

This master's thesis investigates on different aspects of the LFMs. Firstly, a classification of the flexibility products that could be traded in those novel distribution markets is done. Secondly, a state of the art is provided in terms of how other authors model those markets, to then propose a simplified but elegant optimization model of the marketplace. Furthermore, centralized and decentralized versions of the problem are presented, providing two different simplistic illustrative examples and a far more realistic use case. For the purpose of this thesis, only congestion and imbalance scenarios are studied. Those simulations will demonstrate the feasibility of the proposed approach to solve issues at distribution level, both in a centralized and decentralized fashion.



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He is an Industrial Engineer with Mecanical and Electrical specialities, born in Castellar, a little village in the north of Jaén. He ends their master's degree in 2021 with the present thesis, in the School of Industrial Engineering of the Univerisity of Málaga. This project complements their knowledge about optimization techniques and electrical markets, opening a line of research for his future PhD.

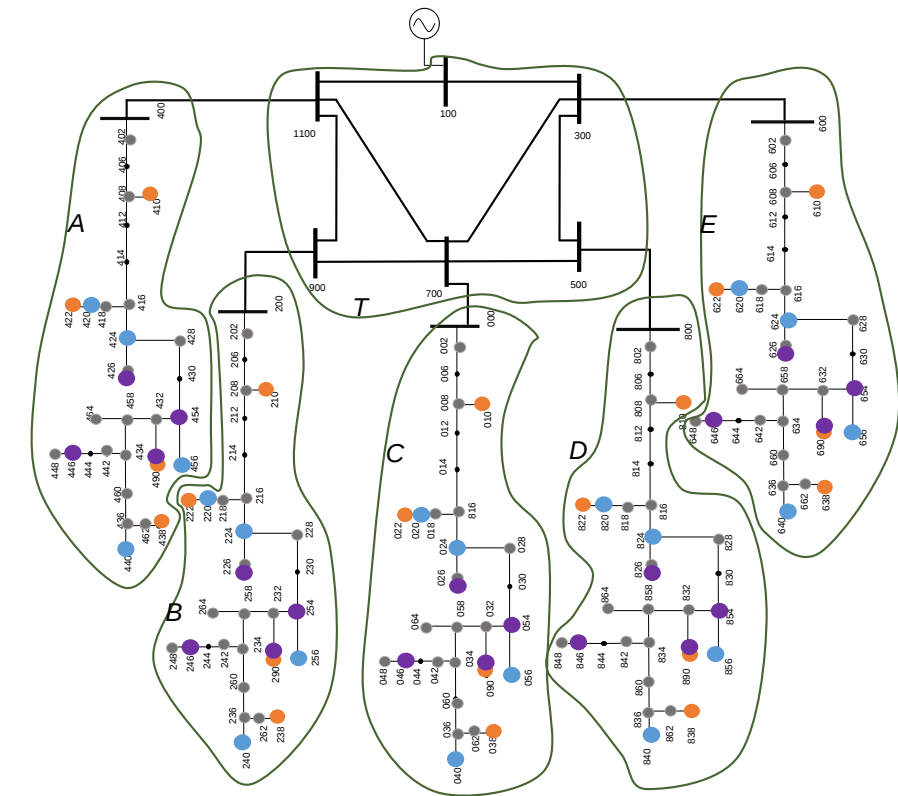
FINAL MASTER'S THESIS

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LOCAL FLEXIBILITY MARKETS: A COORDINATE AND DECENTRALIZED APPROACH FOR ANCILLARY SERVICES

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**MERCADOS LOCALES DE FLEXIBILIDAD: UNA APROXIMACIÓN
COORDINADA Y DESCENTRALIZADA PARA SERVICIOS AUXILIARES**

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Abstract

Local Flexibility Markets (LFMs) are defined as a type of marketplace where electricity products are traded between agents with the intention to solve specific problems in the distribution networks. Agents included in these emerging markets are Distribution System Operator (DSO), Balance Responsible Parties (BRPs), Aggregators (Aggs) and prosumers, who are the final providers of flexibility products.

In the literature, this problem is solved considering only energy products provided by end-users and with complex mathematical formulations, making the problem, not only non-linear but also not convex. Non-linearity and non-convexity characteristics of a problem make difficult finding the global optimal solution and to interpretate the dual version of the problem. In terms of the products included in the market, the adequacy of the introduction of both energy and capacity product is an interesting topic to be investigated as it increases market liquidity. Furthermore, a simplification of the market formulation will also be an intriguing aspect to be explored.

Besides of that, due to the inherent decentralized nature of the distribution networks, it is highly interesting to investigate the implications that a decentralized formulation of the problem would have. This topic is in currently growing interest in the literature, where different formulations of the LFM are addressed from this decentralized perspective. In this matter, although several techniques could be used to obtain a decentralized solution of the problem, Alternating Direction Method of Multipliers (ADMM) stands out as a promising candidate due to its fast-convergence properties.

Thus, this master's thesis investigates on different aspects of the LFMs. Firstly, a classification of the flexibility products that could be traded in those novel distribution markets is done. Secondly, a state of the art is provided in terms of how other authors model those markets, to then propose a simplified but elegant optimization model of the marketplace. Furthermore, centralized and decentralized versions of the problem are presented, providing two different simplistic illustrative examples and a far more realistic use case. For the purpose of this thesis, only congestion and imbalance scenarios are studied. Those simulations will demonstrate the feasibility of the proposed approach to solve issues at distribution level, both in a centralized and decentralized fashion

Keywords

Local Flexibility Markets, flexibility products, Alternating Direction Method of Multipliers, congestion, imbalance, distribution networks.

Resumen

Los mercados locales de flexibilidad se definen como un tipo de mercado donde se comercia entre los agentes participantes con productos eléctricos con la intención de resolver problemas específicos en las redes de distribución. Los agentes incluidos en este tipo de mercados emergentes son el distribuidor (DSO), los agentes responsables del equilibrio o “Balance Responsible Parties (BRPs)”, agregadores (Aggs) y los prosumers, quienes son los proveedores finales de los productos de flexibilidad.

En la literatura, este problema se ha resuelto considerando solamente productos de energía proveídos por los usuarios finales con formulaciones matemáticas complejas que hacen que el problema, no solo no-lineal si no también no convexo. No linealidad y no convexidad son características que hacen difícil encontrar la solución óptima del problema e interpretar la versión dual del mismo. En términos de los productos incluidos en el mercado, la adecuación de la introducción de productos tanto de capacidad como de energía es un tema interesante para ser investigado ya que incrementa la liquidez del mercado. Además, una simplificación de la formulación del mercado es también un aspecto intrigante a ser explorado.

Además de eso, debido a la inherente naturaleza descentralizada de las redes de distribución, es altamente interesante investigar las implicaciones que una formulación descentralizada del problema tendría. Este tema es de creciente interés en la literatura, donde diferentes formulaciones de los LFM son investigadas desde una perspectiva descentralizada. En este aspecto, aunque se pueden utilizar varias técnicas para obtener una solución descentralizada del problema, el método Alternating Direction Method of Multipliers (ADMM) sobresale como un candidato prometedor debido a sus propiedades de rápida convergencia.

Por lo tanto, este Trabajo Final de Máster investiga diferentes aspectos de los mercados locales de flexibilidad. Primeramente, se realiza una clasificación de los productos de flexibilidad que pueden ser comerciados en estos novedosos mercados para, después, proponer una versión simplificada, pero elegante del problema, proporcionando dos ejemplos ilustrativos simples y diferentes y un mucho más realista caso de uso. A efectos de este TFM, solamente serán estudiados escenarios de congestión y desequilibrio. Estas simulaciones demostrarán la viabilidad del enfoque propuesto para resolver problemas a nivel de distribución, tanto desde una perspectiva centralizada como descentralizada.

Palabras clave

Mercados locales de flexibilidad, productos de flexibilidad, Alternating Direction Method of Multipliers, congestión, desequilibrio, redes de distribución.

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“A large part of mathematics which becomes useful developed with absolutely no desire to be useful, and in a situation where nobody could possibly know in what area it would become useful; and there were no general indications that it ever would be so.”

- John Von Neuman, 1954

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(2.10).....	14	(3.7).....	29	(3.49).....	48
(2.11).....	14	(3.8).....	30	(3.50).....	48
(2.12).....	14	(3.9).....	30	(3.51).....	48
(2.13).....	14	(3.10).....	30	(3.52).....	48
(2.14).....	14	(3.11).....	30	(3.53).....	48
(2.15).....	14	(3.12).....	30	(3.54).....	48
(2.16).....	15	(3.13).....	30	(3.55).....	48
(2.17).....	15	(3.14).....	30	(3.56).....	48
(2.18).....	15	(3.15).....	31	(3.57).....	48
(2.19).....	15	(3.16).....	31	(3.58).....	48
(2.20).....	15	(3.17).....	31	(3.59).....	48
(2.21).....	15	(3.18).....	31	(3.60).....	48
(2.22).....	16	(3.19).....	31	(3.61).....	48
(2.23).....	16	(3.20).....	31	(3.62).....	48
(2.24).....	16	(3.21).....	31	(3.63).....	54
(2.25).....	16	(3.22).....	31	(3.64).....	54
(2.26).....	16	(3.23).....	31	(3.65).....	54
(2.27).....	16	(3.24).....	32	(4.1).....	71
(2.28).....	16	(3.25).....	32	(4.2).....	71
(2.29).....	16	(3.26).....	32	(4.3).....	72
(2.30).....	17	(3.27).....	32	(4.4).....	72
(2.31).....	17	(3.28).....	32	(4.5).....	73
(2.32).....	17	(3.29).....	32	(4.6).....	73
(2.33).....	17	(3.30).....	32	(4.7).....	73
(2.34).....	17	(3.31).....	32	(4.8).....	73
(2.35).....	17	(3.32).....	32	(4.9).....	73
(2.36).....	18	(3.33).....	32	(4.10).....	73
(2.37).....	18	(3.34).....	33	(4.11).....	73
(2.38).....	18	(3.35).....	33	(4.12).....	74
		(3.36).....	33	(4.13).....	74
		(3.37).....	33	(4.14).....	74
		(3.38).....	34	(4.15).....	74
		(3.39).....	34	(4.16).....	74

(4.17).....	74	(4.59)	83	(II.15)	114
(4.18).....	75	(4.60)	83	(II.16)	114
(4.19).....	75	(4.61)	83	(II.17)	114
(4.20).....	75	(4.62)	83	(II.18)	114
(4.21).....	76	(4.63)	83	(II.19)	115
(4.22).....	76	(4.64)	83	(II.20)	115
(4.23).....	76	(4.65)	84	(II.21)	115
(4.24).....	76	(4.66)	84	(II.22)	115
(4.25).....	76	(4.67)	85	(II.23)	115
(4.26).....	76	(4.68)	85	(II.24)	115
(4.27).....	76	(4.69)	85		
(4.28).....	78	(I.1)	103		
(4.29).....	78	(I.2)	104		
(4.30).....	78	(I.3)	104		
(4.31).....	78	(I.4)	104		
(4.32).....	78	(I.5)	105		
(4.33).....	78	(I.6)	105		
(4.34).....	78	(I.7)	106		
(4.35).....	78	(I.8)	107		
(4.36).....	79	(I.9)	107		
(4.37).....	79	(I.10)	107		
(4.38).....	79	(I.11)	107		
(4.39).....	79	(I.12)	108		
(4.40).....	79	(I.13)	108		
(4.41).....	79	(I.14)	108		
(4.42).....	79	(I.15)	108		
(4.43).....	79	(I.16)	108		
(4.44).....	82	(I.17)	108		
(4.45).....	82	(II.1)	111		
(4.46).....	82	(II.2)	112		
(4.47).....	82	(II.3)	112		
(4.48).....	82	(II.4)	112		
(4.49).....	82	(II.5)	112		
(4.50).....	83	(II.6)	112		
(4.51).....	83	(II.7)	112		
(4.52).....	83	(II.8)	113		
(4.53).....	83	(II.9)	113		
(4.54).....	83	(II.10)	114		
(4.55).....	83	(II.11)	114		
(4.56).....	83	(II.12)	114		
(4.57).....	83	(II.13)	114		
(4.58).....	83	(II.14)	114		

CHAPTER 1: INTRODUCTION

1. Background, motivation, and objectives

1.1. Background

Affordable, reliable, sustainable and modern energy for all is the Sustainable Development Goal 7 (SDG 7) of the United Nations. This goal pretends to accelerate the use of modern renewable energy, as well as an improvement in the energy efficiency of the systems used. The current innovations and deployment of renewable energy resources in the grid will help to fulfil this remarkable objective. However, in order to boost the introduction of modern renewable energy sources enhancing also the security of the electricity provision, it is necessary to bring into consideration the uncertainties of those systems, that could cause issues in the network.

The penetration of renewable energy in the system is on growing tendency from the last several years, both in Europe, where the renewable capacity installations have increased over a 89% in the last 10 years [1], and in Spain, reaching the 50% of renewable production share in 2020 [2]. Part of this deployment of renewable resources is made at distribution level, and considering the uncertainties inherent to those systems, issues as voltage deviations, imbalances and congestions will arise.

In this aspect, Local Flexibility Markets (LFMs) are emerging at organizational level as a reliable and highly efficient solution to the issues faced by Distribution System Operators (DSOs) who are the final managers of the distribution networks. They are also aligned with the SDG 7 in several aspects, firstly they enhance the energy security of the distribution networks, solving the issues that could appear related with the introduction of new renewable energy assets; secondly, they delete the traditional barriers to the deployment of those modern distributed generators, as LFMs are capable of managing their uncertainties; and thirdly, LFMs also promote energy efficiency as they reduce the power losses associated to the solving of grid issues because of the locality properties of the markets.

LFMs trade with local assets of the grid where it is organized. Those flexibility assets have a lower minimum size as entry level (100 kW) in the current pilot project deployed, compared to TSO balancing services, with a minimum threshold of 1 MW, what lies in lower values of investments. Additionally, the importance of the locality of the asset responding to a local flexibility demand and the short periods involved, suggests that local assets will be most likely to respond, enhancing energy security. Besides of that, this platform where flexibility is traded will require to be simple and to reduce costs of specifying and procuring flexibility at a local level to make it attractive to potential providers or prosumers. [3]

Local flexibility can provide efficient solutions to network constraints as congestions of the power lines, voltage deviations or power outages. Distribution Network Operator could be capable to act as neutral market facilitator, paying to the Distributed Energy Resources (DERs) for:

- Unlocking new network capacity.
- Real timing response to network events.
- Enhancing network operation limits.

Furthermore, understanding the nature of the network constraints Distributed Network Operators, Balance Responsible Parties (BRPs) and Distribution System Operators (DSOs) are addressing will be key to understanding the operating conditions of the local flexibility providers [4].

In any kind of marketplace, there are, at least, three entities present:

- The buyer: in LFM, DSOs with the intention to manage local networks; energy suppliers seeking to manage their electricity volume imbalance or DNO/TSOs looking to balance the wider electricity system.
- The seller. In LFM, DERs, whose flexibility could come from: generators assets owners, large energy users shifting its demand, ESS charging or discharging or through aggregation of smaller local remotely controlled loads.
- The market facilitator. Depending on the nature of the LFM considered, if a centralized version is considered, then a Local Market Operator (LMO) should act as the market facilitator, else, if a decentralized version is applied, then the platform where the market is cleared is considered to be the market facilitator.

The interaction between agents in a can be established not only considering a local marketplace where flexibility products can be matched easily, but also through open tenders or extensions to existing contracts. [5]. One approach taken by National Grid of UK in their balancing services is to provide a standard contract for successful bidders with a stringent testing to ensure capability of the flexibility providers. However, the actual process of calling on flexibility considering standard contracts has not been tested outside some trials developed inside national projects.

On the other hand, LFM has far more potential for solving balancing issues if are designed to be a adaptable entity where multiple parties will be accommodated [6]. The potential of these local markets lies in the possibility of solving with local resources, local issues. For local flexible generators, if there are a ramp down call, they could obtain financial rewarding for voluntary curtailing its generation. For large energy users LFM are an opportunity to provide flexibility adapting their consumptions to markets calls. Flexibility sources could be based on thermal inertia, load shifting or adaptive manufacturing processes, what is also known as Demand Side Response (DSR) techniques. For energy storage operators, LFM are a fundamental basis for their operativity and business model, as they could absorb overgeneration and overdemands with their stored energy.

Furthermore, community and domestic flexibility is the most interested party, due to lower thresholds levels for market entry. However, directly enabling households to the active participation in LFM may require the presence of aggregators or a stronger grid digitalization whose costs of hardware control and administration may be carefully considered. [7]

LFM in early stages with only several trials and innovation funded projects to meet the facilitation of them. Also, it is highly difficult to assign value to the flexibility, with Distribution Network Operators asking themselves question abouts costs of infrastructure, control and administration.

The stochastic nature of LFM would lead TSO and DSO or even between DSOs interests to differ, due to different needs in areas where they operate. For these reasons, a successful marketplace for local flexibility need to meet dynamic and easy bids and offer for multiple procurers and flexibility providers.

In the literature, there are several LFM definitions. Reference [8], considers the Agg as the Local Market Operator (LMO), providing different services to DSO and BRPs, in terms of congestion management and imbalances. Conversely, the LFM defined in [9], does not include BRP figure, enhancing Aggs as the one who oversees the market, collecting bids and asks from the DSO and the flexibility providers. Another approach is considered in [10] where a market structure without the figure of the Agg is presented. In this reference flexibility is traded considering only end users in a decentralized peer-to-peer viewpoint. Moreover, authors of reference [11] acknowledge energy communities as a key player when defining market structure.

1.2. Motivation, scope and objectives

As previous section concludes, there are an extensive background on LFM matters, however, efforts must be made in the field of the mathematical definition of the market. There are several literature gaps to be considered. The first one is the non-linearity and non-convexity properties of the LFM proposed in the literature, all of them use binary variables and complex mathematical definitions to model it, what motivates

a further investigation of the statement of the problem. The second one is the definition of the flexibility products, all the references work with energy-only markets, not considering capacity products in the LFM formulation. The third one is the decentralization of the market, although there exist some contributions in the field of peer-to-peer, the decentralization process of a regional marketplace, diving it into several areas managed by an independent aggregator is a perspective that has not been considered in the literature to the best of the author's knowledge.

Leaving apart the technical and scientific reasons, this topic is selected motivated by the current position as a predoctoral student inside the programme of Electrical Energy Systems of the University of Málaga stood by the author. Additionally, this thesis is the final work of the master's in Industrial Engineering offered by the University of Málaga that give access to the regulated profession "Industrial Engineer" in Spain.

The scope of this investigation will be narrowed to three main aspects. The first one is an analysis of the state of the art of the flexibility products that could be present on a distribution network. Secondly, the definition of the LFM from a centralized perspective using only positive variables. Lastly, the decentralization process of the proposed centralized version of the problem.

Bearing this in mind, the following general and specific objectives are declared:

1. Analysis of the state of the art related to flexibility products and markets definition. If a clear and concise definition of a LFM is ought to be obtained, there is a need of revise what authors have done in this aspects and how it is reflected in an European context. Then, the following specific objectives related to this matter are set:
 - a. Investigation of the current European regulation that harness flexibility in low voltage distribution networks.
 - b. Description of the assets connected to the network that could provide flexibility for solving network issues as congestions or imbalances.
 - c. Classification of the flexibility products that the assets present in the distribution network could easily provide in a market framework for solving the principal issues of the Distribution Network Operators.
2. Definition of a centralized version of the LFM considering positive variables in a simplified and elegant mathematical definition. For that matter, the following specific objectives are descripted:
 - a. Investigation of the characteristics the LFM must have in terms of the european regulation and the pilot projects developed at national and European level.
 - b. Analysis of the modelling techniques of the participants of the market, the distribution network, and the overall objective seek in the LFM present in the literature.
 - c. Description of the clearing mechanisms that could be used to solve a local marketplace.
 - d. Mathematical definition of the proposed market formulation providing also illustrative examples that introduces the reader to the problem, to then solve a more realistic use of case.
3. Decentralization of the previously described centralized version of the LFM mathematical problem. With this intention, the following specific objectives are seek:
 - a. Investigation on the current trends when considering decentralized markets in Distribution Networks both from a regulatory and organizational perspective.
 - b. Analysis of the state of the art of the technical methodologies present when considering decentralized markets definitions, focusing on the coordination techniques and mathematical definition.
 - c. Mathematical description of the proposed decentralization methodology providing also decentralized solutions to the previously solved problems in a centralized fashion. It will be of remarkable importance to obtain the same solution than the one obtained in the centralized problem.
4. Analysis of the results obtained, comparing the centralized and decentralized solutions. Providing also future lines of research that needs to be further investigated for being beyond of the scope of this thesis.

2. Outline of the document

Considering the objectives presented in the previous section, this master's thesis is divided in five chapters and four annexes, detailed in the following list:

- Chapter 1: Introduction
- Chapter 2: Flexibility products.
- Chapter 3: Centralized Local Flexibility Market
- Chapter 4: Decentralized Local Flexibility Market
- Chapter 5: Results Analysis and conclusions
- Annex I: Mathematical Background
- Annex II: Linearization of non-linear constraints.
- Annex III: GAMS code for the centralized solution.
- Annex IV: GAMS code for the decentralized solution.

The objectives of this work are fulfilled with the five introduced chapters. Additionally, four annexes are presented where the knowledge obtained while researching in mathematical optimization techniques and programming the centralized and decentralized versions of the problem is collected and presented. The results explained in Annex II were presented as an article in the "IEEE International Smart Cities Conference 2021" organized by IEEE in Manchester from the 7th to the 10th of September [12].

3. Gantt Diagram

The Gantt Diagram of the works presented in this thesis is depicted in Table 1.

	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.
1. Initial phase of the project													
1.1. Description of the scope and objectives													
1.2. Viability of the approach													
1.3. Initial Analysis of the art													
2. Analysis of the state of the art													
2.1. Electricity markets analysis													
2.2. Flexibility products													
2.3. Centralized formulations													
2.4. Decentralized formulations													
3. Centralized market formulation													
3.1. First illustrative examples													
3.2. Linear approximation													
3.3. Use case simulations													
4. Decentralized market formulation													
4.1. First illustrative examples													
4.2. Decentralization of the use case													
5. Project Documentation													
5.1. Writing of the thesis													
5.2. Writing of the conference													
5.3. Presentation preparation													

Table 1. Gantt diagram of the work done in this master's thesis.

CHAPTER 2: FLEXIBILITY PRODUCTS

1. Introduction

Traditionally, flexibility products were offered to TSO by generation units to solve grid issues (power balancing and frequency control). Conventional electricity plants, (for example thermal, hydro and nuclear), interconnections with other areas or large consumers usually were service providers to the TSO. Those assets generate products that are usually divided into three main groups depending on the type of market that they are traded: Capacity products, energy products and ancillary products. This chapter will focus on those three types of products from a distribution level viewpoint, acknowledging their differences with traditional transmission level products.

Distribution level products differentiate from transmission level products in many aspects. As a first fact, just the size of the assets makes a remarkable difference, as an Aggregator figure is usually needed for operational efficiency in distribution level. Also, types of assets connected to the distribution network greatly differs from those connected at transmission level. What it is more, policies associated with climate change and clean energy production are providing incentives for using small renewable assets in distribution level, what makes production stochasticity of those little generators play a role on products offering.

Having reviewed those reasons, it is necessary to make a profound revision of the types of assets connected to low voltage distribution networks. Clarifying what kind of products can be offered, which prices they would have, in what types of markets they could participate and if they could be object of an aggregation process for external agents.

There is little doubt that electric systems are currently under a deeply process of transformation and digitalization what leads to new types of services and business opportunities. Those related changes are leaded mainly by [13]:

- *The intermittent and uncontrollable generation units installed*: not only PV systems are increasingly being installed in the roof of consumer's houses, but also new modest wind and tidal installations are deployed all over the grid.
- *Controllable and flexible consumers*: with the irruption of the new electricity tariffs, people are getting interested in how to participate in the electricity market with the intention of saving some money. Thus, Demand Side Response (DSR) techniques are increasing their share, converting traditional static consumer to a more flexible and proactive consumer, also known as prosumer.
- *Intelligent and smart systems*: the transition from traditional grid to smart grids are witnessed worldwide with EU, EEUU, China and Asia trials. This is allowed due to advances in Information and Communications Technologies (ICTs) what enable monitoring, control and manage the whole system.
- *Bi-directional and decentralized systems*: a more sparkling scenario with more people participating in it will cause some problems to happen. As a matter of fact, bi-directional elements could cause some voltage deviations, and decentralized systems could lead to misunderstandings between peers, causing balance deviation or overloading of the feeders.

- *Multi-energy systems*: besides of all electric flexibility, when more than one energy vector is playing a role, the services it could provide increase exponentially. Synergies among electricity and other vectors as gas, heat pumps and HVACs could be used as an interesting source of flexibility.

Bearing all of these in mind, the most usual characteristics when describing flexibility are:

- *Direction (up or down)*: upward direction if conventionally used when demand is rising or generation is decreasing while downward direction is used otherwise.
- *Rate of change*: the maximum amount of capacity that could be changed from one period to another.
- *Starting time and its trigger*: when and what causes the flexibility to be activated.
- *Duration*: the maximum and/or the minimum amount of time that a flexibility source should be activated.
- *Location*: the node or nodes of the grid where the flexibility is deployed once it has been activated.

The analysis of the products available at distribution level is also referenced considering H2020 European projects DOMINOES [14], MERLON [15], InterFlex[16], EMPOWER[17], DREAM-GO[11], and the Spain National Project IREMEL[18]. Those projects assets in the creation of local energy markets at distribution level, providing use cases with different assets and type of products. However, main trends of EU projects only consider flexibility as an effect of the Demand Side Response (DSR) of assets controlled by DSO, Aggregators, or large consumers.

This chapter is divided into several sections. Firstly, how to harness flexibility in low voltage distribution networks is explained, considering both the European regulation and the assets connected to the network viewpoint. Secondly, a description of the flexibility products that could be traded in a LFM is presented. The classification of the products is made attending to its direction, to the type of the product itself, whether the product use active or reactive power and, lastly, attending to the type of restrictions associated to the product.

2. Harnessing flexibility in low voltage distribution networks

2.1. European regulation

Flexibility procurement is a powerful tool for DSO and energy providers. It can be used to avoid grid reinforcements leading to a better utilization and development of the network capacity. Council of European Energy regulators (CEER) instate for balanced incentives, adequate neutrality, and clear technical prerequisites as recommended principles in the regulatory framework for flexibility procurement.

Also, is considered that DSO using flexibility will benefit the grid, but it will not provide it. Flexibility arise from those end-users wanting to actively participate a flexibility market [19]. However, those prosumers will only seek individual's profit. For that reason, European legislator should start up consistent and efficient regulation to avoid grid malfunctioning.

European legislators, on article 32 of directive 2019/944 aims to improve efficiencies in the operation and development of the distribution network when considering flexibility procurement both economically and technically efficient not leading to network distortion neither to declaration of losses by providers. Thus, an adequate remuneration and cost recovery should be secured for flexibility providers.

CEER divides the coordination mechanisms, that enable DSOs to gain access to flexibility products, into four categories: rules-based approach, connection agreements, network tariffs and market-based procurement. Prioritization among them depends on the national regulation context as well as to which extent implicit and explicit flexibility mechanisms are merged, signalled, and compensated. On the first hand, Implicit flexibility mechanisms are characterized for a fixed signal price which flexibility providers respond to. On the other hand, using explicit flexibility mechanisms in the definition of a local flexibility market, actors will bid their products considering their economic and technical constraints, responding to a flexibility ask in the market. Economic theory affirms that explicit mechanisms work better for non-regulated markets, kicking most inefficient technologies out of the market.

2.1.1. Rules-based approach

The first coordination mechanism identified by CEER is the rules-based approach. CEER demonstrate that this mechanism should not create high running costs, and the overall costs should be minimised for

effectiveness. However, it is not broadly used in non-regulated markets, and it is eventually used when the rest of alternatives do not provide satisfactory enough results.

2.1.2. Connection agreements

Second method for coordination is reaching connection agreements between the network owner and the users interested in using it. With this modality prices are varied from time to time, depending on the specific situation of the grid in each moment. Changes in connection charges, also known as locational price signals, might be favourable to certain large users, but it could lead to discrimination to the latest connected and limited users. Thus, behavioural changes need to be accounted, with the aim of reaching an equal opportunity agreement between end users, no matter its size or if they enter after or before in the market.

2.1.3. Network tariffs

In this type of coordination methodology, network users are exposed to a price-based signal to change its behavioural consumption contributing to a better utilization of the grid. Network tariffs should be oriented to reduce both the system and individual peak. It should be noted that the effect of the network tariffs is dependent of the availability of flexible resources in origin, depending its effectiveness lastly on end-users.

Using a fully dynamic tariff may arise problems in location decisions for different network users, as they need both consider the spatial and temporal variation of the energy price. A combination of static and dynamic tariffs should be considered, as, under that situation, the procurement of flexibility forms a beneficial instrument.

2.1.4. Market-based procurement

In a market-based configuration, DSO can participate in bilateral transactions among users that are offering their flexibility or, even interact with service providers acting on their behalf. Under those circumstances, a legal framework to allocate responsibilities and rights should be provided by National Regulatory Authorities (NRAs). An important characteristic of the commented legal frameworks is that congestion management is considered to be a monopolistic approach by the CEER. For that reason, NRAs should oversee marketplaces establishing guideline or requirements to reach the right circumstances that provide an efficient and reliable way to manage congestions.

2.2. Assets in low voltage distribution networks

European projects as MERLON worked on models of Distributed Energy Resources (DERs). Devices, together with the projects, that allows to address specific levels of flexibility. Derivable 6.1 (D6.1) of the Working Package 6 (WP6) of MERLON focused on the behaviour of small generators, storage facilities and loads with the intention of obtain flexibility of them [15].

Although MERLON project focused on Integrated Energy Systems (IES), it reflects how possible is to harness flexibility from distribution network using small assets. This project will be used as base for the description of the different assets in low voltage distribution networks. In the following sections, passive elements, generators and storage systems will be analysed. Furthermore, two sections are dedicated to analysing specific devices for voltage control and IES.

2.2.1. Passive Elements

Passive elements, usually known as loads, consume active and reactive power from the grid. In distribution level they are usually identified with final end-users. Mainly, loads can be divided into two groups attending to its controllability: static loads and flexible loads.

Static loads behaviour does not respond to any market signal, they consume what they want for each time slot, no matter if the price of the electricity is higher or lower. From a market perspective they have only interest in terms of designing traditional energy markets, as no flexibility can be extracted from them.

On the contrary, flexible loads do respond to market signals, and it is possible to aggregate them and to control them. Essentially, as flexible loads it is possible to include heat systems, flexible industry processes that can be postponed, demand response techniques, etc. Those systems can offer flexibility products, both upward and downward directed.

MERLON project consider several types of domestic loads from which flexibility can be obtained. This project considers two different approaches for load modelling. Firstly, measurement-based approach what

correlate individual behaviour of the components, and secondly, component-based approach which disaggregates loads into separate parts to model them individually, obtaining a global model.

Lights, HVAC individual systems and domestic hot water are domestic loads considered when harnessing flexibility. Obviously, they are not the unique types of loads at distribution level from which flexibility is obtained. For illustrate some of the approaches to harness flexibility from domestic assets, three examples are provided, involving: lighting systems, HVAC systems and domestic water heating systems.

Lighting systems

Lighting instantaneous power consumption can be approximated considering the dimming level (d_l), its nominal power (P_l^N) and its status considered as a binary variable (s_l):

$$P_l = s_l \cdot d_l \cdot P_l^N \quad (2.1)$$

Dimming level of the lighting can be automatically controlled by means of using a sensor that measure the level of illuminance $w(d, u)$ from the working surface. Mathematically, level of illuminance can be modelled considering dimming levels of the surround light sources (d_l) and the illuminance due to daylight (u):

$$w(d_l, u) = \sum_{l=1}^n P_l^N \cdot d_l + u \geq w_{sp} \quad (2.2)$$

Where w_{sp} is the setpoint level of illuminance for a given zone, measuring $w(d_l, u)$ it is possible to control dimming level of the lighting with market signals.

HVAC systems

Between HVAC modelling techniques it is possible to distinguish between: physics-based models, grey-box models, and black-box models. Due to high complexity of thermal physical modelling and difficult to set characteristics parameters for grey box parameters, black-box models are highlighted for its simplicity and variety (ARIMAX, SARIMAX, ANN, SVM...). Black-box models stand out for they low complexity that allows to reduce the amount of calculation time when considering many scenarios. Also, they provide an interesting approach to take advantage of the data produced by all connected devices in a IoT environment.

Domestic water systems

Lastly, contrary to HVAC systems, domestic water systems are easily modelled thermodynamically, providing a differential equation in terms of the power consumption rate (Q), density and heat capacity of the water (ρ, c_p), volume of the tank (V), consumption rate ($\dot{V}$) and input and output water temperature (T_{in}, T) respectively and thermal losses (Q_{loss}).

$$\rho c_p V \frac{dT(t)}{dt} = Q(t) + \rho c_p V(t)[T_{in} - T(t)] + Q_{loss} \quad (2.3)$$

Tank losses are estimated considering the ambient temperature and the overall heat transfer rate (U).

$$Q_{loss} = UA(T(t) - T_{\infty}) \quad (2.4)$$

Supposing known all parameters, domestic water systems can contribute with flexibility as can storage an amount of thermal energy and serves as little storage systems. Measuring output water temperature, objective temperature and ambient temperature is possible to obtain flexibility from load shifting of these systems.

2.2.2. Generators

Generation at the distribution level is characterized for being low powered and sparse along feeders. In literature, this type of generation is commonly named Distributed Energy Resources (DERs) and cover a wide type of generators that goes from PV systems to tidal generation, passing through wind generators, even diesel generators and gas turbines if microgrids are considering. Attending to the renewable nature of the generation source, the most extended technologies deployed in distribution level are classified:

- Renewable energy resources:
 - o PV generators (Rooftop, PV power plants, and solar concentrators).
 - o Wind generators (from small to medium size).
 - o Micro hydro power plants (run-of-river power plants, usually).

- Fuel cells ($H_2 - O_2$ redox reactions).
- Non-renewable energy resources:
 - Diesel generators (backup generators in case of blackouts in sensible processes).
 - Gas generators (from small to medium size).
 - Combined Heat and Power plants (CHP).

Generally, renewable systems in distributions networks consists in solar photovoltaics or wind power generation. Although non-renewable systems are not very common on distribution networks in Spain, European countries may use Combined Heat and Power plants in the context of integrated energy systems.

Renewable systems, although difficult to model and forecast, are usually the only possible alternative to obtain flexibility. Forecasting renewable generators considering environmental conditions is being object of study by the academic research. Different techniques may be used as: Artificial Neural Network predictors (ANN), Support Vector Machines (SVM) predictors, SARIMAX, ARIMAX, ARIMA analysis, and so on. Contrary, non-renewable energy systems having significantly more stable load profiles, thermodynamical modelling is possible considering topology of considered system. Conventional generators can be modelled using static models, what reduce complexity and boost precisions as stochasticity is nearly reduced to zero.

Ordinarily, PV systems and wind generators are the most common installed DERs. Strategies for DERs deployment at distribution level can consist in the installation of small rooftop assets in households or the creation of medium power projects of PV and wind in the field. As a general behaviour, DERs only provide energy products as their variable costs are near to zero. However, DERs could also provide flexible products, more concisely, downward flexibility, as it can be curtailed, or even stored if the manager of the systems installs a battery.

Secondly, Micro Hydro Power plants can also play a role in distribution level for balancing, or for providing flexibility services, as their power are usually between 5 to 100 kW and can be controlled if are designed as a run-of-river type or has some water storage. This results in an increase of effectiveness when providing flexibility services.

Thirdly, Fuel Cells are increasingly installed with the irruption of green techniques to obtain H_2 in a free CO_2 emission manner. Notwithstanding the possibilities it offers, its share in the generation mix is very low as their installation costs are relatively high compared with other sources.

Lastly, gas turbines and diesel generators usually actuate in microgrids, and provide a wider range of products, as those systems can be controlled under market signals. Nevertheless, they are not so common at distribution level, as they variable costs are high in addition to their carbon emissions. Usually, they are used as tertiary reserve in case of emergency for non-stoppable systems. Combined Heat and Power systems (CHP) are nearly uncommon in southern of Europe, as Heat distribution systems are not deployed in almost any city. Nevertheless, they offer a good opportunity for flexibility in northern and colder countries, where CHP supposes a considerably part of the energy consumption mix.

2.2.3. Storage systems

Batteries and storage systems provide flexibility at distribution level as they serve as a useful component while shifting loads and flattening the demand profile. In general, those types of assets can be modelled using empirical, electrical and ageing techniques. Their principal objective is to know and model the battery charge/discharge rate as a function of its actual state of charge, its charge/discharge efficiencies, and its state of health.

Empirical modelling considers batteries as an ideal storage system but consider an empirical obtained parameter called capacity fade per cycle. The model is function of its state of charge (SoC), charge and discharge efficiencies (η_c, η_D), state of health (SoH) and capacity fade per cycle of charging (c_F). This modelling technique needs of some experiments to setting up a precise model of the SS.

Electrical modelling represents a compromise solution between computational complexity and a reliable and precise model. Battery can be modelled as a circuit with capacitors and resistances. Capacitors represent the component that storage the electrical energy, while resistances model the inefficiencies of the system.

Ageing Modelling is used in practice to determine State of Health (SoH), as this parameter will determine the performance of the battery though its lifetime. SoH is directly related with the depth of the discharge in

terms of the percentage of SOC (DoD) and the number of cycles suffered by the battery up to the moment of measurement (N_c).

Generally, battery energy level at time step t , ($E_b(t)$) can be determined in terms of charging and discharging power flows (E_b^{ch}, E_d^{ch}), energy losses at standby (E_b^{stb}) and the efficiencies of charging and discharging (η_{ch}, η_{dis}):

$$E_b(t) = E_b(t-1) - E_b^{stb} + E_{ch}(t)\eta_{ch} - \frac{E_{dis}(t)}{\eta_{dis}} \quad (2.5)$$

To model battery degradation due to the depth of discharge (DoD) of a single discharge, following equation is written down:

$$\Delta E = E_0 \cdot K_D \cdot |soc(t-1) - soc(t)| \quad (2.6)$$

Where E_0 is the available energy at the begging of the operation, and K_D is a coefficient obtained from battery manufacturer derived for Achievable Cycle Count (ACC).

$$K_D = \frac{1}{2D} \left(1 - 0.8^{\frac{1}{N_{ACC}(D)}} \right) \quad (2.7)$$

$N_{ACC}(D)$ denotes the number of cycles for the battery to reach its end of life when operating at a DoD of D . Generally, battery is substituted when reaching its 80% of exploitable initial capacity. Generally, for simple models this parameter is reduced to $K_D = 1,04 \cdot 10^{-5} \text{ kWh/charging kWh}$. However, this parameter must be modelled considering technical specifications of the manufacturer.

Electric Vehicles

Electric vehicles will play a fundamental role in the distribution grid in the following 10 years, for that reason, they are object of study by many researchers. They compute the EV aggregated power capacity using Markov models based on the stochasticity of the driving behaviour (taking into account traffic information, human's charge preferences or accessibility to charging infrastructures)

Their principle of working is the following: while charging, electric vehicles can sell electricity products such as ramping, capacity or flexibility, obtaining profits by their participation in the market. From a power system perspective, they play the same role than a traditional battery, being possible to offer capacity service for its connection power, ramping services (in both directions) when state of charge is near to the set point previously established, or even flexibility services for a given time slot (upward and downward flexibility) if there exists some contingency. Those offers need to account the battery degradation when offering it, as the lifetime of the battery may be lowered.

In the top of that, it is also possible to aggregate those systems, what boosts its capabilities. This aggregation phenomena could be both physical and virtual, depending on the spatial situation of the different EV charging stations. Location of the EV charges can also play a fundamental role in the type of products an aggregator could offer.

BESS

Combination of photovoltaic system with battery is quite common for households determined to install a rooftop DERs. End users take advantage of protocols as HEMS (House Energy Management System) that provide an optimal usage of the installation, minimizing consumption from the grid while enabling injection of the energy surplus that cannot be no longer stored.

Under this scenario, batteries can provide multiple services at distribution level, especially when they are aggregated, as previously explained with the electric vehicles. Aggregation phenomena is especially interesting when different distributed batteries are joint together, however, this approach incurs in higher uncertainties while forecasting.

Batteries can offer capacity, ramping, flexibility and even energy products controlling its State of Charge (SOC) and the requirements of the end-user. BESS is one of the most flexible devices connected to the distribution network and its entry in the local markets can produce lower products and deter grid reinforcements.

Ultra-capacitors

An ultra-capacitor is a type of capacitor which had its capacity increased at the expense of having a lower voltage limit. They have higher specific power than usual batteries, however, lower specific energy than other technologies. For that reason, they are potential candidates for providing capacity products in the future smart grids.

Although their current applications are limited to energy recovery, regenerative braking technologies, wind turbine pitch control and uninterruptible power supplies, they can also provide useful and valuable services for a DSO if high power its need for a short period of time.

Ultra-capacitors achieve very high capacitance values by eliminating the solid dielectric layer of traditional capacitors. In contrast of the traditional capacitors, that storage the energy inside of an electrical field, ultracapacitors storage the energy inside of an electric double layer, also known as the Helmholtz double layer. Also, the electrodes are rough and permeable, increasing the contact surface with the dielectric material.

Traditional capacitors barely have their capacity superior to 1 mF . However, ultracapacitors brings capacities up to 1 kF . Those values are the responsible of consider them as an active element of the future smart grids, key in transferring high powered but low energized products in the grid.

2.2.4. Devices for voltage control in AC networks

Flexible AC transmissions systems (FACTS) is a system that can enhance operational conditions of a node or line of the network. FACTS where initially developed for enhance controllability in transmissions networks. However, nowadays they are being developed in low voltage distribution network as part of the digitalization process. According to the type of connection of the FACTS with the network controlled, they can be classified as [20]:

- Series Connected Controller: Series controllers are connected in series with the line voltage, a impedance device is used to modify line voltage level. There exist four types of SCCs:
 - o Thyristor Controlled Series Capacitor (TCSC)
 - o Thyristor Controlled Series Reactor (TCSR)
 - o Thyristor Switched Series Capacitor (TSSC)
 - o Static Synchronous Series Compensator (SSSC)
- Shunt Connected Controller: Shunt controllers are connected in parallel with the line. They are used to inject current to the line, modifying its voltage magnitude.
 - o Static Synchronous Compensator (STATCOM) and Distributed Synchronous Compensator (D-STATCOM).
 - o Static VAR Compensator (SVC):
 - Thyristor Controlled Reactor (TCR).
 - Thyristor Switched Capacitor (TSC).
 - Thyristor Switched Reactor (TSR).
- Combined Series-Series Controller (UPFC): consists in a combination of separate series controllers used to control several line voltages. They are equipped with a DC link joining both controllers that can be used for active power transfer.
- Combined Shunt-Series Controller (IPFC): Modify both current and voltage quantities of the transmission line via a pair of shunt and series controllers operated with coordination.

Other devices are present at distribution level with the objective of voltage control of the network nodes. Inverters and step-voltage regulators are examples of this type of devices. Nowadays they provide a certain control level of the node voltage. However, they can require manual operations (in the case of step-voltages) or are not full deployed (in the case of FACTSs) and do not offer a complete control of all nodes of the grid.

On the other hand, those devices provide crucial support to voltage control, and if present, they can offer products to the market, especially in the case of FATCSs and inverters, that can modulate voltage directly, e.g., are responsible to market signals.

2.2.5. Integrated Energy Systems

Integrated energy systems (IES) are complex structures where several types of energies are linked together with the result of higher energy efficiency, security, and lower general costs. These systems typically include natural gas, electricity, thermal and cooling energy [21].

While managing IES several figures may appear, such as, electric grid owner, gas grid owner, different energy suppliers (at least one for each participant energy type) and the final consumers. Involving a huge number of parts, complexity of IES rises rapidly. There are several integrated alternatives to clear such complicated market:

- Newton-Raphson technique for coupled equations at district level. This approach link equations relating different participants and solve iteratively up to their convergence. However, as the number of equations and iterations increase, larger models may be difficult to be solved in a reasonable amount of time.
- Multi-objective optimization algorithm to obtain a Pareto-optimal solution set to find optimal daily operation solutions. This last algorithm allows to obtain a set of Pareto efficient solutions between energy involved in the IES.

These models need simplified equations of the IES, due to usually large, non-convex even not convergent systems. Thus, inaccurate solutions may be found. Also, traditional centralized optimization method which needs all information about the system are not applicable for solving optimal operation problem of IES, because of the operational independence and information privacy issues that may arise. In this trend, hierarchical models with decentralized optimization advantage over traditional integrated model with centralized optimization on two points. Firstly, more accurate solutions – ideal for heterogeneous energies. Secondly, benefits of different system individuals are considered separately, and information privacy and independent operation are respected.

3. Flexibility products

Flexibility products are numerous in the nowadays state of the art. So, Figure 1 is presented, where they are classified attending to their direction, type of product, type of power and type of restriction applied.

Assets in low voltage distribution networks can offer different products to trade in Local Flexibility Markets to alleviate voltage deviations, peak-shaving, line congestions, imbalance issues and power quality. Flexibility products in the context of future low voltage distribution networks can be mainly offered by assets described in previous sections. Those products will solve voltage deviations, line congestions, peak-shaving, imbalance issues and power quality from a distribution level. These issues have been usually solved by Ancillary market at transmission level. Nevertheless, with the future change of paradigm of distribution networks, assets at this level will gain importance and presence in new markets.

This section will quantify mathematically how those products are considered in the market approaches proposed in following chapters. Bearing this intention in mind, following sections will address each type of flexibility products of those classified in Figure 1.

3.1. Attending to the direction

Depending on whether assets can increase or decrease its consumption or injection, it is possible to distinguish between upward flexibility and downward flexibility. If an asset can freely increase consumption, or decrease generation, it can offer upward flexibility. Contrary, if an asset can freely decrease consumption or increase generation, downward flexibility could be offered.

3.1.1. Upward products

In distribution level ESS, controllable loads and EV can offer this type of product under specified circumstances. For example, ESS and EV can increase its rate of charging or discharging if it has not reached its maximum limit within safety conditions. Same principle can be induced for controlled loads. Generators, on the other hand, may be restricted while offering it. Commonly, renewable energy sources cannot be directly controlled due to the stochasticity of the nature, but some non-renewable energy sources as CHP, diesel generators or gas turbines can be directly controlled.

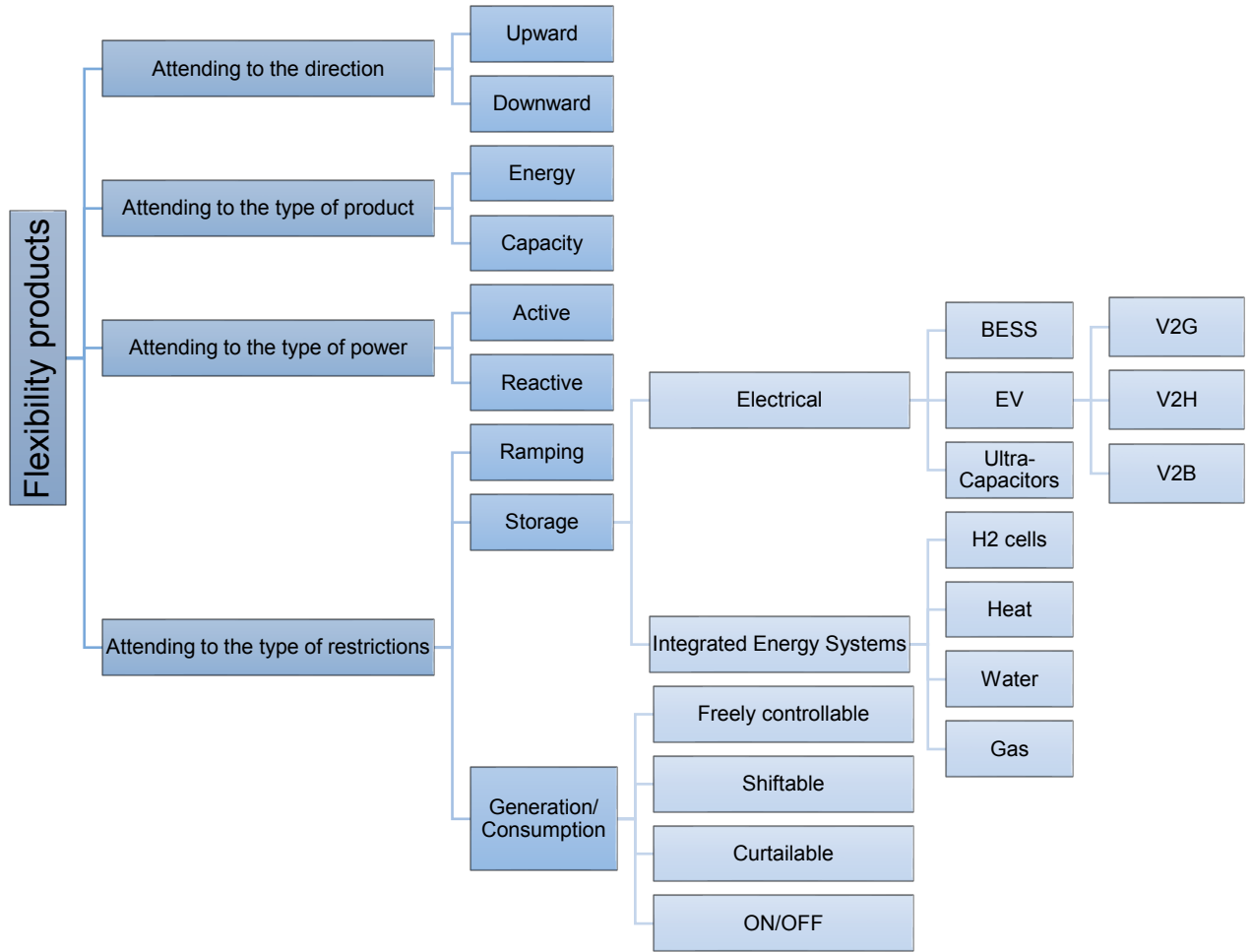


Figure 1. Classification of the flexibility products in distribution networks.

Considering $\omega_{t,n}^u$ as the amount of product offered in the upward direction for the asset n , at a price $\lambda_{t,n}^u$. For a distribution system with n assets offering upward flexibility, utility function of the whole upward products is defined in equation (2.8).

$$U_t^u = \sum_n \lambda_{t,n}^u \cdot \omega_{t,n}^u \quad \forall t \quad (2.8)$$

3.1.2. Downward products

In distribution networks ESS, controllable loads, RE DERs, EV could offer downward flexibility. RE DERs are included in this category as far as they can curtail its production if they are paid for doing it.

Considering $\omega_{t,n}^d$ as the amount of product offered in the downward direction for the asset n , at a price $\lambda_{t,n}^d$. For a distribution system with n assets offering upward flexibility, utility function of the whole upward products is defined in equation (2.9).

$$U_t^d = \sum_n \lambda_{t,n}^d \cdot \omega_{t,n}^d \quad \forall t \quad (2.9)$$

3.2. Attending to the type of product

3.2.1. Energy products

Energy products offered in flexibility market can effectively allocate surplus of the agents connected to the market, enabling trade between users, microgrids or energy communities, usually they are associated with future spot market, for a period of, at least, 1h.

Generally, those products are defined considering a utility function, and when considering a marketplace, then both sellers (s) and buyers (b) are considered. Let λ_t^b be the price that a buyer is aiming to pay for p_t^b

quantity of power, while λ_t^s is the price offered by a seller for its surplus p_t^s . Then, under these conditions, utility function is defined as the social welfare function in equation (2.10). Equation (2.11) represents equality between bought products and sold products in the market.

$$U_e = \sum_b \lambda_t^b p_t^b - \sum_s \lambda_t^s p_t^s \quad \forall t \quad (2.10)$$

$$\sum_b p_t^b = \sum_s p_t^s \quad \forall t \quad (2.11)$$

Restrictions of this product can also consider forecast confidence bands for the case of renewable generators and DERs, and expectancy of consumption in the case of loads. For this thesis purposes the model will be deterministic, and this stochasticity will be considered as bid limits.

3.2.2. Capacity products

As previously mentioned, capacity products are offered by those assets that can modify its output power consumption or injection. More precisely, in the case of distribution networks: EV, storage systems and controllable loads. Considering λ_t^s as the price for energy product at time slot t in the wholesale market, upward capacity product $S_{c,t}^\uparrow$ is compensated at a price $\lambda_t^\uparrow$, while downward capacity product $S_{c,t}^\downarrow$ at a price $\lambda_t^\downarrow$. This compensation is related to the cost assumed by the asset in the wholesale market, what provide incentives for offering capacity products. Furthermore, capacity payment is also made considering σ_t^c as the price for the capacity product. With this information, cost function $C_{c,t}$ and utility function $U_{c,t}$ can be written as equations (2.12) and (2.13) show.

$$C_{c,t} = S_{c,t}^\uparrow (\lambda_t^s - \lambda_t^\uparrow + \sigma_t^c) + S_{c,t}^\downarrow (\lambda_t^\downarrow - \lambda_t^s + \sigma_t^c) \quad (2.12)$$

$$U_{c,t} = S_{c,t}^\uparrow \lambda_t^\uparrow - S_{c,t}^\downarrow \lambda_t^\downarrow \quad (2.13)$$

$$\underline{S_{c,t}} \leq S_{c,t} + S_{c,t}^\uparrow - S_{c,t}^\downarrow \leq \overline{S_{c,t}}, \quad \forall c, \forall t \quad (2.14)$$

Previous equations considers that $\lambda_t^\uparrow < \lambda_t^s$ and $\lambda_t^\downarrow > \lambda_t^s$ so as to incentive assets to provide capacity products. Also, it is assumed linear relationship between capacity and utility function. The difference between wholesale price and the upward and downward product will defines the adversity of the agent to offer capacity products.

Besides, it must be noted that restriction (2.14) ensure that upward, or downward, capacity product offered by the asset will not exceed minimum or maximum limits. That is, for example, if energy bought in the wholesale market is equal to the maximum limit ($S_{c,t} = \overline{S_{c,t}}$), then no upward capacity product can be offered to the market ($S_{c,t}^\uparrow = 0$), and only downward product ($S_{c,t}^\downarrow$) can be offered.

3.3. Attending to the type of power

3.3.1. Active products

Active flexibility products can be defined as a difference between two active power states:

$$\omega_{f,t} = \Delta p = p_{f,t} - p_t^* \quad (2.15)$$

Following this definition, $\omega_{f,t}$ will be positive for upward flexibility and negative for downward flexibility product. For the sake of clarity this section will treat in the same way both upward and downward flexibility products considering them as a unique variable. However, in the reality, those directions are accounted separately.

Active powered products can provide different services, as can modify the amount of active power that flows through a line, or a node injects. For that reason, it can be employed for line congestion management, node voltage control and balancing of the grid.

3.3.2. Reactive products

Voltage or reactive power control services uses flexibility of the LFM participants to avoid exceeding the voltage limits. This service is offered to DSO by the LFM or any of the market participants. TLC can be used also while providing these service among others to prioritize service procuring [8].

Voltage control could also be reached by the usage of a sensitivity analysis in the network. This method provides the zones where flexibility activation is technically and economically feasible for the DSO. Authors in [22] define heuristic optimization methods to perform this analysis with less computational effort.

Several types and sub-types as well as market and sub-markets are defined in [23]. Power system control is defined as a quick activation of locally quickly and securely balancing capacity in a, as defined, cell system, little regions of the grids controlled by a Cell System Operator (CSO). Voltage control is then defined as one of the multiple services offered by this platform.

Other Voltage Control services include Multi-Energy Virtual Power Plants as flexibility services procurators. Economic analysis is conducted in this VPPs to determine the effectiveness of the service considering loss of revenues associated with the curtailment of the injected power [24]

Flexibility can also be defined in terms of reactive power, the only difference with the active power is the limits on which the assets can operate when considering it. For example, a PV generator can theoretically be injected active power to the grid while consuming reactive power [25]. Under this situation, maximum power rate injection must be considered as a quadratic function $s_{f,t} = \sqrt{p_{f,t}^2 + q_{f,t}^2} \leq \bar{s}_f$.

As a general formulation, where $\tau_{f,t}$ represents price for reactive power products, reactive power utility function can be defined as follows:

$$U_{f,t} = \tau_{f,t} q_{f,t} \quad \forall f, \forall t \quad (2.16)$$

$$Q_f \leq q_{f,t} \leq \bar{Q}_f \quad \forall f, \forall t \quad (2.17)$$

However, considering these types of products while constructing a local flexibility market can create additional inconveniences, as the need of a more precise model for the network. DC approach is no longer valid, and other approaches as the conic-based approach or AC approach are needed.

3.4. Attending to the type of restrictions

3.4.1. Ramping products

In this section, two different approaches when defining ramping products are addressed. In the first approach, the viewpoint of the DSO is chosen, where it submits penalties to those generators that are continuously introducing peaks. On the contrary, the second approach consider a market perspective, not introducing penalties but a product that DSO can use to smooth other sources of peaks and imbalances.

Ramp services can be used as a control strategy to smooth the stochastic behaviour of DERs, in terms of reduction of power variations and its related issues (voltage issues, grid instability, etc.). Using penalization fees for solar and wind power, extreme slopes are reduced as it is included in the minimization objective function. This penalization fee ($F(p_{g,t})$) is defined for each generator g and a given time t , considering a time slot $[t - j, t]$ and a fee of λ [26].

$$F(p_{g,t}) = \lambda \sum_{g \in \Omega_g} \sum_{t \in \Omega_t} \left| \frac{p_{g,t} - p_{g,t-j}}{T} \right| \quad (2.18)$$

Also, in literature, ramping products are also called load following products, as the producer of this type of products get paid for increase or decrease its production or consumption following load curve. The formulation of this product can besides be viewed as a complex bid as it needs, at least two time slots for its definition. As can be interpreted the definition in the following approach is very similar to the first one, however, with this definition, no penalization fee is used as the definition is seen from a market perspective.

$$U_r(P_{r,t}) = \sum_{t=1}^{T-1} \lambda_{r,t} (p_{r,t+1} - P_{r,t}) \quad \forall r \quad (2.19)$$

$$\underline{RL}_r \leq p_{r,t+1} - p_{r,t} \leq \overline{RL}_r \quad \forall r, \forall t \quad (2.20)$$

$$\underline{P}_r \leq p_{r,t} \leq \overline{P}_r \quad \forall r, \forall t \quad (2.21)$$

Ramping products are not accounted in the power flow balance equations but $p_{r,t}$ does, as must be an equilibrium between energy injected and consumed.

Besides of that, flexible ramping products have been proposed by California ISO grid (CASIO) and MISO market design. These approaches use a deterministic real-time unit commitment model with flexible ramp constraint, defining ramping products to meet deviations and provide services to the TSO/DSO. However, its effectiveness has been overhauled by the stochastic model considering these services [27].

3.4.2. Storage products

This type of product is especially interesting in the case of distribution grids, as can effectively allocate surplus of RE DERs, and supply energy in the case of a deficit of DERs generation. Depending on the type of storage products included in the market, some differences may appear. In this context, Battery Energy Storage Systems (BESS) together with IES and ultracapacitors are defined as from their definition most of the types of storage product could be inferred.

Battery Energy Storage Systems (BESS)

This type of storage system is the one considered as the simplest model. It is based on the state of charge (SOC) and maximum rates of charging and discharging (RD_s, RC_s), maximum and minimum SOC ($\overline{SOC}_s, \underline{SOC}_s$), charge and discharge efficiencies, (η_s^c, η_s^d) and quantity of energy products offered in the market ($\omega_{s,t}^u, \omega_{s,t}^d$). Thus, the utility function (U_s) and associated restrictions can be written as equations (2.22) to (2.26) show:

$$U_s = \sum_{t=1}^T \lambda_{s,t}^u \omega_{s,t}^u + \lambda_{s,t}^d \omega_{s,t}^d \quad (2.22)$$

$$soc_{s,t} = soc_{s,t-1} + \frac{1}{\Delta t} (\eta_s^c \omega_{s,t}^u - \frac{1}{\eta_s^d} \omega_{s,t}^d) \quad \forall s, \forall t \quad (2.23)$$

$$\underline{SOC}_s \leq soc_{s,t} \leq \overline{SOC}_s \quad \forall s, \forall t \quad (2.24)$$

$$p_{s,t} = \omega_{s,t}^u - \omega_{s,t}^d \quad \forall s, \forall t \quad (2.25)$$

$$RD_s \leq p_{s,t} \leq RC_s \quad \forall s, \forall t \quad (2.26)$$

Integrated Energy Systems (IES)

Depending on the technology used to construct the IES, several storage facilities may be present. From an electric market perspective, no matter what the technology is, their commonplace is that energy could be relocated from one time slot to another in the future. Considering s_t as the apparent power scheduled for a determined asset of the IES, s_t^u and s_t^d are defined as the upward and downward direction flexibility products of those assets. Then, aftermarket apparent power consumed by an asset of the IES:

$$s'_t = s_t + s_t^u - s_t^d \quad \forall t \quad (2.27)$$

For a given time period, the total amount of energy consumed before and after market clearing should be the same:

$$\sum_t s'_t = \sum_t s_t \quad (2.28)$$

However, those systems are restricted depending on the type of asset used for storage the energy between the initial and final time periods. For storing based on thermodynamic properties, temperature of the asset under study should be maintained between $\bar{T}$ and $\underline{T}$. Considering a volume V filled with a fluid with density ρ and specific heat c_p , that is heated considering an energy flow $E_T(t)$ of which Q_{loss} is considered as a fixed amount of energy that is lost due to inefficiencies of the system. The global restriction is showed up in equation (2.29).

$$\rho c_p V \frac{dT(t)}{dt} = E_T(t) + \rho c_p V [T_{in} - T(t)] + Q_{loss} \quad (2.29)$$

If the storing is based on other IES, as kinetic energy systems, then energy stored can be described as equation (2.30), where η_k is defined as the efficiency of the kinetic energy system, Ω is the rolling inertia of the system and ω is the angular speed. For example, flywheels, can works between a minimum and maximum angular velocity $\underline{\omega}$ and $\bar{\omega}$.

$$E_k = \eta_k \frac{1}{2} \Omega \omega^2 \quad (2.30)$$

Other types of energy could also be used for energy stored as the traditional storage water pumps plants.

Ultracapacitors

In previous sections, general functioning of the ultracapacitors is described, they can storage small quantities of energy but delivered in high power rates. Energy stored inside of a ultracapacitor (E_c) follow the same principle that a conventional one. As equation (2.31) shows, it is function of its capacitance (C) and voltage between its poles (V)

$$E_c = \eta_c \frac{1}{2} CV^2 \quad (2.31)$$

Then, the global functioning is quite similar to a conventional BESS, its main difference is the necessity of using a DC/DC link, what increase the cost of the installation and reduce the efficiency of the global system. Most sophisticated modelling of these systems accounts the ageing of the capacitor, using the cycle-to-fail curves provided by manufactures. Those curves, usually takes into account only two parameters: Deep of Discharge (DoD) and C-rate [28]. The relation between those parameters and the ageing as a non-linear from depending on three parameters provided also by the manufacturer (α, β, γ) in each cycle n :

$$fol_c = \sum_n \frac{1}{2\gamma} (Crate_n)^\alpha (DoD_n)^\beta \quad (2.32)$$

Also, ultracapacitors see their maximum power flow limited due to a conservative C-rate ($\overline{Crate}$) that safeguard battery life. Then, restriction (2.33) is defined as a function of the duration of the cycle (ΔT_n), the energy that flows through the capacitor e_n^c as the energy rate of the capacitor ($\overline{e^c}$) and the maximum amount of C-rate.

$$\frac{e_n^c}{\Delta T_n} \leq \overline{Crate} \cdot \overline{e^c} \quad (2.33)$$

3.4.3. Consumption and generation products

Demand Response services provide mechanism for power balancing for BRPs and congestion management for DSOs and TSOs, as it provides flexible units able to modify their power consumption or generation for a given moment.

Also, Demand side response (DSR) modify end-user's behaviour to meet a condition that improve overall distribution system state. End-user's consumption are modified by agents in [29], using a scheme DSO - Aggregator - End-user, they offers contracts base on the change in its consumption. DR is defined as flexibility in [30] to avoid congestions, considering two directions: positive and negative. In the first one, prosumers increase their energy consumption or decrease their energy production while in the second, prosumers are required to decrease their energy consumption or increase their energy production.

DSR services for DSO is also illustrated in [31], where different end-users are modelled (as flexible storage, generation and loads units) within the context of a SESP, a central LFM.

DR services can be also provided in a multiservice context. This situation is proposed to be managed by aggregators in [8], where DSR is offered to BRPs and DSO to meet their requirements of portfolio optimization and grid improvement, respectively.

In terms of mathematical modelling, considering a time slot (Δt), the utility function for active flexibility products can be written as follows in equation (2.34). Let be $\lambda_{f,t}$ the price set for the active flexible product $\omega_{f,t}$:

$$U_{f,t} = \lambda_{f,t} \omega_{f,t} \forall f \forall t \quad (2.34)$$

Considering full controllability product restrictions remaining:

$$\underline{p}_f \leq p_{f,t} \leq \overline{p}_f \forall f, \forall t \quad (2.35)$$

Considering a product that the upward and downward flexibility cannot increase and decrease freely but increase within integer steps ($\psi_{f,t}$), it can be modelled considering a set of Ψ buffered loads of power p_f . Then $\omega_{f,t}$ represent flexibility offered by load:

$$\omega_{f,t} = p_f(\psi_{f,t} - \psi_{f,t}^*), \quad \forall t, \forall f, \psi_{f,t} = 0, 1, \dots, \Psi \quad (2.36)$$

$$\underline{\psi}_{f,t} \leq \psi_{f,t} \leq \bar{\psi}_{f,t}, \quad \forall f \forall t \quad (2.37)$$

Considering a shiftable flexible product, restrictions are same than the buffered load, but with another restriction that set equality between the total amount of product delivered.

$$\omega_{f,t} = p_f(\psi_{f,t} - \psi_{f,t}^*), \quad \forall t, \forall f, \psi_{f,t} = 0, 1, \dots, \Psi \quad (2.38)$$

$$\underline{\psi}_{f,t} \leq \psi_{f,t} \leq \bar{\psi}_{f,t}, \quad \forall f \forall t \quad (2.39)$$

$$\sum_{t=1}^T \psi_{f,t} = \sum_{t=1}^T \psi_{f,t}^*, \quad \forall f \quad (2.40)$$

Lastly, a curtailable flexible product is considered to have an on/off behaviour. Defining binary variable $\alpha_{f,t}$, curtailable flexible product is defined as follows:

$$\omega_{f,t} = p_f \cdot \alpha_{f,t}, \quad \forall f \forall t \quad (2.41)$$

Those flexibility products must be also accounted in power flow balance equations, and sold quantity needs to meet bought.

CHAPTER 3: CENTRALIZED LOCAL FLEXIBILITY MARKET

1. Introduction

Local Flexibility Markets appears in a context where energy has been exchanged in an upper-level market, the wholesale market, where the initial scheduling is elaborated. Energy and electrical services begin to be delivered considering clearing contracts of the superior levels markets. Thus, LFM appears in a different situation, where due to a random increase of distributed generation, or change in consumption pattern, previous accorded contracts cannot be maintained. Several grid issues can occur, from network congestion to voltage deviations or demand imbalance. The locality characteristic of the LFM allow to solve those issues with assets connected to the same distribution network.

Those local markets act in yellow state conditions, according to the Traffic Light Concept, defined by the German Association of Energy and Water. Their actions are directed to restore green state conditions into the distribution network, where no issues are present. Flexibility providers sends flexibility offers to the market facilitator which also collect the asks for flexibility from the interested parties.

In a context where the situation of the network is of highly importance, as depending on the problems that may appear on the network, a different description of the network itself will be needed. If voltage deviations are present, then the reactive power dynamics of the network must be modelled using a full AC-PF or a conic approximation. However, if the problems detected is related with balance issues, an approximate linear model can be used, modelling only the active behaviour of the network.

