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Fatigue crack growth analysis based on energy parameters: A literature review

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ABSTRACT

Fatigue crack growth (FCG) in metallic materials has been studied using non-linear parameters, which permit a better understanding of crack tip damage. The objective here is to make a literature review about the use of energy parameters in this context. Fundamental concepts are presented, namely the different types of energy that can be identified (the external work, the macroscopic elastic energy, the plastic dissipation, the internal potential energy and the thermal energy). FCG rate has been related with the dissipated energy measured externally, with the dissipated energy in the reversed plastic zone, with a punctual value of dissipated density energy at a critical location ahead of crack tip and with the thermal energy. The links between FCG mechanisms and energy parameters are exploited and guidelines for their use are proposed.

1. Introduction

The stress intensity factor range, ΔK, is probably the most widely used parameter for conducting fatigue crack growth (FCG) predictions (Paris and Erdogan, 1963). The stress intensity factor is an elastic parameter which quantifies the magnitude of stress and strain singular fields around the crack tip. However, the inability of da/dN-ΔK models to explain the effect of stress ratio and the odd results observed for short cracks or for variable amplitude loading, led to the appearance of the crack closure concept (Wolf, 1970). According to this approach, during a portion of the load cycle the crack is closed, therefore the effective load range is $\Delta K_{eff} = K_{max}\text{-}K_{open}$, where K_{max} and K_{open}, are the maximum and the crack opening values of the stress intensity factor, respectively. This is an interesting concept, which was able to explain most of the trends of da/dN observed experimentally. Therefore, since 1970 a huge experimental, numerical and analytical effort was developed to understand and quantify crack closure. Twenty-three years went by before anyone questioned its validity, which demonstrates the strength of this concept. However, once again, some limitations were found, which promoted the appearance of new approaches to the problem. After 1993, different authors proposed alternative concepts to ΔK_{eff}, namely the Unified approach (Vasudevan and Sadananda, 1999; Vasudévan et al., 1993) and the UniGrow approach (Noroozi et al., 2005). In both cases, a two-parameter driving force was proposed, based on ΔK and K_{max}, assuming the relevance of residual stresses ahead of the crack tip and of the upper part of load cycle.

However, the FCG studies based on ΔK, K_{max} and ΔK_{eff} also have several limitations, namely: (i) they are only valid in the small-scale yielding range and the definition of the limits of Linear Elastic Fracture Mechanics is not straightforward; (ii) the models are completely phenomenological, not derived from physics; (iii) the relations between da/dN and ΔK usually have dimensional problems, which are accommodated by the fitting parameters; (iv) The inclusion of crack closure is complex, since this depends on many parameters, namely material properties and stress state; (v) they are not able to give information about the underlying damage mechanisms. These methods are not able to provide information about the mechanism that take place at the very crack tip and that cause the progressive fatigue damage. Despite these limitations, ΔK-based approaches have been widely used in component design.

However, for a better understanding and modelling of FCG phenomenon, it is important to look directly to the crack tip region, where the fatigue damage effectively occurs. Fortunately, the numerical methods and the experimental techniques have advanced in such a way that made this possible. This permitted a clear progress towards a new level of understanding of FCG. As the crack tip phenomena are non-linear and irreversible, alternative crack driving parameters are recommended. Therefore, different non-linear parameters have been proposed over time, namely, the crack tip plastic strain (Borges et al., 2020;

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Nomenclature

a	Crack length
β	Taylor–Quinney coefficient
B	Thickness of the specimen
BD	Brazilian Disc
BRS	Blunting-resharpening mechanism
CT	Compact Tension specimen
CTOD	Crack Tip Opening Displacement
δ	Displacement (of the testing machine)
D	Damage parameter
DBC	Double cantilever beam
DIC	Digital Image Correlation
E_c	Critical plastic energy density
E_k	Kinetic energy (macroscopic)
$E_{p,elas}$	Elastic potential energy (macroscopic)
$E_{p,ext}$	Potential energy of external forces
FEM	Finite Element Method
F_{ext}	External force (applied by the grips of testing machine)
FCG	Fatigue Crack Growth
FCGR	Fatigue Crack Growth Rate
G	Energy release rate
ΔK	Stress Intensity Factor range
K_e	Kinetic energy (macroscopic)
ΔK_{eff}	Effective Stress Intensity Factor range
ΔK_{th}	Threshold stress intensity factor range
K_{max}	Maximum Stress Intensity Factor
K_{open}	Crack opening Stress Intensity Factor
LCF	Low Cycle Fatigue
L_{cd}	Critical distance
MT	Middle crack Tension specimen
MVC	Microscale void coalescence
Q	Heat (energy flow due to temperature difference)
$\Delta\sigma_0$	Fatigue limit
TCD	Theory of Critical Distances
U	Internal energy (sum of microscopic kinetic and potential energies)
U_k	Internal kinetic energy
U_p	Internal potential energy
U_s	Energy associated with crack surface
VHCF	Very high cycle fatigue
W	Work of external forces
w_e	Elastic strain energy density
W_c	Energy per thickness dissipated in each load cycle
W_{plas}	Plastic deformation work
w_{plas}	Density of plastic deformation work ($=\int_{cycle}\sigma_{ij}\times d\varepsilon_{ij}^{p}$)
$\dot{W}_{plas}$	Rate of plastic work, or energy dissipation rate
π	Potential energy ($=E_{p,elas}+E_{p,ext}$)

