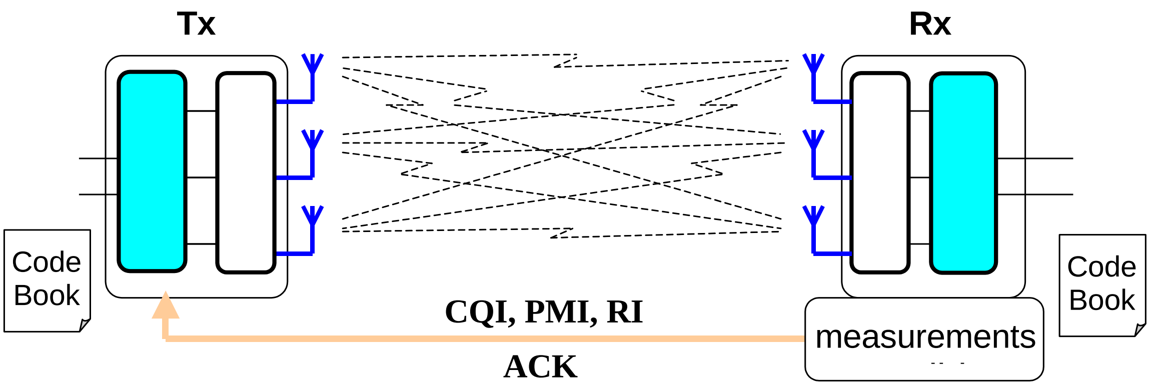
Universidad de Málaga
Escuela Técnica Superior de Ingeniería de Telecomunicación
Departamento de Ingeniería de Comunicaciones
Máster en Ingeniería de Telecomunicación

Signal processing and communications techniques

6 Link adaptation



José Tomás Entrambasaguas

2016

Link adaptation

Adaptation of modulation

- Adaptive modulation system
- Probability of error in Rayleigh channel - Approximate formula
- Inversion of approximate formula - SNR gap
- Calculation of thresholds for integer bits/symbol constellations

Adaptation of coding rate

- Concept
- Increasing coding rate: puncturing
- Reducing coding rate: repetition
- Joint adaptation of modulation and coding

Performance of adaptative systems

- Bits/symb: even integer values
- Bits/symb: non-integer values
- Comparison with interleaving without adaptation
- Examples

MIMO adaptation

- STBC/SFBC
- Beamforming
- Spatial multiplexing (MIMO mux)
- Codebooks

Retransmission

- ARQ and H-ARQ
- Bloks combining
- Incremental redundancy

Link adaptation

Adaptation of modulation

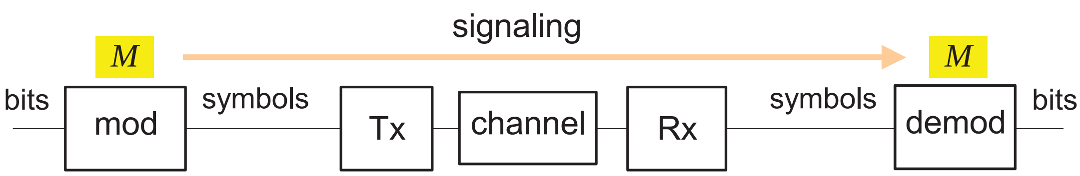
Adaptation of coding rate

Performance of adaptative systems

MIMO Adaptation

Retransmission

Configurable modulation system



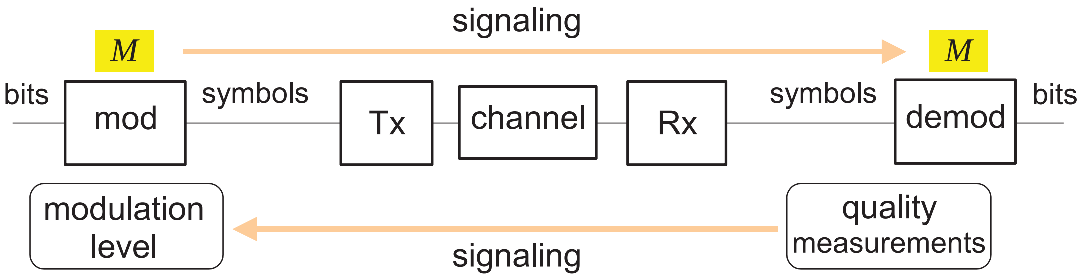
- Transmitter and receiver can work with different modulations
- Transmitter must indicate (signal) the modulation level used
- Signaling channel multiplexed with the data

Examples:

	constelations	signaling
DVB	16-QAM a 256-QAM	periodically
Wi-Fi	BPSK, 4-QAM a 64-QAM	at the beginning of the packet

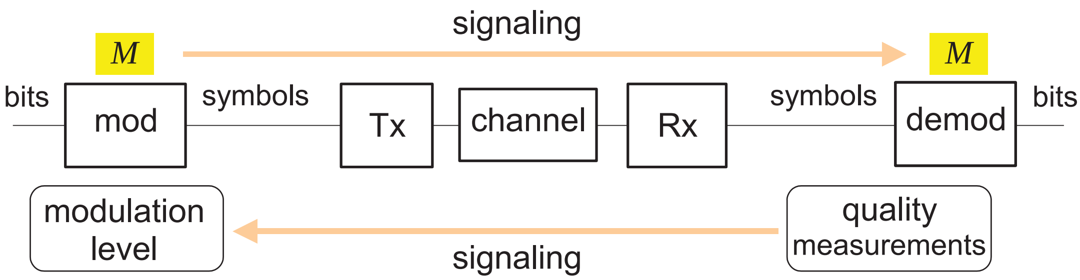
- Signaling uses a simple fixed constellation (BPSK or 4-QAM)

Adaptive modulation system



- Configurable modulation system with signaling channel Tx → Rx +
- Receiver performs quality measurements of the received signal and ...
  sends them to transmitter (through a signaling channel Rx → Tx)
- The transmitter can change the modulation to the most suitable for the quality of the channel (signaling it to the receiver)
- It is useful with channels of unknown or changing quality over time.

Adaptive modulation system



- Typical adaptation criterion: not exceed a target BER
- Can be implemented by measuring the BER or SNR at the receiver

Gaussian channel. *M*-QAM. Probability of error

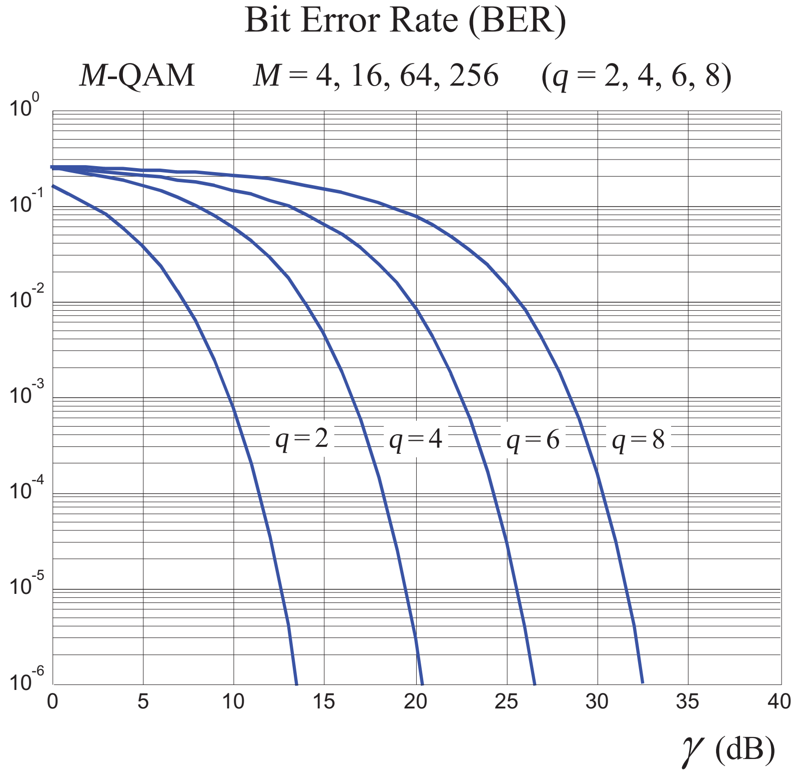
$$P_s = \alpha_M Q\left(\sqrt{\beta_M \cdot \gamma}\right)$$

$$\alpha_M = 4 - 4/\sqrt{M}$$
$$\beta_M = 3/(M-1)$$

$$P_b = \frac{1}{\log_2 M} \cdot P_s(\gamma)$$

Modulation level:

$$q = \log_2 M$$



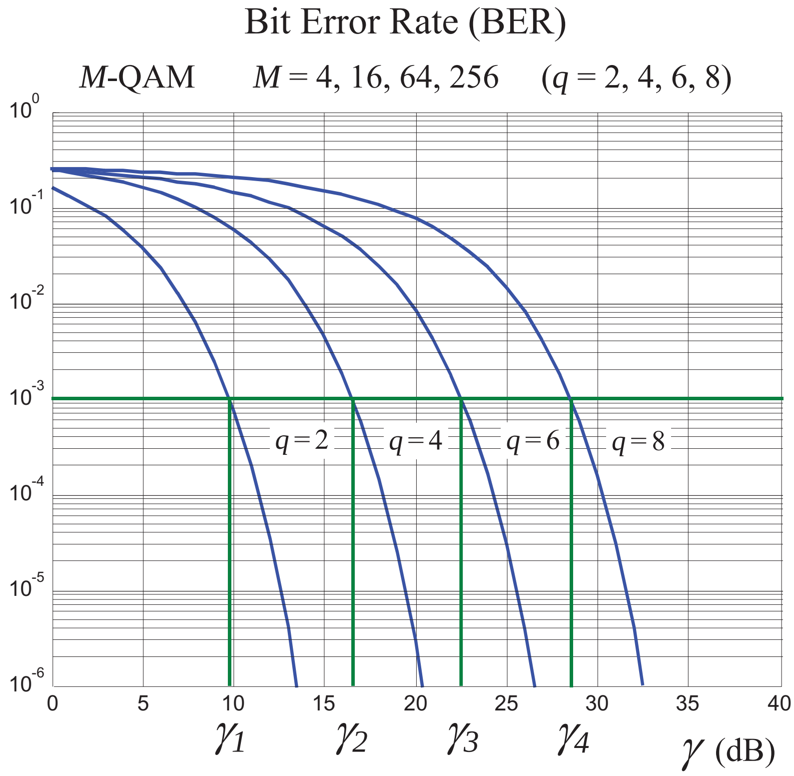
Gaussian channel. *M*-QAM. Adaptation of modulation

Giben a target P_b, ...
if γ is known ...

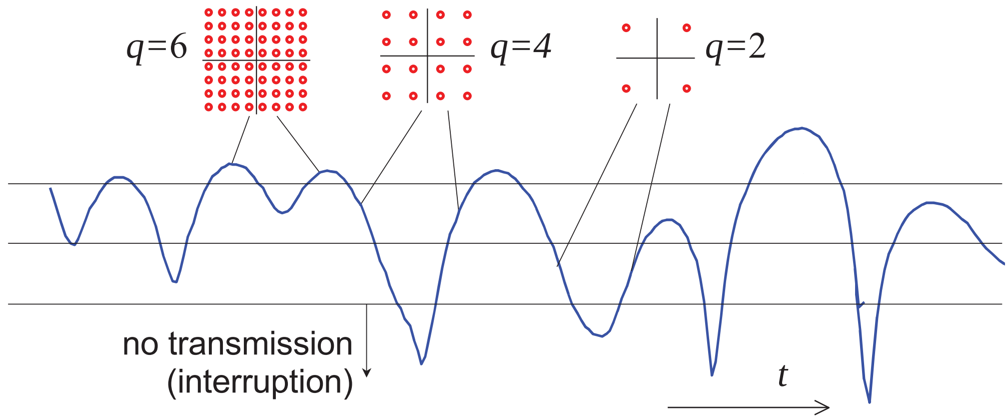
q can be decided
to not exceed the target P_b

Eg.: target $P_b = 10^{-3}$

4 regions:	q	
$\gamma < \gamma_1$	0	no Tx
$\gamma_1 < \gamma < \gamma_2$	2	4-QAM
$\gamma_2 < \gamma < \gamma_3$	4	16-QAM
$\gamma_3 < \gamma < \gamma_4$	6	64-QAM
$\gamma_4 < \gamma$	8	256-QAM

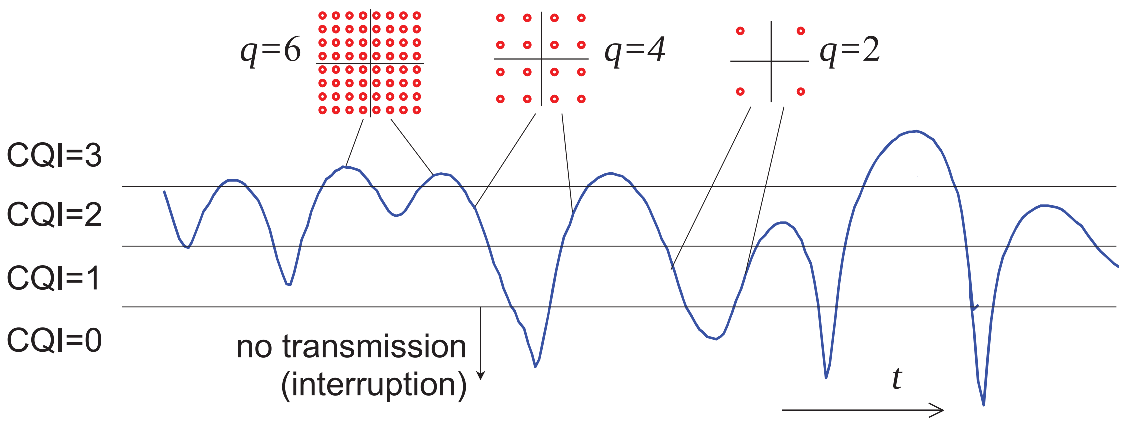


Rayleigh channel - slow variant. Adaptive modulation



- The <u>modulation is modified</u> to not exceed a target P_b
- The receiver <u>measures</u> the SNR and <u>indicates</u> it to the transmitter (signaling)
- The transmitter <u>adjusts</u> the <u>modulation level</u> q
  - There are transmission <u>interruptions</u> (when SNR is too low)

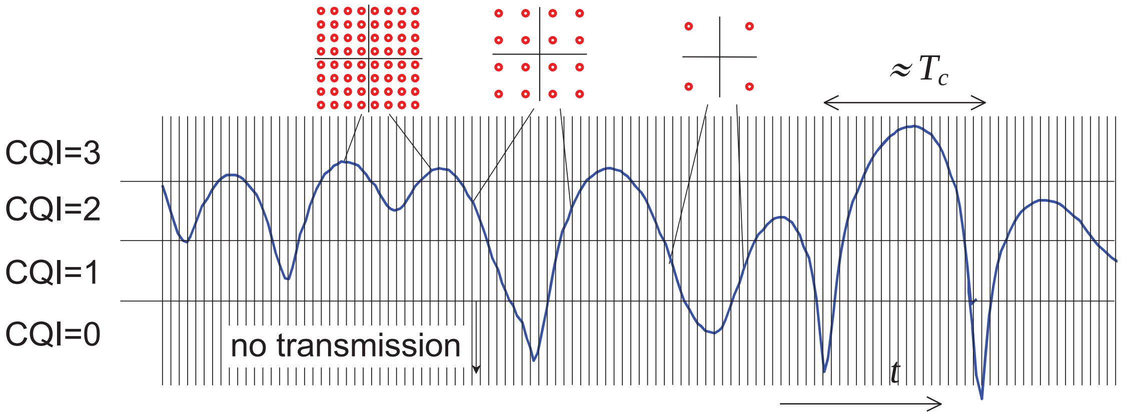
Rayleigh channel - slow variant. Adaptive modulation



- To reduce the signaling the receiver sends only one <u>indication of the region of SNR</u>: **CQI**: ***Channel Quality Indicator***:

0	$\gamma < \gamma_1$
1	$\gamma_1 < \gamma < \gamma_2$
2	$\gamma_2 < \gamma < \gamma_3$
3	$\gamma_3 < \gamma$

Rayleigh channel - slow variant. Adaptive modulation

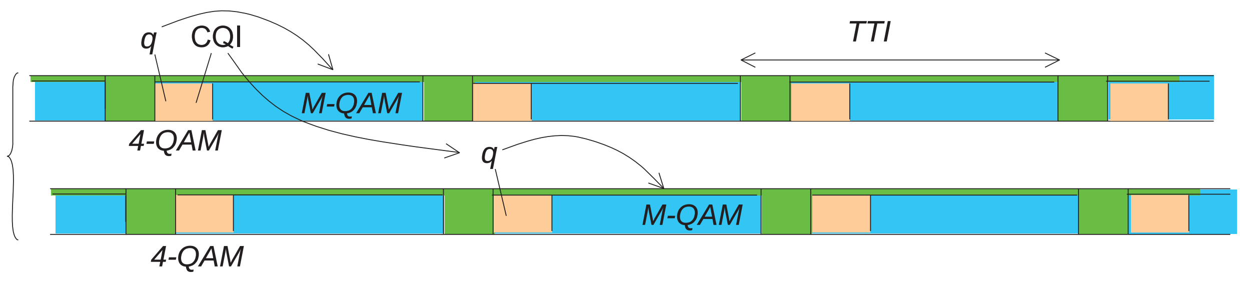


- Transmission is done in **blocks of symbols**
- Blocks of duration $< T_C/20$ (channel gain varies little in block)
- Eg.: $F_c = 1800$ MHz, $v = 5$ km/h (pedestrian) $T_c = 120$ ms
- With a fast variation channel it is not possible to do adaptive modulation

