A data-based approach for solving the Rank Pricing Problem

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Outline

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Rank Pricing Problem

Assumptions

- Set of products, $\mathcal{I} = \{1, \ldots, I\}$.
- Set of customers, $\mathcal{K} = \{1, \ldots, K\}$.
- Budgets, b^k, $\forall k \in \mathcal{K}$.
- Preference values, s_i^k, $\forall k \in \mathcal{K}, \forall i \in \mathcal{I}$.
- Customer preferences, $s_{i_1}^k > s_{i_2}^k \Rightarrow$ customer k prefers to buy product i_1 rather than product i_2.

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- **Objective:** determine the **optimal prices** for the products so that the **benefit** of the company and the **satisfaction** of the customers is **maximized** keeping in mind the **budget constraints**.

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- **Mixed-Integer Bilevel Optimization Problem.**

Rank Pricing Problem

Upper level

$$\begin{cases} \max\limits_{\boldsymbol{p}} \sum\limits_{k \in \mathcal{K}} \sum\limits_{i \in \mathcal{I}} p_i x_i^k & \text{(benefit)} \\ \text{s.t. } p_i \geq 0,\ i \in \mathcal{I} & \text{(non-negativity)} \end{cases}$$

Lower level, for each customer k

$$\begin{cases} \max\limits_{\boldsymbol{x^k}} \sum\limits_{i \in \mathcal{I}} s_i^k x_i^k & \text{(satisfaction)} \\ \text{s.t. } \sum\limits_{i \in \mathcal{I}} x_i^k \leq 1 & \text{(one product at most)} \\ \qquad \sum\limits_{i \in \mathcal{I}} p_i x_i^k \leq b^k & \text{(affordability)} \\ \qquad x_i^k \in \{0,1\},\ i \in \mathcal{I} & \text{(binary)} \end{cases}$$

Rank Pricing Problem

- NP-complete, *Rusmevichientong et al. [2006]*.
- Exact approaches, *Bucarey et al. [2021]; Calvete et al. [2019]; Domínguez [2021]*. Valid inequalities and Benders decomposition.
- Outstanding results but sometimes difficult and time-consuming.

- VNS-heuristic.
- No optimality guarantees but yields good results in a reasonable time.

Outline

Why VNS?

- Neighborhood structure + discrete nature RPP.
- Prices are continuous but optimal solution attained in budgets, *Domínguez [2021]*.
- Explore neighborhoods where closer points may not have similar objective values.

Illustrative example

k	s_1^k	s_2^k	b^k
1	2	1	18
2	2	1	66
3	1	2	27
4	1	2	34
5	1	2	66
6	2	1	50
7	2	1	42
8	1	2	42

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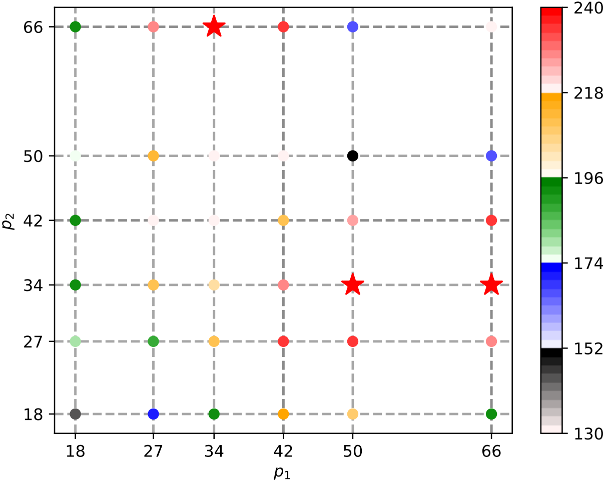

Neighborhood definition

Assume given $\boldsymbol{p} = (b^{m_1}, \ldots, b^{m_I})$, we defined $N_r(\boldsymbol{p})$ as follows:

$$N_r(\boldsymbol{p}) = \left\{(b^{n_1}, \ldots, b^{n_I}) : n_i = m_i - r, \ldots, m_i - 1,\, m_i,\, m_i + 1, \ldots, m_i + r, \forall i \in \mathcal{I}\right\}.$$

