Comparison Matrix Geometric Index: A Qualitative Online Reputation Metric

Gustavo Vaccaro[1,2] Francisco E. Cabrera[1,2] Jose Ignacio Pelaez[1,2] L.G. Vargas[2,3]

[1] Dept. Languages and Computer Sciences. Biomedical Research Institute (IBIMA), University of Malaga, Spain.

[2] Applied Social Research Center (CISA). University of Malaga., Spain.

[3] Joseph M. Katz Graduate School of Business, University of Pittsburgh, USA.

Corresponding author: Jose Ignacio Peláez. jipelaez@uma.es.

Abstract. Previous scientific studies as well as consulting firms have developed numerous Online Reputation Indices (ORIs), i.e. custom-tailored metrics intended to measure the emotions that people express towards a brand, product, or service in social media. These ORIs can provide useful information to assess the impact of marketing campaigns, social approval, and viral behavior of news and memes, among others. However, traditional ORIs are isolated metrics; thus, they are not suitable for determining the relative preference between two alternatives; e.g.: twice the number of "likes" does not imply that a given product is *preferred* two times as much as its competitor. This is an important constraint for ORIs, because stakeholders are driven to make relative and qualitative comparisons to weigh the alternatives' value. Furthermore, relative comparisons are crucial for the systematic evaluation of alternatives in social decision making processes. The aim of this paper is to present a novel qualitative online reputation metric, a Comparison Matrix Geometric Index (CMGI), that considers both the accumulated emotions and the direct comparisons expressed in social media communications to infer the relative preferences among a set of alternatives.

Keywords: Intelligent Information Retrieval, Social Media Information Analysis, Decision Analysis, Multiple Criteria Analysis, Reputation Index, Opinion Analysis, Corporate Reputation.

1. Introduction

Since the dawn of marketing, the business world has been aware of the importance of reputation for achieving good sales results. Customer experience has always been one of the most powerful tools for building reputation and attracting new business. However, traditional word of mouth has undergone major changes thanks to the impact of Social Media in our society. Historically, neighbors would chit-chat on how the new shop on the corner of their street is better than the old one. Nowadays the "neighborhood" is an online forum, the "neighbors" are a community with hundreds of thousands of demanding Internet users and they would compare the ever growing range of products available to them [1–4].

In order to increase sales, both in online and traditional markets, it is necessary to know not only your reputation, as an isolated metric, but its relative positioning among your direct competitors. A customer's bad experience published online could have a widespread negative effect and affect sales. Moreover, bad experiences of consumers, with any of the companies and brands pertaining to a particular business sector, have a direct or indirect impact on each and every known competitor in this sector. Reputation crises have a negative effect on the expectation of consumers about a particular brand being able to provide products that can satisfy their needs. Thus, as the needs themselves are affected by a crisis, consumers will gravitate towards competitors in the same sector that are less affected by it [5].

For this reason, we need effective methods that enable us to measure: (a) our reputation, the reputation of our services and products, and (b) the relative preference of consumers for our products/services in relation to our competitors.

Even though the reputation of a person/brand/service can be tracked from multiple sources, there is a growing tendency on talking about online reputation. This is a consequence of multiple factors. First, the increasing shift of attention towards social media as a data source for reputation analyses, gradually replacing the use of surveys that usually are too long, complex to answer and not in real time, i.e., there is a time delay between the responses and the results of the analyses. Second, the dynamic nature of user generated content, allows the detection of emerging trends on consumers earlier than the traditional approach of using face-to-face surveys, which are heavily restricted on the point of view of the survey designers [6,7].

Lastly, as a consequence of the non-solicited nature of the obtained data, the strategic aspect of decisions is avoided, i.e., we do not have to ask individuals about their opinions in order to make collective decisions.

Because, individuals are prone to realize that collective decisions are made considering their opinion, when they are directly asked about theirs, they usually chose the answer that, from their point of view, can result in the most favorable outcome, instead of just expressing their true opinion [8,9].

In sum, traditional surveys usually express information on what respondent perceive as the “best” answer, rather than their true intentions or opinions. A very clear example of this effect can clearly be seen whenever presidential elections are carried out. Political polls often present results quite dissimilar from actual election results precisely because survey respondents are trying to guess the correct answers instead of stating their opinion.

Online reputation can be defined as the sentiment towards a business/brand/person on the Internet. It is a composite of the image projected by its own brand, news, opinions and comments willingly expressed by third parties in Social Networks, forums, blogs and online media [10].

There are different sets of tools, indices and monitors, that enable us to measure the perceived reputation of brand/services/products by both the public and experts [11,12]. However, these measures pose several design problems, such as the strategic aspect of decisions; the impossibility of obtaining real time valuations; and dynamic problems, since the value of reputation of an alternative must vary according to the reputation obtained by the other alternatives with which it is compared, or the time of exposure of the alternative. In order to solve these problems, companies and researchers have developed different indices, named Online Reputation Indexes or (ORIs), using symbols such as stars, number of visits, likes, etc., which are used to measure the performance of services, products or advertising campaigns. However, to determine whether or not a method is able to properly rank elements, according to preferences expressed in digital ecosystems, two questions must be answered: (1) Do these ORIs truly convey the meaning of what they are being used for, namely measuring preferences between alternatives?; (2) Could these indices be also affected by the strategic aspect of decisions?

The answer to the first question is NO, given that users are not performing a process of comparison between alternatives. Although these indicators do provoke a reflection from the users, and it forces them to consciously evaluate their consumed products and services, this interaction does not necessarily reflect user preferences since, in order to obtain meaningful ranks, comparisons are needed [13]. The answer to the second question is YES. Even after the change on the representation of the same information, scores based on likes or stars are inherently a way of performing surveys, thus including all of their aforementioned shortcomings [14].

When making a purchasing decision between groups of commercial alternatives, two aspects need to be considered. The first one is the limited resources of the buyer. This is the reason forcing a choice from the group of alternatives in any purchasing process from the buyer’s perspective, that can potentially solve a need. This means that any index for the valuation of brand reputation should work with groups of comparable commercial alternatives, rather than the whole universe of commercially available products. The second aspect is the fact that reputation has a great effect on the expected utility of products but not so much on the consumer's need to be solved [15].

Because of that, in the event of a reputational crisis of a brand, consumers have a tendency to label said brand as a poor choice for their need, but the need persists, nonetheless. In these situations, customers have to deal with a decreased probability to satisfy their needs from said brand's potential. This is known as Ellsberg's paradox. It represents a preference shift by consumers towards an option perceived as more secure or with less adverse consequences. The opposite can be said of crazes, which artificially increase the perceived benefit of a given brand or product to the detriment of the competition, making consumers gravitate towards the most popular alternative [16,17].

The current competitive environment, with an increasing number of companies taking risks and making decisions driven by their reputation, makes qualitative reputation indices a pressing necessity. These indices must be capable of directly ascertain preferences within groups of alternatives from unsolicited opinions of the general public. The existence of these indices would be extremely useful for enterprises, since they could benefit from this information when designing products, advertising campaigns, detecting customer behavior patterns or evaluating the performance products and services.

The aim of this paper is to present a novel qualitative online reputation metric, a Comparison Matrix Geometric Index (CMGI). This index considers both accumulated emotions and direct comparisons expressed in unsolicited communications to infer the relative preferences among sets of alternatives. The main advantage of this new indicator is that it directly obtains a preference ranking from large sets of valuations willingly performed by consumers on online platforms such as Social Media platforms. Moreover, this index is designed with the aim of ranking comparable alternatives, and as such allows a parametric adjustment which leads not only to the ranking of these alternatives, but also the valuation of the intensity of said preferences.

This paper has been organized as follows: In the second section, we present the notation and terminology of the new index; in the third section we define CMGI; in the fourth section we give a complete example of its use, and we finish with conclusions and future directions.

2. Notation and Terminology

Let $A = \{a_1, a_2, \dots, a_m\}$ a set of comparable alternatives and let $O = \{o_1, \dots, o_n\}$ a set of opinions expressed by decision makers on alternatives, where o_k represents the *k*th decision maker's emotions/preferences towards elements in A.