Rayleigh channel - slow variant. Adaptive modulation

Example. Bidireccional continuous transmission

- Transmission is divided into **Block Transmission Intervals**
  TTI: *Tranmission Time Interval*
- At the beginning of each block there is a signaling data in 4-QAM indicating:
  - the modulation level q used in the rest of the block
  - the CQI measured for the transmission in the reversed direction
- Each transmitter can use the CQI it receives to change its modulation q
  - But it can only do it in a following block, indicating q at the beginning



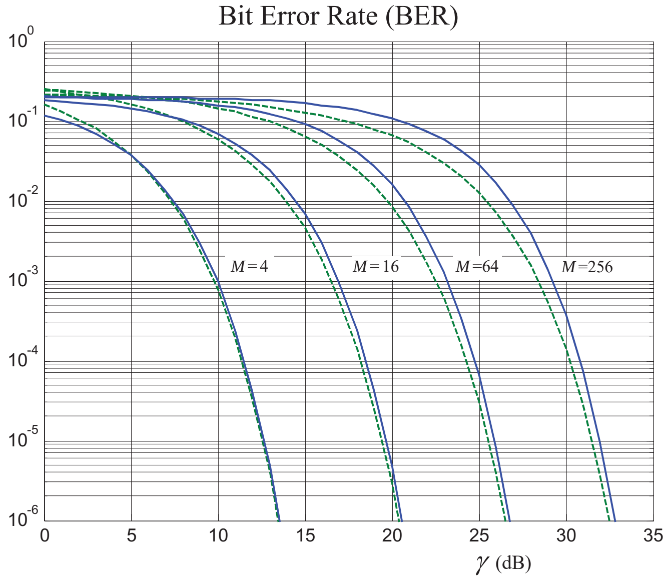
Probability of error - QAM - Approximate formula

$$P_s = \alpha_M Q\left(\sqrt{\beta_M \cdot \gamma}\right)$$

$$\alpha_M = 4 - 4/\sqrt{M} \qquad \beta_M = 3/(M-1)$$

$$P_b = \frac{1}{\log_2 M} \cdot P_s(\gamma) = \frac{\alpha_M}{\log_2 M} \cdot Q\left(\sqrt{\frac{3}{M-1} \cdot \gamma}\right)$$

$$P_b \approx \frac{1}{5} \exp\left(\frac{-1{,}6\gamma}{M-1}\right)$$



- Approximate formula: error < 1dB for P_b<10^{-3})

Probability of error - Inversion of the approximate fomula

$$P_b \approx \frac{1}{5} \exp\left(\frac{-1{,}6\gamma}{M-1}\right)$$

Inversion of the fomula

- Calculation of M as a function of γ
- Depends on target P_b

Solving for M:

$$\ln(5P_b) = \frac{-1{,}6\gamma}{M-1} \quad \rightarrow \quad M-1 = \frac{-1{,}6\gamma}{\ln(5P_b)} \quad \rightarrow \quad M = 1 + \frac{-1{,}6\gamma}{\ln(5P_b)}$$

$$M = 1 + \frac{\gamma}{\Gamma(P_b)} \quad \text{con} \quad \Gamma(P_b) = \frac{-\ln 5P_b}{1{,}6}$$

$$b = \log_2 M = \log_2\left(1 + \frac{\gamma}{\Gamma(P_b)}\right)$$

Probability of error - Inversion of the approximate fomula

Comparison with capacity

Capacity (Shannon)

$$C = \log_2(1+\gamma)$$

Binary rate M-QAM with restriction of P_b

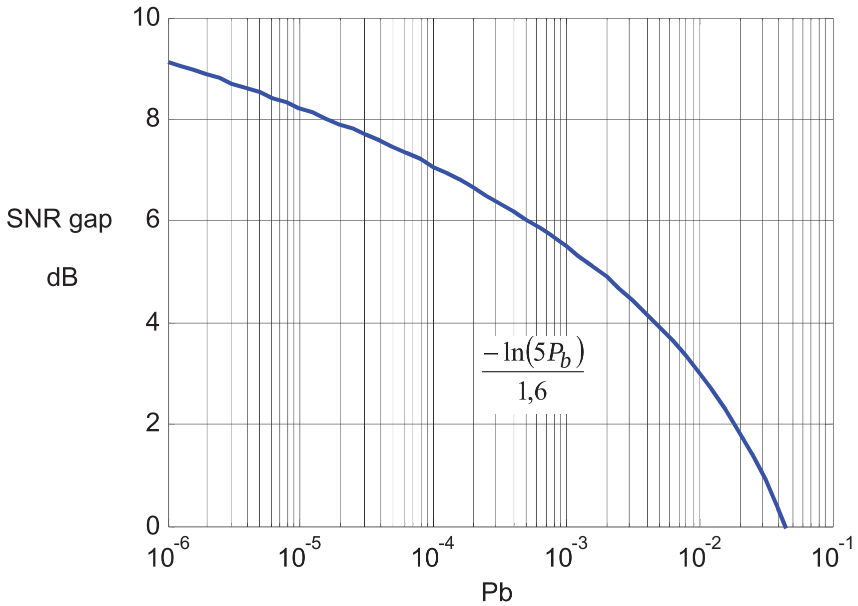
$$b = \log_2\left(1 + \frac{\gamma}{\Gamma(P_b)}\right)$$

$$\Gamma(P_b) = \frac{-\ln(5P_b)}{1{,}6}$$

difference with respect to capacity (***SNR gap***)

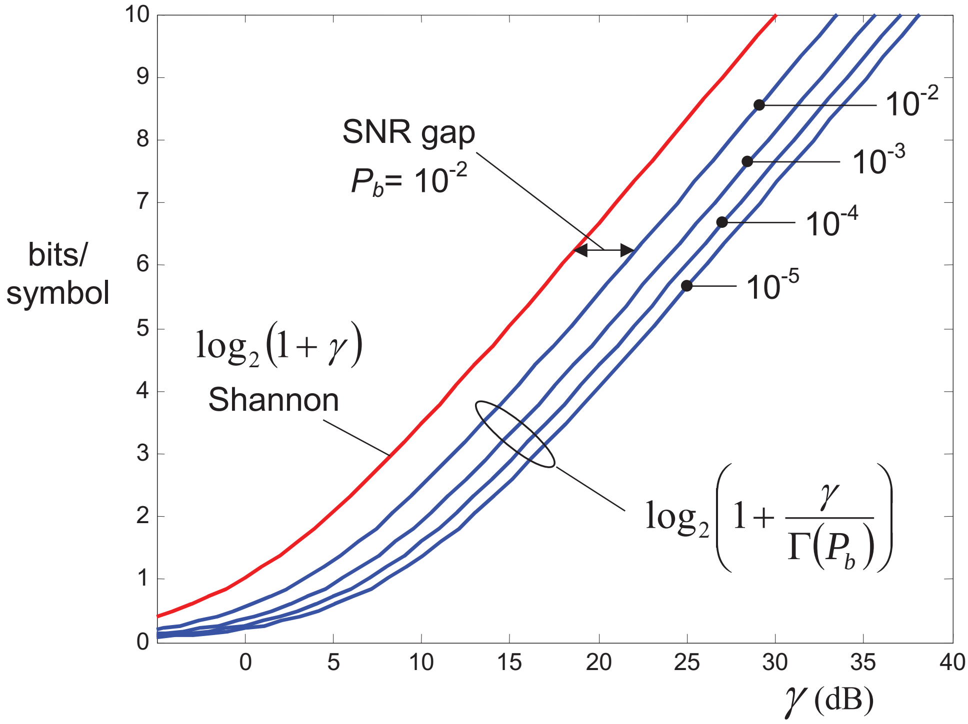
bigger for better P_b

Probability of error - SNR gap



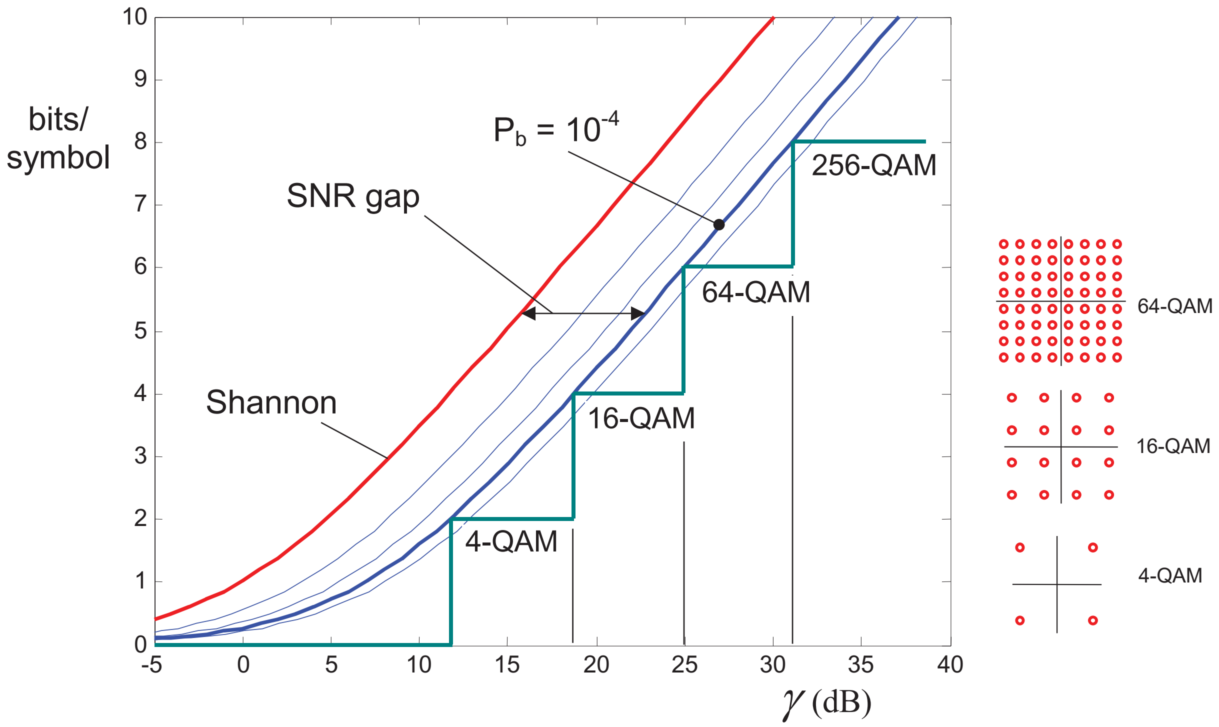
- ***SNR gap*** higher if required better P_b

Probability of error - Inversion of the approximate fomula



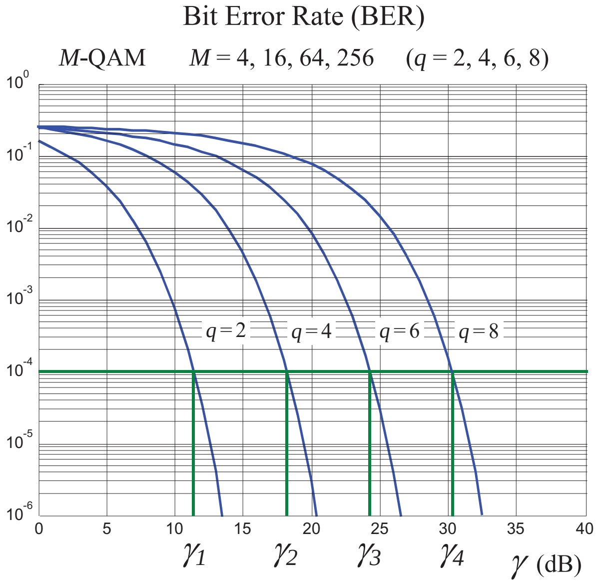
- ***SNR gap*** higher if required better P_b

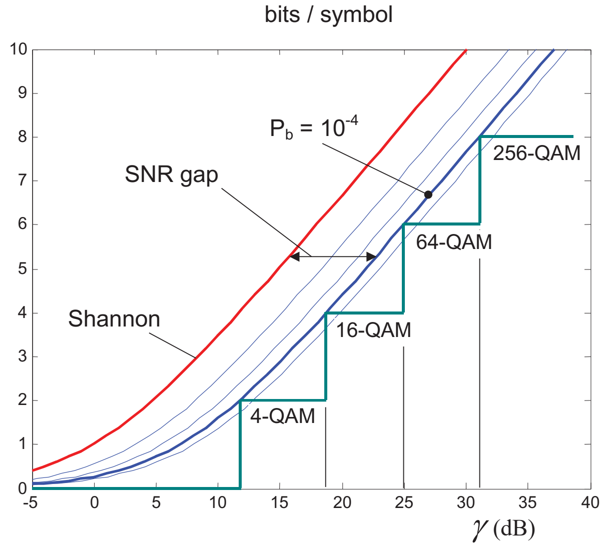
Inversion of Prob_{err} formula - Quantization (even integer)



- For square constellations with integer number of bits in each dimension

Inversion of Prob_{err} formula - Quantization (even integer)





$$\Gamma(P_b) = \frac{-\ln(5P_b)}{1,6}$$

Inversion of Prob_{err} formula - Quantization

Calculation of thresholds between regions

$$M = 1 + \frac{\gamma}{\Gamma(P_b)} \qquad \text{with} \quad \Gamma(P_b) = \frac{-\ln 5P_b}{1,6}$$

Solving for γ

$$\gamma = (M-1)\cdot\Gamma(P_b)$$

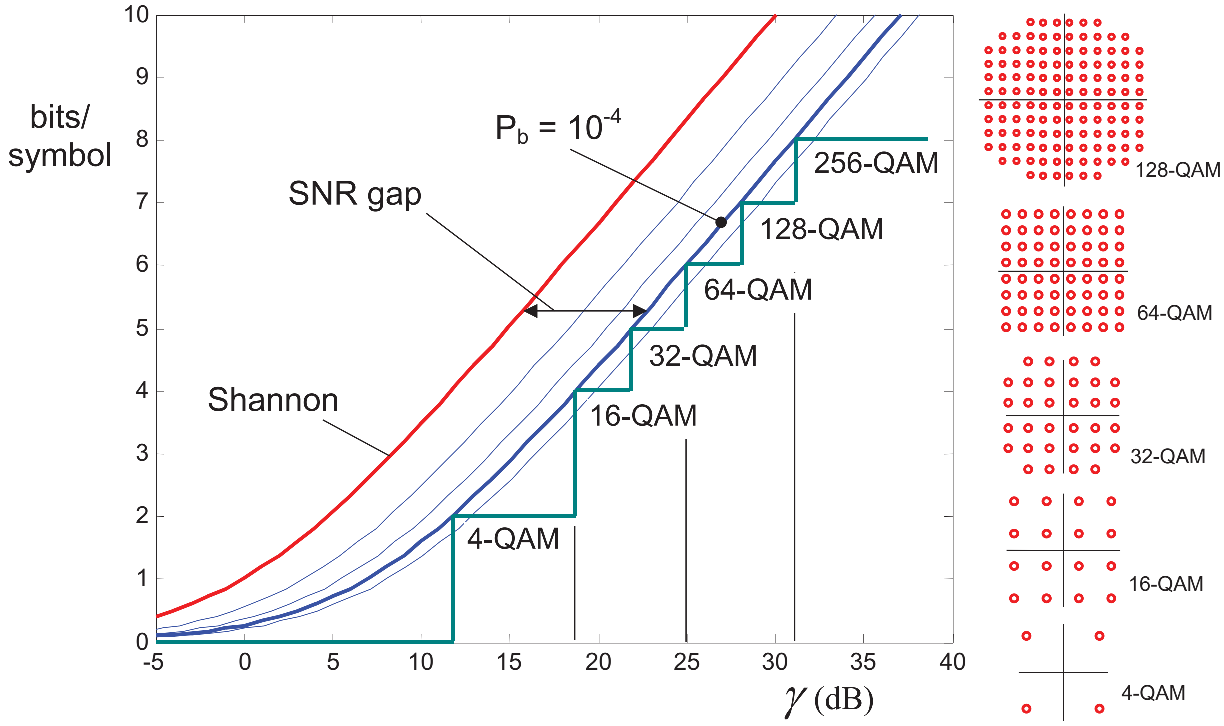
With a modulation M_i ...

