Structural Behaviour, Geometric and Constructive Analysis of an Exceptional Ancient Timber Dome

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ABSTRACT

The wooden dome that covers the Hall of Ambassadors of the Alcazar of Seville (1427 AC) can be considered an exceptional architectural and constructive element in which visible ornamentation and underlying structure are inextricably linked. This article proposes a methodological approach integrating different disciplines (engineering, architecture and history) to discern comprehensively the genesis of this specific architectural heritage. Combined with an in-depth constructive and structural assessment, this method leads to define optimal conservation actions. Thus, based on historical references on the construction of wooden domes together with knowledge that the carpentry guild applied at that time, a detailed geometric and constructive analysis of the dome has enabled us to establish that its construction must have been carried out through the prefabrication of sectors. In addition, a structural model has been generated, with which, through various hypotheses, its mechanical behaviour can be understood. The profusion of noggings, which follow the strict geometry derived from patterns of 10-point stars, plays a fundamental role in the stability and resistance of the set.

Keywords: ancient domes; timber domes; timber structural analysis; carpentry joints; Alcazar of Seville; interlacing patterns

INTRODUCTION

Subject and Purpose of the Research

The object of this study involves the wooden dome in the Hall of Ambassadors of the Real Alcázar of Seville (Spain) built in 1427 AC. Three objectives can be indicated in this research: first, the completion of an in-depth description of the geometry of the dome and its constructive configuration, thereby complementing the current descriptions that adhere to its merely formal and stylistic aspects; second, the establishment of a hypothesis of the procedure carried out for the construction and assembly of the dome; and third, an analysis of its structural behaviour, for which a three-dimensional model is developed. Before tackling these objectives, an overview of the historical construction of wooden domes is presented and, specifically, a description is given of the cultural and knowledge context in which the construction of this dome is situated.

Preliminary Description, Situation, and Historical Context

The Real Alcázar of Seville comprises a walled enclosure within which buildings constructed mainly between the 9th and 15th centuries are preserved, although they do have Roman antecedents and underwent extensions and reforms up to the 19th century. The main building is known as the *Palacio de Don Pedro* wherein the Hall of Ambassadors is located, whose dome is the object of this research (Figure 1 A and B). Since 1987, the complex has been inscribed onto the UNESCO World Heritage List. This palace was built between the years 1364 and 1369 CE. Later, during the reign of Juan II (1405-1454), the construction of emblematic pieces, such as the Hall of Ambassadors and its dome, was carried out. According to an inscription found on the dome itself, its author was the Master craftsman of King Diego Ruiz in 1427 (González de León, 1844).

This is a dome built entirely with wooden elements that includes a complex interlacing pattern, based on the geometry of 10-point stars, that runs along the entire surface of the dome (Figure 1C). Its design and construction form part of the long-standing traditional Spanish structural carpentry. Stylistically, it can be ascribed to the so-called “Mudejar” style, whose meaning alludes to the identification of a specific artistic style, and not as a term ascribable to a specific ethnic or social authorship.

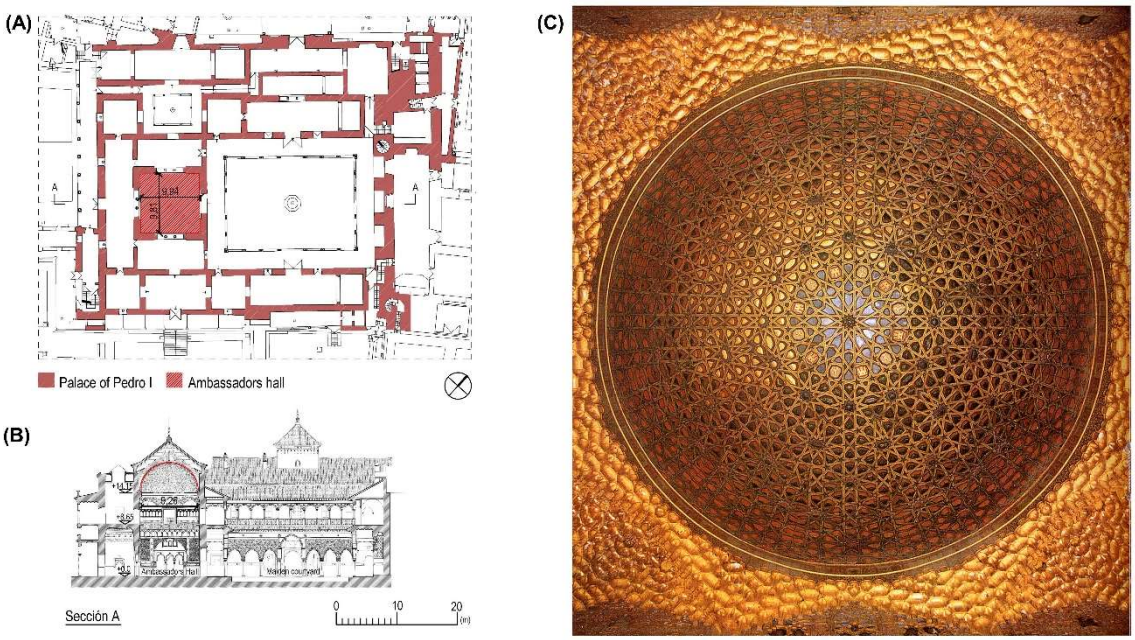


Figure 1: (A) Ground floor of the Real Alcazar of Seville with the Palace of Pedro I and Ambassadors Hall highlighted. (B) Section A: Palace of Pedro I through the Ambassadors Hall and the Maiden courtyard. (C) Wooden dome over the Ambassadors Hall. Overhead view.

The dome is located on a space of approximately square proportions, 9.25 m on each side. The walls of this space, built with solid brick, have an average thickness of 0.90 m. The spring line of the dome is produced at an elevation of +14.15 m above the ground. For the transition between the square ground plan of the wall structure and the circular ground plan of the dome, wooden squinches are created in the corners decorated with clusters of muqarnas.

The structure of the dome is protected from the elements by a covering, probably not original, comprised of a wooden octagonal roof framing covered with ceramic tiles.

WOODEN DOMES: BACKGROUND AND REFERENCE STUDIES

References and Studies on Wooden Domes

Since ancient times, the construction of domes has utilised discrete elements, brick or stone, with numerous references available in the scientific literature regarding their configuration and structural behaviour. However, knowledge regarding timber as a structural element in domes remains scarce, owing to its limited usage therein. This low level of utilisation may have been due to the constructive complexity of the processes involved (bending of the wood, assembly, difficulty of geometric control) or to the lack

of forested land, and hence the lack of any constructive tradition in carpentry in certain geographical areas (Almagro, 2001).

An indication of the existence of domes and vaults built with timber which have since disappeared can be found in classical Greece. Misztal (2018, p. 7) refers to the Aeschylus Temple in Epidaurus (370 BCE) and indicates that *"the slight thickness of masonry, stone walls allows to presume that light, wooden roofs were resting on them. The spacing of walls and pillars shows that it could not be simple beam roofs"*.

A large group of disappeared domes built in wood deserve mention, which are precisely the temporary frameworks destined to become an auxiliary means for the construction of stone or brick domes. The few articles that refer to old domes featuring timber classify them primarily according to their number of layers (simple or multi-shell domes) (Misztal, 2018; Ashkan and Ahmad, 2010). From the point of view of its structural functioning, in domes built with timber, differentiation should be made between membrane-type domes and truss-type domes; and intermediate states (hybrid double domes) can be considered where wood, despite being a secondary element, plays a major role (Tavakoli et al., 2019).

Perhaps the oldest wooden dome for which there are specific references is the Qubbet al-Sakhrah (The Dome of the Rock) in Jerusalem, built circa 687-692 CE. Few references to the constructive composition of its dome are in existence; most references reproduce the magnificent drawing by De Vogue (1864), which represents the existence of two layers of wooden elements that remain joined at the dome crown. This, however, could not be its initial configuration. Indeed, Necipoğlu (2008, pp.31) states that "*the collapsed cupola was rebuilt in 1022-23*", and, more recently, al-Ratrout (2017) has provided further data. Along these lines, the dome over the al-Aqsa mosque (Jerusalem, Israel, 715 CE), also rebuilt circa 1035 CE, deserves mention. The structure of its outer layer stands out with a simple and effective system of ribs arranged according to the meridians of the sphere, which are strengthened with cross bracing elements arranged according to the diagonals of the panels. As in the Dome of the Rock Mosque, the interior structure hangs from the upper structure through a connection in its crown.