Definition 1. $P = \{\uparrow, \downarrow, \leftrightarrow\}$ is defined as a set of valuation polarity artifacts, where $\uparrow$ represents a positive polarity, $\downarrow$ represents a negative polarity, and $\leftrightarrow$ a neutral polarity, that can be expressed with regard to a_j in any o_k.

Definition 2. Strong Positive Preference (sp). Given an opinion o_k and two alternatives $a_i, a_j \in A$, a Strong Positive Preference exists of a_i over a_j if a positive polarity preference of a_i over a_j is explicitly expressed.

For example, given an opinion o_k= "The service X provided by hotel a_i is better than that of hotel a_j". There is a Strong Positive Preference of a_i over a_j, since in this opinion, the user states explicitly that, for a given service, hotel a_i is better than a_j.

Definition 3. Weak Positive Preference (wp). Given an opinion o_k, and two alternatives $a_i, a_j \in A$, there is a Weak Positive Preference of a_i over a_j if any of the two following conditions meet:

- A positive assessment is explicitly expressed about a_i but no explicit assessment is made with regards to a_j.
- No assessment is explicitly expressed about a_i but there is an explicit negative assessment with regards to a_j.

For example, given the opinion o_k= "The service provided by hotel a_i is excellent", there is a Weak Positive Preference of a_i over a_j, since an explicit positive assessment about a_i is made but no assessment is made about a_j.

In order to build a valuation index, we need a relationship function f between sp and wp, $f(sp) = wp$, that models the distance between weak and strong preferences. This function depends on multiple factors, such as the relationships among the alternatives within A or the nature of the unsolicited communications expressed in O. Furthermore, the distance between preference levels depends on the perception of the consumers; also, this perception derives from the experiences and emotions of the consumers [18,19]. This function f can be constructed by considering Weber-Fechner's law, that states that the intensity with which a sensation is perceived is proportional to the logarithm of the intensity of the stimulus causing it [17,20]. With this in mind, we propose that the relationship between $wp\ and\ sp$ is quadratic, where $wp = \sqrt{sp}$.

Definition 4. Neutral Preference (np). Given an opinion o_k and two alternatives $a_i, a_i \in A$, there is a Neutral Preference of a_i over a_j, if any of the three following conditions meet:

- There are positive assessments explicitly expressed about both a_i and a_j.
- There are negative assessments explicitly expressed about both a_i and a_j.

- There are no assessments explicitly expressed about neither a_i nor a_j, but there are explicit assessments about any other $a_k \in A$.

The np relation is the identity element with respect to the aggregation of preferences by means of a multiplicative function.

For example, given the opinion o_k = "The service provided by both hotel a_i and a_j are excellent", there is an Equal Preference of a_i over a_j since an explicit positive assessment is made about both a_i and a_j.

Definition 5. Negative Preferences. Negative preferences are the multiplicative inverse of a corresponding positive preference. That is, given an opinion o_k, and two alternatives $a_i, a_i \in A$, there is a Strong Negative Preference (np/sp) of a_i over a_j if there is a Strong Positive Preference (sp) of a_j over a_i; and there is a Weak Negative Preference (np/wp) of a_i over a_j if there is a Weak Positive Preference (wp) of a_j over a_i.

Definition 6. An opinion tuple s is a transformation from the natural language space of the opinions in O to a m-dimensional tuple on the space of preference valuation artifacts P.

Each communication o_k contains explicit and implicit information on the preferences about the elements of the set of alternatives A. This information can be extracted and represented in a *n*-tuple S^m, where m is the number of alternatives and each element s_j represents the polarity of the communication o_k with regards to alternative a_j. For this purpose, we will employ the artifacts previously defined in P as follows:

$$s_j = \begin{cases} \uparrow & \text{if } o \text{ expresses preferences favorable to } a_j. \\ \downarrow & \text{if } o \text{ expresses preferences against } a_j. \\ \leftrightarrow & \text{otherwise} \end{cases} \quad (1)$$

Definition 7. The Preference Matrix W^i of the tuple s_i is defined as:

$$W^i = (w_{kl}) = \begin{pmatrix} np & w_{12} & \cdots & w_{1m} \\ \frac{np}{w_{12}} & np & \cdots & w_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{np}{w_{1m}} & \frac{np}{w_{2m}} & \cdots & np \end{pmatrix} \quad (2)$$

with:

$$(w_{kl}) = \psi(s_k, s_l) = \begin{cases} np & \text{:if} \quad s_k = s_l \\ sp & \text{:if} \quad s_k = \uparrow \wedge s_l = \downarrow \\ \text{wp} & \text{:if} \quad s_k = \uparrow \wedge s_l = \leftrightarrow \vee s_k = \leftrightarrow \wedge s_l = \downarrow \\ \dfrac{np}{sp} & \text{:if} \quad s_k = \downarrow \wedge s_l = \uparrow \\ \dfrac{np}{\text{wp}} & \text{:if} \quad s_k = \downarrow \wedge s_l = \leftrightarrow \vee s_k = \leftrightarrow \wedge s_l = \uparrow \end{cases} \tag{3}$$

Definition 8. $OPM_{m \times m \times n}$, the Opinion Preference Matrix, is defined as the $m \times m \times n$ matrix consisting of all the W^i Preference Matrices of the tuple s_i. A graphical representation of this matrix is shown on figure 1:

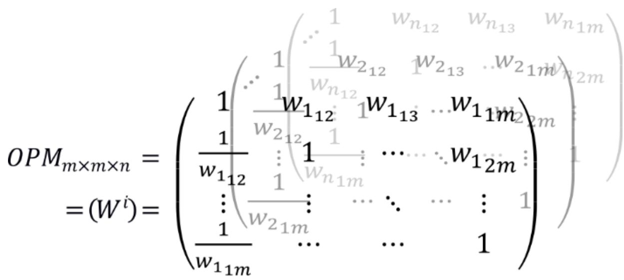

Figure 1: Graphical representation the Opinion Preferences Matrix.

Definition 9. The Aggregated Preferences Matrix $APM_{m \times m}$ is defined as the aggregation of the n layers of the matrix $OPM_{m \times m \times n}$:

$$APM_{m \times m} = \Phi(OPM_{m \times m \times n}) \tag{4}$$

where Φ is a geometric aggregation function of reciprocal matrices.

We have defined the concepts of Preferences, Preference Tuples (s_i), Preference Matrix (W^i), the Opinion Preference Matrix (OPM), and the Aggregated Preferences Matrix (APM). Now we can define a valuation scale analogous to the fundamental scale of the AHP method [21]. In Table 1, this scale is shown next to its meaning and its equivalence with the AHP scale for comparison.

Table 1: Valuation Scale

Notation	**Interpretation**	**AHP Value**
np	*i* and *j* are **equal preference**	1
wp	*i* is **slighty more important** than *j*	3
sp	*i* is **absolutely more important** than *k*	9

Theorem 1: Each of the *n* layers of the $OPM_{m \times m \times n}$ matrix is a subset of the possible AHP matrices with order *m*.

Proof: By construction, each $W^i{}_{mxm} \in OPM_{m\times m\times n}$ is a reciprocal matrix and, as shown in Table 1, each element can only take a value from a set equivalent to $\{1/9, 1/3, 1{,}3, 9\}$, which is a subset of the AHP numerical scale. Therefore, each layer of the $OPM_{m\times m\times n}$ is a valid AHP matrix.

Theorem 2: If the relationship function f between sp and wp yields $\mathrm{wp}^2 = sp$, then each of the n layers of the $OPM_{m\times m\times n}$ matrix is consistent. That is, $\mathrm{wp}^2 = sp \Rightarrow \nabla W^i{}_{mxm} \in OPM_{m\times m\times n}; \lambda_{max}(W^i{}_{mxm}) = m$.

Proof: As defined by Saaty, any positive reciprocal matrix $B_{mxm} = b_{ij}$ is said to be consistent if and only if $b_{ij}b_{jk} = b_{ik}$ for all $i, j, k = 1, \dots, m$ [21].