... to ensure a probability of error $< P_b$

... SNR must be above this threshold

$$\gamma_i = (M_i - 1)\cdot\Gamma(P_b)$$

Inversion of $Prob_{err}$ formula - Quantization (integer)



thresholds $\gamma_i = (M_i - 1) \cdot \Gamma(P_b)$

Link adaptation

Adaptation of modulation

Adaptation of coding rate

Performance of adaptative systems

MIMO Adaptation

Retransmission

Link adaptation

Adaptation of modulation

Adaptation of coding rate

Concept

Increasing coding rate: puncturing

Reducing coding rate: repetition

Joint adaptation of modulation and coding

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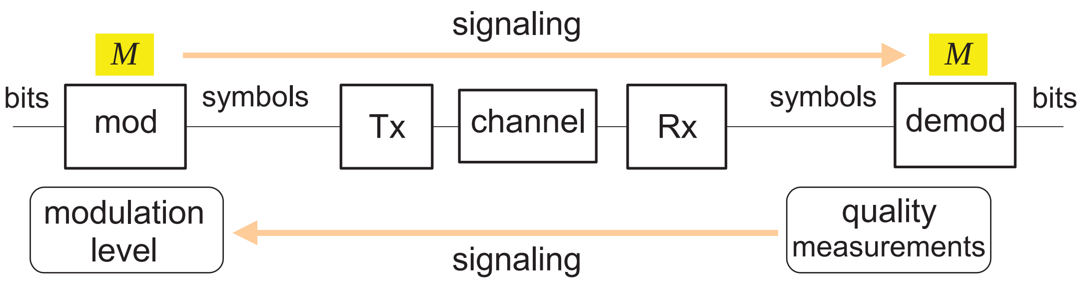
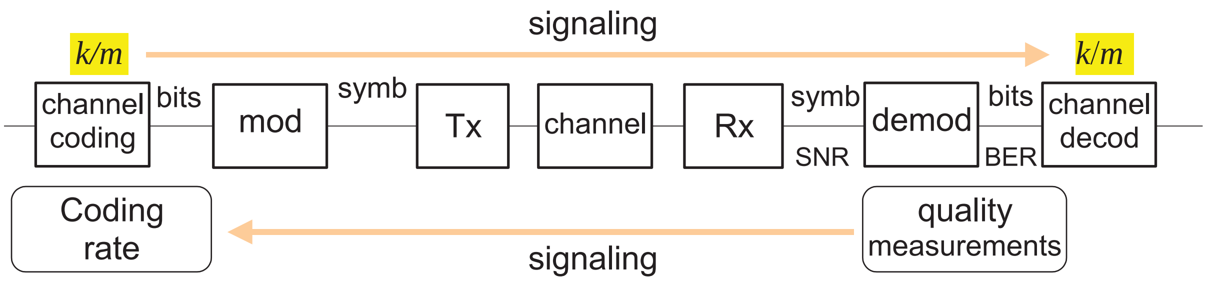
Adaptive modulation system



- Receiver performs quality measurements of the received signal and ...
  can suggest the proper modulation
- Increasing the SNR reduces the BER at the output...
  ... modulation level can be increased
- Typical adaptation criteria: do not exceed a target BER at the output
- Requires a signaling channel in the reverse direction

Adaptive coding system



- Receiver performs quality measurements of the received signal and ...
  can suggest the proper channel coding
- • Increasing the SNR reduces the BER at the demodulator ...
  ... coding rate can be increased
  (less redundancy, less error correction, more data rate)
- Requires forward and reverse signaling channels
- It is the transmitter that decides the encoding (and signals it to the receiver)

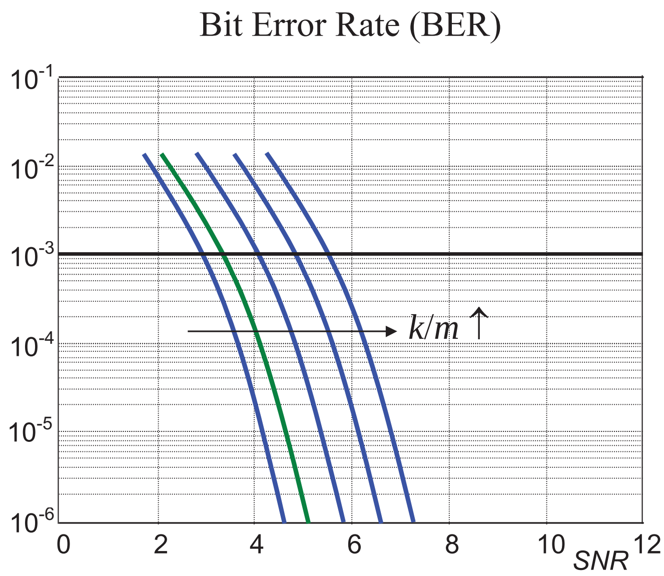
Adaptation of coding rate

Given a target P_b, ...
if *SNR* is known ...

... k/m can be decided
to not exceed the target P_b

Ej.: target $P_b = 10^{-3}$

when *SNR* increases
fewer demodulation errors occur ...
the encoding rate can be increased
(max: $k/m = 1$)



But...

How to build an adaptive rate encoder and decoder?

Link adaptation

Adaptation of modulation

Adaptation of coding rate

Concept

Increasing coding rate: puncturing

Reducing coding rate: repetition

Joint adaptation of modulation and coding

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Convolutional coding. Puncturing

- **Puncturing** reduces redundancy and error correction capability ...

  ... in exchange for increasing the transmitted bit rate

Example

- Coder rate 1/3, puncturing period p = 12 bits

patrón	*bits 1 a 12*												*(q) bits perforados*	*(p-q) bits transmitidos*	*tasa de codificación*	
1	d			d			d			d			8	4	1	1,00
2	d			d			d			d			7	5	4/5	0,80
3	d			d			d			d			6	6	2/3	0,67
4	d			d			d			d			5	7	4/7	0,57
5	d			d			d			d			4	8	1/2	0,50
6	d			d			d			d			3	9	4/9	0,44
7	d			d			d			d			2	10	2/5	0,40
8	d			d			d			d			1	11	4/11	0,36
9	d			d			d			d			0	12	1/3	0,33

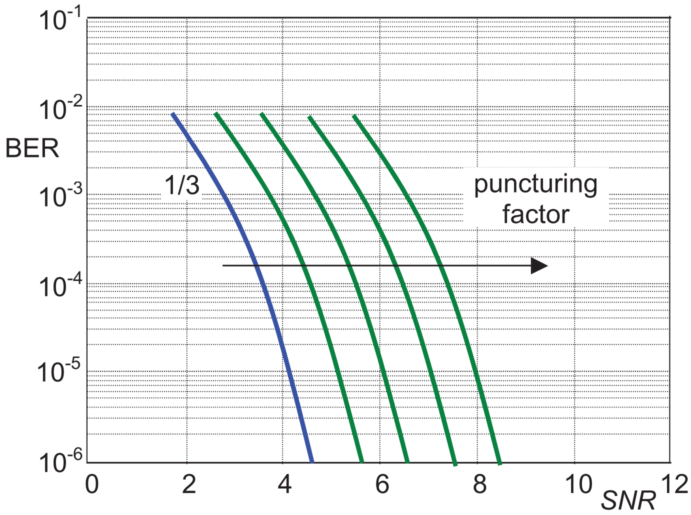
punched bits   d message (uncoded) bits

Adaptation of coding rate

How to build an adaptive rate encoder and decoder?

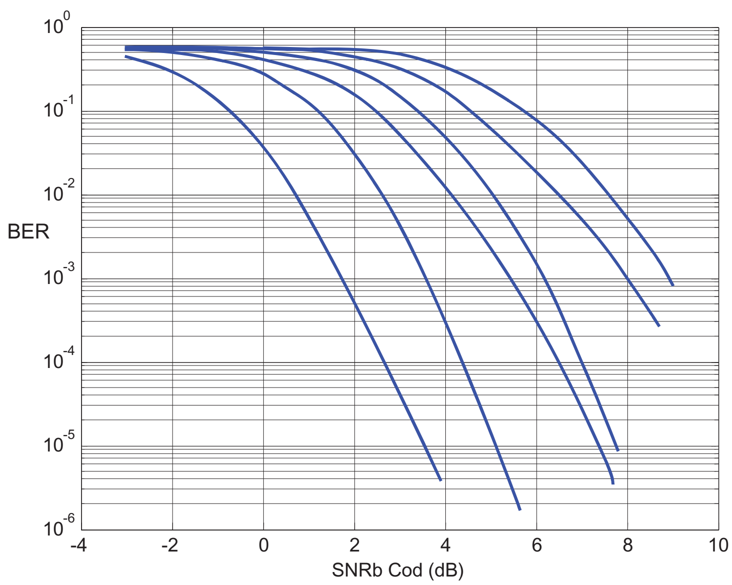
- Use a simple fixed rate encoder (1/3 or 1/2)
- Increase the rate by **puncturing** coded bits and a **sequence decoder** in receiver

coding rate	
1	1,00
4/5	0,80
2/3	0,67
4/7	0,57
1/2	0,50
4/9	0,44
2/5	0,40
4/11	0,36
1/3	0,33



Adaptation of coding rate

- Coder with adjustable coding rate
- Practice 43

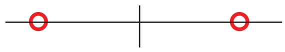


puncturing		coding rate	
q/p	*p/(p–q)*	*p/(p–q)* × 1/2	
0/1	1/1	1/2	0,50
1/4	4/3	2/3	0,67
2/6	6/4	3/4	0,75
4/10	10/6	5/6	0,83
7/16	16/9	8/9	0,89
15/32	32/17	16/17	0,94

Adaptation of coding rate

Example GPRS

GMSK



1 bit / symbol

	coding rate	*kbit/s*
GMSK	0,53	8,8
	0,66	11,2
	0,85	14,8
	1	17,6

Link adaptation

Adaptation of modulation

Adaptation of coding rate

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Increasing coding rate: puncturing

Reducing coding rate: repetition

Joint adaptation of modulation and coding

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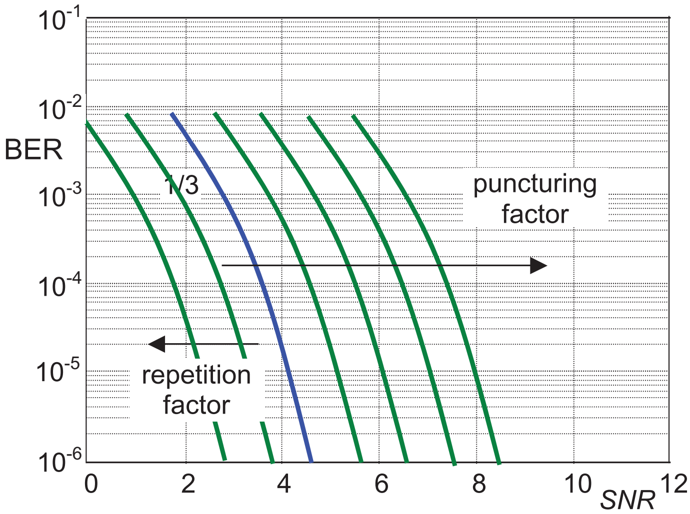
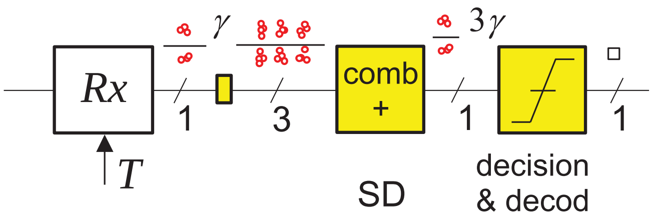
Adaptation of coding rate

How to build an adaptive rate encoder and decoder?

- Use a simple fixed rate encoder (1/3 or 1/2)
- Increase the rate by **puncturing** coded bits and a **sequence decoder** in receiver
- Reduce the rate by **repetition** and **combination**



Channel coding: hard/soft decision



Repetition codes with soft decision (combination before decision)

- **Repetición** increase redundancy and error correction capability ...

... in exchange for reducing the transmitted bit rate

- repetition factor = SNR increase = bit rate reduction

repetition factor	SNR increase	bit rate ×
× 2	3 dB	0,5
× 3	4,7 dB	0,33
× 4	6 dB	0,25
× 5	7 dB	0,25

Adaptation of coding rate

Example UMTS – LTE

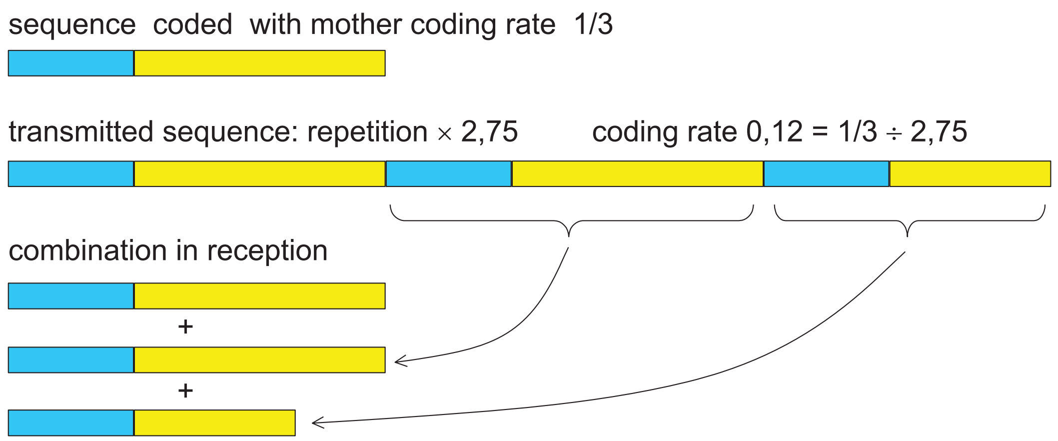
Coding scheme	Modulation 2 bits/symb	Coding rate k/m	Rate ajustement	10log(rep)	bits/symb (uncoded)
1	4-QAM	0.076	rep × 4,34	6,3dB	0,152
2		0.12	rep × 2,75	4,4 dB	0,24
3		0.19	rep × 1,73	2,3 dB	0,38
4		0.33	mother code rate		0,66
5		0.44	punc 25%		0,88
6		0.59	punc 44%		1,18

- *Mother* code with rate 1/3
- Coding rates less than 1/3 can be obtained with repetition
- For low speed transmission on very low SNR channels
- SNR steps: ~ 2 dB

Adaptation of coding rate

Example UMTS – LTE

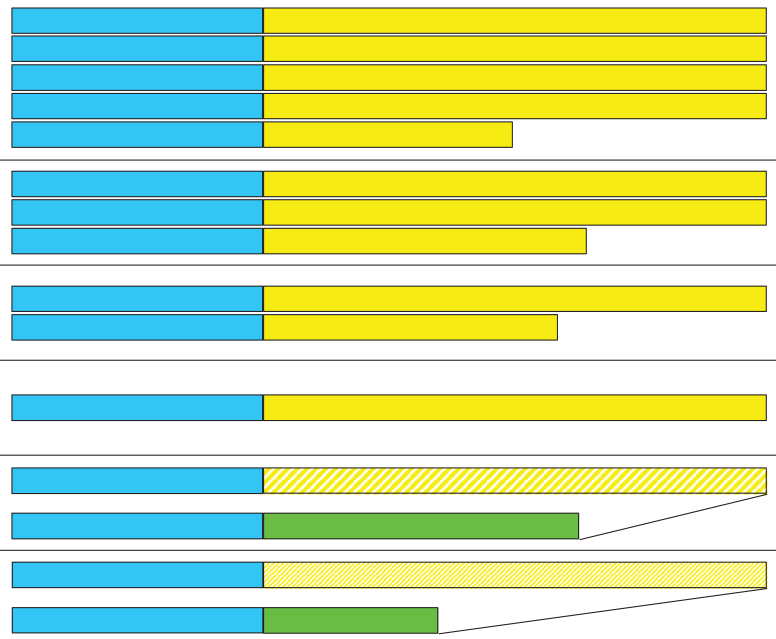
Coding scheme	Modulation 2 bits/symb	Coding rate k/m	Rate ajustement	10log(rep)	bits/symb (uncoded)
2	4-QAM	0.12	rep × 2,75	4,4 dB	0,24



- Receiver combines the repeated bits before passing to the sequence decoder

Adaptation of coding rate

Example UMTS – LTE



	k/m
repetition × 4,34	0,076
repetition × 2,75	0,12
repetition × 1,73	0,19
mother code 1/3	0,33
puncturing 25%	0,44
puncturing 44%	0,59

- Receiver combines the repeated bits before passing to the sequence decoder

Link adaptation

Adaptation of modulation

Adaptation of coding rate

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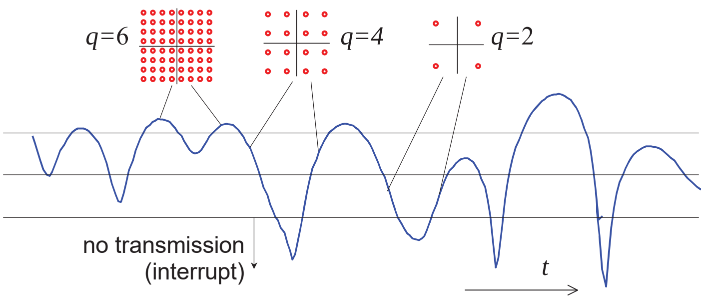
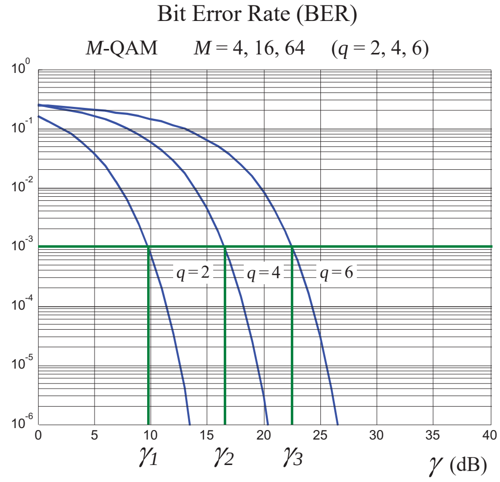
Modulación adaptativa

- Using M-QAM $M = 4, 16, 64,$ adaptation is done ...
  - in 6 dB steps of SNR that correspond to
  - bit rates: R_1 (4-QAM), $2\,R_1$, $3\,R_1$ ...
- It is difficult to use intermediate values





- For each modulation level, **channel coding** could be used....
  ... with **different coding rates**

