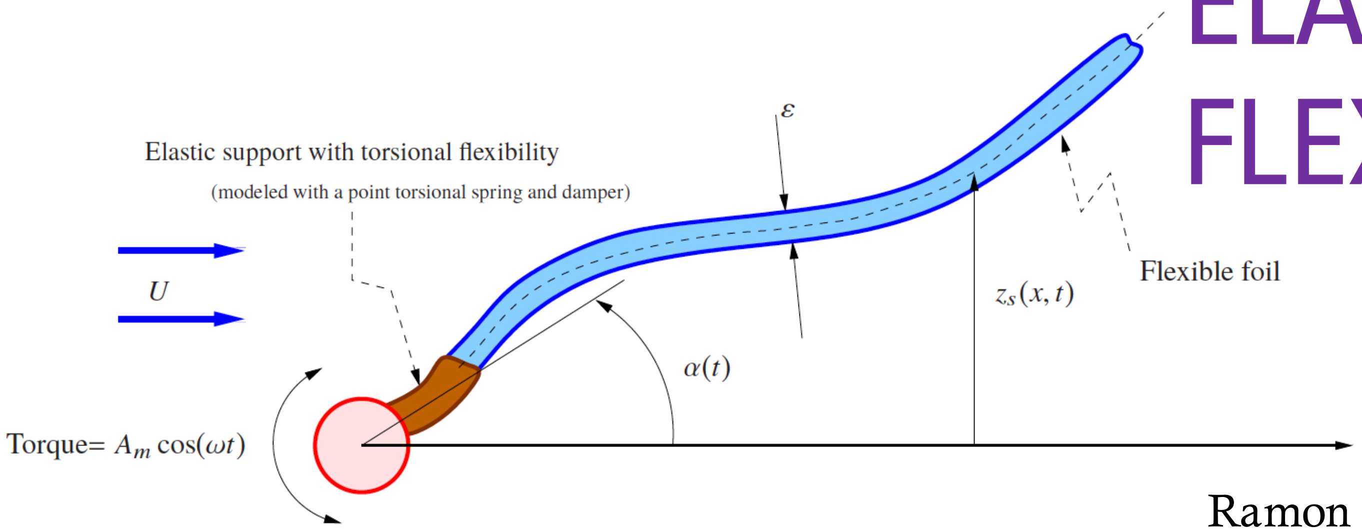
OPTIMAL PROPULSION PERFORMANCE OF AN ELASTICALLY MOUNTED FLEXIBLE PITCHING FOIL



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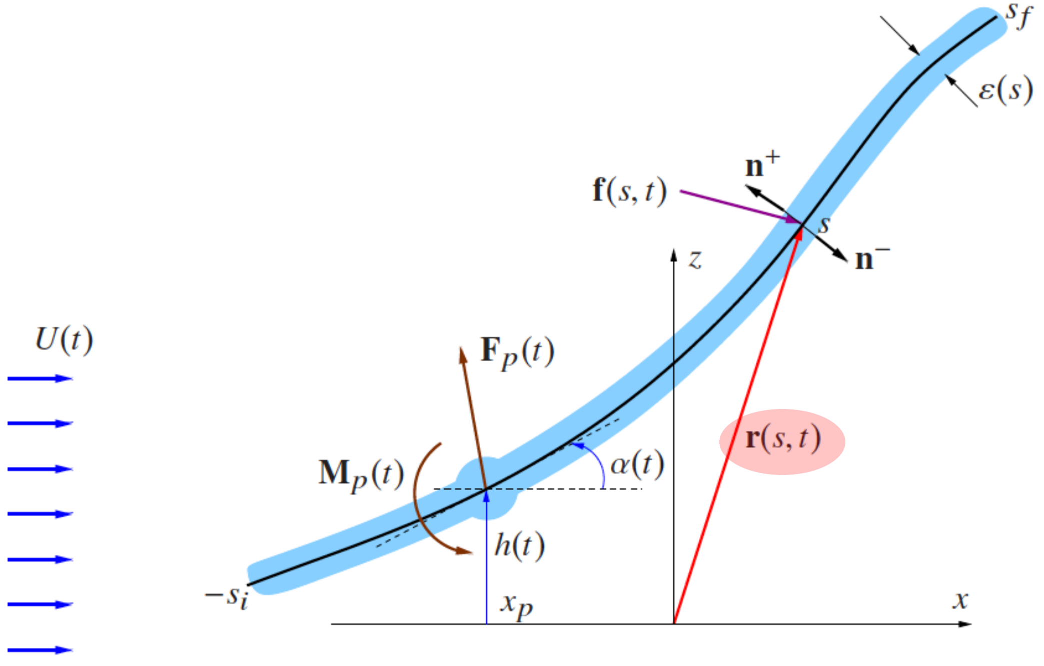
APS-DFD 2025 HOUSTON

NEW ANALYTICAL METHOD TO ANALYZE THE FSI OF FLEXIBLE PLATES

Based on the *moments* of the Euler-Bernoulli equation:

$$\int_{-s_i}^{s_f} s^n \left[\rho_s \varepsilon \frac{\partial^2 \mathbf{r}}{\partial t^2} - \frac{\partial}{\partial s}\left[T\frac{\partial \mathbf{r}}{\partial s}\right] + \frac{\partial^2}{\partial s^2}\left(EI\frac{\partial^2 \mathbf{r}}{\partial s^2}\right) = \mathbf{f}(s,t) + \mathbf{F}_p\delta(s) - \mathbf{g}\delta'(s)\right] ds\,, \qquad n = 0, 1, 2, 3, \ldots$$



Fluid forces:

$$\mathbf{f} = \boldsymbol{\tau}^+ \cdot \mathbf{n}^+ + \boldsymbol{\tau}^- \cdot \mathbf{n}^-\,, \qquad \boldsymbol{\tau} = -p\mathsf{I} + \mu\left(\boldsymbol{\nabla}\mathbf{u} + (\boldsymbol{\nabla}\mathbf{u})^T\right)$$

Actuating point forces and moments at pivot (including e.g. springs and dampers):

$$\mathbf{F}_p(t)$$

$$\mathbf{M}_p(t) \equiv -M_p(t)\mathbf{e}_y = \mathbf{e}_\alpha(t) \wedge \mathbf{g}(t)$$

J. Fluid Mech. **910,** A43 (2021)
J. Fluids Structures, **120**, 103907 (2023)

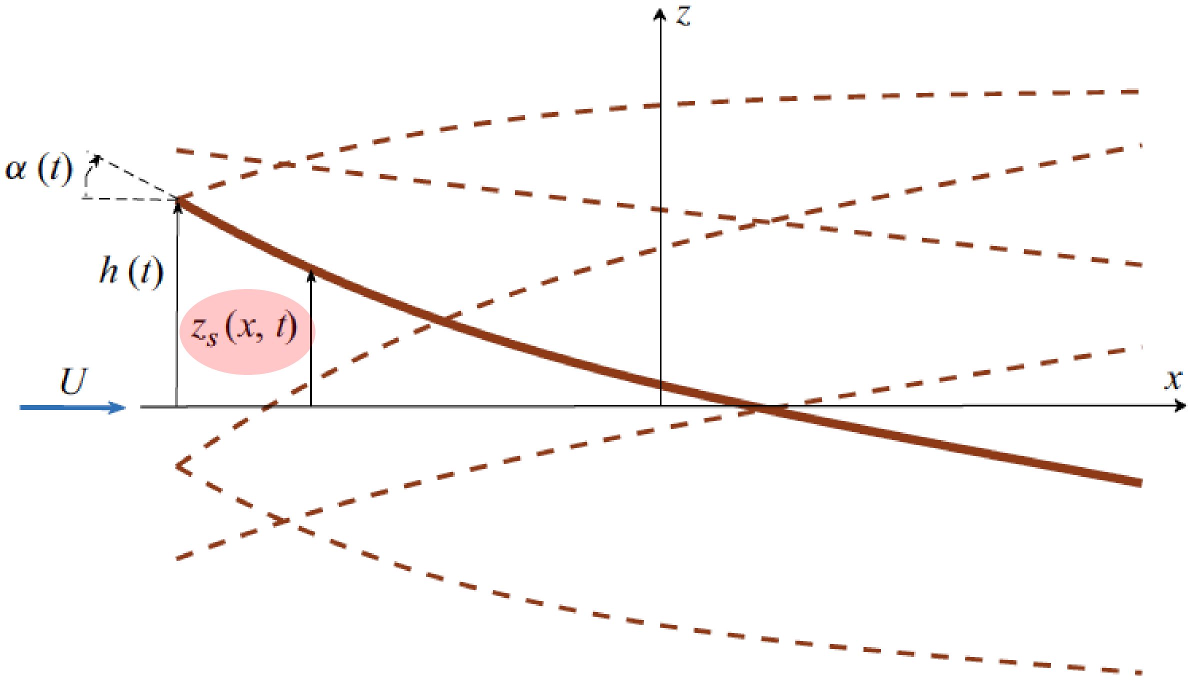
LINEAR APPROXIMATION

- Small-amplitude pitching, heaving and flexural deformation
- Pivot axis close to the leading edge
- Inextensible plate $(E\varepsilon/(\rho U^2 c) \gg 1)$
- Potential flow