Considering that, by definition 8, $OPM_{m\times m\times n} = (W^i)$ and by definition 7 every element of $w_{kl} \in W^i = \psi(s_k, s_l)$, for any $i, j, k = 1, \dots, m$,$w_{ij} = \psi(s_i, s_j)$, $w_{jk} = \psi(s_j, s_k)$and $w_{ik} = \psi(s_i, s_k)$.

Furthermore, as np is the multiplicative identity of preference aggregations by multiplicative functions, by the definition of operator ψ:

$$\psi(s_i, s_j)\psi(s_j, s_k) = \psi(s_i, s_j)\frac{np}{\psi(s_k, s_j)} = \frac{\psi(s_i, s_j)}{\psi(s_k, s_j)} \qquad (5)$$

Therefore, depending on the values of s_i,$s_j$and$s_k$there are several possibilities:

- If s_i=s_j, then, since $\psi(x, x) = np$ for any $x \in P$,$\psi(s_i, s_j)\psi(s_j, s_k) = np\psi(s_j, s_k) = \psi(s_j, s_k) = \psi(s_i, s_k)$.

- If, $\psi(s_i, s_j) = sp$ then $s_i =\uparrow$ and $s_j =\downarrow$ and the above equation valuates as follows:

$$\psi(\uparrow,\downarrow)\psi(\downarrow, s_k) = \frac{sp}{\psi(s_k,\downarrow)} \qquad (6)$$

  The only possible values that $\psi(s_k,\downarrow)$ can take are sp if $s_k =\uparrow$, wp if $s_k =\leftrightarrow$ or np if $s_k =\downarrow$, so the three possible solutions for this case are:

$$\psi(\uparrow,\downarrow)\psi(\downarrow, s_k) = \begin{cases} \dfrac{sp}{sp} = np = \psi(\uparrow,\uparrow) & \text{:if} \quad s_k =\uparrow \\ \dfrac{sp}{\mathrm{wp}} = \mathrm{wp} = \psi(\uparrow,\leftrightarrow) & \text{:if} \quad s_k =\leftrightarrow \\ \mathrm{spnp} = sp = \psi(\uparrow,\downarrow) & \text{:if} \quad s_k =\downarrow \end{cases} \qquad (7)$$

  Consequently $\psi(\uparrow,\downarrow)\psi(\downarrow, s_k) = \psi(\uparrow, s_k)$.

- If $\psi(s_i, s_j) = \mathrm{wp}$, then there are two possibilities: $s_i =\uparrow$ and $s_j =\leftrightarrow$ or $s_i =\leftrightarrow$ and $s_j =\downarrow$.

- If $s_i = \uparrow$ and $s_j = \leftrightarrow$, then

$$\psi(\uparrow, \leftrightarrow)\psi(\leftrightarrow, s_k) = \frac{\text{wp}}{\psi(s_k, \leftrightarrow)} \tag{8}$$

 The only possible values that $\psi(s_k, \leftrightarrow)$ can take are wp if $s_k = \uparrow$, np if $s_k = \leftrightarrow$ or np/wp if $s_k = \downarrow$. This gives us three possible solutions for this case:

$$\psi(\uparrow, \leftrightarrow)\psi(\leftrightarrow, s_k) = \begin{cases} \dfrac{\text{wp}}{\text{wp}} = np = \psi(\uparrow, \uparrow) & :\text{if} \quad s_k = \uparrow \\ \text{wp}np = \text{wp} = \psi(\uparrow, \leftrightarrow) & :\text{if} \quad s_k = \leftrightarrow \\ \text{wpwp} = sp = \psi(\uparrow, \downarrow) & :\text{if} \quad s_k = \downarrow \end{cases} \tag{9}$$

 Consequently, for each of them, $\psi(\uparrow, \leftrightarrow)\psi(\leftrightarrow, s_k) = \psi(\uparrow, s_k)$.

- If $s_i = \leftrightarrow$ and $s_j = \downarrow$, then

$$\psi(\leftrightarrow, \downarrow)\psi(\downarrow, s_k) = \frac{\text{wp}}{\psi(s_k, \downarrow)} \tag{10}$$

 The only possible values that $\psi(s_k, \downarrow)$ can take are sp if $s_k = \uparrow$, wp if $s_k = \leftrightarrow$ or np if $s_k = \downarrow$. The three possible solutions for this case are:

$$\psi(\leftrightarrow, \downarrow)\psi(\downarrow, s_k) = \begin{cases} \dfrac{\text{wp}}{\text{sp}} = \dfrac{np}{\text{wp}} = \psi(\leftrightarrow, \uparrow) & :\text{if} \quad s_k = \uparrow \\ \dfrac{\text{wp}}{\text{wp}} = \text{np} = \psi(\leftrightarrow, \leftrightarrow) & :\text{if} \quad s_k = \leftrightarrow \\ \text{wpnp} = \text{wp} = \psi(\leftrightarrow, \downarrow) & :\text{if} \quad s_k = \downarrow \end{cases} \tag{11}$$

 Consequently $\psi(\leftrightarrow, \downarrow)\psi(\downarrow, s_k) = \psi(\leftrightarrow, s_k)$.

- Lastly, since for any positive and neutral valuation of $\psi(s_i, s_j)$, the operator ψ has been proven transitive, and negative preferences are the multiplicative inverse of positive preferences, the operator ψ is also transitive when $\psi(s_i, s_j)$ is valuated as a negative preference.

Therefore, the operator ψ introduced in definition 7 used to build the reciprocal Preference Matrix W^i, is transitive for any possible value of $\psi(s_i, s_j)$. Consequently, $w_{ij}w_{jk} = w_{ik}; \forall w_{ij}, w_{jk}, w_{ik} \in W^i$, with $i, j, k \in \{1, \dots, m\}$. Thus, W^i always satisfies consistency as defined by Saaty.

Corollary. The Aggregated Preferences Matrix $APM_{m\times m}$ is reciprocal and consistent if the aggregation operator Φ is a geometric function. Since each of the n layers of the $OPM_{m\times m\times n}$ is a subset of the possible consistent AHP matrices with order m, and preference matrices used by AHP preserve reciprocity and consistency when aggregated by a geometric function [22], $APM_{m\times m}$ is always a consistent reciprocal matrix.

3. Comparison Matrix Geometric Index (CMGI)

When people express unsolicited opinions online about services, products or companies they are basically comparing within a set of comparable alternatives A, either explicitly or implicitly. Therefore, each opinion can be expressed as a tuple of preference polarities S assigning a polarity $p \in P$ to each comparable alternative. From any opinion o_k on A, a Preference Matrix W^ican be built from S, representing the preferences for each pair of alternatives within A expressed on o_k as a pairwise comparison matrix.

The Opinion Preference Matrix OPM provides this same representation when evaluating for more than one opinion, and its aggregation by using the geometric mean as the operator Φ results in an Aggregated Preference Matrix APM.

$$APM_{m\times m} = \Phi(OPM_{m\times m\times n}) \tag{12}$$

Where

$$\Phi(M_{m\times m\times n}) = {M'}_{m\times m};\ {m'}_{ij} = \left(\prod_{k=1}^{n} m_{ijk}\right)^{\frac{1}{n}} \tag{12}$$

Since, this APM is a reciprocal consistent matrix in the form of an AHP pairwise comparison matrix, the Comparison Matrix Geometric Vector ($CMGV$) of the APM can be calculated as the right Perron–Frobenius eigenvector of APM,

$$CMGV = PF(APM) \tag{13}$$

This $CMGV$ is a vector of weights of the preferences on the alternatives defined in A, which can be expressed as $CMGV = \left(sp^{d_1}, sp^{d_2}, \dots, sp^{d_n}\right)$ with $d_i \in [-1,1]$. This is a consequence of $np = sp^0$, given that np is the multiplicative identity element, and $\mathrm{wp}^2 = sp$.