Joint adaptation of modulation and coding rate

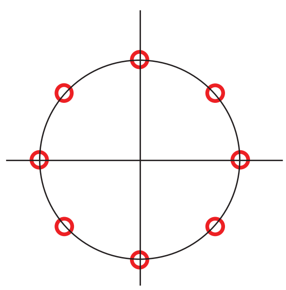
coding scheme combination of a modulation and a coding rate

Example 1 EGPRS

GMSK

1 bit / symbol

8-PSK



3 bits / symbol

coding scheme		
modulation	*coding rate*	*kbit/s*
GMSK	0,53	8,8
	0,66	11,2
	0,85	14,8
	1	17,6
8-PSK	0,38	22,4
	0,49	29,6
	0,76	44,8
	0,82	54,4
	1	59,2

Example 2 UMTS – LTE

Coding scheme	Modulation	Modulation level q bits/simb (coded)	Coding rate k/m	bits/simb (uncoded)	SNR_{min} (dB) $BLER_T = 10^{-1}$	ΔdB
	4-QAM	2	0.076	0,152	-6,9	
2			0.12	0,24	-4,9	2,0
3			0.19	0,38	-2,6	2,3
4			0.33	0,66	-0,8	1,8
5			0.44	0,88	0,9	1,7
6			0.59	1,18	2,9	2,0
7	16-QAM	4	0.37	1,48	4,8	1,9
8			0.48	1,92	6,8	2,0
9			0.6	2,4	8,7	1,9
10	64-QAM	6	0.45	2,7	10,4	1,7
11			0.55	3,3	12,2	1,8
12			0.65	3,9	14,0	1,8
13			0.75	4,5	16,1	2,1
11			0.85	5,1	17,7	1,6
15			0.95	5,7	20,0	2,3

Link adaptation

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Adaptation of coding rate

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Joint adaptation of modulation and coding

Performance of adaptative systems

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Adaptation of coding rate

Performance of adaptative systems - Rayleigh Channel

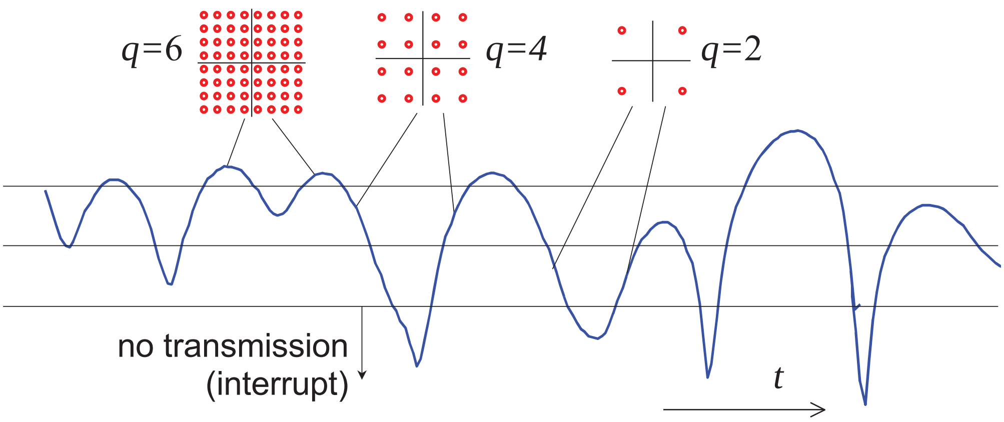
- **Bits/symb: even integer values**
- **Bits/symb: non-integer values**
- **Comparison with interleaving without adaptation**
- **Examples**

MIMO Adaptation

Retransmission

Rayleigh channel with slow variation. Adaptive modulation

Performance



¿Mean bit rate?

$$F_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}}}$$

bit/simb	region
0	$\gamma < \gamma_1$
2	$\gamma_1 < \gamma < \gamma_2$
4	$\gamma_2 < \gamma < \gamma_3$
6	$\gamma_3 < \gamma$

Rayleigh channel with slow variation. Adaptive modulation

Performance

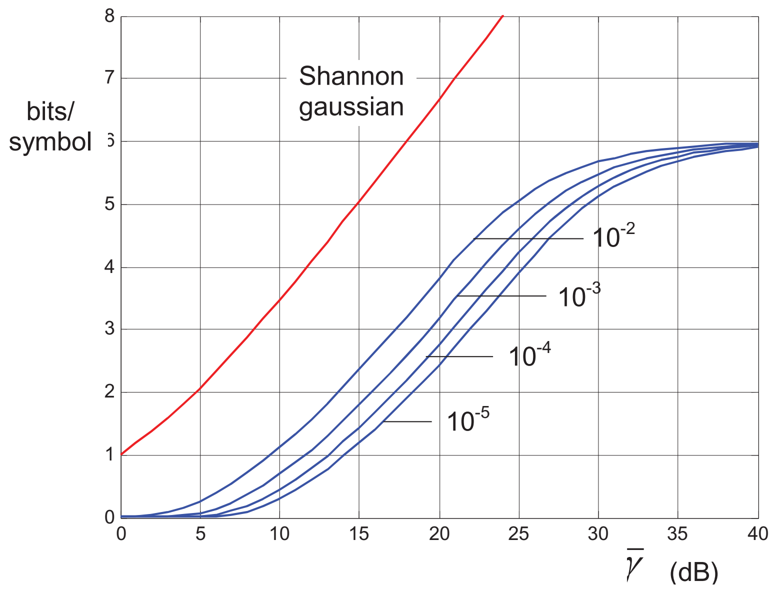
Thresholds	$\gamma_i > (M_i - 1)\cdot\Gamma(P_b)$
Rate (bits/symb) for region i	$b_i = \log_2 M_i$
Probability of being in the region i	$P(\gamma_i < \gamma < \gamma_{i+1}) = F_\gamma(\gamma_{i+1}) - F_\gamma(\gamma_i) = e^{-\frac{\gamma_i}{\bar{\gamma}}} - e^{-\frac{\gamma_{i+1}}{\bar{\gamma}}}$
Mean rate (bits/symb)	$R = \sum_i b_i \cdot P(\gamma_i < \gamma < \gamma_{i+1})$
In each region $(b_i - b_{i-1})$ bits are added	$R = \sum_i (b_i - b_{i-1})\cdot e^{-\frac{\gamma_i}{\bar{\gamma}}} \quad (b_0 = 0)$

Rayleigh channel with slow variation. Adaptive modulation

Performance

Modulations:
4-QAM, 16-QAM, 64-QAM

$$R = \sum_i b_i \cdot P(\gamma_i < \gamma < \gamma_{i+1}) = \sum_i (b_i - b_{i-1})\cdot e^{-\frac{\gamma_i}{\bar{\gamma}}}$$



Link adaptation

Adaptation of modulation

Adaptation of coding rate

Performance of adaptative systems - Rayleigh Channel

Bits/symb: even integer values

Bits/symb: non-integer values

Comparison with interleaving without adaptation

Examples

MIMO Adaptation

Retransmission

Rayleigh channel with slow variation. Adaptive modulation

Performance continuous values of b

How much is lost by using only square constellations?

Assume that a value γ (non-integer) of the number of bits can be used

$$b(\gamma) = \log_2\left(1 + \frac{\gamma}{\Gamma(P_b)}\right)$$

Mean rate (bits/symb): $R = \int b(\gamma) f_\gamma(\gamma) d\gamma$ with $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \cdot e^{-\frac{\gamma}{\bar{\gamma}}}$

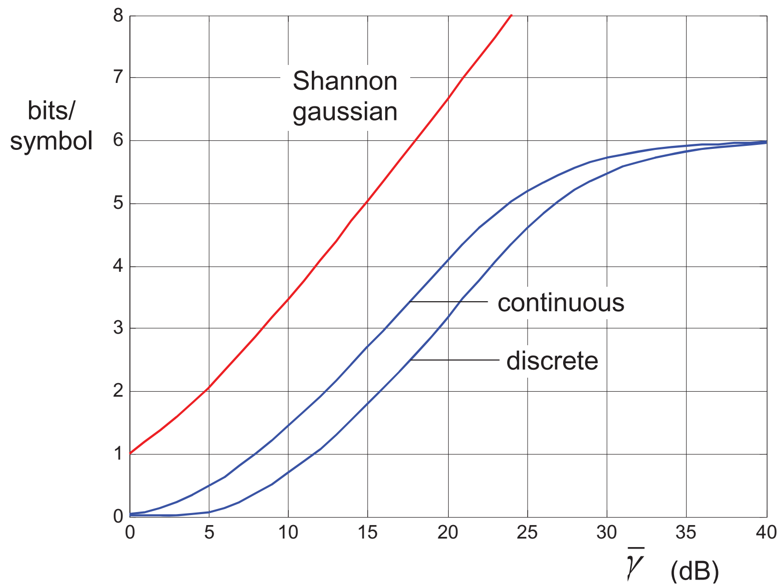
$$R_{cont} = \frac{1}{\bar{\gamma}} \int \log_2\left(1 + \frac{\gamma}{\Gamma(P_b)}\right) \cdot e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma$$

Rayleigh channel with slow variation. Adaptive modulation

Performance

Discrete: 4-QAM, 16-QAM, 64-QAM
P_b target 10^{-3}

Continuous: $b = 2$ to 6
P_b target 10^{-3}



~3dB lost

Link adaptation

Adaptation of modulation

Adaptation of coding rate

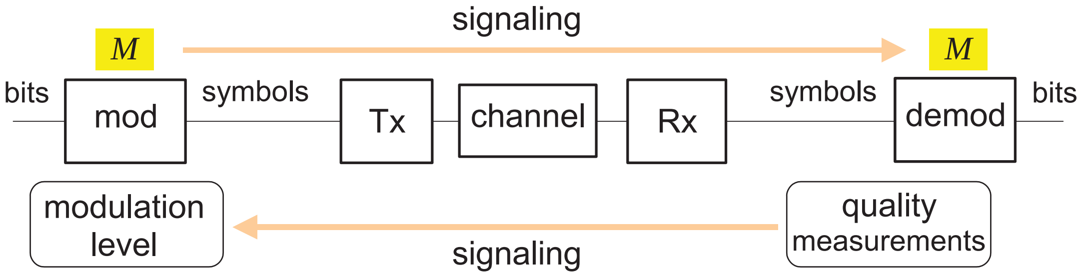
Performance of adaptative systems - Rayleigh Channel

- **Bits/symb: even integer values**
- **Bits/symb: non-integer values**
- **Comparison with interleaving without adaptation**
- **Examples**

MIMO Adaptation

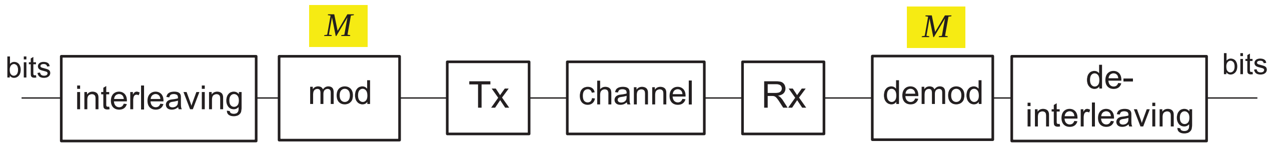
Retransmissionón

Adaptive modulation



M adapts to instantaneous value of γ (with constraint of $P_b(\gamma)$)

Fixed modulation with interleaving



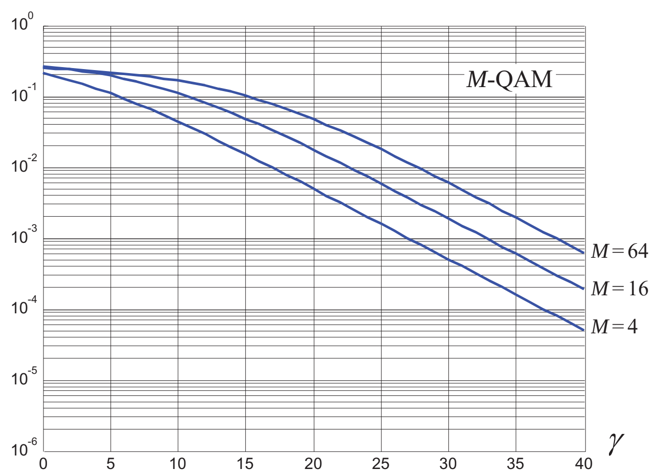
M is selected for target mean P_b, based on mean γ

Rayleigh channel with interleaving

Approximate formula

Asymptotic behavior $\alpha_M/\log_2 M \approx 1$ (0,5 a 1 for $b = 2$ to 8)

$$\bar{\gamma} \to \infty \qquad P_b \to \frac{1}{\log_2 M} \cdot \frac{\alpha_M}{2\beta_M} \cdot \frac{1}{\bar{\gamma}} \approx \frac{1}{2\beta_M} \cdot \frac{1}{\bar{\gamma}}$$



Rayleigh channel with interleaving

Approximate formula

Asymptotic behavior $\alpha_M / \log_2 M \approx 1$ (0,5 a 1 for $b = 2$ to 8)

$$\bar{\gamma} \to \infty \quad \boxed{P_b \;\to\; \frac{1}{\log_2 M} \cdot \frac{\alpha_M}{2\beta_M} \cdot \frac{1}{\bar{\gamma}} \;\approx\; \frac{1}{2\beta_M} \cdot \frac{1}{\bar{\gamma}}}$$

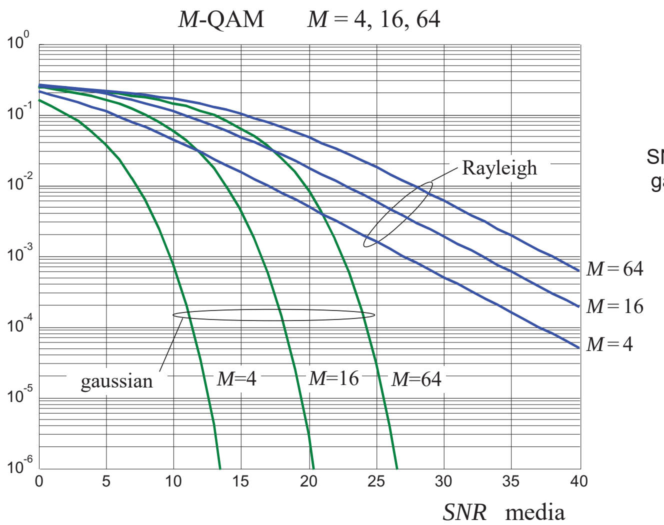
$$P_b = \frac{1}{2} \cdot \frac{M-1}{3} \cdot \frac{1}{\bar{\gamma}}$$ solving for M: $M = 1 + 6 P_b \, \bar{\gamma}$

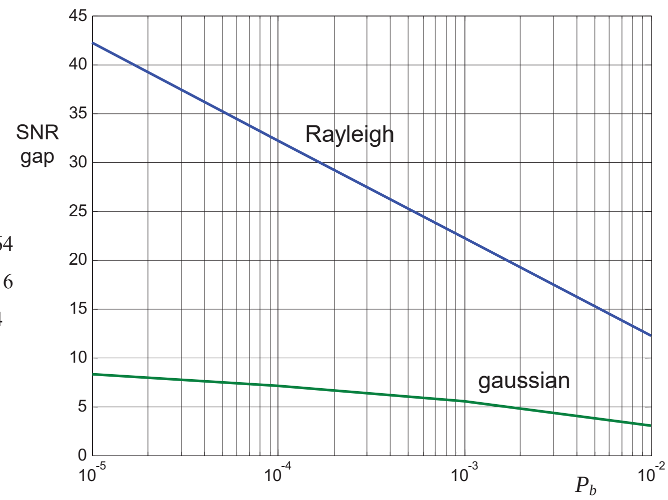
$$b = \log_2 M = \log_2 \left(1 + 6 P_b \, \bar{\gamma}\right) = \log_2 \left(1 + \frac{\bar{\gamma}}{\Gamma_R(P_b)}\right)$$

Rayleigh channel with interleaving ***SNR gap*** $\Gamma_R(P_b) = \dfrac{1}{6\overline{P_s}}$

Rayleigh channel with interleaving

SNR gap

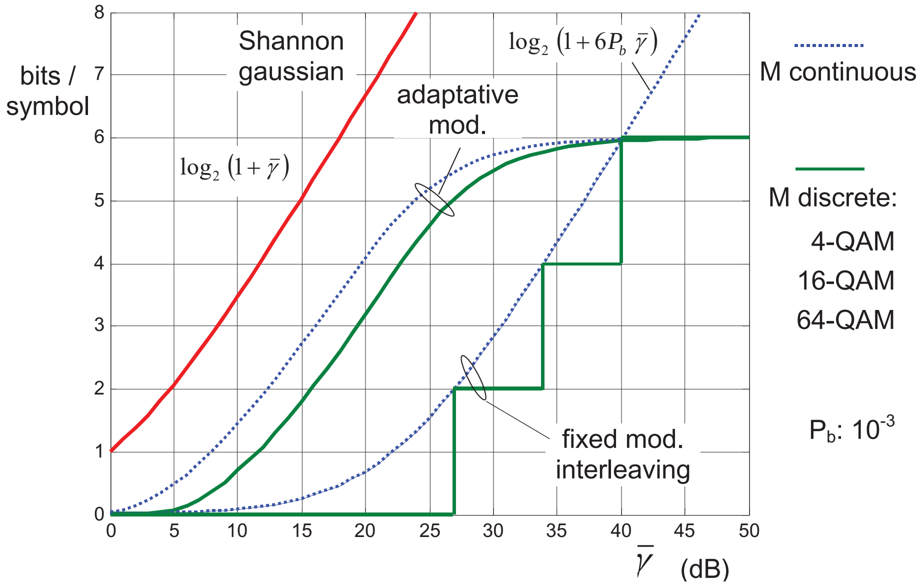




$$\Gamma_G(P_b) \approx \frac{-\ln(5P_b)}{1{,}6} \qquad \Gamma_R(P_b) \approx \frac{1}{6\overline{P_s}}$$