In this chapter, as a first approach to the LFM definition, a centralized fashion is proposed. Considering a distribution network operated by the DSO where a set of active end users is presented managed by aggregators. Between those prosumers, flexible loads (FLs), flexible generators (FGs) and Battery Energy Storage Systems (BESSs) will be modelled.

Each aggregator can manage their end-user's utility, considering the willingness of each one to participate in the market. On the other hand, DSO must maintain grid within safety limits in terms of voltage deviations and network overload. LFM operator, or Local Market Operator (LMO), naturally appears in centralized market definition as an entity that collects all the asks and bids of the participants and clear the market.

This chapter is organized in several sections, firstly describing the LFM characteristics, then analysing the state of the art in term of modelling of the agents, the network and the objective function seek in the mathematical definition. After that, different clearing mechanisms are investigated. Finally, the mathematical modelling of the grid is presented along with two illustrative examples and a use case. The introduction of the illustrative examples will provide a stylised first approach to the final market formulation, helping the reader to fully understand what is happening where a LFM is cleared.

However, previously to start explaining LFM characteristics, a overview of national and European electricity markets is done, with the objective to fully understand the situation of the LFMs an how they are related with the rest of the markets.

1.1. Overview of national and European electricity markets.

Firstly, in order to set a clear and organized view about how Spain electricity markets works, a description of the cleared markets is done from a sequential point of view with the help of reference [32]. The first market to be cleared is the Day-Ahead market, which set most of the energy schedule for the following day, as it solves 24 different hourly scenarios. Once the energy market is solved, then the TSO run a congestion analysis, using the Congestion Management market to modify generation dispatch to guarantee a secure operation considering network constraints.

Then, the Secondary Reserve market (associated with the Automatic Generation Control (AGC)) is carried out. Providing upward and downward regulation for the next 24 hours, each modified schedule is assigned to their respective generator. After this market, the intraday markets are called six time each day, allowing loads and generators to adjust their schedules before the energy is delivered. In the Spain electricity market sector, the most important one is the first intraday market, as solve all the infeasible schedules and it allows to apply strategic modifications, accounting most of the energy traded.

Then, in the fifth position, Tertiary regulation market is found. In practice, this market is only called if the secondary reserve is not enough to solve an infeasible situation or schedule, i.e., the secondary reserve is exhausted. Lastly, the Balancing Markets are used to dump significant energy deviations between generation and demand for the hours not covered by intraday market.

Figure 2 provides a scheme for the sequence of markets within a day. Additionally, future markets, also known as long-term markets, set agreements between agents with a time perspective of, weeks, years or even decades. In those markets, reservation of capacity or agreements to install new power plants and transmission lines are elaborated.



Figure 2. Sequence of market clearing according to Spain regulation.

A European classification of the electricity markets is summarized in Figure 3, where markets are organized considering the products traded, the time sequence and the type of payments.

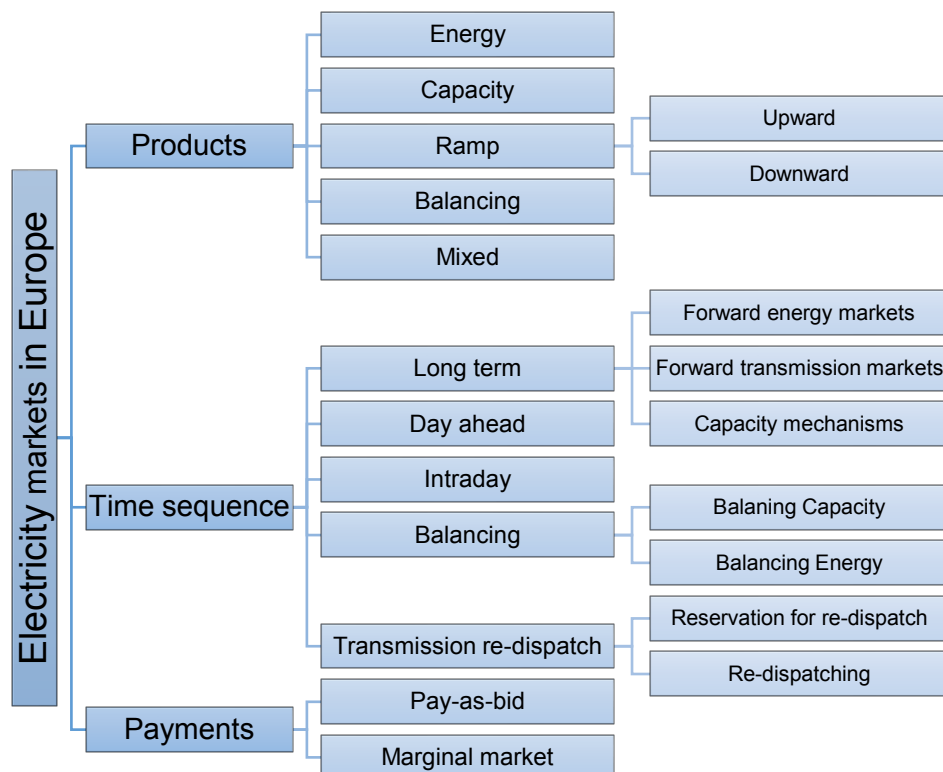


Figure 3. Types of European markets. Source: [33] and own production.

Depending on the products traded in the market, energy, capacity, and balancing markets could be distinguished. Energy markets set agreements between energy providers and consumers, usually in a day advance fashion (wholesale market). Secondly, capacity markets usually serve as a reservation of the capabilities of generators to solve future issues. Ramping products are usually traded in the energy markets as complex energy offers, this product consider the capabilities of the generators to modify their generation from one period to the following. Because of that, there are two different directions of those products, an upward direction and a downward, depending on the tendency of the generator. Additionally, mixed markets are also present in the European context, especially in balancing markets.

Like in the case of Spain, the time sequence of the contracts goes from long-term to balancing markets, as the Figure 3 shows. Long term markets involve three different types of mechanisms to create new infrastructure. Forward energy markets deliver energy contracts with weeks or months in advance, ensuring availability of energy when needed. Also, forward transmission markets put prices on transmission capacity of the power systems. Besides of that, there are also capacity mechanisms that put prices to the reserve of assets capabilities. Considering the daily operational viewpoint, the day-ahead and intraday markets behaviour is the same than the Spanish. However, in the case of the balancing markets, there are huge difference among member states, being possible to distinguish between Balancing capacity and Balancing energy markets. Lastly, as the last step before the real-time, Transmission re-dispatch markets modify the power flow and the schedule of the loads to avoid grid issues as congestions, voltage deviations or frequency issues. Inside this type of market, two different mechanisms are identified: reservation for re-dispatch and re-dispatch, depending on the knowledge the agents have about the situation [34].

In terms of the market payments, two different configurations are present: marginalist markets and pay-as-bids markets. The first one is typical of the wholesale market and balancing markets, as they compute the aggregated curves for demand and generation (in the case of energy markets), obtaining the crossing point where the prices and the quantity are set. In this type of market, the agents are incentivized to bid at their marginal cost if no market power is present. Secondly, the pay-as-bid markets usually rely on bilateral agreements set by a buyer and a seller. Generally, neither the price nor the quantity are widely known, as they are a type of industrial contract between parts.

Focusing on the organization of Ancillary Services (AS), Figure 4 shows a classification of those services. They can be unfolded in frequency control, voltage control services and black-start provision. Frequency control services are obtained from active power products while voltage control from reactive.

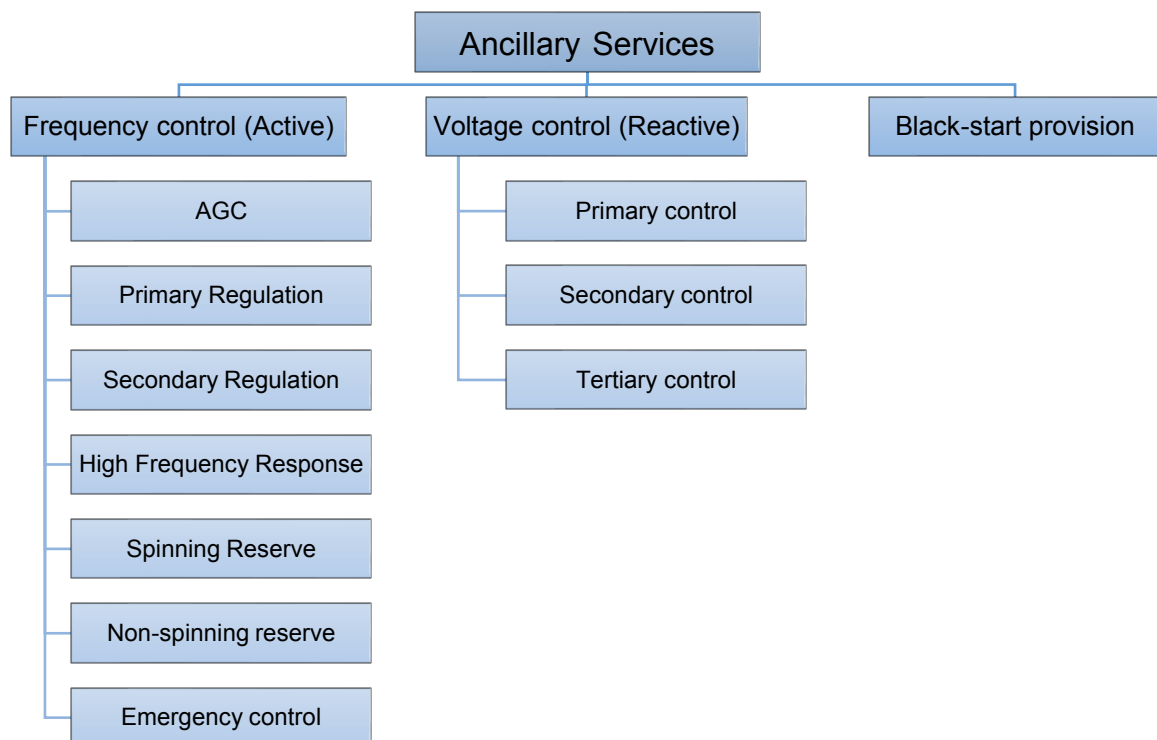


Figure 4. Classification of the traditional Ancillary Services in Europe. Source: [35]

Under the name of frequency control techniques, several AS can be englobed. Automatic Generator Control (AGC) is a system necessary in the generator output interface which adapts moment by moment the generation output to the load variations. This AS is compulsory to maintain grid stability and it acts in a time horizon no longer than 15 s. Primary and secondary regulation services are sequentially activated if the AGC is not sufficient to stabilize the system in frequency. The primary reserve can be activated automatically or manually, depending on the time horizon considered, activating the previously purchased reserve of generation to avoid system collapse. There are other services associated to the frequency, like high frequency response service for regulation of the generators in case of a high frequency variation appears momentarily. Other frequency services are spinning and non-spinning reserves, depending on the nature of the generator considered. If an electrical machine is present, its rotor inertia could be traded as a product as is possible to use it to maintain grid stability. Additionally, there are emergency control mechanisms are present in case of a catastrophe appears and affect to the grid stability.

In terms of the voltage control of the grids, three different mechanisms appear as the figure relates, depending on the type of activation of the reactive power asset. Primary control of the generators allows to maintain their voltage levels between a band, automatically. This system is highly investigated in the field of electrical machine drives. Secondary control is a close loop dedicated to maintaining the voltage of some nodes constant, a minimum set of nodes which ensure the full controllability of the whole system and the representative generators that could modify the voltage in those nodes are selected prior to the control [36]. Lastly, Tertiary control techniques optimizes the power flow reducing the active power losses [37].

Lastly, black-start provision services allow the system recovery from a blackout. This service is provided by the own power plants where no off-site power generation is available. Depending on the type of the power plant considered, different assets are used: from diesel generators in conventional power plants, to the own hydro-power plants if their electrical drives did not lose their excitation.

In the previously described AS markets, electricity products are traded, Figure 5 describe the main ones, considering if they are used for frequency control or voltage control [38].

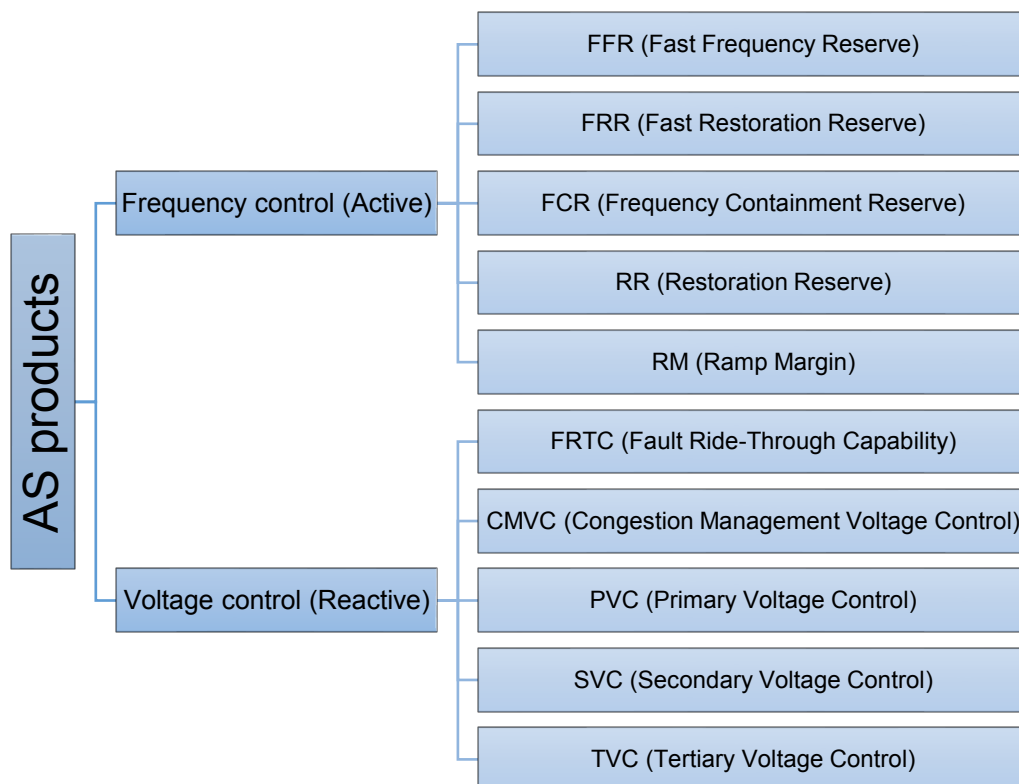


Figure 5. Classification of products traded in AS markets. Source: [38]

In the field of frequency control, FFR, FRR, FCR, RR and RM products are the main products traded among European markets. Firstly, FFR products are used when the inertia of the system is low, and a quick response is needed. Secondly, according to Article 3(2)(7) of the Network Code on System Operation, FRR product uses the active power reserves available to restore system frequency to the nominal frequency

and, to restore power balance to the scheduled value. Inside this denomination, automatic FRR (aFRR) and manual FRR (mFRR) products are distinguished, depending on the type of activation of the product and their time horizon. Besides of that, FCR intervenes automatically within seconds in the entire synchronous area to restore the balance between supply and demand. Additionally, according to Article 3(2)(8) of the Network Code on System Operation, RR are the active power reserves available to restore or support the required level of frequency to be prepared for additional system imbalances. Lastly, RM is defined as the gap present between the capabilities of a determined generation to modify their consumption and the current schedule. This gap is also used as a source of operational flexibility.

Considering voltage control products, PVC, SVC and TVC products are well aligned with the markets describes in the Figure 4. However, two additional products could be described as FRTC and CMVC. The first one is defined as the capability of a power plant or a generator to maintain connected to the grid during a fault in the public grid [39]. The second one oversees the voltage during a congestion management procedure, allowing to maintain the voltage within bounds.

1.2. Where do LFM appear?

Once a European perspective is described, it is timely interesting to understand where, inside all of this different and complex products and markets, do LFMs appear. LFMs are a type of AS market that provide services such as voltage control, congestion management and balancing services to DSO and BRPs from a local perspective, i.e. LFMs are organized in distribution feeders.

The locality properties of the markets are the most important ones because of their implications. A local market allows to solve problems locally, reducing power losses, and being possible to mitigate the problems associated to the introduction of more Renewable energy sources of generations without the usage of traditional centralized markets.

Their main objective is to serve as a secondary market, solving the remaining issues of the wholesale market schedule using local assets with little capacity requirements for participation. In this way, LFMs overcome one of the main shortcomings of the traditional electricity markets, allowing small scale assets to actively participate in the distribution network management. Those small-scale assets are incentivized to participate in this market in the same way as generators are in the traditional markets. LFMs provide a trade-off to agents aiming to trade their products and give services to BRPs and DSOs.

Then, LFMs appear inside a decentralized managing philosophy of the distribution network, after an initial schedule of upstream markets.

2. LFM characteristics

In a LFM several actors may appear, as previously described main agents are the DSO, the LMO, BRPs, Aggs and prosumers or end users. The LMO is the one who oversees the market in a centralized fashion, collecting asks and bids and solving the market. The DSO is typically, the owner of the grid where the LFM is cleared, and along with BRP, are the main buyers of the products exchanged in the market. Lastly, prosumers are the responsible of providing the products traded in the market, they can use Aggs as an interface between their assets and the market.

Since market is defined and cleared locally to solve DSO and BRPs issues, the principal characteristics of this type of market are described in the following list:

- Local market: market is developed in a distribution grid without any access restriction, empowering end users with flexible assets to freely participate, following directives of the European Union.
- Secondary market: LFMs are defined as secondary markets, cleared after wholesale market to solve network issues in the distribution feeders. Provide fundamental support to operation of the DSO in terms of reliability of the system.
- Forward market: market anticipate the state of the grid, solving voltage deviations by making future agreements between DSO, BRP and end-users.
- Pool market: market is cleared considering asks from BRPs and DSO, and bids from prosumers and aggregators. Inside this pool market, several product types could be solved together, the only aspects necessary to this end is the product price and the quantity willing to offer. Those products characteristics are collected by the market facilitator along with the asks of the BRPs and DSO to finally clear the market.

In a centralized version, LMO collects asks and bids from stakeholders and run an optimization problem to clear the market, matching transactions between buyers and sellers. Hence, buyers can acquire different flexibility products with different characteristics to solve network problems. As network is modelled, then nodal prices can be determined, and same product may differ in price depending on its location due to network congestions.

Considering reference [13] an example of how a LFM is organized is presented in Figure 6 where a DSO and a BRP and several aggregators trade flexibility locally under supervision of the LMO. In this framework the DSO buy flexibility products for voltage control, congestion management and power losses reduction, while BRPs buy those products for optimizing their portfolios and reduce their imbalance volumes. In this situation, Aggs sell flexibility for profits by modifying the electricity usage patterns of the assets they manage.

In order to understand deeply how the market works, a summarized list of the actions that occurs during the LFM clearing is described below:

1. Contracting and bidding process: DSO, BRP, aggregators and LFM operator communicates with each other: DSO runs loads analysis for constraints, if positive, demands to LFM flexibility, similarly does BRP. Using forecasts LFM will send necessities of flexibility to aggregators that will offers flexibility bids to the LFM operator. Having received all bids and offers market is cleared.
2. Activation process: DSO and BRP activate their procured flexibility by sending a request to the LFM operator.
3. Settlement process: flexibility transactions are completed through settlement arrangements and payment among the DSO, BRP, LFM operator and aggregators.

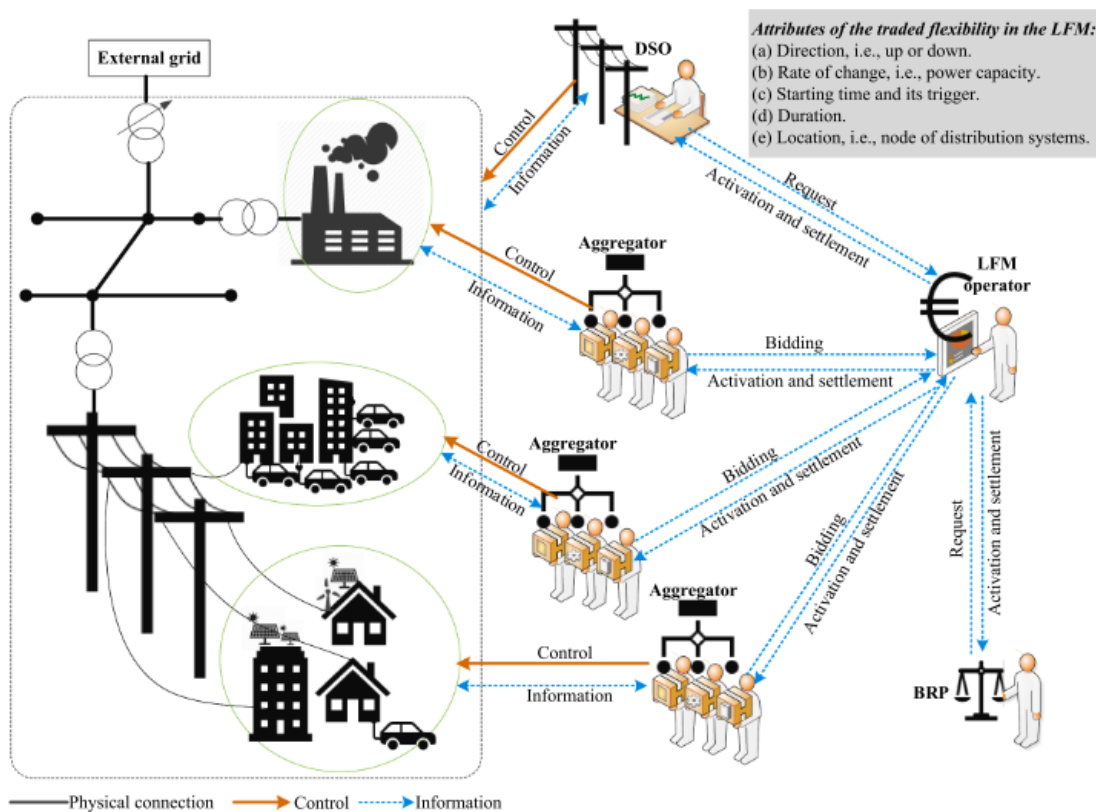


Figure 6. Scheme of the LFM with DSO, BRP and Aggregators. Source: [13]

Regarding the operational viewpoint, LFMs are characterized for being organized in a four-layer scheme. This scheme is composed by the market layer (determines how flexibility is locally traded), the control layer (control strategies are facilitated in this layer to control and optimize the operation of various DERs) the ICT layer (composed by communications tools and protocols to support information flow in the LFM) and the power grid layer (composed by all physical components of the grid).

After the general definition of the characteristics of the LFMs, in the rest of the section the European regulatory framework and some national and European pilot projects are explained.

2.1. European Regulation on local market designs

From European perspective, National Regulatory Authorities (NRAs) carefully evaluate the interactions when implementing new forms of access to flexibilities or when enhancing existing ones.

Guiding principles has been published in CEER report [19], non-discriminatory principle prevails thought the report, for that reason, monopolistic activities must be avoided. For instance, congestion management could lead to those practices, as DSO unilaterally modify network state for its own benefit.

For CEER, the market design should facilitate an optimal valuation of flexibility from a system point of view, enhancing free willingness of those who participate creating the highest benefit, avoiding blocking, or hindering access. DSOs should be able to access to flexibility services when flexibility solution is the most economical solution [40]. These principles have been also identified in directive 2019/943 of the European Union, equity, transparency and non-discriminatory principles are the basis of that EU directive, considering them as a vehicle to a more reliable, decarbonized and sustainable electricity network [41].

Although market-based design for flexibility procurement is identified as the most efficient way of LFM some regulatory tools must be considered. CEER consultant found Price or Revenue Control, Economic Incentive Schemes for DSO, Smart Metering, Regulatory Framework for Tariff Structures and Contractual Arrangements to be useful tool for removing barriers and facilitate flexibility exchanges in the distribution network [19].

LFM is considered as a balancing market as long as it resolves issues of the grid as congestions, imbalances, voltage deviations or even generation allocation. Rules of the balancing markets are defined in directive 2019/943 of the EU, being the most important ones:

- Market participants should determine its service price using marginal pricing; thus, they cannot sell at loss, and imbalance prices should reflect the real-time value of energy.
- Balancing prices cannot be pre-determined.
- Market participant shall be allowed to bid as close to real time as they possible can (before closure of intraday market gate). Considering Decision n. 18/2020 of 15 July 2020, Imbalance Settlement period will be unified to 15 min across EU Members before Q2 2023 [42].
- Balancing capacity shall be based on a primary market.

2.1.1. Pricing for balancing energy

LFMs must define prices for balancing energy exchanged or for operating the imbalance netting process. Based on European regulation (Art. 30 of Directive 2017/2195), price definition must address several rules:

- a. Be based on marginal pricing.
- b. Reflect market congestion or any other network issue.
- c. Technical price limits are needed for efficient functioning. In those cases, based on regulation 2015/1222, maximum and minimum clearing price for day-ahead and intraday timeframes are used as limits.
- d. No apply any additional charges except the one that account the losses.

Based on European technical requirements, a dynamic price mechanism for losses allocation could be on public interest. In this aspect, several techniques are available in the literature. Starting from Grid Usage Prices (GUP) based on the account of technical losses, to more sophisticated techniques that may require solve additional optimization problems.

2.1.2. Traffic Light Concept

Traffic Light Concept was an approach elaborated by German Association of Energy and Water Industries for a better utilization of Smart Grids, avoiding grid reinforcement and widely used by literature. This concept relates to the use of network flexibility, determine whether it may be necessary.

Mainly, there are three network phases, depending on the level of congestion or the presence or not of grid issues as voltage deviations or congestions.

- Green phase: no critical network situation exists, network operators do not participate in the market and transactions take place exclusively between non-regulated market participants (as end users, or aggregators).

- Amber or yellow phase: In this phase, there is a potential or actual network shortage and network operator must remedy the situation by the usage of operational flexibility offered by market participants. In distribution networks, DSO would ask for flexibility in a local market, where different offers from participants will solve grid issues.
- Red phase: there is a direct risk to network stability and TSO and DSO interact in order to guarantee the integrity of the system. In the process some loads may be disconnected or even generators if necessary. As soon as the network operator must use non-market-based control tools, state changes from yellow to red.

Usually, most of the time grid will operate in green phase. But, due to a random variation of the consumption or distribution generation situation may change to amber state. Amber state is a very interesting phase for LFM as can offer flexibility to DSO or BRP to take the grid back to green state [43].

2.2. Pilot projects

In this section pilot projects that investigate the implications of developing a LFM from an operational viewpoint are explained.

Aligned with Horizon2020 objectives of the European Union, several pilot projects were founded by European, and Spanish funds. Currently, Spain is developing IREMEL project (*Integración de Recursos Energéticos a través de Mercados Locales de electricidad*). Also, European Union has found other different project but with similar characteristics. Notably DOMINOES, MERLOW, InterFlex, EMPOWER and DREAM-GO.

2.2.1. IREMEL project

Spanish project IREMEL aims to integrate renewable energy resources through the local energy markets. Use cases of the projects aims to organized both pool market and continuous market with shorter clearing periods. Market address three main mechanisms for DSO to obtain flexibility [18]:

1. Congestion management thought limitations to schedules.
2. Local flexibility products that would not modify generation and consumption.
3. Creation of long-term services for recursive problems. In this situation, its associated products will be hired considering a fixed rate (from pool price) and a variable cost associated to the activation of the flexibility.

2.2.2. DOMINOES

Main objective of the DOMINOES project is to introduce a continuous local marketplace for energy and flexibility products trading. One of the key features for unleashing the flexibility potential at distribution level resides in demand side response (DSR), having been identified by ENTSO-E [14], allowing the network loads to modify their consumptions for the sake of system stability and security.

Regarding products being traded, energy, flexibility products and frequency containment reserves were selected. Additionally, flexibility was mainly obtained by means of DSR, with no distinction between sources.

Payments were configured as pay-as-cleared, embracing the importance of the marginal costs when considering market techniques. Another characteristics of this market is that flexibility offers cannot be partially accepted. Those flexibility products were defined with 7 parameters: earliest start time, start time, duration of the flexibility, maximum profile, minimum profile, latest start time and latest end time. However, from market perspective, the parameters used in the market were:

- Minimum and maximum bid size (kWh & EUR).
- Location in the grid (node).
- Maximum delay between signal arriving and activation.
- Maximum price (EUR/kWh).
- Date and time of delivery.

2.2.3. MERLON

This project currently investigates in the integration of IES (Integrated Energy Systems) in local marketplaces, focusing on households' assets: lightning, storage systems, HVAC, water heaters and photovoltaic production of energy [15]. As it is on industrial development little information is revealed.

2.2.4. InterFlex

Main objective of the InterFlex project is to create a local energy market, enabling both centralized and decentralized versions with a peer-to-peer approach. Actors that participate in this market are Aggs, Industrial customers, and Local Energy Communities, being possible to create P2P platform in this context. Market is solved considering three different time intervals: 2 min, 15 min and 60 min, depending on the type of product and services offered. However, not all the assets can freely participate in the market. For example, EV are incentivized to relax its behaviour through Time of Use (ToU) tariffs instead of participate in the market [16].

2.2.5. EMPOWER

This project studied the implications of having the Agg figure as the center of the LFM, enabling different services at distribution level [8]. Mainly, products traded in this market were obtained from DSR services offered by flexible loads, storage, and generators. DSO was the only buyer in this project, buying flexibility products when a congestion or a voltage deviation were presented in the network.

2.2.6. DREAM-GO

DREAM-GO project define local energy markets with DSR considering a stochastic viewpoint [11]. Transactive energy is provided from curtailable, shiftable, interruptible loads and batteries and electric vehicles. Stochastic modelling considering probability of availability of flexibility products to solve the market. This project is slightly different from the others considered, as it what is organized is a energy market, not an LFM, however, some of the lessons learned from this project could be applied to LFMs definition.

3. Modelling of the agents involved in the LFM

As stated in the previous section, the main agents involved in the LFM operation are the DSO, LMO, the BRPs, the Aggs and the end-users (EU). In addition, for correctly solve market operation, network definition will be considered depending on the necessities of the region considered, being necessary a more complex definition if voltage deviations are present.

The LFM is cleared by means of using a mathematical description of agents and assets involved in the market, i.e., a market model. For that reason, in addition of the products defined in the previous chapter, network and behaviour function of rationale agents must be addressed.

In this section, LMO, DSO, Aggs, BRPs and EUs modelling techniques are analysed from a theoretical perspective considering the published literature.

3.1. Local Market Operator (LMO)

Local market operator is an entity able to coordinate the market in a centralized fashion. It collects asks and bids from stakeholders, organizing them and clearing the market. Owing to that, it will have access to all information shared with the market. However, this information will not be shared with the rest of the participants, as LMO is an independent entity with confidential agreements with each participant.

Central entities are proposed as a traditional approach for clearing LFM. Market operator responsibilities are defined as providing a bidding platform, clearing the market and coordinate bidding direction from BRPs, DSO and Aggregators [30]. Similar responsibilities are defined thought the Smart Energy Service Provider (SESP) whose definition is based on the Smart Grid Architecture Model (SGAM) developed by CEN, CENELEC and ETSI. This SESP uses the Traffic Light Concept (TLC) and the market is only enabled in yellow light situation, when DSO needs flexibility to overcome a network issue and go back to green light situation [31].

3.2. Distribution System Operator (DSO)

The DSO is the owner of the grid and is responsible for its operation within safety limits. Voltage deviations and grid congestions will eventually occur, and DSO is responsible for fixing them. Grid reinforcement will be classical approach for solving those problems. However, recent approaches place local markets as a forward way of solving grid issues without grid reinforcement.

In this context, DSO is incentivized to asks for local market products. In short-term operation, those products will solve grid constraints. From a large term perspective, DSO can hire flexibility services of assets

connected to distribution network to avoid grid reinforcements through the programmed purchase of products associated with those services. This approach is interesting for repetitive problems, as could be congestion at peak hours.

The DSO is also responsible of the maintenance of the power system and to transport energy to consumers in an efficient, secure, and sustainable and cost-effective way. DSO can be modelled as a function of the costs that are involved in:

$$C_{DSO}^t = \beta_d \cdot |\rho_{d,DSO} \times q_{d,DSO}^t|, \quad \forall t \in \Omega_t \quad (3.1)$$

Considering β_d as a binary variable that activate bid of a given Programable Time Unit (PTU), $q_{d,DSO}^t$ is the amount of flexibility purchased at price $\rho_{d,DSO}$ for a given bid (b) and time slot (t) in the time horizon (Ω_t). Multiplication product denotes the total budget of the DSO, and market will not clear if offers from aggregators overcame it [30]. As DSO is in charge of the maintenance of the voltage within safety limits, grid-less approaches force this entity responsible of the validation of the asks and bids when matched [29].

DSO will ask market for flexibility products to solve its grid issues. For that reason, utility function of the DSO is necessarily defined as a monotonously decreasing function. Considering b_{DSO} as the maximum cost willing to pay for products and $a_{DSO} \in \mathbb{R}^-$ the slope of the utility function, for a quantity of p_b , utility function:

$$C_{DSO}(p_b) = b_{DSO} + a_{DSO}p_b \quad (3.2)$$

3.3. Aggregators (Agg)

Generally, at distribution level, single user cannot offer enough flexibility to solve grid problems or balance the grid. For that reason, aggregation is an opportunity for those small users who want to participate in the market, to obtain benefit of its flexible assets, providing them opportunities to act as a single entity with capabilities of solving network issues. In this context, the figure of the independent aggregator (IA), as defined for the European and Spain regulation, - or simply aggregator (Agg)- appears. Aggregator will collect offers from its subscribed prosumers in different location of the grid to package their offers in a single distributed product able to solve grid issues.

Aggregators are used as intermediators when providing flexibility in [29], as long as they collect flexibility offers from end-users and can receive flexibility requests from DSO in a aggregator centric approach. Similar scheme is proposed in [44], where aggregator buys energy from different types of Electricity Suppliers (ESs) and sell it to traditional users. Also, other roles are defined, as it can engage bilateral transactions between BRPs, the DSO or other parties if defined [30]. Using this definition, aggregator utility could be defined as a function of the energy sold and bought from ESs:

$$U_A = \sum_{i \in TC} R_i - \sum_{j \in ES} P_j \quad (3.3)$$

Considering R_i as the revenue of the transaction i in traditional consumers (TC) and P_j as the price of the energy brought to ESs. Other constraint related to flexibility procured is addressed in [30], enforcing to be higher than or equal to the flexibility requested by the DSO:

$$f_d^t \leq \sum_{a \in \Omega_A} \sum_{p_a \in \Omega_a^P} b_a^{p_a} \cdot q_a^{p_a,t} \quad (3.4)$$

Where Ω_A is the group of all aggregators, Ω_a^P is the group of offers (p_a) emitted by the aggregator (a), $b_a^{p_a}$ is the binary variable associated with the activation of the offer p_a and $q_a^{p_a,t}$ is the amount of flexibility requested by DSO from LFM in a given time t .

Aggregator could also be used as a central entity in terms of services providing and procuring different services to other LFM participants [8]. This approach enables aggregators to provide congestion management, voltage control, portfolio optimization, ToU tariffs, maximum power consumption control and even self-balance services to DSO, BRPs and Prosumers.

However, Aggregators can effectively develop a role even if they are another stakeholder of the market through management of assets of the small prosumers and provide distributed products to the market.

3.4. Balance Responsible Parties (BRPs)

BRPs are those agents responsible of the instantaneous balance of the generation and consumption of the grid. These agents will interact with the market in those moments where, due to imprecision of the forecasts or a random fluctuation of the consumption and generation, grid balance cannot be maintained anymore.

BRPs will interact with the market in an independent way of the DSO, what could end with bids in opposite directions. Market will necessarily handle those situations and clear the market under safety conditions.

They are stakeholders of LFM as they can improve its portfolio by actively participating in the market. BRPs operation costs is defined in a similar way than DSO's but considering related to the fact that this entity could request flexibility in the DSO opposite direction. Then:

$$C_{BRP}^t = \sum_{b_o \in \Omega_{Bo}} \sum_{p_{b_o} \in \Omega_{b_o}^P} \beta_{b_o}^{p_{b_o}} C_{b_o}^{p_{b_o},t} + \sum_{b_s \in \Omega_{Bs}} \sum_{p_{b_s} \in \Omega_{b_s}^P} \beta_{b_s}^{p_{b_s}} C_{b_s}^{p_{b_s},t} \quad \forall t \in \Omega_t \quad (3.5)$$

s subscript denote that BRP's bid is in the same direction than DSO's, and o , that is in the opposite direction, $\beta_{b_s}^{p_{b_s}}$ is the binary variable associated to a given PTU for the p_{b_s} amount of flexibility, and $C_{b_s}^{p_{b_s},t}$ is the associated cost to it for a given time slot (t). Lastly, Ω denotes the variable space domain [30].

3.5. End users (EUs)

End users are the agents that are connected in the power distribution network, willing or not to participate in the market. On the one hand, static users are those who do not contribute with flexibility products, and its behaviour cannot be modified. On the other hand, those end users who are willing to actively participate in the market are named prosumers. Prosumers can modify the behaviour of their assets in response of market signals (prices, bids, asks), enabling them to offer product and services to DSO and BRPs.

End users are considered the owner of assets connected to the distribution grid: renewable generators, batteries, electric vehicles, flexible and inflexible loads. Those assets could be directly managed by themselves or aggregated with other assets by means of an aggregator. Depending on the type of assets it will provide different products to the market, from flexibility to storage products.

End-users may be classified attending to their interactions with the network as end-users with only consumption, with consumption and generation and with consumption generation and storage. Those types are covered in different manner in the literature.

In [45] two types of prosumers are modelled: one that have only PV and other with PV and battery storage that uses HEMS (Household Energy Management System). HEMS is obtained considering a mixed-integer linear programming (MILP) problem. Considering p_t^+ as the price of purchased energy for a given time slot t , p_t^- as the price of energy sold, and e_t^+ , e_t^- as the buying and selling energy for a given time slot t :

$$\min \sum_{t \in T} (p_t^+ e_t^+ - p_t^- e_t^-) \quad (3.6)$$

Subject to device operation constraints and energy balance constraints for each time slot (t).

Energy Management Systems (EMS) are required to successfully implement price based DSR programs in practice. Diversity of the Rational End Users responses when receiving the time varying electricity prices are modelled using power function to model the costumer's willingness ($a_{t,j}^\alpha$) [46].

$$a_{t,j}^\alpha = \begin{cases} \bar{a}_j^\alpha \left(\frac{\Delta \lambda_t^\alpha - \Delta \lambda_j^\alpha}{\Delta \lambda_j^\alpha - \Delta \lambda_j^\alpha} \right)^\gamma & \underline{\Delta \lambda_j^\alpha} \leq |\Delta \lambda_j^\alpha| \leq \overline{\Delta \lambda_j^\alpha} \\ \bar{a}_j^\alpha & |\Delta \lambda_j^\alpha| \geq \overline{\Delta \lambda_j^\alpha} \end{cases} \quad (3.7)$$

The six parameters defining the dead-band and saturation limits are modelled as normal distributed random variables, as they represent aggregated human behaviour. This function is showed in Figure 7.

End-users' flexible installations typically have generation and/or storage systems, i.e., PV system with batteries to storage their energy surplus. If this type of facilities can be controlled automatically, they could provide flexibility services to a flexibility market design.

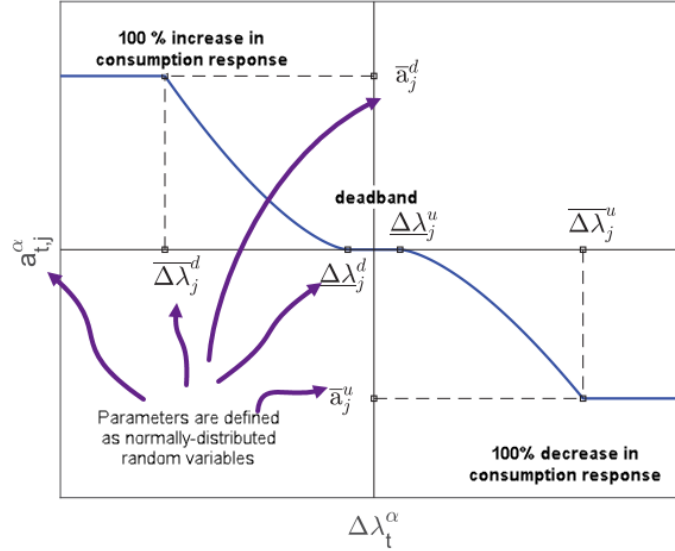


Figure 7. Willingness parameters for time-varying electricity price. Source: [46] with author notes.

Flexible generators may be divided into curtailable ($g \in \Omega_{G_c}$) and disconnectable generators ($g \in \Omega_{G_d}$). First type provides on-demand generation within a limit, while the second one can only be switched on and off. They can be defined as showed in equations (3.8):

$$0 \leq p_{g,t}^{\Omega_{G_c}} \leq \bar{p}_{g,t}^{\Omega_{G_c}} \quad \forall g \in \Omega_{G_c}, \forall t \in \Omega_t \quad (3.8)$$

$$p_{g,t}^{\Omega_{G_d}} = \beta_{g,t}^{\Omega_{G_d}} \cdot \bar{p}_{g,t}^{\Omega_{G_d}} \quad \forall g \in \Omega_{G_d}, \forall t \in \Omega_t \quad (3.9)$$

Considering p as the amount of power delivered from a generator g and a time t , $\bar{p}$ as the maximum limit of its output for a given generator g and a time t , β as the binary variable for a disconnectable generator. Flexible generators curtailment needs to be paid using a curtailing fee agreed by contract [31].

On the other hand, battery modelling follows classical State of Charge (SOC) equations. Considering charging and discharging decision variables $v_{g,t}^c, v_{g,t}^d$, state of charge for a given moment t as $soc_{g,t}$, battery capacity (B_g) and charging and discharging performance as η_g^c, η_g^d :

$$soc_{g,t} = soc_{g,t-1} + \eta_g^c \cdot v_{g,t}^c - \eta_g^d \cdot v_{g,t}^d, \quad \forall t > 1 \in \Omega_t, \forall g \in \Omega_b \quad (3.10)$$

$$soc_{g,1} = soc_{g,0} + \eta_g^c \cdot v_{g,1}^c - \eta_g^d \cdot v_{g,1}^d, \quad \forall g \in \Omega_b \quad (3.11)$$

Limits of operations should be defined as the maximum possible energy storage, and maximum charge and discharge amounts (MC_g, MD_g):

$$b_g \leq soc_{g,t}, \quad \forall g \in \Omega_b, \forall t \in \Omega_t \quad (3.12)$$

$$v_{g,t}^c \leq MC_g, \quad \forall g \in \Omega_b, \forall t \in \Omega_t \quad (3.13)$$

$$v_{g,t}^d \leq MD_g, \quad \forall g \in \Omega_b, \forall t \in \Omega_t \quad (3.14)$$

End-users' loads can also provide flexibility. Flexible loads can effectively adapt their demand to requirements. Considering CENELEC classification [47] based on its controllability, loads can be divided in 5 groups:

1. Freely controllable: This type of load can be modified on-demand without limits. Non-realistic.
2. Buffered: Loads that have inertia and its consumption can be modified within limits.
3. Shiftable: load profile that can be shifted for a given time period without varying its behaviour.
4. Curtailable: load profile that can be decreased or disconnected on demand.
5. Uncontrollable: inflexible load profile.

Shiftable and curtailable loads are used in [31]. Curtailable loads power ($p_{l,t}^c$) is the decision variable that curtails or disconnect the load. These loads need to meet maximum power constraints ($\bar{p}_l$) and curtailable limits ($\bar{p}_{l,t}^c, \underline{p}_{l,t}^c$):

$$0 \leq p_{l,t}^c \leq \bar{P}_l, \forall t \in \Omega_t, \forall l \in \Omega_l^c \quad (3.15)$$

$$\underline{P}_{l,t}^c \leq p_{l,t}^c \leq \bar{P}_{l,t}^c, \forall t \in \Omega_t, \forall l \in \Omega_l^c \quad (3.16)$$

Shiftable load profile needs a more elaborated model to be operated. For the sake of clarity, only one shift is allowed. Considering start and end time of the appliance ($T_{l,s}^{start}, T_{l,s}^{end}$) for a given shiftable period s , $\rho_{l,s}$ as decision variable that define starting time for shiftable load and $\beta_{l,t}$ as a binary variable corresponding to starting period:

$$\sum_{t=T_{l,s}^{start}}^{T_{l,s}^{end}-\rho_{l,s}} \beta_{l,t} = 1, \forall l \in \Omega_l^s, \forall t \in \Omega_s(t) \quad (3.17)$$

Considering $n_{l,t}$ as the new load profile and $P_{l,t}^{SP}$ as the starting profile for a given load l and a time t . Same profile is ensured using following constraint:

$$n_{l,t} = \sum_{i=0}^{T_{l,s}^{start}-T_{l,s}^{end}} \beta_{l,t} \cdot P_{l,T_{l,s}^{start}+i}^{SP} \quad \forall t \in [T_{l,s}^{start}, T_{l,s}^{end}], \forall l \in \Omega_l^s, \forall s \in \Omega_s(t) \quad (3.18)$$

Buffered loads (p_l^f) can be modelled as a curtailable load that has an inertia modelled in terms of the energy stored. Thus, equations of curtailable loads and battery state-of-charge needs to be joined to be modelled. Typical buffered loads are thermal loads, as water heaters, cooling, and heating systems.

4. Network modelling

When modelling distribution network, linear or non-linear approaches could be made. Depending on the magnitudes of interest of the network three different models could be used:

- Linear modelling: Considering no undervoltage or overvoltage problems, it is possible to assume that voltage is near to one ($V_i \approx 1$). If, also, network losses are assumed to be negligible, thus line susceptance can be assumed to be much greater than line resistance ($R \ll X_L \Rightarrow G_{ij} \approx 0$).
- Non-linear modelling: If voltage deviations and network losses must be addressed, then linear modelling of the network will not be sufficient. Then, complete AC or a conic power flow is necessary.
- Modelling based on sensitivity parameters: LFMs are secondary markets operating near the set point defined by wholesale markets. Owing to that, sensitivities can offer useful information about the behaviour of the network with respect to the introduction of a product in the secondary market.

4.1. AC Power Flow equations

Considering π -model of the lines, AC power flow equations can be described as follows. Let $i = \{1, 2, \dots, N\}$ be the set of nodes of the network, V_i node voltage, θ_i node voltage angle, G_{ij} , B_{ij} , b_{ij}^s conductance, susceptance and shunt susceptance associated to the line $i - j$, respectively, and P_j , Q_j active and reactive power injections at node j , respectively. Using polar form of the power flow problem, then:

$$V_i \sum_{j \in i} V_j \cdot (G_{ij} \cos(\theta_{ij}) + B_{ij} \sin(\theta_{ij})) = \sum_j P_j, \quad \forall i \in \Omega_i \quad (3.19)$$

$$V_i \sum_{j \in i} V_j \cdot (G_{ij} \sin(\theta_{ij}) - B_{ij} \cos(\theta_{ij})) = \sum_j Q_j, \quad \forall i \in \Omega_i \quad (3.20)$$

$$\theta_{ij} = \theta_i - \theta_j, \quad \forall i \in \Omega_i \quad (3.21)$$

Considering power flows through the lines, P_{ij} , Q_{ij} flows are calculated for line $i - j$:

$$P_{ij} = V_i V_j (G_{ij} \cos(\theta_{ij}) + B_{ij} \sin(\theta_{ij})) - G_{ij} V_i^2, \quad \forall i \in \Omega_i \quad (3.22)$$

$$Q_{ij} = V_i V_j (G_{ij} \sin(\theta_{ij}) - B_{ij} \cos(\theta_{ij})) - (B_{ij} - b_{ij}^s) V_i^2, \quad \forall i \in \Omega_i \quad (3.23)$$

Line conductance and susceptance can be calculated from cable parameters:

$$G_{ij} = \begin{cases} -\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}, & i \neq j \\ \sum_{j \in i} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}, & i = j \end{cases}, \quad \forall i \in \Omega_i \quad (3.24)$$

$$B_{ij} = \begin{cases} \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}, & i \neq j \\ \sum_{j \in i} \frac{b_{ij}^s}{2} - \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}, & i = j \end{cases}, \quad \forall i \in \Omega_i \quad (3.25)$$

Also, network is subject to node voltage and line capacity transmission restrictions. Defining $\bar{V}_i$, $\underline{V}_i$ as the maximum allowed node voltage, $\bar{\theta}_i$, $\underline{\theta}_i$ as maximum and minimum allowed voltage angle for node i , and $\bar{S}_{ij}$ as the maximum transmission capacity of the line $i - j$:

$$S_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2} \leq \bar{S}_{ij}, \quad \forall i \in \Omega_i \quad (3.26)$$

$$\underline{V}_i \leq V_i \leq \bar{V}_i, \quad \forall i \in \Omega_i \quad (3.27)$$

$$\underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i, \quad \forall i \in \Omega_i \quad (3.28)$$

This modelling defines a non-convex and non-linear restrictions. Primal-dual methods needs a convex problems in order to be applicable [48]. This process is proven to be exact for radial networks, for that reason second-order-cone AC OPF is used in [49]. Radial distribution network is also considered in [45]. Second-order conic constraint that convexifies AC-OPF problem is:

$$\frac{p_l^2 + q_l^2}{I_l^2} \leq v_{s,l} \quad (3.29)$$

Considering p_l and q_l as the active and reactive power flow of distribution line l , I_l current flow of distribution line l and $v_{s,l}$ as the voltage at the sending bus of distribution line l .

4.2. Linear network modelling

Considering voltage and power losses simplifications, linear network modelling can be achieved by using equations (3.30) and (3.31):

$$\sum_{j \in i} B_{ij}(\theta_i - \theta_j) = \sum_{j \in i} P_j, \quad \forall i \in \Omega_i \quad (3.30)$$

$$P_{ij} = B_{ij}(\theta_i - \theta_j), \quad \forall i \in \Omega_i \quad (3.31)$$

Voltage restrictions are not defined as they are assumed to be constant, due to the assumption that the active power losses are negligible ($G_{ij} \approx 0 \rightarrow V_i = 1 \text{ p.u.}$). However, voltage angle and line capacity constraints remain invariable if equations (3.32) and (3.33) are considered:

$$S_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2} \leq \bar{S}_{ij}, \quad \forall i \in \Omega_i \quad (3.32)$$

$$\underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i, \quad \forall i \in \Omega_i \quad (3.33)$$

4.3. Sensitivity based modelling.

In order not to solve an Optimal Power Flow (OPF) problem for each iteration of the LFM, Voltage Sensitivity Coefficients (VSCs), Power Transfer Distribution Factors (PTDFs) and Loss Sensitivity Factors (LSFs) can be incorporated in the market mechanism to block high risk operations and allocate losses in the transactions using these parameters. These coefficients ensure operate network within safety limits as demonstrated in reference [45]. Those sensitivity parameters are defined in equations (3.34), (3.35) and (3.36), respectively, considering the previously settled wholesale market.

VSCs: is obtained using Jacobian Matrix after solving Newton-Raphson power flow as:

$$\Delta|V_i| = \frac{\Delta P_k}{|V_i|} \operatorname{Re} \left(V_i \cdot \frac{\partial V_i}{\partial P_k} \right) \quad (3.34)$$

PTDFs: PTDFs give an idea of the amount of change in the power flow respect to the injection at bus i and a withdrawal of the same amount at the bus j for branch (k, l) .

$$\Phi_{kl}^{ij} = (\tilde{B}_{kl}AB^{-1})_i - (\tilde{B}_{kl}AB^{-1})_j \quad (3.35)$$

LSFs: LSFs give an idea of the variation of the power losses for a given change in power injected at node k . Considering an exchange between node i and j , bilateral exchange coefficient (BEC) is defined as:

$$BEC^{ij} = \frac{\partial P_{loss}}{\partial P_i} - \frac{\partial P_{loss}}{\partial P_j} = 2 \left[\operatorname{Re} \left(V^{*T} G \frac{\partial V}{\partial P_i} \right) - \operatorname{Re} \left(V^{*T} G \frac{\partial V}{\partial P_j} \right) \right] \quad (3.36)$$

4.4. Transaction Losses

Allocation of losses in each transaction is one of the major problems while studying decentralized transactions, as buyers will not receive exactly what they have bought due to transmissions losses. This allocation could be computed using power flow, assuming that there are no generation, measuring the total power imported in a first stage. Later, a power flow study with the hypothetical decentralized markets solutions is performed and difference in the imported power is the total losses associated with all P2P transactions [50].

Other technique includes Distributed Location of Marginal Prices (DLMPs) as a mechanism to settle transaction prices among sellers and buyers. Anthony Papavasiliou proposes three methods to allocate DLMPs by decomposing them: using a duality analysis of the PF problem, using duality analysis of a second-order cone program relaxation and through an analysis of the impact of marginal losses on price [51]. An application of DLMPs is shown in [49], where network usage charge for each trade (ω) between buyer (b) and seller (s), is computed using dual variables (λ) and it is equally allocated between both agents:

$$c_{\omega}^n = \frac{\lambda_{b(\omega)} - \lambda_{s(\omega)}}{2} \quad \forall \omega \in \Omega \quad (3.37)$$

5. Objective function

In this section the different approaches studied by the literature in terms of defining the objective function of the LFM formulation are described. Social Welfare, Cost minimization and, multi-objective optimization definitions are the most remarkable definitions in relation with these markets, and are the ones selected for further investigation.

5.1. Social Welfare

Social Welfare maximization problems are used in optimal-contract calculation problems where several types of agreements are previously defined and activated when flexibility is needed [52].

In a peer-to-peer (P2P) energy trading problem, social welfare is maximized in the system-centric configuration as the difference between the utility functions of consumers and cost functions of producers. This maximization is proposed in a new paradigm of energy business, shifting from the volumetric-centric approach of the current markets to a service-based business model [49].

Another approach seeking to maximize social welfare of the participants is proposed in [10], where a P2P market architecture is proposed parallel to the Ancillary Service Market. This approach is coordinated by DSO considering Grid Usage Pricing that incentivizes the P2P market to meet restrictions imposed.

Social Welfare maximization is performed in local day-ahead and intraday markets, as the difference between benefits and costs [30]. Other approaches seeking to determine price for voltage management also propose social welfare maximization problem in a context of local energy trading [53].

Broadly, energy markets maximize social welfare of the agents involved. Social welfare can be defined as the difference between the utility function of the buyer side and the utility function of the producer side. When DERs are included in the market, objective function can be formulated as a social welfare maximization as equation (3.38) described [45].

$$\max SW = \sum_{i \in N} C_i^c P_i^c - \sum_{i \in N} C_i^p P_i^p \quad (3.38)$$

Considering C_i^c as the marginal benefit of consumers, C_i^p the marginal costs of prosumers, P_i^c active power consumed and P_i^p active power generated by prosumers. It is also subject to restrictions of power flow, power balance, and operation of DERs.

5.2. Cost minimization

Other types of energy markets, and LFMs, minimize flexibility procurement costs. For example in [9] an optimization problem is proposed with the intention of minimize the DSO's total cost of acquiring Demand Side Flexibility considering cost of procuring it.

In the context of the European project "EMPOWER", a cost minimization problem is stated in [31] and [8]. Through the Smart Energy Service Provider and using traffic light approach, flexible bids are collected from energy cooperatives and prosumers, seeking to meet the request of the DSO during following periods. Minimizing total cost of procuring flexibility, control signals of the assets are calculated after market execution.

In the context of transactive energy markets, simultaneous minimization of the costs and thermal discomfort is achieved in [11] and [54], where consumers' flex bids are reschedule depending on the market and grid conditions.

Prediction-Integrated Strategy Optimization (PISO) model is object of cost minimization approach in [55], where instead of constructing the market model based on clearing process, a data-driven method is proposed to learn of the data collected by growing metering and communication infrastructure of the distribution feeders.

Considering λ_t^s price of energy from wholesale market. Then, if an asset is about to reduce its consumption, it should be paid $\lambda_t^d > \lambda_t^s$ to compensate costs. On the other hand, if it is about to increase its consumption, then $\lambda_t^u < \lambda_t^s$. Considering $\Delta\lambda_t$ as the difference between prices of flexibility market and wholesale market, flexibility procurement costs for upward (P_i^u) and downward (P_i^d) products can be defined as depicted in equation (3.39).

$$\min costs = \sum_{i \in N} \Delta\lambda_t^d \cdot P_i^d + \Delta\lambda_t^u \cdot P_i^u \quad (3.39)$$

5.3. Multi objective optimization

Previous sections assess on problem with a unique objective seek by all market participants. However, a different approach may incur in the definition of multiple objective functions that are maximized at the same time obtaining Pareto Efficiency.

Pareto efficiency is achieved when the situation where no individual can be better off without making at least one individual worse off or without any loss thereof. This situation is represented by a frontier of solutions that fulfil the previous condition. Then, if one solution is ought to be selected, weighting criterion should be used for that purpose.

In [56] local energy market is solved in an iterative manner, players adjust their offering/bidding strategies after market clearance, and it is solved again with adjusted parameters. The process is repeated until no change in bids is observed. This process results in a social welfare maximization problem respecting the merit order principle. In that case a study on the interactions between a hydrogen and an electricity network are investigated.

Another proposal that provides a multi objective optimization framework considering a market perspective is studied in reference [57], where a market is proposed in terms of a day-ahead operation for reactive power ancillary services, minimizing the total voltage deviations and the costs under wind generation uncertainties. The authors firstly defined the optimization functions for each agent considered in the market, and then use the Artificial electric field algorithm (AEFA) to perform the multi objective problem and obtain the pareto frontier.

The AEFA algorithm considers agents as a charged particles attracted between each other in directions where their performance or charge is maximized. Then, after obtaining a sufficient representative number

of solutions inside of the pareto frontier, a compromise solution must be selected. To this aim, the authors use a fuzzy decision maker. The membership function (μ_j^i) is described in equation (3.40).

$$\mu_j^i = \begin{cases} 1 & F_j \leq F_j^{min} \\ \frac{F_j^{max} - F_j}{F_j^{max} - F_j^{min}} & F_j^{min} < F_j < F_j^{max} \\ 0 & F_j \geq F_j^{max} \end{cases} \quad (3.40)$$

Where $j \in [0, N_{obj}]$, representing N_{obj} the number of objective functions, F_j^{max} and F_j^{min} the maximum and minimum values of the j^{th} objective function and F_j the value of the objective function considered. The efficacy of the computed non-dominated solutions can be computed considering the normalized version of the membership function as equation (3.41) shows.

$$\mu^i = \frac{\sum_j \mu_j^i}{\sum_i \sum_j \mu_j^i} \quad \forall i \in [1, 2, \dots, M] \quad (3.41)$$

Then, the maximum value of the normalized membership function will be considered to be the best solution.

Multi objective optimization has greater advantages when considering the interaction of different elements and where is the need of finding a compromise solution. This is the case of multi objective planners [58], communities management [59] or energy risk consultants [60]. However, little contributions could be made in this aspect from a deterministic point of view of LFM.

6. Clearing mechanisms

In this section the clearing mechanisms that appear in the literature are collected and explained. Based on the research done, it is possible to organize them in based on how agents interact between them while clearing the market and based on the time horizon that the market is looking forward.

6.1. Classification based on the organization of the agents while clearing

Based on the organization of the agents while the market is clearing, three main algorithms could be used. They are centralized optimization, decomposition methods and bi-level optimization [13].

6.1.1. Centralized optimization

This clearing mechanism uses a central entity that collects all the asks and the bids of the agents involved in the market. Then, the market facilitator, in the case of LFM the LMO, runs an optimization program over a specified time horizon. This mathematical program analyses the characteristics of the asks and the bids with a specific algorithm to search for an optimal value to minimize or maximize the objective function.

The characteristics of the market, i.e., market horizon, objective function, gate closure time, and those similar characteristics are known previously the market is organized. Those characteristics compose the market rules, that are openly known by all the agents involved in the process.

Depending on the type of algorithm used by the optimization problem, direct algorithms and indirect algorithms can be differentiated.

- *Direct algorithms:* optimization programs are classified as direct algorithms in the case that they can be solved directly with commercial solvers (e.g. CPLEX, GUROBI, IPOPT, etc.). Depending on how the problem is formulated, different solvers may be used (e.g. Linear Programming (LP) and Quadratic Programming (QP) convex problems are solved with CPLEX, while Non-Linear Programming problems (NLP) are solved using other as IPOPT, BARON, etc.). The usage of one solver or another strictly depends on the restriction body of the optimization problem, that is affected by the nature of the bids, the problems of the grid that are solved and the complexity of the bids.
- *Indirect algorithms:* This is the case of highly non-convex quadratic equations, where there is no solver able to find an optimal solution. In these cases, indirect algorithms are needed as they convert the highly non-convex problem in other that it is not that is possible to be solved with traditional solvers. In practice, the main technique used is convex relaxation. For LFMs solving voltage deviations, AC-OPF is needed, so, techniques as Second Order Conic Programming (SOCP) or Semi-Definite Programming (SDP) have crucial importance in LFMs.

6.1.2. Decomposition methods

Decomposition methodologies may appear under two different situations. The first one is a large-scale system that takes huge amounts of time to be solved all at once, so a decomposition of the problem imply share the computational burden to several points, enhancing a faster parallel solution. The second one is in a decentralized environment where the information is not freely share among participants, the decomposition methodology is used to safeguard the individual preferences of the agents.

In the field of market clearing techniques, centralized optimization problems often imply high computation overheads. Therefore, decomposition methods are wildly applied on market clearing. These decompositions methods break the original problems into smaller sub-problems solved independently. In the literature there are mainly two different types of decomposition methodologies, being the first one based on the Lagrangian relaxation and the second one based on the Karush-Kunh-Tucker (KKT) conditions.

Lagrangian relaxation is a technique that use the dual problem of the one aiming to decompose to bring the non-convex and complicating constraints to the objective function. There could be distinguished four different methods based on this technique: Alternating Direction Method of Multipliers (ADMM), Analytical Target Cascading (ATC), Proximal Message Passing (PMP) and Auxiliary Problem Principle (APP).

- *Alternating Direction Method of Multipliers (ADMM)*: Combines the decomposability of the dual ascent method and the convergence property of upper bound of method of multipliers.
- *Analytical target cascading (ATC)*: The entire system is divided into a set of subsystems that are hierarchically connected.
- *Proximal Message Passing (PMP)*: in opposition to ADMM, PMP is completely decentralized and highly scalable. PMP distributed problems allow all the sub-problems to be solved in parallel.
- *Auxiliary Problem Principle (APP)*: It based on the usage of coupling variables in the different subproblems, the consistency is then verified by imposing equality between coupling variables. The model was used to solve a decentralized optimization model which enables local energy trading between the DSO and the microgrid operators.

The decomposition based on the KKT conditions decomposes the problem into subproblems, solving each one using the KKT conditions of each area and exchanging the information in the frontier with their adjacent neighbours. For example, for an AC-OPF, the network is divided into different areas, fixing the complicating variables in each iteration, and solving the system of equations defined by the KKT conditions using, for instance, the Newton-Rapshon methodology.

In Chapter 4, further explanations are given to the decentralization process of LFMs describing also the mathematical framework needed for solving the problem using this fashion.

6.1.3. Bi-level optimization

This clearing mechanism is defined as a mathematical program where an optimization problem contains another optimization problem as a constraint. This nested structure is typical of hierarchical structures. Related to LFM, this structure corresponds to a game theory-based local market model, where a leader participant is modelled in an upper level, and follower participants are defined in a lower level.

Mainly, there are two different methods to solve bi-level optimization: single level optimization and nested methods. Additionally, inside the single level optimization, depending on the linearity properties of the problem modelled, two different methods are implemented:

- *Single-level reduction through KKT conditions directly*: The bi-level optimization problems can be transformed into LP single-level problems directly in the case that both of their lower-level and upper-level problems are linear using KKT conditions.
- *Single-level reduction through KKT conditions and dual theory*: For bi-level optimization problems where the lower-level problems are linear while the upper-level problems are non-linear, can be transformed to a single level optimization able to be solved by CPLEX.

In the case of nested methods, they are used to solve lower-level and upper-level non-linear problems, using one optimization algorithm at each level in an iterative and nested manner. Because of the fact that two different methods are used in total for nested methods, they are also named hybrid methods in the literature consequently.

6.2. Classification based on the time horizon

Ahead-markets scheduling mechanisms solve LFM for a given time horizon. Different time horizons cover different necessities of the stakeholders as the uncertainty of the network condition decrease as the time goes by. In this section, a description of different market clearing structures depending on the time horizon considered is presented.

Authors in [30] define a market framework considering two types of markets: day-ahead market and intraday market. Day-ahead scheduling (DA) allows the participants to maximize their profit from the wholesale energy market by modifying energy consumption of its prosumers in a time horizon of a day. Intra-day markets (ID) allow to minimize deviations of the portfolio scheduled, minimizing associated costs for BRPs and solving grid issues for the DSO. ID markets allow participants to offer bids in both directions, clearing bids and offers several times a day. Both markets share great similitudes but differ in terms of time horizon and the time elapsed between market closure and energy delivery. In terms of participants, BRPs are included in ID to optimize their portfolio but not in DA, DSO, Aggregators and Prosumers could participate in both. A scheme of the DA and ID market is shown in Figure 8.

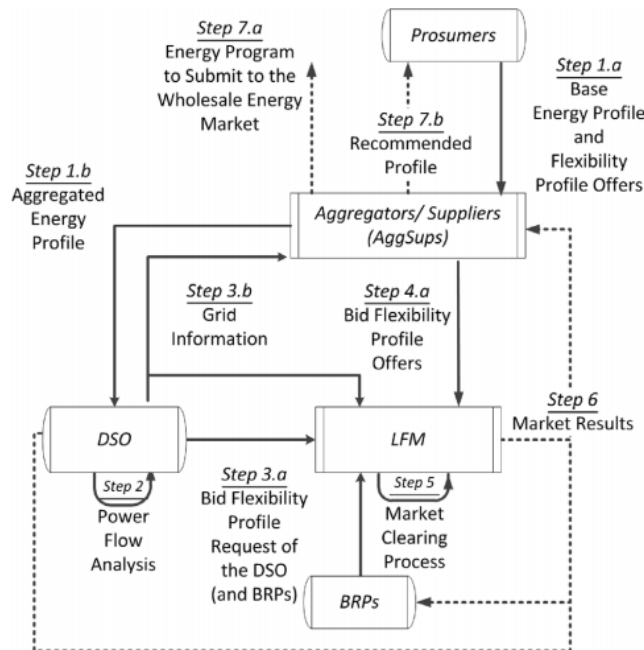


Figure 8. Schematic diagram of the interactions in the ID and DA markets. Source: [30].

A similar structure of a LFM is also proposed in [31], where optimization problem is settled from the DSO point of view. Market objective function minimizes flexibility costs. In this case, DSO determine the need of flexibility for the entire day and asks LFM for flexibility. When several services are provided, adaptative DA and ID market is enabled, providing different services within different time slots, that is, Flexibility proving could be offered in a DA market while portfolio optimization is offered in a ID market. [8].

Other scheme addressed in the literature is the Continuous Double Auction (CDA), based on auction theory, where bids and asks are matched continuously, in an iterative process. Given a time slot, for example 15 min, two periods are defined. Firstly, from minute 0 to 15, asks and bids are accepted, and market is solved. Then, from minute 15 to 30, energy is transferred, and next bidding period starts. In this clearing structure the not matched bids and offer are storage in an order-book during the time the periods lasts.

Furthermore, it is possible to model CDA market from a data-driven perspective using a market prediction model based on Extreme Learning Machine (ELM) to overcome mathematical modelling non-linearities and approximations. For that perspective, objective function (as overall costs of the participants) is minimized and network constraints are linearized to clear the market [55].

7. Mathematical modelling

In this section is explained the mathematical modelling of the proposed centralized version of the LFM. To do so, two illustrative examples are proposed and solved, assessing the most important characteristics of the problem, with the aim to provide a step-by-step comprehension of it. After that, a more realistic use case is presented. Illustrative examples are tested on a simple network composed by two microgrids, while the realistic use case involves five microgrids interconnected by a transmission network.