$$\rho_s \varepsilon \frac{\partial^2 z_s}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 z_s}{\partial x^2}\right) = \Delta p + F_{pz}\delta(x - x_p) - g\delta'(x - x_p)$$

$$\Delta p = p^- - p^+$$

MOMENTS



- Transversal momentum conservation equation

$$\int_{-c/2}^{c/2} \rho_s \varepsilon \frac{\partial^2 z_s}{\partial t^2} \mathrm{d}x = F_z + F_{pz}$$

$$F_z = \int_{-c/2}^{c/2} (\Delta p)\mathrm{d}x \qquad \text{(Lift)}$$

MOMENTS (CONT.)

- Moment about the leading edge conservation (torsional balance equation)

$$\int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)\rho_s\varepsilon\frac{\partial^2 z_s}{\partial t^2}\mathrm{d}x = M + M_p$$

$$M = \int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)(\Delta p)\mathrm{d}x$$ (Fluid moment)

- Flexural moments conservation equations

$$\int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)^2\rho_s\varepsilon\frac{\partial^2 z_s}{\partial t^2}\mathrm{d}x + \int_{-c/2}^{c/2}2EI\frac{\partial^2 z_s}{\partial x^2}\mathrm{d}x = D_1,$$

$$D_1 = \int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)^2(\Delta p)\mathrm{d}x$$

(Fluid flexural moments)

$$\int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)^3\rho_s\varepsilon\frac{\partial^2 z_s}{\partial t^2}\mathrm{d}x + \int_{-c/2}^{c/2}6\left(x+\frac{c}{2}\right)EI\frac{\partial^2 z_s}{\partial x^2}\mathrm{d}x = D_2$$

$$D_2 = \int_{-c/2}^{c/2}\left(x+\frac{c}{2}\right)^3(\Delta p)\mathrm{d}x$$

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FOIL MOTION AND DEFORMATION APPROXIMATION TO OBTAIN ANALYTICAL EXPRESSIONS FOR THE FLUID FORCE AND MOMENTS

Non-dimensional (scaled with c/2 and U)

Fifth-order polynomial approximation for flexible pitching and heaving foil with two dregrees of freedom for the flexural deformation (passive – unknown):

$$z_s(x,t) = h(t) - \alpha(t)(x+1) + d_1(t)[24(x+1)^2 - 8(x+1)^3 + (x+1)^4] + d_2(t)[160(x+1)^2 - 40(x+1)^3 + (x+1)^5], \qquad -1 \leqslant x \leqslant 1$$

Leading edge: $z_s = h(t) \qquad \partial z_s/\partial x = -\alpha(t) \qquad \text{at} \quad x = -1$ (heave and pitch)

Trailing edge: $\partial^2 z_s/\partial x^2 = \partial^3 z_s/\partial x^3 = 0 \text{ at } x = 1$ (free trailing edge)

Captures accurately up to the first two natural bending modes J. Fluid Mech. **1015,** A35 (2025)

MOMENTS

$$R\left(\ddot{h}-\ddot{\alpha}+\frac{96}{5}\ddot{d}_1+\frac{416}{3}\ddot{d}_2\right)=C_L+C_{L_p},$$

$$R\left(\frac{1}{2}\ddot{h}-\frac{2}{3}\ddot{\alpha}+\frac{208}{15}\ddot{d}_1+\frac{704}{7}\ddot{d}_2\right)=C_M+C_{M_p},$$

$$R\left(\frac{4}{3}\ddot{h}-2\ddot{\alpha}+\frac{4544}{105}\ddot{d}_1+\frac{944}{3}\ddot{d}_2\right)+S\left(\frac{32}{3}d_1+80d_2\right)=C_{F_1},$$

$$R\left(2\ddot{h}-\frac{16}{5}\ddot{\alpha}+\frac{496}{7}\ddot{d}_1+\frac{32512}{63}\ddot{d}_2\right)+S\left(16d_1+128d_2\right)=C_{F_2},$$

$$C_L=\frac{F_z}{\frac{1}{2}\rho U^2 c}=\int_{-1}^{1}(\Delta P)\mathrm{d}x\,, \quad \text{with} \quad \Delta P=\frac{\Delta p}{\rho U^2},$$

$$C_M=\frac{M}{\frac{1}{2}\rho U^2 c^2}=\frac{1}{2}\int_{-1}^{1}(x+1)(\Delta P)\mathrm{d}x=\frac{1}{2}C_L+\frac{1}{2}\int_{-1}^{1}x(\Delta P)\mathrm{d}x,$$

$$C_{F_1}=\frac{D_1}{\rho U^2\left(\frac{c}{2}\right)^3}=\int_{-1}^{1}(x+1)^2(\Delta P)\mathrm{d}x=-C_L+4C_M+\int_{-1}^{1}x^2(\Delta P)\mathrm{d}x$$

$$C_{F_2}=\frac{D_2}{\rho U^2\left(\frac{c}{2}\right)^4}=\int_{-1}^{1}(x+1)^3(\Delta P)\mathrm{d}x=C_L-6C_M+3C_{F_1}+\int_{-1}^{1}x^3(\Delta P)\mathrm{d}x$$

Non-dimensional parameters: mass ratio and stiffness

$$R=\frac{4\rho_s\varepsilon}{\rho c} \quad \text{and} \quad S=\frac{4E\epsilon^3}{\rho U^2 c^3}$$

Point force and moment (forced or/and passive)

$$C_{L_p}=\frac{F_{pz}}{\frac{1}{2}\rho U^2 c}, \quad C_{M_p}=\frac{M_p}{\frac{1}{2}\rho U^2 c^2}$$

HARMONIC MOTION

Reduced frequency:

$$k = \frac{\omega c}{2U} = \frac{\pi f c}{U}$$

$$h(t) = h_0 e^{ikt}, \quad \alpha(t) = \alpha_0 e^{ikt} \quad \text{with} \quad \alpha_0 = a_0 e^{i\phi},$$

$$d_1(t) = d_{10} e^{ikt}, \quad d_2(t) = d_{20} e^{ikt}, \quad \text{with} \quad d_{10} = d_{1m} e^{i\psi_1}, \quad d_{20} = d_{2m} e^{i\psi_2}$$

Analytical expressions for the fluid force and moments using von Kármán and Sears decomposition of vorticity density distribution $\varpi_s(x,t) = u^+ - u^-$

$$\varpi_s(x,t) = \varpi_0(x,t) + \varpi_{se}(x,t)$$ added-mass and wake contributions

Pressure in terms of vorticity distribution:

$$\Delta P = \frac{\partial}{\partial t} \int_{-1}^{x} \varpi_s(\xi, t) \mathrm{d}\xi + \varpi_s(x,t)$$

$$\int_{-1}^{1} x^n (\Delta P) \mathrm{d}x = \frac{\mathrm{d}}{\mathrm{d}t} \int_{-1}^{1} \frac{1 - x^{n+1}}{n+1} \varpi_s(x,t) \mathrm{d}x + \int_{-1}^{1} x^n \varpi_s(x,t) \mathrm{d}x$$

For instance, quasisteady bound circulation

$$\Gamma_0(t) = \pi \left[-2\dot{h} + 3\dot{\alpha} + 2\alpha - \frac{263}{4}\dot{d}_1 - 59 d_1 - \frac{3831}{8}\dot{d}_2 - \frac{1755}{4} d_2 \right]$$

Alll the fluid force and moment coefficientes contain a added-mass and a circulatory contributions

LIFT

$$C_L(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} x\varpi_0(x,t)\mathrm{d}x - \int_1^{\infty}\frac{x\,\varpi_e(x,t)}{\sqrt{x^2-1}}\mathrm{d}x \equiv C_{L_0}(t) + C_{L_e}(t)$$