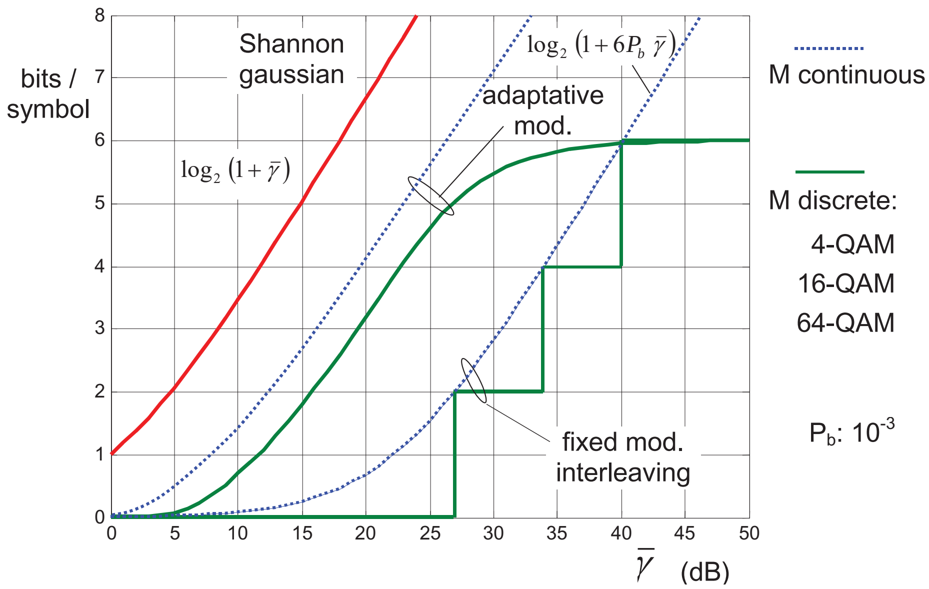
Adaptive modulation vs Fixed modulation with interleaving

Rayleigh channel



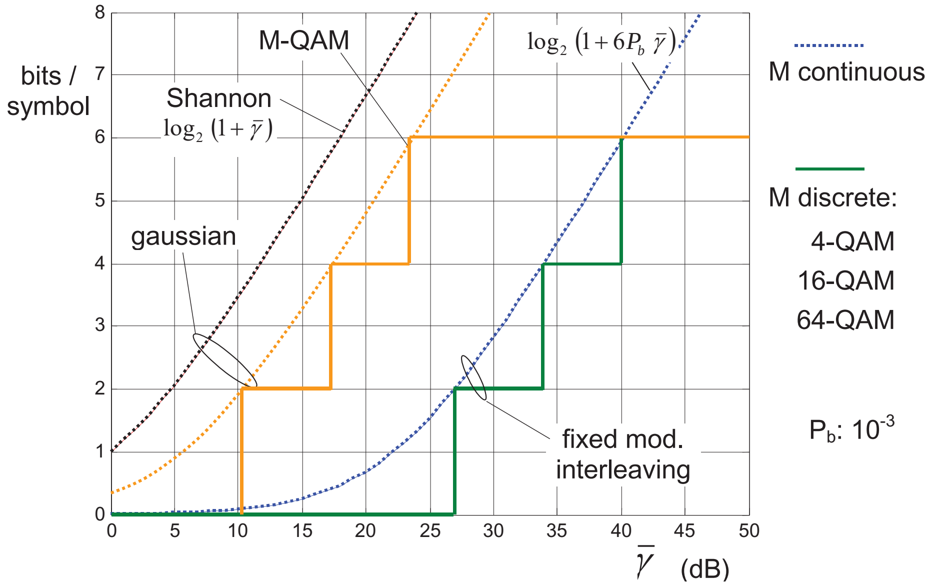
Adaptive modulation vs Fixed modulation with interleaving

Rayleigh channel



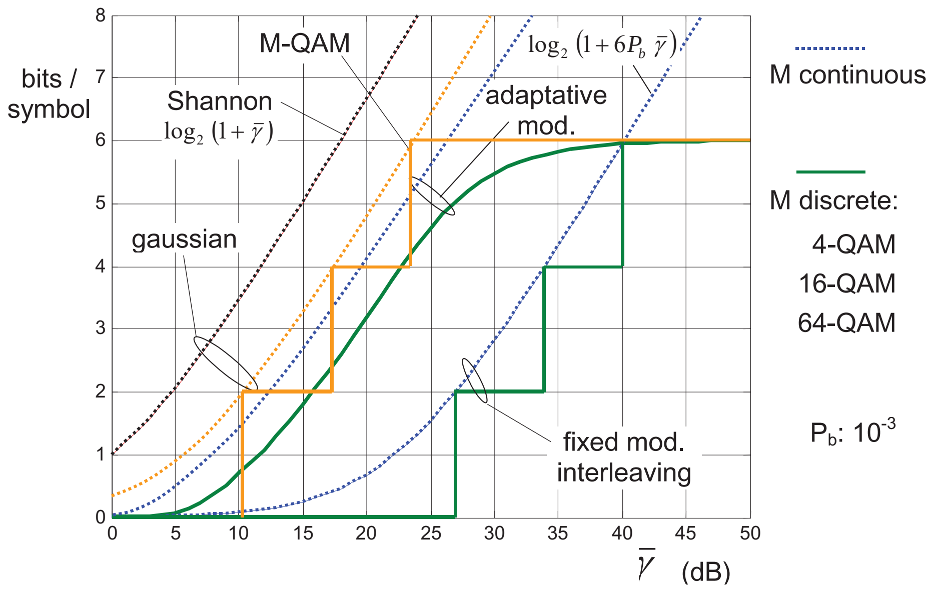
Adaptive modulation vs Fixed modulation with interleaving

Rayleigh channel - Gaussian channel



Adaptive modulation vs Fixed modulation with interleaving

Rayleigh channel - Gaussian channel



- Rayleigh channel - continuous AM ~2 dB from gaussian channel (discrete: ~5 dB)

Link adaptation

Adaptation of modulation

Adaptation of coding rate

Performance of adaptative systems - Rayleigh Channel

Bits/symb: even integer values

Bits/symb: non-integer values

Comparison with interleaving without adaptation

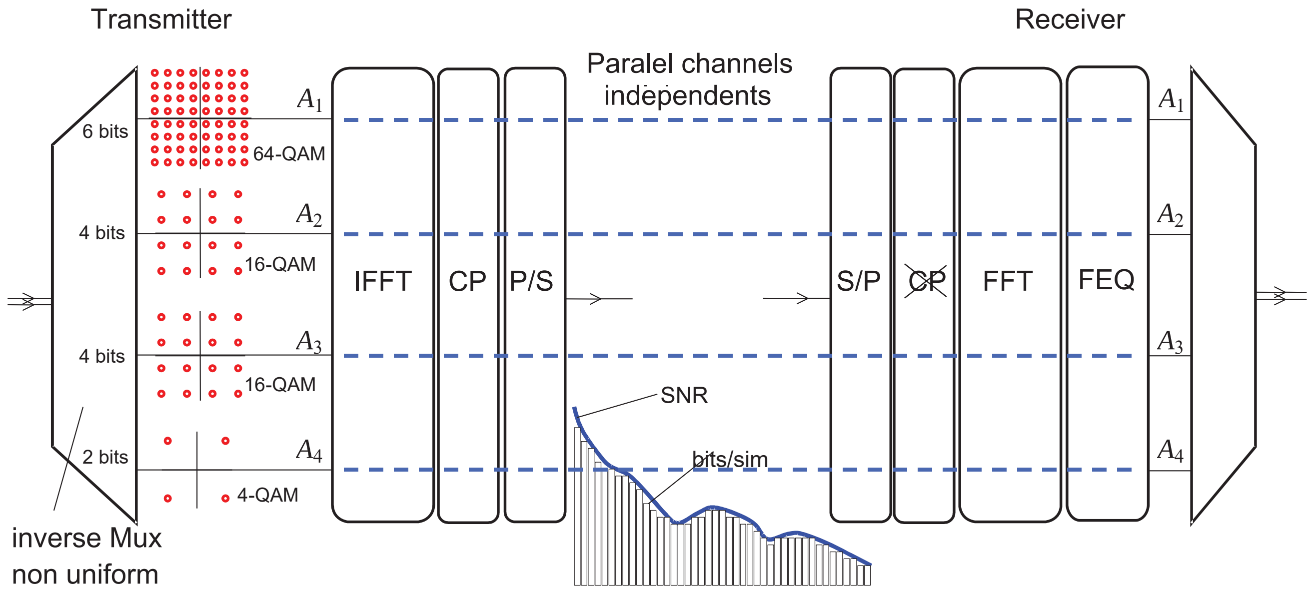
Examples

MIMO Adaptation

Retransmission

Adaptation of modulation - Multicarrier - OFDM

Invariant channel - ADSL

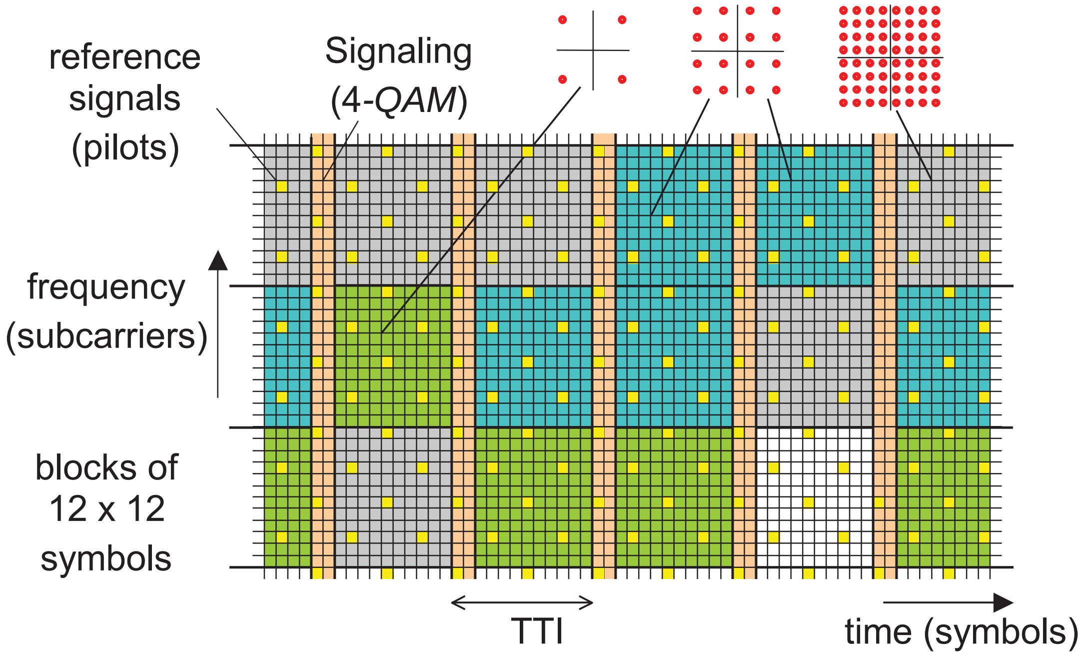


Modulation: M-QAM bit loading (1 a 15)

Channel coding: innner: convolutional 3/4 , 8 states **outer:** RS (255,223)

Adaptation of modulation - Multicarrier - OFDM

Variant channel - LTE



- For each new block (TTI) it is possible to change
  - modulations and coding rates **(coding schemes)**

UMTS – LTE

Coding scheme	Modulaton	Modulation level q bits/simb (coded)	Coding rate k/m	bits/simb (uncoded)	SNR_{min} (dB) $BLER_T = 10^{-1}$	ΔdB
1	4-QAM	2	0.076	0,152	-6,9	
2			0.12	0,24	-4,9	2,0
3			0.19	0,38	-2,6	2,3
4			0.33	0,66	-0,8	1,8
5			0.44	0,88	0,9	1,7
6			0.59	1,18	2,9	2,0
7	16-QAM	4	0.37	1,48	4,8	1,9
8			0.48	1,92	6,8	2,0
9			0.6	2,4	8,7	1,9
10	64-QAM	6	0.45	2,7	10,4	1,7
11			0.55	3,3	12,2	1,8
12			0.65	3,9	14,0	1,8
13			0.75	4,5	16,1	2,1
11			0.85	5,1	17,7	1,6
15			0.95	5,7	20,0	2,3

Link adaptation

Adaptation of modulation

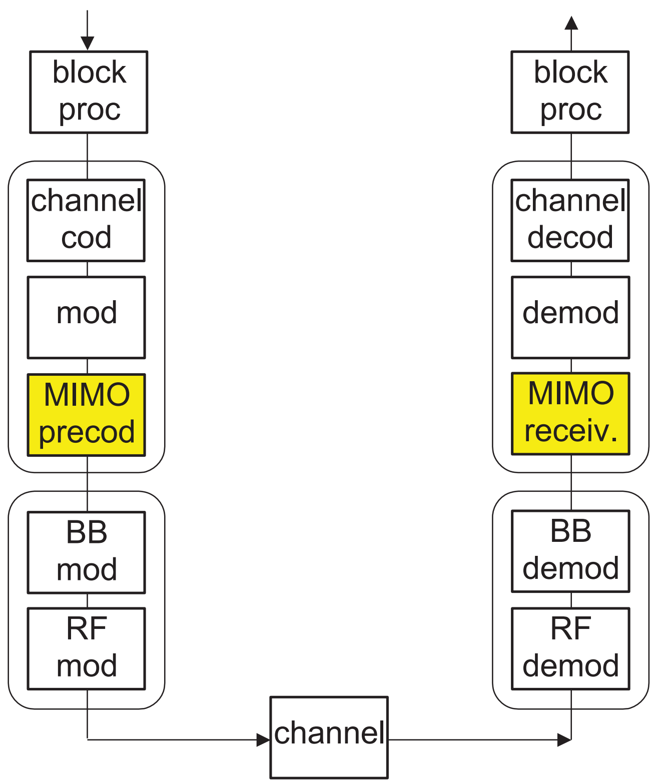
Adaptation of coding rate

Performance of adaptative systems

MIMO Adaptation

Retransmission

Digital transmission system



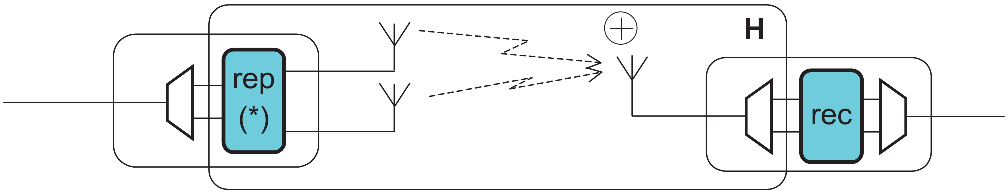
Link adaptation

Precoding adaptation

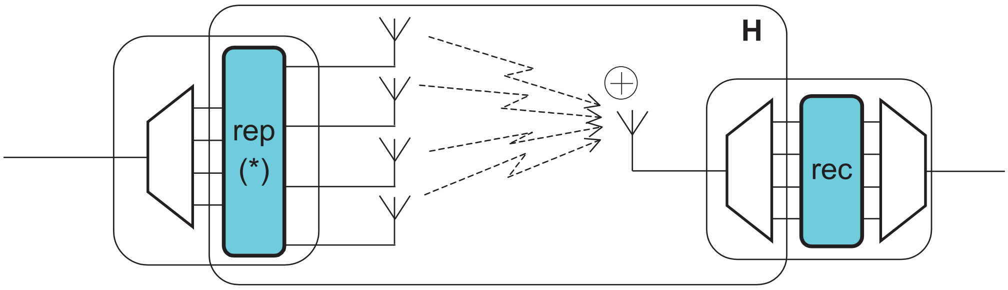
- STBC/SFBC
- Beamforming
- Spatial multiplexing (MIMO mux)
- Codebooks

Spatial diversity in transmission   without return channel

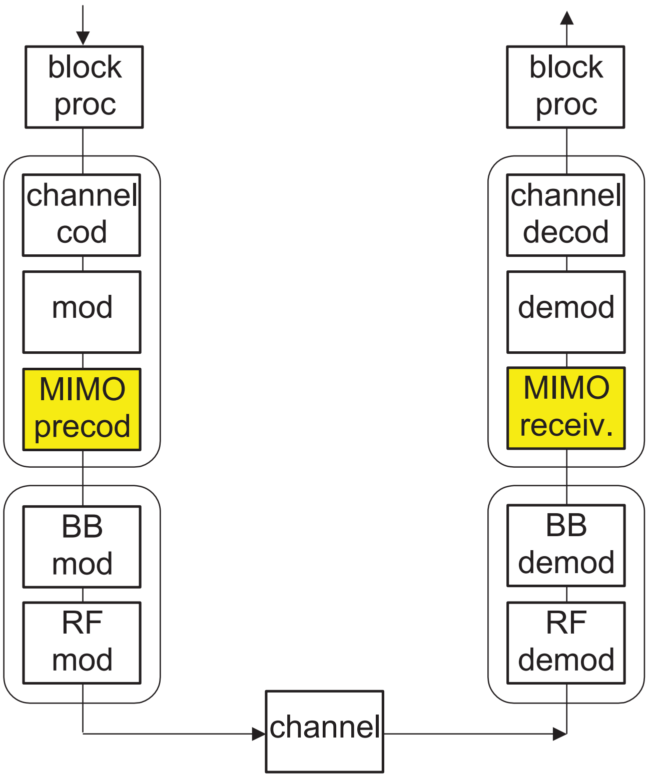
- Several symbols are grouped in a block (in time or in frequency)
- Symbols and their conjugates are repeated by the antennas (***Alamouti*** precoding)
- By jointly processing the received block, coherent combinations of the transmitted signals can be obtained