To the best of our knowledge, there are no other examples of complete wooden domes in the Islamic world. In Iran, where there is a long tradition of building domes, wood can be found as a secondary element in double-layer domes for the purpose of stiffening or connecting the interior and exterior layers built with brick (Ashkan and Ahmad, 2010). One exception could be the dome of the Esfahan Shah

Mosque (Iran). In Misztal (2018), this dome appears as a large wooden load-bearing structure that supports layers of the interior and exterior fabric. However, a more recent and detailed analysis (Tavakoli et al., 2019) defines it as a hybrid double dome, where the timber, despite being indispensable, appears only as a connecting element.

In the West, the five domes of the Basilica of San Marco in Venice, built in the mid-13th century, could constitute the oldest references, despite having been at least partially rebuilt after the fire of 1419. In the work of Fregonese et al. (2009), the structure of the Pentecost dome is clearly shown. This involves a framework of wood in which three levels of horizontal rings are of note, attached to a central king post and large inclined straight beams that span from the key structural compression ring to the base. On the set of rings, thin ribs form the domed outer surface.

Subsequent to this antecedent, no dome construction took place the mid-15th century when several domes were built in Veneto with wooden structures. Piana (2009) provides an interesting account of these constructions: most continue with systems similar to the domes of San Marco, such as that of San Giovani e Paolo (1430) and Santa Maria dei Miracolli (c. 1485). An exceptional case is that of the dome of the Church of San Pietro in Castello (mid-16th century), with its single-layer membrane formed by thin wooden meridian ribs very close to each other connected transversely by numerous noggings that follow the parallels of the dome. This structure is the only construction that is found to be similar in any way to our dome in the Hall of Ambassadors. Later, Palladio, with greater efficiency, recovers the initial system of San Marco, with double-layered domes with two parallel chords joined together by perpendicular counterbraces in San Giorgio Maggiore (1576) and with a hinted triangulation in the Church of the Santissimo Redentore (1592).

Along these lines, close in time, but with very different constructive approaches, Philibert de L'Orme's “Nouvelles inventions” appeared in 1561, wherein, although in the context of vaults, the possibility of their execution with a membrane behaviour is initiated. The double-layer parallel system is later rarely used and gives way to the appearance of systems characterised by the location of trusses in the meridians, where the timber chords located in the upper and lower layers together with cross bracing between them form a stable and efficient structure. Examples of this are given by the domes of Santa Maria della Salute in Venice (1687), the dome in the Church of the Invalides (Paris, France, 1693–1706), the new dome of St. Paul's Cathedral (London, England, 1710), or the Schönborn Chapel (Würzburg,

Germany, 1722). Antecedents of these systems are collected in various treatises, such as those by Jousse (1627) and Le Muet (1681).

In Spain, evidence shows that, from the 17th century, the constructive configuration of domes has followed the same trends as in the rest of Europe, highlighted by the texts of García Berruguilla (1747) and Bails (1796), in which vaults built with truss systems are drawn. However, another system had been developed in Spain in the Middle Ages and at the beginning of the Renaissance, with no historical reference to another era or region: domes featuring interlacing patterns. These are single-layer domes made up of wooden elements that follow a precise geometry governed by an interlacing design based on regular stars. Their structure and ornamentation are inextricably fused with great precision to form the supporting structure, a unique and distinctive feature of the Hispanic carpentry tradition and characteristic of Mudejar art. Only four domes of this type exist in the whole world, of which the largest and oldest is discussed in this article. The other three domes are the dome of the Duques de Maqueda Palace in Torrijos, Toledo, built at the end of the 15th century and currently housed at the National Archaeological Museum of Madrid (Nuere Matauco, Candelas-Gutierrez, and De Mingo García, 2020), the dome over the main staircase of the "Casa de Pilatos" in Sevilla (1538), and the dome built in the convent of San Francisco de Lima (Peru). In the latter case, the original was destroyed in the Trujillo earthquake of 1725. Later rebuilt, due to its poor state of conservation, it collapsed again in the 1940 Lima earthquake. The current dome is from the late 20th century, built by the Peruvian architect Alberto Barreto Arce.

Cultural Context and Previous Constructive Knowledge

Knowing how to build a wooden dome was the highest degree to which a member of the powerful medieval guild of traditional Spanish structural carpenters could aspire. Whoever reached such a level could hold the rank of *Iumetrico*. This is stated in the ordinances of the guild that were later included in the city ordinance of Sevilla in 1527 (Varela de Salamanca, 1527). The knowledge utilised by medieval carpenters in Spain, transmitted orally, came to light in later texts by López de Arenas (1619 and 1633), in a manuscript by Fray Andrés de San Miguel (early 17th century), and in the manuscript by Rodrigo Alvarez (1699). Diego López de Arenas, in his text of 1633, and Fray Andrés de San Miguel (c. 1640), both in the final part of their writings, address the construction of these domes.

The text by López de Arenas contains the procedures with which the carpenters were able to build resistant frameworks based on the rafter and collar-beam system, into which widely different interlacing

configurations could be incorporated as an integral element of the structure. All this provides an extremely interesting contribution: the possibility of prefabricating large areas of the structure in order, on the one hand, to facilitate the complex insertion of interlacing patterns into the framework, and, on the other hand, to be able to carry out an efficient assembly in situ. This was achieved on roofs with three or more planes by duplicating the hip rafter, so that each sector had a precise physical limit and sufficient stiffness and stability prior to lifting and mounting.

The case of these wooden domes is in fact a particular case of the evolution of the rafter system, in which a few main curved rafters serve as a fundamental continuous element. A series of secondary curved rafters are placed in the strict position required by the interlacing geometry that is being incorporated into the structure, all joined with a mesh of noggings that, following the interlacing pattern, join the main and secondary rafters, thereby forming a membrane with a highly effective structural behaviour.

The text by López de Arenas (1633) includes several geometric descriptions for the execution of what he calls a “half orange” that is, a hemispherical dome with an expansion somewhat greater than the semicircle. Its cross-section resembles a horseshoe arch and gives the name “bolsor” to the sector of the sphere that exceeds the line of the equator.

For its construction, López de Arenas proposes the division of the dome into sectors, and exemplifies the process with a sphere divided into ten spherical triangles, and includes the following drawing (Figure 2A), in which, with rudimentary knowledge, he strives to transfer spherical geometry into plane geometry. This author also contributes another interesting drawing in which a section of the dome is represented (Figure 2B). The drawing presents two definitions: on the one hand the length of the "bolsor" and, on the other hand, an attempt to describe the length of the main rafters. He defines the “bolsor” with a cant length equal to 1/6 of the diameter. To obtain the length of the main rafters, he indicates in the text that he is going to apply *“proposition 32 of the first book of Archimedes”* while utilising the relation 22/7 as an approximation to the number Pi. The figure incorporates other important details. On the one hand, the tenon that appears in the lower part of the beam that joins the curved rafter with the wall plate can clearly be seen. On the other hand, there are two joints: an abutting joint at the upper purling structural ring, and a stop-splayed scarf joint at an intermediate point of the arc. In another drawing of the same text (Figure 2C), López de Arenas represents a sector of the sphere with the introduction of interlacing patterns. In Figure 2D, based on this drawing, the location of the underlying structural elements has been represented, in which the duplication of the secondary rafter is utilised, each

sector thereby providing a solid outline that would allow its on-site assembly and its subsequent lifting into its final position.