Pokluda, 2013), the plastic intensity factor (Lopez-Crespo and Pommier, 2008; Pommier et al., 2009), the CTOD (Pelloux, 1970; Pippan and Grosinger, 2013), the plastic CTOD (Antunes et al., 2018), the size of crack tip plastic zones (Park et al., 1996; Zhang et al., 2001) and the dissipated energy (Rice, 1967).

From an energetic standpoint, fatigue can be conceptualized as a continuous and irreversible process of energy dissipation (Wang et al., 2018). Energy is a transversal concept, with great potential in this context, therefore it is important to organize the information about the different approaches that have been used. The present paper provides a comprehensive review of the literature on how robust parameters based on energy concepts can be used for the prediction of the FCG. The review is performed in a critical manner in the sense that pros and cons of the different methods are discussed. But, first of all, it is important to review fundamental concepts, namely the different types of energy, which is done in the next point.

2. Fundamental concepts of energy

The concept of energy is not easily defined, being usually considered as an ability to produce work (Rankine, 1881). There are only two fundamental types of energy: kinetic energy and potential energy. The **kinetic energy** is the energy associated with the movement of matter, i.e. with the change of the position of the body in relation to a certain referential (Coriolis, 1829; Leibniz, 1863). The macroscopic kinetic energy is given by $m \times v^2/2$, being m the quantity of matter and v the speed of the body. Bodies also have an internal (microscopic) kinetic energy, which is due to the movement of their elementary particles (atoms and molecules). In fact, all particles move incessantly at considerable speeds. Different variants can be identified, associated with different movements: translational energy of molecules; rotational kinetic energy (in polyatomic molecules); vibrational energy (when atoms and molecules vibrate around a common centre of mass); kinetic energy of electrons, spin energy of electrons, etc. This disordered energy of elementary particles constitutes the thermal energy. The increase in temperature increases the internal energy. On the other hand, the macroscopic kinetic energy has a higher level of organization, as all particles move in the same sense at the same time.

The potential energy is due to the interaction of a body with the surrounding bodies. It represents the capacity of the system to perform work as it changes from one configuration to another (Kelvin and Tait, 1879; Roche, 2003). Its definition is: a scalar quantity equal to the work done by forces to transport bodies to the position where, by convention, it is assumed that there is no potential energy. Normally the positions for which there is no potential energy correspond to positions of stability of the particles or to positions in which there is no interaction between them because they are sufficiently far apart. At a macroscopic level, it is possible to identify the gravitational potential energy and the elastic deformation energy. The gravitational potential energy is associated with the gravitational interaction between bodies. The elastic deformation energy is felt in mechanically stressed bodies that deform elastically, such as springs. At a microscopic level, several potential energies can be identified: associated with the molecular bonding forces between atoms (electromagnetic); associated with the forces between electrons and the nucleus (electromagnetic), and nuclear energy (associated with nuclear forces and much bigger than molecular energy). This microscopic energy is negative because if the particles move apart, the interaction forces would produce negative work. The increase of body's internal defects (dislocations, voids, etc.) increase the body's energy, that is, the potential energy reduces in absolute value. Chemical energy is the binding energy between molecules, which is released in chemical reactions. Thus, food or gasoline have microscopic potential energy (which has to do with the bonds established between atoms), which is transformed into macroscopic mechanical energy in humans and in an internal combustion engine, respectively. In conclusion, potential energy is stored energy, that is, "in potential", as opposed to kinetic energy, which is energy "in action".

Let us now focus on structural components, which can be seen as closed systems, i.e. having no mass transfer with surroundings. The energy balance, defined in the First Law of Thermodynamics (Adkins, 1968), is:

$$dW + dQ = dE_k + dE_p + dU \tag{1}$$

where dW is the work of external forces, dQ is the system energy flow (heat) due to temperature difference, dE_k is the macroscopic kinetic energy, dE_p is the macroscopic potential energy (gravitational and elastic) and dU is the internal energy (sum of microscopic potential and kinetic energies). Heat is considered positive when is transferred to the system and the work is positive when is done by the surroundings of the system.

3. Energy concepts in cracked components cyclically loaded

FCG occurs in cracked components submitted to cyclic loading. Fig. 1 shows CT and MT specimens, which are the standard geometries usually employed in FCG studies (Quan and Alderliesten, 2022a). Looking at Equation 1, the macroscopic kinetic energy, dE_k is not relevant in this context, because usually there are no significant mass movements. Also, the gravitational component of the potential energy does not change during FCG testing. The other terms will be discussed next.