First two illustrative examples aim to provide an initial understanding of the problem, showing a simplistic view of it. With this purpose in mind, the first examples describe a LFM where only FLs and FGs solve congestions and imbalance issues of the grid after DSO asking. In this first case, only energy products are traded, and only one period is considered. After that, the second illustrative example asses on the influence of the introduction of capacity products acting as a previous stage of the energy products in a multi-period perspective. It is demonstrated that considering energy and capacity products of FLs, FGs and BESSs have a positive influence in the liquidity of the market. To this aim, two time periods are simulated on the two microgrid network. After the introduction of those illustrative examples, a more complex and realistic use case is presented. In this use case the interaction between FLs, FGs and BESSs while solving congestion and imbalance issues is studied.

At this point, the market structure of the LFM and its general functioning are explained. Figure 9 shows a general view of the market structure with consists in the DSO asking for flexibility to the LMO who receives flexibility bids from n Aggs which manage FLs, FGs and BESSs.

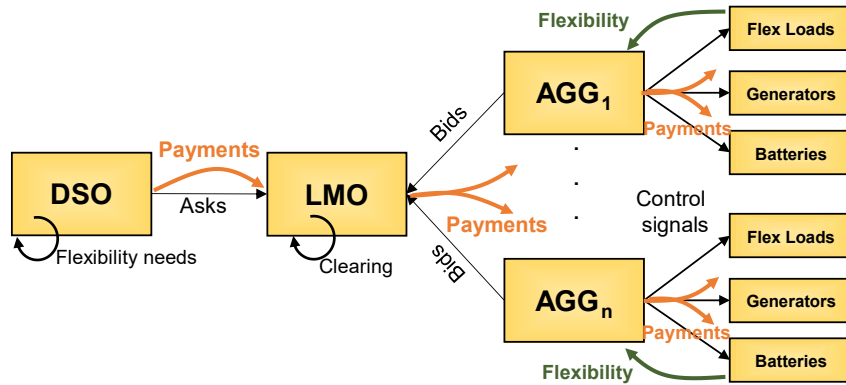


Figure 9. Market structure of asks, bids, control signals and payments of the centralized local flexibility market

Figure 10 presents the timeline sequence and interactions among market participants. As shown, main steps involve; firstly, DSO and Aggs forecasting and planning their energy asks and bids for the current period and the capacity asks and bids for the next period; secondly, LMO, clearing the market and, sending results to agents; lastly, Aggs sending control signals to assets while LMO looking the market out. In each market clearing, energy products are matched for time slot t and capacity products are cleared for time slot $t+1$. Pre-allocation of the capacity products ensures liquidity of energy products in the following period. This increase in the liquidity will be demonstrate in the first illustrative example.

In the remaining of this section the centralized proposed approach is explained. However, instead of trying to cover directly the full complex approach, two illustrative examples are previously explained. Intending to provide a more comprehensive approach or a reader, illustrative examples are lighter versions of the final use case. Several strong hypotheses are made to provide a first simplistic approach, what will help to fully understand the final use case after that.

First illustrative example will assess on the general functioning of the market, assuming that FLs and FGs are able to communicate directly with the LMO and provide only energy flexibility products. Then, the second illustrative example will assess on the influence of including capacity products in the market, highlighting the impact on the market liquidity if two periods of time are considered. Lastly, the final use case is proposed with the introduction of the figure of the Agg as the manager of modest prosumers.

IEEE-34 distribution test network will be used as a base model for the market, enabling building the different illustrative examples and the final use case.

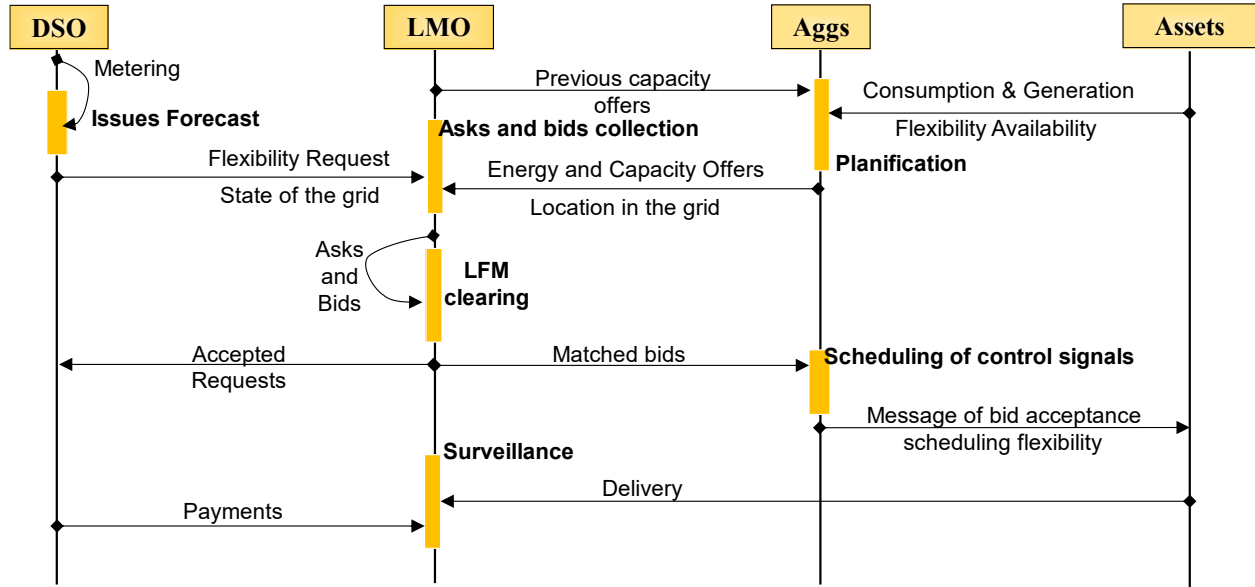


Figure 10. Timeline of the proposed LFM for energy and capacity products.

7.1. First illustrative example

In this first illustrative example, it is investigated how the introduction of energy flexibility products from FLs and FGs influence on the grid issues. As it is the first approach, no time dimension is considered, and only energy products will be traded. The prosumers will directly cooperate with the LMO, bidding for their flexibility products. DSO will ask for flexibility with the aim to solve forecasted power congestions or imbalances.

The market proposed act as a posterior stage of the wholesale market. For that reason, and intending to be cleared each 15 min, all consumptions and generations parameters will be supposed to be known. This approach it is not far from the real functioning of real electricity markets, where markets generally do not consider any type of uncertainty, as the errors produced by error in the forecast will be solved using markets closer to real-time operation, as for example the type of market in this thesis is being proposed.

In terms of the network modelling, as has been covered in previously sections, there are different approaches to effective take into account the restrictions imposed by the network. Some are accurate than others, for instance the AC approach. However, an accurate modelling of the grid introduces highly complex non-linearities as well as non-convexity into the problem. In the present approach, being the main objective to propose an effective and simple market definition that help the DSO to tackle with congestions and imbalances, the introduction of high complex network definition provides is no longer a problem, as the linear approach is enough. Linear modelling of the grid will assume that no voltage issues are presented in the grids ($V_i \approx 1 p.u.$) and that losses are negligible ($G_i \approx 0 p.u.$). Then, only active power flow is considered.

Main sets, indexes, parameters, and variables are listed in this section, other derivatives are introduced in the text.

$l, f, g, s, t, slk, i \in \mathbb{N}$	Sets of static loads, flexible loads, generation assets, storage systems, time, slack node, and grid nodes.
$x \in \{f, g, s\}$	Set of flexible assets connected to the grid.
$j \in i$	Set of nodes connected between them.
B_{ij}	Susceptance of line ij (S).
θ_{ij}	Difference of voltage angle between node i and j (rad).
ω	Product quantity (kW or kWh).
S	Product price (€/kW or €/kWh).
$C_{x,t}$	Cost function for asset x in interval t (€).

soc_s	State of Charge of storage system s (kWh).
η_s^C, η_s^D	Charging and discharging efficiencies of the battery s (p.u.).
p	Active power injection (kW).

Variables are small letters, e.g., ω . Upper and lower limits of ω are noted by $\bar{\omega}$ and $\underline{\omega}$, respectively. Parameters are noted by capital letters, e.g., B_{ij} . Energy and capacity products are noted by superscript e and c , respectively, and direction is noted by superscript u for upward and d for downward directions, e.g., $\omega_{x,t}^{c,d}$ or $\omega_{x,t}^{e,u}$. Scheduled power injection and aftermarket injection are noted by PR and AM , respectively, e.g., $P_{l,t}^{PR}$ or $p_{f,t}^{AM}$. Then, LFM is modelled from equations (3.42) to (3.47).

$$\min z = \sum_f (\Delta S_f^{e,u} \omega_f^{e,u} + \Delta S_f^{e,d} \omega_f^{e,d}) + \sum_g \Delta S_g^{e,d} \omega_g^{e,d} \quad (3.42)$$

$$p_f^{AM} = P_f^{SCH} + \frac{1}{\Delta t} (\omega_f^{e,u} - \omega_f^{e,d}) \forall f \quad (3.43)$$

$$p_g^{AM} = P_g^{SCH} - \frac{\omega_g^{e,d}}{\Delta t} \forall g \quad (3.44)$$

$$\sum_j B_{ij} (\theta_i - \theta_j) = p_i + p_g^{AM} - P_l^{AM} - p_f^{AM} \forall i \quad (3.45)$$

$$-\bar{P}_{ij} \leq B_{ij} (\theta_i - \theta_j) \leq \bar{P}_{ij} \forall i, \forall j, i \neq j \quad (3.46)$$

$$\sum_l (P_l^{SCH} - P_l^{AM}) + \sum_f (P_f^{SCH} - p_f^{AM}) + \sum_g (P_g^{SCH} - p_g^{AM}) = 0 \quad (3.47)$$

Objective function is showed in equation (3.42) and minimize the total cost for procuring flexibility from assets involved in the market (f and g). FLs sell both upward and downward products ($\omega_f^{e,u}$ and $\omega_f^{e,d}$, respectively), at a price $S_f^{e,u}$ and $S_f^{e,d}$. On the other hand, FGs can only sell downward products, due to the fact that they can just curtail their generation. The reason behind this is that in distribution networks the majority of the generators installed obtain energy from renewable energy sources, what make impossible to increase the generation on demand. Then, FGs will sell $\omega_g^{e,d}$ energy flexibility product at a price $S_g^{e,d}$.

The reason behind using delta prices for products costs is that, due to the fact that proposed LFM is an ancillary market cleared before the wholesale market, marginal price of that market should be accounted in the formulation (S_t). Then, for upward products of FLs $\Delta S_f^{e,u} = S_t - S_f^{e,u}$, for downward products $\Delta S_f^{e,d} = S_f^{e,d} - S_t$. Lastly, $\Delta S_g^{e,d} = S_t - S_g^{e,d}$ for all FGs.

Considering that the wholesale market prices are previously cleared at a price S_t . FLs will only be incentive to participate in the market if their bids at a price $S_f^{e,u} < S_t$ for upward products, i.e. increasing their demands, or by selling downward products at a price $S_f^{e,d} > S_t$ that compensate them for the energy previously paid for that they will have no longer access to.

In the case of FGs, the determination of the prices for their bids is different. FGs sell their electricity production in the wholesale market at a price S_t , then, to participate in the LFM, they bid their flexibility products at a price $S_g^{e,d} < S_t$. This price allows the FGs to obtain benefits from their participation in the market, it could be seen as if the FGs sell at a price S_t and then buy back his own production at lower prices, leaving the FGs with a profit for participating in the market.

Equations (3.43) stand for the demand of the FLs after the market clearing (p_f^{AM}), it is formulated as a function of the scheduled consumption in the wholesale market (P_f^{SCH}) and the energy products sold in the LFM ($\omega_f^{e,d}$ and $\omega_f^{e,u}$). Those products are divided between the time slot between of which the market is organized (Δt). As a general approach, 15 min closure gate is the direction in which European regulation

is seeking for future local energy markets. Similarly to the FLs, FGs injection after the market clearing is formulated in equation (3.44) in terms of the predicted generation (P_g^{SCH}) and the presence of downward product sold in the market $\omega_g^{e,d}$.

FLs are considered able to modify its consumption in a range of 30% of its scheduled power demand using DSR techniques. Mathematically it is translated considering that $p_f^{AM} \in [0,70 \cdot P_f^{SCH}, 1,30 \cdot P_f^{SCH}]$ for all FLs. For FGs it is considered that they can modify their injections to the grid considering the following bounds for them $p_g^{AM} \in [0, P_g^{SCH}]$.

Equations (3.45) and (3.46) stands for the linear modelling of the network. First equation considers voltage angles when modelling the grid, while the second one considers the thermal limit of the feeders that join the different nodes together. For this illustrative case the test feeder showed in Figure 11 which is composed by two microgrids connected using the transmission line 400-800. The slack node and the point of common coupling are set in node 490. This model is based on the IEEE 34 test-feeder. In Figure 11 blue circles represent FLs, orange circles FGs and grey circles SLs. The total quantity of SLs is 735,77 kW shared between 18 SLs, while FLs accounts for 467,24 kW in eight FLs, and lastly, distributed generation of FGs sum a total of 163,22 kW for five FGs.

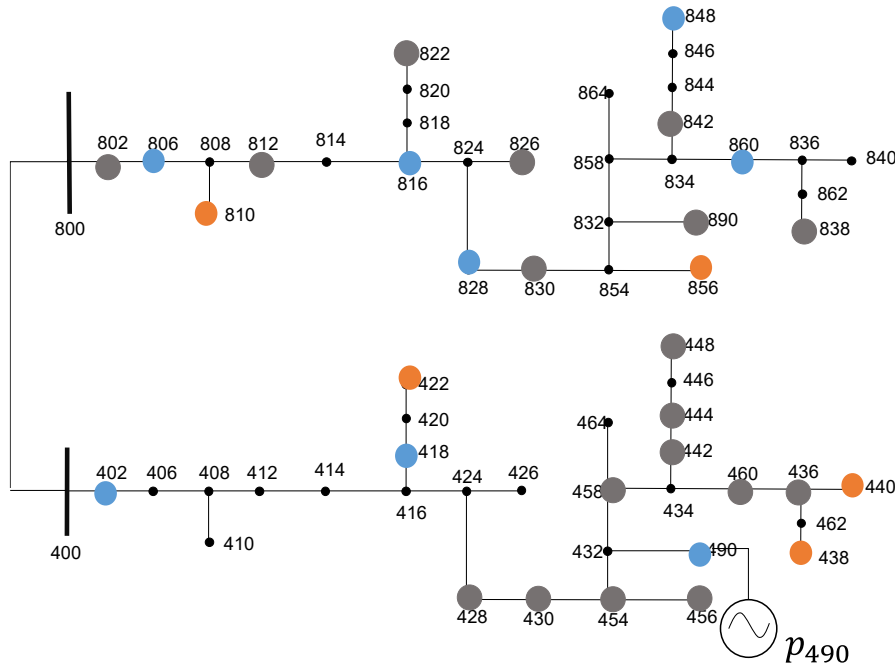


Figure 11. Test feeder for illustrative example 1. Grey circles represent SLs, blue circles FLs, orange circles FGs. Slack node: 490. PCC in 490.

Scheduled power demands for FLs settled in the wholesale market and bids for upward and downward energy products traded in the LFM are shown in Table 2. Power demands were randomly generated as well as the connection node, prices were randomly generated in an interval of 0,05€/kWh of the price 59,68€/MWh obtained from OMIE for 22nd March 2021.

f	node	P_f^{SCH} (kW)	$S_f^{e,d}$ (€/kWh)	$S_f^{e,u}$ (€/kWh)
f_1	806	104.63	0.05997	0.05938
f_2	816	9.22	0.08916	0.03020
f_3	828	63.64	0.10562	0.01374
f_4	848	7.75	0.10388	0.01548
f_5	860	104.63	0.08514	0.03421
f_6	402	9.22	0.09320	0.02616
f_7	418	63.64	0.08731	0.03205
f_8	490	104.63	0.10175	0.01760

Table 2 Flexibility bids for flexible loads both in congestion and imbalance scenario. Prices and power were randomly generated considering that $S_t = 59,68€/MWh$ that was obtained from OMIE for 22nd March 2021. The interval considered was 50 €/MWh.

Flexible generators forecasted injection sold in the wholesale market and their bids for the LFM are shown in Table 3. Both prices and power generations are randomly generated considering the same day and wholesale market clearing price.

g	node	$P_g^{SCH} (kW)$	$S_g^{e,d} (€/kWh)$
g_1	810	13.97	0.03356
g_2	856	116.40	0.04115
g_3	422	11.90	0.02794
g_4	440	6.98	0.04625
g_5	438	13.97	0.04056

Table 3. Flexibility bids for flexible generators both in congestion and imbalance scenario. Prices and power were randomly generated considering that $S_t = 59,68€/MWh$ that was obtained from OMIE for 22nd March 2021. The interval considered was 50 €/MWh.

Lastly, equation (3.47) ensures that congestion and imbalances are solved considering only local resources not affecting assets upstream the network considered. It ensures that the general power balance of the distribution grid considered remains stable before and after the LFM clearing. For congestion solving, it will consider the amount of imbalance created while deviating of the programmed schedule to not break thermal limits and will force another asset to compensate it. For imbalance solving, this equation will model the DSO asking for balance to the LMO. The imbalance will be created by a SL, considering that $P_l^{AM} \neq P_l^{SCH}$, then if the deviation is positive ($P_l^{AM} > P_l^{SCH}$) downward flexibility will be used to maintain the balance equal to the one computed in the wholesale market. However, if the deviation is negative ($P_l^{AM} < P_l^{SCH}$) upward flexibility will be used to solve such problem.

7.1.1. Congestion scenario

In this scenario it is investigated the effectiveness of the proposed approach to solve congestions by means of local assets. For this scenario power demands of the SLs are shown in Table 4. Scheduled and aftermarket power is considered constant as no imbalance of those loads are considered.

l	node	$P_l^{SCH} (kW)$	$P_l^{AM} (kW)$	l	node	$P_l^{SCH} (kW)$	$P_l^{AM} (kW)$
l_1	802	53,94	53,94	l_{10}	428	14,32	14,32
l_2	812	29,61	29,61	l_{11}	454	63,55	63,55
l_3	822	33,81	33,81	l_{12}	456	36,89	36,89
l_4	826	38,75	38,75	l_{13}	458	21,70	21,70
l_5	830	17,34	17,34	l_{14}	442	29,43	29,43
l_6	842	20,59	20,59	l_{15}	444	135,52	135,52
l_7	846	10,38	10,38	l_{16}	448	53,94	53,94
l_8	890	13,83	13,83	l_{17}	460	29,61	29,61
l_9	838	60,00	60,00	l_{18}	436	33,81	33,81

Table 4. Scheduled and aftermarket power consumption from static loads for the congestion scenario. The power schedule was randomly generated in this first scenario, considering a total quantity superior to 700 kW for the considered time slot.

Considering given SLs, FLs and FGs consumptions and injections, the wholesale market cleared resulting in the power flow that Figure 12 shows. As it is highlighted, the power flow through line 414-416 is 446,98 kW after the wholesale market clearing. It is considered that, due to a technical issue, the thermal limit of that line is reduced to 400 kW.

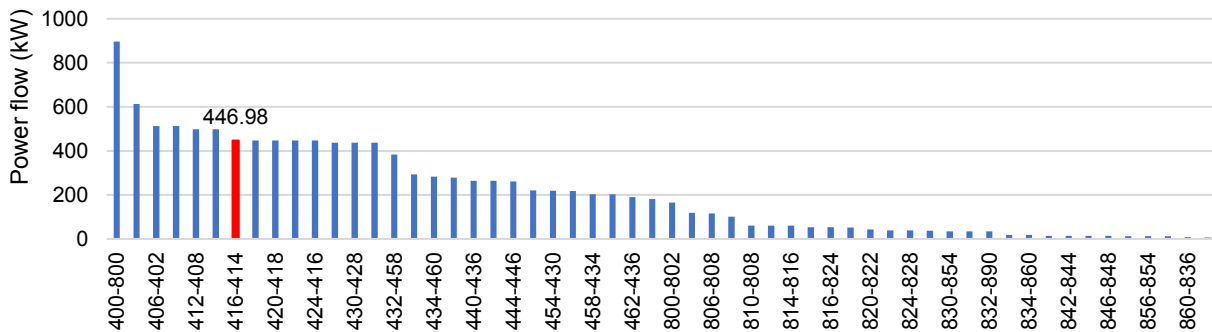


Figure 12. Power flow after wholesale market clearing. Power flow of line 416-414 is marked in red, accounting for 446,98 kW.

Under these conditions, market is cleared. The total cost of flexibility procuring is 0,37862 €. Flexibility procuring for each asset is summarized in Table 5. Downward products of the generators are written in the upward column in order to compare them with the products of the FLs. In other words, downward products of the FGs make the same effect in the grid that upward products of the FLs. Moreover, as can be seen in the table, the total amount of upward and downward flexibility is the same, what ensures the requirement imposed by equation (3.47). This equality between upward and downward products is what create two zones in the network as Figure 13 shows.

<i>Asset</i>	<i>C (€)</i>	$\omega^{e,d} (kWh)$	$\omega^{e,u} (kWh)$
f_1	0,002310	7,8469	
f_5	0,099245	3,8971	
f_7	0,131878		4,7727
g_3	0,055004		1,7333
g_4	0,023441		1,7460
g_5	0,066741		3,4920
TOTALS	0,378618	11,7440	11,7440

Table 5. Summary of the bids matched in the LFM. Generators bids are presented in the upward column to be consistent with the FL upward direction. Total quantity traded: 11,7440 kWh in each direction.

In Figure 13 is shown the upward and downward area created considering the congestion of the line 414-416, as well as the assets involved in the market operation. Products are represented by an arrow indicating their direction. Then, FLs connected to 806 and 860 procure downward flexibility indicated by a downward blue arrow, FLs connected to node 418 is procuring upward flexibility. FGs provide their downward products that contributes to the general upward direction of the area. FGs connected to 422, 440 and 438 provide upward flexibility for the congestion solving.

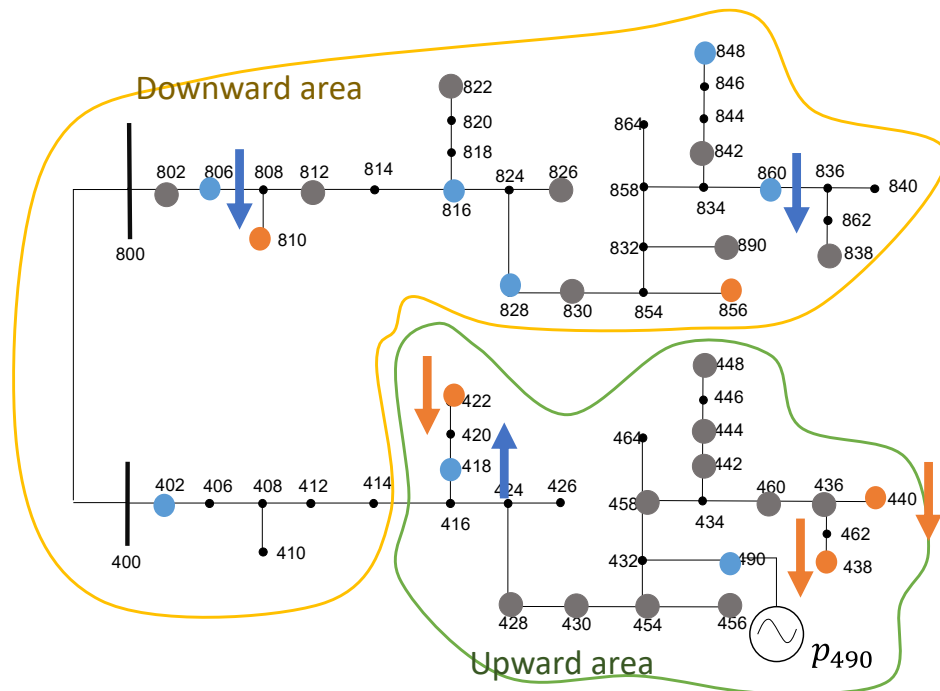


Figure 13. LFM market clearing representation for the congestion scenario. Upward area is marked inside green line, while downward area in marked inside yellow line.

Power flow before (presented in blue) and after the market clearing (presented in orange) is shown in Figure 14. As is shown, the limit in power flow in the line 414-416 limits those lines downstream node 416. A reduction in the consumption of the zone is achieved, what it is compensated by an increase in consumption upstream node 414, as previous Figure 13 shows.

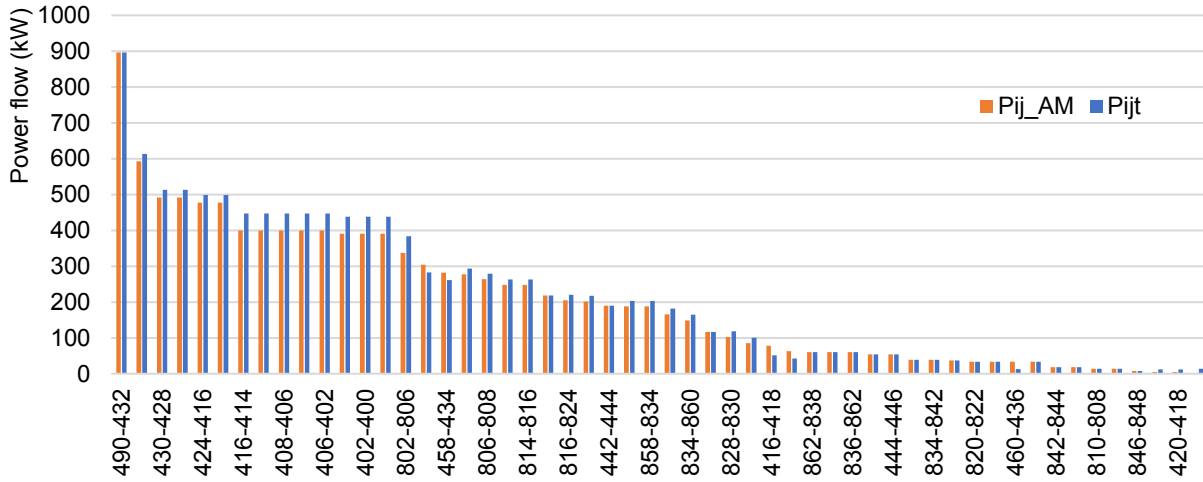


Figure 14. Power flow before (blue) and after (orange) the LFM market clearing solving the 400-kW congestion in the line 414-416.

In terms of the dual problem analysis, the behaviour of the two multipliers presented in Table 6 is analysed.

λ_P (€/kW)	0,01429995
λ_B (€/kW)	0,00793337

Table 6. Lagrange multipliers for the congestion scenario in the first illustrative example.

The value λ_B is the amount that will cost another kW of imbalance in the grid. In the conditions of this scenario, it is coincident with the value of the marginal asset matched in the clearing of the LFM. Indeed, the marginal asset is g_3 connected to node 422.

The generator bids its flexibility product at a price $S_{g_3}^{e,d} = 0,027944$ €/kWh, considering the price of the wholesale market, the benefits for that generator per unit of energy is $S_t - S_{g_3}^{e,d} = 0,0317334$ €/kWh, what in terms of power, considering a $\Delta t = 0,25$ h: $\lambda_B = (S_t - S_{g_3}^{e,d}) \cdot \Delta t = 0,00793337$ €/kW.

The Lagrange multiplier λ_P represents the cost of another kW of congestion in the Line 414-416. Considering that $\lambda_P = \Delta z / \Delta b$, then, if the congestion were 399 kW, the total cost for congestion reduction would be: $z = 0,37862$ € + $0,01429995$ € = $0,392917$ €. Same for the other direction, if the power flow in that line were limited to 401 kW, then, the cost would be: $z = 0,037862$ € - $0,01429995$ € = $0,3643205$ €. As shown, Lagrange multipliers can be shown as sensibilities for the equations they are associated to. Those values have been checked realizing simulations with the stated power flow limit, results are coincident with the given interpretation of the Lagrange multiplier.

7.1.2. Imbalance scenario

In the imbalance scenario it is investigated the behaviour of the local market to solve imbalances. An imbalance is defined as a change in the portfolio established by the wholesale market, that is, a change in the scheduled consumption of the SLs, FLs or FGs. Those situations create different grid situations what could lead, if nothing is done, to destabilize the production of upstream generators and power lines. Owing to that, from the DSO point, it is important to have an efficient and complete control of those imbalances.

In this scenario it is considered that the SLs connected to node 444 and 826 have a technical issue, what forces them to disconnect from the grid for security reasons. In such scenario, a downward imbalance of 43,568 kWh is created. Then, DSO ask for that quantity to the LMO who organizes a LFM accepting bids from the flexible assets. The grid used is the same that the one used in the congestion scenario as the Figure 15 shows. SLs demands before (P_t^{SCH}) and after the market clearing (P_t^{AM}) are shown in Table 7, showing a zero for the SL 4 for the period considered.

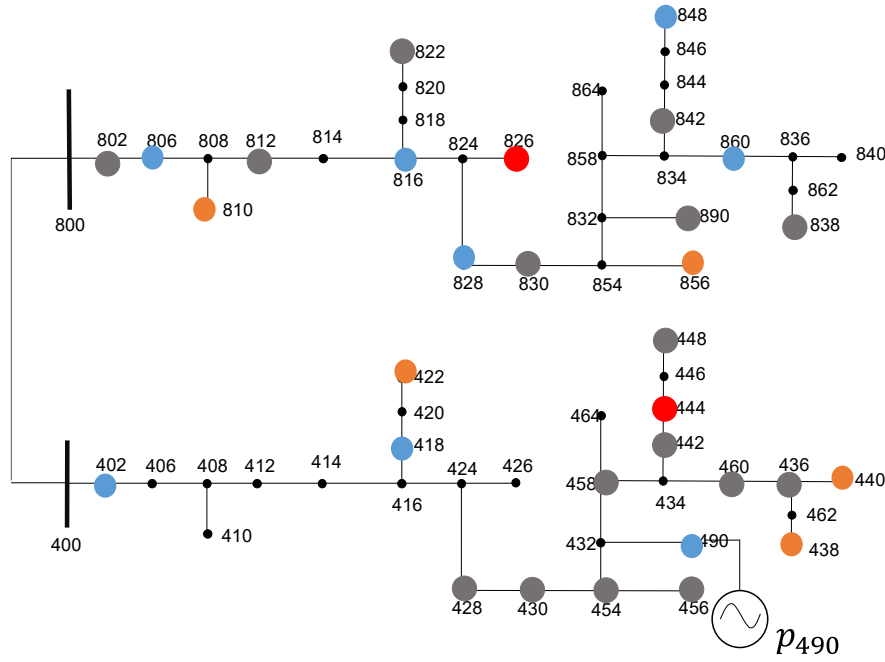


Figure 15. Distribution network for the imbalance scenario. SLs connected to nodes 444 and 826 are marked in red as are those who create the imbalance in the grid.

l	node	$P_l^{SCH} (kW)$	$P_l^{AM} (kW)$	l	node	$P_l^{SCH} (kW)$	$P_l^{AM} (kW)$
l_1	802	53,94	53,94	l_{10}	428	14,32	14,32
l_2	812	29,61	29,61	l_{11}	454	63,55	63,55
l_3	822	33,81	33,81	l_{12}	456	36,89	36,89
l_4	826	38,75	0	l_{13}	458	21,70	21,70
l_5	830	17,34	17,34	l_{14}	442	29,43	29,43
l_6	842	20,59	20,59	l_{15}	444	135,52	0
l_7	846	10,38	10,38	l_{16}	448	53,94	53,94
l_8	890	13,83	13,83	l_{17}	460	29,61	29,61
l_9	838	60,00	60,00	l_{18}	436	33,81	33,81

Table 7. Scheduled and aftermarket power consumption from static loads for the imbalance scenario. The power schedule was randomly generated in this first scenario, considering a total quantity superior to 700 kW for the considered time slot.

The LMO collects the offers of the assets. For the sake of clarity and simplicity, those bids are considered the same than those defined in the congestion scenario in Table 2 and Table 3. Figure 16 shows a graphical representation of the situation described. The total imbalance accounts for a total of $\omega_B^* = 43,57 \text{ kWh}$, with a total cost of 0,66685 €.

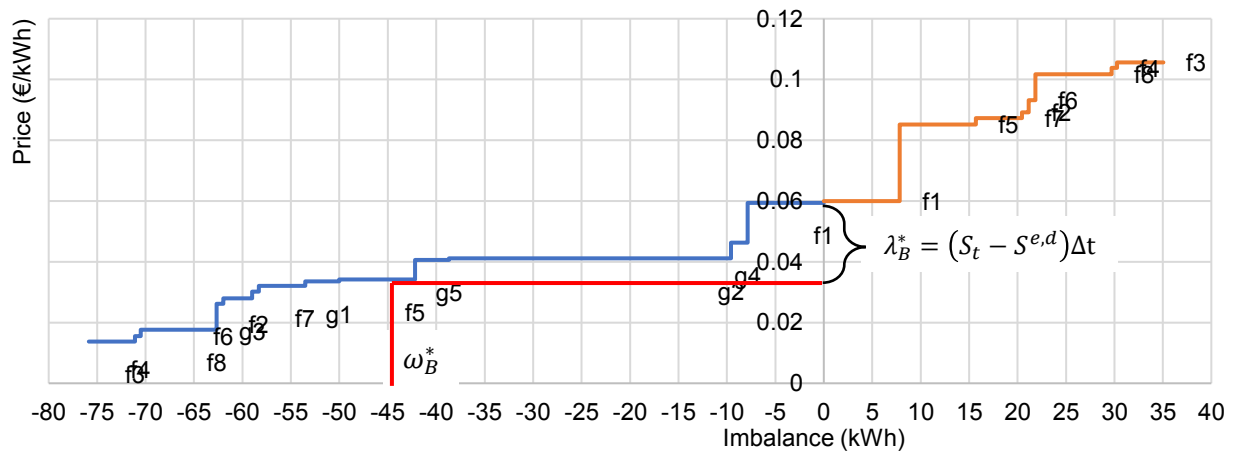


Figure 16. Representation of the collected bids submitted by assets in the LFM. Blue bids represent upward flexibility offers, while orange line represents downward products submitted by flexibility loads. $\omega_B^* = 43,568 \text{ kWh}$, $\lambda_B^* = 0,00636658 \text{ €/kWh}$.

A summary of the matched bids in the market is shown in Table 8. As presented in Figure 16 and in that table, the marginal asset matched in the clearing is the f_5 , what establish the value of the Lagrange multiplier $\lambda_B^* = (S_t - S_{f_5}^{e,d}) \cdot \Delta t = 0,00636658 \text{ €/kW}$. The rest of multipliers are zero, as they do not affect to the overall solution, as there is any congestion. The value of the previously commented Lagrange multiplier is obtained from the marginal value of the equation (3.47), likewise the congestion scenario.

	$S^{e,d} (\text{€}/kWh)$	$\omega^{e,u} (kWh)$	c (€)
g_2	0,041151	29,09947	0,539119
g_4	0,046252	1,745968	0,023441
g_5	0,040565	3,491937	0,066741
f_1	0,059383	7,846875	0,00231
f_5	0,034211	1,38381	0,035241
TOTALS		43,56806	0,66685

Table 8. Results of the LFM for the imbalance scenario.

A visual representation of the use case is presented in Figure 17, where it is possible to observe that the downward areas in this scenario are reduced to the environment of those nodes of the disconnected SLs. The rest of the distribution feeder is defined as an upward area, as it balances the grid with upward flexibility of the FLs and downward flexibility of FGs.

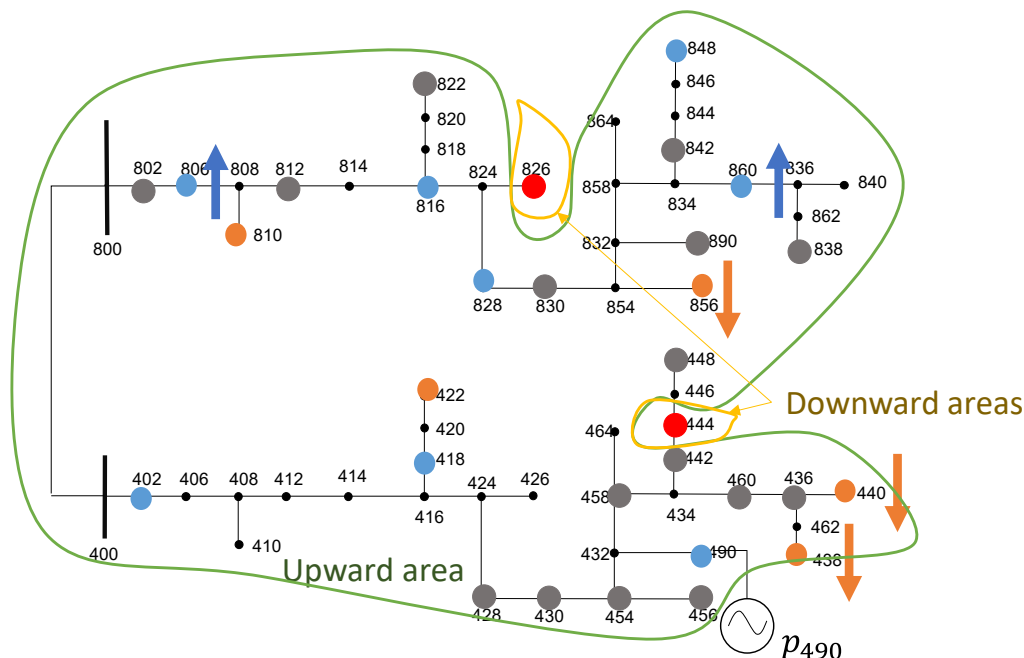


Figure 17. LFM market clearing representation for the imbalance scenario. Upward area is marked inside green line, while downward area in marked inside yellow line

As can be extracted from the two presented illustrative examples, the approach is of utility to solve both congestion and imbalance issues using local resources. The notation avoids the use of binary variables to activate or deactivate assets, what make of interest for its simplicity, as it is possible to achieve the global optimal solution.

7.2. Second illustrative example

In the second illustrative example, the introduction of capacity products into the market is explained. Bearing this objective in mind, a simple problem is formulated to assess on the capabilities of these products. The main idea under the introduction of the capacity products in the market is to use them as a previous stage of energy products. Considering a short-term perspective, they are useful to anticipate future grid issues, as they are cleared in time period t for the following period $t + 1$.

Likewise the previous illustrative example, FLs are assumed to freely modify their consumption in a limited range in both directions. Cost for procuring those products is detailed in equations (3.49) and (3.50) for

each FL. These equations consider the wholesale energy price showing costs incurred by DSO when purchasing flexibility products.

Energy and capacity products are related as no asset can trade energy products far than previously set capacity product. Then, $\omega_{f,t}^{e,u} \leq \omega_{f,t}^{c,u} \cdot \Delta t$ and $\omega_{f,t}^{e,d} \leq \omega_{f,t}^{c,d} \cdot \Delta t$ for all $t > t_1$. Increase of liquidity using capacity products is achieved, limiting energy products bounds in time t in prediction of what will happen in period $t + 1$ [61]. As Figure 18 (a) shows, consumption of FL f is limited in period t using capacity products. Then $\underline{p}_{f,t} + \omega_{f,t+1}^{c,d} \leq p_{f,t} \leq \bar{p}_{f,t} - \omega_{f,t+1}^{c,u}$, what ensures availability of energy products in $t + 1$.

FGs are assumed to only provide flexibility in downward direction. They can only curtail their production; in any case can they increase it. So, the only products they can trade with are $\omega_{g,t}^{e,d}$ for energy and $\omega_{g,t}^{c,d}$, considering that $\omega_{g,t}^{e,d} \leq \omega_{g,t}^{c,d} \Delta t$.

Consistently with previous presented products, BESSs energy and capacity products are related between each other considering $\omega_{s,t}^{e,u} \leq \omega_{s,t}^{c,u} \cdot \Delta t$ and $\omega_{s,t}^{e,d} \leq \omega_{s,t}^{c,d} \cdot \Delta t$ for all BESSs. Previous reasoning explained for FLs, can be also applied to BESSs. Considering a need of energy in $t + 1$ of $\omega_{s,t+1}^{e,u}$, in time $t + 1$, BESSs should have a SOC lower than $\overline{SOC}_s - \omega_{s,t+1}^{c,u} \Delta t$. If the need is in the downward direction, minimum level of SOC will be $\underline{SOC}_s + \omega_{s,t+1}^{c,d} \Delta t$. Then, pre-allocating capacity products will ensure availability of energy products, increasing liquidity of LFM as explained in Figure 18 (b).

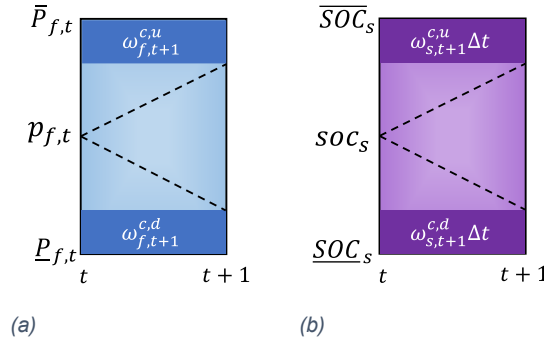


Figure 18. Ensuring availability of energy flexibility products in $t + 1$ using capacity products.

The proposed formulation for the capacity and energy market is developed from equation (3.48) to (3.62). As it will addressed, there are some similitudes and some differences with respect to the energy-only LFM defined in the section 7.1 of this chapter.

Starting for the objective function, in equation (3.48) can be seen that it minimizes the total cost for DSO acquiring energy products for all assets ($C_{x,t}^e$) and the cost of acquiring capacity products ($C_{x,t}^c$). Capacity products are pre-allocated in time t for time period $t + 1$. Thus, in the objective function their costs are considered in advance.

$$\min z = \sum_t \sum_f (C_{f,t}^e + C_{f,t+1}^c) + \sum_t \sum_g (C_{g,t}^e + C_{g,t+1}^c) + \sum_t \sum_s (C_{s,t}^e + C_{s,t+1}^c) \quad (3.48)$$

The definition of the energy products both for FLs, FGs and SSs is the same that the one considered in the energy-only LFM, but has been extended for a multitemporal view in equations (3.49), (3.52) and (3.55), respectively. In terms of the capacity products, they are not related with the wholesale market prices, so, there is no necessity of defining them using delta prices. Then, capacity products costs are defined in equations (3.50), (3.53) and (3.56) for FLs, FGs and BESSs, respectively. The capacity prices are related with the energy products prices for the aim of this thesis. It is considered that the capacity products prices are a 10% of the energy products of the wholesale market prices. This 10% was obtained after analysing several days of the components of the market, comparing the overrun for restrictions to the final price of the energy [62].

As previously explained, the capacity products are used as a previous stage for energy products, ensuring their availability in advance. Energy products are the only one that can modify the power consumption aftermarket of the assets, as equations (3.51), (3.54) and (3.57) show for FLs, FGs and BESSs respectively.

Their consumption or injection aftermarket are function of the wholesale schedule ($P_{x,t}^{SCH}$) and the energy products cleared.

$$C_{f,t}^e = \Delta S_{f,t}^{e,u} \omega_{f,t}^{e,u} + \Delta S_{f,t}^{e,d} \omega_{f,t}^{e,d} \quad \forall f, \forall t \quad (3.49)$$

$$C_{f,t}^c = S_{f,t}^{c,u} \omega_{f,t}^{c,u} + S_{f,t}^{c,d} \omega_{f,t}^{c,d} \quad \forall f, \forall t \quad (3.50)$$

$$p_{f,t}^{AM} = P_{f,t}^{SCH} + \frac{1}{\Delta t} (\omega_{f,t}^{e,u} - \omega_{f,t}^{e,d}) \quad \forall f, \forall t \quad (3.51)$$

$$C_{g,t}^e = \Delta S_{g,t}^{e,d} \omega_{g,t}^{e,d} \quad \forall g, \forall t \quad (3.52)$$

$$C_{g,t}^c = S_{g,t}^{c,d} \omega_{g,t}^{c,d} \quad \forall g, \forall t \quad (3.53)$$

$$p_{g,t}^{AM} = P_{g,t}^{SCH} - \frac{\omega_{g,t}^{e,d}}{\Delta t} \quad \forall g, \forall t \quad (3.54)$$

$$C_{s,t}^e = \Delta S_{s,t}^{e,u} \omega_{s,t}^{e,u} + \Delta S_{s,t}^{e,d} \omega_{s,t}^{e,d} \quad \forall s, \forall t \quad (3.55)$$

$$C_{s,t}^c = S_{s,t}^{c,u} \omega_{s,t}^{c,u} + S_{s,t}^{c,d} \omega_{s,t}^{c,d} \quad \forall s, \forall t \quad (3.56)$$

$$p_{s,t}^{AM} = \frac{\omega_{s,t}^{e,u} - \omega_{s,t}^{e,d}}{\Delta t} \quad \forall s, \forall t \quad (3.57)$$

State of charge restrictions are also written for BESSs in equations (3.58) and (3.59). It is considered that battery SOC will remain between a superior and an inferior limit ($\overline{SOC}_s, \underline{SOC}_s$). Charging (η_c) and discharging (η_d) efficiencies are also considered constant for the whole range of operation and equals to 95%. The SOC for a time period t is modified only by the energy products the BESS trade in the LFM, the influence of other markets or algorithms are not included in the model.

$$soc_{s,t} = SOC_s^0 + \eta_s^c \omega_{s,t}^{e,u} - \frac{1}{\eta_s^d} \omega_{s,t}^{e,d} \quad \forall s, \forall t \quad (3.58)$$

$$\underline{SOC}_s \leq soc_{s,t} \leq \overline{SOC}_s \quad \forall s, \forall t \quad (3.59)$$

Referring to the network modelling, a strong hypothesis is considered when using the linear approach described in equations (3.60) and (3.61), for a multitemporal power flow formulation. It is supposed that the grid losses are negligible ($G_{ij} \approx 0$) and that the node voltages are near to the basis, ($V_i = 1 \text{ p.u.}$). Then, only susceptance of the tie-lines (B_{ij}) and the voltage angles (θ_i) are considered in the model.

$$\sum_j B_{ij} (\theta_{i,t} - \theta_{j,t}) = p_{i,t} + p_{g,t}^{AM} - P_{l,t}^{AM} - p_{f,t}^{AM} - p_{s,t}^{AM} \quad \forall i, \forall t \quad (3.60)$$

$$-\bar{P}_{ij} \leq B_{ij} (\theta_{i,t} - \theta_{j,t}) \leq \bar{P}_{ij} \quad \forall i, \forall j, i \neq j, \forall t \quad (3.61)$$

Lastly, similarly to the definition of the energy-only LFM, with the aim to use only local resources to solve congestions and imbalances, equation (3.62) is stated. It ensures that the scheduled profile considering an upstream viewpoint it is maintained after the market clearing. This equation will force to compensate any deviation of the scheduled program, equilibrating upward and downward flexibility.

$$\sum_l (P_{l,t}^{SCH} - P_{l,t}^{AM}) + \sum_f (P_{f,t}^{SCH} - p_{f,t}^{AM}) + \sum_g (P_{g,t}^{SCH} - p_{g,t}^{AM}) - \sum_s p_{s,t}^{AM} = 0 \quad \forall t \quad (3.62)$$

For test this approach, the test feeder showed in Figure 19 is used. It is slightly different than the first test feeder. The connection with the upstream networks is in node 800, as well as the slack node. In terms of the SLs, they are considered the same than those presented in Table 4. However, to simplify the scenario, only two FLs and two BESSs are considered as the figure shows. Lastly, the issue to be solved is a congestion in line 814-816 of 150 kW, that will divide the network in two areas, a downward area downstream node 816, and an upward area in the rest of the grid. Then, BESSs are forced to provide downward flexibility products while FLs are force to provide upward ones.

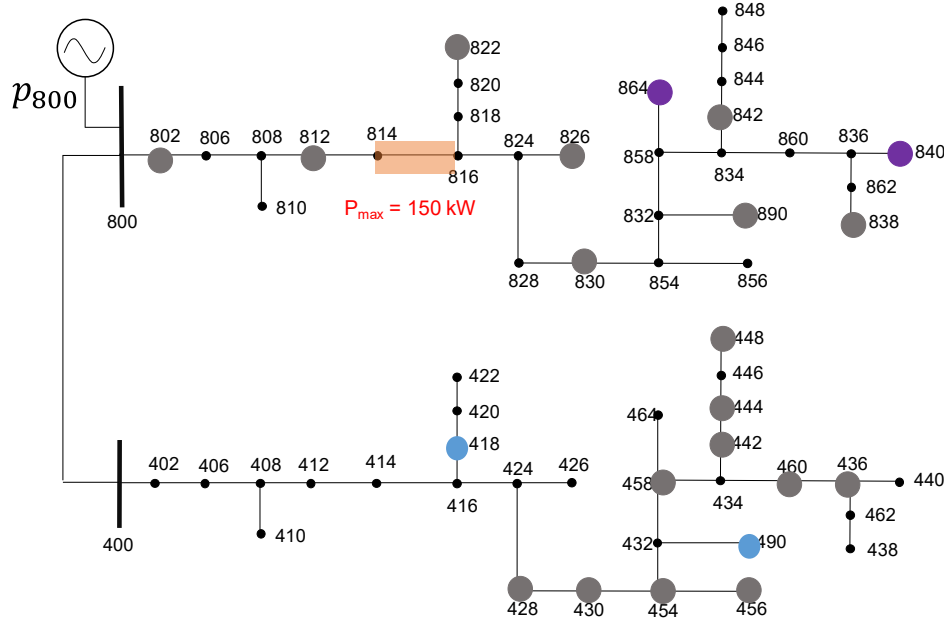


Figure 19. Test feeder for illustrative example 2. Grey circles represent SLs, blue circles FLs, and purple circles BESSs. Slack node: 800. PCC in 800.

Apparently, there is no necessity of capacity products under this situation, however, with the aim of prove the increase in liquidity of the market when capacity products are considered, SOC of BESSs are considered on their lower bound in the initial time. Then, it is expected that an energy-only market, focused only on one time period, results in unfeasible situations if a grid constraint appears. However, a mixed capacity and energy LFM, could pre-allocate capacity products, forcing batteries to prevent that situation to happen.

Figure 20 represents the situation to solve with the LFM. There are two time periods, the first one does not have any grid issue, however, the second one has a congestion in the line 814-816 as can be seen in the picture. It a situation with all the BESSs discharged, it is expected that energy market results in an unfeasible situation, as it does only focus on the current period t . On the other hand, the mixed energy and capacity LFM will be able to anticipate that and solve the congestion pre-allocating capacity products and enhancing BESS to charge in t_1 to provide downward flexibility in t_2 .

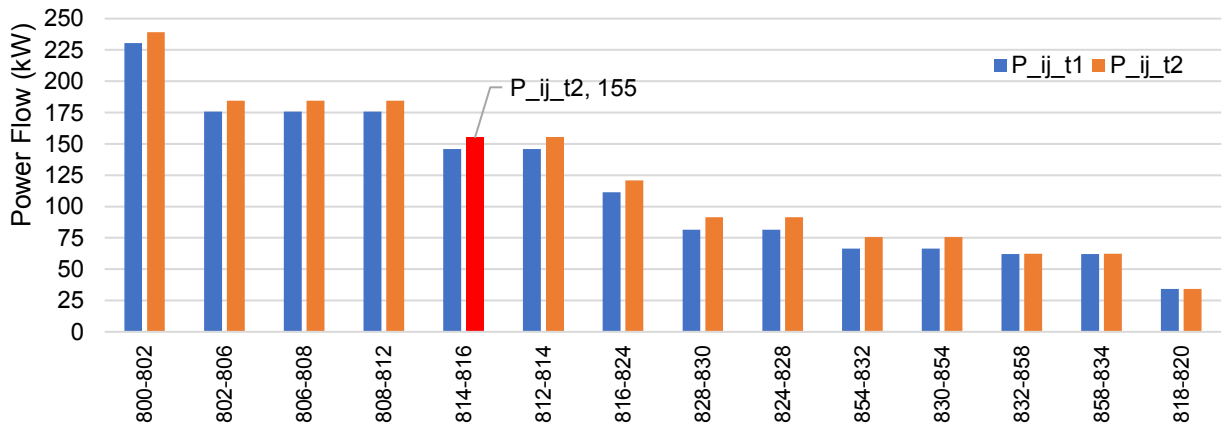


Figure 20. Initial status of the power flows in the grid before market clearing. Line 814-816 has a congestion in t_2 with $P = 155$ kW.

The following section will present the results of an energy-only LFM for the previously explained situation, and then, the results for a mixed capacity and energy LFM.

7.2.1. Results considering the energy only LFM

In this subsection the results of the energy only LFM scenario is explained. The situation and the bids of the FLs and BESSs are showed in Table 9 and Table 10, showing the scheduled demands for the FLs ($P_{f,t}^{SCH}$), and the upward and downward energy prices ($S_{f,t}^{e,u}$ and $S_{f,t}^{e,d}$ respectively); in the case of the BESSs additionally to the prices for the energy products ($S_{s,t}^{e,u}$ and $S_{s,t}^{e,d}$) the systems characteristics and the situation of their respective state of charge at the initial time (SOC_0) are also depicted. BESSs are in their lower bounds in terms of state of charge as referred in the previous section. Energy bids are also detailed in those tables considering downward and upward products for those assets.

		$P_{f,t}^{SCH} (kW)$	$S_{f,t}^{e,d} (€/kWh)$	$S_{f,t}^{e,u} (€/kWh)$
f_1	t_1	81	0.049844	0.049256
	t_2	79,31	0.049642	0.049053
f_2	t_1	9,56	0.079032	0.020068
	t_2	8,91	0.078829	0.019866

Table 9. FLs bids for the energy only LFM in the illustrative example 2.

		$S_{s,t}^{e,d} (€/kWh)$	$S_{s,t}^{e,u} (€/kWh)$	$SOC_0 (kWh)$	$P_s^{INV} (kW)$	$\overline{SOC}_s (kWh)$	$\underline{SOC}_s (kWh)$
s_1	t_1	0.086679	0.012421	36	12	120	36
	t_2	0.086476	0.012219	36	12	120	36
s_2	t_1	0.061635	0.037465	105	35	350	105
	t_2	0.061433	0.037262	105	35	350	105

Table 10. BESSs bids for the energy only LFM in the illustrative example 2.

It should be born in mind that the line 814-816 has a thermal power limit of 150 kW. Considering this situation, the energy market considering only energy products is solved. The power flow after the market clears is showed in Figure 21. As it is possible to observe, the power restriction is fulfilled, however, the overall optimization problem results in an infeasible situation as the BESSs is discharged under their lower limits.

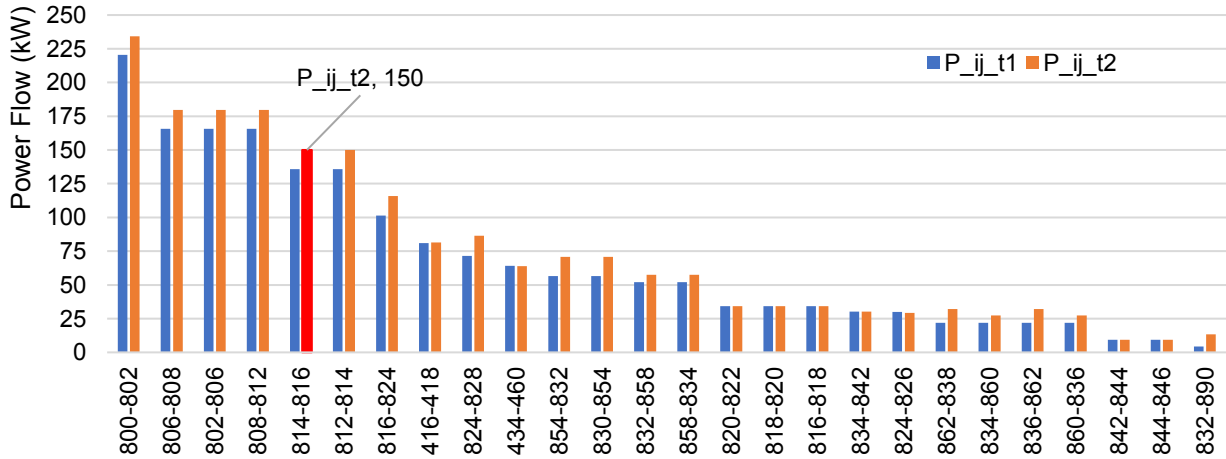


Figure 21. Power flow in the microgrid downstream node 800 for the energy only LFM.

Figure 22 shows the state of charge situation after the LFM clears. The SOC in the period t_2 is solved below its lower limit, represented by the dark purple line. Aiming not to overpass the thermal limit of the line 814-816, the BESS s_1 would have to inject a total of $p_{s_1,t_2}^{AM} = -4,9 kW$, what would discharge the battery up to a state of charge of $soc_{s_1,t_2} = 103,7 kWh$ what is far below the $\underline{SOC}_{s_2} = 105 kWh$. Then, the problem results infeasible, and it is proved that the energy only LFM could not have offer a solution to this grid situation.

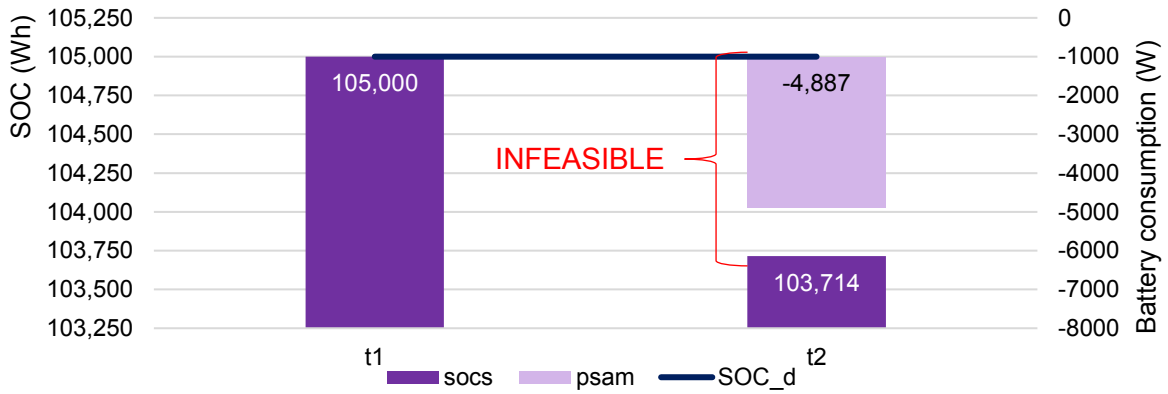


Figure 22. SOC situation after the energy only LFM clearing.

In Figure 23 and Figure 24 it is showed the resulting aftermarket power consumption and the upward energy products matched for FLs f_1 and f_2 , respectively. Both FLs shows a similar behaviour, providing only upward flexibility to match the downward flexibility bid of the BESS s_2 . Energy products of the FLs are sold in the LFM to fulfil balance equation presented in (3.62). The p_{f,t_2}^{AM} results increased considering the matched upward energy bid $\omega_{f,t_2}^{e,u}$, as those figures show.

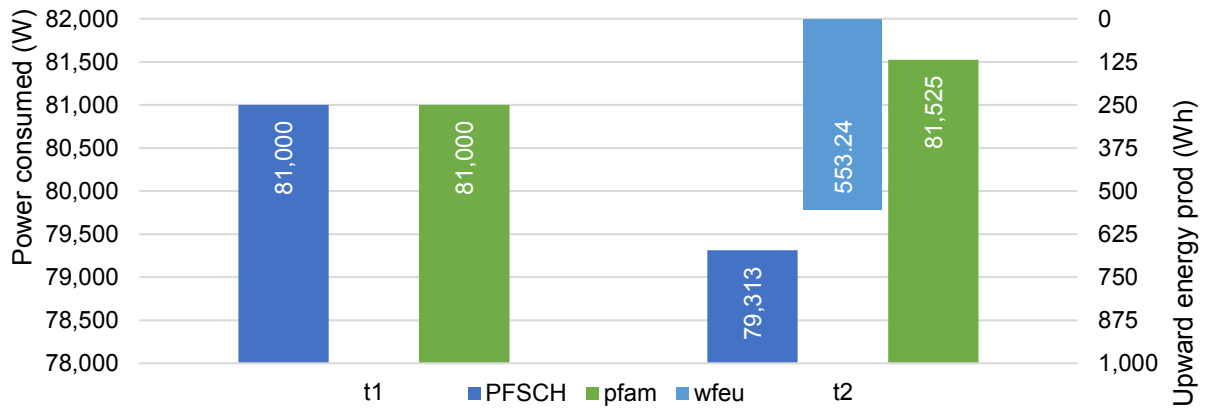


Figure 23. FL 1 cleared bids after the market clearing considering only energy products

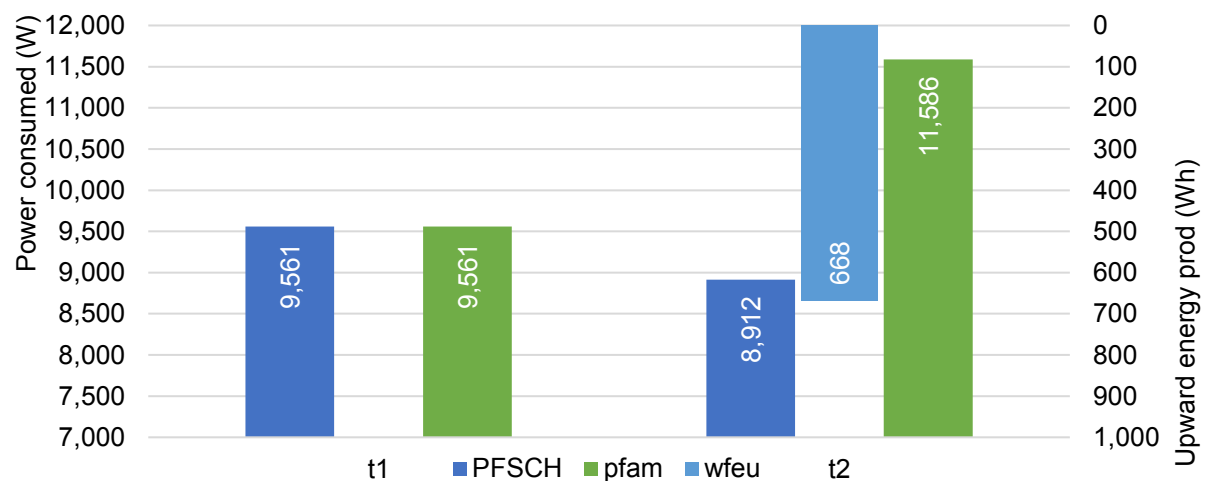


Figure 24. FL 2 cleared bids after the market clearing considering only energy products

A summary of the results of this energy only LFM is presented in Table 11. The total cost would amount to 0,034633 € with a total of $\omega = 1,221 \text{ kWh}$ of flexibility product matched in each direction. FL f_2 accounts for the majority of the cost of the solution.

	c (€)	$\omega^{e,u}(Wh)$	$\omega^{e,d}(Wh)$
f_1	0,000163	553,23	
f_2	0,019706	668,43	
s_2	0,014764		1221,66
TOTALS	0,034633	1221,66	1221,66

Table 11. Energy products traded in the market and costs for the infeasible energy only market.

To sum up, there has been analysed that the energy only market it is not an enough framework to provide a solution for the illustrative example proposed. Then, the introduction of an energy and capacity LFM it is assessed, showing its main benefits compared to the energy only approach.

7.2.2. Results considering mixed energy and capacity LFM

In this section, the results of the mixed energy and capacity LFM are showed. Firstly, it is important to show the energy and capacity bids of the assets involved in the market. For that reason, Table 12 and Table 13 are written, showing the energy and capacity bids for upward ($S_{f,t}^{e,u}, S_{s,t}^{e,u}, S_{f,t}^{c,u}, S_{s,t}^{c,u}$) and downward ($S_{f,t}^{e,d}, S_{s,t}^{e,d}, S_{f,t}^{c,d}, S_{s,t}^{c,d}$). Similarly to the previous scenario, the FLs are allowed to modify its consumption in a range of $0,70 P_{f,t}^{SCH} \leq p_{f,t}^{AM} \leq 1,30 P_{f,t}^{SCH}$, providing both upward and downward products. Then BESS are allowed to offer flexibility products as long as they could maintain their SOC between their lower (SOC_s) and upper ($\overline{SOC}_s$) bounds, and the power they demand or inject do not overpass the power rate of the inverter (P_s^{INV}).

		$P_{f,t}^{SCH}(kW)$	$S_{f,t}^{e,d}(\text{€/kWh})$	$S_{f,t}^{e,u}(\text{€/kWh})$	$S_{f,t}^{c,d}(\text{€/kW})$	$S_{f,t}^{c,u}(\text{€/kW})$
f_1	t_1	81	0,049844	0,049256	0,004984	0,004926
	t_2	79,31	0,049642	0,049053	0,004964	0,004905
f_2	t_1	9,56	0,079032	0,020068	0,007903	0,002007
	t_2	8,91	0,078829	0,019866	0,007883	0,001987

Table 12. FLs for the mixed energy and capacity market of the illustrative example 2.

The characteristics of the BESS limits and inverter ratings are the same than those presented in Table 10, so it is omitted in this section for the sake of clarity.

		$S_{s,t}^{e,d}(\text{€/kWh})$	$S_{s,t}^{e,u}(\text{€/kWh})$	$S_{s,t}^{c,d}(\text{€/kW})$	$S_{s,t}^{c,u}(\text{€/kW})$
s_1	t_1	0.08668	0.01242	0.00867	0.00124
	t_2	0.08648	0.01222	0.00865	0.00122
s_2	t_1	0.06164	0.03746	0.00616	0.00375
	t_2	0.06143	0.03726	0.00614	0.00373

Table 13. BESSs bids for the mixed energy and capacity LFM in the illustrative example 2. SOC and inverter data is the same that the one showed in Table 10.

The power flow results after the mixed energy and capacity LFM is solved, are presented in Figure 25. In this situation, the thermal limit of the line 814-816 is not violated, remaining just in the upper bound of it. The power flow in period t_1 though this line has also been increased, due to the irruption of upward energy products of the BESS s_2 , as it is following explained.

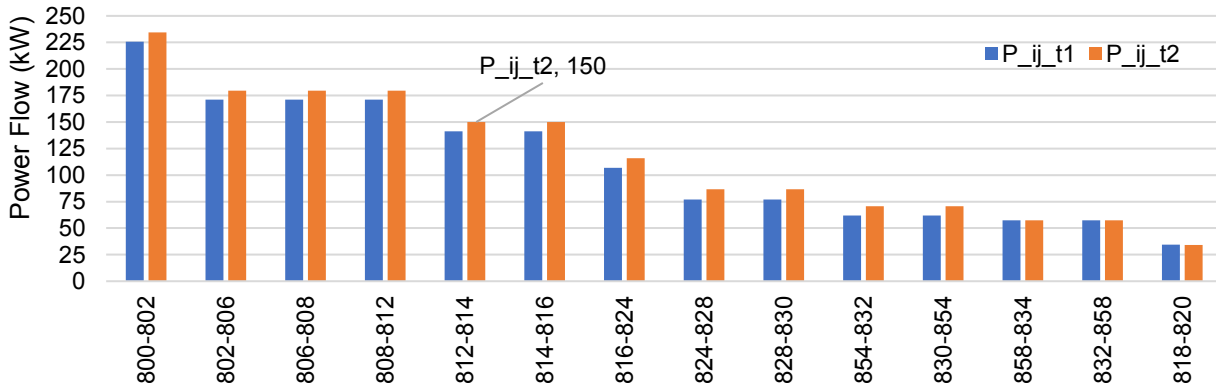


Figure 25. Power flow using a mixed energy and capacity LFM.