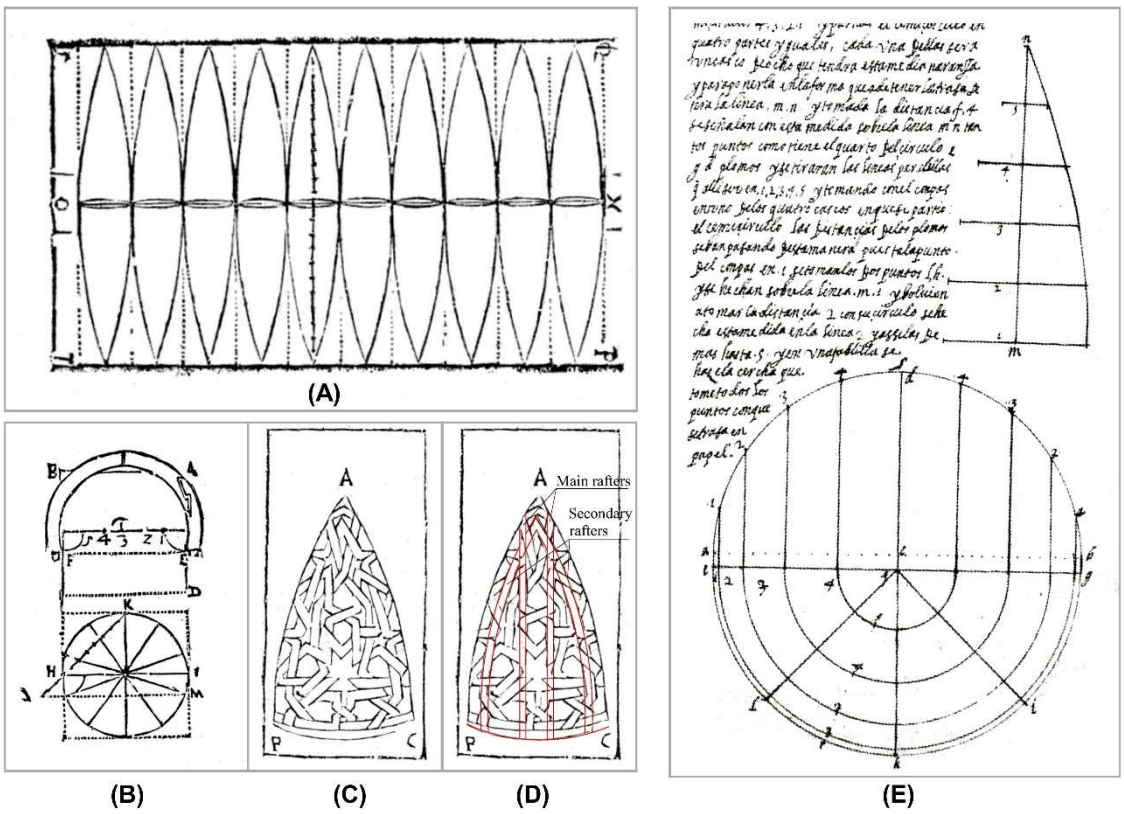


Figure 2: (A, B, C, D) Drawings from the 1633 text of Lopez de Arenas. (E) Drawings from the manuscript of Fray Andres de San Miguel.

Fray Andrés de San Miguel dedicates the last pages of the part of the manuscript on carpentry to the vaulted forms. That manuscript was fully studied in Nuere Matauco (1990). The approach by Fray Andrés is more focused on the geometric description of these domes than on their constructive materialisation, although in this regard, data of interest is also included. The objective pursued by Fray Andrés in the spherical vault involves obtaining a flat surface upon which to develop the interlacing pattern. In the same way as by López de Arenas, this approximation to a flat shape is carried out by displaying a sector of the sphere between meridians. To this end, Fray Andrés uses a division of the sphere into 8 sections, instead of the 10 utilised by López de Arenas. His proposal is very simple, since it is based on obtaining the longitude of a meridian, dividing it into *n* parts, and transferring to these divisions the length of the arcs corresponding to the height of each division.

The geometric description of the process described by Fray Andrés is much more exact than the incomplete proposal by López de Arenas. However, Fray Andrés fails to include the drawing of any sample of the interlacing pattern in his assertions, nor does he introduce any description of the materialisation of a framework constructed in this way. In fact, the drawing provided (Figure 2E) remains totally detached from any reference to construction elements.

DESCRIPTION OF THE DOME OF THE HALL OF AMBASSADORS

Geometry and Constructive Configuration

The section of the dome is ultrasemicircular; its profile extends 4.5º from the equator, and hence its cross-section is, in fact, a horseshoe arch. In this way, the inside diameter of the 9.28 m sphere is slightly reduced at the springline to 9.25 m. The length of the "bolsor" (basilar superelevation) is 0.36 m.

Its configuration can be understood as the repetition of 12 spherical triangles (sectors) around the zenith of the sphere. Each of these spherical triangles is made up of 3 spherical trapezium-shaped zones that are joined together (stacks A, B, and C in Figure 3). These sectors do not converge at the crown of the dome but end at a distance of 1.22 m from the pole of the theoretical sphere. For this reason, for the closure of the spherical surface in its upper part, a last sector is utilised in the shape of a spherical cap (stack D in Figure 3), which, in the form of a keystone, completes the dome. Each sector is made up of 2 main rafters of geometric continuity, although not physically present all the way up to the upper stack. In this way, only 12 rafters start from the wall plate and reach the upper stack, while another 60 secondary rafters start from the base but end up converging with each other.

The arrangement of the rafters is closely related to the interlacing pattern tracing that runs across the entire surface of the sphere. It is a design based on the concatenation of 10-point stars with their corresponding wheels that converge in the upper spherical cap, thereby generating a large central 12-point star.

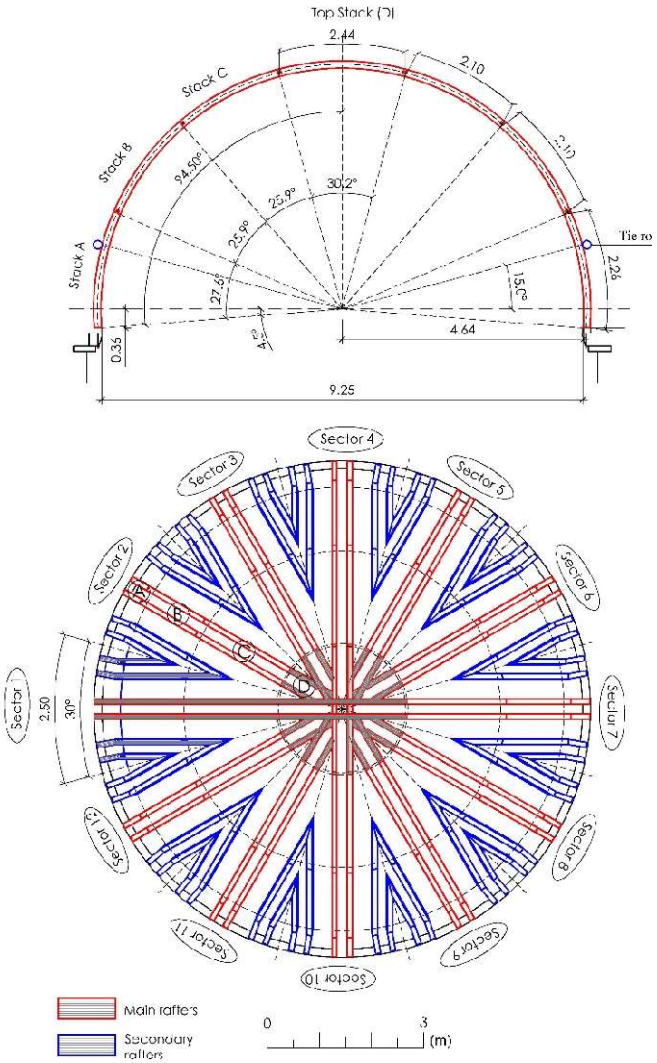


Figure 3: Geometrical diagram of the wooden structure of the dome. Middle section and plan. The main and secondary rafters are indicated.

The carpenter who built this work faced three major difficulties: first, the handling of spherical geometry; second, the introduction therein of the rigid geometry of the interlacing pattern; and third, the need for structural capacity. The interlacing pattern of 10-point stars, in plane geometry, should follow the rhythm as indicated in Figure 4A, where *w* is the width of the rafter. In the rules of traditional Spanish structural carpentry, it was common for the distance between the boundary rafters of the stars to be equal to twice the width of these rafters. From there, and inherent to the geometry of the interlacing pattern of 10-point stars, the distance between the side rafters to the stars had to be 3.854 times the thickness, whereby this measure was obviously obtained through the geometrical layout itself.

In order to adapt this pattern to spherical geometry, the carpenter maintained the distance of twice the thickness in the separation between bordering rafters of stars and modified the canonical separation

between stars, with distances between 3.7 to 4.3 times the thickness. As for the upper stack, the 12-point star is generated by the extension of the main rafters of each sector of the dome.

It can be presumed that the decision to build the dome was made once the walls of the Ambassadors Hall had been completed, that is, the dome had to be adapted to fit a pre-established dimension. The problem entailed translating such a geometric schema to a spherical shape while taking into account these pre-established dimensions (Figure 4B). To this end, the carpenter had to choose the thickness of the wood to be employed and from there to begin the interlacing pattern. Such a procedure is common in traditional Spanish structural carpentry.