(i) Work of external forces, dW

The external loads, applied remotely to the crack tip, introduce mechanical energy into the body:

$$dW = F_{ext} \times d\delta \tag{2}$$

with F_{ext} being the forces applied by the grips of testing machine on the specimen and $d\delta$ being the respective displacement. Usually one of the grips is fixed, which means that its displacement is zero, therefore only the other grip is responsible for the external work. The energy introduced in the cracked body during one load cycle, dW/dN, is obtained by integration of Equation 2.

(ii) Macroscopic potential energy, $dE_{p,elas}$

The elastic deformation of the cracked body results in macroscopic potential energy, $dE_{p,elas}$, which greatly depends on the crack length. The increase of crack length decreases the potential energy for the same external applied displacement. Quan and Alderlestien (Quan and Alderliesten, 2022) calculated the elastic energy stored in a MT specimen from its rigidity.

A second consequence of the elastic deformation is the thermoelastic energy. In the presence of a volume variation, there is a change of temperature, which is called the Joule–Thomson effect (Thomson, 1878). In other words, the microscopic potential energy is transformed into microscopic kinetic energy, keeping the internal energy, U, constant. This change in temperature, i.e. this change of microscopic kinetic energy, is directly proportional to the hydrostatic component of the stress tensor. Nevertheless, the elastic deformation process of solid metallic materials results in negligible volume changes. Consequently, the temperature variation, as defined by the Joule–Thomson effect for solids is insignificant (Kuo et al., 2005). Moreover, it entails a reversible interchange of mechanical and thermal energies, which, in turn, results in no net energy dissipation or absorption during cyclic loading. Anyway, these small variations of temperature have been used to study the elastic stress field around a crack tip (Díaz et al., 2004). Small surface temperature variations, of the order of tens of milliKelvin, were detected using infrared detectors and used to calculate the stress intensity factor of metallic materials.

(iii) Plastic dissipation, dU

The presence of a crack has a significant effect on the stress and strain fields in the crack tip region. Fig. 2 displays the four distinct zones that can be identified ahead of a fatigue crack tip (Paul and Tarafder, 2013). The first two regions (I and II) make up the elastic zone, where the material is deformed in an entirely elastic manner. Region I is the far-field stress region. The magnitude of stress and strain fields in region II is determined by the stress intensity factor. The transition from region II to region I occurs when the K-based crack tip stress field is no longer valid. Region III is called the monotonic plastic zone, where plastic deformation occurs during monotonic loading, followed by elastic loading–unloading. In the fourth region (IV), located close to the fatigue crack-tip and known as the reversed or cyclic plastic zone, a hysteresis loop occurs. Crack advance in metals is primarily influenced by the damage state and the failure behaviour of a small region of material, located ahead of the crack tip. This region, known as the process zone, is usually assumed to be the cyclic plastic zone.

In contrast to the thermo-elastic energy, the plastic work, associated with dislocation movement, generates a significant amount of energy (Farren and Taylor, 1925). At each point, the plastic dissipation per load cycle is:

$$\frac{dU}{dN} = \int_{cycle} \sigma_{ij} d\varepsilon_{ij} \tag{3}$$

This energy has two components: the energy dissipated into heat, which increases the temperature, i.e. the microscopic kinetic energy of the fundamental particles, U_k; and the energy used to increase the internal potential energy, U_p. The plastic heat generation rate is usually expressed as a fraction of the plastic work rate:

$$\frac{dU_k}{dt} = \beta \times \frac{dW^{plas}}{dt} \tag{4}$$

where β is named the Taylor–Quinney coefficient (Taylor and Quinney, 1934). The Taylor–Quinney coefficient is known to exhibit dependency on both the plastic strain and on the plastic strain rate. Macdougall (Macdougall, 2000) showed that the Taylor–Quinney coefficient increases from 0.5 to 0.9, with plastic strain in an aluminium alloy. Zhang *et al.* (Zhang et al., 2018) found that increasing the strain rate of the AA7075-T651 aluminium alloy from 1100 to 4200 s^{-1} resulted in an increase of the Taylor–Quinney coefficient from 0.4 to 0.9. According to Neto *et al.* (Neto et al., 2020) the fraction of plastic work that gets converted into heat increases with the accumulation of plastic deformation. The thermal energy moves rapidly within the metallic material due to the high thermal conductivity of the metals. The remaining energy, i.e. (1-β) × dW^{plast}/dt, is responsible for changing the internal potential energy, through the increase of dislocation density and surface formation. The accumulation of fatigue damage and eventual final fracture can be attributed to this relatively small portion of plastic work, that is stored as internal energy. Therefore, the FCG should be linked to the variation of internal potential energy, which is not observed in the literature. Note that the increase in dislocation density reduces the absolute value of internal potential energy, and the same happens with the formation of a new surface. It is generally accepted that, for ductile materials, the plastic dissipation is much bigger than the surface energy (Quan and Alderliesten, 2022; Ranganathan et al., 2008). The energy dissipated through plastic deformation can be directly linked to the accumulated plastic strain (Nittur et al., 2014). The dissipation of energy can also be studied by adopting the perspective of entropy increase, as proposed by Hajshirmohammadi and Khonsari (Hajshirmohammadi and Khonsari, 2020).

