the Ox axis. By virtue of (22) and (24), we obtain

$$W^{+}(\xi) - W^{-}(\xi) = 0, \qquad -1 < \xi < 1.$$

which means that $W(\zeta)$ is a holomorphic function on the circle $|\zeta| < 1$.

Considering (25), we get

$$\Re e^{-i\alpha(\sigma)}W(\sigma) = \frac{K}{2}\Re e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0, \tag{26}$$

$$\Re e^{-i\alpha(\sigma)}W(\sigma) = -C(\sigma) + \frac{K}{2}\Re e^{-i\alpha(\sigma)}A(\sigma), \qquad \sigma \in \gamma_0^{*}, \tag{27}$$

Conditions (26) and (27) can be rewritten in the following form

$$\Re e^{-i\alpha(\sigma)}W(\sigma) = f(\sigma), \qquad \sigma \in \gamma', \tag{28}$$

where

$$\gamma' = \gamma_0 \cup \gamma_0^{*}, \qquad \alpha(\sigma) = \alpha(\bar{\sigma}), \qquad \sigma \in \gamma_0^{*},$$

$$\alpha_1 = \frac{\pi}{3}, \qquad \alpha_2 = \pi, \qquad \alpha_3 = \frac{3\pi}{2},$$

$$f(\sigma) = \frac{K}{2}\Re e^{-i\alpha(\sigma)}A(\sigma) = \begin{cases} \frac{Ka\sqrt{3}}{4} & \sigma \in (a_2, a_3), \\ 0 & \sigma \in (a_1, a_2) \cup (a_3, a_4) \end{cases} \tag{29}$$

$$f(\sigma) = \frac{K}{2}\Re e^{-i\alpha(\sigma)}A(\sigma) - C(\sigma) = \begin{cases} \frac{Ka\sqrt{3}}{4} + \frac{\sqrt{3}}{2}P & \sigma \in (\bar{a}_3, \bar{a}_2) \\ 0 & \sigma \in (a_4, \bar{a}_3) \cup (\bar{a}_2, a_1) \end{cases}$$

Thus, the problem in question has been reduced to the Riemann-Hilbert problem with piecewise-constant coefficients. The solution of this problem was obtained in [11] (by reducing it to a linear-conjugation problem). Here we reduce the problem to the Dirichlet problem for a circle and represent its solution in terms of the Schwarz integral, which is computationally convenient.

Function $e^{2i\alpha(\sigma)}$ is given by

$$e^{2i\alpha(\sigma)} = \frac{X(\sigma)}{\overline{X(\sigma)}}, \qquad |\sigma| = 1, \tag{30}$$

where $X(\zeta)$ is given as

$$X(\zeta) = \frac{\sqrt{(\zeta - \bar{a}_3)(\zeta - a_3)}(\zeta - a_2)^{\frac{1}{3}}(\zeta - \bar{a}_2)^{\frac{2}{3}}\sqrt[6]{a_2}}{\zeta}, \qquad |\zeta| < 1,$$

By virtue of (30) condition (28) becomes

$$\frac{W(\sigma)}{X(\sigma)} + \frac{\overline{W(\sigma)}}{\overline{X(\sigma)}} = \frac{2fe^{i\alpha(\sigma)}}{X(\sigma)}. \tag{31}$$

Condition (31) represents the boundary condition of the Dirichlet problem, whose solution is presented by Schwarz formula

$$\frac{W(\zeta)}{X(\zeta)} = \frac{1}{2\pi i}\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}(\sigma + \zeta)}{X(\sigma)\sigma(\sigma - \zeta)}\,d\sigma \tag{32}$$

Since function $X(\zeta)$ has a simple pole at the point $\zeta = 0$, the function $\frac{W(\zeta)}{X(\zeta)}$ will have the first-order zero at this point $\zeta = 0$. Hence from (32) we get

$$\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}}{X(\sigma)\sigma}\,d\sigma = 0 \tag{33}$$

Taking into account (29) and (8), the (33) has the form:

$$-\frac{Ka\sqrt{3}}{4}\int_{a_2}^{a_3} \frac{1}{X(\sigma)\sigma}\,d\sigma - \left(\frac{Ka\sqrt{3}}{4} + \frac{\sqrt{3}}{2}P\right)\int_{\bar{a}_3}^{\bar{a}_2} \frac{1}{X(\sigma)\sigma}\,d\sigma = 0 \tag{34}$$

Thus we obtain the equation with respect to two unknown parameters a_2, K.

We could choose some value of K and then determine the parameter a_2, but then the problem would be very complicated. For the purpose of computations, it is more convenient to calculate K for each fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \frac{\pi}{2}$ and given P. From (34) one obtains:

$$K = \frac{-\sqrt{3}PB}{a\sqrt{3}(A + B)}, \tag{35}$$

where

$$A = \int_{a_2}^{a_3} \frac{d\sigma}{X(\sigma)\sigma}, \qquad B = \int_{\bar{a}_3}^{\bar{a}_2} \frac{d\sigma}{X(\sigma)\sigma}.$$

Remark 1. The solution of the problem exists if

$$A + B \neq 0$$

By virtue of (33) the formula (32) has the form:

$$W(\zeta) = \frac{\zeta X(\zeta)}{\pi i}\int_{\gamma'} \frac{f(\sigma)e^{i\alpha(\sigma)}\,d\sigma}{X(\sigma)\sigma(\sigma - \zeta)} \tag{36}$$

i.e.:

$$W(\zeta) =$$

$$= \frac{\zeta X(\zeta)}{\pi i}\left(-\frac{Ka\sqrt{3}}{4}\int_{a_2}^{a_3} \frac{d\sigma}{X(\sigma)\sigma(\sigma - \zeta)} - \left(\frac{Ka\sqrt{3}}{4} + \frac{\sqrt{3}}{2}P\right)\int_{\bar{a}_3}^{\bar{a}_2} \frac{d\sigma}{X(\sigma)\sigma(\sigma - \zeta)}\right).$$