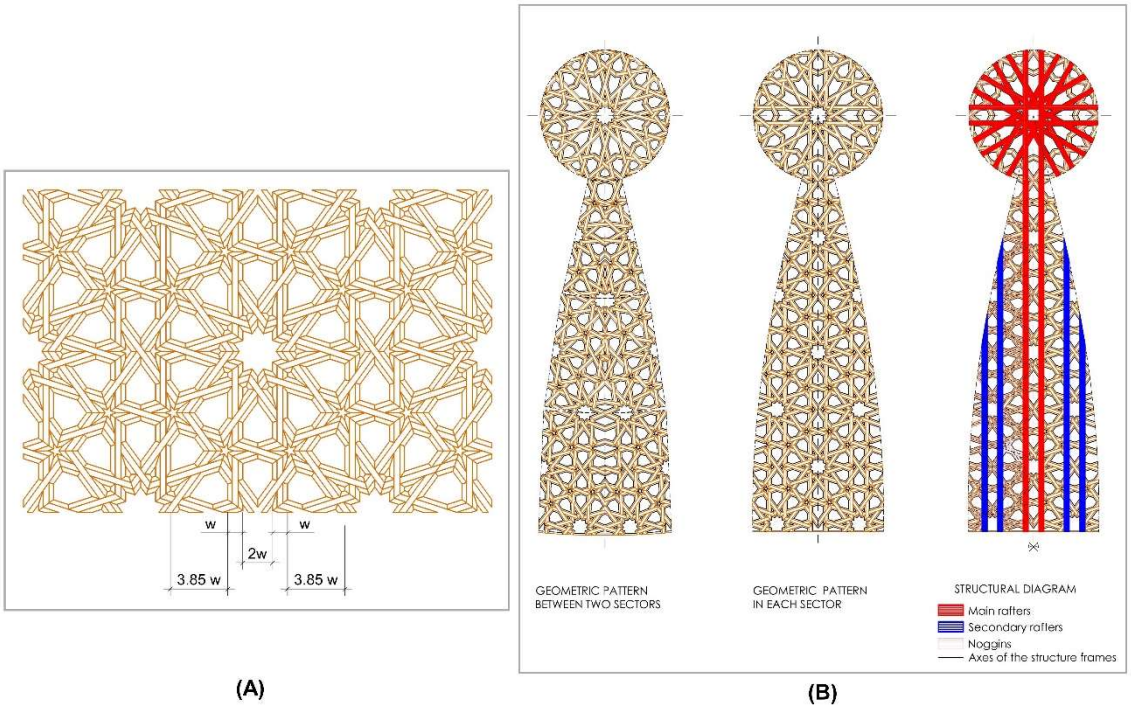


Figure 4: (A) Geometrical pattern of a 10-point interlocking star. (B) Geometric pattern and underlying structure in a sector and in upper stack.

Regarding their dimensions, all the pieces that make up the set (both rafters and noggings) have a thickness of approximately 10 cm (the width of the different sections, measured in situ, yield dimensions that range between 9.7 and 10.2 cm). The height of the beams is approximately 14 cm. In addition to the wooden elements, the dome has a wrought-iron tie rod on the exterior arranged along a parallel and located at approximately 15º from the equator of the dome.

Finally, it should be borne in mind that currently the keystone of the dome is attached to the covering structure. It remains unknown whether this was its original configuration or the product of an early reform. Elucidating this question will also be the object of structural analysis.

The Supporting Base

The load transfer from the dome to the base walls is performed with 3 orders of elements. In the first place, in the corners, 4 angle ties (35 x 28 cm) are placed at 45º and rest directly on the walls at their respective extremes, thus generating 8 support points. On the walls, there are 4 wall plates (17 x 25 cm). Between these wall plates and the angle ties there are another 8 timbers (secondary angle braces) with a smaller cross-section (21 x 17 cm) connected by metal joints (nails) to both the angle tie and the wall plates. This superposition of elements generates a 16-sided platform on which a circular plate (13 x 10 cm) is directly supported upon which the main rafters and secondary rafters of the dome structure rest.

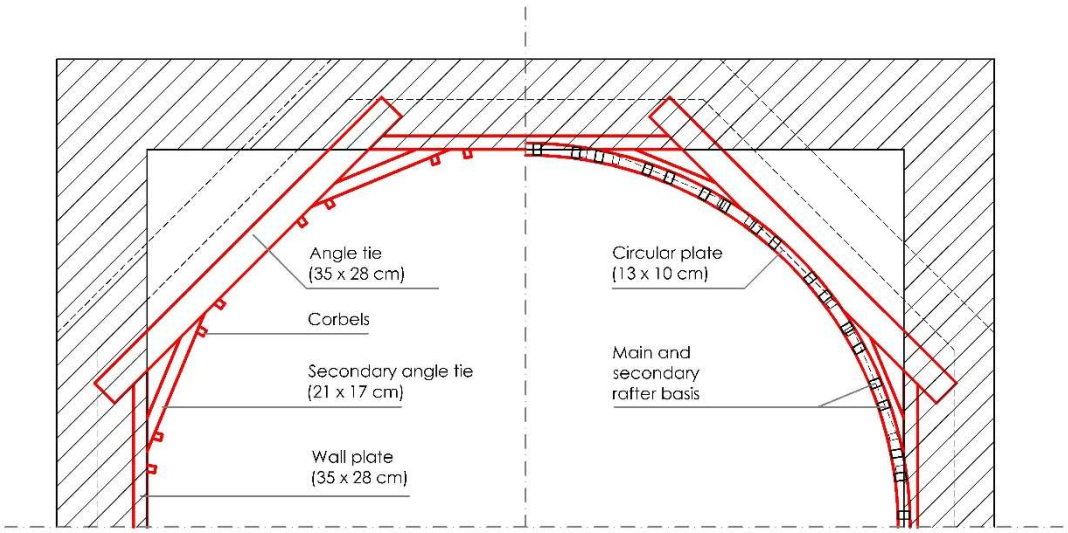


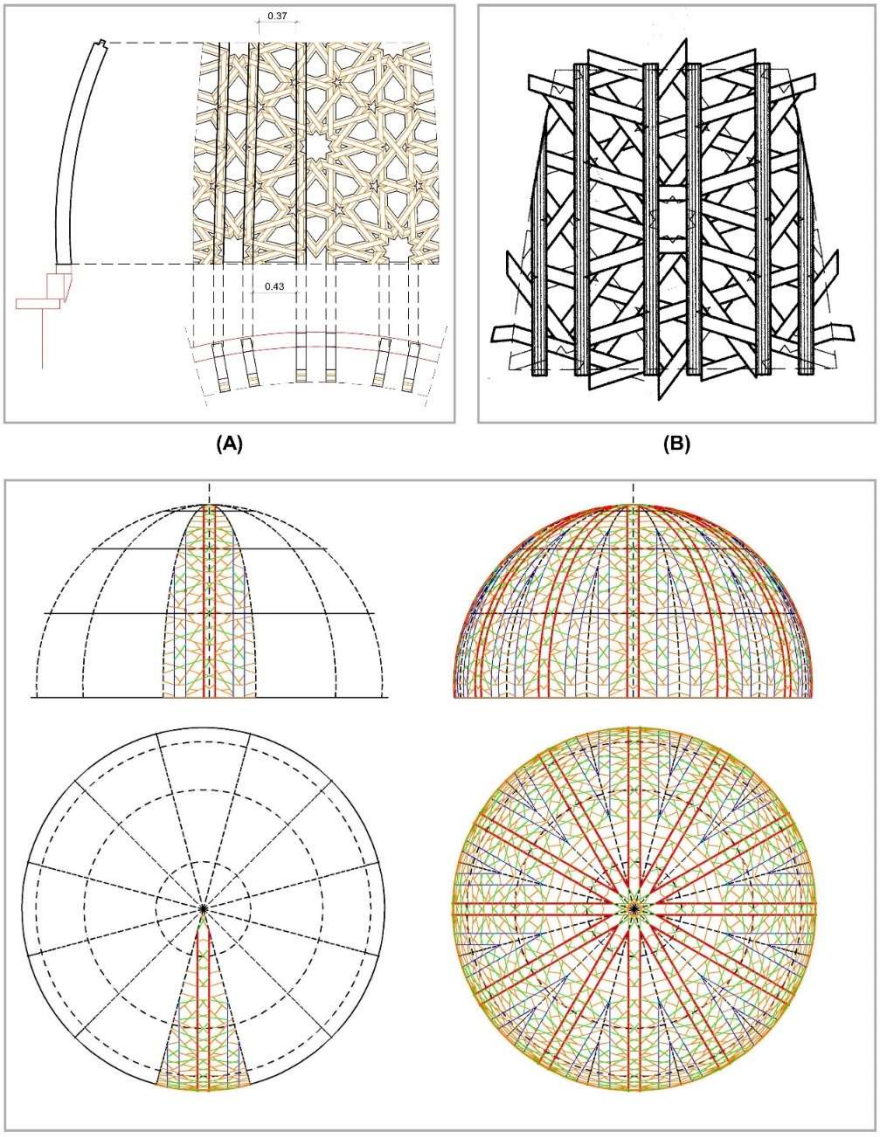
Figure 5: Configuration of the supporting base of the dome.

This circular plate has a projection with respect to the hexadecagonal structure of the base along practically its entire length, and becomes completely in projection, without support, in the area where the wall plate and the angle tie meet. Hence, in order to strengthen the base of the framework, several small corbels have been added (Figure 5).

The Construction of the Stacks of the Dome

The choice of stacks to be prefabricated is based on the geometry of the interlacing pattern. In each stack, it can be appreciated that the lower and upper limits coincide with the mean line of a star, leaving a complete star in the centre of the stack. The definition should have started with the conservation of the canonical layout in the centre of the stack, in the star, and from there the carpenter carried out skilful operations in modifying the distances between adjacent rafters to adapt to the progressive reduction of stack width towards the zenith.