STBC / SFBC



Digital transmission system



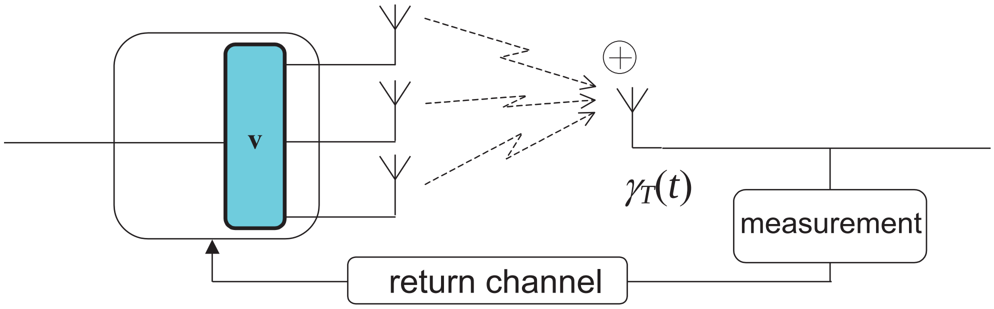
Link adaptation

Precoding adaptation

- STBC/SFBC
- Beamforming
- Spatial multiplexing (MIMO mux)
- Codebooks

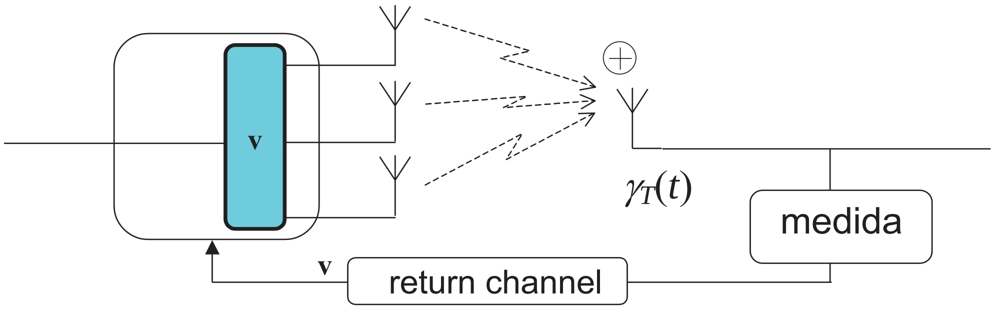
Spatial diversity in transmission

- The same signal is sent by all the antennas. Equal (EGC) or with gains (MRC)
- The only receiving antenna adds all the signals



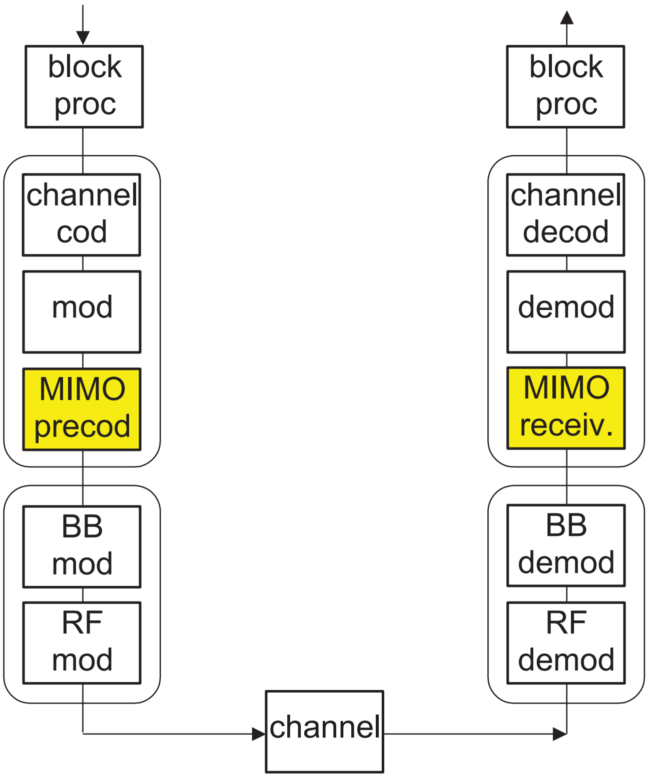
- The transmitter needs to know the measurements of the channels ...
  ... to compensate the **phases** or **weight** the signals
- Signal **measurement** in receiver. **Return channel** to send measurements
- ***Beamforming***: combine the signals to "aim" the power to the receiving antenna

Beamforming adaptation



- To **estimate** the **channel**, **pilot** signals are sent by the antennas,
  - **separated** in time or frequency (OFDM)
- The receiver calculates the **precoding vector** (complex gains)
- The receiver sends the precoding vector on the **return channel**

Digital transmission system

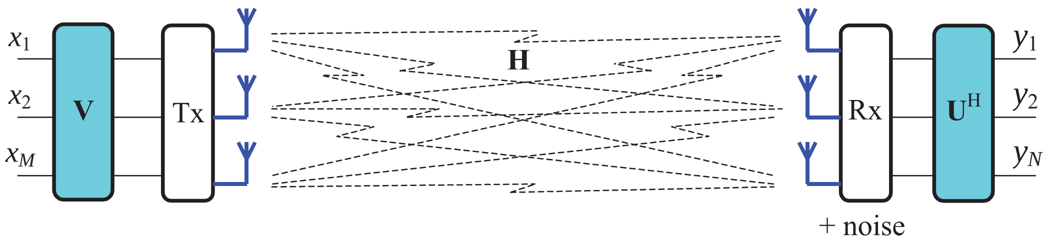


Link adaptation

Precoding adaptation

- STBC/SFBC
- Beamforming
- Spatial multiplexing (MIMO mux)
- Codebooks

MIMO systems with precoding - SVD

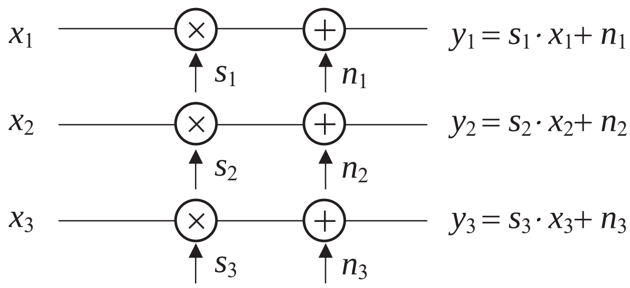


Singular Value Descomposition (SVD)

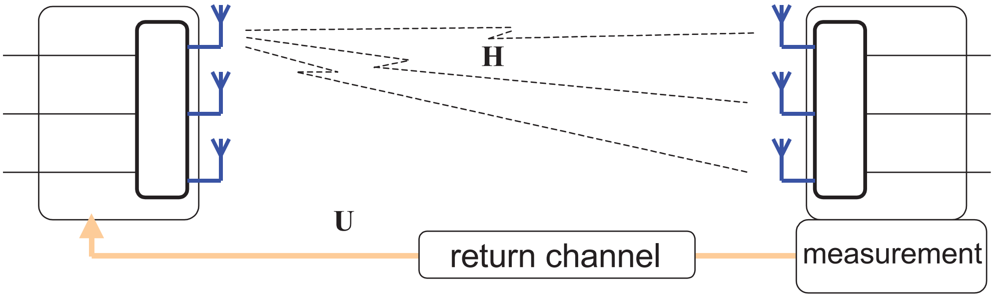
$$\mathbf{H} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^H$$

U V unitary orthogonal matrices $\mathbf{U}^H\mathbf{U} = \mathbf{I}_M$ $\mathbf{V}^H\mathbf{V} = \mathbf{I}_N$

Σ singular values $M \times N$



Spatial multiplexing (MIMO) adaptation

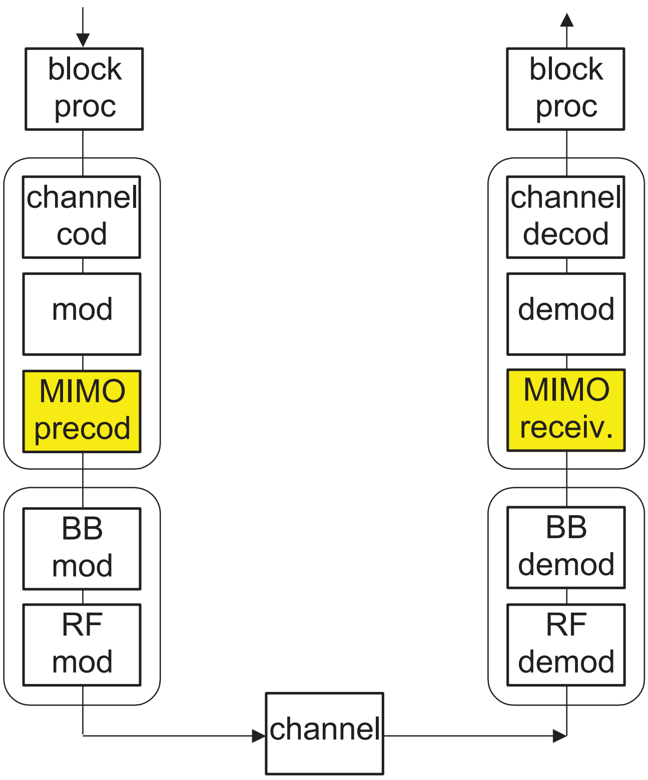


- To **estimate the channel (H)**, **pilots** are sent by the antennas,
  - **separated** in time or frequency (OFDM)
- The receiver calculates the channel matrix **H**

Ideal precoding

- The receiver does the decomposition $\mathbf{H} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^{H}$
- The receiver sends the precoding matrix **U** through the **return channel**

Digital transmission system



Link adaptation

Precoding adaptation

- STBC/SFBC
- Beamforming
- Spatial multiplexing (MIMO mux)
- Codebooks

Precoding adaptation. Complexity

- MIMO: sending matrix **U** is very expensive signaling:
  (matrix of complex numbers)
- Beamforming: sending the vector is also very expensive
  (vector of complex numbers)

- In **adaptive modulation/coding** the signaling is much less
  - The SNR is not sent but the **CQI**: an integer e.g. from 0 to 15 (**4 bits**)
  - The integer indicates a **range of SNR values**: CQI is the "**quantized**" SNR
  - With more CQI bits (smaller quantization), the better the fit ...
    ... but the greater the signaling

- Example. LTE
  - CQI: 4 bits. 15 SNR values spaced $\approx$ 2 dB apart
  - The 15 values correspond to 15 given coding schemes

Precoding adaptation. Complexity reduction

Use of code book

- A **limited set** of data can be defined.
  - a set of beamforming vectors
  - a set of beamforming matrices

- Tx and Rx know that set of vectors and matrices (**codebook**) and...
  ... agree on an **index** for each of their elements:
  PMI *Precoding Matrix Indicator*

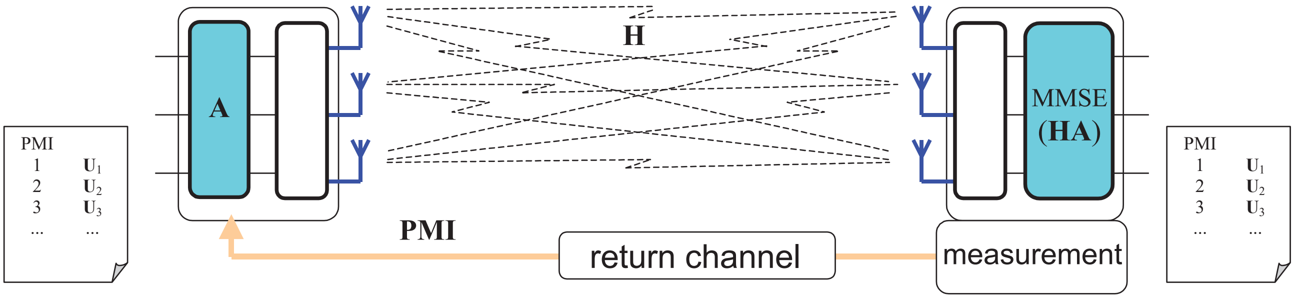
- Based on the measurements, the receiver...
  ... decide the **most suitable** precoding
  ... **from** those **available** in the **code book** and...
  ... **sends its index (PMI)** to the transmitter

PMI	
1	$\mathbf{U}_1$
2	$\mathbf{U}_2$
3	$\mathbf{U}_3$
...	...

- This is equivalent to a **quantization** of the precoding matrices and vectors

Precoding adaptation. Complexity reduction

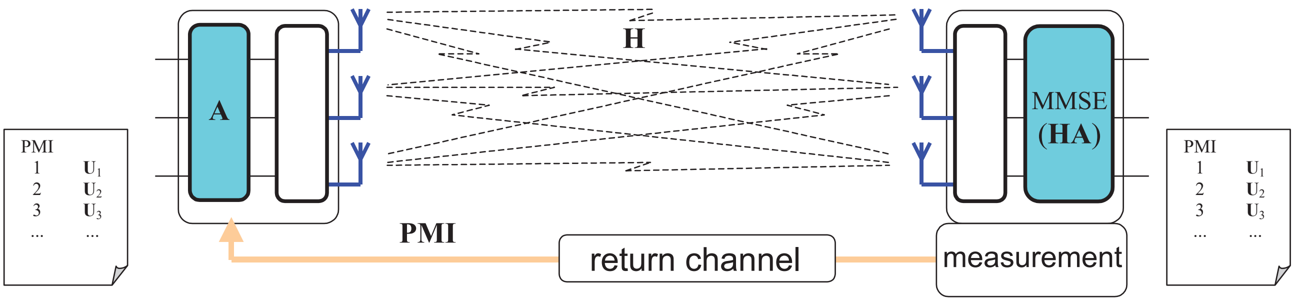
Use of code book



- Equivalent to a **quantization** of the precoding matrices and vectors
- As the ideal precoding is not used, the receiver will be MMSE
- The receiver decides the PMI with which the MMSE reception would be optimal

Precoding adaptation. Complexity reduction

Use of code book

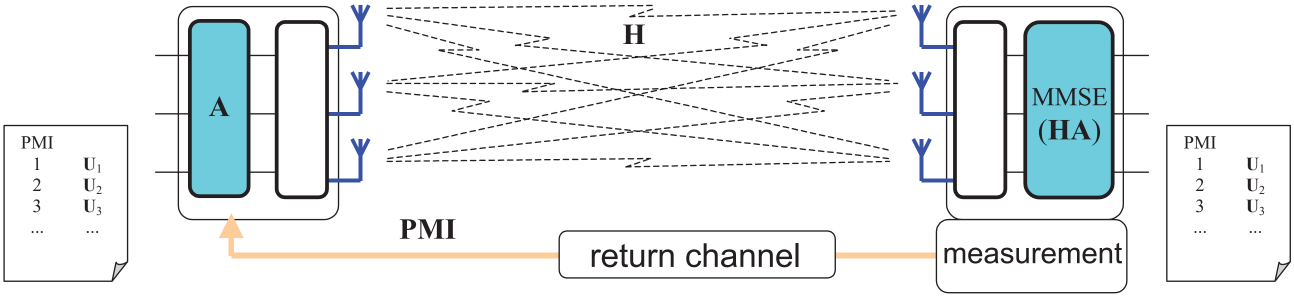


Rank adaptación

- The receiver suggests the <u>number of streams</u> (according to the estimated <u>range</u> of **H**)
- If the **Σ** matrix have very small singular values, it is better to discard them and lower the matrix rank, transmitting fewer streams.
- The receiver sends a **Range Indicator**: **RI**

Precoding adaptation. Complexity reduction

Use of code book



Rank adaptación

- The receiver suggests the <u>number of streams</u> (according to the estimated <u>range</u> of **H**)
- If the **Σ** matrix have very small singular values, it is better to discard them and lower the matrix rank, transmitting fewer streams.
- The receiver sends a **Range Indicator**: **RI**
- Precoding with rank 1 (one stream) is equivalent to beamforming

Precoding adaptation. Use of codebooks

Example: 2 transmitting antennas

Codebook		*streams*	*precodificación*	
RI-PMI	*technique*		*matrix format*	*book elements*
1-0	only one antena	1	1x1	no precod
1-1	STBC/SFBC	1	2x2	el código de *Alamouti*
1-2	Beamforming	1	2x1	vector 2
1-3		1	2x1	vector 3
1-4		1	2x1	vector 4
. . .		1	. . .	. . .
2-1	Multiplexing	2	2x2	matrix 1
2-2		2	2x2	matrix 2
2-3		2	2x2	matrix 3
. . .		2	. . .	. . .