The BESS s_2 sold products are presented in Figure 26. It has to be considered that the capacity products are cleared in a period of time before it appears in the figure. For each capacity product, there are associated an energy one that is what really influence on the SOC of the battery, allowing it to increase the SOC in t_1 to provide downward products in period t_2 , avoiding overcoming the thermal limit of the line 814-816. As it is showed, the lower bound of the state of charge restriction it is neither violated, what results in a feasible market situation.

As the capacity products are allocated one period before the energy products allocation, the $\omega_{s,t_1}^{c,u}$ is considered as known from period t_0 .

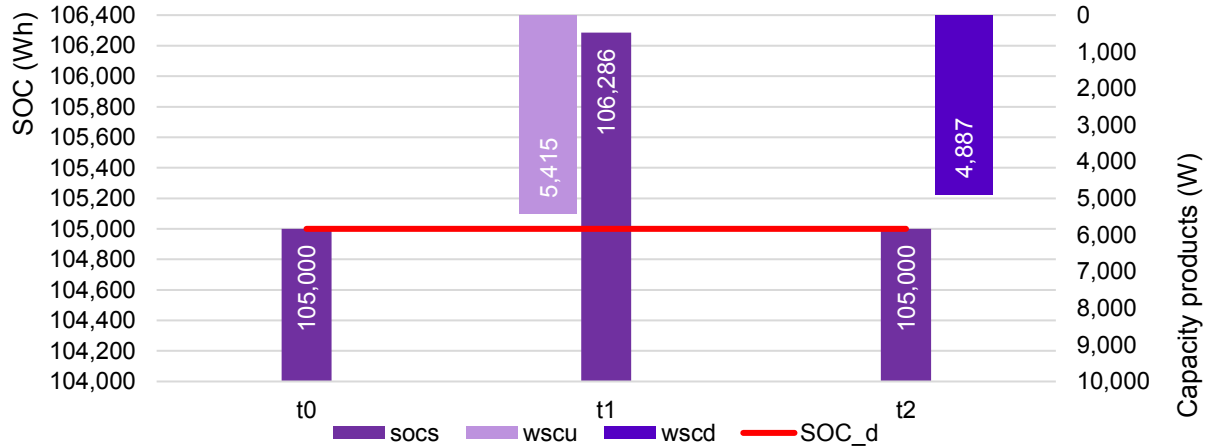


Figure 26. SOC of BESS 2 and capacity products traded in the market for the energy and capacity market.

In this scenario, the only FL required to complete the LFM clearing is f_1 . As Figure 27 shows, that asset provides both downward and upward products to balance the grid, solving the congestion considering only local assets. As Table 14 also shows, the balance is achieved for both energy and capacity products. In that situation, the upstream assets would not have to modify their consumption as a result of the flexible products traded in the LFM.

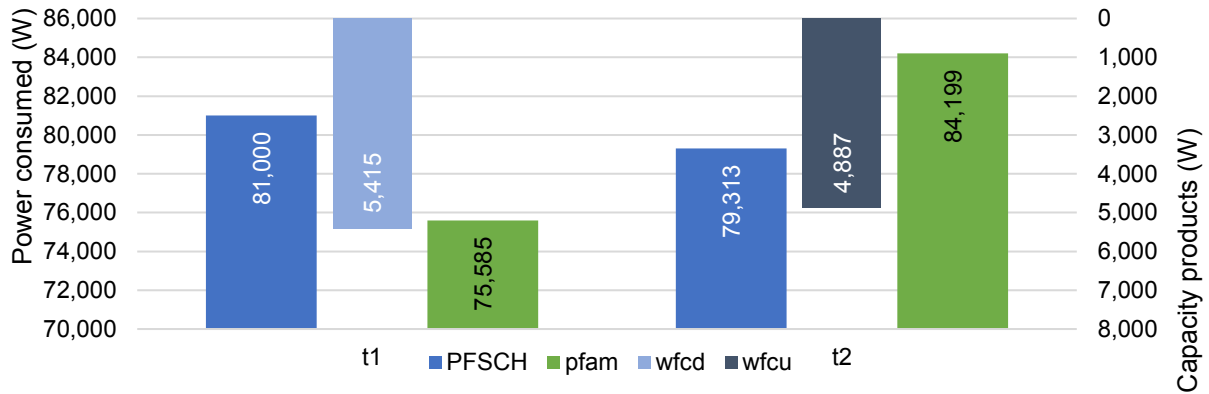


Figure 27. FL 1 products and power demand in the mixed energy and capacity LFM.

The results presented in Table 14 determine that the total cost for alleviating the line 814-816 is 0,1331 €, considering all the products needed for prepare the BESS and balance the grid. The total energy products add to 2,575 kWh and a total of 10,301 kW for capacity products.

		$c_t(\text{€})$	$c_t^e(\text{€})$	$c_t^c(\text{€})$	$\omega_t^{e,d}(\text{Wh})$	$\omega_t^{e,u}(\text{Wh})$	$\omega_t^{c,u}(\text{W})$	$\omega_t^{c,d}(\text{W})$
f_1	t_1	0,0274 €	0,0004 €	0,0270 €	1.353,64			5.414,57
	t_2	0,0243 €	0,0004 €	0,0240 €		1.221,66	4.886,65	
s_2	t_1	0,0366 €	0,0164 €	0,0203 €		1.353,64	5.414,57	
	t_2	0,0448 €	0,0148 €	0,0300 €	1.221,66			4.886,65
TOTALS		0,1331 €	0,0319 €	0,1013 €	2.575,30	2.575,30	10.301,21	10.301,21

Table 14. Results of the mixed energy and capacity market.

It is important to notice that the ratio between energy and capacity products is 0,25, as it is the Δt consider for this approach. This is due to the fact that any uncertainties are considered, and all the grid data is known before the market clears, allowing the DSO to take advantage of it, reducing to the minimum the necessity of buy any product just in case the situation is worse than expected.

7.3. Use case

To finalize with the simulations of the centralized approach, in this section, a realistic use case is presented, which involves five microgrids interconnected with a transmission system, as Figure 28 presents. Each microgrid feeder is managed by an Agg, i.e., Agg 1 will manage assets downstream node 000, Agg 2, downstream node 200, and so on. It is considered that there is only a PCC with the rest of the network, in node 100. There is a total of 75 SLs, 20 FLs, 20 FGs and 20 BESSs along with 5 Aggs. SLs are represented by grey dots in the figure, FLs by blue dots, BESSs by purple ones, and FGs by orange dots. The total static demands sums to 60,5 MW.

In this use case it is minimized the total cost for each aggregator for energy products ($C_{a,t}^e$) and for capacity products considering they are cleared in advance ($C_{a,t+1}^c$). Those cost are expressed as a function of the rest of the costs of each asset as equations (3.63), (3.64) and (3.65) shows.

$$\min z = \sum_t \sum_a (C_{a,t}^e + C_{a,t+1}^c) \quad (3.63)$$

$$C_{a,t}^e = \sum_f C_{f,t}^e + \sum_g C_{g,t}^e + \sum_s C_{s,t}^e \quad \forall a, \forall t \quad (3.64)$$

$$C_{a,t}^c = \sum_f C_{f,t}^c + \sum_g C_{g,t}^c + \sum_s C_{s,t}^c \quad \forall a, \forall t \quad (3.65)$$

The rest of the restrictions of the problem has been defined in the previous illustrative example, concisely from equation (3.49) to (3.62), so they are not defined in this section again for the sake of conciseness.

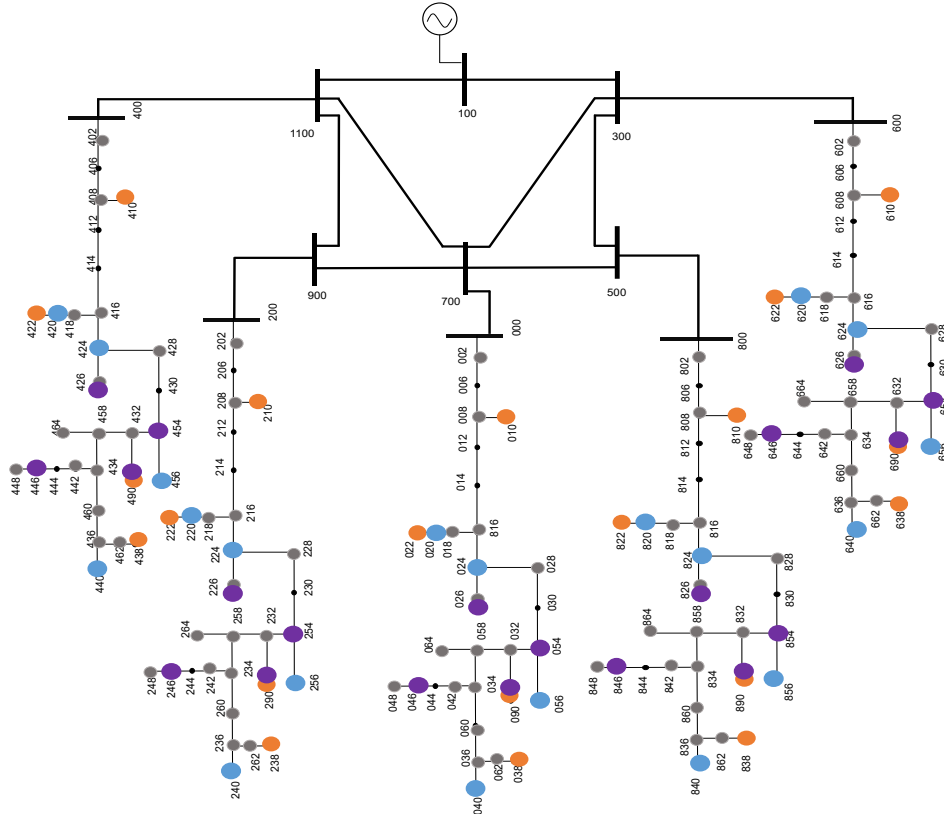


Figure 28 Test feeder for the use case. Grey circles represent SLs, blue circles FLs, orange circles FGs and purple circles BESSs. Slack node: 100. PCC in 100.

The problem is solved for a whole day, in time slots of 15 min. For that reason, as multiperiod simulation need multiperiod data, some examples of the consumption of SLs are shown in Figure 29. Those consumption profiles are made randomly considering several generation functions. Some of them represent electro-intensive product-oriented industries, which profile is near constant, as SL 15. Other, on the other hand, are considered to present a more domestic consumption profile, with speaks about 12:00 and 21:00, as SL 13. There can be also highlighted that some SLs are considered to have installed PV systems downstream its connection with the network, showing a duck curve profile, as SL 4.

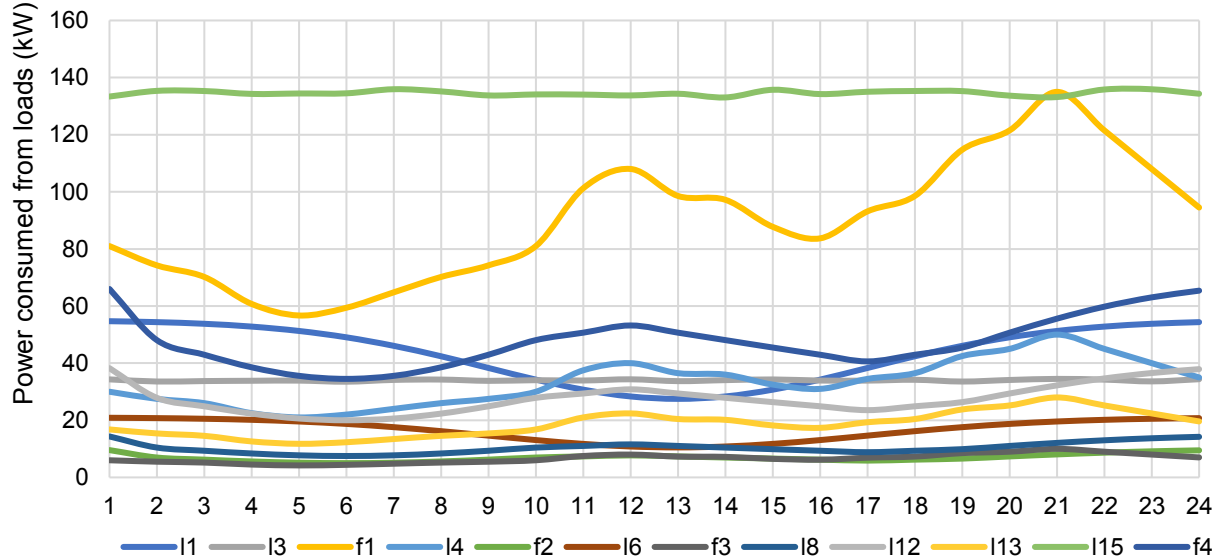


Figure 29. Examples of power consumed by static and flexible loads from 00:00 to 24:00.

Also, considering that all flexible generators are PV systems, generation profiles are computed for a typical sunny day in Málaga, as Figure 30 shows. There are some of them with higher installed power, as FG 3, others are considered to have lower profiles, typical of non-extensive PV installations.

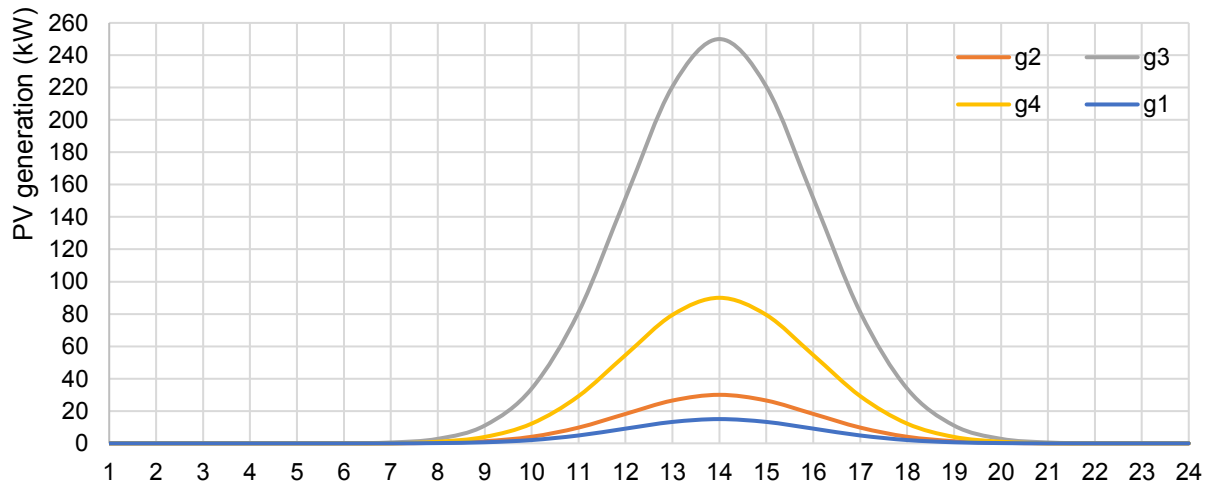


Figure 30. PV generation profiles considering a typical sunny day for generators g1 to g4.

In terms of the BESSs, their main characteristics are written in Table 15, power of the inverter is considered as 10% of the total capacity of the BESS, and both charging and discharging efficiencies are set constant to 95%. Also, all the batteries are forced to maintain its initial state of charge ($SOC_{s,0}$) at the end of the day, meaning that ($SOC_{s,0} = SOC_{s,T|}$). This restriction tries to model the usual behaviour of the BESSs, as if a long enough period is considered, their power balance is zero, as they cannot generate electricity, just storage it.

s	$SOC_{s,0}(kWh)$	$P_s^{INV}(kW)$	η_u	η_d	$\overline{SOC}_s(kWh)$	$SOC_s(kWh)$
s_1	72	12	0.95	0.95	120	36
s_2	210	35	0.95	0.95	350	105
s_3	270	45	0.95	0.95	450	135
s_4	360	60	0.95	0.95	600	180
s_5	72	12	0.95	0.95	120	36
s_6	210	35	0.95	0.95	350	105
s_7	270	45	0.95	0.95	450	135
s_8	360	60	0.95	0.95	600	180
s_9	72	12	0.95	0.95	120	36
s_{10}	210	35	0.95	0.95	350	105
s_{11}	270	45	0.95	0.95	450	135
s_{12}	360	60	0.95	0.95	600	180
s_{13}	72	12	0.95	0.95	120	36
s_{14}	210	35	0.95	0.95	350	105
s_{15}	270	45	0.95	0.95	450	135
s_{16}	360	60	0.95	0.95	600	180
s_{17}	72	12	0.95	0.95	120	36
s_{18}	210	35	0.95	0.95	350	105
s_{19}	270	45	0.95	0.95	450	135
s_{20}	360	60	0.95	0.95	600	180

Table 15. BESSs characteristics for the Use Case.

For sake of simplicity bids are maintained constant though all hours for all types of products with the same characteristics of previous examples. Storage systems are configured to stop discharge if their state of charge is less than 40%, considering a half-full battery at initial time.

In Figure 31 prices of some of the products are represented to show how they vary depending on the wholesale market price. As $S_t^{e,d} > S_t$ and $S_t^{e,u} < S_t$, lines drawn above black line of the wholesale market price, represents downward flexibility products while those drawn below represents upward flexibility products. Wholesale prices were obtained from OMIE for Spanish spot market cleared in 22/03/21.

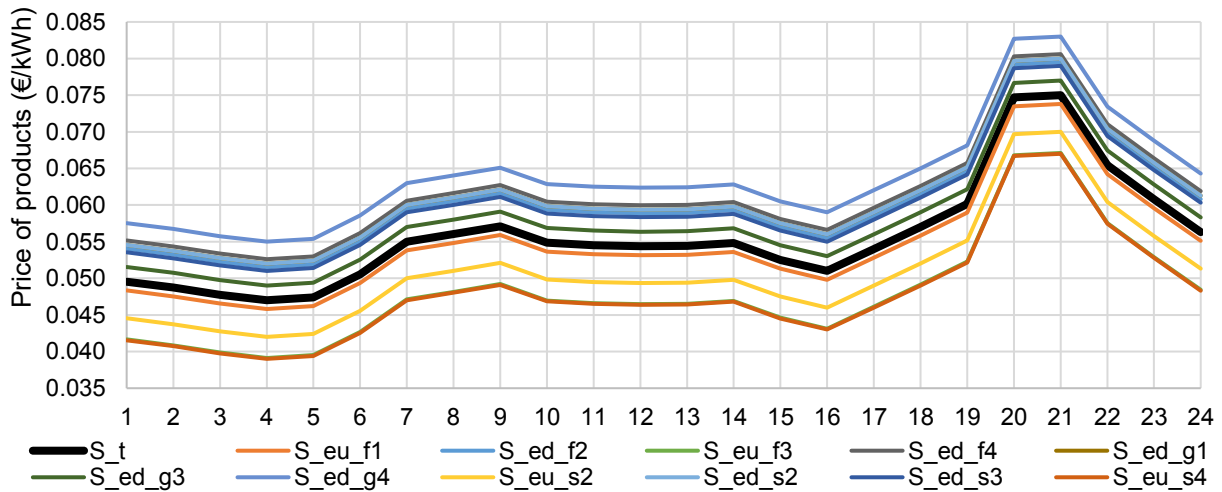


Figure 31. Prices for bids of the different assets. Wholesale prices were obtained from OMIE for Spanish market (day 22/03/21).

Previous examples are limited because of the amount of assets connected to the grid. With this approach, possibilities increase exponentially, as more assets may harness higher quantities of flexibility. It also allows to solve issues in the PCC of the microgrid with the transmission system, as opposed to the illustrative example were problem result infeasible as a result of the impossibility of maintain balance creating an upward region and downward region.

In this use case several scenarios are illustrated using grid proposed in Figure 28. The system has a total static load of 60,5 MW. Test scenarios aims to demonstrate the effectiveness of the of the proposed LFM market for distribution operation. For all test scenarios bids of market participants are described in Table 16.

FLs	Node	$\Delta S_{f,t}^e$ (€/kWh)	FGs	Node	$\Delta S_{g,t}^e$ (€/kWh)	BESSs	Node	$\Delta S_{s,t}^e$ (€/kWh)
f_1	20	0,000294	g_1	10	0,026121	s_1	20	0,037129
f_2	24	0,029482	g_2	22	0,018527	s_2	54	0,012085
f_3	40	0,045939	g_3	90	0,031733	s_3	90	0,041499
f_4	56	0,044199	g_4	38	0,013426	s_4	46	0,006196
f_5	220	0,025466	g_5	210	0,019113	s_5	220	0,020576
f_6	224	0,033518	g_6	222	0,024987	s_6	254	0,028503
f_7	240	0,027631	g_7	290	0,005625	s_7	290	0,045643
f_8	256	0,006366	g_8	238	0,044024	s_8	246	0,033373
f_9	420	0,042077	g_9	410	0,008155	s_9	420	0,021564
f_{10}	424	0,029675	g_{10}	422	0,015202	s_{10}	454	0,035534
f_{11}	440	0,045399	g_{11}	490	0,045977	s_{11}	490	0,032682
f_{12}	456	0,007641	g_{12}	438	0,043019	s_{12}	446	0,042566
f_{13}	620	0,020943	g_{13}	610	0,010344	s_{13}	620	0,001739
f_{14}	624	0,023488	g_{14}	622	0,008079	s_{14}	654	0,043272
f_{15}	640	0,040479	g_{15}	690	0,037335	s_{15}	690	0,040940
f_{16}	656	0,038477	g_{16}	638	0,038053	s_{16}	646	0,028216
f_{17}	820	0,010154	g_{17}	810	0,045510	s_{17}	820	0,019854
f_{18}	824	0,027453	g_{18}	822	0,028131	s_{18}	854	0,019210
f_{19}	840	0,011069	g_{19}	890	0,020115	s_{19}	890	0,007944
f_{20}	856	0,024060	g_{20}	838	0,023505	s_{20}	846	0,001014

Table 16. Delta prices for market participants (FLs, FGs and BESSs) in the use case.

Bids for markets participants are generated randomly with the consideration that no negative prices will arise, i.d. that delta prices must be lower or equal than the minimum price of wholesale market. As the previous examples, for the sake of clarity, delta prices are maintained both for upward and downward flexibility products during all day. Capacity products prices are generated assuming they are a 10% of the costs of energy products, and also a 5% random variation are applied to them. In this situation the shape of the prices is similar to those showed in Figure 31.

With all this information, two different scenarios are proposed, the first one solves a congestion maintained in time due to a technical issue, while the second one present an imbalance issue due to the disconnection of a load.

7.3.1. Congestion scenario

In this section the results of the congestion scenario are described. Results are presented considering that the power flow through line 228-230 is limited to 500 kW. In Figure 32 the results of the power flow are presented for the scheduled program (orange line), and after the market clearing (blue line). In the same figure the energy products sold during the day. In grey the summatory of the upward energy products for all aggregators is presented, while in yellow it is presented the downward products. As it is possible to see, the amount of upward and downward products cancels each other, due to the fact that the grid issues are solved considering only local resources.

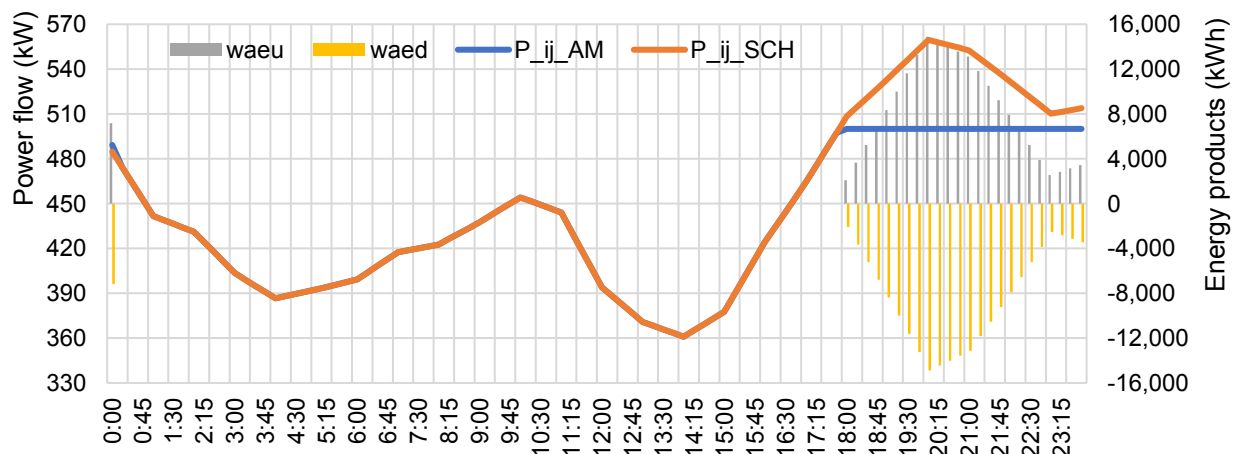


Figure 32. Power flow through line 228-230 and upward and downward energy products traded in the market.

In Figure 33 the distribution of the products among Aggs is presented. It is possible to see that the Agg 1 and 2 are those who manage the assets who greatly participate in the market, with 170,52 kWh and 198,14 kWh of flexibility products respectively. Others Aggs contributes to the global share in a minor portion.

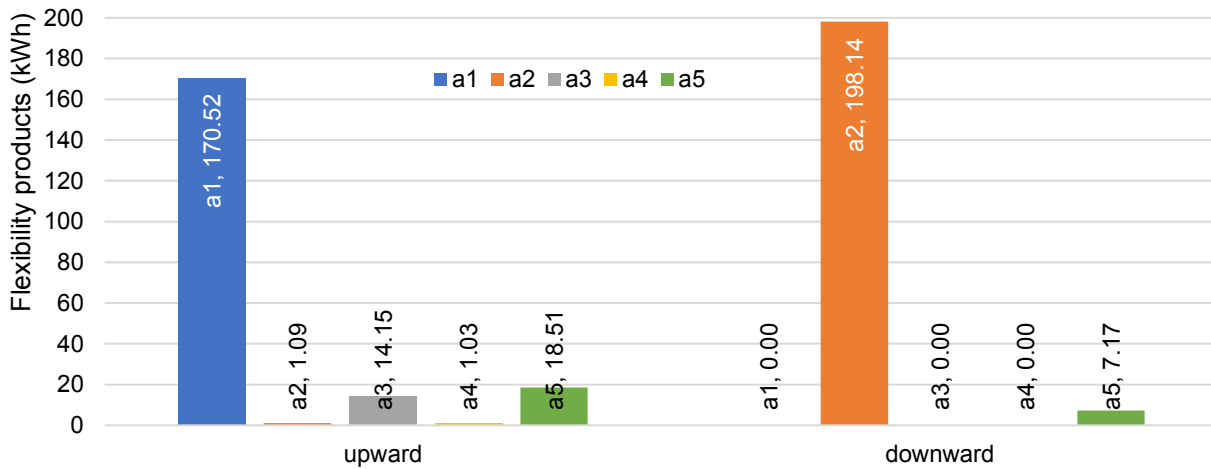


Figure 33. Aggregators share of flexibility products traded in the market in the congestion solving scenario.

In Figure 34 it is presented a pie chart of the products sold by each aggregator in the LFM. Agg 1 and Agg 2 dominate a major part of the trading, as previous figure suggested. Besides of that, in this figure it is represented in blue the contribution of FLs, in orange, FGs and in purple BESSs. FLs dominate the trading, with more than the 96% of the share, BESSs have a total of 3% and FGs less than 1%.

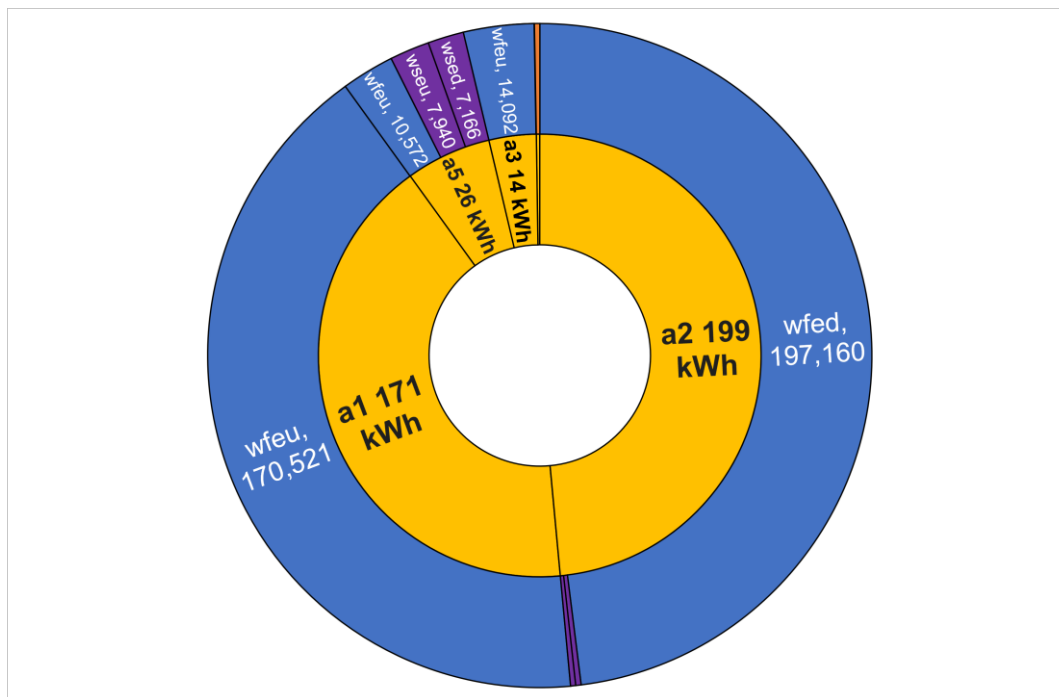


Figure 34. Pie chart of the aggregators share of FLs products (in blue), FGs products (in orange) and BESSs products (in purple) for the congestion scenario.

The hourly distribution of the products traded in the LFM is shown in Figure 35. In the positive semiaxis of the figure the upward flexibility products of the FLs involved in the traded are presented, while in the negative semiaxis, downward flexibility products are drawn. FL 1 is the one which sold more products, in the second place is FL 8. The trading is concentrated from 18:00 to 23:45 in the night, when the congestion is localized.

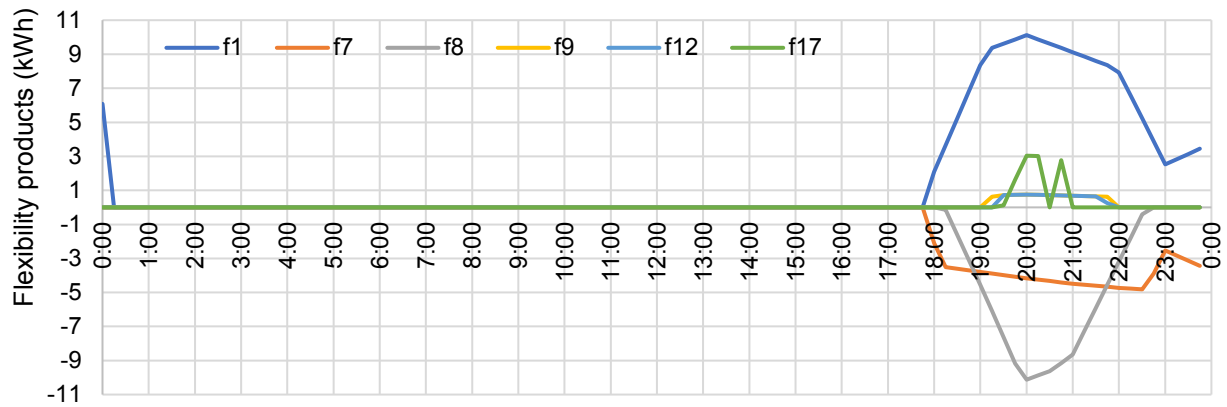


Figure 35. Flexibility products for FLs. Positive semiaxis represent upward direction and negative semiaxis downward direction.

In Figure 36 it is presented the downward products traded by the FGs, the value of the products trades is very low, representing less than 1% in the total of the market as it is possible to see in the figure.

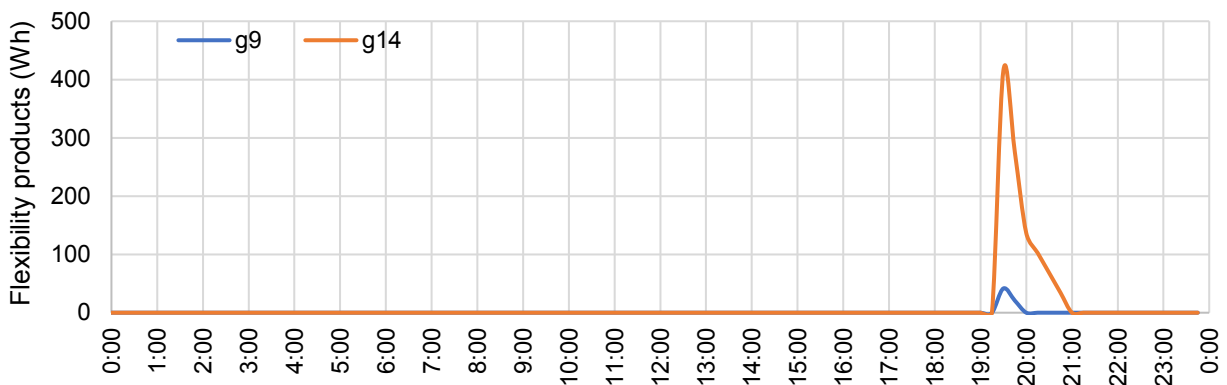


Figure 36. Flexibility products for FGs. Positive semiaxis represent downward products of FGs.

Upward and downward flexibility products from BESSs are presented in the positive and negative semiaxis of the Figure 37. Only two BESSs are present in this scenario, being activated in the first period of the market and in the ones between 20:00 and 22:00.

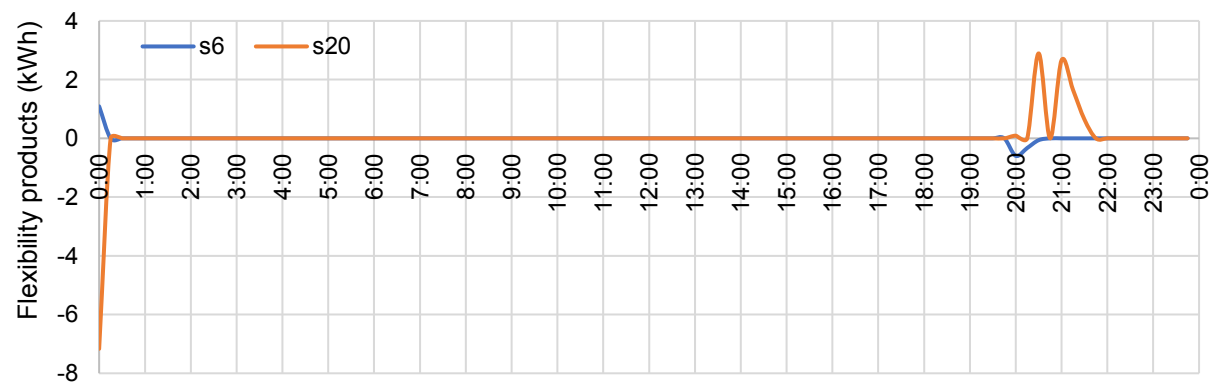


Figure 37. Flexibility products for BESS in the congestion scenario. Positive semiaxis represent upward products while negative downward products.

Besides of the analysis of the interaction of the flexible assets given, it is also of interest of showing the hourly distribution of the multipliers, as they give a sense of how a change in the thermal limit of the line modify the total cost of the solution given in the LFM. Depending on the assets activated to provide flexibility products, the value of the multipliers changes, as the Figure 38 show. In that figure, it is presented the hourly distribution of the multiplier value of the balance equation in node 230, downstream the congestion.

As it is possible to see, the hourly distribution is related with the difference between the thermal limit and the baseload power flow. Between 20:00 and 21:00 where the congestion is higher, the multiplier price is also.

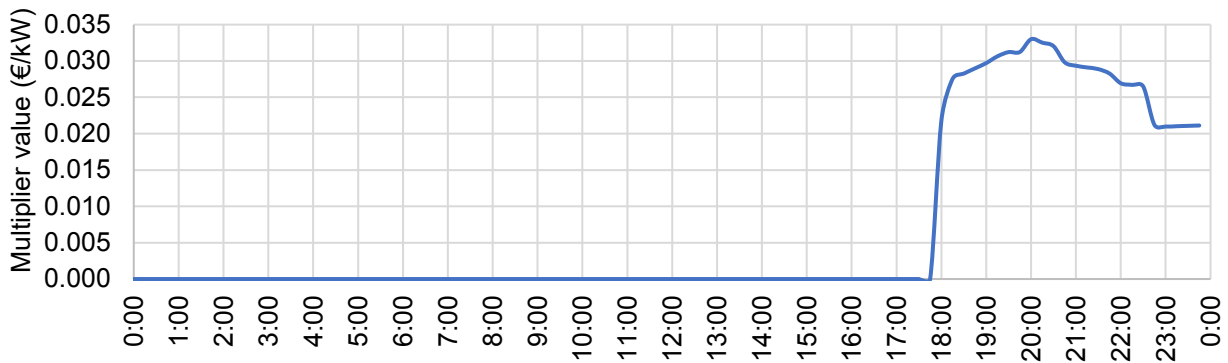


Figure 38. Multipliers value in the balance equation on node 230, downstream the line with congestion issues.

Lastly, the hourly distribution of the costs is presented in Figure 39, representing a similar shape than the multiplier value and the congestion value. It is represented in blue the total cost for the FLs, in orange the cost for the flexibility of FGs and in purple the values of the BESSs. The total cost added is 20,969 € for a total amount of 392,344 kWh of energy products and 1,64247 MW of capacity products.

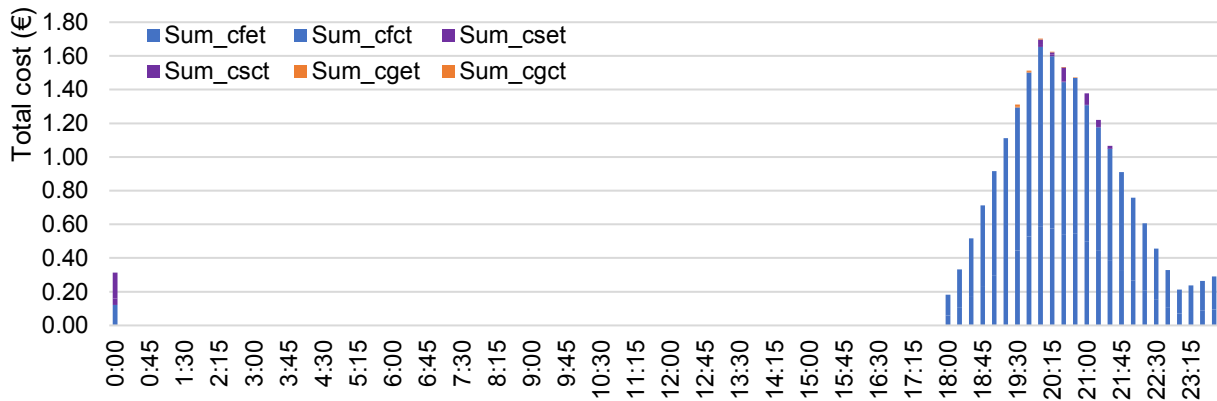


Figure 39. Distribution of the periods costs of the flexibility products traded in the market in the congestion scenario.

7.3.2. Imbalance scenario

In the imbalance scenario it is presented the behaviour of the grid if the SL 75 is disconnected for the whole day. Figure 40 presents the situation of the upward flexibility products in the semipositive axis and the downward flexibility products and the SL imbalance on the negative semiaxis. Blue lines present the flexibility products of the FLs, orange the ones of the FGs and purple the ones associated to the BESSs. Grey charts present the imbalance generated by the SL 75.

As is possible to see, the upward and downward axis remains equilibrated, solving the imbalance with local resources. FGs products are solved in the central hours of the day, when their production can be curtailed.

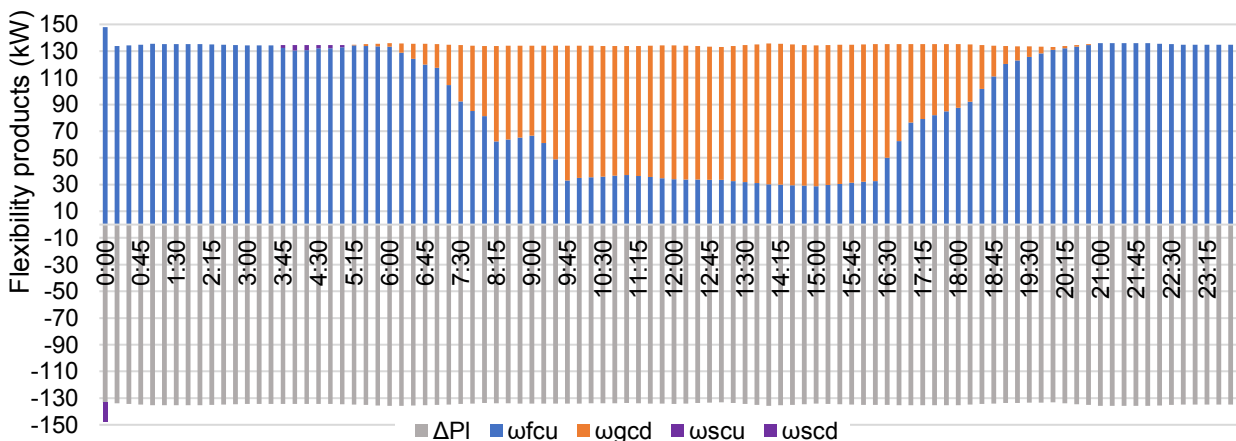


Figure 40. Imbalance solving using flexibility products of FLs (blue), FGs (orange) and BESSs (purple). Imbalance created by SL 75 is presented in grey.

In the Figure 41 the flexibility products share among aggregators is presented. As the imbalance created by SL 75 is in the downward direction, the vast majority of the products sold in the market is in the upward direction. The only exception is the 14,55 kW of energy products of the Agg 5, that are needed in order to maintain the state of charge at the end of the day with the initial value. Besides, as is presented, Agg 4 is the one with highest share of flexibility products (3.283,92 kW).

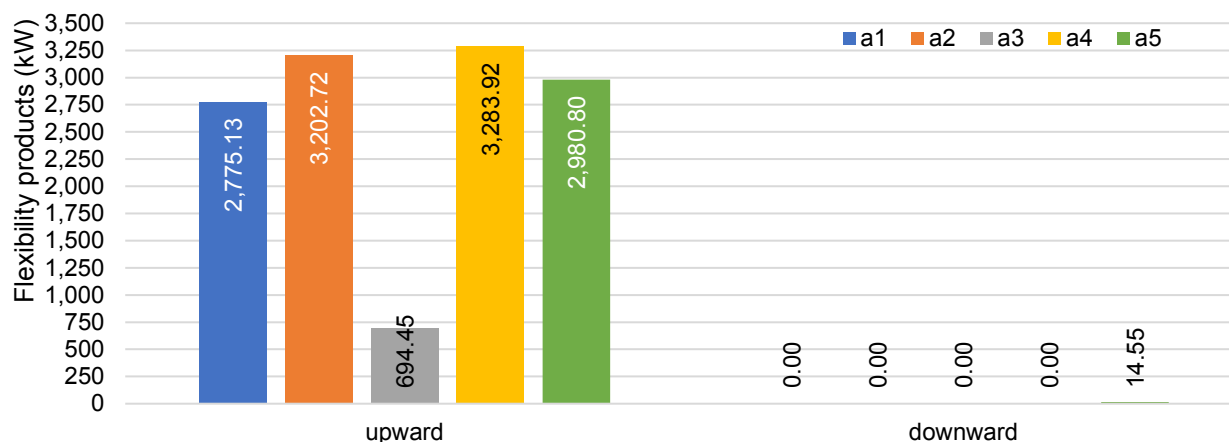


Figure 41. Share of the upward and downward products sold in the market in the imbalance scenario.

The distribution of the flexibility products it is also presented in Figure 42, the Aggs share of Agg 1, Agg 2, Agg 4 and Agg 5 is near the 25%. In blue is presented the flexibility products of the FLs which are the ones that dominates the trading. In second place, the products of the FGs are presented in orange. Lastly, the BESSs products are presented in purple, however those products are residuals, as could be extracted from the representation.

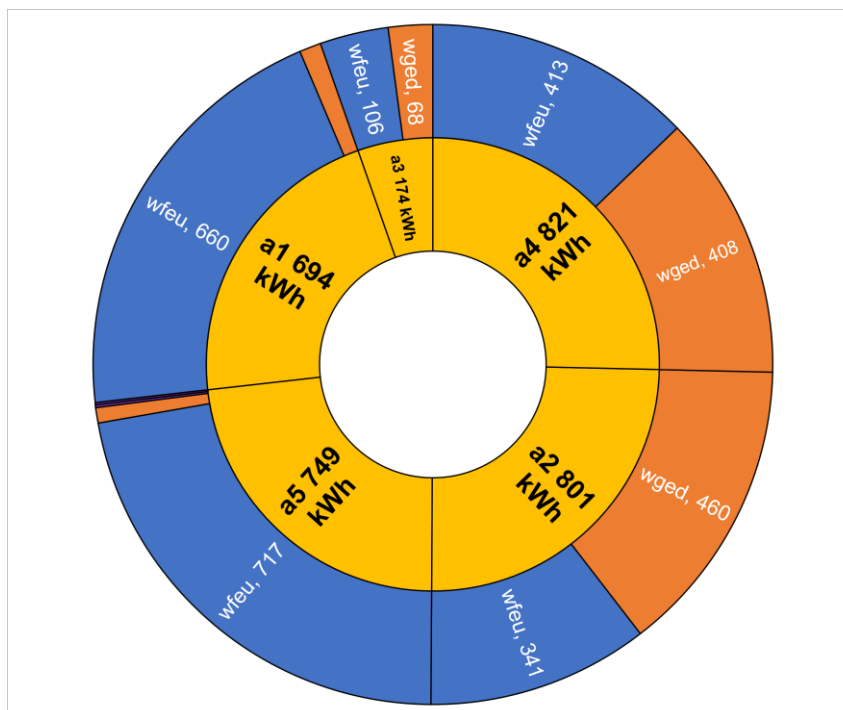


Figure 42. Pie chart of the aggregators share of FLs products (in blue), FGs products (in orange) and BESSs products (in purple) for the imbalance scenario

In Figure 43 is presented the energy flexibility products for the FLs. All products traded in the market are set in the upward direction as the imbalance created is in the other direction.

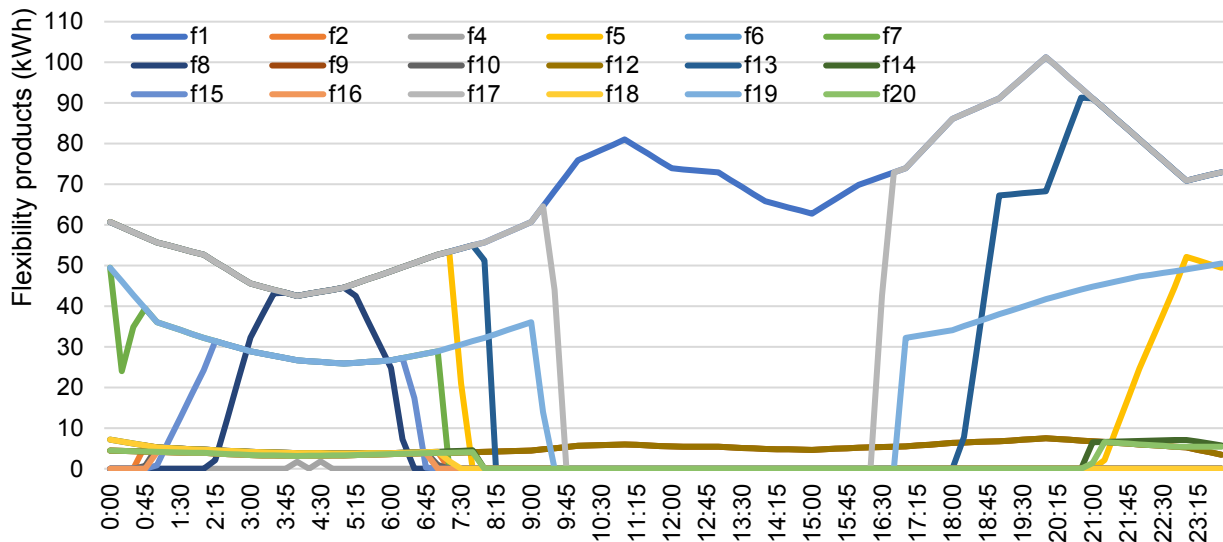


Figure 43. Upward flexibility products sold by FLs in the imbalance scenario.

In Figure 44 is presented the flexibility products traded in the market, as the imbalance created in the downward direction, FGs can offer their products to solve that issue. There are some assets that offer all of their energy production (for instance, g7) in order to solve the imbalance issue.

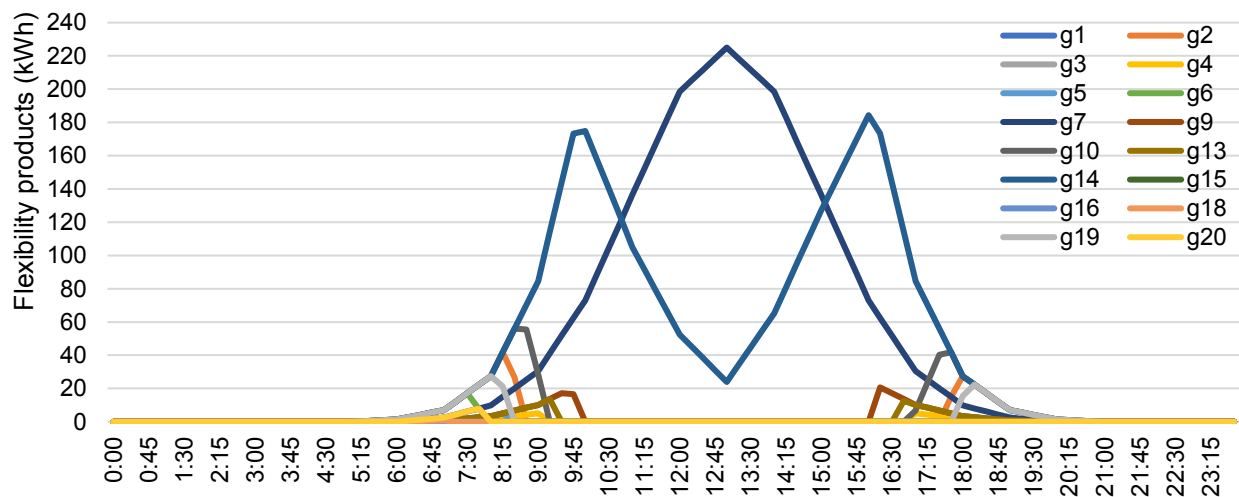


Figure 44. Downward flexibility products offered by FGs in the imbalance scenario.

Lastly, in Figure 45 it is presented the BESS traded in the market. Only BESS 20 trades their flexibility products in the market, as shown. Besides, it is possible to observe that the initial SOC is maintained after the market clearing.

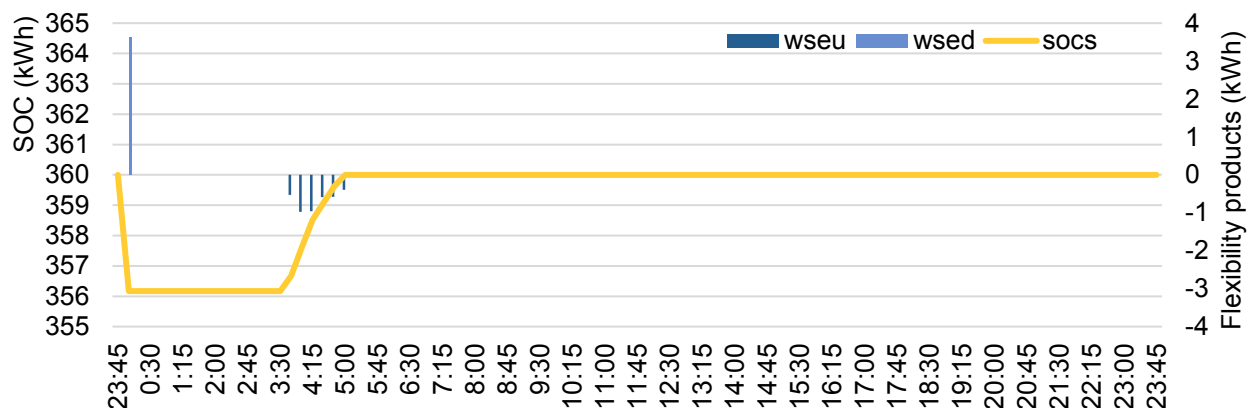


Figure 45. State of charge evolution and products sold in the market by BESSs 20 in the imbalance scenario.

Besides of the description of the flexibility product done, a cost analysis is also carried out. Figure 46 shows the Lagrange multiplier associated to the balance equation. There are some hours when the marginal value of balancing the grid is higher than others. For example, between 00:30 to 7:00 is where the marginal cost has their maximum values. Then, it follows valley with lower prices to increase again from 18:00. Then, the dual interpretation of those values is that in those periods when the price is higher the cost of increasing 1kW the imbalance higher. Or, in other words, if the imbalance could be reduced by other means, reducing it in the periods when the marginal value is higher will obtain the highest savings.

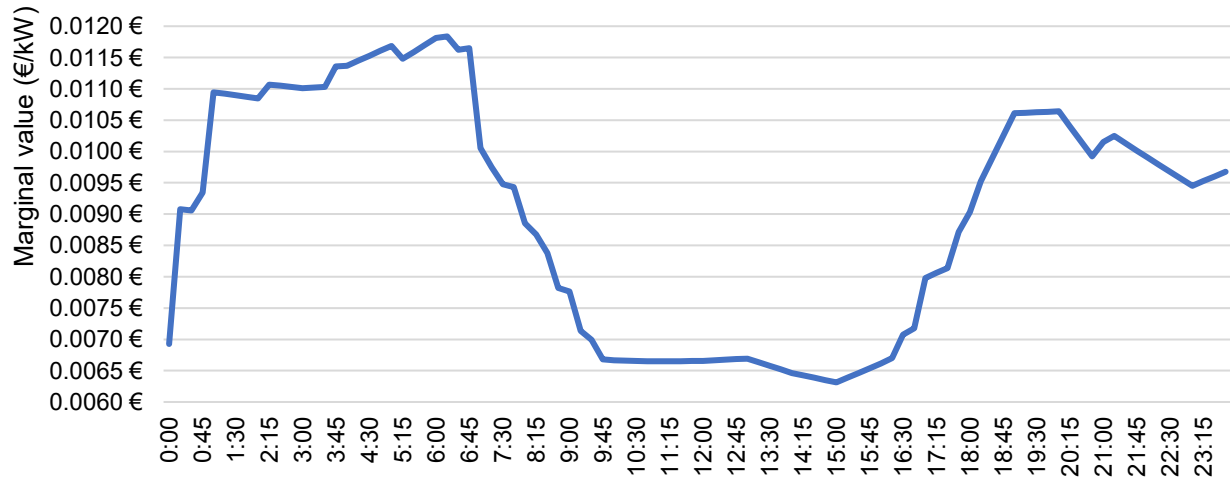


Figure 46. Marginal value evolution of the balance equation in the imbalance scenario.

In Figure 47 it is presented the hourly cost to solve the imbalance issue presented in the grid. In blue is presented the cost of the FLs, in orange the cost of the FGs and in purple the cost of the BESSs. Most of the costs come from the products offered by FLs. In central hours, the costs are concentrated by the FGs. Only a little part of the costs came from the BESSs, that is residual compared with the rest. The total cost of the scenarios amounts to 94,8548 € for a total of 3.237,89 kW of flexibility products.

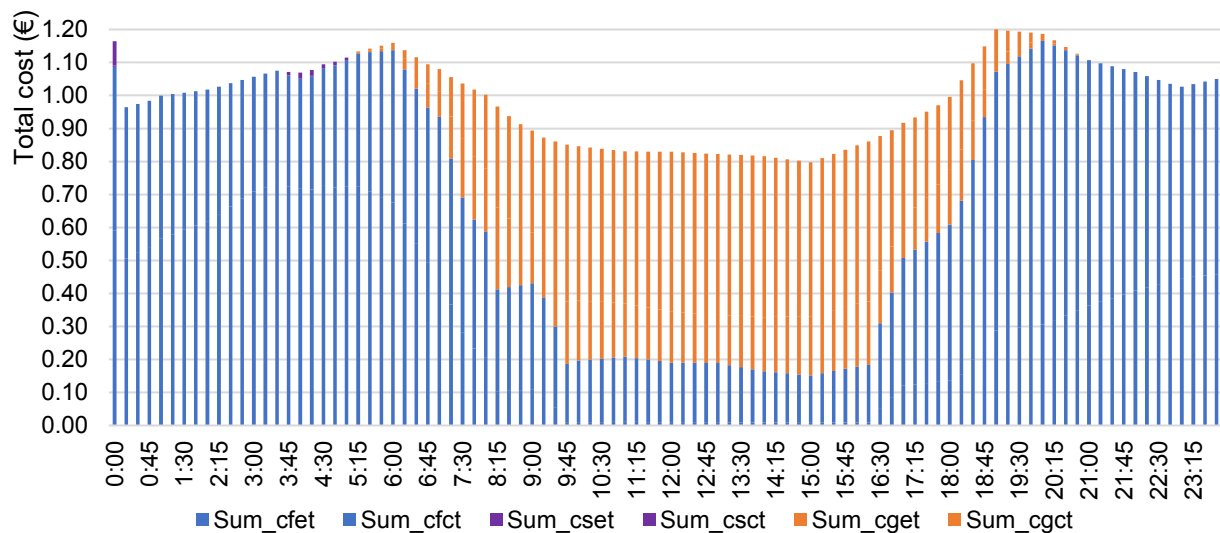


Figure 47. Aggregated hourly cost for each flexible asset.

CHAPTER 4: DECENTRALIZED LOCAL FLEXIBILITY MARKET

1. Introduction

After the proposal of a centralized LFM, it is timely interesting to address the same problem from a decentralized point of view. The reasons behind this studied, has been briefly discussed in section 6.1.2 of the previous chapter, where the computational burden and the information privacy issues were identified as the main problems that doom to a decentralized formulation of the initial centralized formulation.

However, completely decentralized trading could be detrimental to maintaining the technical limits of the network within the safety range. Thus, it is important to develop methodologies and clear formulations that enable a flexibility trading in the market without compromising the network's security needs [63].

Under this situation, the main challenges that needs to be further investigated in this matter are unfold in this section. Those hurdles can be organized depending on they can be attributed to the virtual or physical layer of the market.

The virtual layer is the part of the problem that is developed as an algorithm in a computer-based environment. Then, this layer collects the asks and bids, balance the grid, set the prices for the final products, in the end, it is in charge of doing all the agreements between agents that will be, after that, deployed in the real world. The main challenges of this layer are listed below:

- *Reducing cost of the energy*: In this sense, small-scale prosumers could sell their surplus of energy at a lower price compared with a traditional scale generation company.
- *Balancing local generation and demand*: Enabling prosumers to coordinate their energy usage and prepare the buy and sell orders with the purpose of balancing the demand while solving issues as voltage deviations or congestions of the network.
- *Incentivizing and engaging prosumers*: If a market mechanism is investigated and performed, it is important to incentivize the prosumers to participate in it. For that reason, it is important to further investigate the pricing mechanisms of the products they would sell in such framework.
- *Developing pricing mechanism*: to design pricing mechanism that are suitable to apply for a decentralized network for ensure fast and frequency trading processes and engage the participants in the market.
- *Identifying uncertainty and asynchronicity*: Identify communication and computation complexity issues for robust system operation. In this sense, a more robust communication infrastructure should be developed and deployed to avoid grid issues related with the communication.
- *Securing transactions*: to provide secure trading platform to enable prosumers seamlessly engage in the trading.

The physical layer is composed by the network itself, where the flexibility products are traded as if it were a traditional market were participants physically go to trade their products. In this sense, the main challenged to be overcome are related with physical and measurable magnitudes, as it is listed below:

- *Voltage and capacity constraint:* Under a scenario of higher uncertainties, unexpected situations that derive in voltage deviations and even in reverse power flow problems could appear.
- *Network power losses:* electrical networks are highly non-linear systems, meaning that depending on the situation, a small change on a variable may be reflected in a huge change in others. In practice, this characteristic difficult the power losses allocation for a determined transaction on the systems, meaning that it is hard to assign the costs of the transactions to each participant.
- *System strength:* the impact of the LFM on the energy security must be addressed, considering also the scenario of uncertainty motivated by a high share of renewable energies.

This chapter presents the work done in the field of coordination and decentralization of the previously proposed LFM. In this sense, firstly a state of the art considering the current situation of the decentralized market is done. After that, it is revised the technical methodologies used in the literature in terms of the decomposition methodologies that can be used for solving the market in a decentralized manner, focusing on the ADMM methodology that is the one finally used for the decentralization process. After that state-of-the-art revision, the mathematical description of the proposal is done.

2. Decentralized markets in Distribution Networks

2.1. European regulatory challenges

European electricity sector regulation aims to enhance the competition between activities without considering the grid infrastructure, what involves the separation of activities. A single company could not operate at the same time in the transportation, generation, and supply activities. Legally, there must exist data isolation between those company if they belong to the same conglomerate. Clearly, the deregulation of the power electricity sector helped to increase in number the agents actively participating in the sector as generators and suppliers of electricity.

Currently, on the one hand, the electricity sector is increasing their DER among distribution networks while in the other, investments are made in the digitalization of the grid. Being aligned with the Sustainable Development Goal 7: “Affordable and clean energy”, this transformation brings new opportunities to the sector. For example, the digitalization process allows end-users to participate actively in the operation of the grid, transforming those traditional considered static consumers to active and flexible prosumers.

However, there are uncertainties to be solved when considering the role of assets in the managing of the network. Main issues are related to the share of the costs and benefits of their participation. For example, when one peer sells active power to another one, there are power losses associated to that transaction, as well as costs of maintenance of the transmission grid, that are non-linear and difficult to account at first glance.

The high costs of the electricity sector make it a capital-intensive sector, being necessary colossal amounts of resources to create a successful company. In practice, this sector is part of the European national policy agendas and legislation. The principal objective of the legislation and the introduction of this matter in the European policy discussion is to offer an affordable and clean product despite of the capital-intensive attribute of the sector. Conversely, the same regulation that facilitates clean and affordable energy, create barriers when considering bringing the final end-user to consideration. Not only due to the minimum quantities of products to be solved, but also, the inexistent framework to make compatible the prosumers participation with the current market infrastructures.

Notwithstanding, efforts were made in the field to create a policy framework that incentive the participation of the small-scale users in the grid governance. One of those trials was deployed in The Netherlands. Aiming to improve the stability and sustainability of the DERs generation, the country launched 12 projects in 2012, which were intended to investigate new models of governance to facilitate the introduction of prosumers into the grid operation.

The results of those projects revealed the regulation difficulties that appeared when trying to deploy a “flexible system operation” that take into account generation, storage, transportation and supply at the same time. That concept would require a complete reformulation of the regulation and most of the market rules [64].

Nonetheless, there are other mechanisms to introduce the consumption into the operation of the grid. For example, in terms of the loads, Demand Side Management (DSR) technique is recognized by the electricity

sector. The modification of the load consumption patterns in benefit of the stability of the system has been used for decades. However, this technique is mainly applied to large industrial consumers that participate taking advantage of their greater commercial capabilities to manage short-term demand [65].

As shown, the regulation and national policies has allowed the participation of large assets, preserving the operation within safety limits, and enhancing an affordable and clean energy. However, regarding the prosumers and small-assets role, the current regulation leaves them out. In this context, a new framework is needed to effectively introduce them in the market operation.

An important aspect of this new framework is the decentralized nature of the prosumers. They are located in the distribution network, inside emerging smart grids and new patterns of consumption. For that reason, the Aggregator figure is of interest for representing them, to facilitate the operation and management. In this novel landscape with Aggregators, prosumers, regional DSOs and BRPs could lead to computation and communications issues if a traditional centralized perspective is considered. Thus, the decentralization of those interactions should be considered if a resilient and efficient regulatory framework is ought to be created.

Generally, decentralized systems cannot be governed by one single central regulator. Their dependence of the individual actions of the agents involved, makes them hard to control from a single and centralized European perspective. Research that links complex theory with this aspect concludes that, although certain elements of the electricity system are controlled by TSOs and DSOs (as physical security and stability), the system as a whole involves from individual choices to market dynamics, being impossible to obtain precise predictions from a model, for reasons such as non-linearities and near non-finite sensitivities to certain changes. A decentralized electricity system has an unpredictable and a self-organizing nature, as it is comprised by nested systems that interact with others, as law and economy. Those characteristics are inherent to their definition, and could not be controlled by any public or private institution. [66].

In this landscape, European regulators face to a difficult challenge, that is to facilitate a regulatory framework that enhance the participation of the small-scale prosumers with their inherent indetermined and uncontrollable characteristics and effectively integrate them with the objectives of a secure and resilient operation of the grid. Any attempt of regulatory framework should be designed with special care and considering any special situation that might appear that will affect to the security of the grid, adopting escape valves to each of them.

2.2. Network elements

In this section the layers that are present in a decentralized LFM are fully explained. In the introduction of the chapter the main hurdles both of the virtual and physical layer were described. Here it is presented a further description of the role of those layers from a decentralized perspective.

The virtual layer provides connection for participants, ensuring that all participants have equal access to the platform where the market takes place. In this layer, buy and sell orders are created, an appropriate market algorithm is used to clear it and financial transactions are carried out. Inside this layer, four different systems can be listed:

- *Information system:* Depending on the market structure the literature has investigated in different mechanisms to facilitate agreements between market participants. It can be listed technologies as blockchain-based smart contracts, consortium blockchain or the novel platform Elecbay.
- *Market Operation:* This infrastructure enables agents to participate in a real-time trading process by matching sell and buy offers with nearly real-time granularity. This element of the market must be carefully designed as the function in the market behaviour is crucial.
- *Pricing Mechanism:* Needs to reflect the state of energy of the grid, a higher surplus of energy should lower the energy price and vice versa. Then, if there are presence of congestions in multiple points of the grid, the price mechanism should reflect this matter on higher prices for flexibility resources than in periods where the power lines are inside their thermal limits.
- *Energy Market System:* EMS has access to the real-time supply and demand information of the prosumer through the transactive meter based on which it develops the generation and consumption profile of the prosumer and subsequently decides the bidding strategy to participate in the trading. This system is owned by the prosumers of the grid that aims to participate in the market. In a decentralized environment, the data managed by this element is protected and no central entity has access to it.

On the other hand, the physical layer is essentially composed by the physical electric network that facilitates the transfer of the electricity from sellers to buyers once the financial settlements between both parties are completed over the virtual platform. It comprises from the loads connected to the grid, passing through the substations and power transmission lines up to the upstream generators. The main elements of the grid are related in the following list:

- *Grid connection:* By using smart meters in the connection points of a distribution network, it is possible to evaluate the performance of the market actuations in terms of costs savings and energy delivered. For islanded microgrids, participants should have enough generation capacity to ensure an appropriate level of security and reliability in the case that the market mechanisms were not enough to ensure grid stability.
- *Metering:* Each prosumer should be equipped with a transactive meter in addition to a traditional energy meter. This additional transactive meter allows the connection between the physical and the virtual layer, enabling assets to deliver their flexibility products.
- *Communication Infrastructure:* The communication infrastructure should guarantee an efficient and equal opportunity access to the marketplace on the virtual layer. It must be able to exchange the necessary information in real-time with the market algorithm, and there should not be communication preferences between the participants.

Apart from the elements contained in the physical and virtual layer of a decentralized market infrastructure, other elements as the market participants and the regulation must be considered:

- *Market participants:* for a effective and secure flexibility trading, the existence of a minimum number of participants in the market is fundamental. Former to the organization of these type of market infrastructure, it is mandatory to incentive the introduction of generation and storage technologies in the network.
- *Regulation:* As any other type of market, there must exist a framework that guarantee the well functioning of the market, and safeguard the interests of the participants, avoiding reckless behaviours that could cause market inefficiencies or non-equal opportunities to the rest of the participants. For that reason, governments must consult and take into consideration all the stakeholders of the markets prior to the regulation of it.