In very high cycle fatigue (VHCF) tests the macroscopic deformation

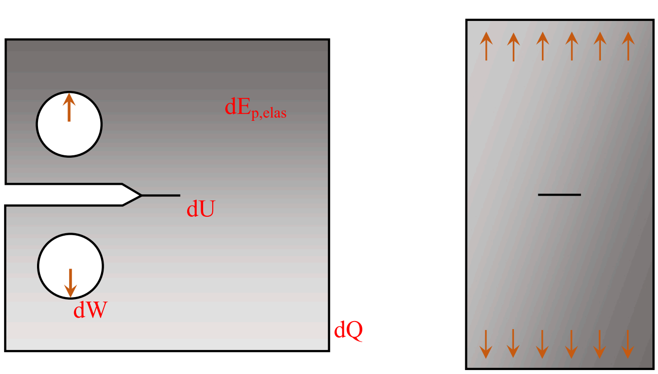

Fig. 1. Standard CT and MT specimens for FCG studies.

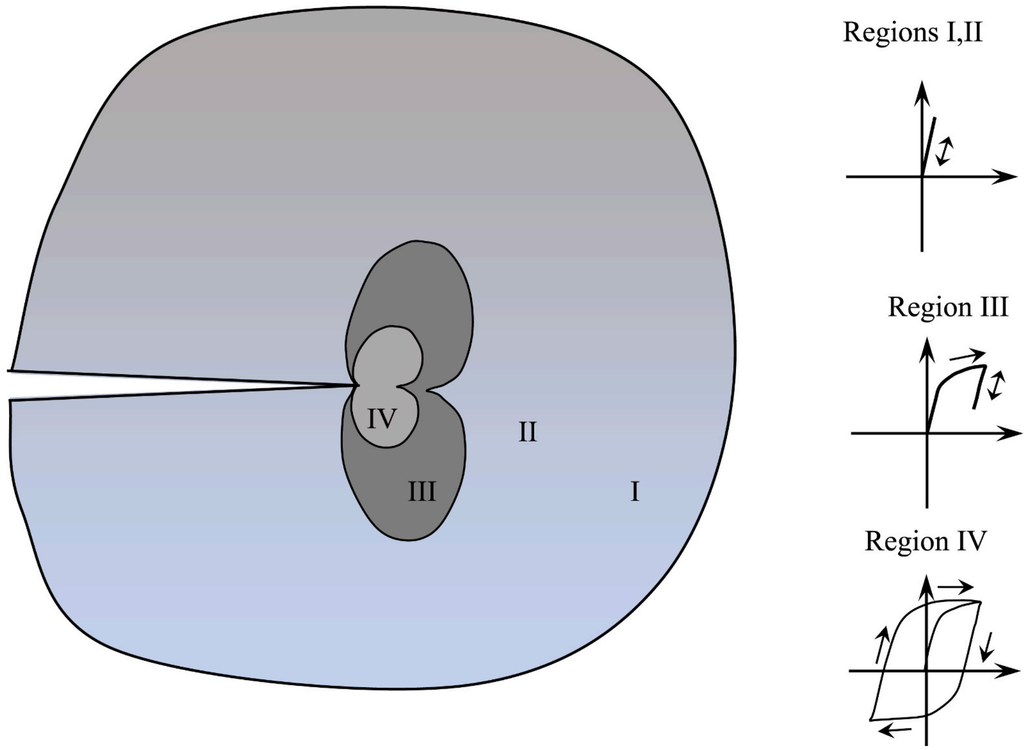

Fig. 2. Schematic representation of a cracked body with crack tip zones and energy fluxes.

is expected to be elastic, however there is a great increase of temperature. This indicates that the plastic dissipation may also occur at a microscopic level (Boussattine et al., 2020). This heat generation at the grain scale originates self-heating phenomena that takes place even in the region outside the crack tip plastic zones. When a polycrystalline material is subjected to external loading, individual grains exhibit distinct deformation behaviour due to the anisotropy of their elastic and plastic (crystal slip) properties. Specifically, grains that possess lower degree of plasticity experience larger stress magnitudes than their counterparts, while softer neighbouring grains exhibit a greater propensity for crystal slip and subsequent plastic strain accumulation (Korsunsky et al., 2007). According to Boulanger *et al.* (Boulanger, 2004), this type of energy was proved by testing non-cracked smooth specimens using a resonant fatigue machine. In VHCF the testing frequency is 20,000 Hz, and the heat generated needs to be extracted rapidly from the specimens. Two solutions are generally adopted to solve this issue: refrigeration with forced air flow; and intermittent testing.

(iv) Energy flow (heat) to surroundings, dQ

The local increase in temperature near crack tip resulting from plastic deformation produces a temperature field. Heat moves from higher temperatures to lower temperatures. A flux of heat is, therefore, expected from the component to surroundings, by convection, radiation and conduction. Conduction occurs through the grips, while convection and radiation occurs at the remaining surfaces of the body.