Precoding adaptation. Use of codebooks

Codebook

Example:

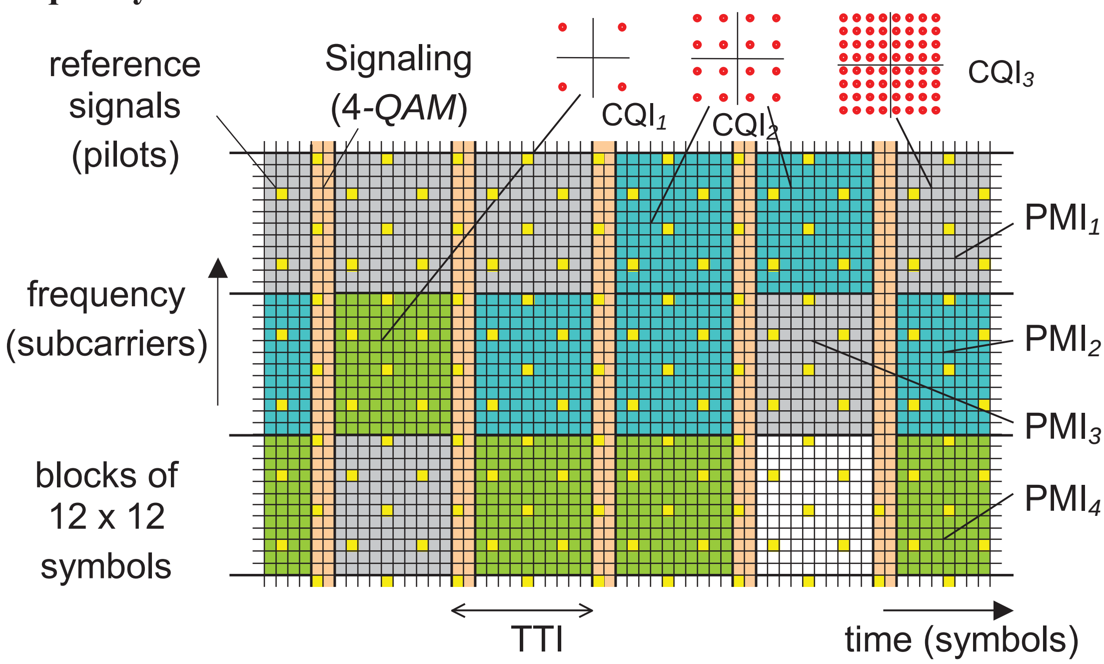
4 transmitting antennas

- Receiver estimates T_c. Adaptation is possible only if TTI << T_c
- If adaptation is not possible: **SFTC/SFBC** is suggested
- If the **adaptation** is possible: a **rank** and a **precoding** vector or matrix are selected

RI-PMI	*technique*	*streams*	*format*	*precod*
1-0	only one antena	1	1x1	no precod
1-1	STBC/SFBC	1	4x4	código STBC
1-2	Beamforming	1	4x1	vector 2
1-3		1	4x1	vector 3
1-4		1	4x1	vector 4
. . .		1	. . .	. . .
2-1	Multiplexing Rank 2	2	4x2	matrix 21
2-2		2	4x2	matrix 22
2-3		2	4x2	matrix 23
. . .		2	. . .	. . .
3-1	Multiplexing Rank 3	3	4x3	matrix 31
3-2		3	4x3	matrix 32
3-3		3	4x3	matrix 33
. . .		3	. . .	. . .
4-1	Multiplexing Rank 4	4	4x4	matrix 41
4-2		4	4x4	matrix 42
4-3		4	4x4	matrix 43
. . .		4	. . .	. . .

Precoding adaptation. Multicarrier - OFDM

Time-frequency variant channel - LTE



- For each new block (TTI) it is possible to change
  - modulations, coding rates and precodings **(PMI, RI)**

Link adaptation

Adaptation of modulation

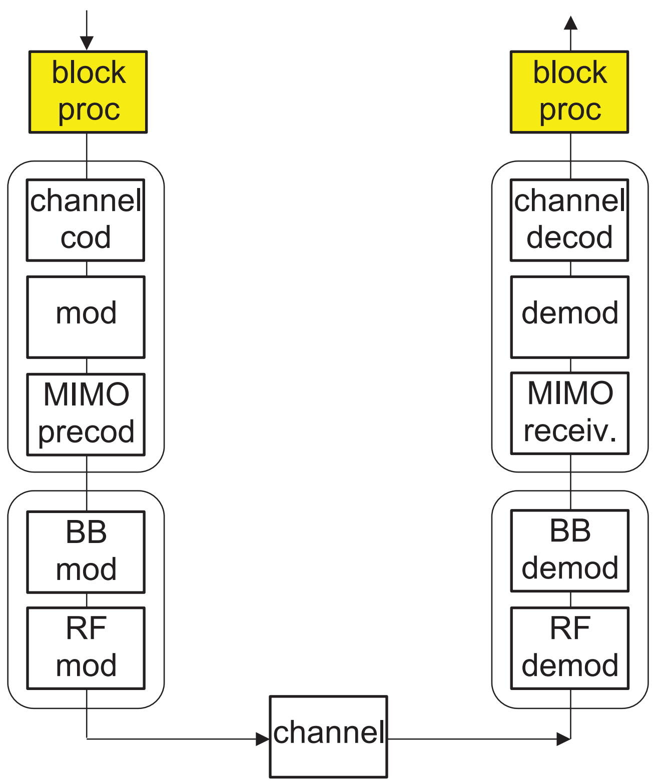
Adaptation of coding rate

Performance of adaptative systems

MIMO Adaptation

Retransmission

Digital transmission system

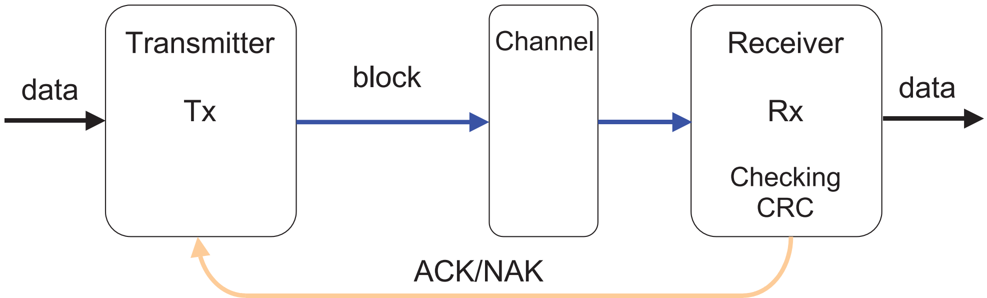


Block transmission with check sequence (CRC)

Retransmission

- ARQ
- H-ARQ

Block transmission: Automatic repeat request (ARQ)



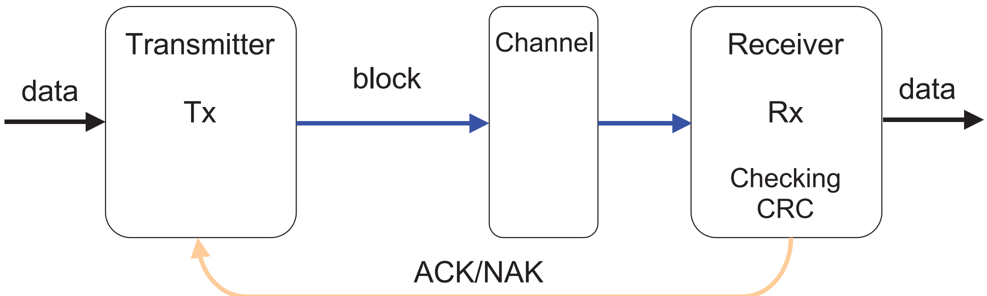
The receiver ...

- can use a check sequence to detect errors in the block and ...
- can **ask** the transmitter to **send** the block **again** (they are numbered)

Automatic repeat request: ARQ *Automatic Repeat Request*

- needs a return channel for **confirmation** signaling
  - NAK: indicates erroneously received block: Retransmission request
  - ACK: indicates block received correctly (Tx can free it from its memory)

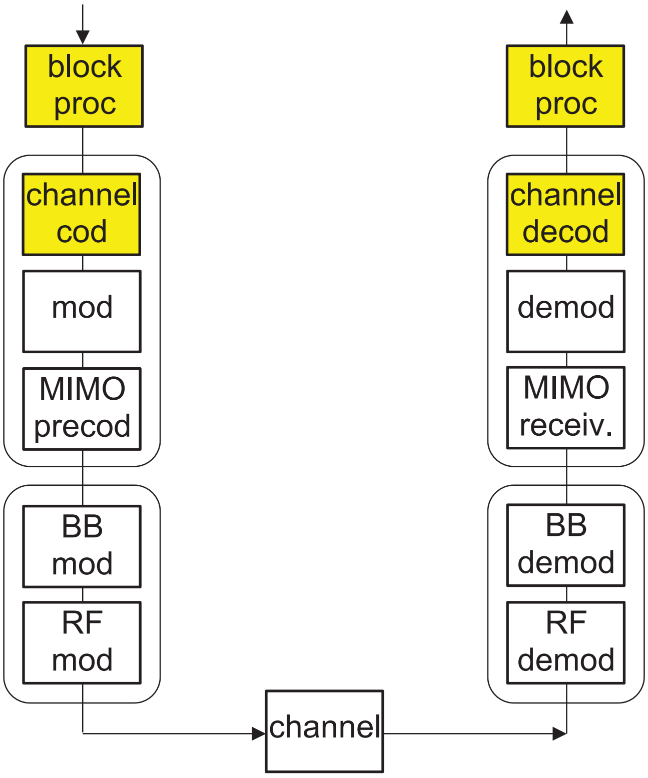
Block transmission: Automatic repeat request (ARQ)



Block error rate (BLER)

- P_B is the probability of receiving an erroneous block
- ARQ can be used up to a maximum of N_{tx} transmissions
- Probability that, finally, the block will not be received correctly: $P_B^{N_{tx}}$
  - Example: $P_B = 10^{-1}$ for $N_{tx} = 2 \rightarrow 10^{-2}$ for $N_{tx} = 3 \rightarrow 10^{-3}$
  - Example: $P_B = 10^{-2}$ for $N_{tx} = 2 \rightarrow 10^{-4}$ for $N_{tx} = 3 \rightarrow 10^{-6}$

Digital transmission system

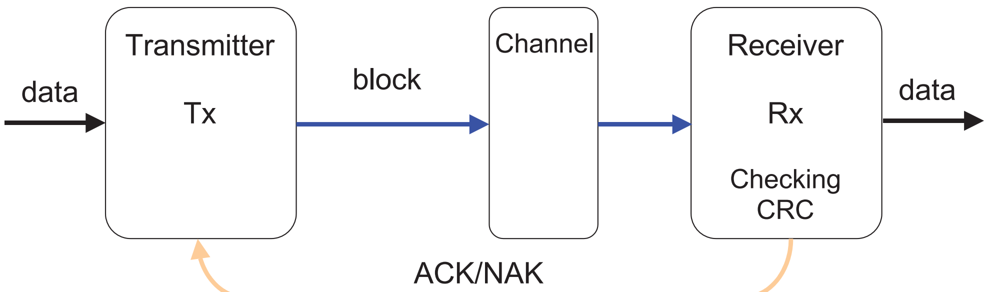


Block transmission with check sequence (CRC)

Retransmission

- ARQ
- H-ARQ

Block transmission: ARQ



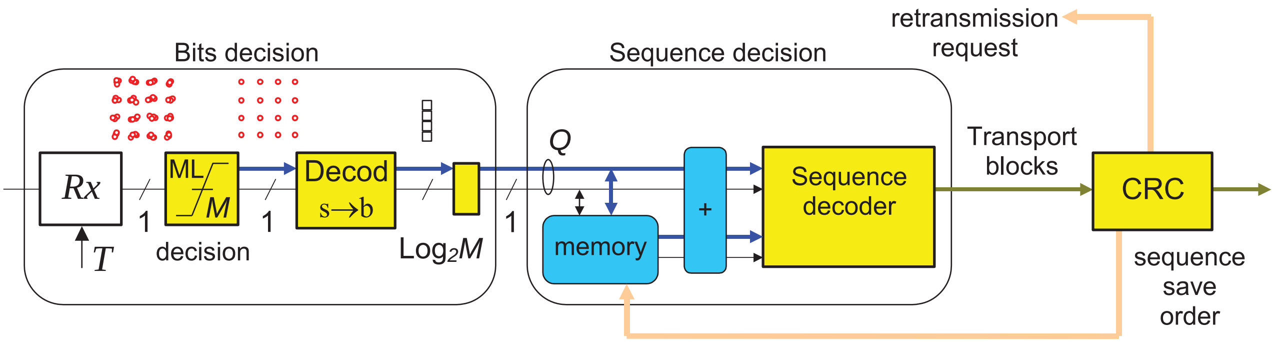
- **ARQ**: when the receiver receives a Retransmission, does not use information from previously received bad blocks

The **more information** you have, the **better decision** you can make

- **Information from previous transmissions** would allow make a **better decision** about which block has been transmitted

¿ How to combine the information received with that of previous transmissions?

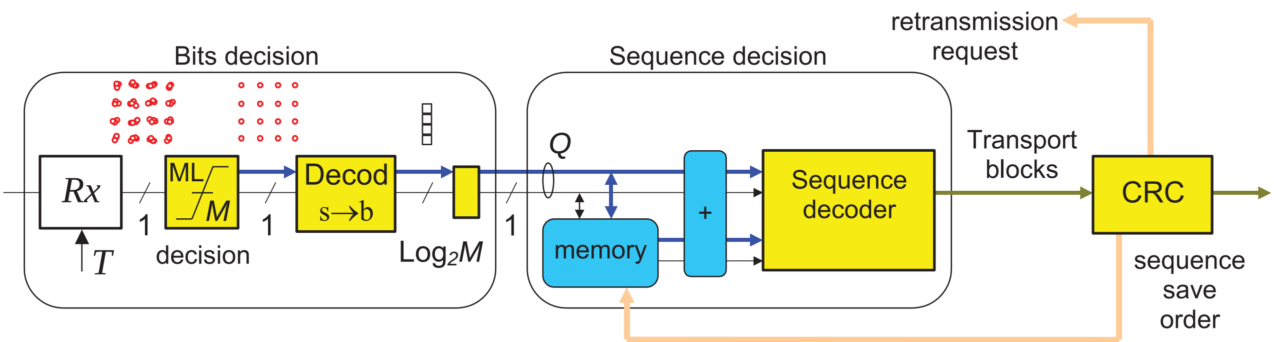
Block transmission: ARQ híbrida (H-ARQ)



The receiver ...

- When it asks for a Retransmission:
  - **saves** the received binary sequence (before the sequence decoder)
- When it receive a Retransmission:
  - **combines** the new sequence received with the previous ones and
  - passes it to the sequence decoder

Block transmission: ARQ híbrida (H-ARQ)



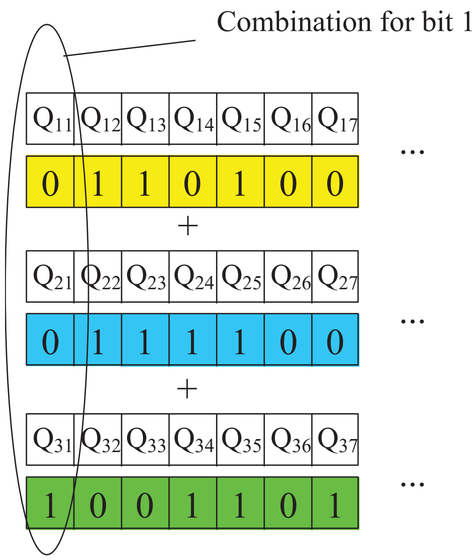
H-ARQ methods:

- **Blocks combining**
  - repetition with soft decision
- **Incremental redundancy**
  - progresive reduction of coding rate
  - (progressive increase of redundancy)

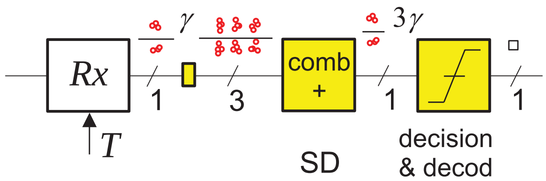
Block transmission: Hybrid ARQ (H-ARQ)

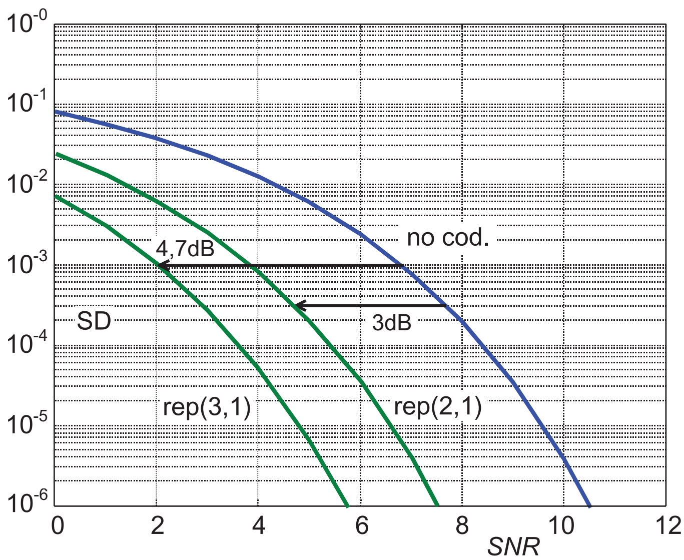
Blocks combining (Chase)

- Binary sequences are saved with soft decisions
- They are added bit by bit weighting the sum with the reliabilities
- For N_t transmissions is similar to use, for **each bit,** an repetition code $(N_t,1)$ with soft decision
- The repetitions are the ones obtained with the retransmissions
- P_b of the N_t bits combination is less than that of each uncombined bit