Non-circulatory or added mass contirbution

$$C_{L_0}(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} x\varpi_0(x,t)\mathrm{d}x$$
$$= \pi\left(\dot{\alpha} + \ddot{\alpha} - \ddot{h} - 25\dot{d}_1 - \frac{149}{8}\ddot{d}_1 - \frac{1465}{8}\dot{d}_2 - \frac{1073}{8}\ddot{d}_2\right)$$

Circulatory contribution

$$C_{L_c}(t) = -\int_1^{\infty}\frac{x}{\sqrt{x^2-1}}\varpi_e(x,t)\mathrm{d}x = \Gamma_0(t)\mathcal{C}(k)$$

Theodorsen's function

$$\mathcal{C}(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i H_0^{(2)}(k)}$$

MOMENT

$$C_M(t) = C_{M_0}(t) + C_{M_c}(t)$$

Added mass contirbution

$$C_{M_0}(t) = \frac{\pi}{256}\left(192\dot{\alpha} + 144\ddot{\alpha} - 128\ddot{h} - 576 d_1 - 5248\dot{d}_1 - 2800\ddot{d}_1 \right.$$
$$\left. -4640 d_2 - 38680\dot{d}_2 - 20213\ddot{d}_2\right),$$

Circulatory contribution

$$C_{M_c}(t) = \tfrac{1}{4}\Gamma_0(t)\mathcal{C}(k)$$

FLEXURAL MOMENTS

$$C_{F_1}(t) = C_{F_{10}}(t) + C_{F_{1c}}(t)$$

$$C_{F_{10}}(t) = \frac{\pi}{192}\big(384\dot{\alpha} + 288\ddot{\alpha} - 240\ddot{h} - 1056d_1 - 10848\dot{d}_1 - 5745\ddot{d}_1$$
$$-8640d_2 - 80190\dot{d}_2 - 41540\ddot{d}_2\big),$$

$$C_{F_{1c}}(t) = \tfrac{1}{2}\Gamma_0(t)\mathcal{C}(k).$$

$$C_{F_2}(t) = C_{F_{20}}(t) + C_{F_{2c}}(t)$$

$$C_{F_{20}}(t) = \frac{\pi}{128}\big(368\dot{\alpha} + 280\ddot{\alpha} - 224\ddot{h} - 912d_1 - 10580\dot{d}_1 - 5680\ddot{d}_1$$
$$-7530d_2 - 78350\dot{d}_2 - 41117\ddot{d}_2\big),$$

$$C_{F_{2c}}(t) = \frac{5}{8}\Gamma_0(t)\mathcal{C}(k)$$

VALIDATION OF THE FLEXURAL DEFORMATION

Passive flexural deformation by solving the system of two linear equations for the flexural deformation amplitudes (two las moment equations) $\mathsf{A} \cdot \boldsymbol{d}_0 = \boldsymbol{b},$

$$\mathsf{A} = \begin{pmatrix} \frac{32}{3} S - \frac{4544}{105} Rk^2 - D_{11} & 80\, S - \frac{944}{3} Rk^2 - D_{12} \\ 16\, S - \frac{496}{7} Rk^2 - D_{21} & 128\, S - \frac{32512}{63} Rk^2 - D_{22} \end{pmatrix},$$

$$\boldsymbol{d}_0 = \begin{pmatrix} d_{10} \\ d_{20} \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} \frac{4}{3} Rk^2 h_0 - 2Rk^2 \alpha_0 + D_{h1} h_0 + D_{a1} \alpha_0 \\ 2Rk^2 h_0 - \frac{16}{5} Rk^2 \alpha_0 + D_{h2} h_0 + D_{a2} \alpha_0 \end{pmatrix},$$

D... are the fluid contributions

Resonant frequencies with FSI:

by minimizing $|\det(\mathsf{A})|$

In vacuum (D... = 0) recovers almos exactly the first two resonant bending frequencies: →

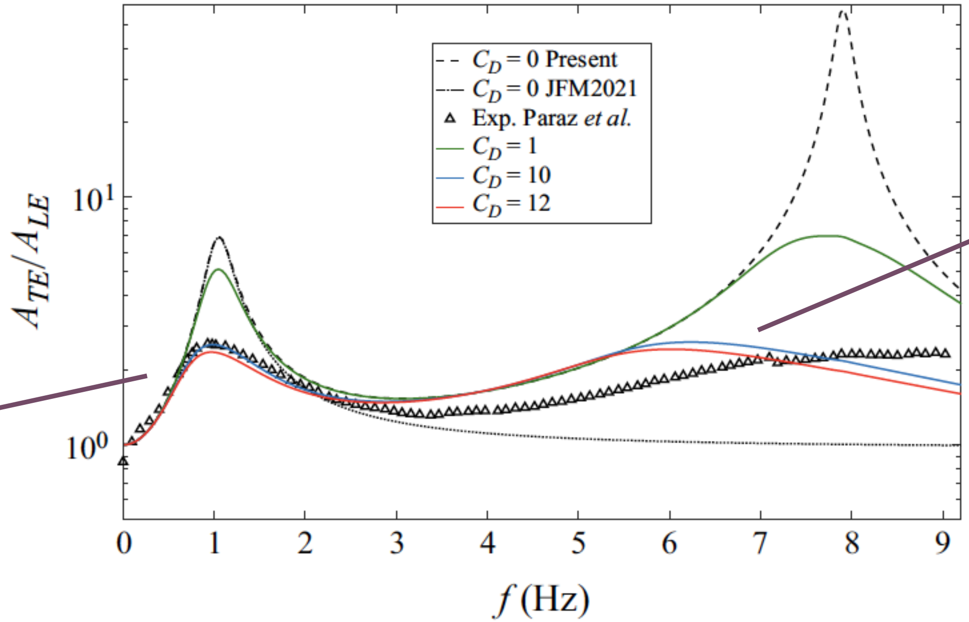
$$\begin{cases} k_{r10} = \sqrt{\frac{7}{953}\left(629 - 2\sqrt{88\,189}\right)\frac{S}{R}} \simeq 0.507521\sqrt{\frac{S}{R}}, \\ k_{r20} = \sqrt{\frac{7}{953}\left(629 + 2\sqrt{88\,189}\right)\frac{S}{R}} \simeq 2.997118\sqrt{\frac{S}{R}}. \end{cases}$$

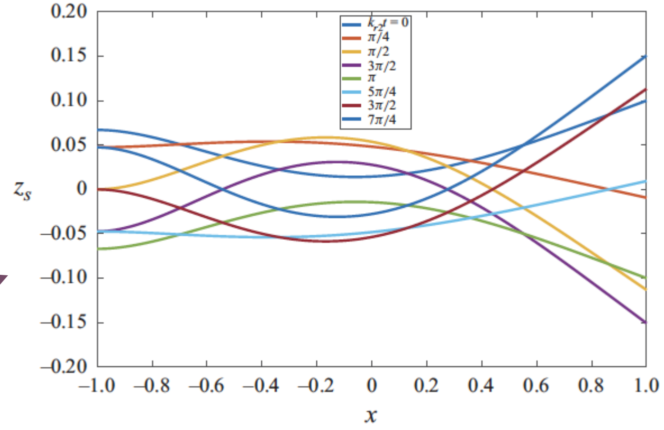
VALIDATION OF THE FLEXURAL DEFORMATION

Comparison with experimental results by Paraz, Eloy and Schouvelier (2014) for pure heave:

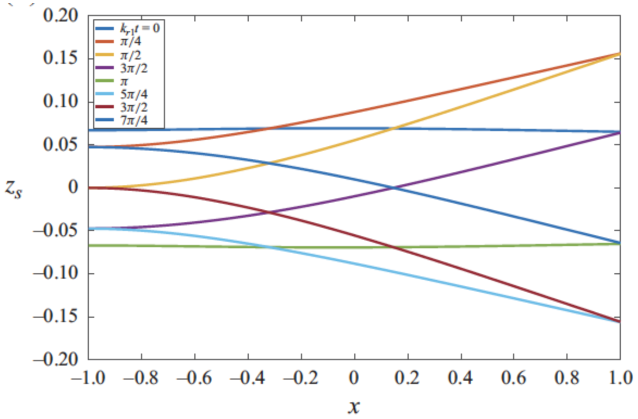
Correction with nonlinear fluid damping

$$\frac{1}{2} C_{Dz} \left| \frac{\partial z_s}{\partial t} \right| \frac{\partial z_s}{\partial t}$$