By virtue of (23), equation of the contour $z = \omega(\xi)$ is presented by

$$\omega(\xi) = \frac{2W(\xi)}{K}, \qquad -1 < \xi < 1 \tag{37}$$

3. Construction of the hexagon's full-strength hole

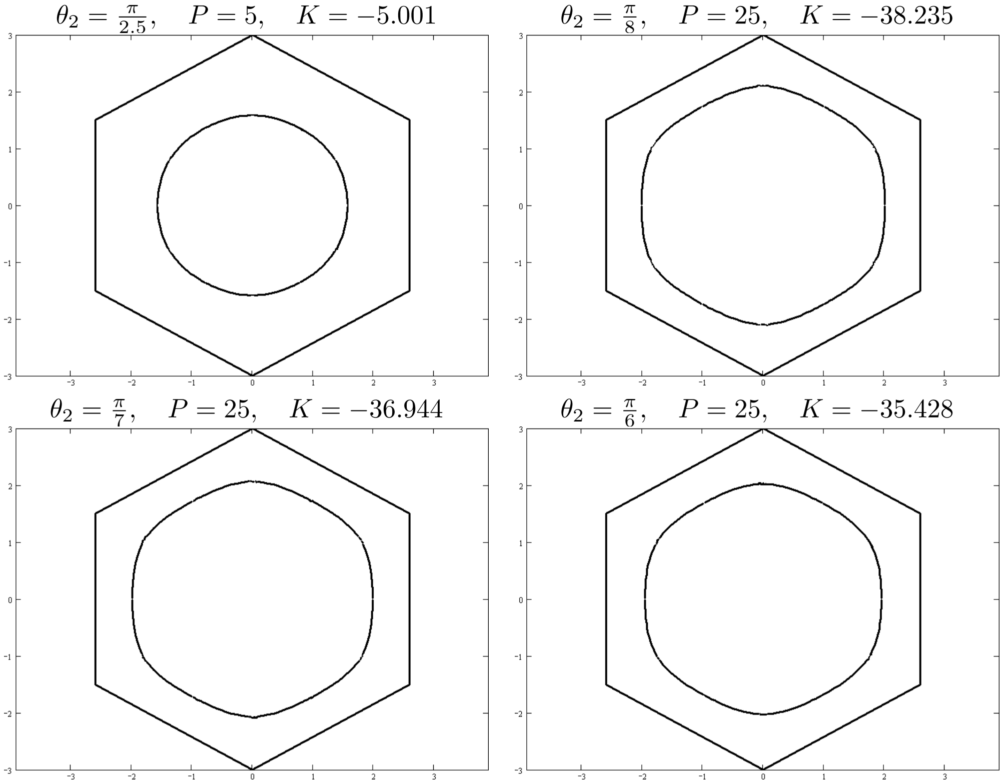
Let us consider some concrete cases. Assume that the side length of a regular hexagon is $a = 3$. To construct the hexagon's full-strength hole, at first we calculate K by virtue of (35) for each fixed point $a_2 = e^{i\theta_2}$, $0 < \theta_2 < \frac{\pi}{2}$ and given P. Then we define $W(\xi)$ by (36) for each pair of parameters a_2 and K. By virtue of relationship (37) the function $\omega(\xi)$ is determined. The image of the function, $\omega(\xi)$, $-1 < \xi < 1$ yields a part of the unknown full-strength boundary. As we considered compressive stress action $P > 0$, it follows that $K < 0$.

To construct the hexagon's full-strength hole, at first, the arc $\gamma = A_4A_1$ of the required full-strength contour is constructed by (37) $w = \omega(\xi)$, $-1 < \xi < 1$. Then by virtue of this function, the next part of required equi-strong contour is constructed by $r = \Re(\omega(\xi)) - i\Im(\omega(\xi))$. Thus one sixth part of the required full-strength hole is constructed (Fig. 1). Since the problem is cyclically symmetric, the other parts of required full-strength hole can be obtained by rotating the graphs w, r through an angle of $\frac{\pi}{3}$.

As an illustration, some graphics of full-strength holes are presented in Fig 2 for different sets of parameters.

4. Conclusion

The most effective methods for studying this problem are those of the theory of analytical functions of complex variable. This problem is both of mechanics and of geometry since the shape of a hole of a plate is required and the conformal mapping function is used to define it. Thus, formulas of Kolosov-Muskhelishvili are used for investigating this problem. Using the conformal mapping, the investigation of the problem is reduced to a Riemann-Hilbert problem. The solution is written in quadratures and the unknown full-strength part of the plate boundary is constructed. The conformal mapping function is a generalization of Christoffel-Schwarz formulas.



$\theta_2 = \frac{\pi}{2.5}$, $P = 5$, $K = -5.001$

$\theta_2 = \frac{\pi}{8}$, $P = 25$, $K = -38.235$

$\theta_2 = \frac{\pi}{7}$, $P = 25$, $K = -36.944$

$\theta_2 = \frac{\pi}{6}$, $P = 25$, $K = -35.428$

FIGURE 2. Graphs of a full-strength Boundary

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