In order to calculate the Comparison Matrix Geometric Index ($CMGI$), sp must be assigned a numerical value of $h > 1$. This value must be chosen carefully, as it represents the intensity of an explicitly stated preference with respect to both implicit and neutral preferences. In most cases, a value of $h = 9$ is recommended, because this makes the $CMGI$ compatible with the resulting scale of aggregating AHP matrices using geometric operators, which has a well-tested efficacy when modeling judgments by pairwise comparisons.

After setting the value of $sp = h$ in $CMGV$, the $CMGI$ is calculated as the normalized $CMGV$:

$CMGI = \frac{CMGV}{\sum CMGV}$ with $sp = h$, i.e.,

$$CMGI = \left(\frac{h^{d_1}}{\sum h^{d_i}}, \frac{h^{d_2}}{\sum h^{d_i}}, \dots, \frac{h^{d_n}}{\sum h^{d_i}}\right) \tag{14}$$

This CMGI calculation process can be summarized as the 5 step process shown in figure 1, with each step providing a partial result that will become the input of the following step. The first step calculates the preference tuples from user opinions expressed in natural language, the second step builds the Opinion Preference Matrices (OPM) from these tuples, the third step aggregates the OPM into an AMP, the fourth step calculates the $CMGV$ from the APM and in the final step a numerical value is assigned to h resulting in the $CMGI$.

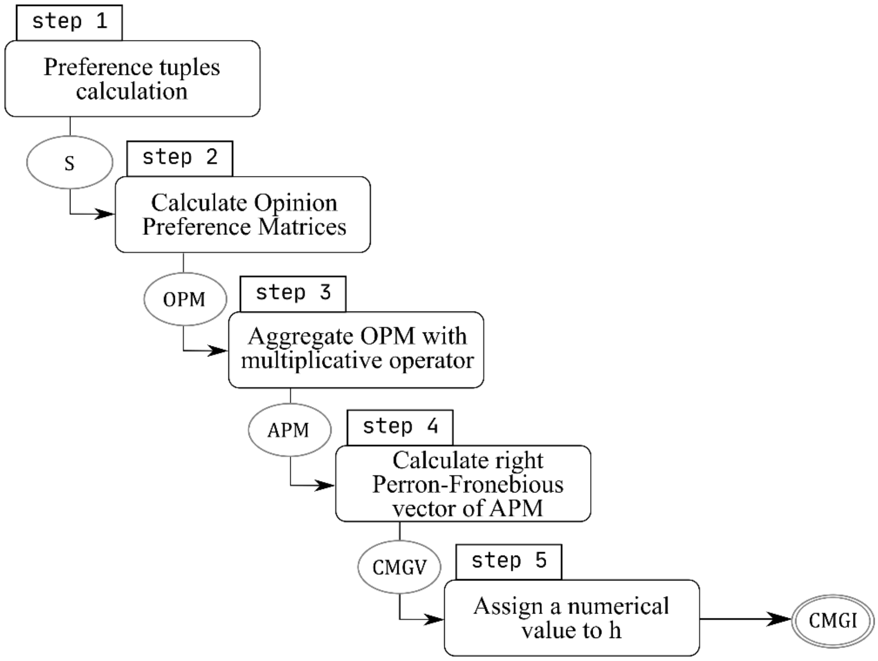

Figure 1: CMGI 5-step calculation process.

4. CMGI Application Example

In this section, we show the use of the proposed comparison index with an example. For this purpose, we consider a set of 4 hotels (we have omitted the real name of the hotels), A = {Hotel 1, Hotel 2, Hotel 3, Hotel 4} as the comparable alternatives. Let's suppose that these four hotels have similar enough characteristics, such as price tier, geographical area and available services, so they are comparable.

Step 1. Preference tuples calculation.

In table 2 a set of communications on some of the hotels within A is shown. These communications have been written by Internet users on their own initiative and can be obtained by using data mining processes. As a result, these communications convey information on their writers' opinions with regards to these alternatives. In the first column, the id of the communication is shown. The second column contains the raw text of the communication as written by its author, and the third column shows the resulting preference tuple for each opinion.

In this case, the third column of table 2 was calculated manually, but a Natural Language Processing (NLP) approach could be implemented to automate when performing this process to a great number of communications [23].

In the first comment, it's author states that Hotel 1 has a fantastic cafeteria, therefore, there is an explicit positive opinion on Hotel 1 which is represented as positive polarity (↑). In this communication, no information is expressed on any of the other alternative, so they are represented by a neutral polarity (↔).

In comment 2, there is an explicit comparison since the user states that the restaurant of hotel 3 is better than that of hotel 4. In this case, since no statement was made about hotels 1 and 2, they will be represented in the tuple by a neutral polarity (↔). However, an explicit preference was stated for hotel 3 and against hotel 4. This will be represented as a positive polarity (↑) for hotel 3 and a negative one for hotel 4 (↓). The remainder of comments are processed in a similar fashion.

Table 2. User comments on alternatives and polarity valuation tuples.

ID	User Comment	Polarity tuples (S)
1	Hotel 1's cafeteria is fantastic.	[↑, ↔, ↔, ↔]
2	The restaurant of hotel 3 is better than the one in hotel 4.	[↔, ↔, ↑, ↓]
3	My room on hotel 2 was very comfortable.	[↔, ↑, ↔, ↔]
4	The swimming pool in hotel 1 is way better than that of hotels 2 and 4.	[↑, ↓ ,↔, ↓]

Step 2. Calculating the Opinion Preference Matrix (OPM).

In this step, the OPM is built for each polarity valuation tuple s_i in S. This results in an OPM_i for each tuple s_i. Each element of the OPM_i matrix corresponds to the comparison of a pair of elements on the tuple s_i. This value is calculated according to definition 7 and results in a Positive, Neutral or Negative preference value for each position in the matrix. The results of this process is shown in table 3.

Table 3. Opinion Preference Matrices for S.

$$OPM_1 = \begin{bmatrix} np & \mathrm{wp} & \mathrm{wp} & \mathrm{wp} \\ \frac{np}{\mathrm{wp}} & np & np & np \\ \frac{np}{\mathrm{wp}} & np & np & np \\ \frac{np}{\mathrm{wp}} & np & np & np \end{bmatrix} \qquad OPM_3 = \begin{bmatrix} np & \frac{np}{\mathrm{wp}} & np & np \\ \mathrm{wp} & np & \mathrm{wp} & \mathrm{wp} \\ np & \frac{np}{\mathrm{wp}} & np & np \\ np & \frac{np}{\mathrm{wp}} & np & np \end{bmatrix}$$

$$OPM_2 = \begin{bmatrix} np & np & \frac{np}{\mathrm{wp}} & \mathrm{wp} \\ np & np & \frac{np}{\mathrm{wp}} & \mathrm{wp} \\ \mathrm{wp} & \mathrm{wp} & np & sp \\ \frac{np}{\mathrm{wp}} & \frac{np}{\mathrm{wp}} & \frac{np}{sp} & np \end{bmatrix} \qquad OPM_4 = \begin{bmatrix} np & sp & \mathrm{wp} & sp \\ \frac{np}{sp} & np & \frac{np}{\mathrm{wp}} & np \\ \frac{np}{\mathrm{wp}} & \mathrm{wp} & np & \mathrm{wp} \\ \frac{np}{sp} & np & \frac{np}{\mathrm{wp}} & np \end{bmatrix}$$

Step 3. Aggregating the OPM into the Aggregated Preferences Matrix.

The APM is the result of aggregating the n Opinion Preference Matrices. In order to maintain reciprocity and consistency, the geometric mean is employed, although any other operator satisfying theorem 2 could be valid. The resulting APM is shown below.

$$APM_{4\times4} = \begin{bmatrix} sp^0 & sp^{\frac{1}{4}} & sp^{\frac{1}{8}} & sp^{\frac{1}{2}} \\ sp^{-\frac{1}{4}} & sp^0 & sp^{-\frac{1}{8}} & sp^{\frac{1}{4}} \\ sp^{-\frac{1}{8}} & sp^{\frac{1}{8}} & sp^0 & sp^{\frac{3}{8}} \\ sp^{-\frac{1}{2}} & sp^{-\frac{1}{4}} & sp^{-\frac{3}{8}} & sp^0 \end{bmatrix}$$

Step 4. Obtaining the Comparison Matrix Geometric Vector (CMGV) from the APM.