Foil motion at the second resonant frequency



Foil motion at the first resonant frequency

PROPULSION PERFORMANCE

THRUST from vortical impulse theory

$$\mathbf{F} = -\rho\frac{d}{dt}\int_{\mathcal{V}_\infty}(\mathbf{x}\wedge\omega)d\mathcal{V}$$

$$C_T(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} z_s(x,t)\varpi_s(x,t)\mathrm{d}x - \frac{\mathrm{d}}{\mathrm{d}t}\int_{1}^{\infty} z_e(x,t)\varpi_e(x,t)\mathrm{d}x$$

(Lift : $C_L(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} x\varpi_s(x,t)\mathrm{d}x - \frac{\mathrm{d}}{\mathrm{d}t}\int_{1}^{\infty} x\varpi_e(x,t)\mathrm{d}x$)

$$C_T(t) = C_{T0}(t) + C_{Tc}(t),$$

Added mass contribution:

$$C_{T0} = -\frac{\mathrm{d}}{\mathrm{d}t}[\alpha\Gamma_a + d_1\Gamma_{0d1} + d_2\Gamma_{0d2}],$$

Circulatory contribution:

$$C_{Tc} = \left(\dot{h} - \dot{\alpha} + \frac{163}{8}\dot{d}_1 + \frac{1183}{8}\dot{d}_2\right)[\Gamma_0\mathcal{C}_h] + \dot{\alpha}[\Gamma_0\mathcal{C}_{a1}] + \alpha[\Gamma_0\mathcal{C}_{a0}]$$
$$+ \dot{d}_1[\Gamma_0\mathcal{C}_{d11}] + d_1[\Gamma_0\mathcal{C}_{d10}] + \dot{d}_2[\Gamma_0\mathcal{C}_{d21}] + d_2[\Gamma_0\mathcal{C}_{d20}],$$

CYCLE-AVERAGED THRUST

$$\overline{C}_T = \frac{k}{2\pi}\int_t^{t+2\pi/k} C_T(t)\mathrm{d}t$$

$t_h, t_a, t_{d_1}, t_{d_2}$ ...

functions of the reduced frequency and the phase shifts

$$\overline{C}_T = \overline{C}_{Tc} = t_h h_0^2 + t_a a_0^2 + t_{d_1} d_{1m}^2 + t_{d_2} d_{2m}^2 + t_{ha} h_0 a_0 + t_{hd1} h_0 d_{1m} + t_{hd2} h_0 d_{2m} + t_{ad1} a_0 d_{1m} + t_{ad2} a_0 d_{2m} + t_{d1d2} d_{1m} d_{2m}$$

Only circulatory contributions

POWER

$$C_P(t) = \dot{h}(t) C_{L_p}(t) - 2\dot{\alpha}(t) C_{M_p}(t).$$

Inertia and fluid contributions: $C_P = C_{P_R} + C_{P_F} = C_{P_R} + C_{P_0} + C_{P_c}$

Inertia: $C_{P_R} = R\left[\dot{h}\left(\ddot{h} - \ddot{\alpha} + \frac{96}{5}\ddot{d}_1 + \frac{416}{3}\ddot{d}_2\right) - 2\dot{\alpha}\left(\frac{1}{2}\ddot{h} - \frac{2}{3}\ddot{\alpha} + \frac{208}{15}\ddot{d}_1 + \frac{704}{7}\ddot{d}_2\right)\right]$

Fluid contributions: added-mass and circulatory: $C_{P_F} = -\dot{h} C_L + 2\dot{\alpha} C_M$

$$C_{P_0} = -\dot{h} C_{L_0} + 2\dot{\alpha} C_{M_0},$$

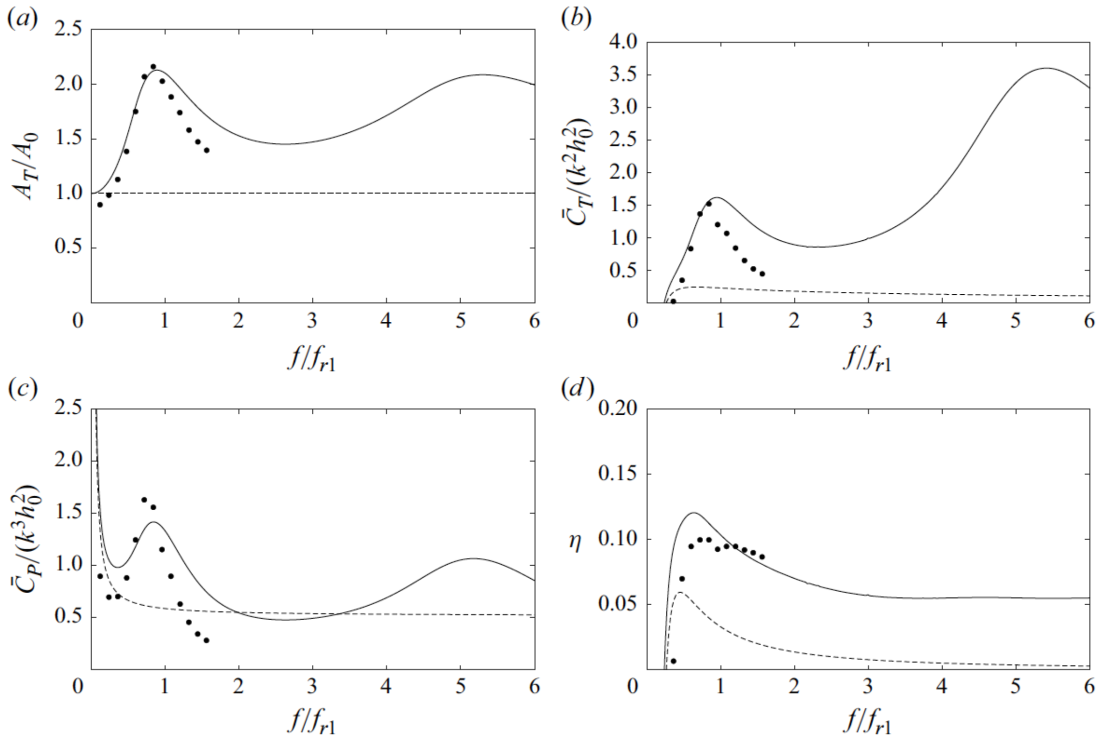
$$C_{P_c} = -\dot{h} C_{L_c} + 2\dot{\alpha} C_{M_c} = \left(-\dot{h} + \frac{1}{2}\dot{\alpha}\right)[\Gamma_0 \mathcal{C}(k)]$$

EFFICIENCY

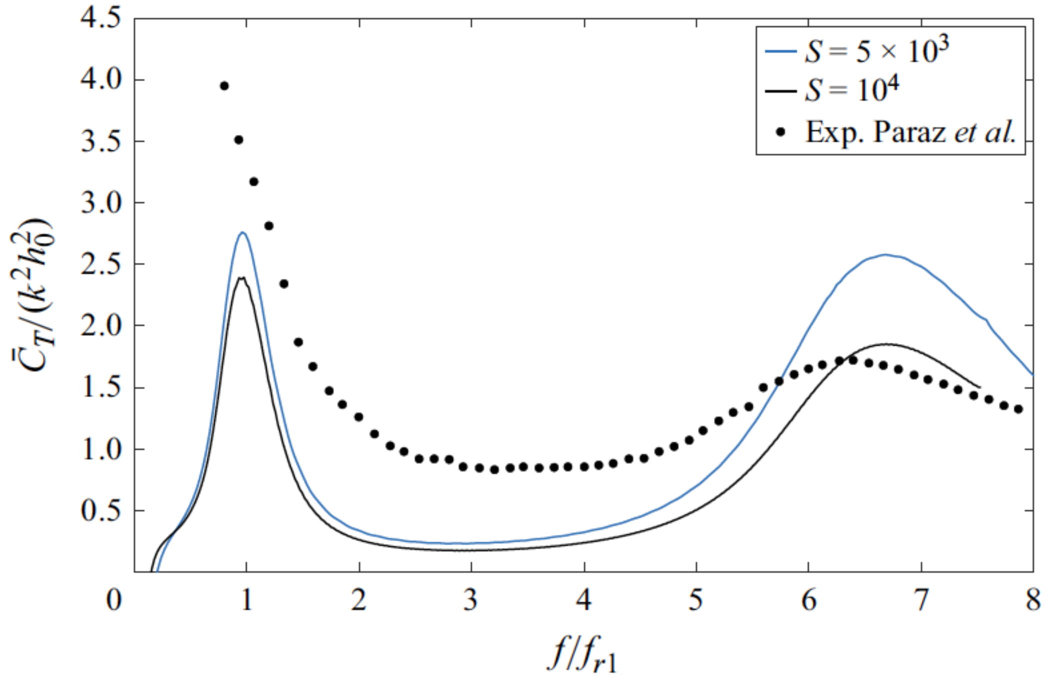
$$\eta = \frac{\overline{C}_T}{\overline{C}_{P+}}$$

