On Intersections of Ideals of Leavitt Path Algebras

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During the 2015 CIMPA Research School in Turkey on "Leavitt path algebras and graph C^*-algebras", Astrid an Huef raised the question whether the statement: *For a given graph E, every (closed) ideal I of $C^*(E)$ is the intersection of all the primitive/prime ideals containing I.*

is true for ideals of Leavitt path algebras.

We first construct examples showing that this statement does not hold in general for Leavitt path algebras, and then prove that, every ideal of the Leavitt path algebra is an intersection of primitive/prime ideals if and only if the graph E satisfies Condition (K).

We examine the uniqueness of factorizing a graded ideal as a product of prime ideals. If I is a graded ideal and $I = P_1 \cdots P_n$ is a factorization of I as an irredundant product of prime ideals P_i, then necessarily all the ideals P_i must be graded ideals and $I = P_1 \cap \ldots \cap P_n$. We get a weaker version of this result for non-graded ideals.

Finally, powers of an ideal I are studied. While $I^2 = I$ for any graded ideal I, for a non-graded ideal I, all I^n are non-graded and distinct, but $\bigcap_{n=1}^{\infty} I^n$ is a graded ideal which is the largest graded ideal contained in I. Hence, $\bigcap_{n=1}^{\infty} I^n = 0$ if and only if I contains no vertices.

(This is joint work with S. Esin and K.M. Rangaswamy)