Thus, in the lower stack (stack A) (Figure 6A), there are 6 curved rafters that are grouped in pairs and coincide with the location of the stars. Each pair always maintains parallelism and it is the area between these pairs of rafters that absorbs the distortion of the transition from planar to spherical geometry.



In the second sector (stack B) (Figure 6B), the location of the main rafters and the entire set of noggings that unite and stiffen each stack are represented. This analysis of the geometry and underlying structure has made it possible to create the 3D model which will provide the structural analysis (Figure 6C).

Configuration of the Joints

The joints between the main rafters are of the mortise-tenon type, with the tenon at the head of the lower element, which fits into a mortise carved into the base of the next rafter (Figure 7a). The joint is secured with a nailed overlay timber that provides continuity to the joint (Figure 7b).

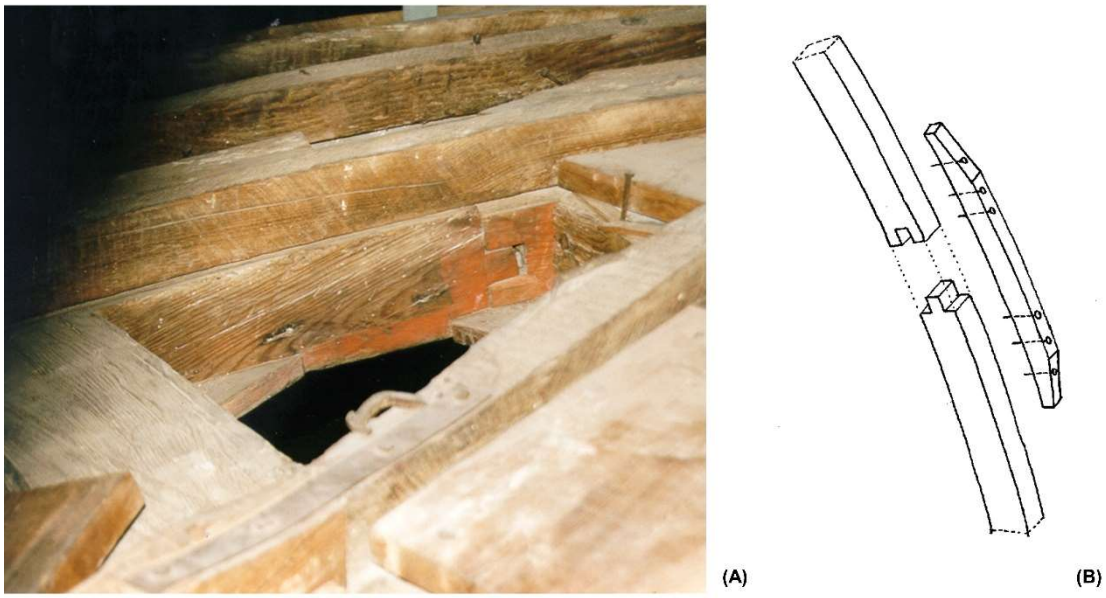


Figure 7: Joints between main rafters in the connection between the stacks.

The union between rafters and noggings is produced through the traditional recessing procedure in the side wall of the rafter into which the carved tenon of the front of the nogging fits.

The joint at the base of the lower rafters is very straightforward, with only a simple support of the base of the rafter on the circular plate, without the existence of a recess or similar, except for connection nails.

The profusion of noggings and their multiple directions due to their accommodation to the interlacing pattern ensures the transverse bracing of the structure on the surface of the dome. The interlocking existing between the noggings, and between the noggings and the rafters, generates a structural assembly with sufficient stiffness to maintain its stability even during the procedure of building the structure.

Hypothesis on the Manufacturing and Assembly Procedure

The analysis of the configuration of the joints between the main rafters indicates that construction had to be carried out from the upper to the lower level, that is, by placing the circular crowning panel first and, once this was fixed (to an auxiliary structure), by incorporating the rest of the stacks successively, in descending order. Indeed, given the configuration of the joints between rafters, in reverse order it would be impossible to place the key stack by fitting its protruding tenons into the mortise at the top of the rafters. To this end, a scarf joint would have had to have been made in order to allow such a fit. For the

same reason, in this descending assembly, it is only possible to join the lower rafters with the circular plate without any tenons, as is the case here. Thus, the assembly procedure that is represented in Figure 8 can be considered.

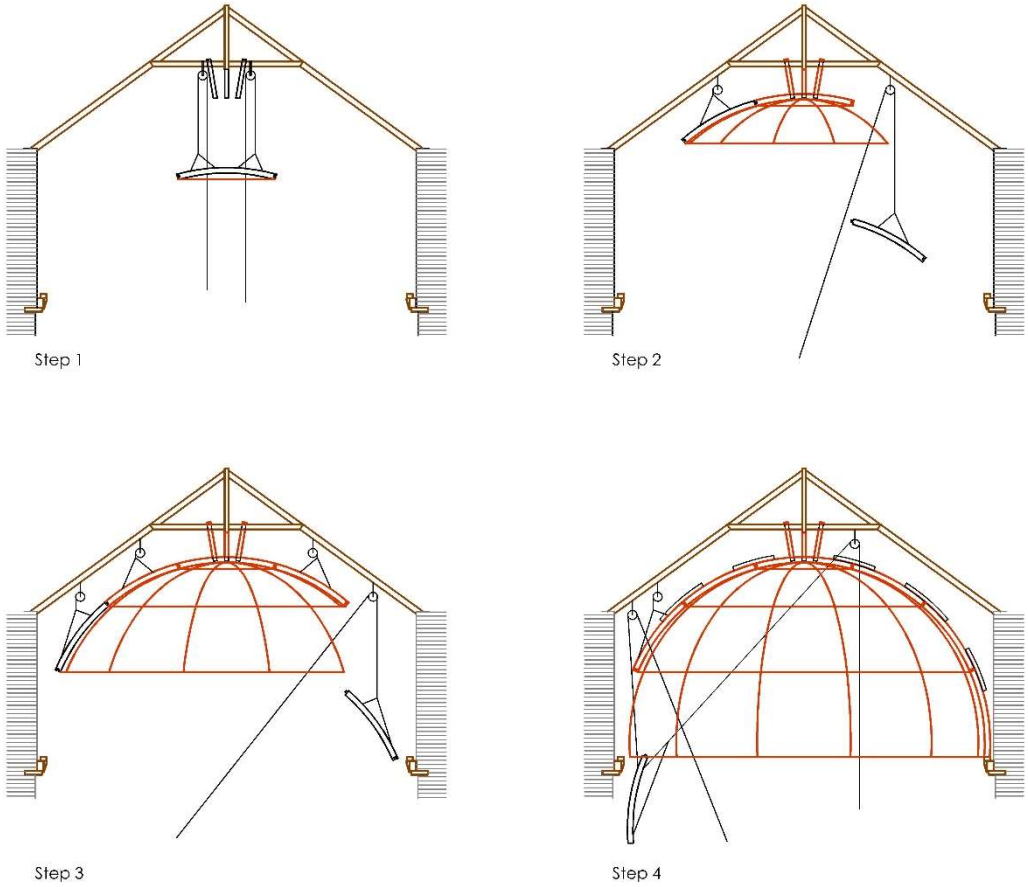


Figure 8: Hypothesis of the construction process of the dome.

STRUCTURAL BEHAVIOUR

Models, Hypotheses, and Calculation Procedure

Based on the aforementioned constructive knowledge, the objectives involve the analysis of what makes this dome stable and resistant, since it is a set of elements that, except for the main rafters, principally present pinned joints and with secondary rafters that do not reach the keystone, without presenting any apparent effective subjection with respect to the main rafters.

In addition to the current configuration of geometry, links, and loads, two other possibilities are discussed: that the dome could function as an independent structure, without the connection that currently

exists with the upper covering; and that the dome could directly support the layers of the roof (waterproofing, covering and wind loads) without the need for said upper structure.

In order to meet these objectives, the calculation has been carried out with two geometric models in which various hypotheses of external links, loads to be supported, and internal joint constraints between frames are considered. Model 1 (M1), or simplified geometry, exclusively includes the main rafters and the tie rod. The load corresponding to the spherical triangles is applied on these rafters, an area that would act exclusively as a plementery. This model seeks to ascertain the overall resistant capacity of these main elements and the possibility of admitting full loads of coverage, wind, and snow. Model 2 (M2), fundamental to this study, includes the main rafters and secondary rafters and all the transverse frame-noggings that generate the interlacing pattern.