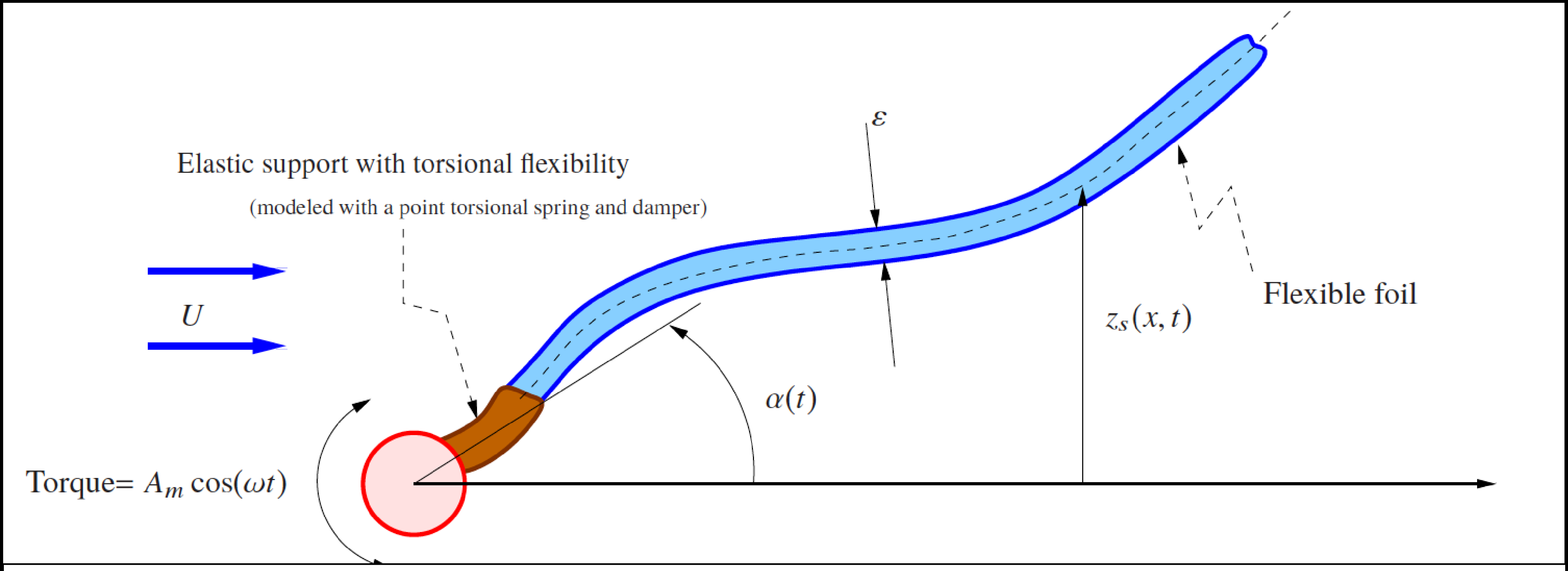
COMPARISON WITH EXPERIMENTAL RESULTS

Quinn, Lauder and Smits (2014)



Paraz, Schouveiler and Eloy (2016) for U=0





APPLICATION TO AN ELASTICALLY MOUNTED FLEXIBLE PITCHING FOIL ACTUATED BY A GIVEN TORQUE AT THE LEADING EDGE

J. Fluids Structures (2025; under review)

PASSIVE HEAVE, PITCH, AND DEFORMATION

$$h(t) = h_0 e^{ikt}, \quad \alpha(t) = \alpha_0 e^{ikt} \quad d_1(t) = d_{10} e^{ikt}, \quad d_2(t) = d_{20} e^{ikt} \qquad k = \frac{\omega c}{2U} = \frac{\pi f c}{U}$$

Resulting from the linear equations:

$$\mathsf{A} \cdot \mathbf{d}_0 \equiv (\mathsf{A}_0 + \mathsf{A}_f) \cdot \mathbf{d}_0 = \mathbf{b} \longrightarrow \mathbf{d}_0 = \begin{pmatrix} h_0 \\ \alpha_0 \\ d_{10} \\ d_{20} \end{pmatrix}$$

$$C_{L_i} = A_l e^{ikt} \qquad \mathbf{b} = \begin{pmatrix} A_l \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad \mathbf{b} = \begin{pmatrix} 0 \\ A_m \\ 0 \\ 0 \end{pmatrix} \qquad C_{M_i} = A_m e^{ikt}$$

Forced oscillating torque

Actuating force or torque at the leading edge

Inertia of the foil and its elastic support (linear and torsional springs and dampers)

$$\mathsf{A}_0 = \begin{pmatrix} -k^2R + k_h + ikb_h & k^2R & -96k^2R/5 & -416k^2R/3 \\ -k^2R/2 & 2k^2R/3 - k_a - ikb_a & -208k^2R/15 & -704k^2R/7 \\ -4k^2R/3 & 2k^2R & 32S/3 - 4544k^2R/105 & 80S - 944k^2R/3 \\ -2k^2R & 16k^2R/5 & 16S - 496k^2R/7 & 128S - 32512k^2R/63 \end{pmatrix}$$

$$C_{L_p} = C_{Li} - k_h h - b_h \dot{h} \qquad C_{M_p} = C_{M_i} + k_a \alpha + b_a \dot{\alpha}$$

FSI

$$\mathsf{A}_f = -\begin{pmatrix} l_h & l_a & l_1 & l_2 \\ m_h & m_a & m_1 & m_2 \\ f_{1h} & f_{1a} & f_{11} & f_{22} \\ f_{2h} & f_{2a} & f_{21} & f_{22} \end{pmatrix}$$

$$l_h = \pi[k^2 - C(k)2ik],$$
$$l_a = \pi[ik - k^2 + C(k)(3ik + 2)],$$
$$l_1 = \pi[-25ik + 149k^2/8 - C(k)(263ik/4 + 59)],$$
$$l_2 = \pi[-1465ik/8 + 1073k^2/8 - C(k)(3831ik/8 + 1755/4)]$$
$$m_h = \pi[k^2 - C(k)ik]/2,$$
$$m_a = \pi[3ik/4 - 9k^2/16 + C(k)(3ik + 2)/4],$$
$$m_1 = \pi[(-576 - 5248ik + 2800k^2)/256 - C(k)(263ik/4 + 59)/4],$$
$$m_2 = \pi[(-4640 - 38680ik + 20213k^2)/256 - C(k)(3831ik/8 + 1755/4)/4]$$

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RESONANT FREQUENCIES by minimizing |det(A)|