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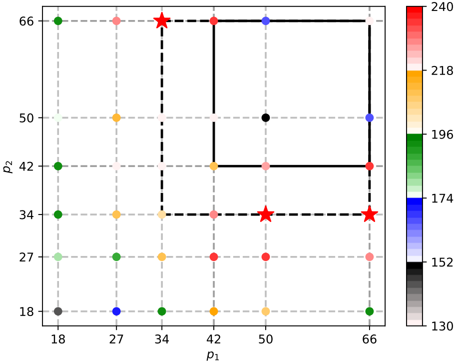
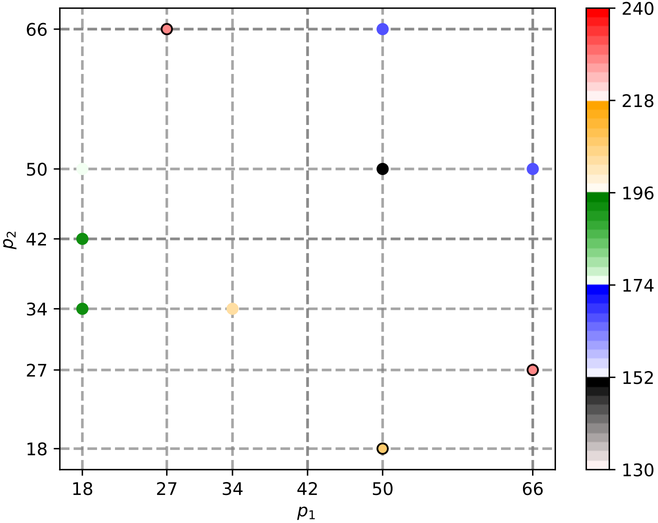


VNS Algorithm



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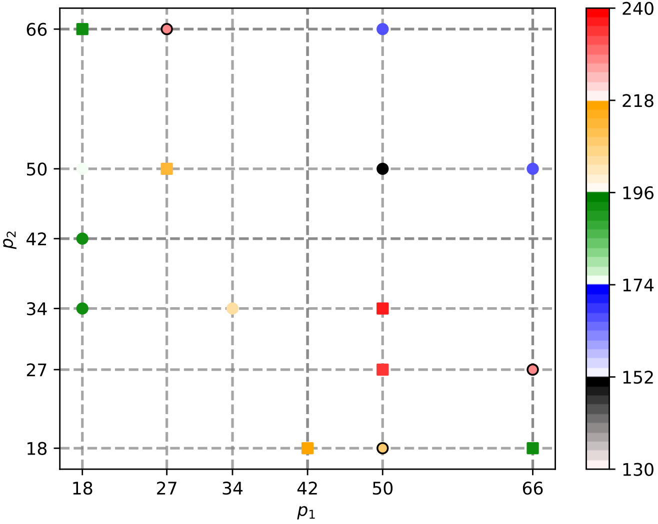
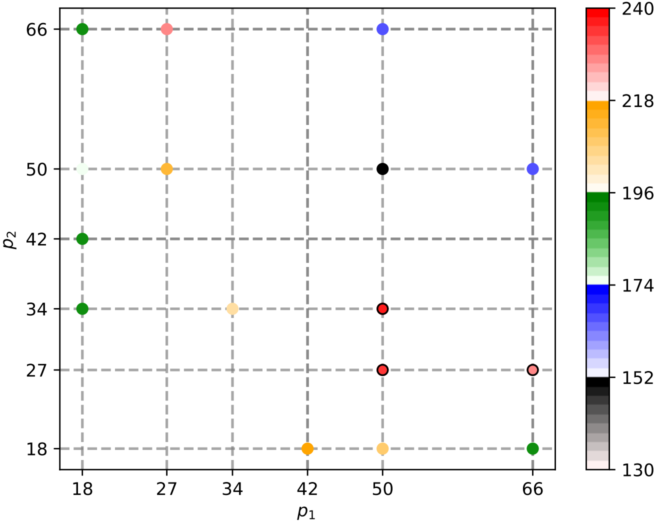


VNS Algorithm



Outline

Setup

- $(K, I) \in \{(30, 5), (30, 25), (60, 50)\}$.
- The data can be downloaded from *OASYS [2024]*.
- Gurobi 10.0.3 (default parameters).
- 1000 runs to get fair conclusions.

Comparative approaches

Exact approach

$$
\begin{cases}
\max\limits_{\boldsymbol{v},\,\boldsymbol{x}} \sum\limits_{k\in\mathcal{K}} \sum\limits_{i\in\mathcal{I}} \left(\sum\limits_{m\in\mathcal{M}} b^m v_i^m \right) x_i^k \\
\text{s.t.} \quad \sum\limits_{m\in M} v_i^m \leq 1, \forall i \in I \\
\qquad \sum\limits_{i\in I} x_i^k \leq 1, \forall k \in K \\
\qquad x_i^k \leq \sum\limits_{m\in M} v_i^m, \forall k \in K, i \in I. \\
\qquad \sum\limits_{j\in I} s_j^k x_j^k \geq s_i^k \sum\limits_{m\in M} v_i^m, \forall k \in K, i \in I \\
\qquad v_i^m, x_i^k \in \{0,1\}, \forall k \in K, i \in I, m \in M.
\end{cases}
$$

- Single-level reformulation of RPP, *[Domínguez, 2021]*.
- Solved with Gurobi.
- Time limit: 600 secs.