(v) Energy balance

The application of energy balance (Equation 1) to a cracked body, submitted to cyclic loading gives:

$$\frac{dW}{dN}+\frac{dQ}{dN}=\frac{dE_{p,elas}}{dN}+\frac{dU}{dN} \tag{5}$$

The change of macroscopic elastic deformation energy, $dE_{p,elas}$, is due to crack propagation. The increase of crack length decreases the elastic energy, therefore this term is negative. It is relatively small, because the crack propagation in each load cycle is also small. The work of external forces produces internal changes in the component, namely its plastic deformation ahead of the crack tip and the formation on new surface. This plastic deformation increases the temperature (i.e. the internal kinetic energy, dU_k), and changes the dislocation density and distribution, hence decreasing the internal potential energy in absolute value, dU_p. The increase of temperature resulting from plastic deformation may produce a flux of heat to outside the cracked body (dQ), which therefore has a negative sign. The microplasticity occurring in the elastic regions, is neglectable in comparison with the plastic work in the reversed plastic zone, except in VHCF tests. The law of conservation of energy applied to cracked bodies is, finally:

$$\frac{dW}{dN}+\frac{dE_{pot,elas}}{dN}=\left(\frac{dU_k}{dN}+\frac{dU_{pot}}{dN}+\frac{dU_s}{dN}\right)+\frac{dQ}{dN} \tag{6}$$

This means that the work of external forces (dW/dN) and the small change of macroscopic elastic deformation resulting from crack increment ($dE_{p,elas}/dN$), are responsible for the heating of the specimen (dU_k/dN), for the modification of dislocation structure (dU_p/dN), for the formation of new surfaces (dU_s/dN) and for a flux of heat to the surroundings (dQ/dN). The creation of new surface could have been included in the change of internal potential energy. However, Quan and Alderlestien (Quan and Alderliesten, 2022), considered a separate term for the creation of a new surface, stating anyway that it is quite small. According to Ranganathan *et al.* (Ranganathan et al., 2008b), the parameter dU_s has to be distinguished from the theoretical energy to separate the atoms. All these are irreversible processes, respecting the Second Law of Thermodynamics, which indicates an increase of entropy, i.e. of disorder.

Equation 6 is able to accommodate different situations that may happen. A phase transformation, for example, changes both the internal potential energy, dU_p, and the internal kinetic energy, dU_k. The presence of residual stresses is also very usual, namely due to the local plastic deformation at the crack tip. From an energetic point of view, residual stresses cause a change in the interatomic distance, and therefore in the internal potential energy. Furthermore, note that there is always a zero global balance of stresses: if there are residual compressive stresses in a given region of the body, there must be residual tensile stresses in the surrounding area.

4. FCG models based on energy parameters

The energy parameters have been linked to FCG by different authors. For this connection to be made on a physical basis, it is important to understand which mechanisms are responsible for FCG. Cyclic plastic deformation is usually assumed to be the main damage mechanism responsible for fatigue crack growth in metallic materials (Hamam et al., 2005; Wasén and Heier, 1998). It is essential to understand how this plastic deformation produces crack propagation and which energy components (dissipation) are involved. Several models have been proposed to relate the occurrence of cyclic plastic deformation at the crack tip with crack propagation and with the formation of striations usually observed on the fracture surface of ductile materials tested in air. The model of striation formation by crack tip plastic blunting of Laird (Laird and Smith, 1963a) is largely accepted as a general description of the propagation mechanism of fatigue cracks in regime II of da/dN-ΔK curves. According Laird's model, plastic deformation at the crack tip is highly concentrated at 45°, producing blunting and creation of new fracture surface. Crack tip compression stresses reverses slipping, the fracture surfaces approach, but the new surface cannot be removed by re-connection of the atomic bonds, which is in accordance with the entropy law of thermodynamics. For most metallic materials, when tested in air condition, the amount of fatigue crack rewelding is small during the unloading part of the fatigue stress cycle. According to Pippan *et al.* (Pippan et al., 2011), the cyclic deformation of the crack flanks in the vicinity of the crack tip does not contribute to the crack extension. In other words, only the dislocation movement in the immediate vicinity of crack tip produces FCG, therefore the most adequate energy parameter should be the energy dissipation close to the crack tip.

An alternative line of research states that FCG is due to damage accumulation, in a process similar to low-cycle fatigue failure. In this case, the failure at a specific point of crack path occurs when the accumulated energy dissipation reaches a critical value. The accumulation of dissipated energy must be calculated only along the crack path.

However, FCG has been linked with different energy parameters (Maquin and Pierron, 2009), namely the total dissipated energy, the energy dissipated in the cyclic plastic zone, the dissipated energy at a specific point ahead of the crack tip and the thermal energy. Fig. 3 is a schematic view of these energy parameters.