In this step the CMGV is calculated. This vector is calculated as the right Perron–Frobenius vector of APM.

$$CMGV = \left[sp^{\frac{1}{2}}, sp^{\frac{1}{4}}, sp^{\frac{3}{8}}, sp^0\right], \text{ with } \lambda_{max} = 4. \tag{15}$$

Step 5. Obtaining Comparison Matrix Geometric Index (CMGI) from CMGV.

In this step, a value for $h > 1$ must be defined. This value affects the distance between weak and strong preferences and, as discussed previously, the recommended value is $h = 9$, since this makes this process analogous to that of computing AHP from a consistent pairwise comparison matrix.

$$CMGV = \left[3, \sqrt{3}, \sqrt[4]{3^3}, 1\right] \tag{16}$$

The CMGI is the normalized CMGV:

$$CMGI = \left[\frac{3}{4+\sqrt{3}+\sqrt[4]{3^3}}, \frac{\sqrt{3}}{4+\sqrt{3}+\sqrt[4]{3^3}}, \frac{\sqrt[4]{3^3}}{4+\sqrt{3}+\sqrt[4]{3^3}}, \frac{1}{4+\sqrt{3}+\sqrt[4]{3^3}}\right] \tag{17}$$

Which is approximately:

$$CMGI = [0.3745, 0.2162, 0.2845, 0.1248] \tag{18}$$

With this index, we have a qualitative metric of the online reputation of the different alternatives. This allows the building of a ranking of alternatives, their priorities and their relative preferences. This is shown in table 4.

Table 4. Alternatives Priorities and Ranking Order.

Alternative	CMGI	Rank Order
Hotel 1	0.3745	1
Hotel 2	0.2162	3
Hotel 3	0.2845	2
Hotel 4	0.1248	4

Now, let's suppose that more unsolicited opinions are obtained, so four more opinions are added to the ones on table 2. These new opinions express positive valuations for some of the hotels, however they do not express any opinion on Hotel 1. The resulting set of unsolicited opinions is shown in table 5.

Table 5. User comments on alternatives and polarity valuation tuples, including 4 newly added comments (IDs from 5 to 8).

ID	User Comment	Polarity tuples (S)
1	Hotel 1's cafeteria is fantastic.	[↑, ↔, ↔, ↔]
2	The restaurant of hotel 3 is better than the one in hotel 4.	[↔, ↔, ↑, ↓]
3	My room on hotel 2 was very comfortable.	[↔, ↑, ↔, ↔]
4	The swimming pool in hotel 1 is way better than that of hotels 2 and 4.	[↑, ↓ ,↔, ↓]
5	The rooms of hotels 2 and 3 are spacious and luminous.	[↔,↑, ↑, ↔]
6	The bed in my room of hotel 4 is amazing. Last night I slept like never before.	[↔,↔, ↔, ↑]
7	Last summer I spent a great time on hotel 2. I'll try to come back this year.	[↔,↑ ,↔, ↔]
8	The receptionist of hotel 3 was very nice to us.	[↔, ↔, ↑, ↔]

Let's calculate the CMGI from the unsolicited opinions expressed on table 6:

Table 6. Opinion Preference Matrices for S, including the new opinions.

$$OPM_1 = \begin{bmatrix} np & \mathrm{wp} & \mathrm{wp} & \mathrm{wp} \\ \frac{np}{\mathrm{wp}} & np & np & np \\ \frac{np}{\mathrm{wp}} & np & np & np \\ \frac{np}{\mathrm{wp}} & np & np & np \end{bmatrix}$$

$$OPM_2 = \begin{bmatrix} np & np & \frac{np}{\mathrm{wp}} & \mathrm{wp} \\ np & np & \frac{np}{\mathrm{wp}} & \mathrm{wp} \\ \mathrm{wp} & \mathrm{wp} & np & sp \\ \frac{np}{\mathrm{wp}} & \frac{np}{\mathrm{wp}} & \frac{np}{sp} & np \end{bmatrix}$$

$$OPM_3 = \begin{bmatrix} np & \frac{np}{\mathrm{wp}} & np & np \\ \mathrm{wp} & np & \mathrm{wp} & \mathrm{wp} \\ np & \frac{np}{\mathrm{wp}} & np & np \\ np & \frac{np}{\mathrm{wp}} & np & np \end{bmatrix}$$

$$OPM_4 = \begin{bmatrix} np & sp & \mathrm{wp} & sp \\ \frac{np}{sp} & np & \frac{np}{\mathrm{wp}} & np \\ \frac{np}{\mathrm{wp}} & \mathrm{wp} & np & \mathrm{wp} \\ \frac{np}{sp} & np & \frac{np}{\mathrm{wp}} & np \end{bmatrix}$$

$$OPM_5 = \begin{bmatrix} np & \frac{np}{\mathrm{wp}} & \frac{np}{\mathrm{wp}} & \mathrm{np} \\ \mathrm{wp} & np & np & \mathrm{wp} \\ \mathrm{wp} & np & np & \mathrm{wp} \\ np & \frac{np}{\mathrm{wp}} & \frac{np}{\mathrm{wp}} & np \end{bmatrix}$$

$$OPM_6 = \begin{bmatrix} np & np & np & \frac{np}{\mathrm{wp}} \\ np & np & np & \frac{np}{\mathrm{wp}} \\ np & np & np & \frac{np}{\mathrm{wp}} \\ \mathrm{wp} & \mathrm{wp} & \mathrm{wp} & np \end{bmatrix}$$

$$OPM_7 = \begin{bmatrix} np & \frac{np}{\mathrm{wp}} & np & np \\ \mathrm{wp} & np & \mathrm{wp} & \mathrm{wp} \\ np & \frac{np}{\mathrm{wp}} & np & np \\ np & \frac{np}{\mathrm{wp}} & np & np \end{bmatrix}$$

$$OPM_8 = \begin{bmatrix} np & np & \frac{np}{\mathrm{wp}} & np \\ np & np & \frac{np}{\mathrm{wp}} & np \\ \mathrm{wp} & \mathrm{wp} & np & \mathrm{wp} \\ np & np & \frac{np}{\mathrm{wp}} & np \end{bmatrix}$$

The APM resulting from aggregating this new OPM is:

$$APM_{4\times4} = \begin{bmatrix} sp^0 & sp^0 & sp^{\frac{-1}{16}} & sp^{\frac{3}{16}} \\ sp^0 & sp^0 & sp^{-\frac{1}{16}} & sp^{\frac{3}{16}} \\ sp^{\frac{1}{16}} & sp^{\frac{1}{16}} & sp^0 & sp^{\frac{1}{4}} \\ sp^{-\frac{3}{16}} & sp^{-\frac{3}{16}} & sp^{-\frac{1}{4}} & sp^0 \end{bmatrix} \tag{19}$$

The associated right Perron–Frobenius vector of APM, with $\lambda = 4$ is:

$$CMGV = \left[sp^{\frac{3}{16}}, sp^{\frac{3}{16}}, sp^{\frac{1}{4}}, sp^0\right] \tag{20}$$

When sp is valuated to the previously defined $h = 9$ and normalized we have:

$$CMGI = [0.2625, 0.2625, 0.3011, 0.1739] \tag{21}$$

In table 7 we show the results of the CMGI after adding the new 4 unsolicited opinions, and in table 8 we compare this result with the previous one. As shown in table 5, in the new unsolicited opinions introduced no statement is made on Hotel 1, and favorable statements are made on the rest of hotels, this results in a decrease of 0.112 on its CMGI. Hotels 2, 3 and 4, on the other hand, experimented an increase on their CMGI, since Internet users kept talking positively on them while ignoring completely Hotel 1. This results on a new ranking order in which Hotel 1 falls from an undisputed first position to a tie in the second position with Hotel 2 reflecting its fall in popularity in comparison with the rest of alternatives.