Comparative approaches

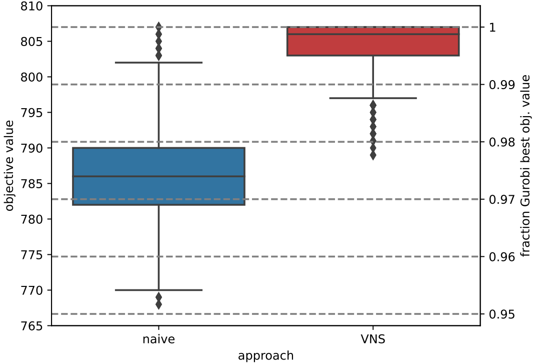
Naive approach

While a stopping criterion is not reached:

1) Randomly generate price vectors.
2) Solve the lower-level problems.
3) Evaluate the upper-level function.

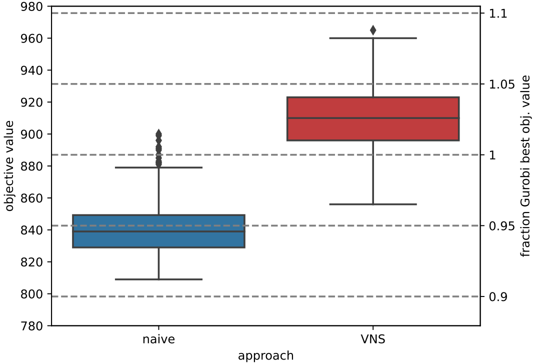
Output: keep the vector with the best objective value.

Instance $(K, I) = (30, 5)$



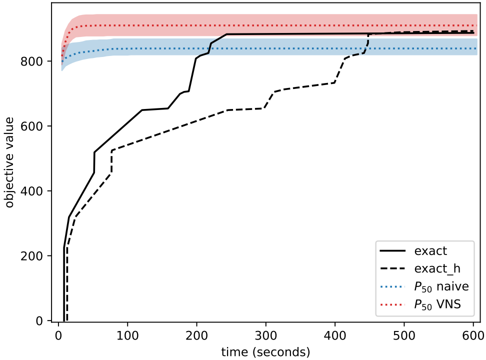
- Stopping criterion: 24000 points.
- Total solutions: $B^I = 27^5 \approx 10^7$.
- Gurobi solves exact in less than 10 seconds.
- VNS: $> 75\%$ runs, objectives > 0.99 times the optimum.

Instance $(K, I) = (30, 25)$



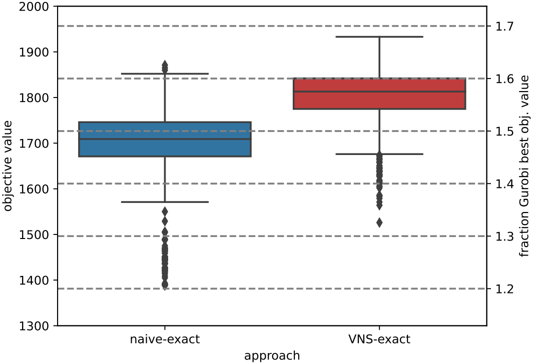
- Stopping criterion: 24000 points.
- Total solutions: $B^I = 23^{25} \approx 10^{34}$.
- Gurobi value in 600 seconds is 887.
- VNS: $> 75\%$ runs, objectives greater than the optimum.

Instance $(K, I) = (30, 25)$



- Evolution in time.
- Exact_h (Gurobi with `Heuristics` $= 1$).
- Range (P_5, P_{95}).
- VNS: better along the time.

Instance $(K, I) = (60, 50)$



- Take solution after 100 seconds.
- Warm-start Gurobi with default parameters in 600 secs.
- Both approaches better than Gurobi.
- VNS: best among the comparative algorithms.

More details

A Variable Neighborhood Search approach for solving the Rank Pricing Problem

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[c] *Department of Electric Engineering, Universidad de Málaga, Málaga, Spain*

Available at:

A. Jiménez-Cordero, S. Pineda, and J. M. Morales, A Variable Neihborhood Search approach for solving the Rank Pricing Problem, Submitted. Link: `https://www.researchgate.net/publication/380895098_A_Variable_Neighborhood_Search_approach_for_solving_the_Rank_Pricing_Problem`.

Outline

Conclusions

- RPP is a challenging combinatorial bilevel problem.
- Develop a VNS-based heuristic for solving it.
- Conveniently define neighborhoods for escaping from local optima.
- Competitive with the comparative algorithms.
- Tested on several instances.

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- RPP is a challenging combinatorial bilevel problem.
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- Competitive with the comparative algorithms.
- Tested on several instances.

Further research

- Combine VNS with other optimization methods.
- Handle more complex bilevel models.
- Integrate some machine learning strategies.

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Thank you very much for your attention!

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