(i) Models based on external work, dW/dN

The experimental measurement of the force applied to the specimen and of the corresponding displacement can be used to calculate the external work (dW/dN in equation 6). This parameter was correlated with FCG rate. Sgambitterra *et al.* (Sgambitterra et al., 2023) studied FCG in a Nickel-Titanium (50.8 % Ni–49.2 % Ti) shape memory alloy using an eccentrically-loaded single Edge Crack tension specimen. They calculated the dissipated energy using a global strategy, evaluating at the loading pin of the sample the area of the hysteresis loop on a cycle in a force versus displacement plot, as follows:

$$\frac{dW}{dN} = \int_{cycle} F \times d\delta \tag{7}$$

and through the integration of volume over a near-crack-tip process/transformation zone local method. A linear relation between da/dN and this dissipated energy was found in log–log scales.

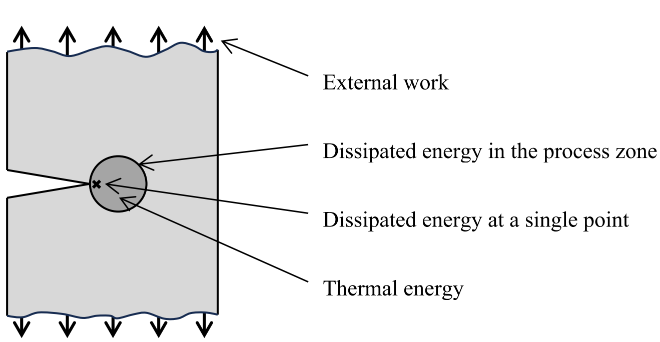

Fig. 3. Schematic view of the energy parameters used as crack driving parameters.

Ranganathan *et al.* (Mazari et al., 2008; Ranganathan et al., 1987) also measured the hysteresis energy by numerically integrating the area under the load versus (amplified) displacement diagrams, measured under the loading line of the specimen.

Note that, in this approach, there is no concern about what is really responsible for FCG, and which fraction of this external work is used to propagate the crack. In actual application, implementing this concept presents a challenge as load–displacement hysteresis cannot usually be observed. This is due to the fact that the loading-displacement curves during both loading and unloading are nearly indistinguishable. In fact, the elastic portion of the displacement is much bigger than the plastic component. Furthermore, the expected occurrence of experimental scatter can introduce significant errors in the calculated external work (Quan and Alderliesten, 2021). Therefore, only very ductile materials can be considered, which explains the limited use of this approach.

(ii) Models based on the energy dissipation in the process zone, dU/dN

The analysis of cyclic plastic deformation energy ahead of the crack tip is a better approach since this is closer to the crack tip. Two strategies have been used: (a) to relate da/dN with the plastic energy dissipated per load cycle, defining a da/dN versus dW_{plast}/dN law; (b) to calculate a critical value of cumulative plastic energy, which is used to predict the number of load cycles required to reach the critical energy and consequently the da/dN from this.

(a) Breitbarth and Besel (Breitbarth and Besel, 2017) combined the use of finite element method with experimental data retrieved from Digital Image Correlation to estimate the accumulation of the plastic strain energy at the crack-tip plastic zone, but no link was made with FCG rate. The displacements measured with DIC were introduced in a FEM model. This model was 2D, assuming plane stress state and bilinear isotropic hardening of the AA2024-T3. The propagation was not simulated, therefore there was no crack closure. According to Daily (Daily, 2004), it was observed that the total plastic dissipation per cycle tends to decrease as the material hardening behaviour and crack tip constraint increase, which aligns with the expected outcomes.

Klingbeil (Klingbeil, 2003) linked da/dN with plastic energy dissipated in the reverse plastic zone per-cycle. 2-D elastic–plastic finite element analysis was employed to predict the energy of a stationary crack subjected to constant amplitude, mode I loading, without accounting for the phenomenon of crack closure. The study thoroughly examined two different constitutive behaviours, elastic–perfectly plastic and bi-linear kinematic hardening. It was assumed that the critical energy for crack propagation is equal to the critical energy release rate, $dW/da = G_c$:

$$\frac{dW}{dN} = \frac{dW}{da}\frac{da}{dN} \Leftrightarrow \frac{da}{dN} = \frac{E}{K_{Ic}^2}\frac{dW}{dN} \tag{8}$$

being $G_c = K_{Ic}^2/E$. Within his proposal, a correlation between the phenomenological fields of fracture and fatigue was drawn through the concept that the amount of energy required to propagate a crack by a specific measure Δa, regardless of the methods in which that energy is dispersed – whether by monotonic or fatigue loading- remains constant. FCG rate was found to be proportional to ΔK^4. Bodner *et al.* (Bodner et al., 1983) postulated that FCG rate is directly proportional to the plastic dissipation per cycle within the reversed plastic zone ahead the crack tip.