- Springs modes
- Two bending modes

In vacuum by minimizing $|\det(A_0)|$

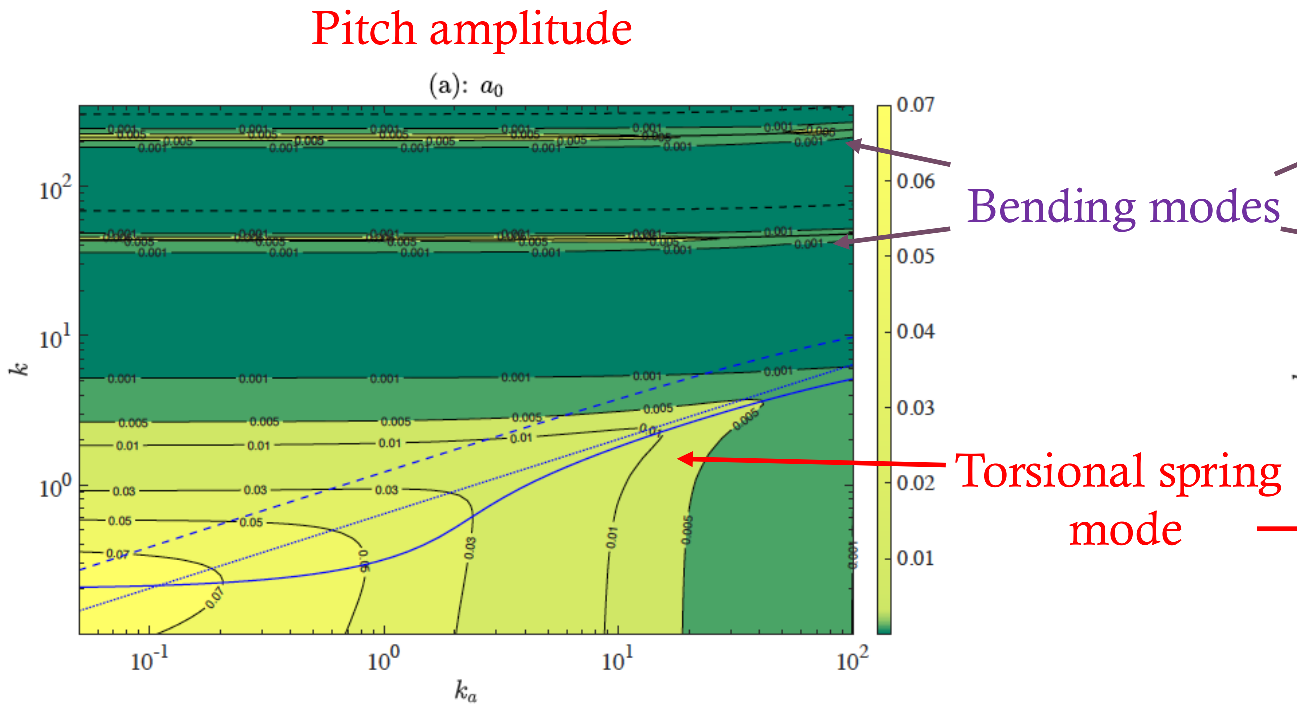
$$|\det(A_0)| = 0 \begin{cases} \text{rigid plate } (S \to \infty) & k_{r0h} = \sqrt{\dfrac{k_h}{R}}, \quad k_{r0a} = \sqrt{\dfrac{3k_a}{2R}} \\ \text{flexible plate} & k_{r01} \simeq 0.50\sqrt{\dfrac{S}{R}}, \quad k_{r02} \simeq 3.0\sqrt{\dfrac{S}{R}} \end{cases}$$

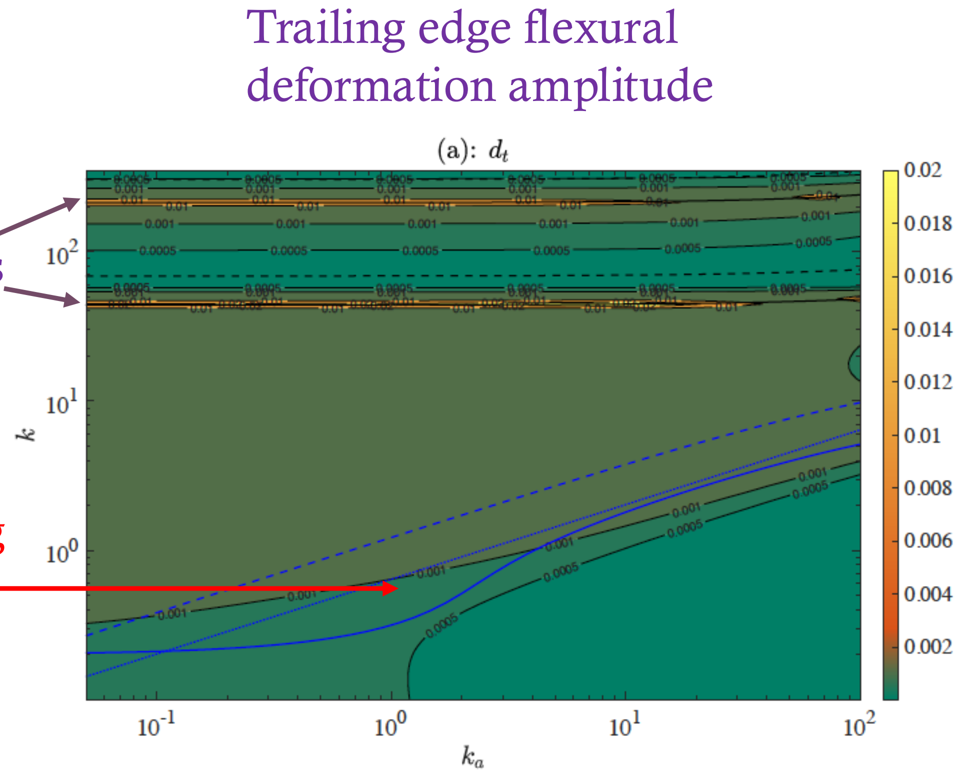
Partially taking into account the FSI (only k^2 terms) for a rigid foil

$$k_{r1ah} = \sqrt{\frac{\pm\sqrt{[48k_a(R+\pi)+k_h(32R+27\pi)]^2 - 192k_ak_h(R+\pi)(8R+3\pi)} + 48k_a(R+\pi)+k_h(32R+27\pi)}{2(R+\pi)(8R+3\pi)}}$$

Pitching only $k_h \to \infty$ $\quad k_a \gg 1 \quad$ $k_{r1a} = \sqrt{\dfrac{k_a}{\frac{2R}{3}+\frac{9\pi}{16}}}$ (Coincide with Moore (2014))

Passive pitch and deformation (no passive heave) $k_h \to \infty$

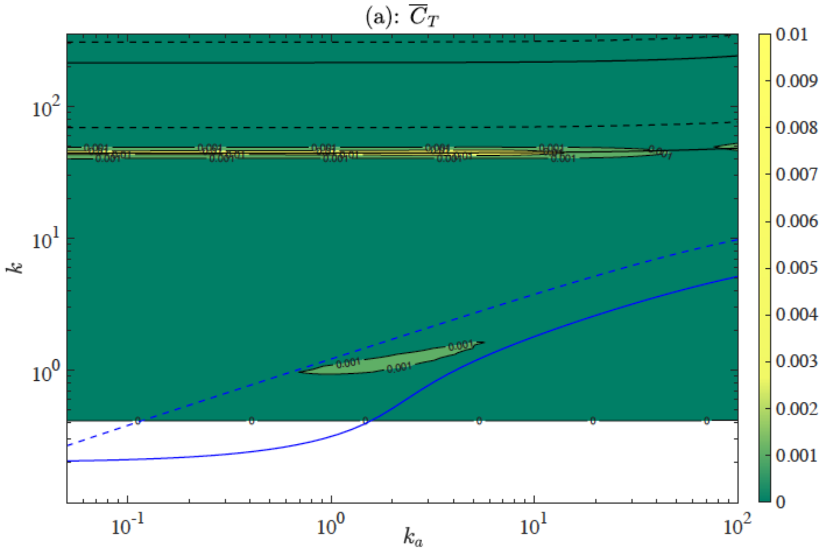




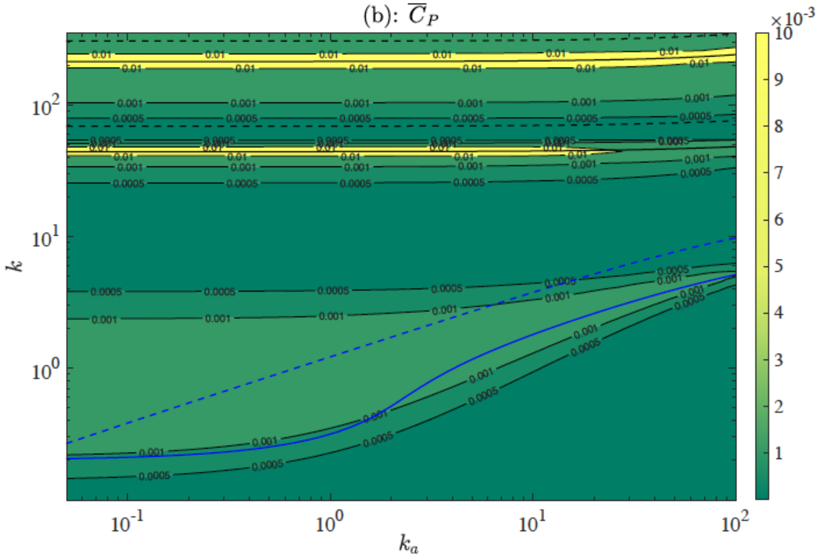
$S = 10^3, R = 1, b_a = 0.05, \quad A_m = 0.1$