In M1, three hypotheses are proposed (M1a, M1b, M1c), which combine the types of actions and links with the upper roof framing (covering): M1a, actions of their self-weight (dead loads) with the connected (hanging) keystone of the upper structure; M1b, actions with the structure supported only at the base; M1c, actions of their self-weight, permanent cover, wind, and snow loads, with the structure supported only at the base.

In M2, four hypotheses are proposed (M2a, M2b, M2c, M2d), which differ regarding whether or not they consider the ability to transmit tensile stresses or moments in the joints between noggings and between these and rafters. In this model, only self-weight loads are taken into account and no connection is considered in the keystone.

For the structural analysis of these models, the finite element method (FEM) has been used, which is a widely contrasted procedure for the numerical analysis of this type of structure (Coz Diaz et al., 2012; Chun et al., 2015; Gao and Ohta, 2007; Galassi et al., 2018). The calculation has been developed using SAP2000 software v.22.

The geometry is referred to in Figure 10 above, defined from the respective axes of the pieces that pass through the centres of gravity of their sections. Due to the geometric pattern that governs the structure, all the axes are concurrent, and therefore no incorporation of additional frames is necessary to model eccentricities.

Material Properties and Actions Considered

Identification tests carried out on the wood of the structure during restoration work at the beginning of the 21st century indicate that the oldest elements are European larch (*Larix decidua Mill.*), whereby Scots pine (*Pinus sylvestris L.*) was used in the renovation repair work of the mid-19th century (Fernández Aguilera and Pérez Ferrer, 2000).

In order to obtain the resistant properties of larch, the UNE EN-1912 (AEN/CTN-56, 2012) has been used. According to this standard, larch wood can be assigned to resistant classes ranging from C14 to C30 depending on its origin and its singularities. In the case of this dome, since it is an element of special importance, it is appreciated that the wood was specifically selected. Proof of this appears in the reduced diameter of all types of knots (simple knots, splay knots, and overlapping knots) in relation to the dimensions of the pieces. Therefore, using the visual grading rules of the various European countries where larch wood could come from, it can be considered that it would always be classifiable to at least an intermediate grade (ST2 of French origin, S2 from Italy, or T2 from the Netherlands), and could be assigned to at least resistant class C22 or C24. In this study, the wood has been conservatively considered as class C22. For the characteristic strength and stiffness properties, the values indicated in UNE EN-338 (AEN/CTN-56, 2017) are used. The permanent and imposed loads, as well as the load combinations and load factors have been determined based on Eurocode 1 (Technical Committee CTN 140, 2019).

Structural Analysis: Hypotheses, Connections, and Stresses.

Simplified model analysis (M1)

First of all, the resistant capacity of the dome is studied, made up exclusively of the twelve pairs of main rafters that have continuity from the base to the keystone. This continuity is guaranteed by the type of connection between stacks as explained in the previous section (Figure 11). In the model, although the noggings are not included, their effect is considered by coercing the lateral instability of the rafters. Furthermore, movement is restricted to the area where the metal tie rod is located. The results obtained for the 3 hypotheses are (Table 1):

Table 1. Results obtained for the hypotheses considered

	M1a: Self-weight only. Structure is connected in the crown.	**M1b:** Self-weight only. Structure is free in the crown.	**M1c:** Self-weight, covering, wind, and snow. Structure is free in the crown.
Nmax	-9.7 kN (upper stack)	-12.6 kN (upper stack)	-25.9 kN (upper stack)
Mmax	-1.02 mkN (upper stack)	0.77 mkN (upper stack)	-14.65 mkN (upper stack)
Vmax	2.25 kN (tie rod)	1.73 kN (tie rod)	4.45 kN (supporting base)
Safety factor	**0.13**	**0.10**	**1.84**

The result shows that the structure is stable. However, the safety factor shows that the design strength is exceeded if it supports the full load of the covering. In contrast, the structure presents sufficient safety factor in the case of supporting only its self-weight and the weight of the upper covering. The values of the maximum stresses obtained are of a similar magnitude in Hypotheses M1a and M1b, and are therefore independent of whether the dome is hanging from an upper structure or not. This analysis thereby reveals that the connection in the crown is not necessary for its stability and resistance, and therefore this connection may have been absent in its original configuration.

Analysis of the complete structural model (M2)

In this model, all the elements of the dome are considered: the main rafters, the secondary rafters, and the noggings distributed on the spherical surface.

A singularity of this structure is that the secondary rafters converge with each other and are only supported at their base. With this configuration, they would be unable to absorb any load, which otherwise would constitute a mechanism, and hence the stability provided by the noggings to the structural assembly is to be analysed.

The joints between noggings and rafters are produced with a mortise into which the tenon is inserted. Observation in situ has revealed no existence of nails or similar that would guarantee a possible transmission of tension stress. Thus, these connections can have compression transmission capacity,

403 practically zero tension transmission, and a certain capacity to prevent rotation. For the analysis, four
404 hypotheses have been made that consider different internal constraints (fixed or pinned) and consider the
405 ability or not to admit tension stress.

406 Table 2 describes the hypotheses and the result, whether it be consistent or not, of the structural
407 behaviour of each model. The external restraints have been considered as free-bearing supports and the
408 structure without connection in the keystone, supporting only its self-weight.

409 *Table 2. Hypothesis for joint connections and frame stresses in Model 2*

Structural hypothesis	Connections (Joint constraints)	Frame stress	Results of the analysis	Comments
M2a	Fixed	No Tension (compression)	Consistent	The result is consistent: the structure is stable.
M2b	Fixed	Tension + Compression	Consistent	The result is consistent: maximum vertical deformation occurs in the upper stack.
M2c	Pinned	No Tension (compression)	Non-consistent	The structure is unstable: its deformation is not as a rigid body.
M2d	Pinned	Tension + Compression	Non-consistent	As in the previous case, the deformation of the structure is not consistent.

410

411 From Table 2 it can be deduced that the dome is only stable if the connections between rafters and
412 noggings, as well as between the noggings themselves, are considered rigid. If the inability of the
413 noggings to transmit tension stress is added to this, then the only viable hypothesis is Hypothesis M2a. It
414 is therefore essential to ensure that the analytical model reproduces physical reality, to determine whether
415 the moments that occur in the joints can be resisted by said joints. The structural analysis indicates that
416 the maximum design shear in the joints between rafters and noggings is produced in the confluence zone
417 between the longest secondary rafters (V_d = 0.43 kN; M_d = 0.015 mkN). The maximum bending moment
418 is generated in the noggings located in the contact between the main rafters and the upper sector (V_d =
419 0.17 kN; M_d = 0.045 mkN).

Recent publications delve into the analysis of the resistance capacity in mortise-tenon-type joints (Li et al., 2015; Yang, Yu, and Law, 2020; Kunecký et al., 2016; Chen, Qiu, and Lu, 2016). Considering exclusively the tenon cross-section (30 x 100 mm) therefore, without considering increases in strength derived from friction and joint operation due to compression of the joint base, a load capacity of 0.21 mkN is obtained for the bending moment and of 9.4 kN for the shear force. This indicates that the tenon itself has the capacity to absorb moments of at least 4.7 times the acting worst bending moment and 5.4 times the maximum shear force.

Based on the results indicated, it can be stated that the existing joints can behave as effective fixings, as indicated in Eurocode 5 (Technical Committee CTN 140 2016, Art. 5.4.2.). This circumstance demonstrates the fundamental contribution of the noggings to the stability and resistance of the whole.

In order to validate the adopted calculation model, for the cases that present coherent behaviour (Table 2, M2a and M2b), the distribution of forces in one of the main rafters has been compared with that of a theoretical simple arch of the same dimension (Figure 9). It is evident that, with the rafters working only in compression, the behaviour of the rafters in our model is similar to that of the simple arch, as was the case in the simplified model M1b. This result is consistent with the type of carpentry joints in the dome nodes.