Smith (Smith, 2011) obtained a linear relation between the dissipated energy per cycle predicted numerically and experimental da/dN values. Only one load cycle was applied between crack increments. This relation was used to predict FCG rates after overloads and an excellent agreement was found with experimental results. The 304 stainless steel was modelled assuming purely kinematic behaviour. Mazari et al. (2008) also proposed a linear relation between da/dN and dissipated energy for the 2024-T351 aluminium alloy. Quan and Alderlestien (Quan and Alderliesten, 2021) proposed that the relation between da/dN and plastic energy dissipation, dW_{plas}/dN, can be fitted to a power law (da/dN = D($dW_{plas}/dN)^n$), and that the relation between the plastic dissipation and ΔK could be fitted in a power law ($dW_{plas}/dN = A(\Delta K)^b$). They also observed that the correlation of da/dN with the plastic dissipation is dependent on the stress ratio.

Szata and Lesiuk (Szata, 2002; Szata and Lesiuk, 2009) proposed a new energy-based parameter as a crack driving parameter:

$$\frac{da}{dN} = D(\Delta H)^k \tag{9}$$

where

$$\Delta H = \frac{W_c}{B(1 - \frac{K_{Imax}^2}{K_{fc}^2})} \tag{10}$$

W_c is the energy per thickness dissipated in each load cycle, K_{Imax} is the maximum value of stress intensity factor, K_{fc} is the critical value of cyclic stress intensity factor and B is the thickness of the component. They pointed out that the dissipated energy included the crack closure phenomenon.

(b) On the other hand, different authors used a critical cumulative energy to predict da/dN. Cojocaru and Karlsson (Cojocaru and Karlsson, 2009) demonstrated that the dissipated energy could be used to predict crack growth retardation after an overload and the stress ratio effect. The energy dissipated as a result of plastic deformation was computed in a specific region adjacent to the crack-tip. Subsequently, this value was compared with the critical threshold. The region of interest was a semi-disk area with a radius r, centred at the current crack-tip and encompassing the cyclic plastic zone.

Nittur *et al.* (Nittur et al., 2014) used a critical cumulative dissipated energy, calibrated with one experimental value of da/dN. A linear-elastic, perfectly-plastic constitutive response was assumed for the AA7475-T7351. A node release scheme was subsequently followed, controlled by the accumulation of energy. The selection of the rectangular dissipation domain was made with the intention of effectively encompassing the reverse plastic zone originated at the end of the initial load cycle. Propagation was made at minimum load and the numerical model assumed a plane strain state. The critical energy dissipated plastically (30 N·mm/mm^2) was scaled proportionally according to the mesh size, that is, the longitudinal size of the crack tip elements. Similarly, values of 86 and 100 N.mm/mm^2 were obtained for Inconel 718 and Ti-6Al-4V. This approach was able to predict the effect of ΔK.

Quan and Alderlestien (Quan and Alderliesten, 2021) indicated that:

$$\frac{dW}{dN} = \frac{dW}{da}\frac{da}{dN} \tag{11}$$

dW/da, i.e. the energy dW required to propagate the crack by da, is analogous to the critical cumulative dissipated energy used by other authors. They studied an MT specimen made of 7075-T6 aluminium alloy with a stationary crack. However, they found that dW/da is not a constant, which may be associated with the crack closure phenomenon, which is not being modelled.

Necemer *et al.* (Nečemer et al., 2022) examined the deterioration of crack tip components. Their methodology involved utilizing a damage parameter, D, which was derived from the inelastic strain energy density (the enclosed area of hysteresis loop). The critical damage variable D_{max}, indicated the point at which the finite elements lost their stiffness due to the accumulation of damage. This approach, which is an alternative to the node release strategy, was applied to a CT specimen (W = 60 mm, t = 4 mm) made of 5083-H111 aluminium alloy. A plane stress state was assumed in the numerical model. The increase of D_{max} was found to decrease the rate of damage propagation, and a good agreement was found with experimental results.

Usually, the process zone is assumed to be the cyclic plastic zone, but alternative approaches were proposed. Cojocaru and Karlsson (Cojocaru and Karlsson, 2009) considered a semi-disk centred at the crack tip, while Nittur *et al.* (Nittur et al., 2014) considered a rectangular domain, both embracing the cyclic plastic zone. On the other hand, Skelton *et al.* (Skelton, 1998) divided the zone ahead of the crack tip in small segments with length ρ in the range of 0.005–0.02 mm. This segment was assumed to fail when the accumulated energy density after N applied cycles reached a critical value, W_c, which was related to LCF results. Upon propagation, the crack advances by a distance ρ, subsequently leading to the accumulation of energy in a new process zone. The energy was calculated analytically from ΔK or J integral. The process zone, with length ρ, was assumed to be much smaller than the cyclic plastic zone. Possible damage accumulation before the crack reaches the process zone was also considered using a magnification factor. Noroozi *et al.* (Noroozi et al., 2005) also considered the material composed of segments with finite dimension, but FCG was regarded as a process of successive crack re-initiations in the crack tip region.