PROPULSION PERFORMANCE (same case)

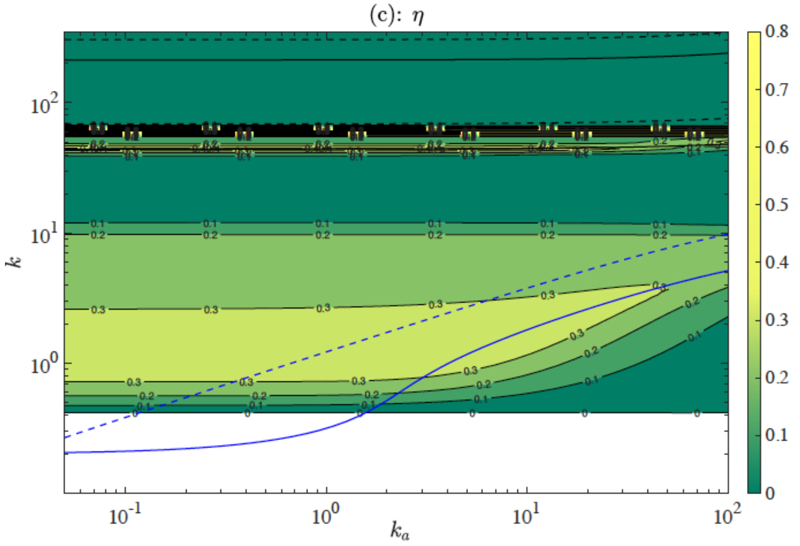
Thrust



Power



Efficiency

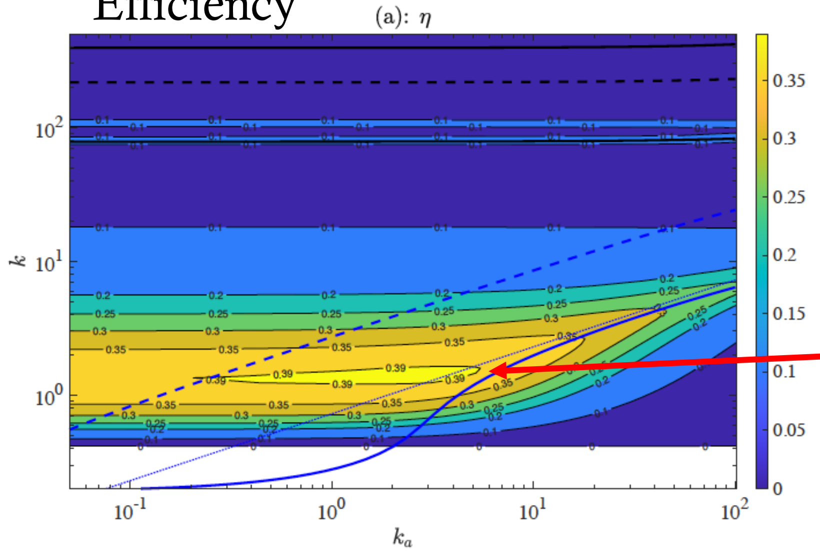


AQUATIC PROPULSION

Mass ratio $R \lesssim 0.5$

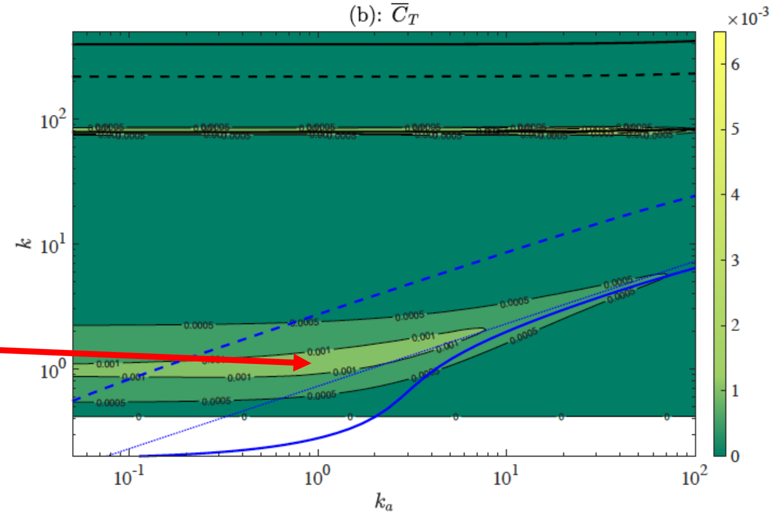
Efficiency

Thrust

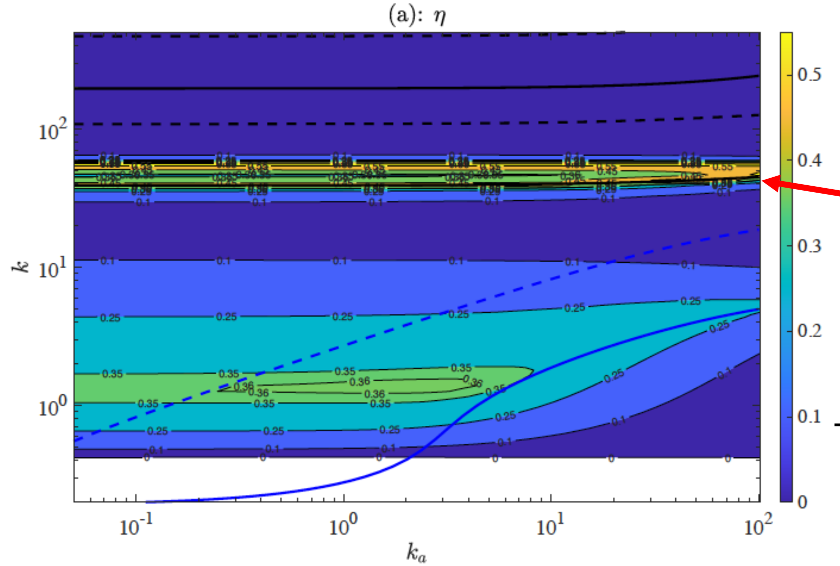


$R = 0.2$

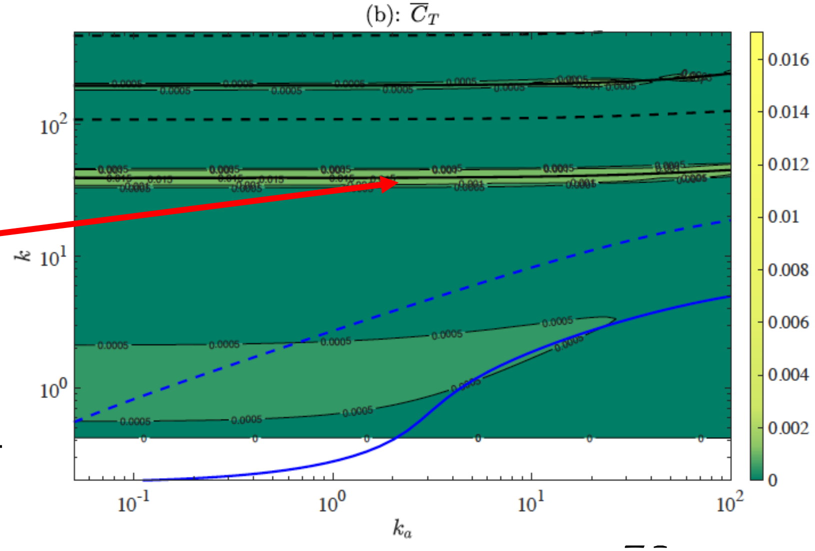
Spring mode



$k_h \to \infty$, $S = 2000$, $R = 0.2$, $b_a = 0.05$, and $A_m = 0.1$.



Bending mode
(too high frequency)



$k_h \to \infty$, $S = 500$, $R = 0.2$, $b_a = 0.05$, and $A_m = 0.1$

AQUATIC PROPULSION

Mass ratio $R \lesssim 0.5$

Nondimensional frequency and spring constant for maximum efficiency at the SPRING MODE remain almost constant as the stiffness is varied.