3.5. Market structures

In this section the principal market structure that may appear under a decentralized market structure are explained. Depending on the integration of the decentralization process on the backbone of the electricity markets, several structures may appear from a fully decentralized market structure to a composite market passing through a community-based market structure.

The fully decentralized market structure is the most decentralized possible case, where all the markets interactions are made under a decentralized framework. In this market structure, the participants only have the information of the matched pair or pairs, and the matching process is done inside a disperse computation network where all the information is distributed among the nodes.

Within this organization, several approaches can be used when finding the matching pairs of the participants. Firstly, bilateral contracts established private and closed agreements between individual market participants and no other agent could know what is happen inside those agreements. Secondly, there are other type of contract called “multi-bilateral economic dispatch” where the prosumers can freely set their preference matching partners depending on the energy source. Finally, there are also innovative methods to coordinate the local generation, the flexible loads and the storage devices, where the matching process follow the merit order and not private contracts as this thesis investigates.

Another decentralized market organization is the community-based market structure. In this organization, the members of the community could interact between them in a collaborative or a competitive manner, making agreements to provide services to the DSO or compete between them to be the one that has the most innovative and interesting product to sold. This market structure allows to interact with other communities in a decentralized environment, what enhance the market possibilities to fit inside the current regulatory framework of the grids.

Lastly, a composite market structure may also appear in a decentralized market framework. This market structure is a combination of both above. In this organization type, one prosumer could interact with another

while maintaining their own market perspective in the community-based market. However, how to integrate this market organization inside a single paradigm remains a challenge inside of the current state of the art.

3. Technical methodologies

In this section the technical methodologies that currently exist in the literature are explained. The methodology allows to develop a mathematical model of the interaction between the agents in the previously describe market structures considering that there are present all the necessary elements both in the virtual and the physical layer of the market.

3.1. Introduction to decentralized market modelling

In this section an introductory view of how decentralized markets can be modelled from a mathematical point of view is detailed. Several mathematical techniques are used in the literature, among which Game theory, auction theory, decomposition techniques and blockchain technology are highlighted as the one with the highest relevance in the field.

3.1.1. Game theory

Game theory is a field of the mathematical primary used in economics, political science and even in psychology. It is the branch of the maths that model the behaviour of the players of any kind of game following previously determined rules. This theory studies how the players may move considering a set of game conditions, and the possible ends of the game. Depending on the type of relationship there exist between the players of the game, non-cooperative and cooperative games can be distinguished.

In a non-cooperative game, players take individual decisions without communicating with one another, each player seek their own objective without considering the implications of their actions on the others. In this type of game, the results could end in two different equilibriums: Nash and Stackelberg equilibriums.

The Nash equilibrium response to a stable state at the end of the game where no player can be better paid off by unilaterally deviation from its action without making other a loss. This situation of equilibrium leaves all the participants with their maximum profits based on the actions they have taken during the course of the game. Conversely, the Stackelberg equilibrium is obtained in a Leader-Follower game, where followers obtain the previously described Nash equilibrium in response of a leader's decision.

Contrary of the previously described games, in cooperative games there exist coalitions and alliances between participants to reach the final state of the game. Depending on the aspect of the game under study, three different investigations could be made:

- *Canonical coalition game*: The main objective is to determine whether a grand coalition is stable and to formulate a fair revenue distribution or not. Depending on the type of the game formulated, these coalitions may lead to unfair distribution of the benefits.
- *Coalition formation game*: the main objective is to study the formation of a coalitional structure through players and inquire the properties of the structure and its adaptability to environmental variations.
- *Coalitional graph game*: Deal with the connectivity and communications between players of the game and tries to derive low complexity distributed algorithms for players.

3.1.2. Auction theory

In a double auction, potential buyers submit their bids to an auctioneer while, at the same time, potential sellers ask prices to the auctioneer. The steps made are described in the following list:

1. Sellers submit their reservation prices in an increasing order.
2. Buyers are arranged in a decreasing order based on their bids.
3. The aggregated supply and demand curves are generated that meet in the intersection point.
4. This point establishes the auction price.

This algorithm is said to possess the property of *individual rationality* if the utility that a prosumer receives for participating in the auction cannot be improved otherwise. Additionally, this mechanism is called *incentive-compatible* if every participant can achieve the best outcome to themselves by acting upon their true preference. This algorithm is widely applied on the electricity wholesale markets, and meanwhile the

participants bids at their marginal costs, the mechanism is incentive compatible. However, if alliances are made, the price of the products auctioned could be artificially increased in benefit of some participants.

In the field of a decentralized environment, this auction process could be done inside a distributed system, where no auctioneer is present, and the asks and bids are submitted to a platform where the auction is developed in a decentralized fashion.

3.1.3. Decomposition methods

In the field of mathematical programming, there exist several techniques that allow to decentralize a mathematical problem. Depending on the complexity of the problem, different algorithm may be used. This aspect is further investigated in the section 3.2 of the present chapter.

3.1.4. Blockchain

Blockchain is a distributed data structure that is replicated and share among the members of a network. This data structure allows to develop from a decentralized and ungoverned virtual coin, to a decentralized and coordinated market. In the literature there are several implementations of this technology in the field of decentralized and P2P trading among an electrical network:

- Smart contracts: contractual clauses are translated into code and embedding them into property that can enforce them. With these types of contracts, the malicious and accidental exceptions can be minimized.
- Elecbay: is software platform dedicated to the development of P2P trading within a microgrid. This platform has the ability to accept or reject orders based on the network constraints.
- Consortium blockchain: is a specific type of blockchain with multiple authorized nodes to establish the distributed shared ledger with moderate cost. Is in these nodes where an optimization technique is executed to audit the transaction and its result is recorder to the shared ledger.
- Hyperledger: is an open software hosted by Linux Foundation where its main core run in Docker. When a peer wants to trade it logs into the system through the appropriate terminal and submits the appropriate transaction. Then a power platform unit analyses and initiates the transaction.
- Ethereum: is a programmable public blockchain with a native cryptocurrency called Ether, although is very similar to that of bitcoin, a big difference is that Ethereum nodes also store the most recent state of each smart contract.

3.2. Decomposition methods

In this section several decomposition techniques for optimization problems are discussed with the aim of choosing a tool for effectively decentralize the formulation of the LFM, enabling peers to introduce themselves in the market without sharing sensible information to competing participants.

Several approaches have been studied in the literature to coordinate and decentralize the operation of the power system. The main decomposition methodologies are presented below [67]–[70]:

1. Adjustment at common interface (ACCI): coupling variables are duplicated and assigned to different areas, then coupling constraint is imposed, enforcing equality of the duplicated variables. Only coupling variables are shared between different areas.
2. Passing adjacent variables using Lagrangian Relaxation (PAVLR): Using PAVLR, coordination between different areas is achieved by passing coupling variables to the neighbour. Considering no duplicated variables, sharing variables are included in so-called complicated constraints of both areas. Lagrangian multipliers are used to relax complicated constraint of foreign area and adding them to the objective function while maintaining the own complicating constraints.
3. Predictor-Corrector Proximal Multiplier Method (PCPM): It is based on properties of Rockafellar's proximal method of multipliers, and its primal-dual application, which needs an augmented lagrangian with an additional quadratic proximal term. This approach uses several multipliers, updating them twice each iteration (prediction-correction). Thus, it is possible to take advantage of parallel implementations.
4. Alternating Direction Method of Multipliers (ADMM). The basic idea underlying this approach is to sequentially perform minimization with respect to different variables while fixing the others, followed by an update of multiplier. ADMM has been extended to solve non-convex operation problems in a distributed manner.

Also, Auxiliary Problem Principle (APP) is commonly used across all decomposition methods to improve convergence performance of Lagrangian relaxation by means of solving auxiliary problems. Following sections will discuss each decomposition method, and in its application to LFMs coordination. It is necessary to define a common problem to solve with each decomposition technique. Considering two areas A, B, the overall optimization problem is given by:

$$\begin{aligned} \min_{x_a, x_b} f(x_a, x_b) &= c(x_a, x_b) \\ \text{s. t. } g(x_a, x_b) &= 0 \\ h(x_a, x_b) &\leq 0 \end{aligned} \quad (4.1)$$

The optimization vectors x_a represent optimization variables as voltage and angle variables for a given area A. Equality constraints represent power flow equation and inequality constraints correspond to capacity, bids limits or voltage limits. The solution space is non-convex thus, convergence and optimal solution are not warranted with numerical techniques.

Literature also includes Analytical Target Cascading (ATC) as a decomposition technique applicable to power systems. However, this method requires a formal hierarchy between each market participant. ATC is a model-based method for multilevel hierarchical optimization problems. Subsystems in higher levels are connected to subsystems in lower levels, with no coupling variables between subsystems in the same level, parents and children have. Parents set the values of target variables and propagate them down to their children. Consistency constraints are then introduced to the model and relaxed which enables convergence. Hierarchy methods are not included in the decentralized analysis as they interfere with European principles of equity and access to the market.

In the rest of the section, the first three methodologies are further described, then the last one (ADMM) will have a own section describing it with more details, as it will be the one selected for the decentralization of the proposed LFM.

3.2.1. Adjustment at the interface

Adjustment at the interface procedure involves sharing common variables at a border, as represented in Figure 48. Defining x_a, x_b as own variables, sharing variables (y) are duplicated in y_a and y_b . Then both areas can solve their subproblems with optimization vectors (x_a, y_a) and (x_b, y_b) .

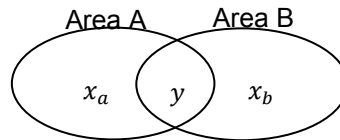


Figure 48. Coordination between two areas A and B with sharing areas represented by variable y .

Constraints are also split and no information except y is shared between areas. Furthermore, it is necessary to incorporate to the overall problem a coupling constraint to impose equality between duplicated variables.

To boost convergence, augmented Lagrangian procedure is implemented, using tuning variable γ to improve convergence velocity. Considering Lagrangian multiplier (λ^T), Lagrangian function is then defined as:

$$\mathcal{L}(x_a, y_a, x_b, y_b) = c(x_a, y_a) + c(x_b, y_b) + \lambda^T (y_a - y_b) + \frac{\gamma}{2} |y_a - y_b|^2 \quad (4.2)$$

Then, as $\mathcal{L}$ is no more decomposable, APP is invoked to solve this objective function by obtaining iteratively the solution of a sequence of auxiliary problems.

<u>Area A</u>	<u>Area B</u>
$\begin{aligned} \min_{(x_a, y_a) \in A} & c(x_a, y_a) + \frac{\beta}{2} y_a - y_a^k ^2 + \gamma y_a^T (y_a^k - y_b^k) \\ & + \lambda^{kT} y_a \\ \text{s. t. } & g_a(x_a, y_a) = 0 \\ & h_a(x_a, y_a) \leq 0 \end{aligned}$	$\begin{aligned} \min_{(x_b, y_b) \in B} & c(x_b, y_b) + \frac{\beta}{2} y_b - y_b^k ^2 + \gamma y_b^T (y_b^k - y_a^k) \\ & + \lambda^{kT} y_b \\ \text{s. t. } & g_b(x_b, y_b) = 0 \\ & h_b(x_b, y_b) \leq 0 \end{aligned}$

Objective function consists in two parts, first one oversees the main objective of cost, and the second one is responsible for the adaptation between areas. Also, multipliers must be updated following the equation (4.3).

$$\lambda^{k+1} = \lambda^k + \alpha(y_a^{k+1} - y_b^{k+1}) \quad (4.3)$$

α, β, γ are positive constants representing the weighting factors for adaption procedure, must be tuned for optimal convergence rates.

3.2.2. Passing Adjacent Variables using Lagrangian Relaxation

This approach is carried out on strictly separated areas rather than on areas sharing a common border. In this approach, coordination is achieved by means of shared variables to the neighbour.

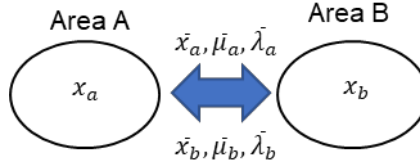


Figure 49. Coordination procedure between two interconnected areas A and B

Complicated constraints prevent areas from independent operation, those constraints contain variables from both areas, what serves to coordinate them. Independent constraints are named $g_a(x_a), h_a(x_a), g_b(x_b), h_b(x_b)$, while complicated constraints are noted by $\tilde{g}_a(x_a, \bar{x}_b), \tilde{h}_a(x_a, \bar{x}_b), \tilde{g}_b(\bar{x}_a, x_b), \tilde{h}_b(\bar{x}_a, x_b)$. This methodology establishes subproblems relaxing the complicating constraints and adding them to the objective function.

Considering $\bar{x}_a, \bar{x}_b, \bar{\lambda}_a, \bar{\lambda}_b, \bar{\mu}_a, \bar{\mu}_b$, as the state variables and multipliers of last iteration, subproblems are solved iteratively, updating those parameters at each iteration with the data obtained from the other area.

<u>Area A</u>	<u>Area B</u>
$\min_{x_a \in A} c_a(x_a, \bar{x}_b) + \bar{\lambda}_b^T \cdot \tilde{g}_b(x_a, \bar{x}_b) + \bar{\mu}_b^T \tilde{h}_b(x_a, \bar{x}_b)$	$\min_{x_b \in B} c_b(\bar{x}_a, x_b) + \bar{\lambda}_a^T \cdot \tilde{g}_a(\bar{x}_a, x_b) + \bar{\mu}_a^T \tilde{h}_a(\bar{x}_a, x_b)$
$s. t. \quad g_a(x_a) = 0 \quad \tilde{g}_a(x_a, \bar{x}_b) = 0$	$s. t. \quad g_b(x_b) = 0 \quad \tilde{g}_b(\bar{x}_a, x_b) = 0$
$h_a(x_a) \leq 0 \quad \tilde{h}_a(x_a, \bar{x}_b) \leq 0$	$h_b(x_b) \leq 0 \quad \tilde{h}_b(\bar{x}_a, x_b) \leq 0$

Resolution stops when optimized variables do not change significantly within two iterations.

3.2.3. Predictor-Corrector Proximal Multiplier method (PCPMP)

The predictor corrector proximal multiplier method involves augmented Lagrangian with an additional quadratic proximal term. Considering scheme of Figure 48, decomposition can be obtained by solving:

<u>Area A</u>	<u>Area B</u>
$(x_a^{k+1}, y_a^{k+1}) = \min_{x_a, y_a \in A} c_a(x_a) + \frac{\beta_k}{2} \ y_a - y_b^k\ _2^2$	$(x_b^{k+1}, y_b^{k+1}) = \min_{x_b, y_b \in B} c_b(x_b) + \frac{\beta_k}{2} \ y_a^k - y_b\ _2^2$
$+ \bar{\lambda}^{k+1 T} y_a$	$+ \lambda^{k+1 T} y_b$
$s. t. \quad g_a(x_a, y_a) = 0$	$s. t. \quad g_b(x_b, y_b) = 0$
$h_a(x_a, y_a) \leq 0$	$h_b(x_b, y_b) \leq 0$

Lagrange multiplier is updated twice each iteration as presented in equation (4.4). The first prediction multiplier is an approximation of what will happen after the resolution of the area B, and for that reason it is corrected once it is know its results.

$$\begin{aligned} \text{prediction:} \quad \bar{\lambda}^{k+1} &= \lambda^k + \frac{1}{\beta_k} (y_a^k - y_b^k) \\ \text{correction:} \quad \lambda^{k+1} &= \lambda^k + \frac{1}{\beta_k} (y_b^k - y_a^k) \end{aligned} \quad (4.4)$$

Those steps can be performed independently, enabling parallel computing of the decentralized problem.

3.3. Alternating Direction Method of Multipliers (ADMM)

In this section ADMM technique is further investigated, defining its main algorithm, its stopping criteria and how complicating variables and constraints are considered in the algorithm. It has been widely demonstrated in the literature the superior convergence properties of this method, which is able to converge to a reasonable approximate solution of the centralized problem within few iterations. As has been stated in the previous section, ADMM methodology tries to take advantage of the decomposability of the dual ascent methodology using also the convergence properties of the method of multipliers. The algorithm solves problems in form of those represented for equation (4.5).

$$\begin{aligned} \min & f(x) + g(z) \\ \text{s. t. } & Ax + Bz = c \quad : \lambda \end{aligned} \quad (4.5)$$

Considering $x \in \mathbb{R}^n$ and $z \in \mathbb{R}^m$, where $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$ and $c \in \mathbb{R}^p$, where $p \in \mathbb{N}$ representing the number of equality restrictions. Assuming that $f(x)$ and $g(z)$ are convex functions the optimal space of solutions is represented in equation (4.6).

$$p^* = \inf\{f(x) + g(z) \mid Ax + Bz = c\} \quad (4.6)$$

On this conditions, Lagrangian function is presented in equation (4.7), where λ represents the multipliers associated to the equality constraints, and $\gamma \in \mathbb{R}^+$ is known as the penalty factor or step, this parameter is the only one that needs to be tuned for the application of the methodology.

$$\mathcal{L}(x, z, \lambda) = f(x) + g(z) + \lambda^t (Ax + Bz - c) + \frac{\gamma}{2} \|Ax + Bz - c\|_2^2 \quad (4.7)$$

Then, the ADMM protocol is implemented considering the following algorithm (4.8):

$$\begin{aligned} x^{k+1} &:= \min \mathcal{L}(x, z^k, \lambda^k) = f(x) + \lambda_k^t (Ax + Bz^k - c) + \frac{\gamma}{2} \|Ax + Bz^k - c\|_2^2 \\ z^{k+1} &:= \min \mathcal{L}(x^{k+1}, z, \lambda^k) = g(z) + \lambda_k^t (Ax^{k+1} + Bz - c) + \frac{\gamma}{2} \|Ax^{k+1} + Bz - c\|_2^2 \\ \lambda^{k+1} &= \lambda^k + \gamma (Ax^{k+1} + Bz^{k+1} - c) \end{aligned} \quad (4.8)$$

The *alternating direction* accounts for the alternative update of the x and the z variables. This method needs to be made in a sequential fashion. Besides, this method is said to be decomposable for the same reason, as the x variables are minimized in a different step that the z variables, what gives the methodology this characteristic. The only parameters that need to be exchanged from one area to the other is those related to the complicating constraints, in other words, those variables of the other area that affect to a determined joint restriction or global restriction.

Convergence properties are well defined in reference [71]. In a nutshell, there are two basic parameters to be aware of while controlling the convergence of the methodology. First one is the residuals of the primal solution ($\|r^k\|_2$) and the residuals of the dual solution ($\|s^k\|_2$). Those parameters are defined in equations (4.9) and (4.10). Demonstration of those equations are provided in the same reference considering primal and dual feasibility.

$$\|r^{k+1}\|_2 = \|Ax^{k+1} + Bz^{k+1} - c\|_2 \quad (4.9)$$

$$\|s^{k+1}\|_2 = \gamma \|A^t B(z^{k+1} - z^k)\|_2 \quad (4.10)$$

Then, the stopping criteria is chosen considering an absolute tolerance ε . Then, for the ADMM to converge, equation (4.11) needs to be satisfied.

$$\max\{\|r^{k+1}\|_2, \|s^{k+1}\|_2\} \leq \varepsilon \quad (4.11)$$

This criterion is chosen as it is the one that has been further investigated in the literature, and it is simple and clear to be implemented.

3.3.1. Complicating constraints

Considering a large problem where there are several constraints some of which involves variables of several areas ($Ax + Bz = c$) that need to be computed in a distributed manner, and other which just need area variables ($Dx = d, Ez = e$). To solve in a coordinate and distributed manner such a problem, it is necessary to include the first type of restriction (complicating constraints) in the Lagrange function, while maintaining the second type of constraints inside the restrictions body of each problem. Such problem is represented in equation (4.12).

$$\begin{aligned} \min & f(x) + g(z) \\ \text{s. t. } & Ax + Bz = c : \lambda \\ & Dx = d \\ & Ez = e \end{aligned} \quad (4.12)$$

Then, the problem that area $\mathcal{A}$ should solve has the form:

$$\begin{aligned} x^{k+1} := \min & f(x) + \lambda^{kt} (Ax + Bz^k - c) + \frac{\gamma}{2} \|Ax + Bz^k - c\|_2^2 \\ \text{s. t. } & Dx = d \end{aligned} \quad (4.13)$$

Passing variables x^{k+1} to area $\mathcal{B}$, this area should solve the following problem:

$$\begin{aligned} z^{k+1} := \min & g(z) + \lambda^{kt} (Ax^{k+1} + Bz - c) + \frac{\gamma}{2} \|Ax^{k+1} + Bz - c\|_2^2 \\ \text{s. t. } & Ez = e \end{aligned} \quad (4.14)$$

And then, multipliers are updated:

$$\lambda^{k+1} = \lambda^k + \gamma (Ax^{k+1} + Bz^{k+1} - c) \quad (4.15)$$

This type of problem assumes that there are no sharing variables between areas $\mathcal{A}$ and $\mathcal{B}$. However, likewise complicating constraints, it is also possible to find problems that share some variables, also known as, complicating variables.

3.3.2. Complicating variables

Complicating variables are those variables that are shared between areas, in other words, they serve as natural barriers between problems, and need to be coordinated while updating. Let $y \in \mathbb{R}^s$ as the set of sharing variables between areas. Then, Lagrangian function is no longer decomposable, and the APP should be invoked.

Let consider problem that equation (4.16) shows:

$$\begin{aligned} \min & f(x, y) + g(z, y) \\ \text{s. t. } & Ax + Bz = c : \lambda \end{aligned} \quad (4.16)$$

Considering two areas, complicating variables should be duplicated (y_a, y_b) and complicating constraint ($y_a - y_b = 0$) should be added to the problem. Then, the sequence of problems to be solved have the form:

$$\begin{aligned} (x^{k+1}, y_a^{k+1}) &:= \min f(x, y_a) + \lambda_k^t \left[\begin{matrix} Ax + Bz^k - c \\ y_a - y_b^k \end{matrix} \right] + \frac{\gamma}{2} \left\| \begin{matrix} Ax + Bz^k - c \\ y_a - y_b^k \end{matrix} \right\|_2^2 \\ (z^{k+1}, y_b^{k+1}) &:= \min g(z, y_b) + \lambda_k^t \left[\begin{matrix} Ax^{k+1} + Bz - c \\ y_a^{k+1} - y_b \end{matrix} \right] + \frac{\gamma}{2} \left\| \begin{matrix} Ax^{k+1} + Bz - c \\ y_a^{k+1} - y_b \end{matrix} \right\|_2^2 \\ \lambda^{k+1} &= \lambda^k + \gamma \left[\begin{matrix} Ax^{k+1} + Bz^{k+1} - c \\ y_a^{k+1} - y_b^{k+1} \end{matrix} \right] \end{aligned} \quad (4.17)$$

3.4. Coordination techniques

The concept of coordination stands for the process to match power flows among areas while solving the OPF in a decentralized fashion. The main objective is to obtain a consistent solution that will be as near as possible as the one the centralized version of the problem would obtain. In the literature this problem is well stated and investigated, and several techniques are used depending on the type of modelling of the grid. In a nutshell, there are two main approaches while coordinating the solution of the decentralized OPF problem [72]: the first one involves coordinating adjacent areas considering only power injections and the second one considers voltage angles. The following subsections will explain those approaches.

3.4.1. Power injection coordination

Considering two adjacent areas, $\mathcal{A}$ and $\mathcal{B}$ joint together with a power line between nodes i and j as Figure 50 shows. Power flow between nodes is noted by $p_{i,j}$ and $p_{j,i}$. In a centralized approach it must be satisfied that $p_{i,j} + p_{j,i} = 0$. That will be the complicating constraint involving variables of both areas in the decentralized version of the problem that Figure 50 (b) represents.



Figure 50. Coordination using power injection between adjacent areas.

Considering the centralized version of the problem stated in equation (4.18), where p_i are the power injections to node i and d_i are the demands to the same node.

$$\begin{aligned} & \min f(x) + g(z) \\ \text{s. t. } & \sum_{j \in \mathcal{I}} p_{i,j} = p_i - D_i \quad \forall i, i \neq j \\ & \underline{p}_{i,j} \leq p_{i,j} \leq \bar{p}_{i,j} \quad \forall i, \forall j, i \neq j \end{aligned} \quad (4.18)$$

Then, the decentralized version of the problem is done by the coordination achieved by the variables presented in Figure 50 (b). Then, considering two areas $\mathcal{A}$ and $\mathcal{B}$, and that there could be several connections between areas, the problem to solve has the form that equation (4.19):

$$\begin{aligned} & \min f(x) + \lambda^{k^t} \cdot \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} (p_{i,j} + p_{j,i}^k) + \frac{\gamma}{2} \cdot \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} (p_{i,j} + p_{j,i}^k)^2 \\ \text{s. t. } & \sum_{j \in \mathcal{I}} p_{i,j} = p_i - D_i \quad \forall i \in \mathcal{A}, i \neq j \\ & \underline{p}_{i,j} \leq p_{i,j} \leq \bar{p}_{i,j} \quad \forall i \in \mathcal{A}, \forall j \in \mathcal{A}, i \neq j \end{aligned} \quad (4.19)$$

Then, the Lagrange multipliers should be updated considering equation (4.20).

$$\lambda^{k+1} = \lambda^k + \gamma(p_{i,j}^k + p_{j,i}^k) \quad (4.20)$$

With this approach, it is possible to give an economic interpretation of the multipliers, as represent the price of the energy exchange between areas. However, with this approach, it is not possible to coordinate voltage angles between areas, power flow is only computed considering power flows between nodes.

3.4.2. Voltage angle coordination

The second approach consider voltage angles while coordinating the decentralized method. Considering the same situation that the previous coordination technique, represented by Figure 51, two areas are connected by nodes i and j , considering a power line with a susceptance of B_{ij} . Voltage angle is noted by θ_i for node i .

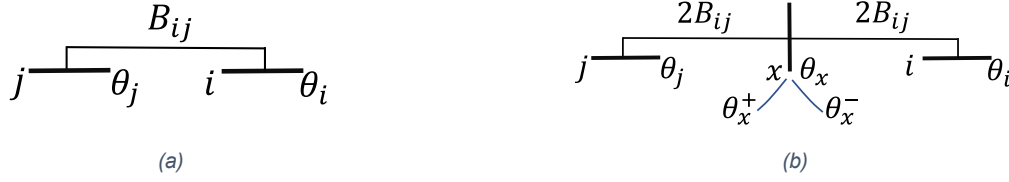


Figure 51. Coordination using voltage angle between adjacent areas.

The centralized version of the problem is presented in equation (4.21) for a OPF with separable objective function.

$$\begin{aligned} & \min f(x) + g(z) \\ \text{s.t. } & \sum_{j \in i} B_{i,j}(\theta_i - \theta_j) = p_i - D_i \quad \forall i, i \neq j \\ & \underline{p}_{i,j} \leq B_{i,j}(\theta_i - \theta_j) \leq \bar{p}_{i,j} \quad \forall i, \forall j, i \neq j \end{aligned} \quad (4.21)$$

In this case, it is necessary to create a fictitious node x in the middle of the power line connected both areas. This situation is represented in Figure 51. The susceptance of the fictitious power lines created are the double of the original power line. In this situation, the coordination is achieved using equations (4.22) to (4.25).

$$\theta_x^+ - \theta_x^- = 0 \quad (4.22)$$

$$p_{x,i} = 2B_{ij}(\theta_x^- - \theta_i) \quad (4.23)$$

$$p_{x,j} = 2B_{ij}(\theta_x^+ - \theta_j) \quad (4.24)$$

$$p_{x,i} + p_{x,j} = 0 \quad (4.25)$$

Both equation (4.22) and (4.25) are complicating constraints that should be relaxed and included into the objective function, while the other two equations can be considered separately in each subarea problem. Then subarea problem remains as equation (4.26) states.

$$\begin{aligned} & \min f(x) + \lambda^{k,t} \cdot \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} \left[p_{x,i} + p_{x,j}^k \right] + \frac{\gamma}{2} \cdot \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} \left\| p_{x,i} + p_{x,j}^k \right\|^2 \\ & \text{s.t. } \sum_{j \in i} p_{i,j} = p_i - D_i \quad \forall i \in \mathcal{A}, i \neq j \\ & \underline{p}_{i,j} \leq p_{i,j} \leq \bar{p}_{i,j} \quad \forall i \in \mathcal{A}, \forall j \in \mathcal{A}, i \neq j \\ & p_{x,i} = 2B_{ij}(\theta_x^- - \theta_i) \quad \forall x \in \{\mathcal{A} \cup \mathcal{B}\} \end{aligned} \quad (4.26)$$

Then, the Lagrange multipliers should be updated considering equation (4.27).

$$\lambda^{k+1} = \lambda^k + \gamma \begin{bmatrix} p_{x,i}^k + p_{x,j}^k \\ \theta_x^{+k} - \theta_x^{-k} \end{bmatrix} \quad (4.27)$$

This approach allows to coordinate both voltage angle and power flows between areas, however, the number of complicating restrictions and equation considered increases. The benefits of this approach are that the final solution of the market is the same that the one that would have been obtained in a centralized fashion, and that enables to model complex restrictions in relation with the voltage angle magnitude between nodes.

Although introducing more variables into the problem the convergence speed is not affected as far as the voltage angle magnitudes play a role in the market formulation.

4. Mathematical modelling

In this section mathematical models proposed in the Chapter 3 will be decentralized using ADMM in a coordinate manner. The two illustrative examples, as well as the use case solved will be analysed and decentralized. For the coordination, techniques described in the previous section are used, considering also the complicating constraint that models the DSO asks for flexibility.

The general procedure for using the ADMM methodology is presented in algorithm of the Figure 52. The first step is to initialize all the variables and parameters involved in the decentralization to zero. Then, the loop begins solving sequentially each sub-area problem. While the stopping criteria is not fulfilled, the loop maintains iterating and updating Lagrange multipliers after each iteration.

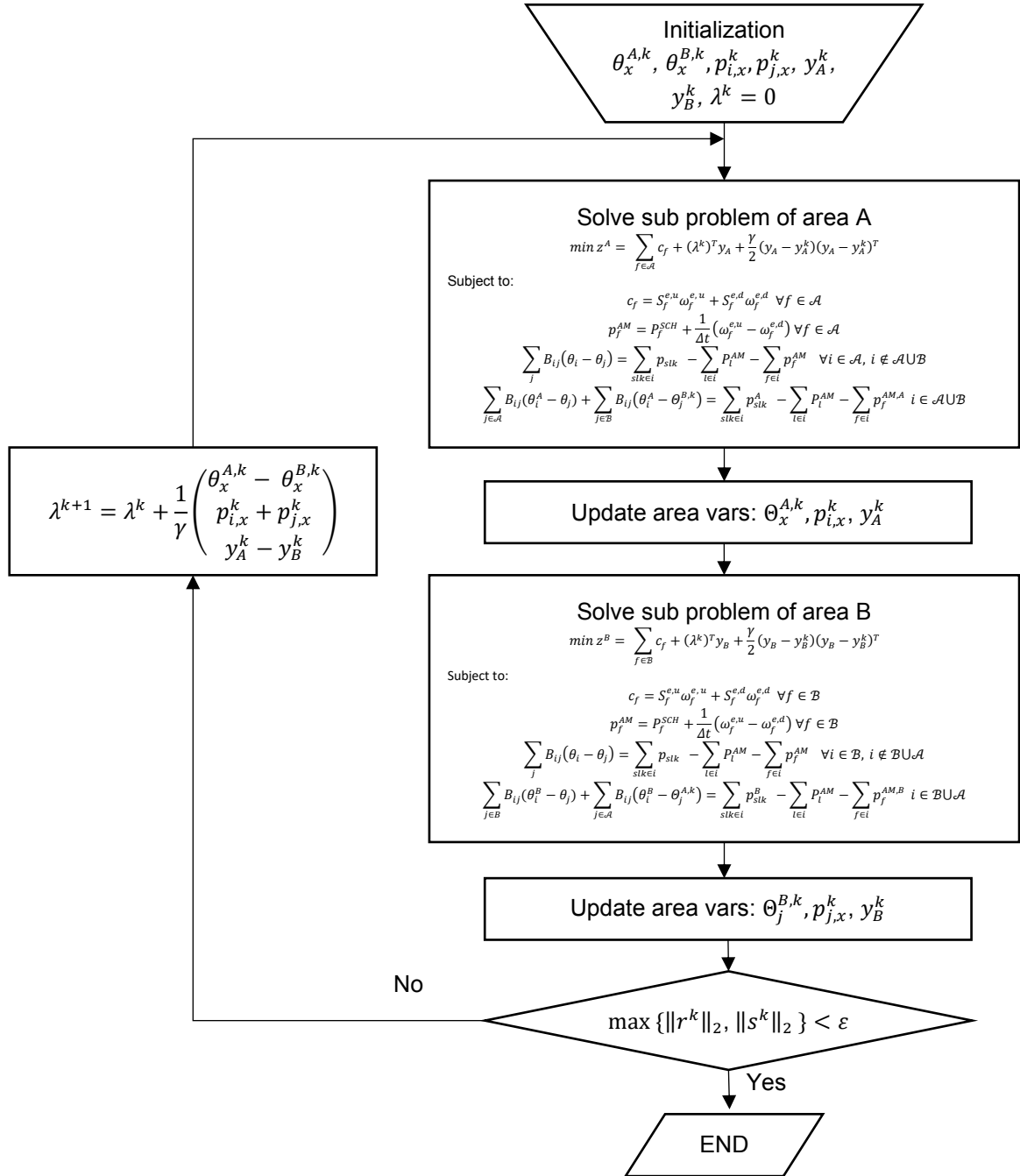


Figure 52. ADMM algorithm for decentralized LFM solving

The rest of the section is organized solving the first use case in first place, considering both congestion and imbalance scenarios. Then the second illustrative example is also decentralized, showing that the increase of liquidity can be also made in a decentralized manner. Lastly, the final use case is tested, showing the real applications and the importance to decentralize the whole methodology.

4.1. First illustrative example

The decentralization process of the first illustrative example can be seen in Figure 53 in a nutshell. This example involves two areas (A and B) marked by an ellipse in the figure. The coordination is done by using the Lagrange multipliers at the frontier ($\lambda_x^k, \lambda_\theta^k$ and λ_B^k) of the areas. The problem is decentralized considering the procedure showed in Figure 52.

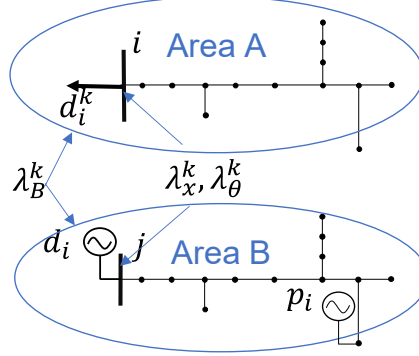


Figure 53. Coordination between areas considering Lagrange multipliers for restrictions of power balance, voltage angle and imbalance.

Mathematical definition of the decentralization follows the equations (4.16) and (4.17) of the previously section. From equations (4.28) to (4.43), first illustrative case is decentralized. Objective function is stated in equation (4.28), minimizing the costs of the FLs and the FGs, while relaxing complicating variables and restrictions. $\tilde{h}_x^{k,A}, \tilde{h}_B^{k,A}$ and $\tilde{h}_\theta^{k,A}$ stands for the residuals of the complicating variables and equations, while λ_x^k, λ_B^k and λ_θ^k represents the Lagrange multipliers associated to those restrictions.

From equations (4.29) to (4.34) the equations are not varied from the centralized solution, in exception of the node balance in equation (4.33), where the term $p_{i,x}$ is added representing the power exchange between areas A and B. This power flow is defined in terms of the voltage angle in equation (4.34), considering the coordination technique presented in section 3.4.2.

Equation (4.35) represent the power flow exchange between the areas through the fictitious node x . As has been explained in sections 3.4.1 and 3.4.2, the susceptance of the half of the line is the double of the complete tie-line. This power flow is computed as a function of the of the complicated variable θ_x^+ of the area A.

$$\min z^A = \sum_{f \in \mathcal{A}} C_f^e + \sum_{g \in \mathcal{A}} C_g^e + \lambda_x^k \tilde{h}_x^{k,A} + \lambda_B^k \tilde{h}_B^{k,A} + \lambda_\theta^k \tilde{h}_\theta^{k,A} + \frac{\gamma}{2} (\tilde{h}_x^{k,A^2} + \tilde{h}_B^{k,A^2} + \tilde{h}_\theta^{k,A^2}) \quad (4.28)$$

$$C_f^e = S_f^{e,u} \omega_f^{e,u} + S_f^{e,d} \omega_f^{e,d} \quad \forall f \in \mathcal{A} \quad (4.29)$$

$$p_f^{AM} = P_f^{SCH} + \frac{1}{\Delta t} (\omega_f^{e,u} - \omega_f^{e,d}) \quad \forall f \in \mathcal{A} \quad (4.30)$$

$$C_g^e = S_g^{e,d} \omega_g^{e,d} \quad \forall g \in \mathcal{A} \quad (4.31)$$

$$p_g^{AM} = P_g^{SCH} - \frac{\omega_g^{e,d}}{\Delta t} \quad \forall g \in \mathcal{A} \quad (4.32)$$

$$\sum_j B_{ij} (\theta_i - \theta_j) = p_i + p_g^{AM} - p_{i,x} - P_i^{AM} - p_f^{AM} \quad \forall i \in \mathcal{A} \quad (4.33)$$

$$-\bar{P}_{ij} \leq B_{ij} (\theta_i - \theta_j) \leq \bar{P}_{ij} \quad \forall i \in \mathcal{A}, \forall j \in \mathcal{A}, i \neq j \quad (4.34)$$

$$p_{i,x} = 2B_{ij} (\theta_i - \theta_x^+) \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\} \quad (4.35)$$

Then, residuals are defined from equation (4.36) to (4.38). Residual for the balance equation is defined considering the computed power flows aftermarket (p_f^{AM}, p_g^{AM}) for the area A, and those that area B passed from previous iteration ($p_f^{AM,k}, p_g^{AM,k}$). Residual for power exported to the Area B is defined in terms of the output power flow of Area A computed ($p_{i,x}$) and the power flow from area B to A known from previous iteration ($p_{j,x}^k$). Lastly, residual for theta in the fictitious node is defined in terms of the angle computed from area A ($\theta_x^{A,k}$) and the one known from area B in the previous iteration ($\theta_x^{B,k}$)

$$\tilde{h}_B^{k,A} = \sum_{l \in \mathcal{A}} (P_l^{SCH} - P_l^{AM}) + \sum_{l \in \mathcal{B}} (P_l^{SCH} - P_l^{AM}) + \sum_{f \in \mathcal{A}} (P_f^{SCH} - p_f^{AM}) + \sum_{f \in \mathcal{B}} (P_f^{SCH} - p_f^{AM,k}) + \sum_{g \in \mathcal{A}} (p_g^{SCH} - p_g^{AM}) + \sum_{g \in \mathcal{B}} (p_g^{SCH} - p_g^{AM,k}) \quad (4.36)$$

$$\tilde{h}_x^{k,A} = p_{i,x} + p_{j,x}^k \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\} \quad (4.37)$$

$$\tilde{h}_\theta^{k,A} = \theta_x^+ - \theta_x^{-,k} \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\} \quad (4.38)$$

After solving this subproblem, the variables of the area are updated and the other area subproblem is solved, following the algorithm previously defined. After solving area B problem, Lagrange multipliers can be updated as equations (4.39) to (4.41) show. They are updated using the penalty factor γ that is considered constant through iterations.

$$\lambda_x^{k+1} = \lambda_x^k + \gamma \tilde{h}_x^k \quad (4.39)$$

$$\lambda_B^{k+1} = \lambda_B^k + \gamma \tilde{h}_B^k \quad (4.40)$$

$$\lambda_\theta^{k+1} = \lambda_\theta^k + \gamma \tilde{h}_\theta^k \quad (4.41)$$

Lastly, primal ($\|r^k\|_2$) and dual ($\|s^k\|_2$) residuals are computed to estimate the error committed in the process. Those residuals are defined considering the definition given in equation (4.9) and (4.10).

$$\|r^k\|_2 = \sqrt{\tilde{h}_\theta^{k^2} + \tilde{h}_x^{k^2} + \tilde{h}_B^{k^2}} \quad (4.42)$$

$$\|s^k\|_2 = \gamma^k \sqrt{\sum_{x \in \mathcal{B}} (p_x^{AM,k+1} - p_x^{AM,k})^2 + \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} (p_{j,x}^{k+1} - p_{j,x}^k)^2 + \sum_{ij \in \{\mathcal{A} \cup \mathcal{B}\}} (\theta_x^{k+1,-} - \theta_x^{k,-})^2} \quad (4.43)$$

The loop will end when the maximum value of residual is less or equal than a fixed epsilon parameter (ε). This criterion is widely used in the literature for a huge range of problems. The rest of the section show the validity of the approach for the congestion and imbalance scenario solved in the previous chapter. For both scenarios, penalty factor is set constant and equal to $\gamma = 10^{-4}$, while accepted residual is set to $\varepsilon = 10^{-3}$.

4.1.1. Congestion scenario

In this scenario, a congestion of the line 414-416 of the distribution feeder showed in Figure 11 is solved in a decentralized manner. The results obtained in this process are the same than those obtained in the centralized approach. For the sake of clarity and intending not to be repetitive those results are omitted since can be checked in section 7.1.1 of the Chapter 3. In this section it will be investigated the convergence properties of the methodology, analysing the evolution of the total cost of the congestion and the values of the Lagrange multipliers used for coordination.

Primal and dual residuals for the decentralization process are shown in Figure 54. In blue is represented the primal residual $\|r^k\|_2$ of the solution, while in orange the dual residual $\|s^k\|_2$. As represented, the primal residual decreases monotonously up to reach the epsilon value in iteration 37. However, the dual residual does not decrease in the same way, depending on the deactivation of different assets while the number of iterations were increasing.

The deactivation of the generators and other assets while the number of iterations is increasing can also be seen in Figure 55. The total cost increasing or decreasing represents the assets activating and deactivating to obtain a solution for the congestion, that directly influence on the dual residual.

The number of necessary iterations to effectively converges under ε value is 37 as can be seen in Figure 54 where dual residuals converge up to iteration 31 while primal residual do not converge up to 37th iteration.

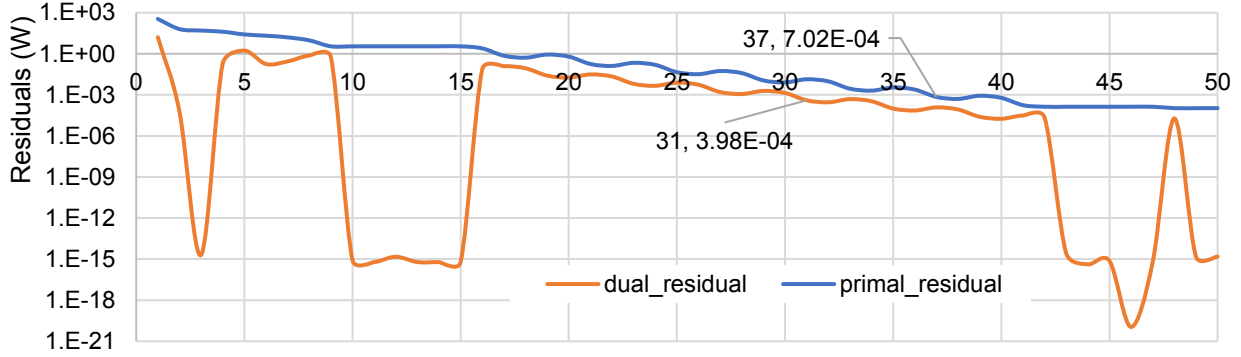


Figure 54. Residuals for the congestion scenario of the first illustrative example.

The total cost is stabilized in the same value than the one presented in the centralized version, as Figure 55 shows. This figure can be compared with the Figure 54, showing its relationship with the convergence of the dual residuals. There are periods of where the dual residual falls, matching with periods of stability in the solution. For example, during iterations 9 to 16 the dual residual converges rapidly, while the cost solution is stable above 0,40€. Eventually, the solution converges to an efficient one, deactivating the flexibility of an asset, what produces an increase in the dual residual while minimizing the total cost.

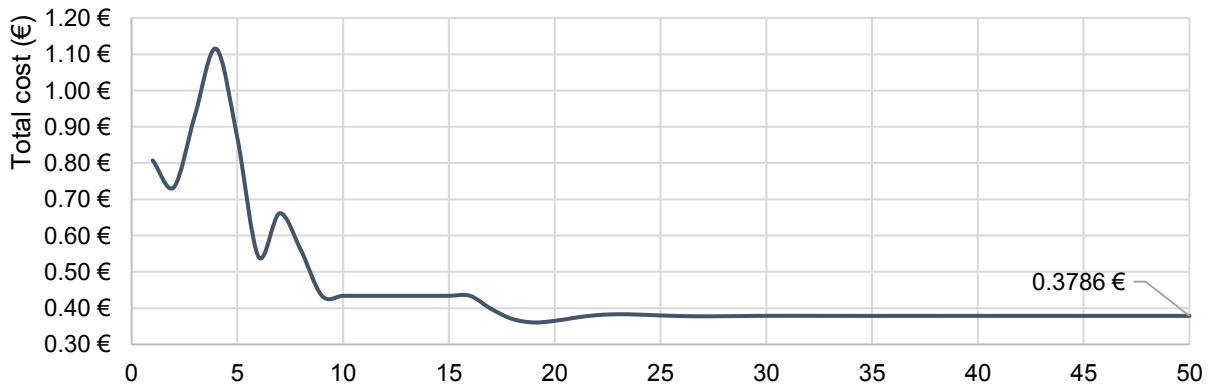


Figure 55. Representation of the evolution of the total costs for congestion solving while the number of iterations increase in the decentralization process.

Likewise the centralized version, the λ_x is finally stabilized to 0,0143 €/kW, also the Lagrange multiplier λ_B is also stabilized to the same value than centralized version, as Figure 56 shows. Those values converge monotonously to the optimal centralized solution.

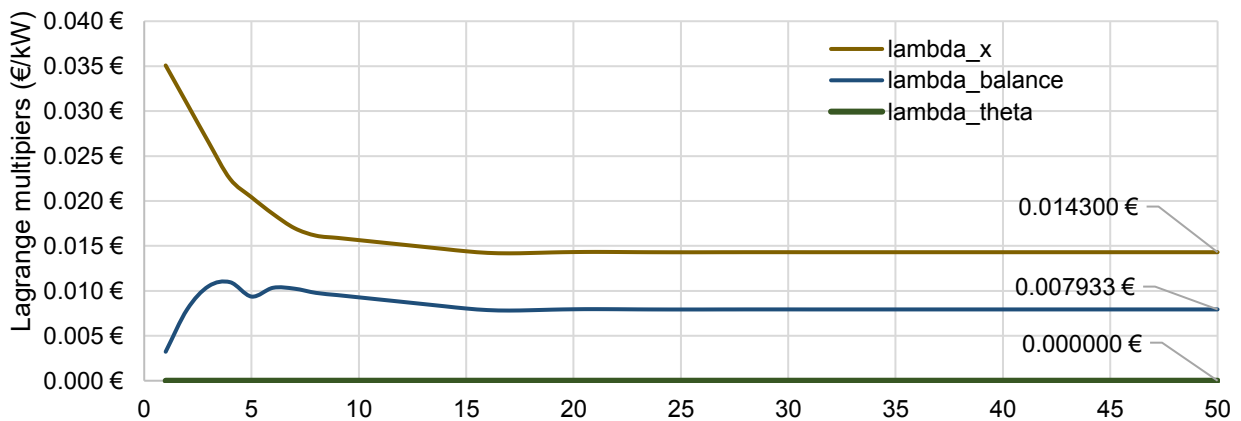


Figure 56. Evolution of the Lagrange multipliers during the decentralization loop for congestion scenario of the first illustrative example.

4.1.2. Imbalance scenario

In this section, the imbalance scenario produces in section 7.1.2, is replicated in a decentralized manner. The input data of the model is the same, and for the sake of clarity and simplicity it is not repeated again. In this section an iteration study of the decentralization process is done, presenting the primal and dual residuals of the decentralized approach, as well as the evolution of the total cost of the imbalance solving and the value of Lagrange multipliers used to coordinate the problem.

Figure 57 shows the monotonously decreasing residuals curves in a semi-logarithmic axis. There is a need of 35 iterations to converge up to the global optimal solution. Likewise the congestion scenario, primal residual maintain in a monotonous decreasing tendency, while the dual residual oscillate in iteration 45 at the end.

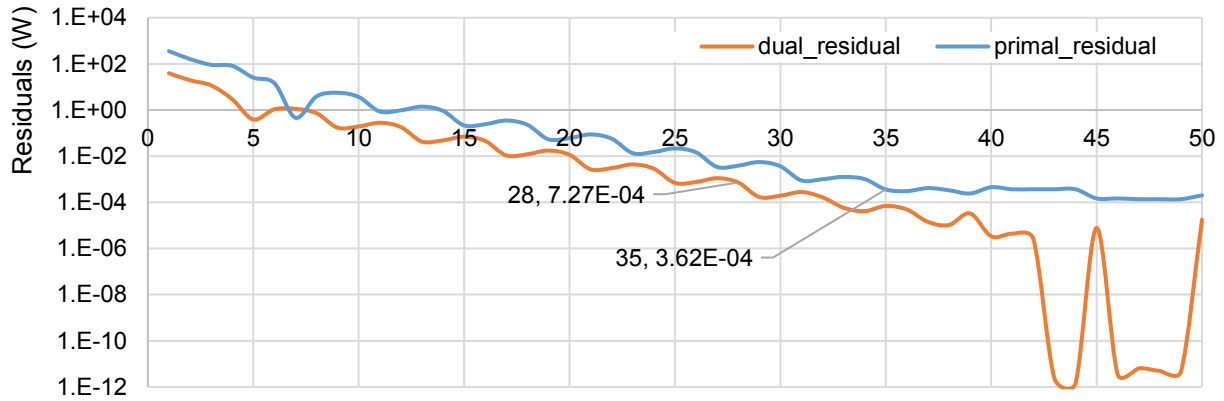


Figure 57. Residuals for the imbalance scenario of the first illustrative example.

Regarding the total cost evolution of the decentralized version, it is presented in Figure 58. Initially it shows an oscillating behaviour, that stabilises when reaching the 15th iteration. Then, the tendency begins to flatten, reaching the same value than the previously obtained in the centralized approach ($z = 0,66685$ €).

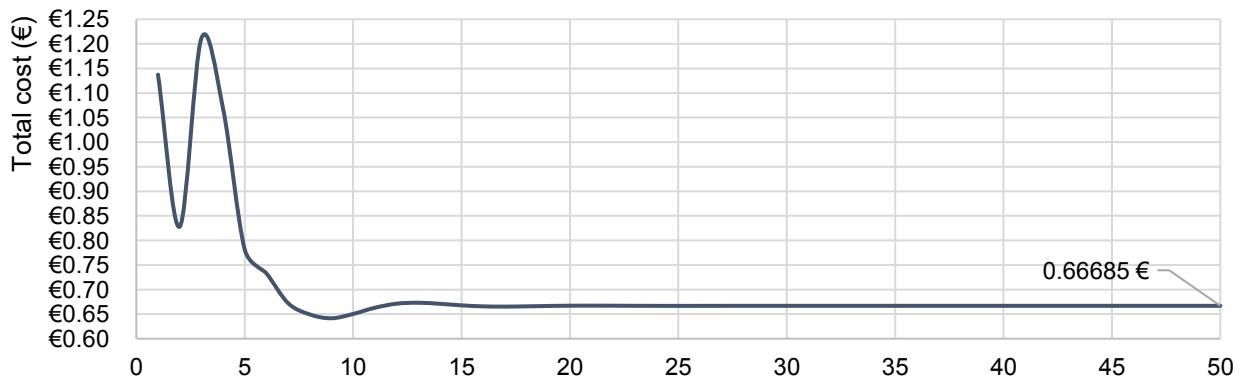


Figure 58. Representation of the evolution of the total costs for imbalance solving while the number of iterations increase in the decentralization process.

Lastly, the evolution of the Lagrange multipliers used for the coordination of the decentralization process is analysed in Figure 59. Beginning with a non-zero value, the Lagrange multiplier associated to the power balance of the grid is able to equals the importing and exporting power flows in the frontier ($p_{i,x}, p_{j,x}$). Once, this is achieved, λ_x reach zero value, the same that the centralized version of the problem. On the other hand, λ_B starts oscillating in the first iterations, to finally stabilize in the last iterations, what it is a probe of the consistency of the proposed methodology for the decentralization process. Considering that the ADMM methodology used is unscaled, the multipliers can be directly interpreted in the same way than the centralized approach. This is the reason behind obtaining the same values no matter of the approach or scenario considered.

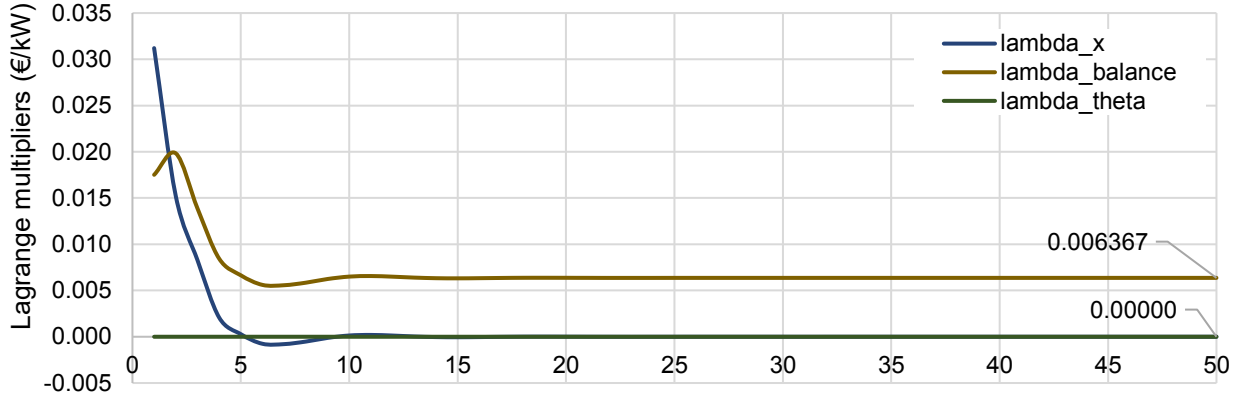


Figure 59. Evolution of the Lagrange multipliers during the decentralization loop in the imbalance scenario for first illustrative example.

4.2. Second illustrative example

In the second illustrative example has been demonstrated the applicability of the proposed energy and capacity LFM for solving situations that a simple energy-only LFM will results in infeasible. In this section the main objective is to present a decentralized approach for that mixed energy and capacity LFM. Likewise previous section, the coordination is done by means of using power flows between areas, using a fictitious node x for passing voltage angle information between areas and using the residual of the centralized balance equation.

From equation (4.44) to (4.61) the decentralization formulation for area A is defined using a standard ADMM algorithm. The formulation for area B is similar to the one described between those equations; however, it includes the updated variable values obtained from area A resolution.

The objective function presented in equation (4.44) is the Lagrange function of the centralized problem considering the ADMM algorithm. The first part involves the first line of the equation, adding the total cost of assets in area A procuring energy and capacity products ($C_{x,t}^e, C_{x,t}^c$, respectively). The second part, relates the Lagrange multipliers (λ^k) and the penalty parameter (γ) with the residuals ($\tilde{h}^{k,A}$) of the area A.

$$\begin{aligned} \min z^A = & \sum_t \sum_{f \in \mathcal{A}} (C_{f,t}^e + C_{f,t+1}^e) + \sum_t \sum_{g \in \mathcal{A}} (C_{g,t}^e + C_{g,t+1}^c) + \sum_t \sum_{s \in \mathcal{A}} (C_{s,t}^e + C_{s,t+1}^c) \\ & + \sum_t (\lambda_{x,t}^k \tilde{h}_{x,t}^{k,A} + \lambda_{B,t}^k \tilde{h}_{B,t}^{k,A} + \lambda_{\theta,t}^k \tilde{h}_{\theta,t}^{k,A}) + \frac{\gamma}{2} \cdot \sum_t (\tilde{h}_{x,t}^{k,A^2} + \tilde{h}_{B,t}^{k,A^2} + \tilde{h}_{\theta,t}^{k,A^2}) \end{aligned} \quad (4.44)$$

From equations (4.45) to (4.55) the flexibility products are defined for FLs, FGs and BESS in terms of positive variables $\omega_{x,t}^{e,u}, \omega_{x,t}^{c,u}$ for upward energy and capacity products, respectively, $\omega_{x,t}^{e,d}, \omega_{x,t}^{c,d}$ for downward energy and capacity products. There is no change with respect of the formulation explained in the section 7.2 of the chapter 3. The equations are included here with the intention of seeing the problem at one glance.

$$C_{f,t}^e = \Delta S_{f,t}^{e,u} \omega_{f,t}^{e,u} + \Delta S_{f,t}^{e,d} \omega_{f,t}^{e,d} \quad \forall f \in \mathcal{A} \quad \forall t \quad (4.45)$$

$$C_{f,t}^c = S_{f,t}^{c,u} \omega_{f,t}^{c,u} + S_{f,t}^{c,d} \omega_{f,t}^{c,d} \quad \forall f \in \mathcal{A} \quad \forall t \quad (4.46)$$

$$p_{f,t}^{AM} = P_{f,t}^{SCH} + \frac{1}{\Delta t} (\omega_{f,t}^{e,u} - \omega_{f,t}^{e,d}) \quad \forall f \in \mathcal{A}, \forall t \quad (4.47)$$

$$C_{g,t}^e = \Delta S_{g,t}^{e,d} \omega_{g,t}^{e,d} \quad \forall g \in \mathcal{A}, \forall t \quad (4.48)$$

$$C_{g,t}^c = S_{g,t}^{c,d} \omega_{g,t}^{c,d} \quad \forall g \in \mathcal{A}, \forall t \quad (4.49)$$

$$p_{g,t}^{AM} = p_{g,t}^{SCH} - \frac{\omega_{f,t}^{e,d}}{\Delta t} \quad \forall g \in \mathcal{A}, \forall t \quad (4.50)$$

$$C_{s,t}^e = \Delta S_{s,t}^{e,u} \omega_{s,t}^{e,u} + \Delta S_{s,t}^{e,d} \omega_{s,t}^{e,d} \quad \forall s \in \mathcal{A} \quad \forall t \quad (4.51)$$

$$C_{s,t}^c = S_{s,t}^{c,u} \omega_{s,t}^{c,u} + S_{s,t}^{c,d} \omega_{s,t}^{c,d} \quad \forall s \in \mathcal{A} \quad \forall t \quad (4.52)$$

$$p_{s,t}^{AM} = \frac{1}{\Delta t} (\omega_{s,t}^{e,u} - \omega_{s,t}^{e,d}) \quad \forall s \in \mathcal{A}, \forall t \quad (4.53)$$

$$soc_{s,t} = SOC_s^0 + \eta_s^c \omega_{s,t}^{e,u} - \frac{1}{\eta_s^d} \omega_{s,t}^{e,d} \quad \forall s, \forall t \quad (4.54)$$

$$\underline{SOC}_s \leq soc_{s,t} \leq \overline{SOC}_s \quad \forall s \quad \forall t \quad (4.55)$$

Then, the network is modelled considering a linear approach, likewise the other approach so as not to blur the proposal with complex and non-convex formulation. Equations (4.56) and (4.57) define the majority of the grid behaviour. However, it is necessary to include equation (4.58) obtained from the fictitious node x used for coordinate both areas. With that equation it is possible to coordinate the voltage angles of both networks, being possible to obtain the same results than those that will arise in a centralized approach.

$$\sum_j B_{ij} (\theta_{i,t} - \theta_{j,t}) = p_{i,t} + p_{g,t}^{AM} - p_{l,t}^{AM} - p_{f,t}^{AM} - p_{s,t}^{AM} \quad \forall i \quad \forall t \quad (4.56)$$

$$-\bar{P}_{ij} \leq B_{ij} (\theta_{i,t} - \theta_{j,t}) \leq \bar{P}_{ij} \quad \forall i, \forall j, i \neq j, \forall t \quad (4.57)$$

$$p_{x,i}^k = 2B_{ij} (\theta_{x,t}^{k,+} - \theta_{i,t}) \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\} \quad (4.58)$$

Then residuals are defined in equations (4.59), (4.60) and (4.61) for each period of time t . The first equation computes the residual for the balance equation of the grid, the second one calculates the power flow difference between the grids, and the last one the difference in voltage angle in the fictitious node x . The main difference with the previous definition is that, in this case, the residuals must be computed for each period of time, defining also a different Lagrange multiplier associated to each one.

$$\begin{aligned} \tilde{h}_{B,t}^{k,A} = & \sum_{l \in \mathcal{A}} (P_{l,t}^{SCH} - P_{l,t}^{AM}) + \sum_{l \in \mathcal{B}} (P_{l,t}^{SCH} - P_{l,t}^{AM}) + \sum_{f \in \mathcal{A}} (P_{f,t}^{SCH} - p_{f,t}^{AM}) + \sum_{f \in \mathcal{B}} (P_{f,t}^{SCH} - p_{f,t}^{AM,k}) \\ & + \sum_{g \in \mathcal{A}} (P_{g,t}^{SCH} - p_{g,t}^{AM}) + \sum_{g \in \mathcal{B}} (P_{g,t}^{SCH} - p_{g,t}^{AM}) + \sum_{s \in \mathcal{A}} -p_{s,t}^{AM} + \sum_{s \in \mathcal{B}} -p_{s,t}^{AM,k} \quad \forall t \end{aligned} \quad (4.59)$$

$$\tilde{h}_{x,t}^{k,A} = p_{i,t} - d_{l,t}^k \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\}, \forall t \quad (4.60)$$

$$\tilde{h}_{\theta,t}^{k,A} = \theta_{x,t}^+ - \theta_{x,t}^{-,k} \quad \forall ij \in \{\mathcal{A} \cup \mathcal{B}\}, \forall t \quad (4.61)$$

Lagrange multipliers are associated to those residuals, and are defined equally than previous illustrative example, as a function of the residual $\tilde{h}^k$ after both areas are simulated and the penalty parameter or step size γ . Equation (4.62) define the multiplier for the difference in power flow between areas, (4.63) the one for the balance in the grid and (4.64) the one for voltage angle difference.

$$\lambda_{x,t}^{k+1} = \lambda_{x,t}^k + \gamma \tilde{h}_{x,t}^k \quad \forall t \quad (4.62)$$

$$\lambda_{B,t}^{k+1} = \lambda_{B,t}^k + \gamma \tilde{h}_{B,t}^k \quad \forall t \quad (4.63)$$

$$\lambda_{\theta,t}^{k+1} = \lambda_{\theta,t}^k + \gamma \tilde{h}_{\theta,t}^k \quad \forall t \quad (4.64)$$

Last but not least, global residuals of the problem are defined in equations (4.65) and (4.66). The first equations define the primal residual as the Euclidean norm of each residual, while the second one defines the dual residual, as a function of the penalty factor and the difference between iterations in area B.

$$\|r^k\|_2 = \sqrt{\sum_t (\tilde{h}_{\theta,t}^k{}^2 + \tilde{h}_{x,t}^k{}^2 + \tilde{h}_{B,t}^k{}^2)} \quad (4.65)$$

$$\|s^k\|_2 = \gamma^k \sqrt{\sum_t \sum_{x \in B} (p_{x,t}^{AM,k+1} - p_{x,t}^{AM,k})^2 + \sum_t \sum_{ij \in \{A \cup B\}} (p_{j,x,t}^{k+1} - p_{j,x,t}^k)^2 + \sum_t \sum_{ij \in \{A \cup B\}} (\theta_{x,t}^{k+1,-} - \theta_{x,t}^{k,-})^2} \quad (4.66)$$

In this case, there will be only obtained the results of the decentralization process for the mixed energy and capacity LFM, as the energy-only LFM results in an infeasible situation unbearable for an ADMM protocol. The network modelled is the one presented in Figure 19, and the conditions of the FLs and BESSs are the same than those explained in section 7.2.2 of the chapter 3.

The ADMM protocol is implemented considering a penalty factor $\gamma = 4 \cdot 10^{-3}$. That value optimizes the number of iterations needed for convergence. The value was obtained after several try and error iterations. However, values around $\gamma = [10^{-3}, 10^{-4}]$ provide similar results.

Figure 60 represents the evolution both of the primal and dual residuals. Generally speaking, it is possible to observe a monotonous decreasing tendency of both residuals, except in the iteration 8 with the dual primal residual. That abrupt decrease in the primal residual relates with the approximation of a local optimal solution, where the solution remains stable for a couple of iterations. Then, the ADMM protocol is able to activate other restrictions, and avoid that local optimality point and continuing looking for the global optimal point. The number of iterations to achieve full converge is 39, being necessary 37 to converge in primal residual, and 39 to achieve full convergence (primal and dual).

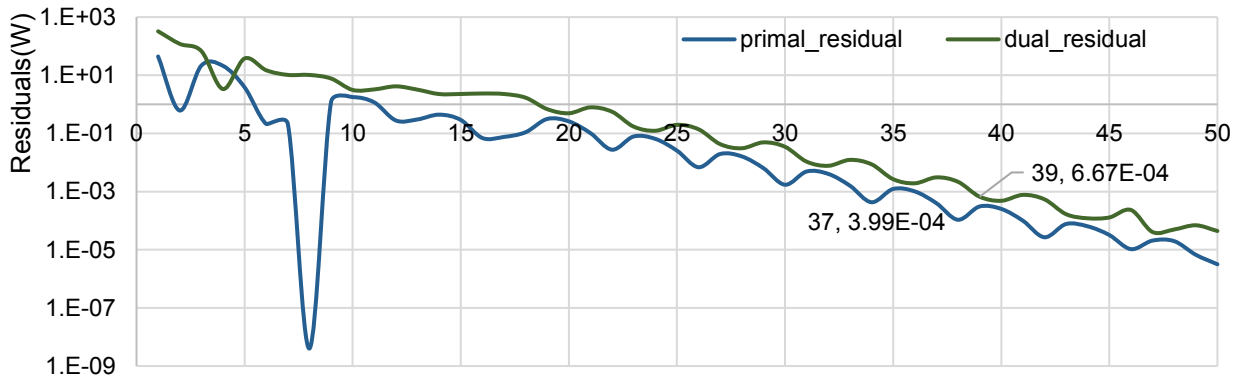


Figure 60. Primal and dual residuals evolution for the illustrative example 2.

The multipliers evolution during the ADMM protocol is presented in Figure 61, both $\lambda_{\theta,t}^k$ and $\lambda_{x,t}^k$ are zero, consistently with the centralized solution. The only non-null multiplier is $\lambda_{B,t}^k$, representing the marginal asset used for the solution.

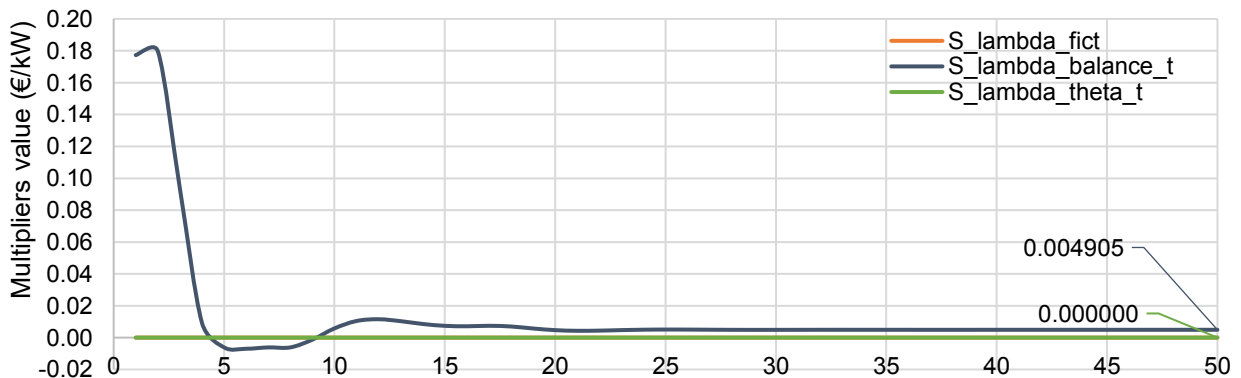


Figure 61. Summatory of the Lagrange multipliers evolution in the illustrative example 2.