Table 7. Alternatives Priorities and Ranking Order.

Alternative	CMGI	Rank Order
Hotel 1	0.2625	2
Hotel 2	0.2625	2
Hotel 3	0.3011	1
Hotel 4	0.1739	4

Table 8. Comparison before and after adding 4 new communications.

Alternative	CMGI (4 opinions)	Rank Order (4 opinions)	CMGI (8 opinions)	Rank Order (8 opinions)
Hotel 1	0.3745	1	0.2625	2
Hotel 2	0.2162	3	0.2625	2
Hotel 3	0.2845	2	0.3011	1
Hotel 4	0.1248	4	0.1739	4

The previous example showed the reactive changes of the CMGI when new unsolicited opinions are added. On the other hand, it is important to notice that the resultant CMGI values are exclusive to this particular set of comparable alternatives. Therefore, a new set of alternatives formed by some of the elements of this example (e.g. Hotels 2 and 4) and new elements (Hotels X and Y) will not necessary yield the same CMGI values. This is one of the main features that differentiate the CMGI to other well-known ORIs, such as the Stars, Likes, Forks, among others; that are independent of the comparable alternatives set, and should not be used to valuate the comparative preferences between alternatives.

4.1 CMGI comparison with the PROMETHEE method

The PROMETHEE method is a Multi-criteria decision making (MCDM) method that provides a framework for understanding and structuring decision problems. This method provides rankings of alternatives by constructing preference indices from information obtained from a set of decision-makers. MCDM methods are usually designed around the availability of decision makers but in this example we will adapt PROMETHEE to be used as an online reputation metric using unsolicited opinion as an input and we will compare its results to the results previously obtained by the CMGI [24–26].

For this purpose, we will use as an input the user comments expressed in Table 2, and later expanded in Table 5, which were the only information needed to calculate the CMGI with the set of alternatives A = {Hotel 1, Hotel 2, Hotel 3, Hotel 4}. However, this information alone is not enough if we want to use methods which require knowledge on criteria, such as PROMETHEE in order to measure online reputation. These methods require knowledge about the criteria used by the decision makers to assess their preferences, which are dependent on the domain of the problem. Therefore, in order to use them to measure reputation, it is necessary to conduct a previous research that identifies these criteria and measures their weights.

Fortunately, the set of comparable alternatives in this example comprises hotels, which is a domain that has been thoroughly studied in scientific literature and there are several published scientific works about the criteria that guests ponder when they are deciding which hotels to book. In order to compare PROMETHEE with CMGI, we will use the work published by W. Poon and K. Lock-Teng Low [27], so let's suppose that the reviews expressed in the example of table 5 are from Malaysian hotels and those reviews were expressed by western travelers.

There are 12 main factors that influence western travelers' satisfaction with Malaysian hotels which are: hospitality (weight: 0.1843), accommodation (weight: 0.1247), food & beverages (weight: 0.1797), recreation & entertainment (weight: 0.1239), supplementary services (weight: 0.1005), security & safety (weight: 0.191), innovation & value added services (weight: 0.0016), transportation (weight: 0.0066), location (weight: 0.0113), appearance (weight: 0.0686), pricing (weight: 0.023) and payment (weight: 0.0035). These 12 factors in this same order, can be considered as the consistent family of criteria $F = \{F_1, \dots, F_{12}\}$ needed to calculate the PROMETHEE ranking.

The next step needed before starting to calculate the PROMETHEE ranking is to transform the information on preferences expressed in natural language to an evaluation table. We performed this task manually by reading each comment and selecting which criteria was being evaluated in each comment, afterwards we used a lexicon-based method to assign an intensity value in a scale from 0 to 9. Very high positive intensity adjectives such as "fantastic" or "amazing" were assigned a value of 9, high positive adjectives such as "very comfortable", "spacious" and "luminous" were assigned a value of 7 and upper medium adjectives such as "great time" were assigned a value of 5 [28].

With the objective of providing the maximum possible information to the PROMETHEE method we had to make a couple of assumptions in this process: Firstly, when direct comparisons were made between alternatives (e.g. "The swimming pool in hotel 1 is way better than that of hotels 2 and 4"), it was considered as a positive valuation on the best alternative and secondly, when a criterion was not mentioned in the comment a value of 0 was assigned to it. The results of this process are shown in table 9.

Table 9: Alternatives valuation on hotel assessment criteria extracted from communications in natural language by translating the linguistic labels using a lexicon-based method.

ID	User Comment	Alt	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12
1	Hotel 1's cafeteria is fantastic.	A_1	0	0	**9**	0	0	0	0	0	0	0	0	0
2	The restaurant of hotel 3 is better than the one in hotel 4.	A_3	0	0	**5**	0	0	0	0	0	0	0	0	0
3	My room on hotel 2 was very comfortable.	A_2	0	**7**	0	0	0	0	0	0	0	0	0	0
4	The swimming pool in hotel 1 is way better than that of hotels 2 and 4.	A_1	0	0	0	**7**	0	0	0	0	0	0	0	0
5	The rooms of hotels 2 and 3 are spacious and luminous.	A_2	0	0	0	0	0	0	0	0	0	**7**	0	0
		A_3										**7**		
6	The bed in my room of hotel 4 is amazing. Last night I slept like never before.	A_4	0	**1**	0	0	0	0	0	0	0	0	0	0
7	Last summer I spent a great time on hotel 2. I'll try to come back this year.	-	0	0	0	0	0	0	0	0	0	0	0	0
8	The receptionist of hotel 3 was very nice to us.	A_3	**7**	0	0	0	0	0	0	0	0	0	0	0

From the information on table 9 we can construct an evaluation table which can be used with the set of criteria F , their weights, and the set of alternatives A to calculate the PROMETHEE ranking [25]. The PROMETHEE multi-criteria preference degree matrix is shown in table 10 as a partial result. In this table we can appreciate that, since there were no comments about criteria F_5, F_6, F_7, F_8, F_9, F_{11} and F_{12} every single value on their columns results in 0.5, which means that there is no information about preferences towards any alternative on these criteria.

Table 10: PROMETHEE multi-criteria preference degree matrix.

	$\boldsymbol{F_1}$	$\boldsymbol{F_2}$	$\boldsymbol{F_3}$	$\boldsymbol{F_4}$	$\boldsymbol{F_5}$	$\boldsymbol{F_6}$	$\boldsymbol{F_7}$	$\boldsymbol{F_8}$	$\boldsymbol{F_9}$	$\boldsymbol{F_{10}}$	$\boldsymbol{F_{11}}$	$\boldsymbol{F_{12}}$
$\pi_k(A_1, A_2)$	0.36	0.31	1	0.89	0.5	0.5	0.5	0.5	0.5	0.31	0.5	0.5
$\pi_k(A_1, A_3)$	0.31	0.5	0.86	0.89	0.5	0.5	0.5	0.5	0.5	0.31	0.5	0.5
$\pi_k(A_1, A_4)$	0.5	0	1	0.89	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\pi_k(A_2, A_1)$	0.64	0.69	0	0.11	0.5	0.5	0.5	0.5	0.5	0.69	0.5	0.5
$\pi_k(A_2, A_3)$	0.44	0.69	0.36	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\pi_k(A_2, A_4)$	0.64	0.19	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.69	0.5	0.5
$\pi_k(A_3, A_1)$	0.69	0.5	0.14	0.11	0.5	0.5	0.5	0.5	0.5	0.69	0.5	0.5

$\pi_k(A_3, A_2)$	0.56	0.31	0.64	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\pi_k(A_3, A_4)$	0.69	0	0.64	0.5	0.5	0.5	0.5	0.5	0.5	0.69	0.5	0.5
$\pi_k(A_4, A_1)$	0.5	1	0	0.11	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\pi_k(A_4, A_2)$	0.36	0.81	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.31	0.5	0.5
$\pi_k(A_4, A_3)$	0.31	1	0.36	0.5	0.5	0.5	0.5	0.5	0.5	0.31	0.5	0.5

After following all the steps of the PROMETHEE method, the final PROMETHEE II ranking is $\phi = \{0.1420, -0.0565, -0.0284, -0.0571\}$, which means that the final order of alternatives obtained from this method is [Hotel 1, Hotel 3, Hotel 2 and Hotel 4] this is shown in table 11. Although we were using the full set of user comments from table 5 as an input for this method, the ranking order is similar to the CMGI ranking obtained when using only the 4 user comments expressed in table 2.