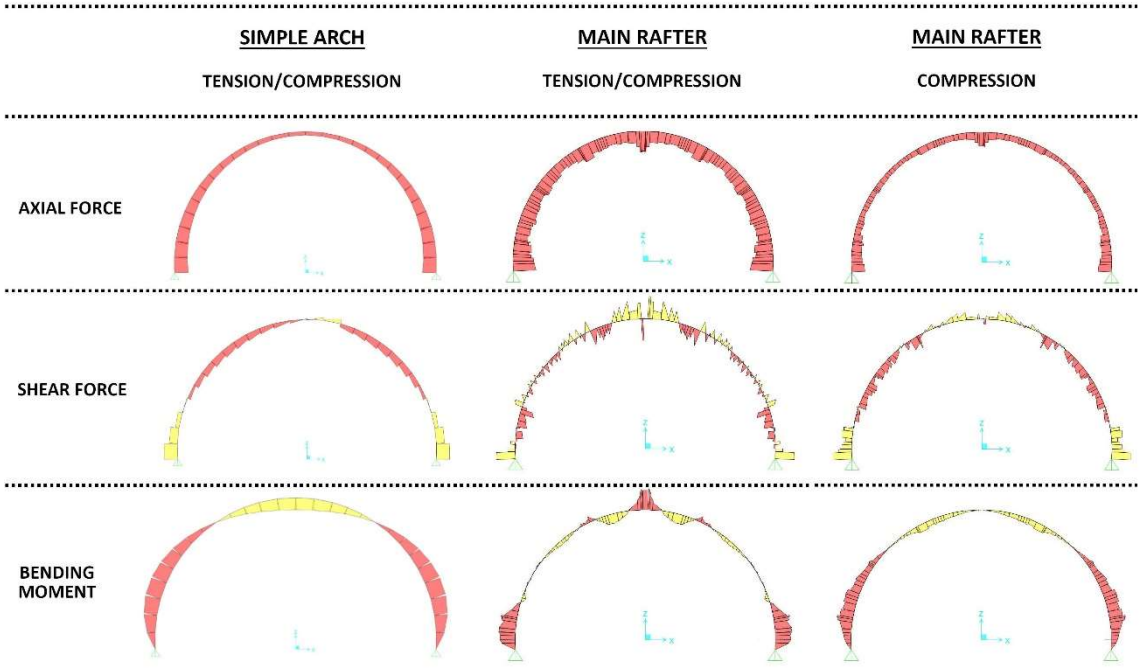


Figure 9: Results obtained in the analysis of the main isolated rafter.

Although the axial forces are similar in the two cases, the model with frames only in compression (M2a) presents higher values for both the shear forces and the bending moments. Figure 10 shows the forces that occur in the main rafters and in the two secondary rafters, in the M2a and M2b models.

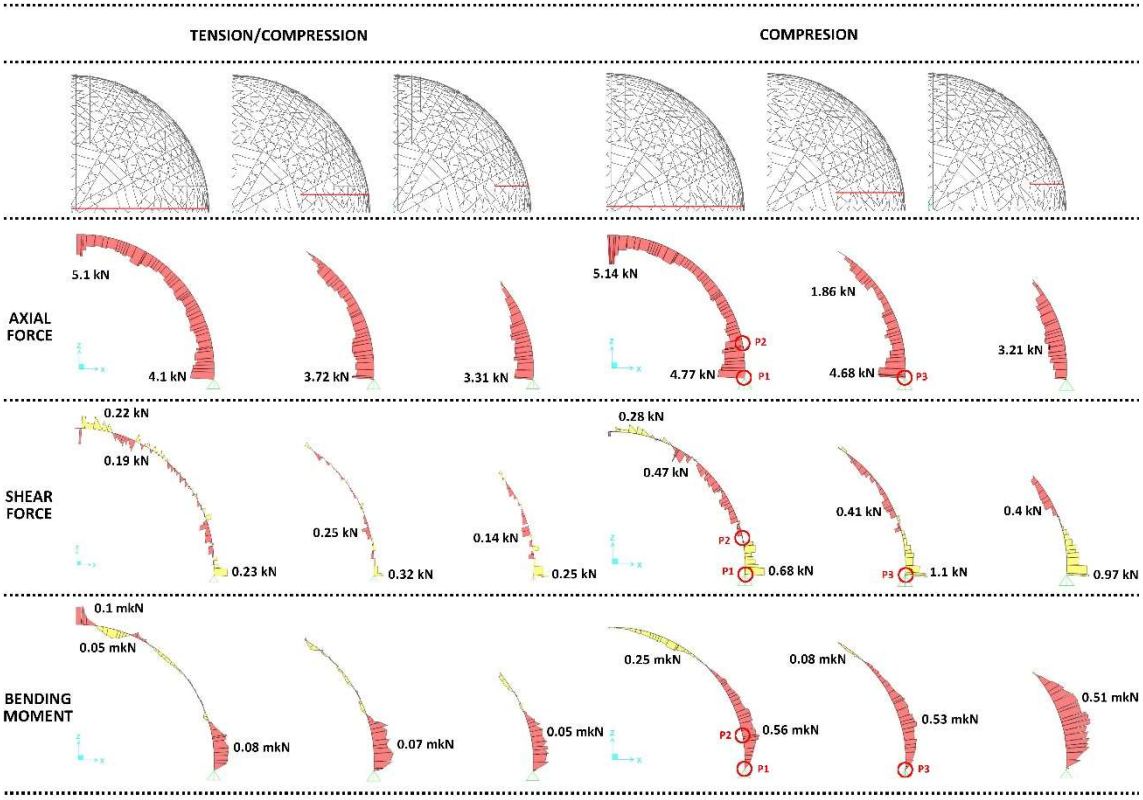


Figure 10. Results obtained in the analysis of M2a (right) and M2b (left) models. Circles highlight points where the safety factor is obtained.

Table 3 indicates the safety factors for the frames with maximum forces in the hypothesis of frames only in compression (M2a), using the Ultimate Limit States procedure of Eurocode 5. The results reflect that the resistance of the sections utilised is far from exhausted, with only 19% of its resistant capacity being employed in the worst case scenario (combination of axial and bending moment in point 2).

Table 3. Safety factor analysis on main rafters

								Safety Factor			
		Forces/Bending Moment			Stresses (kN/m^2)			N_d, M_d	V_d	N_d, M_d	V_d
Point	**K_{mod}**	**N_d (kN)**	**V_d (kN)**	**M_d (mkN)**	**$\sigma_{c,0,d}$**	**$\sigma_{m,y,d}$**	**τ_{max}**	**C14**		**C22**	
1	0.6	-4.77	0.68	0	-340.71	0	108.74	0.0021	0.08	0.0005	0.04
2	0.6	-3.11	0	0.56	-222.14	1714.29	0.00	**0.1866**	0.00	0.0701	0.00
3	0.6	-4.68	1.1	0	-334.29	0	175.91	0.0020	0.13	0.0005	0.06

The three-dimensional analysis of the forces (Figure 11) shows how, with frames only under compression (M2a), all the rafters work as a succession of arches, collaborating together to drive the gravitational loads to the base of the structure.

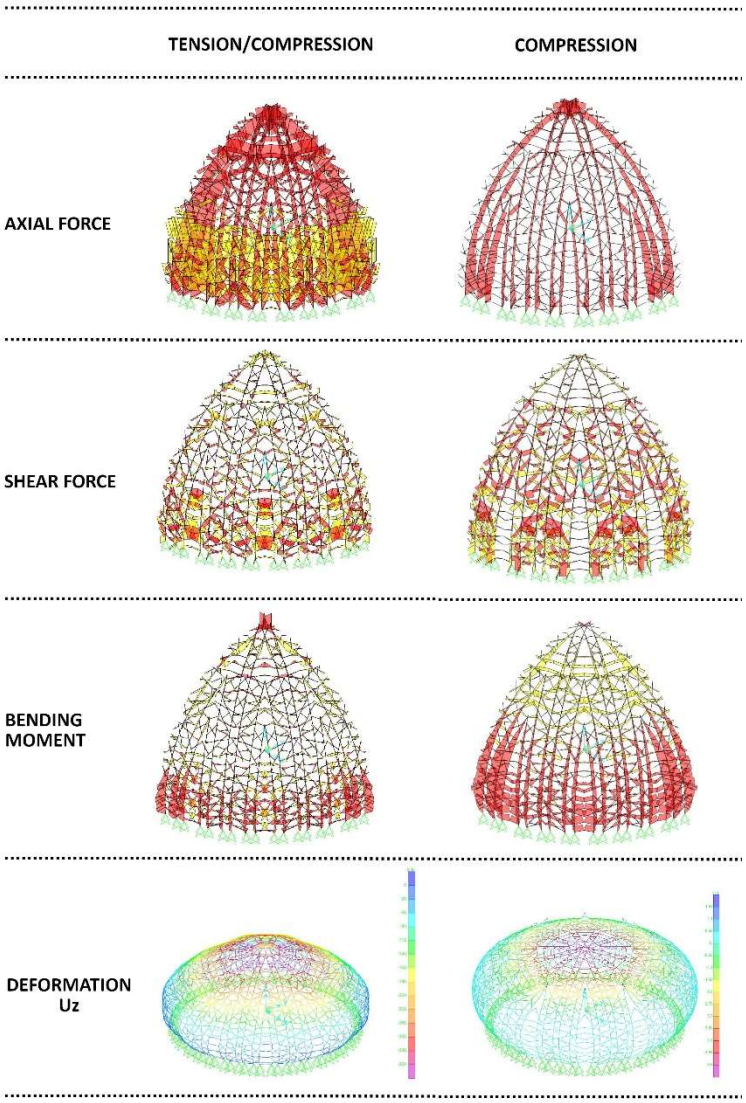


Figure 11: 3D internal forces diagram on a dome sector for M2a (right) and M2b (left) models

Likewise, Hypothesis M2a shows that the maximum transverse tension stresses are generated between the 22° parallel and the spring line of the structure (Figure 12A). In the dome of the Hall of Ambassadors there is a tie rod placed at the height of the 15° parallel, in the area where the greatest horizontal deformation occurs (Figure 11). This position of the tie rod coincides with point P2 (Figure 10) in which the maximum bending moment occurs in the rafters. The mitre joint between the noggings makes it impossible to transmit tension stresses in the direction of the parallels since there are no metal connectors. The tie rod absorbs these tension stresses and maintains the stability of the dome by preventing the separation between the different sectors. In every sector, the interlacing of the noggings increases the stiffness in their surface, which allows a unitary behaviour as solid-rigid, thereby enabling the assembly process by sectors as explained in Figure 8.