Case study: $R = \frac{4m}{\rho c^2 s} \simeq 0.37$ steel ($E = 2 \times 10^{11}$ Pa, $\rho_s = 7700$ kg m^{-3})

$S = \frac{512}{U^2}$ $f_c = \frac{U}{0.31416}$, $K_a = 1.65\, U^2$ $f = f_c k$, $\tilde{k}_a = K_a k_a$

U (in m/s)

f (in Hz), $\tilde{k}_a$ (in Nm)

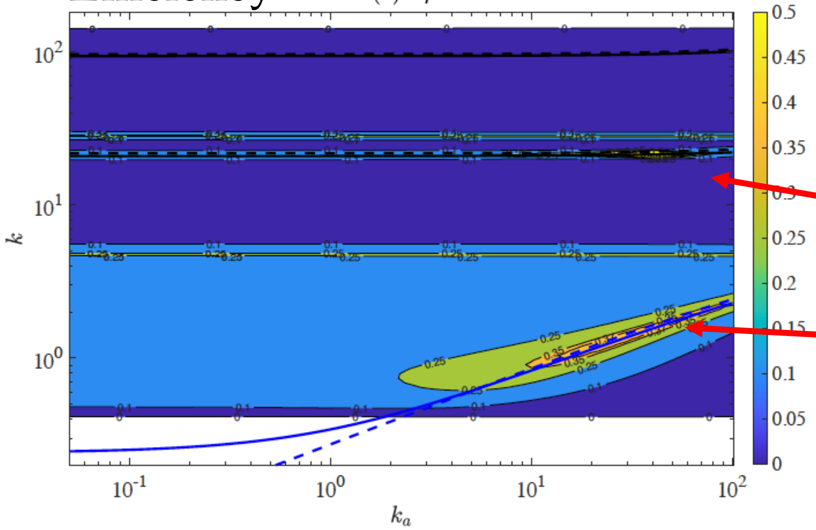
U	S	η_{max}	f	$\tilde{k}_a$	k	k_a	$\bar{C}_{T_{max}}$	f	$\tilde{k}_a$	k	k_a
0.1	5120	0.400	0.432	0.0426	1.358	2.582	0.111	0.398	0.029	1.250	1.758
0.2	1280	0.387	0.860	0.170	1.350	2.576	0.107	0.738	0.105	1.160	1.591
0.4	320	0.339	1.729	0.619	1.358	2.345	0.091	1.354	0.287	1.064	1.087

Optimal propulsion: $k_{opt} \approx 1.4$ and $k_{a_{opt}} \approx 2.5$ JUST ABOVE THE SPRING MODE

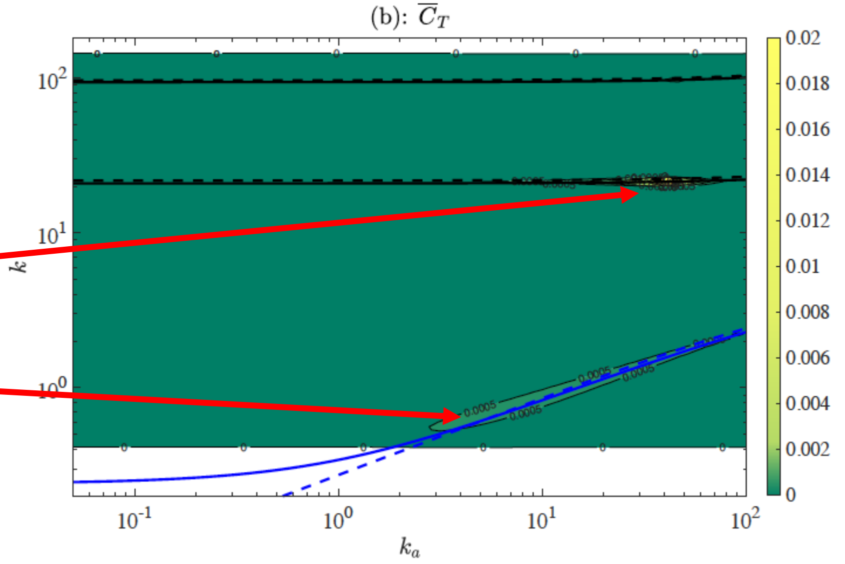
AERIAL PROPULSION

Mass ratio $R \gtrsim 20$

Efficiency



Thrust

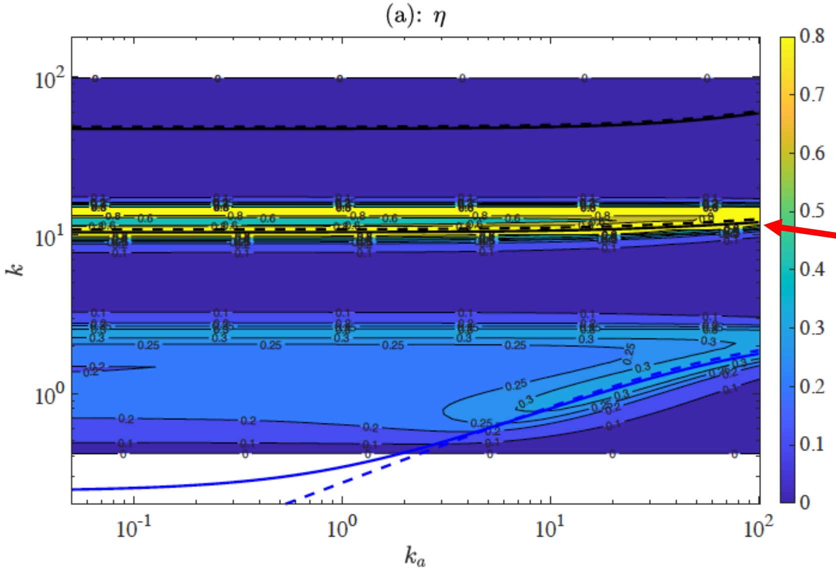


$R = 20$

Bending mode
(not too high frequency)

Spring mode

$k_h \to \infty$, $S = 2000$, $R = 20$, $b_a = 0.05$, and $A_m = 0.1$

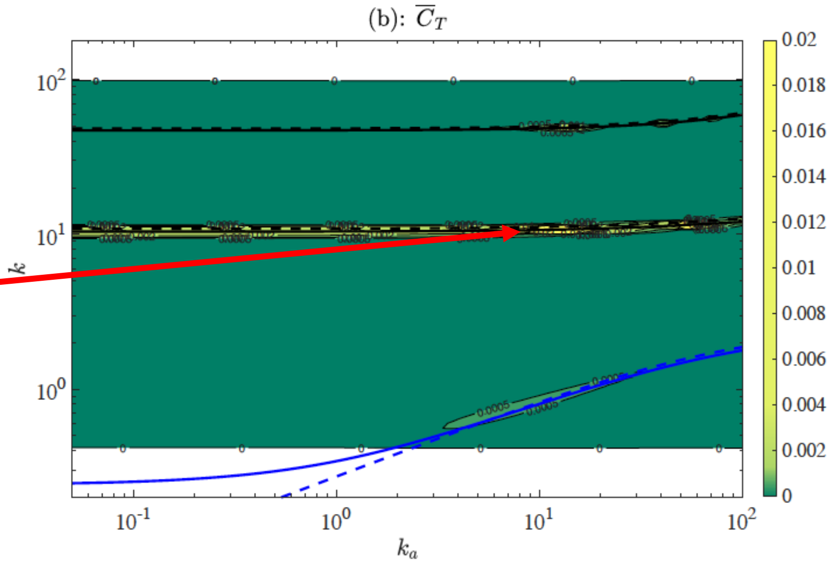


Optimal propulsion:

$k_{opt} \approx k_{a_{opt}} \approx 10$

Bending mode
(not too high frequency)

$k_h \to \infty$, $S = 500$, $R = 20$, $b_a = 0.05$, and $A_m = 0.1$



CONCLUSION

- The FSI of a flexible pitching foil elastically supported at its leading edge and actuated by a given torque is analyzed, characterizing analytically its propulsion performance for small-amplitude of the oscillation and deformation

- Values of the nondimensional parameters for optimal propulsion are obtained for both aquatic and aerial propulsion

  The optimal dimensional parameters have to be tuned with the freestream speed. For instance, for aquatic propulsion

$$f_{opt} \approx \frac{1.4}{\pi}\frac{U}{c}, \quad \tilde{k}_{a,opt} \approx \frac{2.5}{2}\rho c^2 U^2$$

- Can serve as a guide for the design of efficient aquatic and aerial thrusters. Experimental validation as a future work