Quasi-Analytical Model for Space-Time Periodic Structures Controlled by Diodes

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***Abstract*—This paper presents a technique to model time-varying periodic structure with two axis of spatial periodicity, x̂ and ŷ. The model is based on circuit-approach techniques, where the knowledge, a priori, of the field at the discontinuity is necessary. Time modulation is invoked via diodes. The fields when the diodes are in ON or in OFF states are extracted from a commercial software. A smart combination of these fields give rise to the space-temporal profile, leading to the resolution of the whole electromagnetic field in the regions surrounding the structure.**

***Keywords*—Time-varying, Space-time, Metasurface, Circuit model, Periodic structure.**

I. Introduction

Time modulation has emerged as a new technology where time is considered as an additional degree of freedom. [1], [2]. Many efforts have been put in the study and development of time-varying metasurfaces. They are considered space-time structures since they have modulation both in time (time periodicity) and space (spatial periodicity). The inclusion of time is groundbreaking due to the possibility of invoking interesting phenomena such as non-reciprocity [3], net amplification [4] or frequency conversion [5]. The main constraint otherwise lies on the significant absent of commercial tools to analyze and design the systems. In this context, researchers revisit well-known resolution techniques in the framework of microwaves and millimeter-waves, and reformulate them by including time. The present work provides a technique based on circuit model approaches. Though some problems have previously been tackled by this same technique [6], [7], this is the first time where it is applied to a 2-dimensional spatial problem including the third dimension (periodicity in time).

II. Modelling of the structure

The structure to be modelled is a 2-dimensional periodic structure with negligible thickness. It has 2 spatial periodicities, p_x and p_y, along the x̂ and ŷ directions respectively. The unit cell is sketched in Fig.1. The grey regions denote metal, while the zones in light blue denote air. A gap with width g_y exists between the vertical metal strips. This gap is the zone where the diode will be placed. The diode has standard dimensions $g_y \times g_x$. It is conceived to work in two possible states: ON state, where the diode is approximately considered as a perfect metal; OFF state, where the diode is considered as a open circuit. Though this is an approximation, space-time structures barely operates over 10 GHz in practice. This leads us to consider the approximation as an effective estimator. The time modulation is induced by modifying the OFF and ON states of the diodes in periodic cycles. The time periodicity of each of these cycles is defined by T_M.

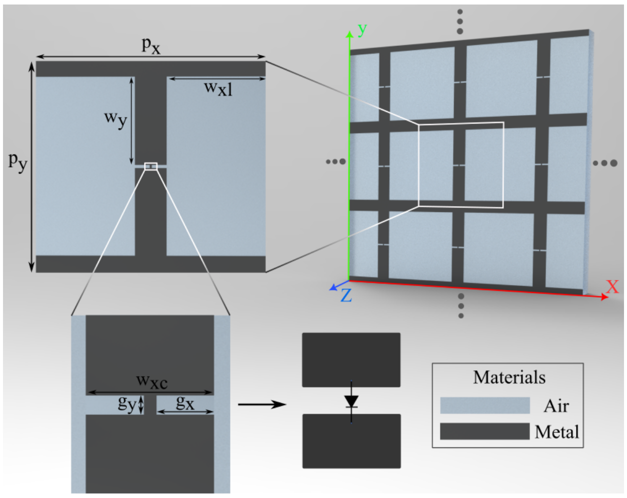
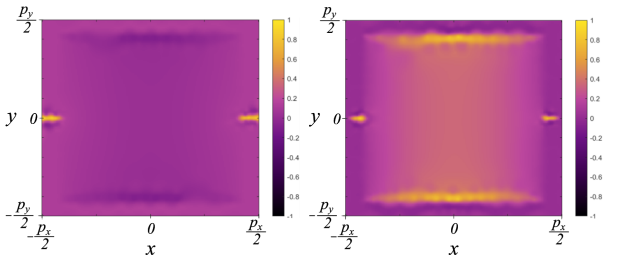


Fig. 1. View in perspective of the cell to analyze and corresponding unit cell including the diode.

If we focus on the unit cell, we call *discontinuity plane* to the plane where the structure is ($z = 0$). The structure is illuminated by a impinging plane wave with angular frequency ω_0 (associated periodicity $T_0 = \omega_0/2\pi$). According to the methodology in [5], the electric field at the discontinuity inside a periodic cycle, $\mathbf{E}_{BF}(x, y, t)$, must be known a priori. As a good approximation, this field could be estimated in the



(a) (b)

Fig. 2. Electric field at the discontinuity. (a): diode OFF. (b): diode ON.