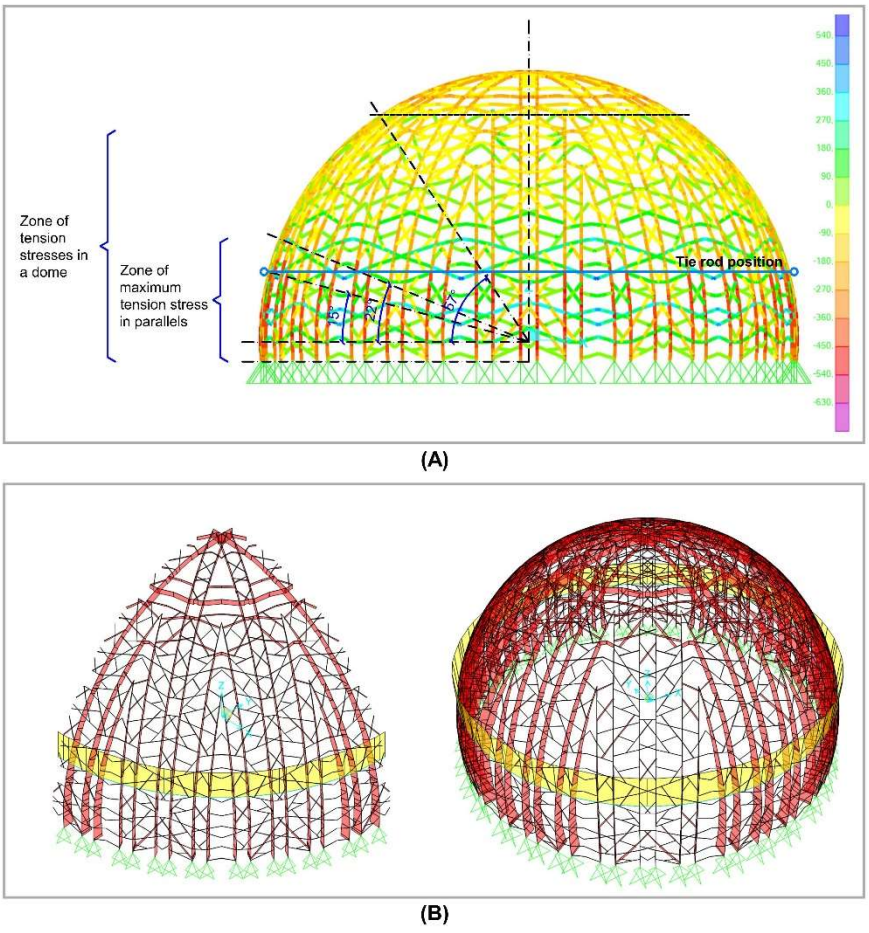


Figure 12: (A) Stresses on the dome for the M2a model and location of the tie rod. (B) Axial forces on Model M2a considering the presence of the tie rod.

In an analysis of the upper stack, the noggings that delimit it should function as a compression ring to ensure the overall stability of the structure. This occurs in the hypothesis with frames only under

476 compression (M2a) when the tie rod is incorporated. The result (Figure 12B) presents total coherence: the
477 frames of the upper spherical stack work under compression, thereby granting the rafters the
478 responsibility of transmitting the gravitational load, with the tie rod resisting the tension stresses
479 generated in the dome. The compressive force in the frames that limit the upper stack, of 1.65 kN, is
480 much lower than their resistance to axial capacity, of 92.3 kN.

481 The maximum reactions in the rafters in the base of the structure are $R_{x,d}$ = 0.30 kN; $R_{y,d}$ = 0.30 kN;
482 $R_{z,d}$ = 4.0 kN, with the global vertical reaction at the base being 253.1 kN. The maximum axial force in
483 the tie is N_d = 9.1 kN. Given the age of the tie rod, assuming a resistance for a steel with practically no
484 carbon content of 180 N/mm^2, a circular section ϕ8 or a square section of 8x8 mm^2 is sufficient to
485 withstand the axial force. Given that the dome was built in 1427, the original solution to resist these
486 tension stresses could have consisted of a series of staggered frames arranged transversely to the main
487 rafters. In this hypothesis, the current metal tie rod would have been incorporated in a subsequent
488 intervention.

489 *Analysis of the model without a covering*

490 Having verified that the only congruent hypotheses are those of M2a and M2b, and that Hypothesis
491 M2a reliably represents its structural behaviour, an analysis is carried out of the possibility that the
492 structure can work autonomously, that is, without an additional upper framework, thereby supporting the
493 dead load of the covering, as well as those of snow and wind. The results indicate that, as long as the
494 joints only transmit compressions as in Hypothesis M2a, then the dome behaves as a mechanism against
495 horizontal actions. Therefore, it is concluded that from its origin there had to be some type of upper
496 covering similar to the one that exists today.

497 **CONCLUSIONS**

498 The wooden dome of the Hall of Ambassadors of the Alcázar of Seville, dating from 1427, is one of
499 the rare examples of wooden domes built following the guidelines of the Spanish medieval carpentry
500 tradition. This, in addition, is the oldest specimen of those that are still preserved. The analysis of the
501 historical evolution of the construction of wooden domes reveals that there is no structure with similar
502 characteristics in other countries.

503 The geometric and constructive analysis has made it possible to ascertain both the configuration of
504 the joints and the construction process that could be followed for the construction of this dome. It is
505 evident that it was performed by means of independent prefabricated sectors, whereby a unique assembly
506 process was followed for its construction. This began by lifting and assembling the upper spherical stack
507 to which the lower stacks were successively assembled until the spring line of the structure was reached.

508 The models and calculation hypotheses adopted, which have been analysed using FEM, enable
509 the structural behaviour of the dome to be determined. It has been verified that the stability of the set can
510 be explained, on the one hand, due to the constructive continuity of the 12 main rafters, and, on the other
511 hand, through the fundamental structural function exercised by the noggings. These elements follow the
512 layout of the lacework generated by the interlacing pattern geometry present in the cupola and unite the
513 main rafters in multiple directions. The interlacing between the numerous noggings provides stability to
514 the set, revealing not only the decorative but also the resistant role of these pieces, which generate a
515 bracing of the structure on the surface of each panel. This function of the noggings would be especially
516 relevant in absorbing the forces generated during the lifting and assembly process of the different stacks
517 during the construction of the framework.

518 The structural analysis has revealed the impossibility of transmission of tension stresses due to the
519 mortise-tenon joints existing between noggings and rafters. Likewise, it is shown that these joints have a
520 load capacity greater than 1.5 times that corresponding to the combination of the applied force and
521 moment, and hence it can be considered that they behave as fixed joints in the face of acting actions. The
522 frames most requested in the structure are the main rafters, the longest (those that reach the zenith of the
523 dome) being those that support the greatest forces. Despite this, the safety factor obtained indicates that
524 they work at 19% of their capacity. Therefore, with all the frames in a good state of preservation, the dead
525 load, which currently corresponds exclusively to the sheathing, could be increased to 5 kN/m^2.

526 The structural behaviour of the spherical framework implies the appearance of maximum tension
527 stresses from the 22° parallel. Inability to absorb the tension stress of the noggings would imply the
528 collapse of the structure, which is why the fundamental role of the metal tie rod (tension ring), capable of
529 absorbing these tension stresses, is demonstrated. This tie rod is placed at the 15° parallel and coincides
530 with the area of greatest transverse deformation of the structure.

To conclude, delving into the constructive and structural study of historical architecture has the value of revealing not only the construction techniques and procedures, but also the knowledge treasured in each age by master builders. Furthermore, a full understanding of the functionality of its different components provides useful information towards perfecting and improving the conservation and maintenance processes necessary to ensure their state of preservation. Likewise, from the point of view of current structural design, given this knowledge, it is possible to design optimal structural typologies based on the adaptation of its construction system, thereby improving those features that are in need of adjustment.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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