Table 11. PROMETHEE Ranking Order.

Alternative	PROMETHEE	Rank Order
Hotel 1	0.1420	1
Hotel 2	-0.0565	3
Hotel 3	-0.0284	2
Hotel 4	-0.0571	4

Although the CMGI is designed to extract information on preferences directly from unsolicited opinions in digital ecosystems and PROMETHEE was built to solve multi-criteria decision problems with multiple decision-makers, in certain cases the later can also be used for extracting information preferences from user comments. However, there are several limitations that we must overcome before using PROMETHEE that are not present when using CMGI.

In order to use PROMETHEE for this task, firstly we need information on criteria and their weights that might not be always available, secondly we need to define the behavior of the method when there are direct comparisons in natural language between alternatives without valuating any of them in isolation, as one alternative's valuation becomes relative to the valuation of the other, thirdly we must decide what to do with valuations performed by the public on alternatives without mentioning, whether explicitly or implicitly, any criteria to be valuated (e.g. "I like Alternative very much.")

The comparison of the results of the PROMETHEE example with the previous CMGI example shows that the order obtained by the former in table 11 are similar to the order obtained by the later in table 4 when only half of the user comments were available, although its results should have been more akin to the final results of the CMGI shown in table 7 with the full set of user comments. This is due to a loss of information caused by the aforementioned limitations of the PROMETHEE method when forcing its usage to measure online reputation.

5. Comparison against direct sentiment analysis

Direct sentiment analysis is a common approach for valuating the preference of people. It relies on natural language processing methods to identify the polarity (positive or negative) and the intensity (low to high) of the written opinions. In this regard, the outcome of the CMGI can be analogous to a traditional direct sentiment analysis. The main difference between the two approaches is that the CMGI is designed to consider the implicit and explicit comparisons that the subject expresses within the opinion.

We conducted two experiments to test whether the CMGI is better that direct sentiment analysis at explaining the overall rating of comparable alternatives.

5.1 Consumer reviews from Amazon.com

For the first experiment we selected a random sample of 200 verified consumer reviews of coffee making machines from Amazon.com that were posted between June 20, 1995 and August 31, 2015 [29]. For each review, we collected the product name, product category, number of ratings of the product, date, title, text and star rating (from 1 to 5).

To perform the direct sentiment analysis, a team of two experts manually rated each review using a 10-point Likert-type scale, where 0 represented "completely negative" and 10 was "completely positive" sentiment towards the product indicated in the review. The experts rated each review twice, at two different working sessions. There was a strong intra-rater agreement, with Cohen's kappa of 0.87; and a strong inter-rater agreement, with Cohen's kappa of 0.81. The final sentiment measurement for each review was computed as the mean of the four valuations.

Simultaneously, the same team of two experts computed the opinion tuples (see Definition 6) of each review. The researchers found that the set of comparable alternatives was comprised of 24 different coffee making machines. The aggregated sentiment score was computed as the mean of the measurements from direct sentiment analysis for each of the reviews related to that alternative. Additionally, the mean star rating per alternative was computed as the predicted variable.

Two linear regression models were calculated to predict the mean star rating based on the aggregated sentiment score and the CMGI respectively. The results confirmed that the CMGI is better at explaining the mean star rating (R2 = 0.94) versus direct sentiment analysis (R2 = 0.72).

5.2 Consumer reviews from Yelp

For the second experiment, we selected a new random sample of 200 consumer reviews of restaurants from Yelp.com. These reviews were posted between 2007 and 2019 and were available in the Yelp Public Dataset as of July 1st, 2020 [30]. For each review, we collected the name of the restaurant, the star rating, date and the body of text. Then, a team of two experts manually rated each review using a 10-point Likert-type scale, where 0 represented “completely negative” and 10 was “completely positive” sentiment towards the restaurant indicated in the review. The experts rated each review twice, at two different working sessions. There was a strong intra-rater agreement, with Cohen’s kappa of 0.89; and a strong inter-rater agreement, with Cohen’s kappa of 0.85. The final sentiment measurement for each review was computed as the mean of the four valuations. Moreover, the same team of two experts computed the opinion tuples of each review.

For this experiment, the set of comparable alternatives was comprised of 5 fast food restaurants in the United States. The main difference between this experiment and the previous one is that, although there are some explicit comparisons between the selected alternatives and other alternatives that could be considered a gold standard (e.g. highly recognizable brands fast-food chains), there are no explicit comparisons between the five selected alternatives.

The aggregated sentiment score was computed as the mean of the direct sentiment analysis measurements. The mean star rating per alternative was computed as the predicted variable. Two linear regression models were calculated to predict the mean star rating based on the aggregated sentiment score and the CMGI respectively. Like the first experiment, the results confirmed that the CMGI is better at explaining the mean star rating (R2 = 0.85) versus direct sentiment analysis (R2 = 0.82). Although it should be noted that the difference between the two approaches in this case is minimal.

This similarity in the results of the CMGI and a direct sentiment analysis on the communications between these five restaurants is caused by the lack of the information that explicit comparisons provide to the model. By using the CMGI on a set of comparable alternatives with no explicit comparisons between them we are not benefiting from one of the main advantages of the CMGI, which are the strong preferences among competitors that people express when they compare brands, products and services. These results show that the CMGI behaves in the same way as a direct sentiment analysis in its worst case scenario.

6. Discussions and Conclusions

One of the main advantages of the Comparison Matrix Geometric Index is that, the resulting priority on a given alternative a_i is not only dependent on the polarity of the valuations performed by online users towards a_i, but also on the relative presence with regards to its competitors. This is extremely important, since, when users valuate our competitors positively but do not express their opinion about ourselves, the CMGI value of said competitor is increased, but our value decreases lightly too as a result. On the other hand, if the unsolicited opinions expressed online shift towards negative valuations of any alternative, this alternative's CMGI value will decrease and their competitors will experiment an increase of their CMGI value.

This is one of the most important benefits of the CMGI, as it replicates the behavior of consumers. When a product or service experiments a boost on popularity and favorable insights, the market tends to gravitate more towards it. However, a crisis on its reputation causes the opposite effect and attracts more consumers towards their comparable alternatives, such as substitute goods and services.

Moreover, the CMGI is the only Multi Criteria Decision Analysis online reputation index which do not requires knowledge about the criteria that people use to assess their satisfaction in the domain of the alternatives. When the alternatives pertain to a broad and very studied domain, such as hotel valuations or restaurants, the criteria used to valuate them can be easily found in the scientific literature and other methods can be used. However, the more niche or specific the topic, the less probable is that information about its criteria is available. In this cases, the CMGI is the only method of its kind that can be used to obtain an appropriate preference ranking.

Even when criteria information is available, there is information loss when using other methods for ranking alternatives using unsolicited opinions as their main input. Multi-criteria decision making methods such as AHP and PROMETHEE are designed for their usage on groups of informed decision-makers that can be directly asked about their preferences between criteria, therefore they are better suited than the CMGI when we can directly ask decision-makers about their preferences. However, the CMGI models the concept of strong and weak preferences in similar fashion to the way in which Internet users express themselves, therefore it is a more suitable approach when working with information extracted directly from digital ecosystems expressed without prompt.

Moreover, the values of the index on each alternative also provide important information, such as the degree of preference of each alternative and can also be used to measure variations when introducing new communications. The intensity of these changes can be regulated by adjusting the value of h. Higher values of h increase the effect that strong preferences have on the index, whereas low values of h mitigate it.