following way:

$$\mathbf{E}_{\mathrm{BF}}(x,y,t)=\sin(\omega_0 t)\begin{cases}\mathbf{E}^{\mathrm{OFF}}(x,y) & 0\le t<\alpha T_{\mathrm{M}}\\ \mathbf{E}^{\mathrm{ON}}(x,y) & \alpha T_{\mathrm{M}}\le t<T_{\mathrm{M}}\end{cases} \tag{1}$$

with $\sin(\omega_0 t)$ being the field of the incident wave, $\mathbf{E}^{\mathrm{OFF/ON}}(x,y)$ being the field when the diode is in OFF/ON state, and $\alpha\in[0,1]$. A schematic representation of both fields is illustrated in Fig. 2. Notice that the cell has been shifted $p_{\mathrm{x}}/2$ with respect the cell shown in Fig. 1. The field profile in (1) is employed for the computation of the whole fields in the surrounding media, which is a superposition of Floquet harmonics (see [5] for more information).

III. Numerical Results

The surrounding fields, defined as a superposition of space-time harmonics, can be computed at any point of the space (x,y,z) and at any time instant t. In fact, if this surrounding field is computed at $(x,y,z=0)$, it should coincide with the field profile defined a priori, $\mathbf{E}_{\mathrm{BF}}(x,y,t)$. Fig. 3(a) shows the field at $(x,y,z=0)$ in a time sequence of slides, each referring to the field at a different instant from $t=0$ to $t=T_{\mathrm{M}}$. The structures parameters are indicated in the caption of the figure. The diode is in OFF in $t\in[0,T_{\mathrm{M}}/3)$ and holds in ON from $t=[T_{\mathrm{M}}/3,T_{\mathrm{M}})$. Fig. 3(b)-(c) shows 2 frames at different instants: $t=T_{\mathrm{M}}/8$ (OFF state); and $t=3T_{\mathrm{M}}/4$ (ON state). As expected, they are quite similar to those in Fig. 2. Some of the individual harmonics employed to reconstruct the field at the discontinuity are plotted in Fig. 4. Each individual harmonic is defined by 3 indexes, n,m,i, each for a different periodicity $p_{\mathrm{x}},p_{\mathrm{y}},T_{\mathrm{M}}$ respectively. Fig. 4(a) shows the amplitude distribution for harmonics with $m=i=0$ and $n\neq 0$, which basically are spatial harmonics. Similarly, Fig. 4(b) plot the amplitude distribution of temporal-like harmonics ($m=n=0$), where we can infer a power split between the harmonics of order $i=0$ and $i=-6$.

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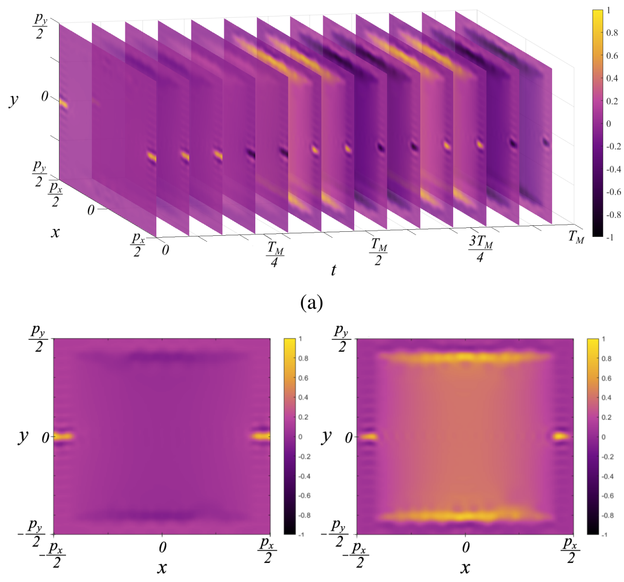
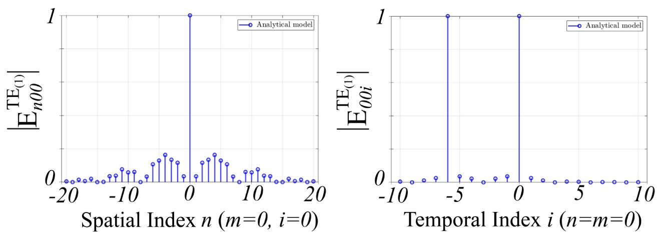


(a) (b) (c)

Fig. 3. (a) Field at the discontinuity reconstructed by considering the unit cell dimensions (spatial periodicity) and along a time periodicity T_{M}; (b) Field at the time instant $t=T_{\mathrm{M}}/8$). Diode in OFF state; (c) Field at $t=3T_{\mathrm{M}}/4$). Diode in ON state. Structure parameters: $p_{\mathrm{x}}=p_{\mathrm{y}}=20\,\mathrm{mm}$, $T_M=3T_0$



(a) (b)

Fig. 4. Space-time harmonics with orders (a) $m=i=0,n\neq 0$; (b) $n=m=0,i\neq 0$.

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