Lastly, the evolution of the total cost of the congestion solving considering energy and capacity products is shown in Figure 62. The total cost amounts for 0,13315€, obtaining the same solution than the centralized version.

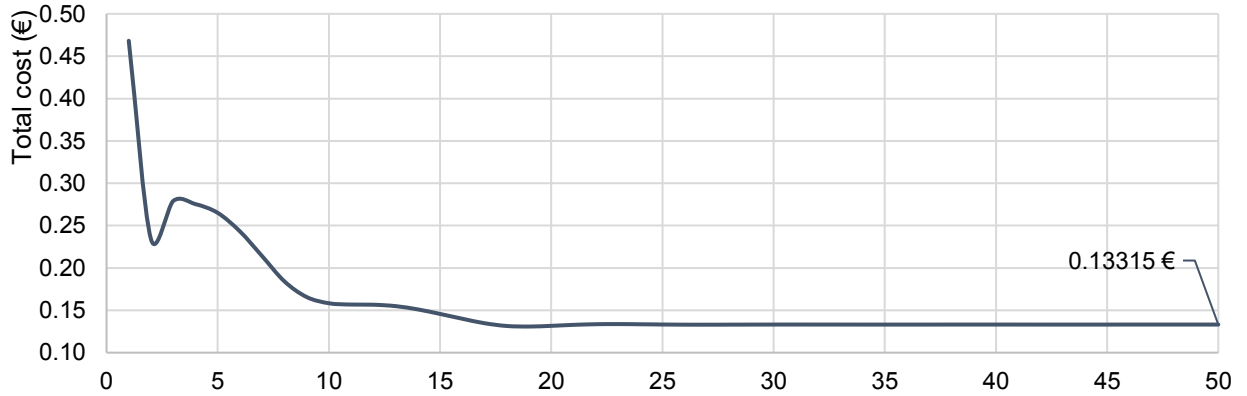


Figure 62. Total costs evolution for the illustrative example 2.

As has been demonstrated in this section, the ADMM protocol also manage well a LFM with energy and capacity products. For that reason, it is timely and technical interesting to investigate how to apply this powerful tool to a more complex use case.

4.3. Use case

In the chapter 3 there has been presented the centralized solution to the use case, in this section a decentralized solution is presented. Firstly, the formulation is presented from equation (4.67) to (4.69). In the first equation a cost minimization objective for all Aggs is seek and the ADMM formulation is also included. In terms of the ADMM formulation, the only difference with the rest of the cases studied is that all interconnections of the areas must be considered. Those interconnections are represented by $\mathcal{I}$.

The second and the third one represents the cost of energy and capacity products for each Agg, respectively. The rest of the restrictions are the same than those considered from equation (4.45) to (4.66) of the previous section.

$$\begin{aligned} \min z^A = & \sum_t \sum_{a \in \mathcal{A}} (C_{a,t}^e + C_{a,t+1}^c) + \sum_t \sum_{ij \in \mathcal{I}} (\lambda_{x,t}^k \tilde{h}_{x,t}^{k,A} + \lambda_{\theta,t}^k \tilde{h}_{\theta,t}^{k,A}) + \sum_t \lambda_{B,t}^k \tilde{h}_{B,t}^{k,A} \\ & + \frac{\gamma}{2} \left[\sum_t \sum_{ij \in \mathcal{I}} (\tilde{h}_{x,t}^{k,A^2} + \tilde{h}_{\theta,t}^{k,A^2}) + \sum_t \tilde{h}_{B,t}^{k,A^2} \right] \end{aligned} \quad (4.67)$$

$$C_{a,t}^e = \sum_{f \in a} C_{f,t}^e + \sum_{g \in a} C_{g,t}^e + \sum_{s \in a} C_{s,t}^e \quad \forall a, a \in \mathcal{A}, \forall t \quad (4.68)$$

$$C_{a,t}^c = \sum_{f \in a} C_{f,t}^c + \sum_{g \in a} C_{g,t}^c + \sum_{s \in a} C_{s,t}^c \quad \forall a, a \in \mathcal{A}, \forall t \quad (4.69)$$

In order to complete the decentralization process, the network is divided into six different areas represented in Figure 63. Each Agg controls one feeder noted by capital letters from A to E. The transmission network interconnecting those feeders is noted by capital letter T.

The order considered while solving the problem was A, B, C, D, E and T before update multipliers. In each interconnection between areas, a fictitious node is defined, they were used to coordinate the voltage angle values through areas, as the slack node is the node 100, what also represent the connection with the rest of the upstream network.

Likewise the centralized solution, two different scenarios are solved: the first one solves a congestion of the line 228-230, which is limited to 500 kW; and the second one manages an imbalance created by the SL 75, which is disconnected during the whole day

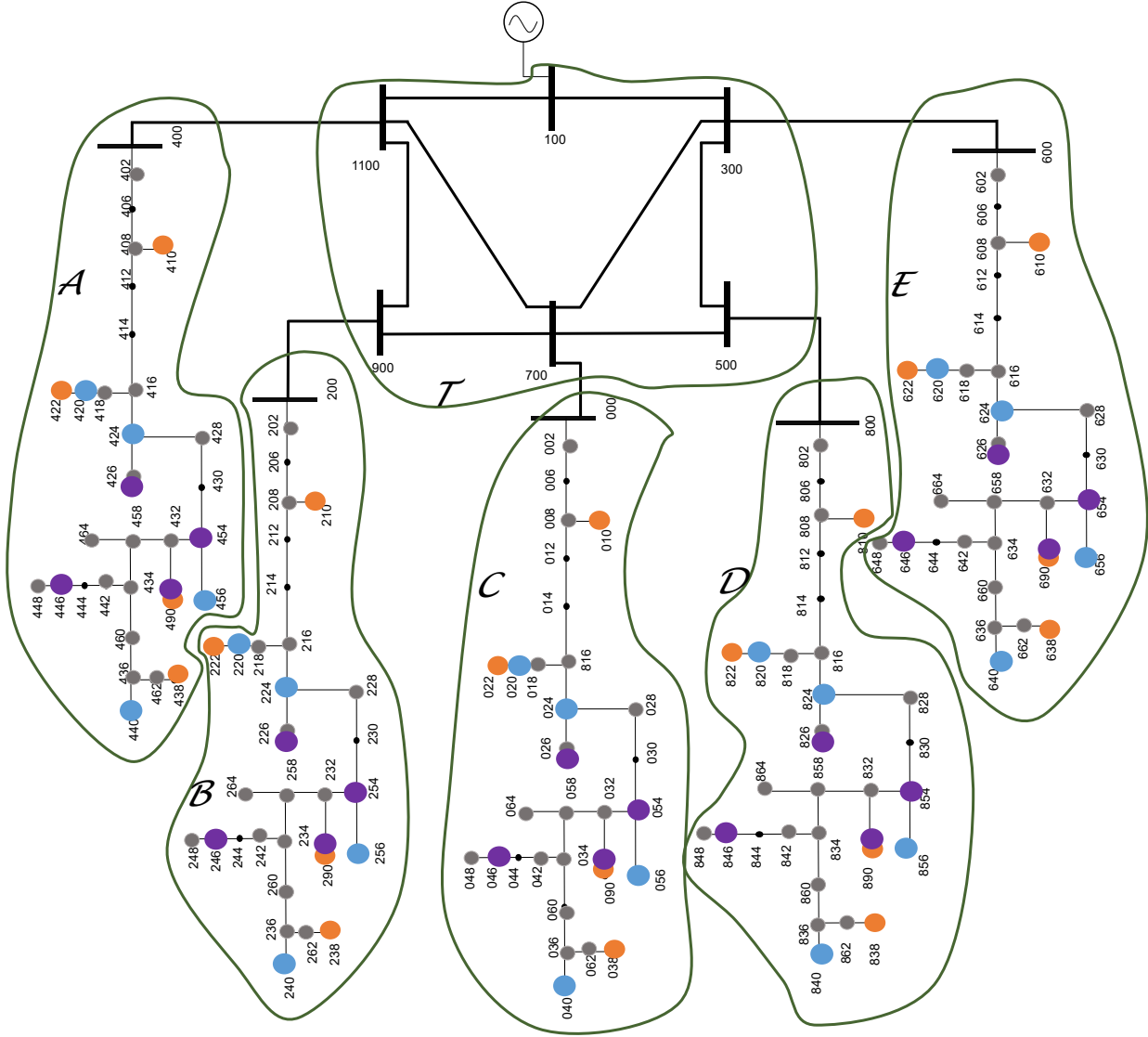


Figure 63. Areas of the decentralized modelling of the network in the use case.

4.3.1. Congestion scenario

In this scenario a congestion is simulated in the line 228-230 of the network presented in Figure 63. In blue is represented the primal residual $\|r^k\|_2$ of the solution, while in orange the dual residual $\|s^k\|_2$. The results obtained in this process are the same than those obtained in the centralized approach. For the sake of clarity and intending not to be repetitive those results are omitted since can be checked in section 7.3.1 of the chapter 3.

The ADMM protocol is implemented considering a penalty factor $\gamma = 4 \cdot 10^{-3}$. That value optimizes the number of iterations needed for convergence. The value was obtained after several try and error iterations. However, values around $\gamma = [10^{-3}, 10^{-4}]$ provide similar results compared to the ones presented. The evolution of the primal and dual residuals is drawn in Figure 64.

In terms of the convergence criterion, for the case of study it is considered that an epsilon lower than 10^{-3} is enough for the level of precision required. Then, as it is possible to see in the figure, the convergence was achieved in iteration 26 both for primal and dual problems. The evolution is monotonously decreasing up to the convergence criteria.

However, in this case, the evolution of the residuals shows a slower tendency to the convergence that the one obtained in the illustrative examples due to the complexity and multiple information exchange between areas in the sequency of resolution of the problem.

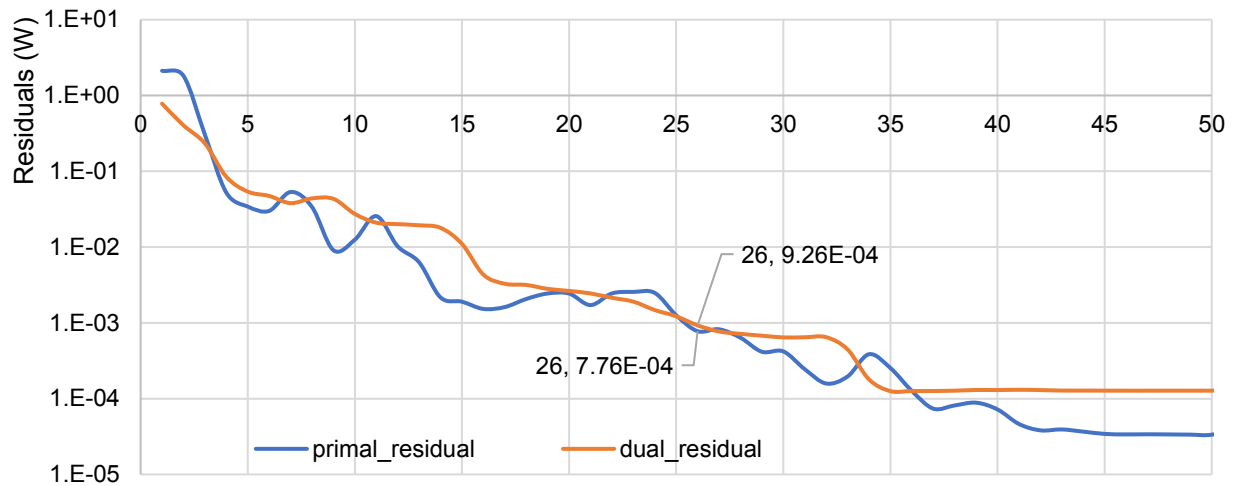


Figure 64. Residuals of the congestion solving scenario for the use case.

The evolution of the total costs of the congestion scenario is presented in Figure 65. The final value obtained in this decentralization methodology is the same that the one obtained in the centralized version of the problem, with a total cost of 20,969 €. The evolution of the costs is monotonously decreasing down to the centralized solution.

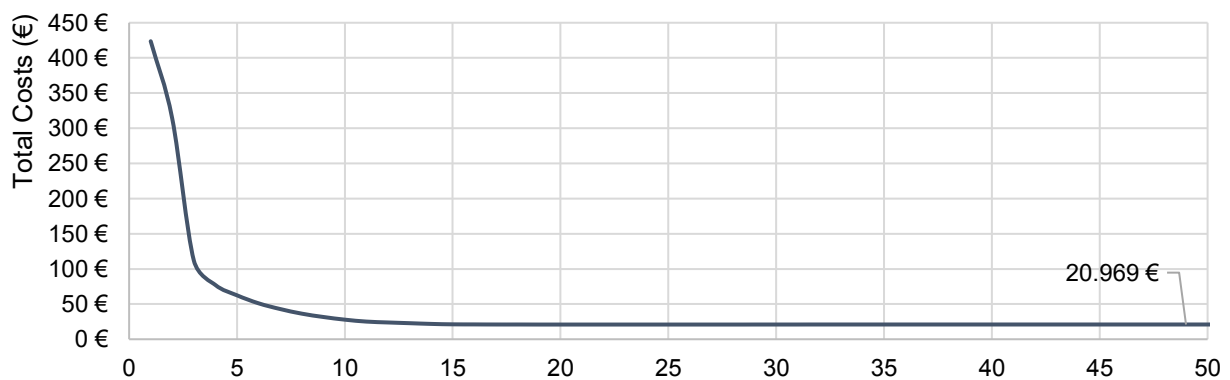


Figure 65. Evolution of the total costs of the congestion scenario in the use case.

Besides of the evolution of the costs, the marginal costs of the complicating constraints evolution are also investigated in Figure 66, where the summatory of the Lagrange multipliers are displayed. The only marginal value that is not equal to zero is the multiplier associated to the balance of the grid, that accounts for 0,1254€/kW if all the values of the day are added. The shape of that multiplier is the same than the one presented in the chapter 3, in Figure 38.

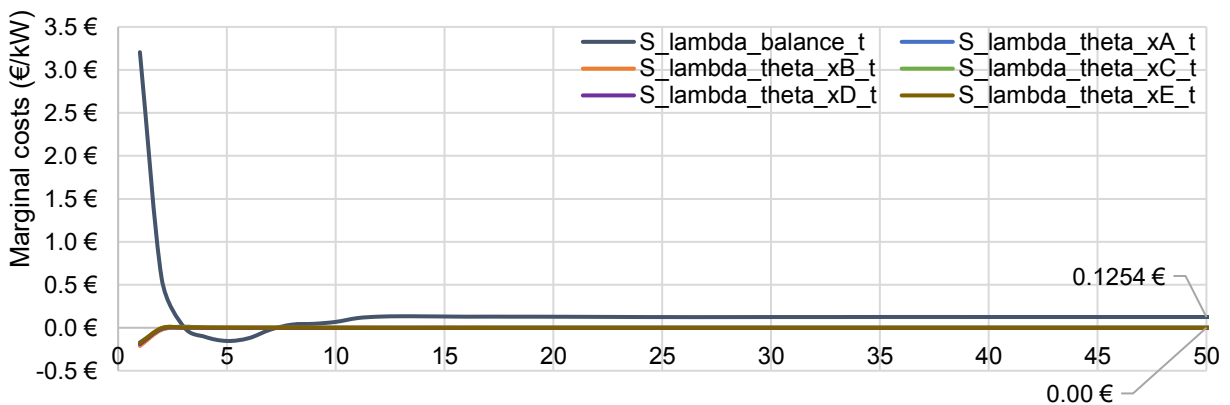


Figure 66. Evolution of the summatory of the marginal values in the congestion scenario of the use case.

4.3.2. Imbalance scenario

In this imbalance scenario is investigated how the LFM formulation will solve the imbalance created by the disconnection of the SL 75 during the whole day considering a decentralized approach. The input data of the model is the same, and for the sake of simplicity it is not repeated. In this section an iteration study of the decentralization process is done, presenting the primal and dual residuals of the decentralized approach, as well as the evolution of the total cost of the imbalance solving and the value of Lagrange multipliers used to coordinate the problem.

The ADMM protocol used for the decentralization of the scenario uses a penalty factor (γ) equals to $4 \cdot 10^{-3}$, value that optimizes the number of iterations needed for convergence. The value was obtained after several try and error iterations, like the congestion solving scenario. The convergence criterion in this case is considered also $\varepsilon = 10^{-3}$.

In Figure 67 the evolution of the primal and dual residuals for the imbalance scenario of the Use Case is presented in semi-logarithmic axis. Both residuals show a decreasing tendency. Stabilizing under the epsilon value after 20 iterations in the case of the dual residual and 63 iterations in the case of primal residual.

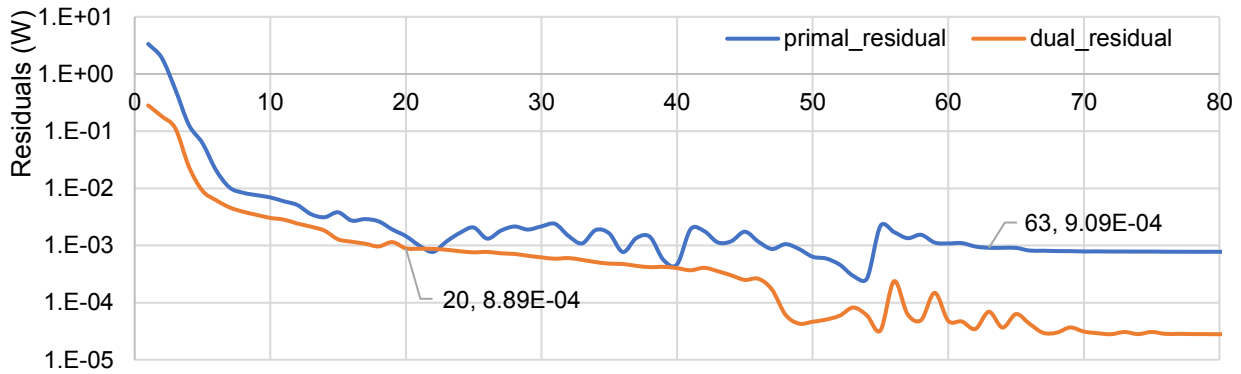


Figure 67. Evolution of the primal and dual residuals in the imbalance scenario of the use case.

Besides of that, the evolution of the total costs for this decentralized LFM is presented in Figure 68, exhibiting a consistent tendency to the centralized optimal solution found of 94,8549€. The solution found in the decentralized version is the same than in the centralized methodology.

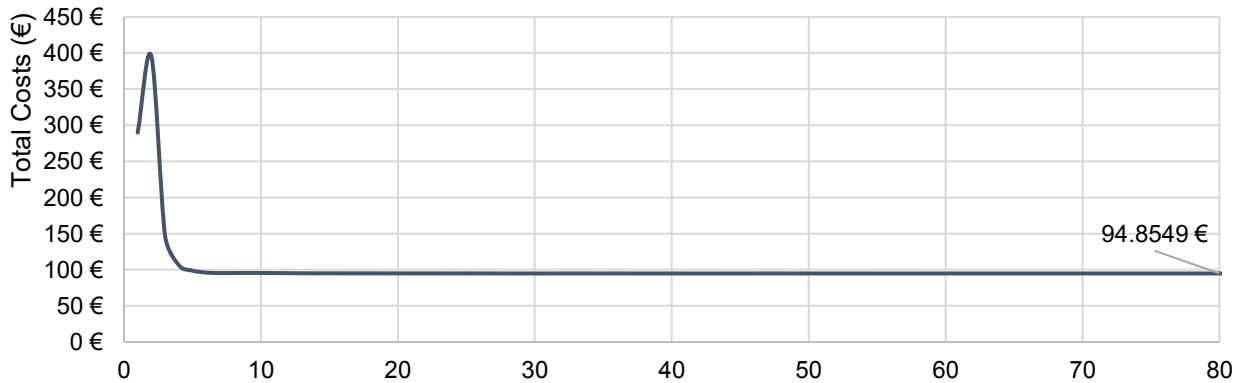


Figure 68. Evolution of the total costs of the imbalance scenario in the use case.

In terms of the marginal costs, Figure 69 is presented. The only marginal cost activated in the grid operation is the one associated to the overall balance of the grid, consistently with the fact that the issue the scenario is solving is the imbalance created by SL 75. The total cost is equal to 0,8650 €, same value than the one obtained in the centralized solution if all the periods are added up. The shape of the multipliers during the day is the same that the one presented in Figure 46.

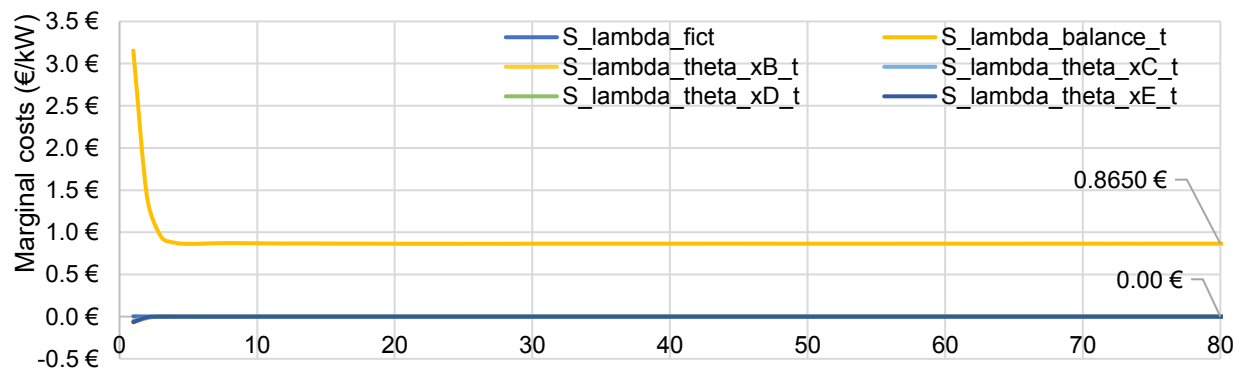


Figure 69. Evolution of the marginal costs for the imbalance scenario in the use case.

CHAPTER 5: ANALYSIS OF RESULTS AND CONCLUSION

1. Introduction

In the present master's thesis is has been proposed an innovative and simple approach for the clearing of a Local Flexibility Market (LFM) that solve both congestions and imbalance issues in distribution grids. This market proposal has solved considering both centralized and decentralized methodologies. To this aim, firstly, the thesis has investigated the type of flexibility products that the literature has develop in recent times. Secondly, research about how the electricity market works was done. With this information, the centralized version of the problem was proposed, analysing two illustrative examples and a use case under congestion and imbalance scenarios. Lastly, a proposal of how to decentralize and coordinate the proposed LFM market formulation has been also carried out.

This chapter ends the major works of the thesis analysing the results obtained in chapters 3 and 4, where the simulations of centralized and decentralized clearings of the proposed market are explained. With this aim, an analysis of the results is done in this chapter, comparing both centralized and decentralized solutions, highlighting the most representative findings of the investigations and presenting the future lines of research.

2. Analysis of results

In this section an analysis of the results of the centralized and decentralized LFM proposed in this thesis is done. With the intention of not being repetitive with the results, in this section, it is analysed the different convergence rates of each complicating variable of the problem.

2.1. Comparison of the first illustrative example

In order to plot the residuals in a semi-logarithm axis, the absolute value of the residuals of each variable is computed. In this case, the studied variables are $\tilde{h}_\theta^k = \theta_x^{k,+} - \theta_x^{k,-}$ for the voltage angle values in the virtual node of both areas, $\tilde{h}_x^k = p_{i,x}^k + p_{j,x}^k$ for the power exchange mismatch between areas and $\tilde{h}_B^k$ as has been defined in equation (4.36).

In Figure 70 the evolution of the residuals of the complicating variables of the congestion scenario is presented. In this graphic a detailed evolution of the convergence is presented. Both the residual at the fictitious node and the balance residual converge at the same pace. On the other hand, the residual of the voltage angle (presented in blue) presents a good performance in terms of the converge speed since the first iteration.

In Figure 71 the evolution of the residuals of the complicating variables and equations is presented for the imbalance scenario. In this case, both the power balance at the fictitious node and the residual of the voltage angle at that node present good convergence properties. On the other hand, the residual of the balance equation shows a different behaviour, and if it is compared with the primal residuals showed in Figure 57 it is possible to infer that is similar, meaning that it strongly influences on the overall convergence of the problem.

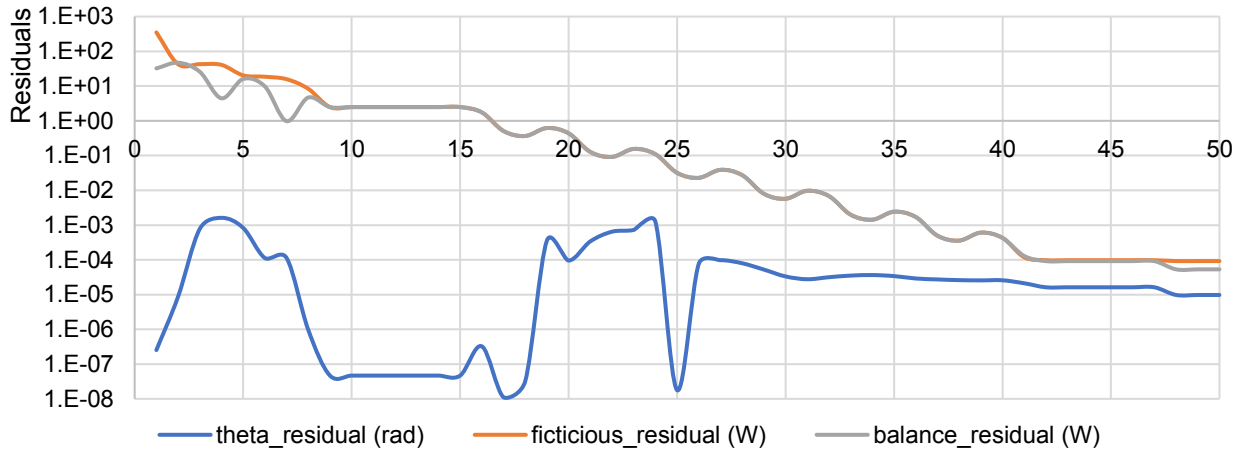


Figure 70. Evolution of the residuals of the complicating variables and equations of the congestion scenario in the first illustrative example

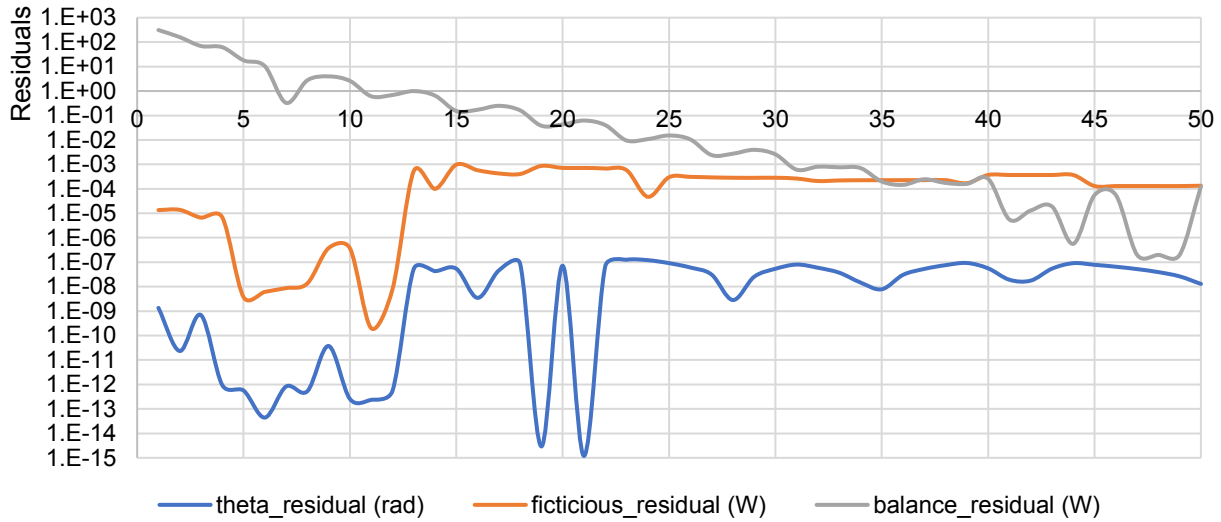


Figure 71. Evolution of the residuals of the complicating variables and equations of the imbalance scenario in the first illustrative example

As it is possible to observe, in the imbalance scenario, the power flow mismatch in the fictitious node does not affect to the overall result of the problem. Conversely, in the congestion scenario, this power flow mismatch does affect it. The reason behind this is that the coordination in the first case is done only by means of the overall balance equation, and in the congestion scenario, the power flow between areas is also used for that purpose.

The proposed methodology can adapt perfectly in those different situations to effectively coordinate the solution between areas. This coordination process is done automatically without the intervention of any additional variable or choose, showing the possibilities of such an adaptative tool as ADMM.

2.2. Comparison of the second illustrative example

Similarly to the previous section, the evolution of the residuals of the voltage angle variable in the fictitious node ($\tilde{h}_\theta^k$), the residual of the power mismatch in the fictitious node ($\tilde{h}_x^k$) and the residual of the balance equation ($\tilde{h}_B^k$) are studied for the second illustrative example, where a LFM where energy and capacity products are exchanged is proposed.

Figure 72 presents the evolution of the described variables. In this case, the residuals at the fictitious interconnecting node present good convergence properties, showing an error less than 10^{-5} during the evolution of the simulation. If this evolution is compared with the one presented in Figure 60, it is found that the residual of the balance equation is the one with the highest influence on the overall convergence of the problem.

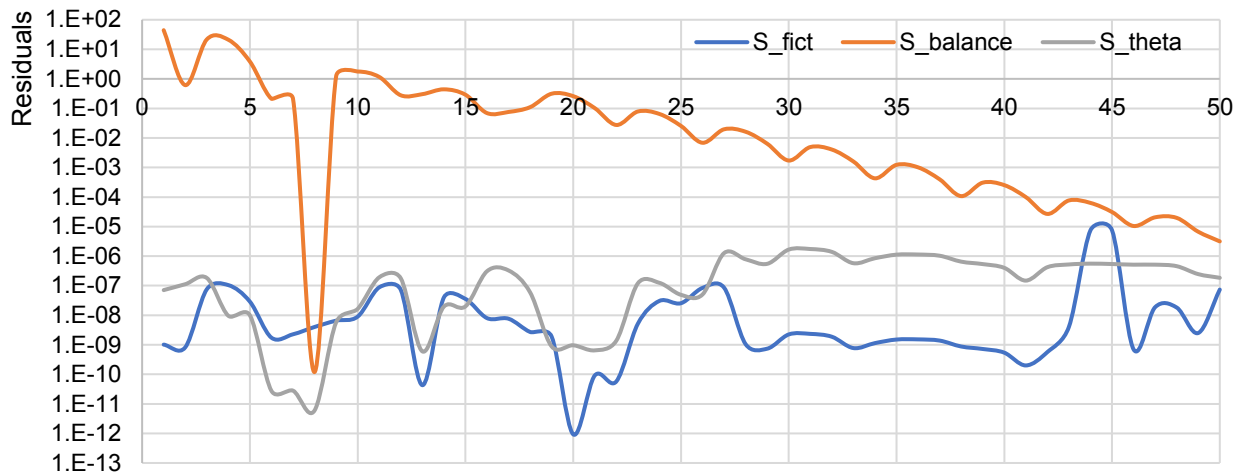


Figure 72. Evolution of the residuals of the complicating variables and equations in the second illustrative example.

Similarly to the previous illustrative example, the coordination is achieved by means of the balance equation, as the evolution of the residual shows.

2.3. Comparison of the use case

In this section, the residuals of each complicating variable and equation are analysed for the final use case, where a LFM is proposed in a grid composed by 5 distribution grids and a transmission system that interconnects all of them.

In Figure 73 and Figure 74 the evolution of the residuals variables and equation that influence on the overall convergence of the problem is presented for the congestion and imbalance scenarios. All the variables show a monotonous decreasing tendency down to the centralized solution. However, there are variables that influences this overall convergence more than others. It is possible to observe that the complicating variables associated to the power flow mismatch in the fictitious node interconnecting the areas C and T, and the balance equation highly influence the overall solution convergence.

In both figures it is possible to observe that there are three different initial points for the convergence. Balance equation is the one with the highest residual in the beginning of the iterative process. Following it, the voltage angle at the fictitious node in the interconnections start in a residual near to 10^0 . Lastly, the power flow mismatch is the one that start the closest to the solution, however, the power mismatch between area C and T, ends with a residual similar to the balance equation in both scenarios.

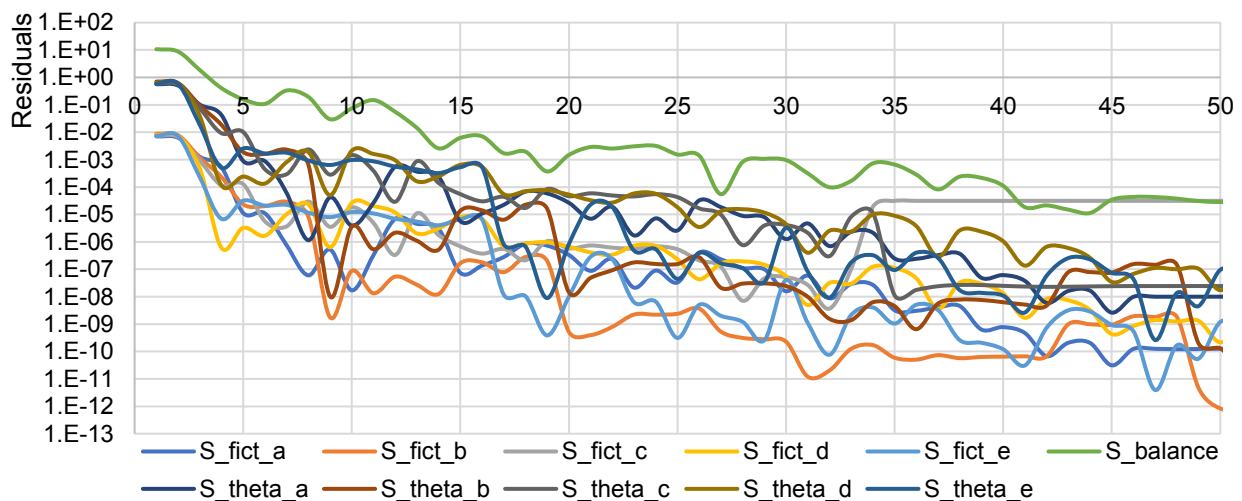


Figure 73. Evolution of the residuals of the complicating variables and equation in the congestion scenario of the use case.

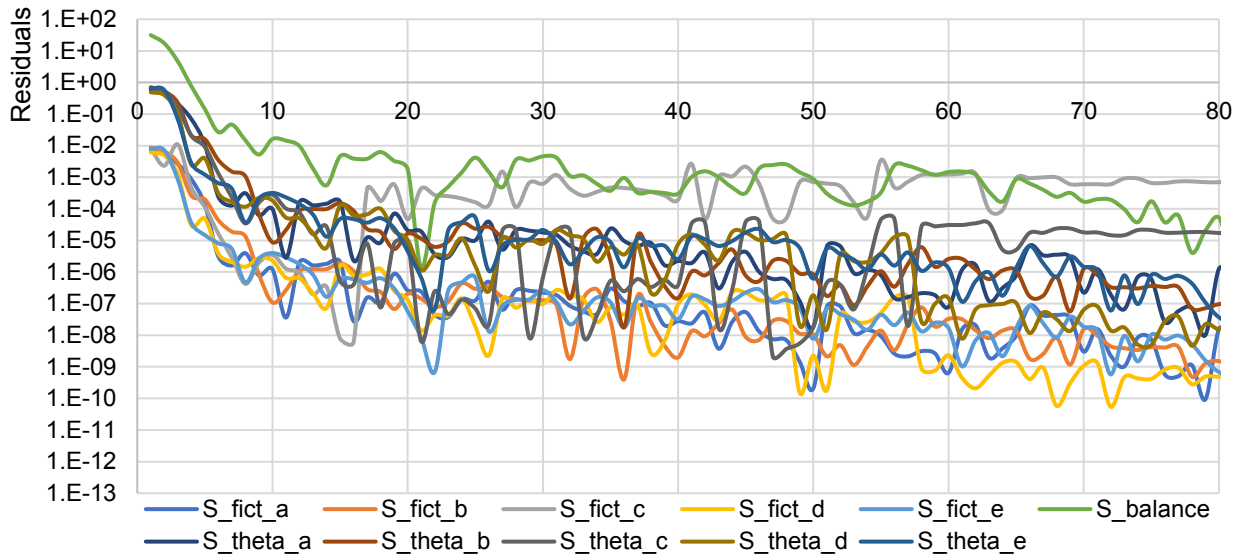


Figure 74. Evolution of the residuals of the complicating variables and equation in the imbalance scenario of the use case.

In both cases, the power flow mismatch in fictitious node connecting areas C and T increases after several iterations. After analysing the formulation of the problem, this is caused to the penalty factor as was set to an order of magnitude of 10^{-3} . This assignation results in a low representative value in the objective function, as the multiplier value associated to that residual is also low.

Different values of the penalty factor were chosen, however, is with a γ inside the range of $[10^{-3}, 10^{-4}]$ when the fastest converge is achieved.

3. Conclusions

In this section the conclusions of the master's thesis are presented. Bearing this objective in mind, the conclusions of the study are presented in this point. Three main aspects are discussed: the proposed LFM formulation, the simulations done, and the programming language used for simulating the marketplace.

3.6. About the proposed LFM

In a nutshell, the proposed marketplace joints together capacity and energy flexibility products in a single auction solving both imbalance and congestion issues in distribution grids. This market proposal has been tested under different conditions and in a centralized and decentralized fashion, showing its effectiveness to solve those problematic situations in a high loaded distribution grid with renewable energy generators, batteries, and flexible loads.

Regarding the formulation of the problem, the fact that the problem has been simplified with respect to the current state of the art is highlighted. In the literature, the problem has been investigated using non-convex and Mixed Integer description. On the other hand, this master's thesis has achieved to describe an equal realistic and useful approximation to the problem using only positive variables.

The market formulation works correctly in centralized and decentralized environments under congestion and imbalance situations, showing the validity of the approach. In addition, the decentralized fashion enables solving much more larger problems, as they are divided into smaller parts solved in a distributed but coordinated manner along the grid.

The coordination between the areas was achieved using three different aspects of the grid: the voltage angles, the power flow in the fictitious nodes that interconnect the areas and the overall grid balance. This coordination procedure allows a minimum information exchange between the areas while maintaining the same solution that if the market was cleared in a centralized manner.

Besides of that, the introduction of the capacity products increased the market liquidity, as the second illustrative example has demonstrated. The proposed mixed energy and capacity LFM is the first in this field, to the best of the author's knowledge, what advances the current state of the art of the field.

3.7. About the simulations

The validity of the proposed market has been extensively tested under two different illustrative examples and a more complex and realistic use case. In this highlighted the adaptability of the proposed approach to different and changing scenarios, being possible to solve both imbalance and congestion issues using only local resources.

The centralized problem showed the simplicity of the approach in mathematical terms with a simple interpretation of the Lagrange multipliers. This explanation it just not possible if the non-linearity and non-convexity of the problem increases, as the Lagrange multipliers move away from their linear and convex explanations. This situation can be detected using the concept of duality gap, that can be defined as the difference between the primal and dual solutions of the problem.

In terms of the decentralized simulations, less than 100 iterations are needed for the convergence of the problem considering a standard ADMM formulation. Considering this algorithm, the solution found strictly corresponded to the centralized one, showing the lawfulness of the distributed coordination procedure.

The penalty factor of the standard ADMM procedure highly influence on the general speed of the converge and the quality of the function. For the marketplace described in this thesis, a γ inside the interval $[10^{-3}, 10^{-4}]$ was selected as those values adapt to all the different situations described. However, in some cases, as in the final use case, using such lower valued penalty factor could end with an inequal convergence among variables, with some of them near values 10^{-8} and others near values 10^{-4} .

3.8. About programming language

General Algebraic Modelling System (GAMS) is the language used for the modelling of the marketplace in this master's thesis. It is a well develop and documented programming language that allows to set up a huge marketplace model within hours.

This modelling language is used to describe both the objective and restriction body of the problem aiming to solve. However, the generation of the dataset and the results visualization must be made by means of other software.

The software has several advantages as related below:

- Fast set up: once the programming language is dominated, it is possible to build up different market models, and simulations really quick. This ends in a productivity increase, as it is possible to simulate different scenarios and formulations rapidly.
- Simple definition of the equations: the mathematical description of the equations are defined really fast, as the progamming language ensures a straightforward definition in the body of the program.
- Wide range of solvers: The software ensures the connection with a wide range of solvers, being possible to use CPLEX, BARON, CONOPT, Gurobi and much others.

On the other hand, there are some shortcomings when using this programming language:

- Steep learning curve: The progamming language have many advantages, however, it is hihgly difficult to learn how to use it. There are only a few manuals that can help with this process, and is specially taught in terms of the data introduction.
- Data managining: The power flow followed in this thesis was: firstly, creating a valid dataset using Microsoft Excel, then feed that data into GAMS, solve the problem and export the solution to Microsoft Excel again.
- Reusability of the code: conversely to other programming languages, it is specially difficult to develop a library of tools in this language. The practical result of this is long programs, for calculus that could be done inside of a tool-box.

4. Conclusiones

En este apartado se presentan las conclusiones del Trabajo Final de Máster. Teniendo en cuenta este objetivo, en este punto se presentan las conclusiones del estudio. Se discuten tres aspectos principales: la formulación del LFM propuesta, las simulaciones realizadas y el lenguaje de programación utilizado para la simulación del mercado.

4.1. Sobre el LFM propuesto

En pocas palabras, el mercado propuesto reúne los productos de capacidad y flexibilidad energética en una única subasta que resuelve los problemas de desequilibrio y congestión en las redes de distribución. Esta propuesta de mercado ha sido probada en diferentes condiciones, de forma centralizada y descentralizada, mostrando su eficacia para resolver estas situaciones problemáticas en una red de distribución con alta carga, con generadores de energía renovable, baterías y cargas flexibles.

En cuanto a la formulación del problema, se destaca el hecho de que se ha simplificado el problema con respecto al estado del arte actual. En la literatura, el problema ha sido investigado utilizando descripciones no convexas y de variable binaria. En cambio, en este Trabajo Final de Máster se ha conseguido describir una aproximación igual de realista y útil al problema utilizando sólo variables positivas.

La formulación del mercado funciona correctamente en entornos centralizados y descentralizados bajo situaciones de congestión y desequilibrio, demostrando la validez de la aproximación. Además, el modo descentralizado permite resolver problemas mucho más grandes, ya que se dividen en partes más pequeñas que se resuelven de forma distribuida pero coordinada a lo largo de la red.

La coordinación entre las zonas se logró utilizando tres aspectos diferentes de la red: los ángulos de tensión, el flujo de potencia en los nodos ficticios que interconectan las zonas y el equilibrio global de la red. Este procedimiento de coordinación permite un intercambio mínimo de información entre las zonas, manteniendo la misma solución que si el mercado se compensara de forma centralizada.

Además, la introducción de los productos de capacidad aumentó la liquidez del mercado, como ha demostrado el segundo ejemplo ilustrativo. La propuesta de un LFM mixto de energía y capacidad es la primera en este campo, hasta donde el autor conoce, lo que hace avanzar el estado actual de la técnica en este campo.

4.2. Sobre las simulaciones

La validez del mercado propuesto se ha probado ampliamente en dos ejemplos ilustrativos diferentes y en un caso de uso más complejo y realista. De este modo, se puso de manifiesto la adaptabilidad del enfoque propuesto a escenarios diferentes y cambiantes, siendo posible resolver tanto los problemas de desequilibrio como los de congestión utilizando únicamente recursos locales.

El problema centralizado mostró la simplicidad del enfoque en términos matemáticos con una interpretación sencilla de los multiplicadores de Lagrange. Esta explicación no es posible si aumenta la no linealidad y la no convexidad del problema, ya que los multiplicadores de Lagrange se alejan de sus explicaciones lineales y convexas. Esta situación puede detectarse utilizando el concepto de brecha de dualidad, que puede definirse como la diferencia entre las soluciones primal y dual del problema.

En cuanto a las simulaciones descentralizadas, se necesitan menos de 100 iteraciones para la convergencia del problema considerando una formulación ADMM estándar. Considerando este algoritmo, la solución encontrada se corresponde estrictamente con la centralizada, mostrando la legalidad del procedimiento de coordinación distribuida.

El factor de penalización del procedimiento ADMM estándar influye mucho en la velocidad general de convergencia y en la calidad de la función. Para el mercado descrito en esta tesis, se seleccionó un γ dentro del intervalo $[10^{-3}, 10^{-4}]$ ya que esos valores se adaptan a todas las diferentes situaciones descritas. Sin embargo, en algunos casos, como en el caso de uso final, el uso de un factor de penalización de valor tan bajo podría terminar con una convergencia desigual entre las variables, con algunas de ellas cerca de los valores 10^{-8} y otras cerca de los valores 10^{-4} .

4.3. Sobre el lenguaje de programación

General Algebraic Modelling System (GAMS) es el lenguaje utilizado para la modelización del mercado en esta tesis de máster. Se trata de un lenguaje de programación bien desarrollado y documentado que permite configurar un enorme modelo de mercado en cuestión de horas.

Este lenguaje de modelización se utiliza para describir tanto el cuerpo de objetivos como de restricciones del problema que se pretende resolver. Sin embargo, la generación del conjunto de datos y la visualización de los resultados deben realizarse mediante otro software.

El software tiene varias ventajas que se relacionan a continuación:

- Rápida puesta en marcha: una vez dominado el lenguaje de programación, es posible construir diferentes modelos de mercado, y simulaciones realmente rápidas. Esto se traduce en un aumento de la productividad, ya que es posible simular diferentes escenarios y formulaciones rápidamente.
- Definición sencilla de las ecuaciones: la descripción matemática de las ecuaciones se define muy rápidamente, ya que el lenguaje de programación garantiza una definición sencilla en el cuerpo del programa.
- Amplia gama de solucionadores: El software asegura la conexión con una amplia gama de solucionadores, siendo posible utilizar CPLEX, BARON, CONOPT, Gurobi y muchos otros.

Por otro lado, existen algunas carencias a la hora de utilizar este lenguaje de programación:

- Curva de aprendizaje pronunciada: El lenguaje de programación tiene muchas ventajas, sin embargo, es muy difícil aprender a usarlo. Sólo hay unos pocos manuales que pueden ayudar en este proceso, y es especialmente exigente en cuanto a la introducción de datos.
- Gestión de datos: El flujo de alimentación seguido en esta tesis fue: primero, crear un conjunto de datos válido usando Microsoft Excel, luego introducir esos datos en GAMS, resolver el problema y exportar la solución a Microsoft Excel de nuevo.
- Reutilización del código: a diferencia de otros lenguajes de programación, es especialmente difícil desarrollar una biblioteca de herramientas en este lenguaje. El resultado práctico de esto son programas largos, para cálculos que podrían hacerse dentro de una caja de herramientas.

4. Future lines of research

This master's thesis has contributed to the current state of the art of the LFM in terms of a new simple mathematical approximation using only positive variables, introducing capacity and energy flexibility products in a single marketplace and with a decentralization algorithm that clear the LFM by means of solving several subproblems in a coordinated manner.

The future lines of research of the current work will investigate:

- The introduction of different flexibility assets in the network definition and how this affect to the mathematical complexity and the quality of the solution.
- How to solve voltage deviations with a similar description of the LFM proposed.
- The resolution of the decentralized LFM using improved ADMM algorithms present in the literature.
- New coordination procedures that improve the convergence.
- A decentralization procedure based on other technologies, as block-chain or P2P based on smart-contracts.

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ANNEX I: MATHEMATICAL BACKGROUND

1. Introduction.

1.1. Formulation

A linear programming problem lies in find optimal point (maximum or minimum) of a lineal function in a set of hyperplanes and semi-spaces defined in $\mathbb{R}^n$. Considering a vector $c \in \mathbb{R}^n$, non-negative vector $b \in \mathbb{R}^m$, and a matrix $A_{m \times n}$, standard linear programming problem as following form:

$$\begin{aligned} \min z &= c^T x \\ \text{st. } Ax &= b \\ x &\geq 0 \end{aligned} \tag{1.1}$$

Considering this formulation, inequality restrictions are transformed into equality ones using dummy, surplus or slack variables. Typically, a linear programming is in standard form, if and only if:

1. Is a minimization problem.
2. Only includes equality constraints.
3. Vector b is non-negative.
4. Variable vector x is non-negative.

With respect to solutions, a point $x = (x_1, x_2, \dots, x_n) / A \cdot x = b$ it is named as feasible solution, and the set of all feasible solutions are named as feasible region (Ω). On the other hand, a feasible point $\bar{x} / f(x) \geq f(\bar{x}) \forall x \in \Omega$ is named as optimal solution.

Generally, the main objective of the linear programming is to find optimal solution, but optimality conditions only grant local optimal points. However, linear programming has several properties that make possible to ensure global optimal point:

1. If feasible region is bounded, problem has solution.
2. Optimal point of a linear programming problem is always global optimal point.
3. If x and y are optimal solutions of a linear programming problem, then every convex combination of those solutions are optimal solutions.
4. Optimal solution is met at an extreme point of the feasible region.

1.2. Basic solutions.

Considering a linear programming problem in its standard form, the range of the matrix A , ($Rg(A)$) provide information about the existence of solution. If $Rg(A) < n$, then some restrictions are duplicated, if $Rg(A) = n$, then problem is infeasible, and if $Rg(A) = m$ and the system $Ax = b$, has solutions, then problem has solution.

Definition

A non-singular submatrix B of dimension $m \times m$ of A it is said to be basic matrix or base. Then, if matrix B is named basic feasible matrix if and only if $B^{-1}b \geq 0$.

Each basic matrix has a vector associated called basic solution. Being x_B vector of the variables associated to columns of the matrix A need for build B matrix. Then, variables x_B are named basic variables and the remaining non-basic variables. Considering non-basic variables as zero:

$$Ax = b, \quad (B \ N) \begin{pmatrix} x_B \\ 0 \end{pmatrix} = b, \quad Bx_B = b, \quad x_B = B^{-1}b \quad (1.2)$$

The number of basic feasible solution of a linear programming problem is always finite and each point match with the extreme points of the feasibility region.

Basic solution theorem

If a linear programming problem has an optimal solution, is also a basic feasible solution.

2. Sensitivity and duality.

2.1. Sensitivities.

Considering x_B^* , B^* optimal variable vector and its associated basic matrix, then, $x_B^* = (B^*)^{-1}b$. Thus, $z^* = c_B^T x_B^*$. Considering a marginal change on the independent vector b , $b^* \rightarrow b^* + \Delta b$, this marginal change will originate a change in the vector of variables and in the optimal value as follow: $x_B^* \rightarrow x_B^* + \Delta x_B$, $z^* \rightarrow z^* + \Delta z$. Then: $\Delta z = c_B^T (B^*)^{-1} \cdot \Delta b = \lambda^{*T} \Delta b$,

$$\lambda_j^* = \frac{\Delta z}{\Delta b_j} \quad (1.3)$$

Where λ_j^* is the sensibility of a coordinate j with respect to the independent term b_j . This analysis gives information about the change in the objective function with respect to the independent term of the linear programming problem. λ_j is also know as the dual variables or Lagrangian multipliers.

2.2. Duality.

Given a linear programming problem named *primal problem*, there exists other problem closely connected with it, named *dual problem*. Under certain hypothesis, those problems arrive to the same optimal solution, and the primal problem can be solved indirectly using dual problem, what might involve a great computational advantage.

Considering linear programming stated in (1.1). Then, its dual problem to maximize is written as follows:

$$\begin{aligned} Z &= b^T y \\ \text{s. t. } A^T y &\leq c \\ y &\geq 0 \end{aligned} \quad (1.4)$$

Where variables of the vector $y = (y_1, y_2, \dots, y_m)^T$ are called dual variables. To obtain the dual problem from a primal, there are some conversion rules that allows the conversion, as Table 17 reflects.

Conversion rules	Primal	Dual
Objective function	Minimizing	Maximizing
Transformations	Constraint: Less or equal than	Variable: less or equal than 0
	Constraint: Bigger or equal than	Variable: greater or equal than 0
	Constraint: Equality	Free variable
Objective function	Minimizing	Maximizing
Transformations	Constraint: Less or equal than	Variable: greater or equal than 0
	Constraint: Greater or equal than	Variable: less or equal than 0
	Equality constraint	Free variable

Table 17. Conversion rules for primal and dual problems in linear programming.

2.3. Duality theorems.

Weak duality theorem

If x is a feasible solution for the primal problem, and λ is a feasible solution for the dual problem, then $\lambda^T b \leq c^T x$.

On the other hand, the strong duality theorem establishes that considering primal and dual problems, they will find optimal solutions simultaneously.

Strong duality theorem

If x^* is the minimizer of the primal problem, there exists a vector λ^* that maximizes the dual problem, such as $\lambda^{*T} b = c^T x^*$.

Lastly, complementary slackness theorem of the linear programming allows to find the optimal solution of the dual problem once optimal solution of the primal problem is known, and vice versa.

Complementary slackness theorem

Consider an arbitrary pair of primal and dual linear programming problems. Let x , and λ be the feasible solutions to the primal and dual linear programming problems, respectively. Then x and λ are both optimal if:

$$\lambda_j^* (A_j x^* - b_j) = 0 \quad \forall i \quad (1.5)$$

$$x_i^* (\lambda^{*T} A_i - c_i) = 0 \quad \forall j \quad (1.6)$$

3. Feasible solution set.

As objective function is independent of the solutions set, this section will only verse about restrictions set. Restrictions conforms feasible solution set for a given Lineal Programming Problem, and have interest itself, as provide information about optimal solution. For example, a PPL is said well-formulated if its solutions are delimited.

Restrictions give rise to different types of feasible solutions set. Depending on its formulation vector spaces, affine spaces, cones, polytopes, and polyhedrons can arise. Considering those 5 categories, it is possible to define them as a function of its restrictions or as an intern representation:

Algebraic structure	Restrictions definition	Intern representation
Vector space	$Hx = 0$	$x = \sum_i \rho_i v_i; \rho_i \in \mathbb{R}$
Affine space	$Hx = a$	$x = q + \sum_i \rho_i v_i; \rho_i \in \mathbb{R}$
Cone	$Hx \leq 0$	$x = \sum_j \pi_j w_j; \pi_j \geq 0$
Polytope	$Hx \leq a$	$x = \sum_k \lambda_k q_k; \lambda_k \geq 0; \sum_k \lambda_k = 1$
Polyhedron	$Hx \leq a$	$x = \sum_i \rho_i v_i + \sum_j \pi_j w_j + \sum_k \lambda_k q_k;$ $\rho_i \in \mathbb{R}; \pi_j \geq 0; \lambda_k \geq 0; \sum_k \lambda_k = 1$

Restriction definition is the most natural way to write down feasible solutions set, but it makes difficult to provide insight about it. On the other hand, intern representation ease obtaining feasible solution by modifying coefficients ρ_i, π_j y λ_k .

3.1. Convex sets.

Convex sets are the easiest sets that appears in mathematical programming naturally. A set S is convex if and only if it is possible to joint any two points with a line of points belonging to the set.

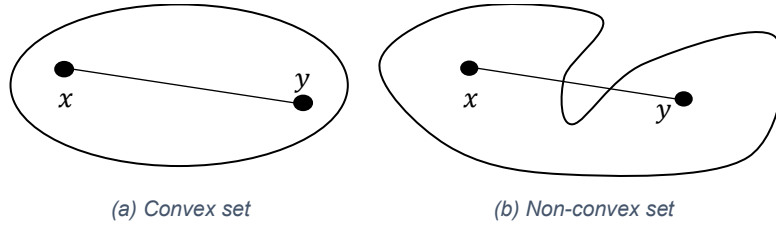


Figure 75. Representation of a convex and non-convex set

Definition of convex sets

Let $S \in \mathbb{R}^n$ convex $\Leftrightarrow \lambda x + (1 - \lambda)y \in S$ for all $\lambda \in [0,1]$ with $x, y \in S$.

As a direct consequence of this definition, following properties are named:

- If $\{S_i\}_{i \in I}$ is a family of convex sets, then set $\bigcap_{i \in I} S_i \equiv \{x | x \in S_i \ \forall i \in I\}$ is also a convex set.
- If S_1 and S_2 are convex sets, then $S_1 + S_2 \equiv \{y = x_1 + x_2 \mid x_1 \in S_1, x_2 \in S_2\}$ is also a convex set.

Owing to that, intersection and addition of convex spaces resulting in other convex spaces.

3.2. Extreme points.

Extreme point definition

Let S be a non-void convex set in $\mathbb{R}^n$. A vector $x \in S$ is named extreme point of S if and only if $x = \lambda x_1 + (1 - \lambda)x_2$, $x_1, x_2 \in S$, $\lambda \in (0,1) \Rightarrow x = x_1 = x_2$.

In a circle the number of extreme points is infinite, while in a square is four and in a triangle is three, and so on.

Representation of convex finite set theorem

If a finite convex set is delimited, any of its interior points can be described using a convex combination of its extreme points.

This theorem is proved if and only if the set is delimited. If not, extreme direction concept is needed.

Extreme direction definition

(Direction) Let S be a convex set, delimited and non-void in $\mathbb{R}^n$. A unity vector d is a direction of S if and only if for each $x \in S \ \exists x + \pi d \in S \ \forall \pi \geq 0$.

(Extreme direction) A direction d is named extreme direction if it can not be expressed as a lineal positive combination of two different directions, that is, if $d = \pi_1 d_1 + \pi_2 d_2$ for $\pi_1, \pi_2 > 0, \Rightarrow d = d_1 = d_2$.

3.3. Vector spaces.

Vector spaces arise as a lineal combination of a set of vectors, as the most known type of convex sets.

Lineal combination definition

Considering matrix A as a set of column vectors $A \equiv \{a_1, \dots, a_k\}$. A vector x , is said to be a lineal combination of vector of A if it can be expressed as:

$$x = \rho_1 a_1 + \dots + \rho_k a_k, \quad \rho_i \in \mathbb{R} \ \forall i = 1, \dots, k \quad (1.7)$$

To clarify notation, the set of all possible lineal combination of the vectors of A matrix is noted as A_p .

Vector space

The set $A_p \equiv \{x \in \mathbb{R}^n | x = \rho_1 a_1 + \dots + \rho_k a_k, \rho_i \in \mathbb{R}; i = 1, \dots, k\}$ of all vectors of the Euclidean space $\mathbb{R}^n$, that can be generated as a lineal combination of set A is named vector space generated by A , and the A set is set of its generators.

Identification of subspaces theorem

Let $\mathcal{V}$ a vector space and $\mathcal{W}$ a subspace of it. Then, $\mathcal{W}$ is a subspace of $\mathcal{V}$ if $a, b \in \mathcal{W} \Rightarrow a + b \in \mathcal{W}$ and $a \in \mathcal{W}, \alpha \in \mathbb{R} \Rightarrow \alpha a \in \mathcal{W}$

Representation of a vector space theorem

Let S be a subset of $\mathbb{R}^n$. Set S is a vector subspace of $\mathbb{R}^n$ if and only if exists a matrix H of dimension $m \times n$, as $S = \{x \in \mathbb{R}^n | Hx = 0\}$.

This theorem demonstrates that the feasible solution set of a homogeneous system of lineal equations is a vector space.

3.4. Convex polyhedric cones.

Non-negative lineal combination definition

A vector x is named non-negative lineal combination of the vectors $A = \{a_1, \dots, a_k\}$ if and only if:

$$x = \pi_1 a_1 + \dots + \pi_k a_k, \quad \pi_i \in \mathbb{R}^+ \forall i \in \mathbb{R}^+ \quad (1.8)$$

Non-negative lineal combination of a A set of vectors is noted as A_π .

Convex polyhedric cone definition

Let $A = \{a_1, \dots, a_k\}$. Set A_π of all non-negative lineal combinations of vectors of A is named convex polyhedric cone. Vectors $a_1, \dots, a_k$ are cone generators.

For the sake of clarity, convex polyhedric cones will be named only cones ($\mathcal{C} = A_\pi$).

Representation of a cone theorem

Let S be a subset of $\mathbb{R}^n$. S cone with origin in $0 \Leftrightarrow \exists H_{m \times n} | S = \{x \in \mathbb{R}^n | Hx \leq 0\}$

3.5. Polytopes.

Lineal convex combination definition

A vector x is a linear convex combination of vectors within $A = \{a_1, \dots, a_k\}$ if:

$$x = \lambda_1 a_1 + \dots + \lambda_k a_k, \quad \lambda_i \in \mathbb{R}^+, \forall i = 1, \dots, k, \sum_{i=1}^k \lambda_i = 1 \quad (1.9)$$

The set of all convex linear combinations of vectors of matrix A is named A_λ .

Polytope definition

Let $A = \{a_1, \dots, a_k\}$. The set:

$$S = A_\lambda \equiv \left\{ x \in \mathbb{R}^n | x = \lambda_1 p_1 + \dots + \lambda_k p_k, \lambda_i \geq 0; \sum_{i=1}^k \lambda_i = 1 \right\} \quad (1.10)$$

of all convex lineal combination of the vectors of A is named polytope or convex enclosure generated by $\{a_1, \dots, a_k\}$. If the set of vectors is minimum, is then the set of k extreme points of S .

Polytope representation theorem

A polytope can be represented as the intersection of a finite set of semi-spaces:

$$S = \{x \in \mathbb{R}^n | Hx \leq b\} \quad (1.11)$$

3.6. Polyhedron.

Polyhedron definition

A polyhedron is the intersection of a number finite of semi-spaces non delimited.

$$S = \{x \in \mathbb{R}^n | Hx < b\} \quad (I.12)$$

If the set S is delimited, is then a polytope.

General theorem of polyhedron representation

Any polyhedron can be represented as the sum of a vectorial space, a cone, and a polytope $S = V_\rho + W_\pi + Q_\lambda$. That is:

$$\begin{aligned} \text{if } x \in S \Leftrightarrow x = \sum_i \rho_i v_i + \sum_j \pi_j w_j + \sum_k \lambda_k q_k; \quad \rho_i \in \mathbb{R}; \pi_j \in \mathbb{R}^+; \\ \lambda_k \in \mathbb{R}^+, \sum_k \lambda_k = 1 \end{aligned} \quad (I.13)$$

4. PPL bounded and unbounded.

Considering problem stated in (I.1), several considerations must be addressed:

Vectorial space

The objective function has following form:

$$\min Z = c^T x = \sum_i \rho_i c^T v_i \quad (I.14)$$

The problem will be bounded if vector c^T is orthogonal to all v_i vectors, in any other case, problem is unbounded.

Cone

In this case, objective function is:

$$Z = c^T x = \sum_j \pi_j c^T w_j, \quad \pi_j \geq 0 \forall j \quad (I.15)$$

The problem will be bounded if c^T is orthogonal to all vectors w_j . In any other case, the problem is unbounded.

Polytope

In this case, the objective function:

$$c^T x = \sum_k \lambda_k c^T q_k; \lambda_k \geq 0 \quad (I.16)$$

Which always originates a bounded problem.

Polyhedron

In this case, the objective function:

$$c^T x = \sum_i \rho_i v_i + \sum_j \pi_j w_j + \sum_k \lambda_k c^T q_k \quad (I.17)$$

Which originates following cases:

1. The problem is bounded if and only if c^T is orthogonal to vectors v_i and w_j .
2. The problem is bounded below if c^T is orthogonal to vectors v_i and $c^T w_j \geq 0$.
3. The problem is bounded above if c^T is orthogonal to vectors v_i and $c^T w_j \leq 0$.
4. The problem is unbounded in any other case.

5. Degenerate solutions

Degeneracy definition

Considering a basic solution $x \in \mathbb{R}^n$ of the linear optimization problem defined in (I.1), is said to be *degenerate* if there are more than n constraints which are active at x .

Considering the standard form of the polyhedron defined in equation (I.17), let x be a basic solution. Let m be the number of rows of matrix A . The solution vector x is a *degenerate* basic solution if more than $n - m$ of the component of x are zero. That is, at least one basic solution is zero given all non-basic solutions being zero.

Degenerated solutions can induce cyclic behaviour in the Simplex method. However, it is not very often to obtain this situation. One technique to reach convergence is to add another non-active restriction contained in the solution.

6. Lineal programming problem resolution

Simplex method and exterior point method are prescribed method for linear problems resolution. Those methods provide optimal solution in a finite number of steps. However, its complexity exponentially increases. Exterior point methods are a particular case of the simplex dual methodology, one of the variations of the simplex method introduced by Dantzig in 1947.

In this section, simplex methodology (SM) and Exterior point method (EPM) are described.

6.1. Simplex method

Basic solution theorem of the linear programming ensures finding optimal solution, if exists, it is reached in a basic feasible solution. Simplex methodology generates a sequence of basic feasible solutions than continuously improve objective function. This method operates in two phases:

- Initial phase in where equality constraints set is transformed in other equivalent associated with a basic solution, and the values of the basic variables are transformed in non-negative values.
- Iteratively phase in where objective function coefficients are transformed into non-negative values and objective value is improved progressively up to obtain optimal solution.

6.2. Exterior point method

Exterior point algorithm is equivalent to solve dual problem with the simplex method. Exterior point method operates outside the feasible region, and it is operated within three phases:

1. Initial phase, where que original equality constraints set is transformed in another equivalent.
2. Regulation phase, where coefficients of the objective function are transformed into non-negative terms.
3. Iterative phase, where objective is improved in each iteration up to reach optimal solution, or non-finding feasible terms or establishment of unbounded problem.

ANNEX II: LINEARIZATION OF NON-LINEAR CONSTRAINTS

1. Introduction

Linearization of non-linear constraints in mathematical programming ensures convergence at the global optimal point of the problem. Linear Programming (LP) problems can be solved using CPLEX solver ensuring reaching the global optimal point. Due this characteristic, it is interesting to investigate linearization of the problem proposed in the body of this thesis.

This linearization could provide different benefits, among of which could be included: being used as a previous stage of a Non-Linear Programming problem, to ensure reaching global optimal point within a reasonable amount of time, or to directly clear the market reaching a sufficient consistent solution.

In this annex, the second approach is considered, assessing about the influence of the linearization granularity on the solution. This linearization has been presented to the “International Smart Cities Conference 2021” organized by IEEE Smart Cities in Manchester inside of the Smart Energy Systems Track.

Bearing this objective in mind and aiming to contribute to the progress of the scientific research, the rest of the annex is organized as follows. Section 2 investigate two linearization techniques for non-linear constraints. Then, third section defines the problem considering the approach described for the conference. Lastly, section 4 brings two scenarios for simulation (one solving imbalance issues and the other one considering a congestion scenario).

2. Linearization of non-linear constraints

In this section investigated techniques for linearization of non-linear constraints are explained. In the literature, there exists two main approaches, the McCormick Envelope and the tangent lines approach. Those techniques are further explained in the following subsections.

2.1. McCormick Envelope

As has been previously stated, agents can sell flexibility products in both upward and downward directions. However, they must choose one direction in detriment of the other. It is just not physically possible to give both products in the same period. For example, for a battery it would imply to charge and discharge at the same time, equivalent to breath in and breath out at the same time. Considering special conditions in the market and the grid, it could result economically interesting to do such thing, but it just impossible. Batteries cannot be charged and discharged at the same time as well as a load cannot simultaneously increase and decrease its consumption.