One of the limitations of this method is the need of a defined set of comparable alternatives A, therefore imposing a domain restriction. Although the users of Social Media and online platforms have an absolute freedom on the topics and opinions expressed on their unsolicited communications, the set of communications O employed has to be a subset of the universe of communications, selecting those referring at least one alternative within A. This is a consequence of Arrow's impossibility theorem. Given that it is impossible to satisfy all the axioms defined by

Arrow, when extracting a preference order from a given set of opinions, we chose to relax the condition of unrestricted domain.

In this work we recommended a value of $h = 9$ as the numerical representation of strong preferences, this is due to the comparability of the results obtained with that of AHP as one of the most widely used methods of Multi Criteria Decision Analysis. Further research should consider the potential effects of variations of this value and its implications on the resulting CMGI derived from different sets of alternatives and unsolicited opinions.

Finally, as discussed at the beginning of this paper, working with a large quantity of unsolicited opinions is one of the main challenges of developing online reputation metrics. Moreover, we have to take into account that, when expressing opinions about products/services, users are actually comparing these items against the set of known competitors, which further complicates the matter of ranking products and services using unsolicited opinions.

This paper presented the Comparison Matrix Geometric Index (CMGI). The CMGI was specially designed to address these problems; and as such, it is the first Multi Criteria Decision Analysis method capable of ascertaining the relative preferences on brands, products and services, solely from a set of raw unsolicited information emanating from Internet users. Therefore, the CMGI works as a completely objective online reputation metric that aggregates this information into an index that is representative of a brand, product and service within its business ecosystem.

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The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

References

[1] J.I. Peláez, E.A. Martínez, L.G. Vargas, Products and services valuation through unsolicited information from social media, Soft Comput. 3 (2019). doi:10.1007/s00500-019-04005-3.

[2] J.I. Peláez, F.E. Cabrera, L.G. Vargas, Estimating the importance of consumer purchasing criteria in digital ecosystems, Knowledge-Based Syst. 162 (2018) 252–264. doi:10.1016/j.knosys.2018.07.023.

[3] D. Voramontri, L. Klieb, Impact of social media on consumer behaviour, 2019. http://creativecommons.org/licenses/by/4.0/ (accessed July 14, 2020).

[4] D. Zraková, M. Demjanovičová, M. Kubina, Online reputation in the transport and logistics field, Transp. Res. Procedia. 40 (2019) 1231–1237. doi:10.1016/j.trpro.2019.07.171.

[5] P.P. Iglesias-Sánchez, M.B. Correia, C. Jambrino-Maldonado, The challenge of linking destination online reputation with competitiveness, Tour. Manag. Stud. 15 (2019) 35–43. doi:10.18089/tms.2019.150103.

[6] C.M.Q. Ramos, A.M. Casado-Molina, J.I. Pelaez, Measuring Air and Terrestrial Transport Company Reputation: Tourism Intangibles Expressed in the Digital Environment, J. Spat. Organ. Dyn. 5 (2017) 443–462.

[7] S.C. Ma, Y. Fan, J.F. Guo, J.H. Xu, J. Zhu, Analysing online behaviour to determine Chinese consumers' preferences for electric vehicles, J. Clean. Prod. 229 (2019) 244–255. doi:10.1016/j.jclepro.2019.04.374.

[8] A. Gibbard, Manipulation of Voting Schemes, Econometrica. 41 (1973) 587–601. doi:10.2307/1914083.

[9] H. Moulin, On strategy-proofness and single peakedness, Public Choice. 35 (1980) 437–455. doi:10.1007/BF00128122.

[10] A.M. Casado Molina, A. Méndiz Noguero, J.I. Peláez Sánchez, The evolution of Dircom: from communication manager to reputation strategist La evolución del DirCom: de gestor de la comunicación a estratega de la reputación, Comun. y Soc. 26 (2013) 47–66. www.unav.es/fcom/comunicacionysociedad/en/.

[11] B. Cankir, M.L. Arslan, S.E. Seker, Web Reputation Index for XU030 Quote Companies, J. Ind. Intell. Inf. 3 (2015) 2013–2016. doi:10.12720/jiii.3.2.110-113.

[12] A.M. Casado, J.I. Peláez, Intangible management monitors and tools: Reviews, Expert Syst. Appl. 41 (2014) 1509–1529. doi:10.1016/j.eswa.2013.08.048.

[13] L.A. Goodman, K.J. Arrow, Social Choice and Individual Values., Yale university press, 1953. doi:10.2307/2087870.

[14] M.A. Satterthwaite, Strategt-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions, J. Econ. Theory. 10 (1975) 187–217. doi:10.1023/A:1021437021108.

[15] R.E. Quandt, A Probabilistic Theory of Consumer Behavior, Q. J. Econ. 70 (1956) 507. doi:10.2307/1881863.

[16] D. Ellsberg, Risk, Ambiguity, and the Savage Axioms, Q. J. Econ. 75 (1961) 643.

doi:10.2307/1884324.

[17] S. Dehaene, The neural basis of the Weber–Fechner law: a logarithmic mental number line, Trends Cogn. Sci. 7 (2003) 145–147. doi:10.1016/S1364-6613(03)00055-X.

[18] J.I. Pelaez, A. Casado, E.R. Yanez, F.E. Cabrera, A.H. Yanez, A model for social assessment of governmental areas through intangible assets, 2017 4th Int. Conf. EDemocracy EGovernment, ICEDEG 2017. (2017) 200–204. doi:10.1109/ICEDEG.2017.7962534.

[19] T.L. Saaty, L.G. Vargas, The possibility of group choice: Pairwise comparisons and merging functions, Soc. Choice Welfare. 38 (2012) 481–496. doi:10.1007/s00355-011-0541-6.

[20] R.N. Shepard, D.W. Kilpatric, J.P. Cunningham, The internal representation of numbers, Cogn. Psychol. 7 (1975) 82–138. doi:10.1016/0010-0285(75)90006-7.

[21] T.L. Saaty, Decision making with the analytic hierarchy process, Int. J. Serv. Sci. 1 (2008) 83. doi:10.1504/IJSSCI.2008.017590.

[22] T.L. Saaty, L.G. Vargas, Models, Methods, Concepts & Applications of the Analytic Hierarchy Process, Springer US, Boston, MA, 2012. doi:10.1007/978-1-4614-3597-6.

[23] A. Moreno, T. Redondo, Text Analytics: the convergence of Big Data and Artificial Intelligence, Int. J. Interact. Multimed. Artif. Intell. 3 (2016) 57. doi:10.9781/ijimai.2016.369.

[24] S. Khiat, H. Djamila, A Temporal Distributed Group Decision Support System Based on Multi-Criteria Analysis, Int. J. Interact. Multimed. Artif. Intell. 5 (2019) 7. doi:10.9781/ijimai.2019.03.002.

[25] J.P. Brans, P. Vincke, Note—A Preference Ranking Organisation Method, Manage. Sci. 31 (1985) 647–656. doi:10.1287/mnsc.31.6.647.

[26] K. Miettinen, J. Hakanen, D. Podkopaev, Multiple Criteria Decision Analysis, Springer New York, New York, NY, 2016. doi:10.1007/978-1-4939-3094-4.

[27] W. Poon, K. Lock-Teng Low, Are travellers satisfied with Malaysian hotels?, Int. J. Contemp. Hosp. Manag. 17 (2005) 217–227. doi:10.1108/09596110510591909.

[28] J. Ruppenhofer, M. Wiegand, J. Brandes, Comparing methods for deriving intensity scores for adjectives, in: Proc. 14th Conf. Eur. Chapter Assoc. Comput. Linguist. Vol. 2 Short Pap., Association for Computational Linguistics, Stroudsburg, PA, USA, 2014: pp. 117–122. doi:10.3115/v1/E14-4023.

[29] Amazon Customer Reviews Dataset, Amazon Customer Reviews Dataset, (2015) 3–5.

[30] Yelp Open Dataset, Yelp Open Dataset, (2020). https://www.yelp.com/dataset/.