Channel coding. Soft decision





repetition factor	SNR enhancement
× 2	3 dB
× 3	4,7 dB
× 4	6 dB
× 5	7 dB

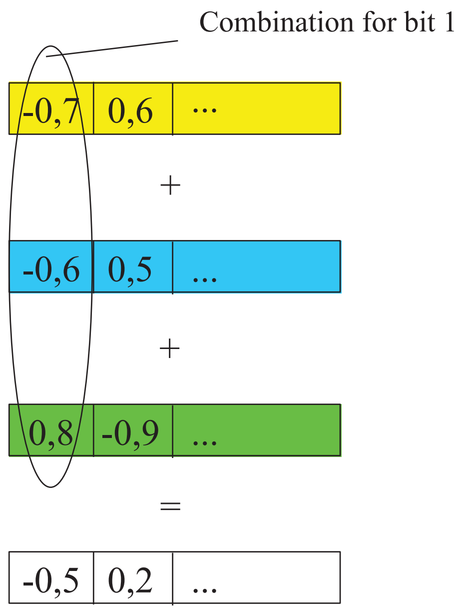
Block transmission: Hybrid ARQ (H-ARQ)

Blocks combining (Chase)

Example PAM-2 {-1, 1}

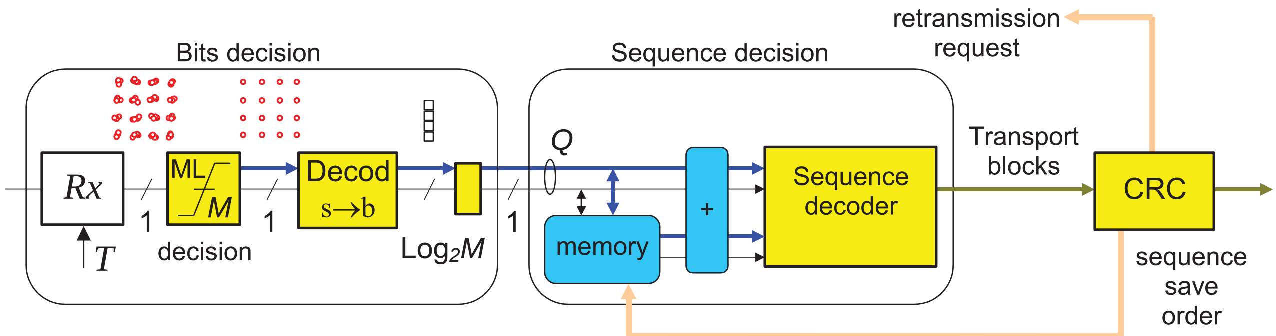
- The symbols are added before the decision
- It is equivalent to adding the energies of the transmitted bits instead of keeping only the last one.
- SNR_{eff} enhance $10\log_{10}(N_t)$

N_t	SNR enhance	BLER
1	0 dB	$P_{B1} = P_B(\gamma)$
2	3 dB	$P_{B2} = P_B(2\gamma) < P_{B1}$
3	4,7 dB	$P_{B3} = P_B(3\gamma) < P_{B2}$
4	6 dB	$P_{B4} = P_B(4\gamma) < P_{B3}$



final BLER: $P_{B1} \cdot P_{B2} \cdot P_{B3}$ ...

Block transmission: Hybrid ARQ (H-ARQ)



H-ARQ métodos:

- **Blocks combining**
  - repetition with soft decision
- **Incremental redundancy**
  - progresive reduction of coding rate
  - (progressive increase of redundancy)

Block transmission: Hybrid ARQ (H-ARQ)

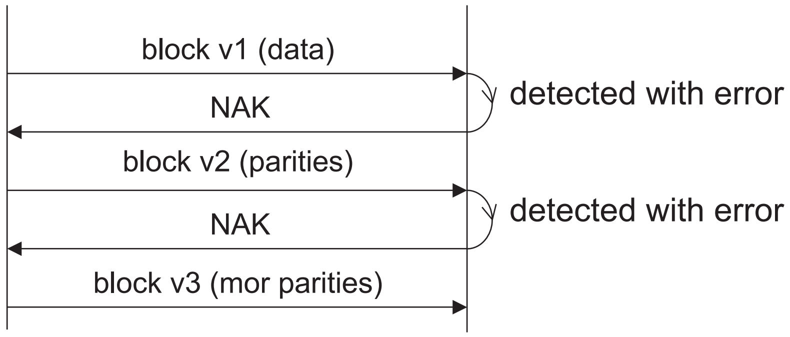
Incremental redundancy

- Convolutional coding is used whith **adjustable rate** (i.e.: with puncturing)
- First transmission
  - **high coding rate**
  - **little redundancy**
  - almost all parity bits punctured
- Successive transmissions (if NAK)
  - (instead of repeating the same)
  - **new information** is sent: bits that had been punctured
  - The receiver has more and more parity bits (greater redundancy)

Block transmission: Hybrid ARQ (H-ARQ)

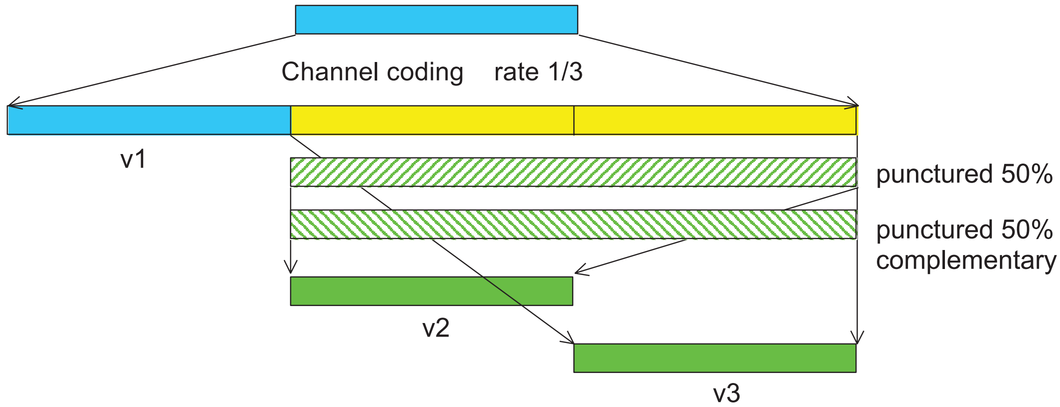
Incremental redundancy

- Convolutional coding whith adjustable rate (i.e.: with puncturing)
- First transmission: high coding rate, almost all parity bits punctured
- Successive transmissions: **new information**: bits that had been punctured
- The receiver has more and more parity bits



Block transmission: Hybrid ARQ (H-ARQ)

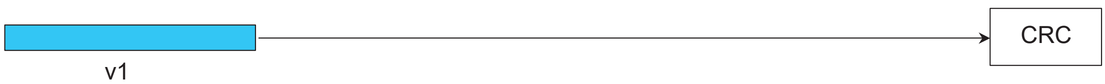
Incremental redundancy. Example from code with rate 1/3



- Two complementary puncturing versions are generated v2 y v3
- v1 is the message without redundancy
- Concatenating v1 and v2 we have a punctured code of rate 1/2
- Combining v2 and v3 we have all the parity bits, ...
  ... which together with v1 form a code of rate 1/3

Block transmission: Hybrid ARQ (H-ARQ)

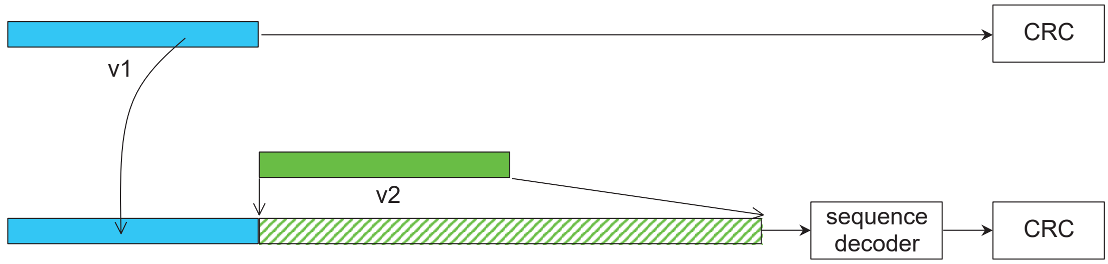
Incremental redundancy. Example from code with rate 1/3



- Transmitter sends v1 (message without redundancy)
- Receiver checks CRC → If error, ask for Retransmission
  (and saves v1)

Block transmission: Hybrid ARQ (H-ARQ)

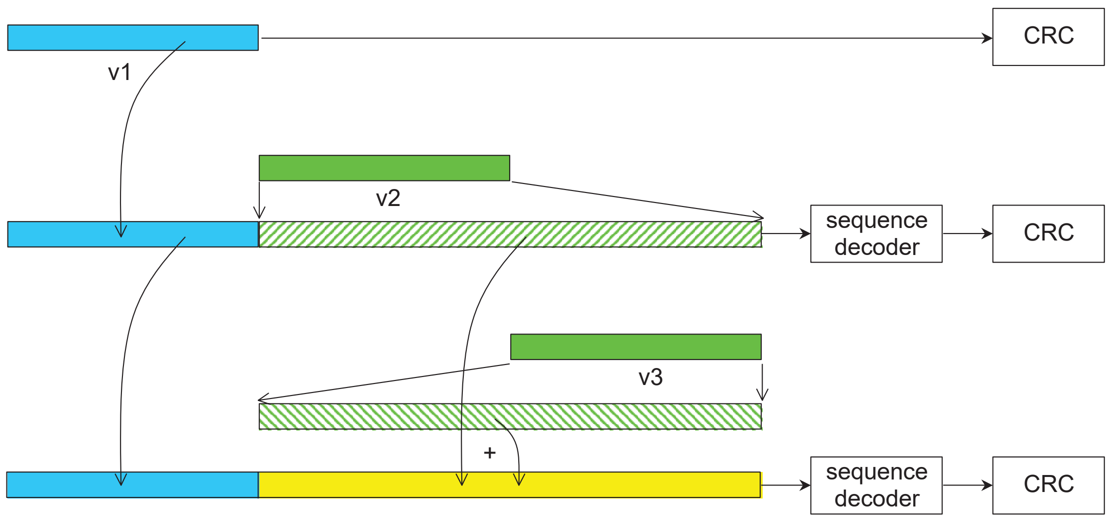
Incremental redundancy. Example from code with rate 1/3



- Transmitter sends v2
- Receiver decodes the combined sequence [v1, v2] (rate 1/2)
- Receiver checks CRC → If error, ask for Retransmission (and saves v1 and v2)

Block transmission: Hybrid ARQ (H-ARQ)

Incremental redundancy. Example from code with rate 1/3



- Transmitter sends v3
- Receiver decodes the combined sequence [v1, v2, v3] (rate 1/3)

Block transmission: Hybrid ARQ (H-ARQ)

ARQ

- In each new Retransmission, the same information is always sent
- The probability of detecting error (CRC) is the same in all retransmissions

Incremental redundancy

- With each new Retransmission, the receiver has encoded sequences of increasing rate:
  - 1st time: 1/1 (no redundancy)
  - 2nd time: 1/2
  - 3rd time: 1/3
- The probability of detecting CRC error is lower with each new Retransmission

Link adaptation

Adaptation of modulation

Adaptation of coding rate

Performance of adaptative systems

MIMO Adaptation

Retransmission

Universidad de Málaga. Escuela Técnica Superior de Ingeniería de Telecomunicación
Departamento de Ingeniería de Comunicaciones

Master en Ingeniería de Telecomunicación

Ejercicios de *Técnicas de tratamiento de señal y comunicaciones*

Tema 6

José Tomás Entrambasaguas

Ed. 3   15-6-2015

6.1

Fill in the following table with the values of the SNRgap (in dB) of the QAM transmission through a gaussian channel, for various values of the bit error probability [use the given graphs and the approximate formula]

SNRgap (dB)	$P_b=10^{-3}$	$P_b=10^{-4}$	$P_b=10^{-5}$
Graphics			
Approximate formula			

6.2

Using the result of exercise 6.1, fill in the following table with the SNR thresholds (in dB) for various constellations and target error probabilities.

	4-QAM	16-QAM	32-QAM	64-QAM	128-QAM	256-QAM	512-QAM
$P_b=10^{-3}$							
$P_b=10^{-4}$							
$P_b=10^{-5}$							

6.3

A single carrier transmission system uses M-QAM adaptive modulation with 0, 2, 4 and 6 bits/symbol and a speed of 1 Mbaud. If the SNR measured by the receiver is in the same proportion within each of the four regions (25% of the time in each CQI), what is the average transmission bit rate?

6.4

An ADSL system sends OFDM symbols at 4 kbaud (4000 OFDM symbols per second). With each OFDM symbol, 224 QAM symbols are sent on carriers 32 to 255. Symbols on each carrier frequency are of a different constellation depending on the SNR at this carrier frequency.

In a particular case, the constellation assignment to the carriers indicated in the following table has been decided. The table includes the number of bits per symbol of each constellation, the initial and final indices of the carriers to which that number of bits is assigned, and the number of carriers in each group.

Assignment of bits to carriers (*bit-loading*)

carrier indices	32-56	57-81	82-106	107-131	132-151	152-171	172-191	192-211	212-221	222-231	232-241	242-251	252-255
number of carriers	25	25	25	25	20	20	20	20	10	10	10	10	4
bits/symb	12	11	10	9	8	7	6	5	4	3	2	1	0

a) Calculate the total number of bits that are sent in an OFDM symbol

b) Calculate the resulting bit rate (in bits per second)

6.5 For the ADSL system of the previous exercise, fill in the following table with the SNR thresholds that a carrier must have in order to assign each of the constellations from 1 to 12 bits per symbol, for a target probability of error. $P_b=10^{-4}$

bits/symb	12	11	10	9	8	7	6	5	4	3	2	1
threshold SNR (dB)												

6.6

Calculate the average bit rate of a QAM adaptive modulation system (4, 16 and 64-QAM) over a Rayleigh channel, for SNR average 20 dB and target $P_b = 10^{-3}$.

6.7

A transmission system uses 4-QAM modulation without channel coding and a H-ARQ procedure by combination. Blocks of 200-bit are sent that include a CRC. When the receiver detects an error in a block, it saves the sequence of symbols (without demodulating) and sends a retransmission request. With each new retransmission, the receiver adds the new symbols received with those saved before demodulating. The channel is Gaussian with SNR = 10 dB.

a) Calculate the BER at the output of the demodulator when the block is first received.

b) Calculate the BLER (Block Error Rate) when the block is first received. [=200×BER since there is no channel coding].

c) Calculate the BLER after receiving the block for the second time and combining it with the first.

6.8

A transmission system uses an H-ARQ procedure by incremental redundancy to send 200-bit messages. It uses three encoded versions of the message that, sent successively and aggregated at the receiver, form encoded sequences with rates 1/1, 1/2 and 1/3 successively. The SNR of the channel is such that the block error rates at the output of the sequence decoder are $p_1 = 2\times10^{-1}$, $p_2 = 2\times10^{-2}$, $p_3 = 2\times10^{-3}$, respectively, for each of the three encoding rates.

a) Calculate the probability that, after using the three versions, the block will be erroneous.

b) For each message, a different number of transmissions will be necessary depending on the probability of receiving the blocks with errors. Calculate the expected value of the number of transmissions for a message.

c) Calculate the expected value of the number of bits to be transmitted for each message (message length × expected value of transmissions).

d) Calculate the mean coding rate (number of bits in the message=200) / (expected number of bits sent for a message)

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Ed. 2   15-6-2017

6.1

SNRgap	P_b=10 $^{-3}$	P_b=10 $^{-4}$	P_b=10 $^{-5}$
Formula	3,31	4,75	6,19
Formula (dB)	5,2	6,8	7,9
Graphics (dB)	5,5	6,6	8,3

6.2

$$10\log_{10}\left((M-1)\cdot\Gamma(P_b)\right)$$

	$\Gamma(P_b)$	4-QAM	16-QAM	32-QAM	64-QAM	128-QAM	256-QAM	512-QAM
P_b=10 $^{-3}$	3,31	10,0	17,0	20,1	23,21	26,2	29,3	32,3
P_b=10 $^{-4}$	4,75	11,5	18,5	21,7	27,8	27,8	30,8	33,9
P_b=10 $^{-5}$	6,19	12,7	19,7	22,8	25,9	29,0	32,0	35,0

6.3

1 Mbaud × (0+2+4+6)/4 bits/symb = 3 Mbit/s

6.4

a) 1670
b) 6,68 Mbit/s

6.5

$$10\log_{10}\left((M-1)\cdot\Gamma(P_b)\right)$$

bits/symb	12	11	10	9	8	7	6	5	4	3	2	1
theshold SNR (dB)	42.9	39.9	36.9	33.9	30.8	27.8	24.8	21.7	18.5	15.2	11.5	6.8

6.6

3.28 bits/symbol

6.7

Use $P_b \approx \frac{1}{5}\exp\left(\frac{-1{,}6\gamma}{M-1}\right)$ with $\gamma = 10$

a) BER = P_b = $9{,}7\times10^{-4}$

b) BLER = 0,19

c) The same but with γ=20: BER = $4{,}7\ 10^{-6}$ BLER = $9{,}3\times10^{-4}$

6.8

a) p1×p2×p3 = 8×10^{-6}

b)

The first is always sent: 200
The probability of sending the second is p1
The probability of sending the third is p1×p2

Expected value of the total number of bits sent:

200 + 200×p1 +200×p1×p2 = 240,8

c) 200/240,8 = 0.83