Thus, the LFM requires a non-linear constraint that avoid simultaneous activation of flexibility in both directions for assets that provide upward and downward products. Considering upward (ω_t^u) and downward (ω_t^d) flexibility products, to avoid such behaviour, restriction (II.1) is defined.

$$\omega_t^u \cdot \omega_t^d = 0, \quad \forall t \in \Omega_t \quad (\text{II.1})$$

In this context, aiming to obtain a linear restriction of the previous non-linear one, McCormick envelopes could be used [73]. McCormick envelopes are a type of convex relaxation used in bilinear Non-Linear Programming, solving relaxed problem will provide a lower bound number on the optimal solution of the original problem. Then, checking feasibility and solving original problem using relaxed solution as starting point it will give global optimal point.

Let $z = x \cdot y$ with decision variables $x, y \in \mathbb{R}$, if continuous decision variables have upper and lower bounds defined as $x_L \leq x \leq x_U$ and $y_L \leq y \leq y_U$, then underestimators of the function are represented by equations (II.2) and (II.3), while overestimators are represented by equations (II.4) and (II.5).

$$z \geq x_L y + x y_L - x_L y_L \quad (\text{II.2})$$

$$z \geq x_U y + x y_U - x_U y_U \quad (\text{II.3})$$

$$z \leq x_U y + x y_L - x_U y_L \quad (\text{II.4})$$

$$z \leq x y_U + x_L y - x_L y_U \quad (\text{II.5})$$

Figure 76 shows a graphical representation of the McCormick envelope showing blue area as overestimators and orange area as underestimators.

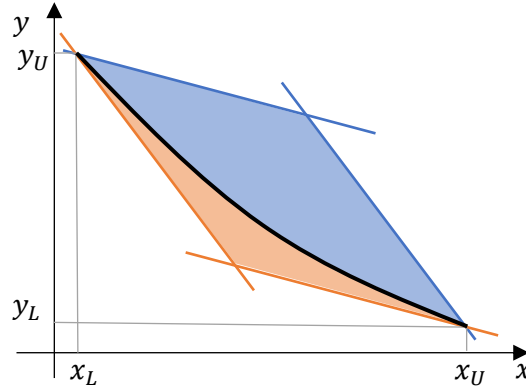


Figure 76. Representation of the McCormick Envelope. Blue area represents convex overestimators and orange are convex underestimators, black line represents product of variables x and y .

At this moment, considering restriction of the equation (II.1), it can be relaxed by means of using McCormick envelope. Considering $0 \leq \omega_t^u \leq \bar{\omega}_t^u$ and $0 \leq \omega_t^d \leq \bar{\omega}_t^d$, underestimators leads to statements $0 \leq \bar{\omega}_t^u \omega_t^d$ and $0 \leq \omega_t^u \bar{\omega}_t^d$ what do not represent any more further lower limits of variables. For overestimators, one equation leads to triviality $0 = 0$, while the other give the convex linearization of the restriction as shows equation (II.6) that can be evaluated for flexible assets and storage systems.

$$0 \geq \bar{\omega}_t^u \omega_t^d + \omega_t^u \bar{\omega}_t^d - \bar{\omega}_t^u \bar{\omega}_t^d \quad (\text{II.6})$$

However, this approach is just an approximation of the non-linear restriction, and simultaneous upward and downward flexibility activation may occur. Thus, this linearization, although efficient because of the scant number of additional restrictions needed, it provides a low-quality approximation if the feasible region of the variables linearized are sufficiently wide.

2.2. Tangent lines approach

Another mathematical approach to approximate NLP restriction is to define a set of tangent lines to the NLP restriction. This method was firstly discovered by Newton, and their applications has been widely used in almost every single mathematical field.

Then, to effectively linearize NLP restrictions defined in (II.1), firstly it is necessary to convert them into quadratic ones. To this aim, a variable change will be used. After that, n tangent lines are used to linearize quadratic restrictions. Non-linear restriction is expressed in term of upward ($\omega_{x,t}^{c,u}$) and downward ($\omega_{x,t}^{c,d}$) directions for a given asset x . Firstly, variable change is satisfied considering $y_{x,t}^{c,u} = 1/2 \cdot (\omega_{x,t}^{c,u} + \omega_{x,t}^{c,d})$ and $y_{x,t}^{c,d} = 1/2 \cdot (\omega_{x,t}^{c,u} - \omega_{x,t}^{c,d})$, resulting equation (II.7).

$$(y_{x,t}^{c,u})^2 - (y_{x,t}^{c,d})^2 = 0 \quad \forall x, \forall t \quad (\text{II.7})$$

As can be noted, upward and downward variables are now separated, and restriction can be relaxed to a set of n regions defined by n tangent lines to the quadratic function, Figure 77 represents. To this end, an auxiliary variable z is defined for each y variable.

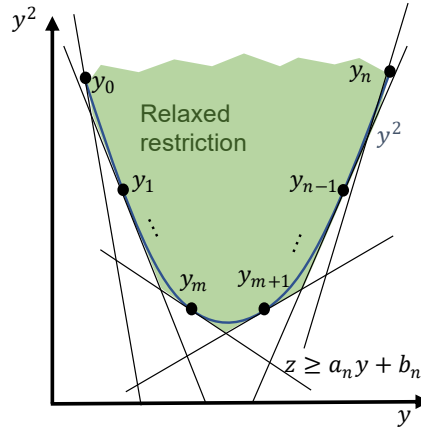


Figure 77. Relaxation of the convex quadratic restrictions obtained after a variable change over non-linear restriction.

Tangent lines are defined with n linear coefficients a_n , as the slope, and b_n , as the independent term. Relaxed feasible region has the form defined in equation (II.8).

$$z_{x,t} \geq a_n \cdot y_{x,t} + b_n \quad (\text{II.8})$$

$z_{x,t}$ variables must be minimized in the objective function also to ensure convergence on the lower boundary, approximating quadratic restriction and, consequently, satisfying non-linear restriction as Figure 77 shows.

3. Formulation of the problem

In this section, the formulation of the problem proposed for the IEEE Conference is explained. Likewise the use case of this thesis, it was considered the presence of FLs, FGs, BESSs managed by Aggs to solve DSO problems such as imbalances and congestions, for the conference.

It was only considered the centralized version of the problem, then, LMO figure is presented, and it oversees the market, clearing it and sending the results to the agents involved, as previous Figure 8 stated. Then, Aggs will send bids, while DSO sends asks to the LMO who is responsible for the clearing of the market and the following control of the agreements made in the market.

The remain of the section is organized as follows, the first subsection will define the flexibility products, the second one, the restrictions of the associated problem and the third and last one defines the objective function of the LFM.

3.1. Flexibility products definition

In this section, flexibility products are defined. Before the LFM, agents have bought and sold energy at the marginal price S_t^s in the wholesale market. In this situation, they have no incentive to participate in a LFM if no benefit is present. However, DSO can offer a trade-off, incentivizing trading among prosumers. Thus, loads will reduce its consumption if they receive an offer of $S_t^{e,d} > S_t^s$ for their products and will increase it if the ask involves energy at a price $S_t^{e,u} < S_t^s$. Same reasoning can be done for BESSs. Nevertheless, for FGs, offering only curtailment of its generation, as their downward direction is equivalent to an increase in the consumption, they bid at a price $S_{g,t}^{e,d} < S_t^s$.

FLs are assumed to freely modify their consumption in a limited range in both directions. Cost for procuring those products is detailed in equations (II.9) and (II.10) for each FL. These equations consider the wholesale energy price showing costs incurred by DSO when purchasing flexibility products.

$$C_{f,t}^e = (S_t^s - S_{f,t}^{e,u}) \cdot \omega_{f,t}^{e,u} + (S_{f,t}^{e,d} - S_t^s) \cdot \omega_{f,t}^{e,d} \quad \forall t, \forall f \quad (\text{II.9})$$

$$C_{f,t}^c = S_{f,t}^{c,u} \cdot \omega_{f,t}^{c,u} + S_{f,t}^{c,d} \cdot \omega_{f,t}^{c,d} \quad \forall t, \forall f \quad (\text{II.10})$$

Energy and capacity products are related as no asset can trade energy products far away than previously set capacity product. Then, $\omega_{f,t}^{e,u} \leq \omega_{f,t}^{c,u} \cdot \Delta t$ and $\omega_{f,t}^{e,d} \leq \omega_{f,t}^{c,d} \cdot \Delta t$ for all $t > t_1$. It is important, also, to clarify that simultaneous trade of upward and downward products are not allowed. For that reason, non-linear restriction (II.11) is established. Moreover, since energy products are dependent of capacity, no simultaneity is also achieved. Proposed restriction ensures that only one of the directions is activated, as the other necessarily needs to be zero to satisfy equation (II.11).

$$\omega_{f,t}^{c,u} \cdot \omega_{f,t}^{c,d} = 0 \quad \forall t, \forall f \quad (\text{II.11})$$

For FGs, PV is the main option when considering DERs in distribution networks. As a result, FGs are assumed to only provide flexibility in downward direction. They can only curtail their production; in any case can they increase it. Costs for flexibility procurement of energy and capacity products are shown in equations (II.12) and (II.13).

$$C_{g,t}^e = (S_t^s - S_{g,t}^{e,d}) \cdot \omega_{g,t}^{e,d} \quad \forall t, \forall g \quad (\text{II.12})$$

$$C_{g,t}^c = S_{g,t}^{c,d} \cdot \omega_{g,t}^{c,d} \quad \forall t, \forall g \quad (\text{II.13})$$

Also, FGs capacity and energy products are related considering $\omega_{g,t}^{e,d} \leq \omega_{g,t}^{c,d} \cdot \Delta t$ for all FGs.

BESSs are key elements for procuring flexibility as are the only participants with the ability of storage energy. Costs of those products are detailed in equations (II.14) and (II.15).

$$C_{s,t}^e = (S_t^s - S_{s,t}^{e,u}) \cdot \omega_{s,t}^{e,u} + (S_{s,t}^{e,d} - S_t^s) \cdot \omega_{s,t}^{e,d} \quad \forall t, \forall s \quad (\text{II.14})$$

$$C_{s,t}^c = S_{s,t}^{c,u} \cdot \omega_{s,t}^{c,u} + S_{s,t}^{c,d} \cdot \omega_{s,t}^{c,d} \quad \forall t, \forall s \quad (\text{II.15})$$

Consistently with previous presented products, BESSs energy and capacity products are related between each other considering $\omega_{s,t}^{e,u} \leq \omega_{s,t}^{c,u} \cdot \Delta t$ and $\omega_{s,t}^{e,d} \leq \omega_{s,t}^{c,d} \cdot \Delta t$ for all BESSs. Moreover, as a matter of fact, BESSs do only store energy, in consequence, their net balance is zero if a sufficient long period is considered. Thus, SOC is assumed to have the same value in the first and last period, $\text{soc}_{s,t_1} = \text{soc}_{s,|T|}$. In the case of BESSs, simultaneous activation of upward and downward directions is not allowed defining restriction (II.16).

$$\omega_{s,t}^{c,u} \cdot \omega_{s,t}^{c,d} = 0, \quad \forall t, \forall s \quad (\text{II.16})$$

Aggs manage assets in a specific area, collecting their flexibility and bidding in the LFM. Costs for energy and capacity products for an Agg is described in equations (II.17) and (II.18), respectively.

$$C_{a,t}^e = \sum_{f \in a} C_{f,t}^e + \sum_{g \in a} C_{g,t}^e + \sum_{s \in a} C_{s,t}^e \quad \forall t, \forall a \quad (\text{II.17})$$

$$C_{a,t}^c = \sum_{f \in a} C_{f,t}^c + \sum_{g \in a} C_{g,t}^c + \sum_{s \in a} C_{s,t}^c \quad \forall t, \forall a \quad (\text{II.18})$$

3.2. Restrictions

Balance restriction is defined considering programmed wholesale market schedule and the injections after clearing LFM. Aftermarket power consumption is defined considering energy products traded as $p_{f,t}^{AM} = P_{f,t}^{PR} + 1/\Delta t \cdot (\omega_{f,t}^{e,u} - \omega_{f,t}^{e,d})$ for all FLs, $p_{s,t}^{AM} = 1/\Delta t \cdot (\omega_{s,t}^{e,u} - \omega_{s,t}^{e,d})$ for all BESSs and $p_{g,t}^{AM} = P_{g,t}^{PR} - 1/\Delta t \cdot \omega_{g,t}^{e,d}$ for all FGs. Then, equation (II.19) states power equality after the market clearing, preventing deviations from the original schedule and ensuring solving issues locally. This restriction represent DSO ask for balance to the local flexibility market.

$$\sum_l P_{l,t}^{PR} - \sum_x P_{x,t}^{PR} = \sum_l P_{l,t}^{AM} - \sum_x P_{x,t}^{AM} \quad \forall t \quad (II.19)$$

On the other hand, BESSs charging and discharging efficiencies (η_s^C, η_s^D) are also considered. For the sake of simplicity, they are set constant equals to 95%. Then, SOC restrictions for energy market ($soC_{s,t}$) are considered.

$$soC_{s,t} = soC_{s,t-1} + \left(\eta_s^C \omega_{s,t}^{e,u} - \frac{1}{\eta_s^D} \omega_{s,t}^{e,d} \right) \quad \forall s \quad \forall t \quad (II.20)$$

$$\underline{SOC}_s \leq soC_{s,t} \leq \overline{SOC}_s \quad \forall s, \forall t \quad (II.21)$$

Output power extracted from battery cannot be greater than the nominal power of the inverter (P_s^{INV}) connected to. Then, $\omega_{s,t}^{c,u} \leq P_s^{INV}$ and $\omega_{s,t}^{c,d} \leq P_s^{INV}$ for all BESSs.

Network restrictions should also be described. Grid will be modelled using a linear approach, considering that no voltage issues are present. Linear modelling considers all node voltages remain stable at voltage basis ($V_i = 1$), and that grid losses are negligible ($G_{ij} \approx 0$). Then, network constraints for linear modelling of the distribution network are presented in equations (II.22) and (II.23).

$$\sum_{j \in i} B_{ij}(\theta_{i,t} - \theta_{j,t}) = p_{slk,t} - P_{l,t}^{AM} + P_{x,t}^{AM} \quad \forall i \quad \forall t \quad (II.22)$$

$$-\bar{P}_{ij} \leq B_{ij}(\theta_{i,t} - \theta_{j,t}) \leq \bar{P}_{ij} \quad \forall i, j \in i \quad \forall t \quad (II.23)$$

3.3. Objective function

The objective function includes two terms, the first one is the total cost of procuring flexibility from the aggregators to the DSO, and second ensures converge in the lower bound of the linearized restrictions as equation (II.24) estates. $C_{a,t}^e$ represents both flexibility direction of energy products, and $C_{a,t+1}^c$ also for capacity products.

$$\min_{vars} \sum_a \sum_t (C_{a,t}^e + C_{a,t+1}^c) + \sum_x \sum_t (z_{x,t}^{c,u} + z_{x,t}^{c,d}) \quad (II.24)$$

4. Simulations

In this section simulations done to demonstrate the validity of the approach are shown. Two different scenarios are considered under the network presented in Figure 28, which involves 5 microgrids connected using transmission system. It is considered that the assets of each microgrid are managed by a different Agg. Then, Agg 1 will manage the asset downstream node 000, Agg 2 those present downstream node 200, and so on up to Agg 5.

4.1. Imbalance scenario

The first scenario considers the situation of an imbalance created by the disconnection of SL 75 (node 848). To demonstrate the effectiveness of the proposed LFM to solve imbalances, Figure 78 is presented. SL 75 has a near constant consumption profile of product-oriented industries. Disconnection is solved using all presented products.

Positive values of the figure axis represent upward direction of the products, and negatives, downward. As batteries are urged to maintain its SOC at the end of the day, they charge while FGs can cover their consumption and sell downward products rest of the time.

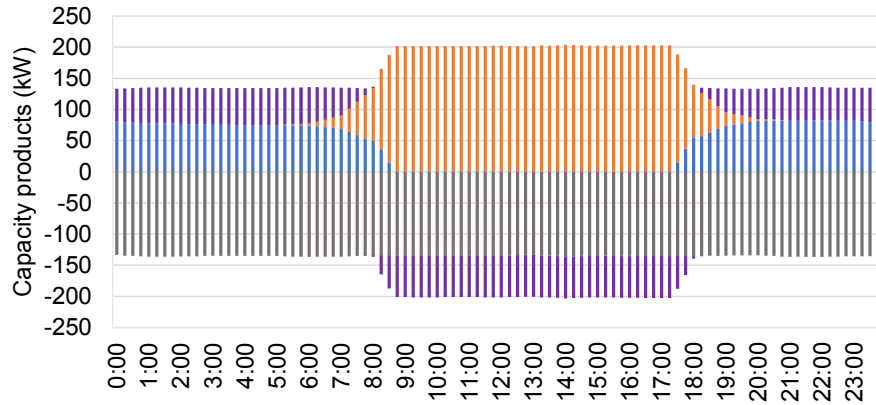


Figure 78. FLs (blue), FGs (orange), BESSs (purple) capacity products traded in the LFM to solve SL75 (grey) imbalance.

An hourly distribution of the costs is presented in Figure 79. Total costs amount to 166€ for a total of 4,491.2 kWh of energy products traded and 17,964.8 kW of capacity products pre-allocated. In this scenario, Aggs share of products is presented in Figure 80. Agg 5 stand out with most of the trade of FGs downward flexibility products, rest of products are similarly shared among Aggs.

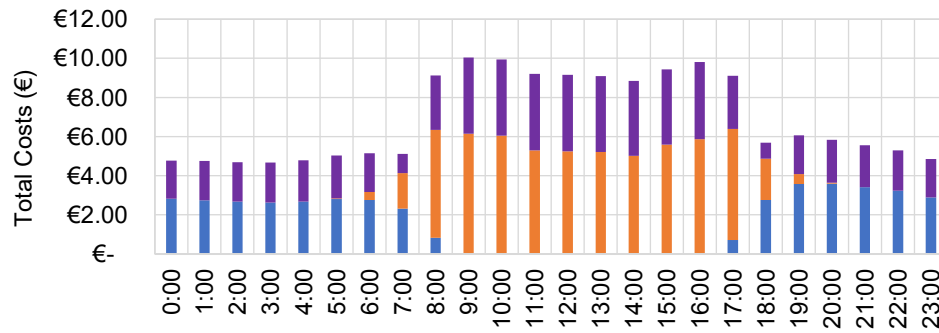


Figure 79. FLs (blue), FGs (orange), BESSs (purple) hourly cost for flexibility dispatch in the imbalance scenario. Total FLs costs: 42.66€; Total FGs costs: 60.33€; Total BESSs costs: 63.01€. Quantity traded: 4,491.2 kWh.

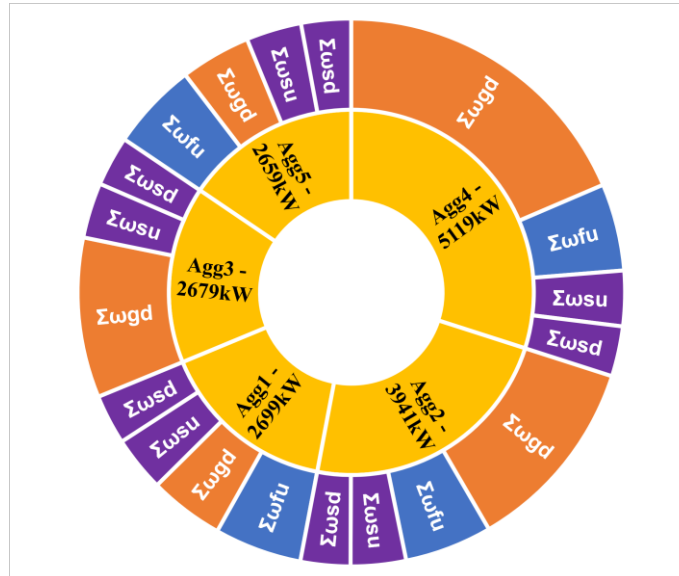


Figure 80. Distribution of the products traded in the market for each aggregator in the imbalance solving scenario. Blue, orange, and purple segments represent FLs, FGs, and BESSs, respectively.

By comparing the total quantity traded in both directions, it is possible to observe that equation (II.19) is satisfied. Note that FGs only trade downward products, but, as they are injecting power to the system, it can be assimilated to a FLs offering products in the upward direction. As a result, $\omega_{g,t}^{c,d}$ and $\omega_{g,t}^{e,d}$ are represented in the positive semi-axis of the figures.

A sensitivity analysis of the influence of the number of tangent lines (n) is also performed. Figure 81 assess on different error and time metrics of the linearized solution. Although all error metrics decrease with n , average, median and standard deviation parameters tend to stabilize when reaching 150 lines after 173.21 s of computation.

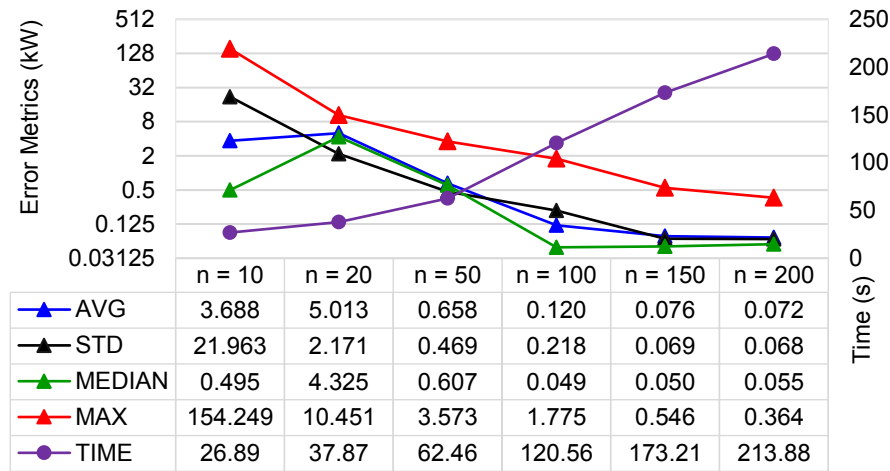


Figure 81. Metrics of the error (kW) and computational time (s) considering different quantity of tangent lines (n) for approximate non-linear restrictions in the imbalance scenario.

4.2. Congestion scenario

The third scenario considers a congestion in the line 400-402. The associated power flow considering the baseload together with the profile after clearing the LFM is represented in Figure 82. It can be noted that the profile of the power flow through line 400-402 after market clearing is slightly higher than baseload. This is due to the BESSs charging processes during the part of the day with no congestion issues to offer downward products when line is near its thermal limit.

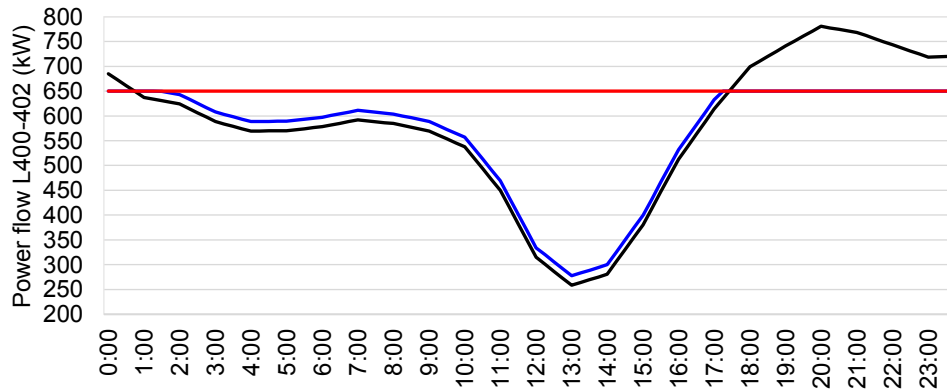


Figure 82. Aftermarket (blue), Forecasted (black) Power Flow through line 400-402 and its Thermal limit (red).

Figure 83 represents energy (lines) and capacity (bars) products traded to solve the congestion in the market. It can be noted that they follow the same profile that capacity products, as all pre-allocated capacity is consumed as energy ($\omega_{x,t}^e = \omega_{x,t}^c \cdot \Delta t$).

The results minimize the total cost of acquiring flexibility in the LFM. As mentioned before, batteries of feeder downstream node 400 is charged between 1:00 and 17:00, while is discharged in the rest of the day. However, to maintain balance of the network, other Aggs offers downward products to globally equilibrate consumption. Only in this way, congestion is fully solved using local assets and products. As congestion is produced mainly at night-time hours, FGs cannot offer as much products as the other assets. Figure 84 represents a distribution of the hourly cost for congestion solving. Total costs amount to 103.85 € for a total of 1,871.2 kWh of energy products and 7,484.8 kW of pre-allocated capacity products.

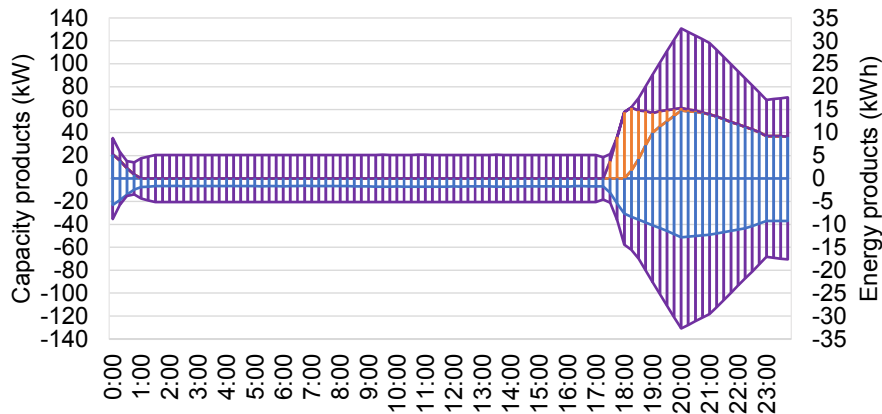


Figure 83. Energy (line) and capacity (bar) products traded in the congestion solving scenario. Blue, orange, and purple bars and line stands for FLs, FGs and BESSs, respectively.

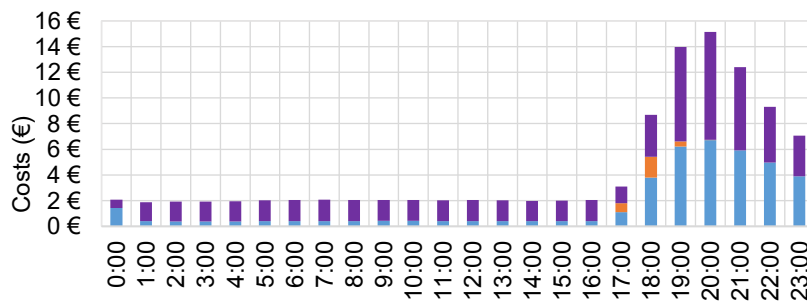


Figure 84. Hourly distribution of the costs for congestion scenario. Total FLs costs: 40.45€; Total FGs costs: 2.74€; Total BESSs costs: 60.66€. Quantity traded: 1,871.2 kWh.

Sensitivity study of the number of tangent lines (n) affecting the overall precision of the linearization is also conducted for the congestion solving scenario. Figure 85 represents the metrics of the remaining imbalance presented after the LFM clearing as well as the computational time metric. Similarly to the imbalance scenario, median, average and standard deviation of the error stabilizes after $n = 150$ with a total of 160.98 s of calculation time. As a result, no further computational effort is needed, due to the remaining imbalance maximum is 0.852 kW over 7,484.8 kW.

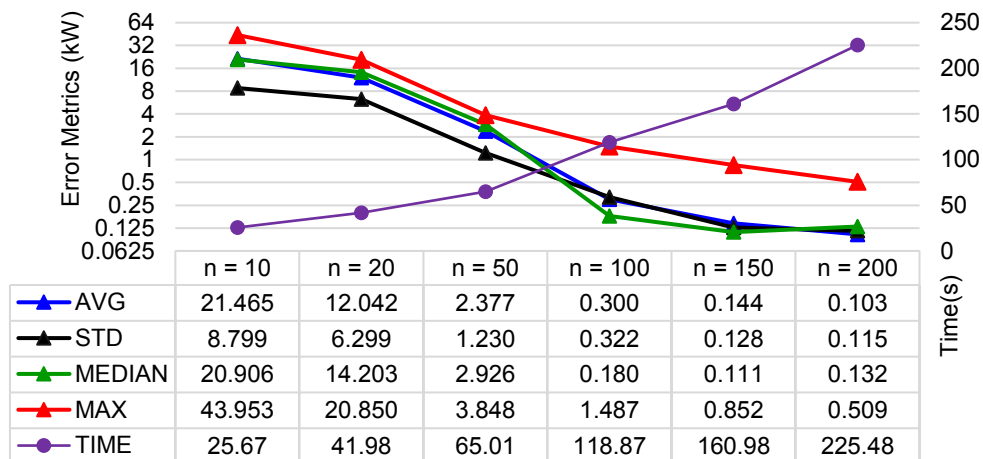


Figure 85. Metrics of the error (kW) and computational time (s) considering different quantity of tangent lines (n) for approximate non-linear restrictions in the congestion scenario.

Agg 3 manages feeder where congestion takes place and, as a result, in Figure 86 can be noticed that half of the trading is carried out by that Agg, accumulating 3,828 kW of capacity products traded. Besides of that, it is perceived that congestion is mainly solved using BESSs products as previously mentioned, as most of the products traded by Agg 3 are from them. FLs also play an important role for solving congestion

as its downward direction is highly traded in the market. FGs represent a near imperceptible amount, but it is indispensable to achieve economic efficiency.

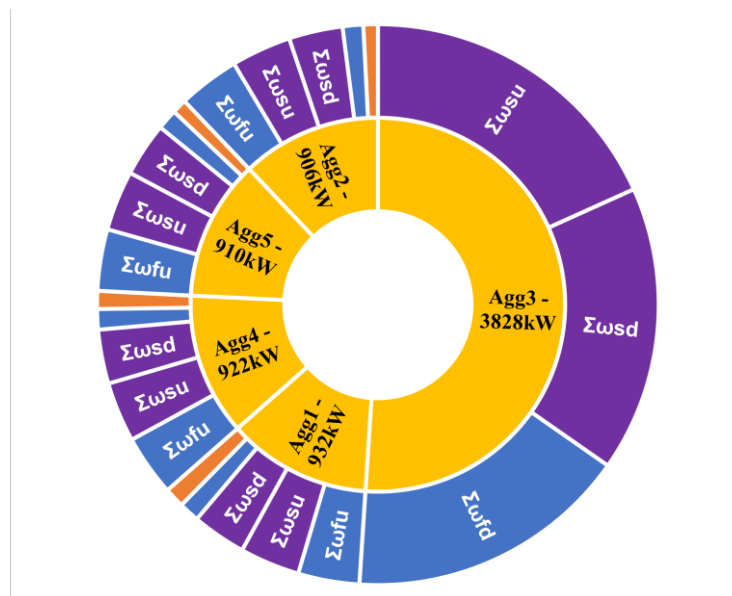


Figure 86. Distribution of capacity products traded in the market for each aggregator in the congestion scenario. Blue, orange, and purple segments represent FLs, FGs, and BESSs, respectively

ANNEX III: GAMS CODE FOR THE CENTRALIZED USE CASE

In the present Annex is presented the code used for simulating the use case presented in Chapter 3. The software used for modelling is GAMS.

SETS

```
i  "Nodes" /0, 2, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30,
32, 34, 36, 38, 40, 42, 44, 46, 48, 54, 56, 58, 60, 62, 64, 90, 100, 200, 202, 206,
208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240,
242, 244, 246, 248, 254, 256, 258, 260, 262, 264, 290, 300, 400, 402, 406, 408, 410,
412, 414, 416, 418, 420, 422, 424, 426, 428, 430, 432, 434, 436, 438, 440, 442, 444,
446, 448, 454, 456, 458, 460, 462, 464, 490, 500, 600, 602, 606, 608, 610, 612, 614,
616, 618, 620, 622, 624, 626, 628, 630, 632, 634, 636, 638, 640, 642, 644, 646, 648,
654, 656, 658, 660, 662, 664, 690, 700, 800, 802, 806, 808, 810, 812, 814, 816, 818,
820, 822, 824, 826, 828, 830, 832, 834, 836, 838, 840, 842, 844, 846, 848, 854, 856,
858, 860, 862, 864, 890, 900, 1000/
```

```
l  "Static loads" /l1*175/
```

```
g  "generators" /g1*g20/
```

```
s  "Storages systems" /s1*s20/
```

```
f  "flexible loads" /f1*f20/
```

```
t  "time variable" /t1*t96/
```

```
a  "aggregators" /a1*a5/
```

```
slack(i) "Slack bus" /100/
```

```
load_connect(i,l) "connection between nodes and end_users"
```

```
$call.gdxrw.exe data.xlsx set=load_connect rng = load_connect!
```

```
$gdxin data.gdx
```

```
$load load_connect
```

```
$gdxin
```

```
flex_load_connect(i,f) "connection between nodes and flexible loads"
```

```
$call.gdxrw.exe data.xlsx set=flex_load_connect rng = flex_load_connect!
```

```
$gdxin data.gdx
```

```
$load flex_load_connect
```

```
$gdxin
```

```
gen_connect(i,g) "connection between nodes and generators"
```

```
$call.gdxrw.exe data.xlsx set=gen_connect rng = gen_connect!
```

```
$gdxin data.gdx
```

```
$load gen_connect
$gdxin

    storage_connect(i,s) "connection between nodes and storage systems"
$call.gdxrw.exe data.xlsx set=storage_connect rng = storage_connect!
$gdxin data.gdx
$load storage_connect
$gdxin

    agg_flex(a,f) "Flexible loads managed by aggregator a"
$call.gdxrw.exe data.xlsx set=agg_flex rng = agg_flex!
$gdxin data.gdx
$load agg_flex
$gdxin

    agg_gen(a,g) "Flexible generators managed by generator g"
$call.gdxrw.exe data.xlsx set=agg_gen rng = agg_gen!
$gdxin data.gdx
$load agg_gen
$gdxin

    agg_storage(a,s) "Storage systems managed by storage system s"
$call.gdxrw.exe data.xlsx set=agg_storage rng = agg_storage!
$gdxin data.gdx
$load agg_storage
$gdxin
;

* nodes can be called as i or j
alias (i,j);

set conex (i,j) "bus connectivity matrix";

SCALARS
    sbase "Power basis kVA" /1000/
    vbase "Voltage basis V" /22000/
    zbase "Impedance basis Ohm"
    DELTA_T "time between clearings" /0.25/
;

TABLE branch(i, j, *)
$call =xls2gms r=net! i=data.xlsx o=data.inc
$include data.inc
;

PARAMETERS
    load(l,t,*)
    flex_load(f,t,*)
    gen(g,t,*)
    storage(s,*)
    storage_prices(s,t,*)
    sigma_s(t,*)
```

```
ST(t)
SFEU(f,t)
SFCU(f,t)
SFED(f,t)
SFCD(f,t)
SGED(g,t)
SGCD(g,t)
PGSCH(g,t)
SSEU(s,t)
SSCU(s,t)
SSED(s,t)
SSCD(s,t)
PFSCH(f,t)
SOC_0(s)
P_s_INV(s)
ETA_u(s)
ETA_d(s)
SOC_u(s)
SOC_d(s)
PLAM(l,t)
PLSCH(l,t)
B(i,j)
PMAX(i,j)
;

* loading parameters from excel file
$call gdxrw.exe data.xlsx par = load rng = load! Cdim=1 Rdim=2
$gdxin data.gdx
$load load
$gdxin

$call gdxrw.exe data.xlsx par = flex_load rng = flex_load! Cdim=1 Rdim=2
$gdxin data.gdx
$load flex_load
$gdxin

$call gdxrw.exe data.xlsx par = storage_prices rng = storage_prices! Cdim=1 Rdim=2
$gdxin data.gdx
$load storage_prices
$gdxin

$call gdxrw.exe data.xlsx par = gen rng = gen! Cdim=1 Rdim=2
$gdxin data.gdx
$load gen
$gdxin

$call gdxrw.exe data.xlsx par = storage rng = storage!
$gdxin data.gdx
$load storage
$gdxin

$call gdxrw.exe data.xlsx par = sigma_s rng = sigma_s!
$gdxin data.gdx
```

```

$load sigma_s
$gdxin

* Computing parameters with the parameters loaded from excel
zbase = vbase*vbase/sbase;

* connectivity matrix is symmetric
conex(i,j) $ (branch(i,j,'x')) = 1;
conex(i,j) $ (conex(j, i)) = 1 ;

* Symmetric network characteristic matrix.
branch (i, j, 'x') $ (branch(i, j, 'x') = 0) = branch (j, i, 'x');
branch (i, j, 'x') = branch(i, j, 'x')/zbase;
branch (i, j, 'r') $ (branch(i, j, 'r') = 0) = branch (j, i, 'r');
branch (i, j, 'r') = branch(i, j, 'r')/zbase;
branch (i, j, 'bs') $ (branch(i, j, 'bs') = 0) = branch (j, i, 'bs');
branch (i, j, 'bs') = branch(i, j, 'bs')*zbase;
branch(i, j, 'P_max') $(branch(i, j, 'P_max') = 0) = branch (j, i, 'P_max');

* Admittance matrix calculus
branch (i, j, 'Bij') $ (conex(i, j)) = branch(i, j, 'x')/(power(branch(i, j, 'x'),2)
+ power(branch(i, j, 'r'),2));
branch (i, i, 'Bij') = sum(j $ conex(j, i), branch(i, j, 'bs')/2 - branch(i, j,
'x')/(power(branch(i, j, 'x'),2) + power(branch(i, j, 'r'),2)));
branch (i, j, 'Gij') $ (conex(i, j)) = - branch(i, j, 'r')/(power(branch(i, j,
'x'),2) + power(branch(i, j, 'r'),2));
branch (i, i, 'Gij') = sum(j $ conex(j, i), branch(i, j, 'r')/(power(branch(i, j,
'x'),2) + power(branch(i, j, 'r'),2)));

* Power lines parameters
B(i,j) = branch(i,j, 'Bij');
PMAX(i,j) = branch(i,j, 'P_max');

* SLs parameters
PLAM(l,t) = load(l,t, 'PLAM');
PLSCH(l,t) = load(l,t, 'PLSCH');

* FLs parameters
SFEU(f,t) = flex_load(f,t, 'SFEU');
SFCU(f,t) = flex_load(f,t, 'SFCU');
SFED(f,t) = flex_load(f,t, 'SFED');
SFCD(f,t) = flex_load(f,t, 'SFCD');
PFSCH(f,t) = flex_load(f,t, 'PFSCH');

* FGs parameters
SGED(g,t) = gen(g,t, 'SGED');
SGCD(g,t) = gen(g,t, 'SGCD');
PGSCH(g,t) = gen(g,t, 'PGSCH');

* SSs parameters
SSEU(s,t) = storage_prices(s,t, 'SSEU');
SSCU(s,t) = storage_prices(s,t, 'SSCU');

```

```
SSED(s,t) = storage_prices(s,t,'SSED');
SSCD(s,t) = storage_prices(s,t,'SSCD');
SOC_0(s) = storage(s,'SOC_0');
P_s_INV(s) = storage(s,'P_s_INV');
ETA_u(s) = storage(s,'ETA_u');
ETA_d(s) = storage(s,'ETA_d');
SOC_u(s) = storage(s,'SOC_u');
SOC_d(s) = storage(s,'SOC_d');
```

```
* S_t
```

```
ST(t) = sigma_s(t,'S_t_s');
```

```
* Free variables definition
```

```
variables
```

```
    z
    p(i,j,t)
    pslk(slack,t)
    theta(i,t)
    psam(s,t)
;
```

```
* Positive variables definition
```

```
positive variables
```

```
    cfe(f,t)
    cfc(f,t)
    wfeu(f,t)
    wfcu(f,t)
    wfed(f,t)
    wfcd(f,t)
    pfam(f,t)
    cge(g,t)
    cgc(g,t)
    wged(g,t)
    wgcd(g,t)
    pgam(g,t)
    cse(s,t)
    csc(s,t)
    socs(s,t)
    wseu(s,t)
    wscu(s,t)
    wsed(s,t)
    wscd(s,t)
;
```

```
* Equations definition
```

```
equations
```

```
    OBJ
    R01
    R02
    R03
    R04
    R05
    R06
```

R07
R08
R09
R10
R11
R12
R13
R14
R15
R16
R17
R18

;

OBJ.. $z = E = \text{sum}((f,t), cfe(f,t) + cfc(f,t+1) \$ (\text{ord}(t) < \text{card}(t)))$
+ $\text{sum}((g,t), cge(g,t) + cgc(g,t+1) \$ (\text{ord}(t) < \text{card}(t))) + \text{sum}((s,t), cse(s,t) +$
 $csc(s,t+1) \$ (\text{ord}(t) < \text{card}(t)))$;

R01(f,t).. $cfe(f,t)*sbase = E = (ST(t) - SFEU(f,t))*wfeu(f,t) +$
 $(SFED(f,t) - ST(t))*wfed(f,t)$;

R02(f,t).. $cfc(f,t)*sbase = E = SFCU(f,t)*wfcu(f,t) +$
 $SFCD(f,t)*wfcd(f,t)$;

R03(f,t).. $wfeu(f,t) = L = wfcu(f,t)*DELTA_T$;

R04(f,t).. $wfed(f,t) = L = wfcd(f,t)*DELTA_T$;

R05(f,t).. $pfam(f,t) = E = PFSCCH(f,t) + 1/DELTA_T*(wfeu(f,t) -$
 $wfed(f,t))$;

R06(i,t).. $\text{sum}(j, B(i,j) * (\text{theta}(i,t) - \text{theta}(j,t)))*sbase = E = +$
 $\text{sum}(\text{slack}(i), \text{pslk}(\text{slack},t)) + \text{sum}(g \$ \text{gen_connect}(i,g), \text{pgam}(g,t)) - \text{sum}(l \$$
 $\text{load_connect}(i,l), \text{PLAM}(l,t)) - \text{sum}(f \$ \text{flex_load_connect}(i,f), \text{pfam}(f,t)) - \text{sum}(s \$$
 $\text{storage_connect}(i,s), \text{psam}(s,t))$;

R07(i,j,t) \$ conex(i,j).. $p(i,j,t) = E = B(i,j) * (\text{theta}(i,t) - \text{theta}(j,t))$;

R08(g,t).. $cge(g,t)*sbase = E = (ST(t) - SGED(g,t))*wged(g,t)$;

R09(g,t).. $cgc(g,t)*sbase = E = SGCD(g,t)*wgcd(g,t)$;

R10(g,t).. $wged(g,t) = L = wgcd(g,t)*DELTA_T$;

R11(g,t).. $\text{pgam}(g,t) = E = \text{PGSCH}(g,t) - wged(g,t)/DELTA_T$;

R12(t).. $\text{sum}(l, \text{PLAM}(l,t)) + \text{sum}(f, \text{pfam}(f,t)) - \text{sum}(g, \text{pgam}(g,t))$
+ $\text{sum}(s, \text{psam}(s,t)) = E = \text{sum}(l, \text{PLSCH}(l,t)) + \text{sum}(f, \text{PFSCCH}(f,t)) - \text{sum}(g, \text{PGSCH}(g,t))$;

R13(s,t).. $cse(s,t)*sbase = E = (ST(t) - SSEU(s,t))*wseu(s,t) +$
 $(SSED(s,t) - ST(t))*wsed(s,t)$;

```

R14(s,t)..          csc(s,t)*sbase =E= SSCU(s,t)*wscu(s,t) +
SSCD(s,t)*wscd(s,t);

R15(s,t)..          wseu(s,t) =L= wscu(s,t)*DELTA_T;

R16(s,t)..          wsed(s,t) =L= wscd(s,t)*DELTA_T;

R17(s,t)..          psam(s,t) =E= 1/DELTA_T*(wseu(s,t) - wsed(s,t));

R18(s,t)..          socs(s,t) =E= SOC_0(s) $ (ord(t) = 1) + socs(s,t-1) $
(ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);

* Upper and lower limit for power after market clearing in FLs
pfam.lo(f,t) = 0.70*PFSCH(f,t);
pfam.up(f,t) = 1.30*PFSCH(f,t);

* lower limit for power after market clearing in FGs
pgam.up(g,t) = PGSCH(g,t);

* Upper and lower limit for power after market clearing in SSs. State of Charge
limits.
psam.lo(s,t) = - P_s_INV(s);
psam.up(s,t) = P_s_INV(s);
socs.lo(s,t) = SOC_d(s);
socs.up(s,t) = SOC_u(s);

* * POWER LIMIT IN SCENARIO
* p.up('814','816',t) = 150;
* p.lo('814','816',t) = -150;
* p.up('816','814',t) = 150;
* p.lo('816','814',t) = -150;

* Flat state at beggining and slack node set as angle reference
theta.l(i,t) = 0;
theta.fx(slack,t) = 0;

socs.fx(s,'t96') = SOC_0(s);

*****
***** MODELLING *****
*****

* Modelling and solving market
model market /all/;

*****
***** SOLVERS *****
*****

* Solving using Non-Linear Programming
solve market minimizing z using LP;

```

```
*****
***** REPORT PARAMETERS OF A GIVEN OPTIMIZATION PROBLEM *****
*****
```

PARAMETERS

```
    report_branches(i,j,t,*)
    report_network(i,t,*)
    report_costs(t,*)
    report_agg(a,t,*)
    report_FLs(f,t,*)
    report_FGs(g,t,*)
    report_SSs(s,t,*)
;

report_branches(i,j,t,'Pijt') = p.l(i,j,t);
report_branches(i,j,t,'PMAX') = PMAX(i,j);

report_network(i,t,'theta') = theta.l(i,t);
report_network(i,t,'injection') = sum(j, p.l(i,j,t));
report_network(i,t,'marg_balance') = R06.m(i,t)*1e3;

report_costs(t,'Sum_cfet') = sum(f, cfe.l(f,t));
report_costs(t,'Sum_cfct') = sum(f, cfc.l(f,t));
report_costs(t,'Sum_cset') = sum(s, cse.l(s,t));
report_costs(t,'Sum_csct') = sum(s, csc.l(s,t));
report_costs(t,'Totals_e') = sum(f, cfe.l(f,t)) + sum(s, cse.l(s,t));
report_costs(t,'Totals_c') = sum(f, cfc.l(f,t)) + sum(s, csc.l(s,t));
report_costs(t,'Totals') = report_costs(t,'Totals_e') + report_costs(t,'Totals_c');

report_agg(a,t,'caet') = (sum(f, cfe.l(f,t) $ agg_flex(a,f)) + sum(g, cge.l(g,t) $
agg_gen(a,g)) + sum(s, cse.l(s,t) $ agg_storage(a,s)))/sbase;
report_agg(a,t,'cact') = (sum(f, cfc.l(f,t) $ agg_flex(a,f)) + sum(g, cgc.l(g,t) $
agg_gen(a,g)) + sum(s, csc.l(s,t) $ agg_storage(a,s)))/sbase;
report_agg(a,t,'wfeu_a') = sum(f $ agg_flex(a,f),wfeu.l(f,t));
report_agg(a,t,'wfed_a') = sum(f $ agg_flex(a,f),wfed.l(f,t));
report_agg(a,t,'wged_a') = sum(g $ agg_gen(a,g),wged.l(g,t));
report_agg(a,t,'wseu_a') = sum(s $ agg_storage(a,s),wseu.l(s,t));
report_agg(a,t,'wsed_a') = sum(s $ agg_storage(a,s),wsed.l(s,t));
report_agg(a,t,'waeu') = report_agg(a,t,'wfeu_a') + report_agg(a,t,'wged_a') +
report_agg(a,t,'wseu_a');
report_agg(a,t,'waed') = report_agg(a,t,'wfed_a') + report_agg(a,t,'wsed_a');
report_agg(a,t,'wfcu_a') = sum(f $ agg_flex(a,f), wfcu.l(f,t));
report_agg(a,t,'wfgd_a') = sum(f $ agg_flex(a,f), wfgd.l(f,t));
report_agg(a,t,'wgcd_a') = sum(g $ agg_gen(a,g), wgcd.l(g,t));
report_agg(a,t,'wscu_a') = sum(s $ agg_storage(a,s), wscu.l(s,t));
report_agg(a,t,'wscd_a') = sum(s $ agg_storage(a,s), wscd.l(s,t));
report_agg(a,t,'wacu') = report_agg(a,t,'wfcu_a') + report_agg(a,t,'wgcd_a') +
report_agg(a,t,'wscu_a');
report_agg(a,t,'wacd') = report_agg(a,t,'wfgd_a') + report_agg(a,t,'wscd_a');
```

```
report_FLs(f,t,'cfet') = cfe.1(f,t);  
report_FLs(f,t,'cfct') = cfc.1(f,t);  
report_FLs(f,t,'wfed') = wfed.1(f,t);  
report_FLs(f,t,'wfcd') = wfcd.1(f,t);  
report_FLs(f,t,'wfeu') = wfeu.1(f,t);  
report_FLs(f,t,'wfcu') = wfcu.1(f,t);  
report_FLs(f,t,'SFED') = SFED(f,t);  
report_FLs(f,t,'SFCD') = SFCD(f,t);  
report_FLs(f,t,'SFEU') = SFEU(f,t);  
report_FLs(f,t,'SFCU') = SFCU(f,t);  
report_FLs(f,t,'pfam') = pfam.1(f,t);  
report_FLs(f,t,'PFSCH') = PFSCH(f,t);
```

```
report_FGs(g,t,'cget') = cge.1(g,t);  
report_FGs(g,t,'cgct') = cgc.1(g,t);  
report_FGs(g,t,'wged') = wged.1(g,t);  
report_FGs(g,t,'wgcd') = wgcd.1(g,t);  
report_FGs(g,t,'PGSCH') = PGSCH(g,t);  
report_FGs(g,t,'pgam') = pgam.1(g,t);  
report_FGs(g,t,'SGED') = SGED(g,t);  
report_FGs(g,t,'SGCD') = SGCD(g,t);
```

```
report_SSs(s,t,'cse') = cse.1(s,t);  
report_SSs(s,t,'csc') = csc.1(s,t);  
report_SSs(s,t,'SSEU') = SSEU(s,t);  
report_SSs(s,t,'SSCU') = SSCU(s,t);  
report_SSs(s,t,'SSED') = SSED(s,t);  
report_SSs(s,t,'SSCD') = SSCD(s,t);  
report_SSs(s,t,'wseu') = wseu.1(s,t);  
report_SSs(s,t,'wscu') = wscu.1(s,t);  
report_SSs(s,t,'wsed') = wsed.1(s,t);  
report_SSs(s,t,'wscd') = wscd.1(s,t);  
report_SSs(s,t,'psam') = psam.1(s,t);  
report_SSs(s,t,'P_s_INV') = P_s_INV(s);  
report_SSs(s,t,'socs') = socs.1(s,t);  
report_SSs(s,t,'SOC_0') = SOC_0(s);  
report_SSs(s,t,'SOC_u') = SOC_u(s);  
report_SSs(s,t,'SOC_d') = SOC_d(s);
```

```
execute_unload 'report.gdx' report_branches  
execute 'gdxrw.exe report.gdx o=report_centralized.xlsx par = report_branches rng =  
Branches!'  
execute_unload 'report.gdx' report_network  
execute 'gdxrw.exe report.gdx o=report_centralized.xlsx par = report_network rng =  
Network!'  
execute_unload 'report.gdx' report_FLs  
execute 'gdxrw.exe report.gdx o=report_centralized.xlsx par = report_FLs rng = FLs!'  
execute_unload 'report.gdx' report_FGs  
execute 'gdxrw.exe report.gdx o=report_centralized.xlsx par = report_FGs rng = FGs!'  
execute_unload 'report.gdx' report_SSs  
execute 'gdxrw.exe report.gdx o=report_centralized.xlsx par = report_SSs rng = SSs!'  
execute_unload 'report.gdx' report_agg
```

```
execute 'gdxxrw.exe report.gdx o=report_centralized.xlsx par = report_agg rng = Agg!'  
execute_unload 'report.gdx' report_costs  
execute 'gdxxrw.exe report.gdx o=report_centralized.xlsx par = report_costs rng =  
Costs!'
```

```
* Delete unuseful files
```

```
execute 'rm -f data.gdx';  
execute 'rm -f data.inc';  
execute 'rm -f report.gdx';
```

ANNEX IV: GAMS CODE FOR THE DECENTRALIZED USE CASE

In this annex is presented the GAMS code used for the simulation of the decentralized version of the use case presented in Chapter 4.

SETS

```
i  "Nodes"                /0, 2, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30,
32, 34, 36, 38, 40, 42, 44, 46, 48, 54, 56, 58, 60, 62, 64, 90, 100, 200, 202, 206,
208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240,
242, 244, 246, 248, 254, 256, 258, 260, 262, 264, 290, 300, 400, 402, 406, 408, 410,
412, 414, 416, 418, 420, 422, 424, 426, 428, 430, 432, 434, 436, 438, 440, 442, 444,
446, 448, 454, 456, 458, 460, 462, 464, 490, 500, 600, 602, 606, 608, 610, 612, 614,
616, 618, 620, 622, 624, 626, 628, 630, 632, 634, 636, 638, 640, 642, 644, 646, 648,
654, 656, 658, 660, 662, 664, 690, 700, 800, 802, 806, 808, 810, 812, 814, 816, 818,
820, 822, 824, 826, 828, 830, 832, 834, 836, 838, 840, 842, 844, 846, 848, 854, 856,
858, 860, 862, 864, 890, 900, 1000/
l  "Static loads"         /l1*l175/
g  "generators"           /g1*g20/
s  "Storages systems"     /s1*s20/
f  "flexible loads"       /f1*f20/
t  "time variable"        /t1*t96/
a  "aggregators"         /a1*a5/
i_a(i) /400, 402, 406, 408, 410, 412, 414, 416, 418, 420, 422, 424, 426, 428,
430, 432, 434, 436, 438, 440, 442, 444, 446, 448, 454, 456, 458, 460, 462, 464, 490/
i_b(i) /200, 202, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228,
230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 254, 256, 258, 260, 262, 264, 290/
i_c(i) /0, 2, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38,
40, 42, 44, 46, 48, 54, 56, 58, 60, 62, 64, 90/
i_d(i) /800, 802, 806, 808, 810, 812, 814, 816, 818, 820, 822, 824, 826, 828,
830, 832, 834, 836, 838, 840, 842, 844, 846, 848, 854, 856, 858, 860, 862, 864, 890/
i_e(i) /600, 602, 606, 608, 610, 612, 614, 616, 618, 620, 622, 624, 626, 628,
630, 632, 634, 636, 638, 640, 642, 644, 646, 648, 654, 656, 658, 660, 662, 664, 690/
i_t(i) /100, 300, 500, 700, 900, 1000/
k  "iters" /k1*k100/
```

```
load_connect(i,l) "connection between nodes and end_users"
```

```
$call gdxrw.exe data.xlsx set=load_connect rng = load_connect!
$gdxin data.gdx
$load load_connect
$gdxin

    flex_load_connect(i,f) "connection between nodes and flexible loads"
$call gdxrw.exe data.xlsx set=flex_load_connect rng = flex_load_connect!
$gdxin data.gdx
$load flex_load_connect
$gdxin

    gen_connect(i,g) "connection between nodes and generators"
$call gdxrw.exe data.xlsx set=gen_connect rng = gen_connect!
$gdxin data.gdx
$load gen_connect
$gdxin

    storage_connect(i,s) "connection between nodes and storage systems"
$call gdxrw.exe data.xlsx set=storage_connect rng = storage_connect!
$gdxin data.gdx
$load storage_connect
$gdxin

    agg_flex(a,f) "Flexible loads managed by aggregator a"
$call gdxrw.exe data.xlsx set=agg_flex rng = agg_flex!
$gdxin data.gdx
$load agg_flex
$gdxin

    agg_gen(a,g) "Flexible generators managed by generator g"
$call gdxrw.exe data.xlsx set=agg_gen rng = agg_gen!
$gdxin data.gdx
$load agg_gen
$gdxin

    agg_storage(a,s) "Storage systems managed by storage system s"
$call gdxrw.exe data.xlsx set=agg_storage rng = agg_storage!
$gdxin data.gdx
$load agg_storage
$gdxin
;

* nodes can be called as i or j
alias (i,j);
alias (i_a,j_a);
alias (i_b,j_b);
alias (i_c,j_c);
alias (i_d,j_d);
alias (i_e,j_e);
alias (i_t,j_t);
```

```
set conex (i,j) "bus connectivity matrix";
```

SCALARS

```
sbase "Power basis kVA" /1000/
vbase "Voltage basis V" /22000/
zbase "Impedance basis Ohm"
DELTA_T "Delta time between time slots" /0.25/
GAMMA "penalty factor" /5e-4/
;
```

```
TABLE branch(i, j, *)
$call =xls2gms r=net! i=data.xlsx o=data.inc
$include data.inc
;
```

PARAMETERS

```
load(l,t,*)
flex_load(f,t,*)
gen(g,t,*)
storage(s,*)
storage_prices(s,t,*)
sigma_s(t,*)
ST(t)
SFEU(f,t)
SFCU(f,t)
SFED(f,t)
SFCD(f,t)
SGED(g,t)
SGCD(g,t)
PGSCH(g,t)
SSEU(s,t)
SSCU(s,t)
SSED(s,t)
SSCD(s,t)
PFSCH(f,t)
SOC_0(s)
P_s_INV(s)
ETA_u(s)
ETA_d(s)
SOC_u(s)
SOC_d(s)
PLAM(l,t)
PLSCH(l,t)
B(i,j)
PMAX(i,j)
;
```

PARAMETERS

```
* Y
```

* power exchanged between areas

p_AT_k(t)
p_BT_k(t)
p_CT_k(t)
p_DT_k(t)
p_ET_k(t)

p_TA_k(t)
p_TB_k(t)
p_TC_k(t)
p_TD_k(t)
p_TE_k(t)

* Fictitious nodes between areas (theta coordination)

theta_xAT_k(t)
theta_xBT_k(t)
theta_xCT_k(t)
theta_xDT_k(t)
theta_xET_k(t)

theta_xTA_k(t)
theta_xTB_k(t)
theta_xTC_k(t)
theta_xTD_k(t)
theta_xTE_k(t)

* X

PFAM_k(f,t)
PGAM_k(g,t)
PSAM_k(s,t)

* LAMBDA

* Multipliers associated to the power btw areas

LAMBDA_A_k(t)
LAMBDA_B_k(t)
LAMBDA_C_k(t)
LAMBDA_D_k(t)
LAMBDA_E_k(t)

* Multipliers associated to the voltage angle in the fictitious node

LAMBDA_theta_xA_k(t)
LAMBDA_theta_xB_k(t)
LAMBDA_theta_xC_k(t)
LAMBDA_theta_xD_k(t)
LAMBDA_theta_xE_k(t)

* Multiplier associated to the power balance of the grid

LAMBDA_BALANCE_k(t)

;

* loading parameters from excel file

```
$call gdxrw.exe data.xlsx par = load rng = load! Cdim=1 Rdim=2
$gdxin data.gdx
$load load
$gdxin
```

```
$call gdxrw.exe data.xlsx par = flex_load rng = flex_load! Cdim=1 Rdim=2
$gdxin data.gdx
$load flex_load
$gdxin
```

```
$call gdxrw.exe data.xlsx par = storage_prices rng = storage_prices! Cdim=1 Rdim=2
$gdxin data.gdx
$load storage_prices
$gdxin
```

```
$call gdxrw.exe data.xlsx par = gen rng = gen! Cdim=1 Rdim=2
$gdxin data.gdx
$load gen
$gdxin
```

```
$call gdxrw.exe data.xlsx par = storage rng = storage!
$gdxin data.gdx
$load storage
$gdxin
```

```
$call gdxrw.exe data.xlsx par = sigma_s rng = sigma_s!
$gdxin data.gdx
$load sigma_s
$gdxin
```

* Computing parameters with the parameters loaded from excel

```
zbase = vbase*vbase/sbase;
```

* connectivity matrix is symmetric

```
conex(i,j) $(branch(i,j,'x')) = 1;
conex(i,j) $ (conex(j, i)) = 1 ;
```

* Symmetric network characteristic matrix.

```
branch (i, j, 'x') $ (branch(i, j, 'x') = 0) = branch (j, i, 'x');
branch (i, j, 'x') = branch(i, j, 'x')/zbase;
branch (i, j, 'r') $ (branch(i, j, 'r') = 0) = branch (j, i, 'r');
branch (i, j, 'r') = branch(i, j, 'r')/zbase;
branch (i, j, 'bs') $ (branch(i, j, 'bs') = 0) = branch (j, i, 'bs');
branch (i, j, 'bs') = branch(i, j, 'bs')*zbase;
branch(i, j, 'P_max') $(branch(i, j, 'P_max') = 0) = branch (j, i, 'P_max');
```

* Admittance matrix calculus

```
branch (i, j, 'Bij') $ (conex(i, j)) = branch(i, j, 'x') /(power(branch(i, j, 'x'),2)
+ power(branch(i, j, 'r'),2));
```

```
branch(i, i, 'Bij') = sum(j $ conex(j, i), branch(i, j, 'bs')/2 - branch(i, j,
'x')/(power(branch(i, j, 'x'),2) + power(branch(i, j, 'r'),2)));
branch(i, j, 'Gij') $ (conex(i, j)) = - branch(i, j, 'r')/(power(branch(i, j,
'x'),2) + power(branch(i, j, 'r'),2));
branch(i, i, 'Gij') = sum(j $ conex(j, i), branch(i, j, 'r')/(power(branch(i, j,
'x'),2) + power(branch(i, j, 'r'),2)));
```

*** Power line parameters**

```
B(i,j) = branch(i,j, 'Bij');
PMAX(i,j) = branch(i,j, 'P_max');
```

*** SLs parameters**

```
PLAM(l,t) = load(l,t, 'PLAM');
PLSCH(l,t) = load(l,t, 'PLSCH');
```

*** FLs parameters**

```
SFEU(f,t) = flex_load(f,t, 'SFEU');
SFCU(f,t) = flex_load(f,t, 'SFCU');
SFED(f,t) = flex_load(f,t, 'SFED');
SFCD(f,t) = flex_load(f,t, 'SFCD');
PFSCCH(f,t) = flex_load(f,t, 'PFSCCH');
```

*** FGs parameters**

```
SGED(g,t) = gen(g,t, 'SGED');
SGCD(g,t) = gen(g,t, 'SGCD');
PGSCH(g,t) = gen(g,t, 'PGSCH');
```

*** SSs parameters**

```
SSEU(s,t) = storage_prices(s,t, 'SSEU');
SSCU(s,t) = storage_prices(s,t, 'SSCU');
SSED(s,t) = storage_prices(s,t, 'SSED');
SSCD(s,t) = storage_prices(s,t, 'SSCD');
SOC_0(s) = storage(s, 'SOC_0');
P_s_INV(s) = storage(s, 'P_s_INV');
ETA_u(s) = storage(s, 'ETA_u');
ETA_d(s) = storage(s, 'ETA_d');
SOC_u(s) = storage(s, 'SOC_u');
SOC_d(s) = storage(s, 'SOC_d');
```

*** S_t (Wholesale electricity prices)**

```
ST(t) = sigma_s(t, 'S_t_s');
```

*** Initialization of the parameters**

```
PFAM_k(f,t) = PFSCCH(f,t);
PGAM_k(g,t) = PGSCH(g,t);
PSAM_k(s,t) = 0;
```

variables

*** Objective variables for each area sub-problem**

za
zb
zc
zd
ze
zt

* Power between lines

p(i,j,t)

* Power exchanged in the slack node (upstream node 100)

p₁₀₀(t)

* Voltage angle definition for each area

theta_a(i_a,t)
theta_b(i_b,t)
theta_c(i_c,t)
theta_d(i_d,t)
theta_e(i_e,t)
theta_t(i_t,t)

* Power consumed/injected by BESSs

psam(s,t)

* Power exchanged btw areas.

* (AT means power flow from x to A in line connecting A-T,

* TA means power flow from x to T in line connecting A-T)

p_AT(t)
p_BT(t)
p_CT(t)
p_DT(t)
p_ET(t)

p_TA(t)
p_TB(t)
p_TC(t)
p_TD(t)
p_TE(t)

* Voltage angle in the fictitious nodes.

* (xAT means theta_x of line A-T passed to area A,

* xTA means theta_x of line T-A passed to area T)

theta_xAT(t)
theta_xBT(t)
theta_xCT(t)
theta_xDT(t)
theta_xET(t)

theta_xTA(t)
theta_xTB(t)

```

theta_xTC(t)
theta_xTD(t)
theta_xTE(t)
;

positive variables
* Costs for FLs
  cfe(f,t)
  cfc(f,t)
* Products for FLs
  wfeu(f,t)
  wfcu(f,t)
  wfed(f,t)
  wfcd(f,t)
* Power demanded by FLs
  pfam(f,t)

* Costs for FGs
  cge(g,t)
  cgc(g,t)
* Products for FGs
  wged(g,t)
  wgcd(g,t)
* Power injected by FGs
  pgam(g,t)

* Costs for BESSs
  cse(s,t)
  csc(s,t)
* State of charge (SOC) for BESSs
  socs(s,t)
* Products for BESSs
  wseu(s,t)
  wscu(s,t)
  wsed(s,t)
  wscd(s,t)
;

* Equations definition
equations
  OBJA, OBJB, OBJC, OBJD, OBJE, OBJT,
  R01A, R01B, R01C, R01D, R01E, R01T,
  R02A, R02B, R02C, R02D, R02E, R02T,
  R03A, R03B, R03C, R03D, R03E, R03T,
  R04A, R04B, R04C, R04D, R04E, R04T,
  R05A, R05B, R05C, R05D, R05E, R05T,
  R06A, R06B, R06C, R06D, R06E, R06T,
  R07A, R07B, R07C, R07D, R07E, R07T,
  R08A, R08B, R08C, R08D, R08E,
  R09A, R09B, R09C, R09D, R09E,

```

```

R10A, R10B, R10C, R10D, R10E,
R11A, R11B, R11C, R11D, R11E,
R12A, R12B, R12C, R12D, R12E,
R13A, R13B, R13C, R13D, R13E,
R14A, R14B, R14C, R14D, R14E,
R15A, R15B, R15C, R15D, R15E,
R16A, R16B, R16C, R16D, R16E,
R17A, R17B, R17C, R17D, R17E,
R18A, R18B, R18C, R18D, R18E,
R19A, R19B, R19C, R19D, R19E
;

* * L_a(x,z,y) = f(x) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2 (ADMM formulation of
the restriction Ax+Bzk-c [NOT INCLUDED IN THE CONSTRAINTS BODY])
OBJA.. za =E= + sum((f,t), (cfe(f,t) + cfc(f,t+1)) $ sum(i_a,
flex_load_connect(i_a,f)))
+ sum((g,t), (cge(g,t) + cgc(g,t+1)) $ sum(i_a, gen_connect(i_a,g)))
+ sum((s,t), (cse(s,t) + csc(s,t+1)) $ sum(i_a,
storage_connect(i_a,s)))
+ sum(t,
+ LAMBDA_A_k(t)*(p_AT(t) - p_TA_k(t))/sbase
)
+ sum(t,
+ LAMBDA_BALANCE_k(t)*(
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_a,
flex_load_connect(i_a,f)) - PFAM_k(f,t) $ (not sum(i_a, flex_load_connect(i_a,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_a,
gen_connect(i_a,g)) - PGAM_k(g,t) $ (not sum(i_a, gen_connect(i_a,g))))
+ sum(s, - psam(s,t) $ sum(i_a, storage_connect(i_a,s)) -
PSAM_k(s,t) $ (not sum(i_a, storage_connect(i_a,s))))
)/sbase
)
+ sum(t,
+ LAMBDA_theta_xA_k(t)*(theta_xAT(t) - theta_xTA_k(t))
)
+ GAMMA/2*sum(t,
+ power((p_AT(t) - p_TA_k(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power((
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_a,
flex_load_connect(i_a,f)) - PFAM_k(f,t) $ (not sum(i_a, flex_load_connect(i_a,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_a,
gen_connect(i_a,g)) - PGAM_k(g,t) $ (not sum(i_a, gen_connect(i_a,g))))
+ sum(s, - psam(s,t) $ sum(i_a, storage_connect(i_a,s)) -
PSAM_k(s,t) $ (not sum(i_a, storage_connect(i_a,s))))
)/sbase,2)
)

```

```
+ GAMMA/2*sum(t,
+ power(theta_xAT(t) - theta_xTA_k(t),2)
);
```

```
R01A(f,t) $ sum(i_a, flex_load_connect(i_a,f)).. cfe(f,t)*sbase =E= (ST(t) -
SFEU(f,t))*wfeu(f,t) + (SFED(f,t) - ST(t))*wfed(f,t);
```

```
R02A(f,t) $ sum(i_a, flex_load_connect(i_a,f)).. cfc(f,t)*sbase =E=
SFCU(f,t)*wfcu(f,t) + SFCD(f,t)*wfcd(f,t);
```

```
R03A(f,t) $ sum(i_a, flex_load_connect(i_a,f)).. wfeu(f,t) =L= wfcu(f,t)*DELTA_T;
```

```
R04A(f,t) $ sum(i_a, flex_load_connect(i_a,f)).. wfed(f,t) =L= wfcd(f,t)*DELTA_T;
```

```
R05A(f,t) $ sum(i_a, flex_load_connect(i_a,f)).. pfam(f,t) =E= PFSCH(f,t) +
1/DELTA_T*(wfeu(f,t)-wfed(f,t));
```

```
R06A(i_a,t).. sum(j_a, B(i_a,j_a) *
(theta_a(i_a,t) - theta_a(j_a,t)))*sbase =E= p_AT(t) $ (ord(i_a) = 1) + sum(g $
gen_connect(i_a,g), pgam(g,t))
- sum(l $ load_connect(i_a,l),
PLAM(l,t)) - sum(f $ flex_load_connect(i_a,f), pfam(f,t)) - sum(s $
storage_connect(i_a,s), psam(s,t));
```

```
R07A(i_a,j_a,t) $ conex(i_a,j_a).. p(i_a,j_a,t) =E= B(i_a,j_a) *
(theta_a(i_a,t) - theta_a(j_a,t));
```

```
R08A(t).. p_AT(t) =E=
2*B('400','1000')*(theta_xAT(t) - theta_a('400',t));
```

```
R09A(s,t) $ sum(i_a, storage_connect(i_a,s)).. wscu(s,t) * wscd(s,t) =E= 0;
```

```
R10A(g,t) $ sum(i_a, gen_connect(i_a,g)).. cge(g,t)*sbase =E= (ST(t) -
SGED(g,t))*wged(g,t);
```

```
R11A(g,t) $ sum(i_a, gen_connect(i_a,g)).. cgc(g,t)*sbase =E=
SGCD(g,t)*wgcd(g,t);
```

```
R12A(g,t) $ sum(i_a, gen_connect(i_a,g)).. wged(g,t) =L= wgcd(g,t)*DELTA_T;
```

```
R13A(g,t) $ sum(i_a, gen_connect(i_a,g)).. pgam(g,t) =E= PGSCH(g,t) -
wged(g,t)/DELTA_T;
```

```
R14A(s,t) $ sum(i_a, storage_connect(i_a,s)).. cse(s,t)*sbase =E= (ST(t) -
SSEU(s,t))*wseu(s,t) + (SSED(s,t) - ST(t))*wsed(s,t);
```

```
R15A(s,t) $ sum(i_a, storage_connect(i_a,s)).. csc(s,t)*sbase =E=
SSCU(s,t)*wscu(s,t) + SSCD(s,t)*wscd(s,t);
```

```

R16A(s,t) $ sum(i_a, storage_connect(i_a,s))..      wseu(s,t) =L= wscu(s,t)*DELTA_T;

R17A(s,t) $ sum(i_a, storage_connect(i_a,s))..      wsed(s,t) =L= wscd(s,t)*DELTA_T;

R18A(s,t) $ sum(i_a, storage_connect(i_a,s))..      psam(s,t) =E=
1/DELTA_T*(wseu(s,t) - wsed(s,t));

R19A(s,t) $ sum(i_a, storage_connect(i_a,s))..      socs(s,t) =E= SOC_0(s) $ (ord(t)
= 1) + socs(s,t-1) $ (ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);

* * L_a(x,z,y) = g(z) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2
OBJB.. zb =E= + sum((f,t), (cfe(f,t) + cfc(f,t+1)) $ sum(i_b,
flex_load_connect(i_b,f)))
+ sum((g,t), (cge(g,t) + cgc(g,t+1)) $ sum(i_b, gen_connect(i_b,g)))
+ sum((s,t), (cse(s,t) + csc(s,t+1)) $ sum(i_b,
storage_connect(i_b,s)))
+ sum(t,
+ LAMBDA_B_k(t)*(p_BT(t) - p_TB_k(t))/sbase
)
+ sum(t,
+ LAMBDA_BALANCE_k(t)*(
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_b,
flex_load_connect(i_b,f)) - PFAM_k(f,t) $ (not sum(i_b, flex_load_connect(i_b,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_b,
gen_connect(i_b,g)) - PGAM_k(g,t) $ (not sum(i_b, gen_connect(i_b,g))))
+ sum(s, - psam(s,t) $ sum(i_b, storage_connect(i_b,s)) -
PSAM_k(s,t) $ (not sum(i_b, storage_connect(i_b,s))))
)/sbase
)
+ sum(t,
+ LAMBDA_theta_xB_k(t)*(theta_xBT(t) - theta_xTB_k(t))
)
+ GAMMA/2*sum(t,
+ power((p_BT(t) - p_TB_k(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power((
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_b,
flex_load_connect(i_b,f)) - PFAM_k(f,t) $ (not sum(i_b, flex_load_connect(i_b,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_b,
gen_connect(i_b,g)) - PGAM_k(g,t) $ (not sum(i_b, gen_connect(i_b,g))))
+ sum(s, - psam(s,t) $ sum(i_b, storage_connect(i_b,s)) -
PSAM_k(s,t) $ (not sum(i_b, storage_connect(i_b,s))))
)/sbase,2)
)
+ GAMMA/2*sum(t,

```

```
+ power(theta_xBT(t) - theta_xTB_k(t),2)
);
```

```
R01B(f,t) $ sum(i_b, flex_load_connect(i_b,f))..      cfe(f,t)*sbase =E= (ST(t) -
SFEU(f,t))*wfeu(f,t) + (SFED(f,t) - ST(t))*wfed(f,t);
```

```
R02B(f,t) $ sum(i_b, flex_load_connect(i_b,f))..      cfc(f,t)*sbase =E=
SFCU(f,t)*wfcu(f,t) + SFCD(f,t)*wfcd(f,t);
```

```
R03B(f,t) $ sum(i_b, flex_load_connect(i_b,f))..      wfeu(f,t) =L= wfcu(f,t)*DELTA_T;
```

```
R04B(f,t) $ sum(i_b, flex_load_connect(i_b,f))..      wfed(f,t) =L= wfcd(f,t)*DELTA_T;
```

```
R05B(f,t) $ sum(i_b, flex_load_connect(i_b,f))..      pfam(f,t) =E= PFSCH(f,t) +
1/DELTA_T*(wfeu(f,t)-wfed(f,t));
```

```
R06B(i_b,t)..                                          sum(j_b, B(i_b,j_b) *
(theta_b(i_b,t) - theta_b(j_b,t)))*sbase =E= p_BT(t) $ (ord(i_b) = 1) + sum(g $
gen_connect(i_b,g), pgam(g,t))
- sum(l $ load_connect(i_b,l),
PLAM(l,t)) - sum(f $ flex_load_connect(i_b,f), pfam(f,t)) - sum(s $
storage_connect(i_b,s), psam(s,t));
```

```
R07B(i_b,j_b,t) $ conex(i_b,j_b)..                  p(i_b,j_b,t) =E= B(i_b,j_b) *
(theta_b(i_b,t) - theta_b(j_b,t));
```

```
R08B(t)..                                              p_BT(t) =E=
2*B('200','900')*(theta_xBT(t) - theta_b('200',t));
```

```
R09B(s,t) $ sum(i_b, storage_connect(i_b,s))..      wscu(s,t) * wscd(s,t) =E= 0;
```

```
R10B(g,t) $ sum(i_b, gen_connect(i_b,g))..          cge(g,t)*sbase =E= (ST(t) -
SGED(g,t))*wged(g,t);
```

```
R11B(g,t) $ sum(i_b, gen_connect(i_b,g))..          cgc(g,t)*sbase =E=
SGCD(g,t)*wgcd(g,t);
```

```
R12B(g,t) $ sum(i_b, gen_connect(i_b,g))..          wged(g,t) =L= wgcd(g,t)*DELTA_T;
```

```
R13B(g,t) $ sum(i_b, gen_connect(i_b,g))..          pgam(g,t) =E= PGSCH(g,t) -
wged(g,t)/DELTA_T;
```

```
R14B(s,t) $ sum(i_b, storage_connect(i_b,s))..      cse(s,t)*sbase =E= (ST(t) -
SSEU(s,t))*wseu(s,t) + (SSED(s,t) - ST(t))*wsed(s,t);
```

```
R15B(s,t) $ sum(i_b, storage_connect(i_b,s))..      csc(s,t)*sbase =E=
SSCU(s,t)*wscu(s,t) + SSCD(s,t)*wscd(s,t);
```

```
R16B(s,t) $ sum(i_b, storage_connect(i_b,s))..      wseu(s,t) =L= wscu(s,t)*DELTA_T;
```

```

R17B(s,t) $ sum(i_b, storage_connect(i_b,s))..      wsed(s,t) =L= wscd(s,t)*DELTA_T;

R18B(s,t) $ sum(i_b, storage_connect(i_b,s))..      psam(s,t) =E=
1/DELTA_T*(wseu(s,t) - wsed(s,t));

R19B(s,t) $ sum(i_b, storage_connect(i_b,s))..      socs(s,t) =E= SOC_0(s) $ (ord(t)
= 1) + socs(s,t-1) $ (ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);

* * L_a(x,z,y) = g(z) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2
OBJC.. zc =E= + sum((f,t), (cfe(f,t) + cfc(f,t+1)) $
sum(i_c,flex_load_connect(i_c,f)))
+ sum((g,t), (cge(g,t) + cgc(g,t+1)) $ sum(i_c,gen_connect(i_c,g)))
+ sum((s,t), (cse(s,t) + csc(s,t+1)) $ sum(i_c,
storage_connect(i_c,s)))
+ sum(t,
+ LAMBDA_C_k(t)*(p_CT(t) - p_TC_k(t))/sbase
)
+ sum(t,
+ LAMBDA_BALANCE_k(t)*(
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_c,
flex_load_connect(i_c,f)) - PFAM_k(f,t) $ (not sum(i_c, flex_load_connect(i_c,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_c,
gen_connect(i_c,g)) - PGAM_k(g,t) $ (not sum(i_c, gen_connect(i_c,g))))
+ sum(s, - psam(s,t) $ sum(i_c, storage_connect(i_c,s)) -
PSAM_k(s,t) $ (not sum(i_c, storage_connect(i_c,s))))
)/sbase
)
+ sum(t,
+ LAMBDA_theta_xC_k(t)*(theta_xCT(t) - theta_xTC_k(t))
)
+ GAMMA/2*sum(t,
+ power((p_CT(t) - p_TC_k(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power((
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_c,
flex_load_connect(i_c,f)) - PFAM_k(f,t) $ (not sum(i_c, flex_load_connect(i_c,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_c,
gen_connect(i_c,g)) - PGAM_k(g,t) $ (not sum(i_c, gen_connect(i_c,g))))
+ sum(s, - psam(s,t) $ sum(i_c, storage_connect(i_c,s)) -
PSAM_k(s,t) $ (not sum(i_c, storage_connect(i_c,s))))
)/sbase,2)
)
+ GAMMA/2*sum(t,
+ power(theta_xCT(t) - theta_xTC_k(t),2)
);

```

```

R01C(f,t) $ sum(i_c, flex_load_connect(i_c,f))..      cfe(f,t)*sbase =E= (ST(t) -
SFEU(f,t))*wfeu(f,t) + (SFED(f,t) - ST(t))*wfed(f,t);

R02C(f,t) $ sum(i_c, flex_load_connect(i_c,f))..      cfc(f,t)*sbase =E=
SFCU(f,t)*wfcu(f,t) + SFCD(f,t)*wfcd(f,t);

R03C(f,t) $ sum(i_c, flex_load_connect(i_c,f))..      wfeu(f,t) =L= wfcu(f,t)*DELTA_T;

R04C(f,t) $ sum(i_c, flex_load_connect(i_c,f))..      wfed(f,t) =L= wfcd(f,t)*DELTA_T;

R05C(f,t) $ sum(i_c, flex_load_connect(i_c,f))..      pfam(f,t) =E= PFSCH(f,t) +
1/DELTA_T*(wfeu(f,t)-wfed(f,t));

R06C(i_c,t)..                                          sum(j_c, B(i_c,j_c) *
(theta_c(i_c,t) - theta_c(j_c,t)))*sbase =E= p_CT(t) $ (ord(i_c) = 1) + sum(g $
gen_connect(i_c,g), pgam(g,t))
                                                    - sum(l $ load_connect(i_c,l),
PLAM(l,t)) - sum(f $ flex_load_connect(i_c,f), pfam(f,t)) - sum(s $
storage_connect(i_c,s), psam(s,t));

R07C(i_c,j_c,t) $ conex(i_c,j_c)..                  p(i_c,j_c,t) =E= B(i_c,j_c) *
(theta_c(i_c,t) - theta_c(j_c,t));

R08C(t)..                                              p_CT(t) =E=
2*B('0','700')*(theta_xCT(t) - theta_c('0',t));

R09C(s,t) $ sum(i_c, storage_connect(i_c,s))..      wscu(s,t) * wscd(s,t) =E= 0;

R10C(g,t) $ sum(i_c, gen_connect(i_c,g))..          cge(g,t)*sbase =E= (ST(t) -
SGED(g,t))*wged(g,t);

R11C(g,t) $ sum(i_c, gen_connect(i_c,g))..          cgc(g,t)*sbase =E=
SGCD(g,t)*wgcd(g,t);

R12C(g,t) $ sum(i_c, gen_connect(i_c,g))..          wged(g,t) =L= wgcd(g,t)*DELTA_T;

R13C(g,t) $ sum(i_c, gen_connect(i_c,g))..          pgam(g,t) =E= PGSCH(g,t) -
wged(g,t)/DELTA_T;

R14C(s,t) $ sum(i_c, storage_connect(i_c,s))..      cse(s,t)*sbase =E= (ST(t) -
SSEU(s,t))*wseu(s,t) + (SSED(s,t) - ST(t))*wsed(s,t);

R15C(s,t) $ sum(i_c, storage_connect(i_c,s))..      csc(s,t)*sbase =E=
SSCU(s,t)*wscu(s,t) + SSCD(s,t)*wscd(s,t);

R16C(s,t) $ sum(i_c, storage_connect(i_c,s))..      wseu(s,t) =L= wscu(s,t)*DELTA_T;

R17C(s,t) $ sum(i_c, storage_connect(i_c,s))..      wsed(s,t) =L= wscd(s,t)*DELTA_T;

```

```
R18C(s,t) $ sum(i_c, storage_connect(i_c,s))..      psam(s,t) =E=
1/DELTA_T*(wseu(s,t) - wsed(s,t));
```

```
R19C(s,t) $ sum(i_c, storage_connect(i_c,s))..      socs(s,t) =E= SOC_0(s) $ (ord(t)
= 1) + socs(s,t-1) $ (ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);
```

```
* * L_a(x,z,y) = g(z) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2
OBJD.. zd =E= + sum((f,t), (cfe(f,t) + cfc(f,t+1)) $
sum(i_d,flex_load_connect(i_d,f)))
+ sum((g,t), (cge(g,t) + cgc(g,t+1)) $ sum(i_d,gen_connect(i_d,g)))
+ sum((s,t), (cse(s,t) + csc(s,t+1)) $ sum(i_d,
storage_connect(i_d,s)))
+ sum(t,
+ LAMBDA_D_k(t)*(p_DT(t) - p_TD_k(t))/sbase
)
+ sum(t,
+ LAMBDA_BALANCE_k(t)*(
+ sum(1, PLSCH(1,t) - PLAM(1,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_d,
flex_load_connect(i_d,f)) - PFAM_k(f,t) $ (not sum(i_d, flex_load_connect(i_d,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_d,
gen_connect(i_d,g)) - PGAM_k(g,t) $ (not sum(i_d, gen_connect(i_d,g))))
+ sum(s, - psam(s,t) $ sum(i_d, storage_connect(i_d,s)) -
PSAM_k(s,t) $ (not sum(i_d, storage_connect(i_d,s))))
)/sbase
)
+ sum(t,
+ LAMBDA_theta_xD_k(t)*(theta_xDT(t) - theta_xTD_k(t))
)
+ GAMMA/2*sum(t,
+ power((p_DT(t) - p_TD_k(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power((
+ sum(1, PLSCH(1,t) - PLAM(1,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_d,
flex_load_connect(i_d,f)) - PFAM_k(f,t) $ (not sum(i_d, flex_load_connect(i_d,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_d,
gen_connect(i_d,g)) - PGAM_k(g,t) $ (not sum(i_d, gen_connect(i_d,g))))
+ sum(s, - psam(s,t) $ sum(i_d, storage_connect(i_d,s)) -
PSAM_k(s,t) $ (not sum(i_d, storage_connect(i_d,s))))
)/sbase,2)
)
+ GAMMA/2*sum(t,
+ power(theta_xDT(t) - theta_xTD_k(t),2)
);
```

```

R01D(f,t) $ sum(i_d, flex_load_connect(i_d,f))..      cfe(f,t)*sbase =E= (ST(t) -
SFEU(f,t))*wfeu(f,t) + (SFED(f,t) - ST(t))*wfed(f,t);

R02D(f,t) $ sum(i_d, flex_load_connect(i_d,f))..      cfc(f,t)*sbase =E=
SFCU(f,t)*wfcu(f,t) + SFCD(f,t)*wfcd(f,t);

R03D(f,t) $ sum(i_d, flex_load_connect(i_d,f))..      wfeu(f,t) =L= wfcu(f,t)*DELTA_T;

R04D(f,t) $ sum(i_d, flex_load_connect(i_d,f))..      wfed(f,t) =L= wfcd(f,t)*DELTA_T;

R05D(f,t) $ sum(i_d, flex_load_connect(i_d,f))..      pfam(f,t) =E= PFSCH(f,t) +
1/DELTA_T*(wfeu(f,t)-wfed(f,t));

R06D(i_d,t)..                                         sum(j_d, B(i_d,j_d) *
(theta_d(i_d,t) - theta_d(j_d,t)))*sbase =E=  p_DT(t) $ (ord(i_d) = 1) + sum(g $
gen_connect(i_d,g), pgam(g,t))
                                                    - sum(l $ load_connect(i_d,l),
PLAM(l,t)) - sum(f $ flex_load_connect(i_d,f), pfam(f,t)) - sum(s $
storage_connect(i_d,s), psam(s,t));

R07D(i_d,j_d,t) $ conex(i_d,j_d)..                  p(i_d,j_d,t) =E= B(i_d,j_d) *
(theta_d(i_d,t) - theta_d(j_d,t));

R08D(t)..                                             p_DT(t) =E=
2*B('800','500')*(theta_xDT(t) - theta_d('800',t));

R09D(s,t) $ sum(i_d, storage_connect(i_d,s))..      wscu(s,t) * wscd(s,t) =E= 0;

R10D(g,t) $ sum(i_d, gen_connect(i_d,g))..          cge(g,t)*sbase =E= (ST(t) -
SGED(g,t))*wged(g,t);

R11D(g,t) $ sum(i_d, gen_connect(i_d,g))..          cgc(g,t)*sbase =E=
SGCD(g,t)*wgcd(g,t);

R12D(g,t) $ sum(i_d, gen_connect(i_d,g))..          wged(g,t) =L= wgcd(g,t)*DELTA_T;

R13D(g,t) $ sum(i_d, gen_connect(i_d,g))..          pgam(g,t) =E= PGSCH(g,t) -
wged(g,t)/DELTA_T;

R14D(s,t) $ sum(i_d, storage_connect(i_d,s))..      cse(s,t)*sbase =E= (ST(t) -
SSEU(s,t))*wseu(s,t) + (SSED(s,t) - ST(t))*wsed(s,t);

R15D(s,t) $ sum(i_d, storage_connect(i_d,s))..      csc(s,t)*sbase =E=
SSCU(s,t)*wscu(s,t) + SSCD(s,t)*wscd(s,t);

R16D(s,t) $ sum(i_d, storage_connect(i_d,s))..      wseu(s,t) =L= wscu(s,t)*DELTA_T;

R17D(s,t) $ sum(i_d, storage_connect(i_d,s))..      wsed(s,t) =L= wscd(s,t)*DELTA_T;

```

```

R18D(s,t) $ sum(i_d, storage_connect(i_d,s))..      psam(s,t) =E=
1/DELTA_T*(wseu(s,t) - wsed(s,t));

R19D(s,t) $ sum(i_d, storage_connect(i_d,s))..      socs(s,t) =E= SOC_0(s) $ (ord(t)
= 1) + socs(s,t-1) $ (ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);

* * L_a(x,z,y) = g(z) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2
OBJE.. ze =E= + sum((f,t), (cfe(f,t) + cfc(f,t+1)) $ sum(i_e,
flex_load_connect(i_e,f)))
+ sum((g,t), (cge(g,t) + cgc(g,t+1)) $ sum(i_e, gen_connect(i_e,g)))
+ sum((s,t), (cse(s,t) + csc(s,t+1)) $ sum(i_e, storage_connect(i_e,s)))
+ sum(t,
+ LAMBDA_E_k(t)*(p_ET(t) - p_TE_k(t))/sbase
)
+ sum(t,
+ LAMBDA_BALANCE_k(t)*(
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_e,
flex_load_connect(i_e,f)) - PFAM_k(f,t) $ (not sum(i_e, flex_load_connect(i_e,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_e,
gen_connect(i_e,g)) - PGAM_k(g,t) $ (not sum(i_e, gen_connect(i_e,g))))
+ sum(s, - psam(s,t) $ sum(i_e, storage_connect(i_e,s)) -
PSAM_k(s,t) $ (not sum(i_e, storage_connect(i_e,s))))
)/sbase
)
+ sum(t,
+ LAMBDA_theta_xE_k(t)*(theta_xET(t) - theta_xTE_k(t))
)
+ GAMMA/2*sum(t,
+ power((p_ET(t) - p_TE_k(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power((
+ sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - pfam(f,t) $ sum(i_e,
flex_load_connect(i_e,f)) - PFAM_k(f,t) $ (not sum(i_e, flex_load_connect(i_e,f))))
- sum(g, PGSCH(g,t) - pgam(g,t) $ sum(i_e,
gen_connect(i_e,g)) - PGAM_k(g,t) $ (not sum(i_e, gen_connect(i_e,g))))
+ sum(s, - psam(s,t) $ sum(i_e, storage_connect(i_e,s)) -
PSAM_k(s,t) $ (not sum(i_e, storage_connect(i_e,s))))
)/sbase,2)
)
+ GAMMA/2*sum(t,
+ power(theta_xET(t) - theta_xTE_k(t),2)
);

R01E(f,t) $ sum(i_e, flex_load_connect(i_e,f))..      cfe(f,t)*sbase =E= (ST(t) -
SFEU(f,t))*wfeu(f,t) + (SFED(f,t) - ST(t))*wfed(f,t);

```

```

R02E(f,t) $ sum(i_e, flex_load_connect(i_e,f))..      cfc(f,t)*sbase =E=
SFCU(f,t)*wfcu(f,t) + SFCD(f,t)*wfgd(f,t);

R03E(f,t) $ sum(i_e, flex_load_connect(i_e,f))..      wfeu(f,t) =L= wfcu(f,t)*DELTA_T;

R04E(f,t) $ sum(i_e, flex_load_connect(i_e,f))..      wfed(f,t) =L= wfgd(f,t)*DELTA_T;

R05E(f,t) $ sum(i_e, flex_load_connect(i_e,f))..      pfam(f,t) =E= PFSCH(f,t) +
1/DELTA_T*(wfeu(f,t)-wfed(f,t));

R06E(i_e,t)..                                         sum(j_e, B(i_e,j_e) *
(theta_e(i_e,t) - theta_e(j_e,t)))*sbase =E=  p_ET(t) $ (ord(i_e) = 1) + sum(g $
gen_connect(i_e,g), pgam(g,t))
                                                    - sum(l $ load_connect(i_e,l),
PLAM(l,t)) - sum(f $ flex_load_connect(i_e,f), pfam(f,t)) - sum(s $
storage_connect(i_e,s), psam(s,t));

R07E(i_e,j_e,t) $ conex(i_e,j_e)..                  p(i_e,j_e,t) =E= B(i_e,j_e) *
(theta_e(i_e,t) - theta_e(j_e,t));

R08E(t)..                                             p_ET(t) =E=
2*B('600','300')*(theta_xET(t) - theta_e('600',t));

R09E(s,t) $ sum(i_e, storage_connect(i_e,s))..      wscu(s,t) * wscd(s,t) =E= 0;

R10E(g,t) $ sum(i_e, gen_connect(i_e,g))..          cge(g,t)*sbase =E= (ST(t) -
SGED(g,t))*wged(g,t);

R11E(g,t) $ sum(i_e, gen_connect(i_e,g))..          cgc(g,t)*sbase =E=
SGCD(g,t)*wgcd(g,t);

R12E(g,t) $ sum(i_e, gen_connect(i_e,g))..          wged(g,t) =L= wgcd(g,t)*DELTA_T;

R13E(g,t) $ sum(i_e, gen_connect(i_e,g))..          pgam(g,t) =E= PGSCH(g,t) -
wged(g,t)/DELTA_T;

R14E(s,t) $ sum(i_e, storage_connect(i_e,s))..      cse(s,t)*sbase =E= (ST(t) -
SSEU(s,t))*wseu(s,t) + (SSED(s,t) - ST(t))*wsed(s,t);

R15E(s,t) $ sum(i_e, storage_connect(i_e,s))..      csc(s,t)*sbase =E=
SSCU(s,t)*wscu(s,t) + SSCD(s,t)*wscd(s,t);

R16E(s,t) $ sum(i_e, storage_connect(i_e,s))..      wseu(s,t) =L= wscu(s,t)*DELTA_T;

R17E(s,t) $ sum(i_e, storage_connect(i_e,s))..      wsed(s,t) =L= wscd(s,t)*DELTA_T;

R18E(s,t) $ sum(i_e, storage_connect(i_e,s))..      psam(s,t) =E=
1/DELTA_T*(wseu(s,t) - wsed(s,t));

```

```
R19E(s,t) $ sum(i_e, storage_connect(i_e,s)).. socs(s,t) =E= SOC_0(s) $ (ord(t)
= 1) + socs(s,t-1) $ (ord(t)>1) + ETA_u(s)*wseu(s,t) - 1/ETA_d(s)*wsed(s,t);
```

```
* * L_a(x,z,y) = g(z) + y^t*(Ax+Bz-c) + gamma/2*||Ax+Bz-c||^2
OBJT.. zt =E= + sum(t,
+ LAMBDA_A_k(t)*(p_AT_k(t) - p_TA(t))/sbase
+ LAMBDA_B_k(t)*(p_BT_k(t) - p_TB(t))/sbase
+ LAMBDA_C_k(t)*(p_CT_k(t) - p_TC(t))/sbase
+ LAMBDA_D_k(t)*(p_DT_k(t) - p_TD(t))/sbase
+ LAMBDA_E_k(t)*(p_ET_k(t) - p_TE(t))/sbase
)
+ sum(t,
+ LAMBDA_theta_xA_k(t)*(theta_xAT_k(t) - theta_xTA(t))
+ LAMBDA_theta_xB_k(t)*(theta_xBT_k(t) - theta_xTB(t))
+ LAMBDA_theta_xC_k(t)*(theta_xCT_k(t) - theta_xTC(t))
+ LAMBDA_theta_xD_k(t)*(theta_xDT_k(t) - theta_xTD(t))
+ LAMBDA_theta_xE_k(t)*(theta_xET_k(t) - theta_xTE(t))
)
+ GAMMA/2*sum(t,
+ power((p_AT_k(t) - p_TA(t))/sbase,2)
+ power((p_BT_k(t) - p_TB(t))/sbase,2)
+ power((p_CT_k(t) - p_TC(t))/sbase,2)
+ power((p_DT_k(t) - p_TD(t))/sbase,2)
+ power((p_ET_k(t) - p_TE(t))/sbase,2)
)
+ GAMMA/2*sum(t,
+ power(theta_xAT_k(t) - theta_xTA(t),2)
+ power(theta_xBT_k(t) - theta_xTB(t),2)
+ power(theta_xCT_k(t) - theta_xTC(t),2)
+ power(theta_xDT_k(t) - theta_xTD(t),2)
+ power(theta_xET_k(t) - theta_xTE(t),2)
);
```

```
R01T(i_t,t).. sum(j_t, B(i_t,j_t) * (theta_t(i_t,t) - theta_t(j_t,t)))*sbase =E=
p_100(t) $ (ord(i_t) = 1) - (p_TA(t) $ (ord(i_t) = 6) + p_TB(t) $ (ord(i_t) = 5) +
p_TC(t) $ (ord(i_t) = 4) + p_TD(t) $ (ord(i_t) = 3) + p_TE(t) $ (ord(i_t) = 2));
```

```
R02T(i_t,j_t,t) $ conex(i_t,j_t).. p(i_t,j_t,t) =E= B(i_t,j_t) * (theta_t(i_t,t) -
theta_t(j_t,t));
```

```
R03T(t).. p_TA(t) =E= 2*B('400','1000')*(theta_xTA(t) - theta_t('1000',t));
R04T(t).. p_TB(t) =E= 2*B('200','900')*(theta_xTB(t) - theta_t('900',t));
R05T(t).. p_TC(t) =E= 2*B('0','700')*(theta_xTC(t) - theta_t('700',t));
R06T(t).. p_TD(t) =E= 2*B('800','500')*(theta_xTD(t) - theta_t('500',t));
R07T(t).. p_TE(t) =E= 2*B('600','300')*(theta_xTE(t) - theta_t('300',t));
```

```
***** MODELLING *****
*****
```

```
* Modelling and solving market
```

```
model market_A /OBJA, R01A, R02A, R03A, R04A, R05A, R06A, R07A, R08A, R10A, R11A,
R12A, R13A, R14A, R15A, R16A, R17A, R18A, R19A/;
model market_B /OBJB, R01B, R02B, R03B, R04B, R05B, R06B, R07B, R08B, R10B, R11B,
R12B, R13B, R14B, R15B, R16B, R17B, R18B, R19B/;
model market_C /OBJC, R01C, R02C, R03C, R04C, R05C, R06C, R07C, R08C, R10C, R11C,
R12C, R13C, R14C, R15C, R16C, R17C, R18C, R19C/;
model market_D /OBJD, R01D, R02D, R03D, R04D, R05D, R06D, R07D, R08D, R10D, R11D,
R12D, R13D, R14D, R15D, R16D, R17D, R18D, R19D/;
model market_E /OBJE, R01E, R02E, R03E, R04E, R05E, R06E, R07E, R08E, R10E, R11E,
R12E, R13E, R14E, R15E, R16E, R17E, R18E, R19E/;
model market_T /OBJT, R01T, R02T, R03T, R04T, R05T, R06T, R07T/;
```

```
*****
***** SOLVERS *****
*****
```

```
*Limits for FLs and FGs
```

```
pfam.lo(f,t) = 0.70*PFSCH(f,t);
pfam.up(f,t) = 1.30*PFSCH(f,t);
pgam.up(g,t) = PGSCH(g,t);
```

```
* Upper and lower limit for power after market clearing in SSs. State of Charge
limits.
```

```
psam.lo(s,t) = -P_s_INV(s);
psam.up(s,t) = P_s_INV(s);
socs.lo(s,t) = SOC_d(s);
socs.up(s,t) = SOC_u(s);
socs.fx(s,'t96') = SOC_0(s);
```

```
* * POWER LIMIT IN LINE 228-230
```

```
* p.up('228','230',t) = 500;
* p.lo('228','230',t) = -500;
* p.up('230','228',t) = 500;
* p.lo('230','228',t) = -500;
```

```
* Flat state at beggining and slack node set as angle reference
```

```
theta_a.l(i_a,t) = 0;
theta_b.l(i_b,t) = 0;
theta_c.l(i_c,t) = 0;
theta_d.l(i_d,t) = 0;
theta_e.l(i_e,t) = 0;
theta_t.l(i_t,t) = 0;
```

```
* Declaration of the slack node
```

```
theta_t.fx('100',t) = 0;
```

```
*****
```

```
***** REPORT PARAMETERS OF A GIVEN OPTIMIZATION PROBLEM *****
```

```
*****
```

PARAMETERS

```
    report_costs(k,t,*)
    report_iter(k,t,*)
    report_network(i,t,*)
    report_branches(i,j,t,*)
    report_agg(a,k,t,*)
    report_FLs(f,k,t,*)
    report_FGs(g,k,t,*)
    report_SSs(s,k,t,*)
    report_convergence(k,*)
```

```
;
```

```
* INITIALIZATION
```

```
* Y
```

```
p_AT_k(t) = 0;
p_BT_k(t) = 0;
p_CT_k(t) = 0;
p_DT_k(t) = 0;
p_ET_k(t) = 0;
```

```
p_TA_k(t) = 0;
p_TB_k(t) = 0;
p_TC_k(t) = 0;
p_TD_k(t) = 0;
p_TE_k(t) = 0;
```

```
theta_xAT_k(t) = 0;
theta_xBT_k(t) = 0;
theta_xCT_k(t) = 0;
theta_xDT_k(t) = 0;
theta_xET_k(t) = 0;
```

```
theta_xTA_k(t) = 0;
theta_xTB_k(t) = 0;
theta_xTC_k(t) = 0;
theta_xTD_k(t) = 0;
theta_xTE_k(t) = 0;
```

```
* lambda
```

```
LAMBDA_A_k(t) = 0;
LAMBDA_B_k(t) = 0;
LAMBDA_C_k(t) = 0;
LAMBDA_D_k(t) = 0;
LAMBDA_E_k(t) = 0;
```

```

LAMBDA_theta_xA_k(t) = 0;
LAMBDA_theta_xB_k(t) = 0;
LAMBDA_theta_xC_k(t) = 0;
LAMBDA_theta_xD_k(t) = 0;
LAMBDA_theta_xE_k(t) = 0;
LAMBDA_BALANCE_k(t) = 0;

loop(k,

* Solving AREA A
solve market_A minimizing za using QCP;
PFAM_k(f,t) $ sum(i_a, flex_load_connect(i_a,f)) = pfam.l(f,t);
PGAM_k(g,t) $ sum(i_a, gen_connect(i_a,g)) = pgam.l(g,t);
PSAM_k(s,t) $ sum(i_a, storage_connect(i_a,s)) = psam.l(s,t);
p_AT_k(t) = p_AT.l(t);
theta_xAT_k(t) = theta_xAT.l(t);

* Reporting cost information of the area a
report_costs(k,t,'Sum_cfet_a') = sum(f, cfe.l(f,t) $ sum(i_a,
flex_load_connect(i_a,f)));
report_costs(k,t,'Sum_cfct_a') = sum(f, cfc.l(f,t) $ sum(i_a,
flex_load_connect(i_a,f)));
report_costs(k,t,'Sum_cget_a') = sum(g, cge.l(g,t) $ sum(i_a, gen_connect(i_a,g)));
report_costs(k,t,'Sum_cgct_a') = sum(g, cgc.l(g,t) $ sum(i_a, gen_connect(i_a,g)));
report_costs(k,t,'Sum_cset_a') = sum(s, cse.l(s,t) $ sum(i_a,
storage_connect(i_a,s)));
report_costs(k,t,'Sum_csct_a') = sum(s, csc.l(s,t) $ sum(i_a,
storage_connect(i_a,s)));
report_costs(k,t,'Sum_totals_e_a') = report_costs(k,t,'Sum_cfet_a') +
report_costs(k,t,'Sum_cget_a') + report_costs(k,t,'Sum_cset_a');
report_costs(k,t,'Sum_totals_c_a') = report_costs(k,t,'Sum_cfct_a') +
report_costs(k,t,'Sum_cgct_a') + report_costs(k,t,'Sum_csct_a');
report_costs(k,t,'Sum_totals_a') = report_costs(k,t,'Sum_totals_e_a') +
report_costs(k,t,'Sum_totals_c_a');

report_FLs(f,k,t,'cfe') $ sum(i_a, flex_load_connect(i_a,f)) = cfe.l(f,t);
report_FLs(f,k,t,'cfc') $ sum(i_a, flex_load_connect(i_a,f)) = cfc.l(f,t);
report_FGs(g,k,t,'cge') $ sum(i_a, gen_connect(i_a,g)) = cge.l(g,t);
report_FGs(g,k,t,'cgc') $ sum(i_a, gen_connect(i_a,g)) = cgc.l(g,t);
report_SSs(s,k,t,'cse') $ sum(i_a, storage_connect(i_a,s)) = cse.l(s,t);
report_SSs(s,k,t,'csc') $ sum(i_a, storage_connect(i_a,s)) = csc.l(s,t);

* Solving AREA B
solve market_B minimizing zb using QCP;
PFAM_k(f,t) $ sum(i_b, flex_load_connect(i_b,f)) = pfam.l(f,t);
PGAM_k(g,t) $ sum(i_b, gen_connect(i_b,g)) = pgam.l(g,t);
PSAM_k(s,t) $ sum(i_b, storage_connect(i_b,s)) = psam.l(s,t);
p_BT_k(t) = p_BT.l(t);

```

```
theta_xBT_k(t) = theta_xBT.l(t);
```

```
* Reporting cost information of the area b
```

```
report_costs(k,t,'Sum_cfet_b') = sum(f, cfe.l(f,t) $ sum(i_b,  
flex_load_connect(i_b,f)));  
report_costs(k,t,'Sum_cfct_b') = sum(f, cfc.l(f,t) $ sum(i_b,  
flex_load_connect(i_b,f)));  
report_costs(k,t,'Sum_cget_b') = sum(g, cge.l(g,t) $ sum(i_b, gen_connect(i_b,g)));  
report_costs(k,t,'Sum_cgct_b') = sum(g, cgc.l(g,t) $ sum(i_b, gen_connect(i_b,g)));  
report_costs(k,t,'Sum_cset_b') = sum(s, cse.l(s,t) $ sum(i_b,  
storage_connect(i_b,s)));  
report_costs(k,t,'Sum_csct_b') = sum(s, csc.l(s,t) $ sum(i_b,  
storage_connect(i_b,s)));  
report_costs(k,t,'Sum_totals_e_b') = report_costs(k,t,'Sum_cfet_b') +  
report_costs(k,t,'Sum_cget_b') + report_costs(k,t,'Sum_cset_b');  
report_costs(k,t,'Sum_totals_c_b') = report_costs(k,t,'Sum_cfct_b') +  
report_costs(k,t,'Sum_cgct_b') + report_costs(k,t,'Sum_csct_b');  
report_costs(k,t,'Sum_totals_b') = report_costs(k,t,'Sum_totals_e_b') +  
report_costs(k,t,'Sum_totals_c_b');
```

```
report_FLs(f,k,t,'cfe') $ sum(i_b, flex_load_connect(i_b,f)) = cfe.l(f,t);  
report_FLs(f,k,t,'cfc') $ sum(i_b, flex_load_connect(i_b,f)) = cfc.l(f,t);  
report_FGs(g,k,t,'cge') $ sum(i_b, gen_connect(i_b,g)) = cge.l(g,t);  
report_FGs(g,k,t,'cgc') $ sum(i_b, gen_connect(i_b,g)) = cgc.l(g,t);  
report_SSs(s,k,t,'cse') $ sum(i_b, storage_connect(i_b,s)) = cse.l(s,t);  
report_SSs(s,k,t,'csc') $ sum(i_b, storage_connect(i_b,s)) = csc.l(s,t);
```

```
* Solving AREA C
```

```
solve market_C minimizing zc using QCP;  
PFAM_k(f,t) $ sum(i_c, flex_load_connect(i_c,f)) = pfam.l(f,t);  
PGAM_k(g,t) $ sum(i_c, gen_connect(i_c,g)) = pgam.l(g,t);  
PSAM_k(s,t) $ sum(i_c, storage_connect(i_c,s)) = psam.l(s,t);  
p_CT_k(t) = p_CT.l(t);  
theta_xCT_k(t) = theta_xCT.l(t);
```

```
* Reporting cost information of the area c
```

```
report_costs(k,t,'Sum_cfet_c') = sum(f, cfe.l(f,t) $ sum(i_c,  
flex_load_connect(i_c,f)));  
report_costs(k,t,'Sum_cfct_c') = sum(f, cfc.l(f,t) $ sum(i_c,  
flex_load_connect(i_c,f)));  
report_costs(k,t,'Sum_cget_c') = sum(g, cge.l(g,t) $ sum(i_c, gen_connect(i_c,g)));  
report_costs(k,t,'Sum_cgct_c') = sum(g, cgc.l(g,t) $ sum(i_c, gen_connect(i_c,g)));  
report_costs(k,t,'Sum_cset_c') = sum(s, cse.l(s,t) $ sum(i_c,  
storage_connect(i_c,s)));  
report_costs(k,t,'Sum_csct_c') = sum(s, csc.l(s,t) $ sum(i_c,  
storage_connect(i_c,s)));  
report_costs(k,t,'Sum_totals_e_c') = report_costs(k,t,'Sum_cfet_c') +  
report_costs(k,t,'Sum_cget_c') + report_costs(k,t,'Sum_cset_c');
```

```
report_costs(k,t,'Sum_totals_c_c') = report_costs(k,t,'Sum_cfct_c') +
report_costs(k,t,'Sum_cgct_c') + report_costs(k,t,'Sum_csct_c');
report_costs(k,t,'Sum_totals_c') = report_costs(k,t,'Sum_totals_e_c') +
report_costs(k,t,'Sum_totals_c_c');
```

```
report_FLs(f,k,t,'cfe') $ sum(i_c, flex_load_connect(i_c,f)) = cfe.l(f,t);
report_FLs(f,k,t,'cfc') $ sum(i_c, flex_load_connect(i_c,f)) = cfc.l(f,t);
report_FGs(g,k,t,'cge') $ sum(i_c, gen_connect(i_c,g)) = cge.l(g,t);
report_FGs(g,k,t,'cgc') $ sum(i_c, gen_connect(i_c,g)) = cgc.l(g,t);
report_SSs(s,k,t,'cse') $ sum(i_c, storage_connect(i_c,s)) = cse.l(s,t);
report_SSs(s,k,t,'csc') $ sum(i_c, storage_connect(i_c,s)) = csc.l(s,t);
```

* Solving AREA D

```
solve market_D minimizing zd using QCP;
PFAM_k(f,t) $ sum(i_d, flex_load_connect(i_d,f)) = pfam.l(f,t);
PGAM_k(g,t) $ sum(i_d, gen_connect(i_d,g)) = pgam.l(g,t);
PSAM_k(s,t) $ sum(i_d, storage_connect(i_d,s)) = psam.l(s,t);
p_DT_k(t) = p_DT.l(t);
theta_xDT_k(t) = theta_xDT.l(t);
```

* Reporting cost information of the area d

```
report_costs(k,t,'Sum_cfet_d') = sum(f, cfe.l(f,t) $ sum(i_d,
flex_load_connect(i_d,f)));
report_costs(k,t,'Sum_cfct_d') = sum(f, cfc.l(f,t) $ sum(i_d,
flex_load_connect(i_d,f)));
report_costs(k,t,'Sum_cget_d') = sum(g, cge.l(g,t) $ sum(i_d, gen_connect(i_d,g)));
report_costs(k,t,'Sum_cgct_d') = sum(g, cgc.l(g,t) $ sum(i_d, gen_connect(i_d,g)));
report_costs(k,t,'Sum_cset_d') = sum(s, cse.l(s,t) $ sum(i_d,
storage_connect(i_d,s)));
report_costs(k,t,'Sum_csct_d') = sum(s, csc.l(s,t) $ sum(i_d,
storage_connect(i_d,s)));
report_costs(k,t,'Sum_totals_e_d') = report_costs(k,t,'Sum_cfet_d') +
report_costs(k,t,'Sum_cget_d') + report_costs(k,t,'Sum_cset_d');
report_costs(k,t,'Sum_totals_c_d') = report_costs(k,t,'Sum_cfct_d') +
report_costs(k,t,'Sum_cgct_d') + report_costs(k,t,'Sum_csct_d');
report_costs(k,t,'Sum_totals_d') = report_costs(k,t,'Sum_totals_e_d') +
report_costs(k,t,'Sum_totals_c_d');
```

```
report_FLs(f,k,t,'cfe') $ sum(i_d, flex_load_connect(i_d,f)) = cfe.l(f,t);
report_FLs(f,k,t,'cfc') $ sum(i_d, flex_load_connect(i_d,f)) = cfc.l(f,t);
report_FGs(g,k,t,'cge') $ sum(i_d, gen_connect(i_d,g)) = cge.l(g,t);
report_FGs(g,k,t,'cgc') $ sum(i_d, gen_connect(i_d,g)) = cgc.l(g,t);
report_SSs(s,k,t,'cse') $ sum(i_d, storage_connect(i_d,s)) = cse.l(s,t);
report_SSs(s,k,t,'csc') $ sum(i_d, storage_connect(i_d,s)) = csc.l(s,t);
```

* Solving AREA E

```
solve market_E minimizing ze using QCP;
PFAM_k(f,t) $ sum(i_e, flex_load_connect(i_e,f)) = pfam.l(f,t);
```

```
PGAM_k(g,t) $ sum(i_e, gen_connect(i_e,g)) = pgam.l(g,t);
PSAM_k(s,t) $ sum(i_e, storage_connect(i_e,s)) = psam.l(s,t);
p_ET_k(t) = p_ET.l(t);
theta_xET_k(t) = theta_xET.l(t);
```

*** Reporting cost information of the area a**

```
report_costs(k,t,'Sum_cfet_e') = sum(f, cfe.l(f,t) $ sum(i_e,
flex_load_connect(i_e,f)));
report_costs(k,t,'Sum_cfct_e') = sum(f, cfc.l(f,t) $ sum(i_e,
flex_load_connect(i_e,f)));
report_costs(k,t,'Sum_cget_e') = sum(g, cge.l(g,t) $ sum(i_e, gen_connect(i_e,g)));
report_costs(k,t,'Sum_cgct_e') = sum(g, cgc.l(g,t) $ sum(i_e, gen_connect(i_e,g)));
report_costs(k,t,'Sum_cset_e') = sum(s, cse.l(s,t) $ sum(i_e,
storage_connect(i_e,s)));
report_costs(k,t,'Sum_csct_e') = sum(s, csc.l(s,t) $ sum(i_e,
storage_connect(i_e,s)));
report_costs(k,t,'Sum_totals_e_e') = report_costs(k,t,'Sum_cfet_e') +
report_costs(k,t,'Sum_cget_e') + report_costs(k,t,'Sum_cset_e');
report_costs(k,t,'Sum_totals_c_e') = report_costs(k,t,'Sum_cfct_e') +
report_costs(k,t,'Sum_cgct_e') + report_costs(k,t,'Sum_csct_e');
report_costs(k,t,'Sum_totals_e') = report_costs(k,t,'Sum_totals_e_e') +
report_costs(k,t,'Sum_totals_c_e');
```

```
report_FLs(f,k,t,'cfe') $ sum(i_e, flex_load_connect(i_e,f)) = cfe.l(f,t);
report_FLs(f,k,t,'cfc') $ sum(i_e, flex_load_connect(i_e,f)) = cfc.l(f,t);
report_FGs(g,k,t,'cge') $ sum(i_e, gen_connect(i_e,g)) = cge.l(g,t);
report_FGs(g,k,t,'cgc') $ sum(i_e, gen_connect(i_e,g)) = cgc.l(g,t);
report_SSs(s,k,t,'cse') $ sum(i_e, storage_connect(i_e,s)) = cse.l(s,t);
report_SSs(s,k,t,'csc') $ sum(i_e, storage_connect(i_e,s)) = csc.l(s,t);
```

*** Solving AREA T**

```
solve market_T minimizing zt using QCP;
```

```
p_TA_k(t) = p_TA.l(t);
p_TB_k(t) = p_TB.l(t);
p_TC_k(t) = p_TC.l(t);
p_TD_k(t) = p_TD.l(t);
p_TE_k(t) = p_TE.l(t);
```

```
theta_xTA_k(t) = theta_xTA.l(t);
theta_xTB_k(t) = theta_xTB.l(t);
theta_xTC_k(t) = theta_xTC.l(t);
theta_xTD_k(t) = theta_xTD.l(t);
theta_xTE_k(t) = theta_xTE.l(t);
```

*** REPORTING THE REST OF THE PARAMETERS**

*** Updating lambdas**

```
LAMBDA_BALANCE_k(t) = LAMBDA_BALANCE_k(t) + GAMMA*(sum(l, PLSCH(l,t) - PLAM(l,t))
+ sum(f, PFSCH(f,t) - PFAM_k(f,t))
```

- $\text{sum}(g, \text{PGSCH}(g,t) - \text{PGAM}_k(g,t))$
- $\text{sum}(s, \text{PSAM}_k(s,t))/\text{sbase};$

```
LAMBDA_A_k(t) = LAMBDA_A_k(t) + GAMMA*(p_AT_k(t) - p_TA_k(t))/sbase;
LAMBDA_B_k(t) = LAMBDA_B_k(t) + GAMMA*(p_BT_k(t) - p_TB_k(t))/sbase;
LAMBDA_C_k(t) = LAMBDA_C_k(t) + GAMMA*(p_CT_k(t) - p_TC_k(t))/sbase;
LAMBDA_D_k(t) = LAMBDA_D_k(t) + GAMMA*(p_DT_k(t) - p_TD_k(t))/sbase;
LAMBDA_E_k(t) = LAMBDA_E_k(t) + GAMMA*(p_ET_k(t) - p_TE_k(t))/sbase;
```

```
LAMBDA_theta_xA_k(t) = LAMBDA_theta_xA_k(t) + GAMMA*(theta_xAT_k(t) -
theta_xTA_k(t));
LAMBDA_theta_xB_k(t) = LAMBDA_theta_xB_k(t) + GAMMA*(theta_xBT_k(t) -
theta_xTB_k(t));
LAMBDA_theta_xC_k(t) = LAMBDA_theta_xC_k(t) + GAMMA*(theta_xCT_k(t) -
theta_xTC_k(t));
LAMBDA_theta_xD_k(t) = LAMBDA_theta_xD_k(t) + GAMMA*(theta_xDT_k(t) -
theta_xTD_k(t));
LAMBDA_theta_xE_k(t) = LAMBDA_theta_xE_k(t) + GAMMA*(theta_xET_k(t) -
theta_xTE_k(t));
```

* Reporting aggregator information

```
report_agg(a,k,t,'caet') = (sum(f, cfe.l(f,t) $ agg_flex(a,f)) + sum(g, cge.l(g,t) $
agg_gen(a,g)) + sum(s, cse.l(s,t) $ agg_storage(a,s)))/sbase;
report_agg(a,k,t,'cact') = (sum(f, cfc.l(f,t) $ agg_flex(a,f)) + sum(g, cgc.l(g,t) $
agg_gen(a,g)) + sum(s, csc.l(s,t) $ agg_storage(a,s)))/sbase;
report_agg(a,k,t,'wfeu_a') = sum(f $ agg_flex(a,f),wfeu.l(f,t));
report_agg(a,k,t,'wfed_a') = sum(f $ agg_flex(a,f),wfed.l(f,t));
report_agg(a,k,t,'wged_a') = sum(g $ agg_gen(a,g),wged.l(g,t));
report_agg(a,k,t,'wseu_a') = sum(s $ agg_storage(a,s),wseu.l(s,t));
report_agg(a,k,t,'wsed_a') = sum(s $ agg_storage(a,s),wsed.l(s,t));
report_agg(a,k,t,'wacu') = report_agg(a,k,t,'wfeu_a') + report_agg(a,k,t,'wged_a') +
report_agg(a,k,t,'wseu_a');
report_agg(a,k,t,'waed') = report_agg(a,k,t,'wfed_a') + report_agg(a,k,t,'wsed_a');
report_agg(a,k,t,'wfcu_a') = sum(f $ agg_flex(a,f), wfcu.l(f,t));
report_agg(a,k,t,'wfcd_a') = sum(f $ agg_flex(a,f), wfcd.l(f,t));
report_agg(a,k,t,'wgcd_a') = sum(g $ agg_gen(a,g), wgcd.l(g,t));
report_agg(a,k,t,'wscu_a') = sum(s $ agg_storage(a,s), wscu.l(s,t));
report_agg(a,k,t,'wscd_a') = sum(s $ agg_storage(a,s), wscd.l(s,t));
report_agg(a,k,t,'wacu') = report_agg(a,k,t,'wfcu_a') + report_agg(a,k,t,'wgcd_a') +
report_agg(a,k,t,'wscu_a');
report_agg(a,k,t,'wacd') = report_agg(a,k,t,'wfcd_a') + report_agg(a,k,t,'wscd_a');
```

* Reporting flexible loads information

```
report_FLs(f,k,t,'wfed') = wfed.l(f,t);
report_FLs(f,k,t,'wfcd') = wfcd.l(f,t);
report_FLs(f,k,t,'wfeu') = wfeu.l(f,t);
report_FLs(f,k,t,'wfcu') = wfcu.l(f,t);
```

```
report_FLs(f,k,t, 'SFED') = SFED(f,t);
report_FLs(f,k,t, 'SFCD') = SFCD(f,t);
report_FLs(f,k,t, 'SFEU') = SFEU(f,t);
report_FLs(f,k,t, 'SFCU') = SFCU(f,t);
report_FLs(f,k,t, 'pfam') = pfam.l(f,t);
report_FLs(f,k,t, 'PFAM_k') = PFAM_k(f,t);
report_FLs(f,k,t, 'PFSCH') = PFSCH(f,t);

* Report flexible generators information
report_FGs(g,k,t, 'cget') = cge.l(g,t);
report_FGs(g,k,t, 'cgct') = cgc.l(g,t);
report_FGs(g,k,t, 'wged') = wged.l(g,t);
report_FGs(g,k,t, 'wgcd') = wgcd.l(g,t);
report_FGs(g,k,t, 'PGSCH') = PGSCH(g,t);
report_FGs(g,k,t, 'pgam') = pgam.l(g,t);
report_FGs(g,k,t, 'PGAM_k') = PGAM_k(g,t);
report_FGs(g,k,t, 'SGED') = SGED(g,t);
report_FGs(g,k,t, 'SGCD') = SGCD(g,t);

* Report storage systems information
report_SSs(s,k,t, 'SSEU') = SSEU(s,t);
report_SSs(s,k,t, 'SSCU') = SSCU(s,t);
report_SSs(s,k,t, 'SSED') = SSED(s,t);
report_SSs(s,k,t, 'SSCD') = SSCD(s,t);
report_SSs(s,k,t, 'wseu') = wseu.l(s,t);
report_SSs(s,k,t, 'wscu') = wscu.l(s,t);
report_SSs(s,k,t, 'wsed') = wsed.l(s,t);
report_SSs(s,k,t, 'wscd') = wscd.l(s,t);
report_SSs(s,k,t, 'psam') = psam.l(s,t);
report_SSs(s,k,t, 'PSAM_k') = PSAM_k(s,t);
report_SSs(s,k,t, 'P_s_INV') = P_s_INV(s);
report_SSs(s,k,t, 'socs') = socs.l(s,t);
report_SSs(s,k,t, 'SOC_0') = SOC_0(s);
report_SSs(s,k,t, 'SOC_u') = SOC_u(s);
report_SSs(s,k,t, 'SOC_d') = SOC_d(s);

* Reporting iteration information
report_iter(k,t, 'p_AT_k (kW)') = p_AT_k(t)/sbase;
report_iter(k,t, 'p_BT_k (kW)') = p_BT_k(t)/sbase;
report_iter(k,t, 'p_CT_k (kW)') = p_CT_k(t)/sbase;
report_iter(k,t, 'p_DT_k (kW)') = p_DT_k(t)/sbase;
report_iter(k,t, 'p_ET_k (kW)') = p_ET_k(t)/sbase;

report_iter(k,t, 'p_TA_k (kW)') = p_TA_k(t)/sbase;
report_iter(k,t, 'p_TB_k (kW)') = p_TB_k(t)/sbase;
report_iter(k,t, 'p_TC_k (kW)') = p_TC_k(t)/sbase;
report_iter(k,t, 'p_TD_k (kW)') = p_TD_k(t)/sbase;
report_iter(k,t, 'p_TE_k (kW)') = p_TE_k(t)/sbase;
```

```

report_iter(k,t,'theta_xAT_k (rad)') = theta_xAT_k(t);
report_iter(k,t,'theta_xBT_k (rad)') = theta_xBT_k(t);
report_iter(k,t,'theta_xCT_k (rad)') = theta_xCT_k(t);
report_iter(k,t,'theta_xDT_k (rad)') = theta_xDT_k(t);
report_iter(k,t,'theta_xET_k (rad)') = theta_xET_k(t);

report_iter(k,t,'theta_xTA_k (rad)') = theta_xTA_k(t);
report_iter(k,t,'theta_xTB_k (rad)') = theta_xTB_k(t);
report_iter(k,t,'theta_xTC_k (rad)') = theta_xTC_k(t);
report_iter(k,t,'theta_xTD_k (rad)') = theta_xTD_k(t);
report_iter(k,t,'theta_xTE_k (rad)') = theta_xTE_k(t);

report_iter(k,t,'p_100 (kW)') = p_100.l(t)/sbase;

report_iter(k,t,'lambda_x') = LAMBDA_A_k(t);
report_iter(k,t,'lambda_balance') = LAMBDA_BALANCE_k(t);
report_iter(k,t,'lambda_theta') = LAMBDA_theta_xA_k(t);

report_iter(k,t,'balance_residual (kW)') = (sum(l, PLSCH(l,t) - PLAM(l,t))
      + sum(f, PFSCH(f,t) - PFAM_k(f,t))
      - sum(g, PGSCH(g,t) - PGAM_k(g,t))
      - sum(s, PSAM_k(s,t)))/sbase;

report_iter(k,t,'fictitious_residual_a (kW)') = (p_AT_k(t) - p_TA_k(t))/sbase;
report_iter(k,t,'fictitious_residual_b (kW)') = (p_BT_k(t) - p_TB_k(t))/sbase;
report_iter(k,t,'fictitious_residual_c (kW)') = (p_CT_k(t) - p_TC_k(t))/sbase;
report_iter(k,t,'fictitious_residual_d (kW)') = (p_DT_k(t) - p_TD_k(t))/sbase;
report_iter(k,t,'fictitious_residual_e (kW)') = (p_ET_k(t) - p_TE_k(t))/sbase;

report_iter(k,t,'theta_residual_a (rad)') = theta_xAT_k(t) - theta_xTA_k(t);
report_iter(k,t,'theta_residual_b (rad)') = theta_xBT_k(t) - theta_xTB_k(t);
report_iter(k,t,'theta_residual_c (rad)') = theta_xCT_k(t) - theta_xTC_k(t);
report_iter(k,t,'theta_residual_d (rad)') = theta_xDT_k(t) - theta_xTD_k(t);
report_iter(k,t,'theta_residual_e (rad)') = theta_xET_k(t) - theta_xTE_k(t);

report_iter(k,t,'TOTAL_COST_e') = report_costs(k,t,'Sum_totals_e_a') +
report_costs(k,t,'Sum_totals_e_b') + report_costs(k,t,'Sum_totals_e_c') +
report_costs(k,t,'Sum_totals_e_d') + report_costs(k,t,'Sum_totals_e_e');

report_iter(k,t,'TOTAL_COST_c') = report_costs(k,t,'Sum_totals_c_a') +
report_costs(k,t,'Sum_totals_c_b') + report_costs(k,t,'Sum_totals_c_c') +
report_costs(k,t,'Sum_totals_c_d') + report_costs(k,t,'Sum_totals_c_e');

report_iter(k,t,'TOTAL_COST') = report_iter(k,t,'TOTAL_COST_e') +
report_iter(k,t,'TOTAL_COST_c');

```

* Reporting convergence

```
report_convergence(k, 'S_lambda_fict') = sum(t, LAMBDA_A_k(t) + LAMBDA_B_k(t) +  
LAMBDA_C_k(t) + LAMBDA_D_k(t) + LAMBDA_E_k(t));  
report_convergence(k, 'S_fict_a') = sum(t, report_iter(k, t, 'fictitious_residual_a  
(kW)'));  
report_convergence(k, 'S_fict_b') = sum(t, report_iter(k, t, 'fictitious_residual_b  
(kW)'));  
report_convergence(k, 'S_fict_c') = sum(t, report_iter(k, t, 'fictitious_residual_c  
(kW)'));  
report_convergence(k, 'S_fict_d') = sum(t, report_iter(k, t, 'fictitious_residual_d  
(kW)'));  
report_convergence(k, 'S_fict_e') = sum(t, report_iter(k, t, 'fictitious_residual_e  
(kW)'));  
  
report_convergence(k, 'S_lambda_balance_t') = sum(t, LAMBDA_BALANCE_k(t));  
report_convergence(k, 'S_balance') = sum(t, report_iter(k, t, 'balance_residual (kW)'));  
  
report_convergence(k, 'S_lambda_theta_t') = sum(t,  
    + LAMBDA_theta_xA_k(t)  
    + LAMBDA_theta_xB_k(t)  
    + LAMBDA_theta_xC_k(t)  
    + LAMBDA_theta_xD_k(t)  
    + LAMBDA_theta_xE_k(t)  
);  
report_convergence(k, 'S_theta_a') = sum(t, report_iter(k, t, 'theta_residual_a  
(rad)'));  
report_convergence(k, 'S_theta_b') = sum(t, report_iter(k, t, 'theta_residual_b  
(rad)'));  
report_convergence(k, 'S_theta_c') = sum(t, report_iter(k, t, 'theta_residual_c  
(rad)'));  
report_convergence(k, 'S_theta_d') = sum(t, report_iter(k, t, 'theta_residual_d  
(rad)'));  
report_convergence(k, 'S_theta_e') = sum(t, report_iter(k, t, 'theta_residual_e  
(rad)'));  
  
report_convergence(k, 'S_p_AT_k') = sum(t, p_AT_k(t)/sbase);  
report_convergence(k, 'S_p_BT_k') = sum(t, p_BT_k(t)/sbase);  
report_convergence(k, 'S_p_CT_k') = sum(t, p_CT_k(t)/sbase);  
report_convergence(k, 'S_p_DT_k') = sum(t, p_DT_k(t)/sbase);  
report_convergence(k, 'S_p_ET_k') = sum(t, p_ET_k(t)/sbase);  
  
report_convergence(k, 'S_p_100') = sum(t, p_100.l(t)/sbase);  
  
report_convergence(k, 'market_A_NumInfes') = market_A.NumInfes;  
report_convergence(k, 'market_B_NumInfes') = market_B.NumInfes;  
report_convergence(k, 'market_C_NumInfes') = market_C.NumInfes;  
report_convergence(k, 'market_D_NumInfes') = market_D.NumInfes;  
report_convergence(k, 'market_E_NumInfes') = market_E.NumInfes;  
  
report_convergence(k, 'SUM_TOTAL_COST') = sum(t, report_iter(k, t, 'TOTAL_COST'));
```

```

report_convergence(k, 'primal_residual') = sqrt(
    + sum(t, power(report_iter(k, t, 'theta_residual_a (rad)'), 2))
    + sum(t, power(report_iter(k, t, 'theta_residual_b (rad)'), 2))
    + sum(t, power(report_iter(k, t, 'theta_residual_c (rad)'), 2))
    + sum(t, power(report_iter(k, t, 'theta_residual_d (rad)'), 2))
    + sum(t, power(report_iter(k, t, 'theta_residual_e (rad)'), 2))
    + sum(t, power(report_iter(k, t, 'fictitious_residual_a (kW)'), 2))
    + sum(t, power(report_iter(k, t, 'fictitious_residual_b (kW)'), 2))
    + sum(t, power(report_iter(k, t, 'fictitious_residual_c (kW)'), 2))
    + sum(t, power(report_iter(k, t, 'fictitious_residual_d (kW)'), 2))
    + sum(t, power(report_iter(k, t, 'fictitious_residual_e (kW)'), 2))
    + sum(t, power(report_iter(k, t, 'balance_residual (kW)'), 2))
);

report_convergence(k, 'dual_residual') = GAMMA*sqrt(
    + sum((f, t), power(report_FLs(f, k, t, 'PFAM_k') -
report_FLs(f, k-1, t, 'PFAM_k') $ (ord(k)>1), 2))
    + sum((g, t), power(report_FGs(g, k, t, 'PGAM_k') -
report_FGs(g, k-1, t, 'PGAM_k') $ (ord(k)>1), 2))
    + sum((s, t), power(report_SSs(s, k, t, 'PSAM_k') -
report_SSs(s, k-1, t, 'PSAM_k') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'p_TA_k (kW)') -
report_iter(k-1, t, 'p_TA_k (kW)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'p_TB_k (kW)') -
report_iter(k-1, t, 'p_TB_k (kW)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'p_TC_k (kW)') -
report_iter(k-1, t, 'p_TC_k (kW)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'p_TD_k (kW)') -
report_iter(k-1, t, 'p_TD_k (kW)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'p_TE_k (kW)') -
report_iter(k-1, t, 'p_TE_k (kW)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'theta_xTA_k (rad)') -
report_iter(k-1, t, 'theta_xTA_k (rad)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'theta_xTB_k (rad)') -
report_iter(k-1, t, 'theta_xTB_k (rad)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'theta_xTC_k (rad)') -
report_iter(k-1, t, 'theta_xTC_k (rad)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'theta_xTD_k (rad)') -
report_iter(k-1, t, 'theta_xTD_k (rad)') $ (ord(k)>1), 2))
    + sum(t, power(report_iter(k, t, 'theta_xTE_k (rad)') -
report_iter(k-1, t, 'theta_xTE_k (rad)') $ (ord(k)>1), 2))
);
);

```

* SAVING RECORDS THAT CAUSES OVERFLOW IN EXCEL BECAUSE OF THEIR LENGTH

* Reporting nodes information

```

report_network(i_a, t, 'theta') = theta_a.l(i_a, t);
report_network(i_b, t, 'theta') = theta_b.l(i_b, t);
report_network(i_c, t, 'theta') = theta_c.l(i_c, t);
report_network(i_d, t, 'theta') = theta_d.l(i_d, t);
report_network(i_e, t, 'theta') = theta_e.l(i_e, t);

```

```
report_network(i_t,t,'theta') = theta_t.l(i_t,t);
report_network(i,t,'injection') = sum(j, p.l(i,j,t));
report_network(i_a,t,'marg_balance') = R06A.m(i_a,t)*1e3;
report_network(i_b,t,'marg_balance') = R06B.m(i_b,t)*1e3;
report_network(i_c,t,'marg_balance') = R06C.m(i_c,t)*1e3;
report_network(i_d,t,'marg_balance') = R06D.m(i_d,t)*1e3;
report_network(i_e,t,'marg_balance') = R06E.m(i_e,t)*1e3;

* Reporting branches information
report_branches(i,j,t,'Pijt') = p.l(i,j,t);
report_branches(i,j,t,'PMAX') = PMAX(i,j);
report_branches(i,j,t,'B') $ (ord(i) <> ord(j)) = B(i,j);

execute_unload 'report.gdx' report_branches
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_branches rng
= Branches!'
execute_unload 'report.gdx' report_network
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_network rng =
Network!'
execute_unload 'report.gdx' report_FLs
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_FLs rng =
FLs!'
execute_unload 'report.gdx' report_FGs
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_FGs rng =
FGs!'
execute_unload 'report.gdx' report_SSs
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_SSs rng =
SSs!'
execute_unload 'report.gdx' report_agg
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_agg rng =
Aggs!'
execute_unload 'report.gdx' report_costs
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_costs rng =
Costs!'
execute_unload 'report.gdx' report_iter
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_iter rng =
iter!'
execute_unload 'report.gdx' report_convergence
execute 'gdxxrw.exe report.gdx o=report_decentralized.xlsx par = report_convergence
rng = convergence!'

* Delete unuseful files
execute 'rm -f data.gdx';
execute 'rm -f data.inc';
execute 'rm -f report.gdx';
```