(iii) Plastic energy density at a single point

In the previous cases the dissipated energy was calculated over all crack tip region. An alternative approach, called the Point method of Theory of Critical Distances (TCD), is based on a specific point ahead of the crack tip. The underlying premise of TCD is that fracture or fatigue failure occurs when a physical quantity, situated at a characteristic distance in advance of the notch root or crack tip, attains a critical value (Taylor, 2008, 2007). It is assumed that the critical point correctly represents the average elastic–plastic field at the crack tip area. One significant advantage of TCD is the possibility of accurately assess the stresses and plastic strains at the critical distance point. On the other hand, at the crack tip there are strong gradients of stresses and strains, which are significantly affected by element size and element type. The position of the critical point is, according to Susmel (Susmel, 2008):

$$L_{cd} = \frac{1}{\pi}(\frac{\Delta K_{th}}{\Delta \sigma_0})^2 \tag{12}$$

where $\Delta\sigma_0$ is the fatigue limit and ΔK_{th} is the threshold stress intensity factor range. This quantity is expected to be of the same order of magnitude as the material's main characteristic microstructural feature. The energy density is normally defined in a volume, but it is possible to define it at a point by considering the limit when the volume tends to zero. For any crack length, the accumulation of plastic energy density at the critical distance is calculated using:

$$\Delta w = \int_{cycle} \sigma_{ij} \times d\varepsilon_{ij}^p \tag{13}$$

He *et al.* (He et al., 2020) used the accumulated plastic energy density at a critical distance to predict FCG rate in the Q345 steel. CT specimens with W = 50.8 mm and a thickness t = 4.2 mm were considered. The numerical model assumed mixed hardening model, plane stress state and had elements with a size of 10 μm. The critical distance was found to be 66.4 μm. FCG rate was calculated as the ratio of the element size to the number of load cycles needed to reach the critical accumulated energy. The numerical predictions were compared to experimental results obtained for overloads, and an excellent agreement was found.

Zheng *et al.* (Zheng et al., 2013) postulated that the progression of a fatigue crack tip can be modelled by the accumulation of plastic energy density at a critical distance ahead of the crack tip. Specifically, it is suggested that the crack tip in the FE mesh would advance by one element length when the accumulated plastic energy density reaches a critical value. Therefore, two quantities are needed: the critical distance,

L_{cd}, and the critical plastic energy density, E_c, at the critical point. The calibration was made using the experimental curve obtained for R = 0.1, and the resulting values were L_{cd} = 17.5 μm; E_c = 0.55 J/mm^3. da/dN was determined by dividing the length of the element under consideration by the number of cycles required to accumulate the fatigue damage to the predetermined critical value at the critical distance point. In fact, only two load cycles are applied, and the dissipated energy is calculated at the second load cycle, which is used to estimate the number of load cycles required to reach the critical energy. The elements ahead of critical distance undergo a summation of their plastic energy density. The approach was able to predict the effect of ΔK with reasonable accuracy, the effect of stress ratio and the effect of overloads and underloads (Zheng et al., 2014, Zheng et al., 2013). However, the use of just two load cycles is questionable. In fact, due to strain ratcheting, the application of more load cycles can reduce the crack closure level. This phenomenon is particularly relevant with kinematic hardening and plane strain state (Antunes et al., 2014).

Alternatively, Antunes *et al.* (Antunes et al., 2015) calculated the dissipated energy immediately ahead of the crack tip, at the first Gauss point. The disadvantage is that this value may be affected by finite element errors. The area of the stress–strain loop is the density of dissipated energy. This dissipation occurs at each point of the cyclic plastic zone, in a non-uniform way, being more important at positions closer to the crack tip.

(iv) Thermal energy

The crack tip damage is supposed to be linked to the movement of dislocations. The thermal dissipation resulting from the plastic deformation is not directly linked to FCG. Anyway, there is a relation between the stored energy and the thermal energy, which justifies the use of this parameter as a crack driving force. This perspective is interesting since the thermal energy can be calculated from experimental measurements of temperature.

Meneghetti and Ricotta (Meneghetti and Ricotta, 2020, 2018) used the thermal energy, averaged in a control volume, having a circular shape embracing the crack tip and a radius of 0.52 mm. This heat energy, experimentally measured using an infrared camera, was successfully correlated with FCG rate. A linear relation was found between *da/dN* and averaged specific heat energy per cycle, in log–log scales. Also, a linear relation was found between the elasto-plastic strain energy in the control volume and the maximum value of J-integral.

Wang *et al.* (Wang et al., 2018), based on thermal measurements, proposed a linear relation between *da/dN* and the energy dissipation rate, $\dot{W}_{plas}$. The dissipation was measured considering all the cyclic plastic zone and a well-defined correlation was found with ΔK. The heat source field was found to be more reliable than the temperature field. However, this energy dissipation is only indirectly linked to crack tip damage, which may explain the problems encountered, namely the stress ratio dependence.

Palumbo *et al.* (Palumbo et al., 2017) studied CT specimens of AISI 422 martensitic stainless steel. During the study, the specimens were closely monitored using a cooled infrared camera to obtain thermographic sequences. They found that the heat dissipation is directly proportional to the fourth power of the stress intensity factor range. Besides, they showed a relation between crack growth rate and the heat dissipated per cycle:

$$\frac{da}{dN} = C_1 Q_d^{C_2} \tag{14}$$

Since the relation between dissipated heat and plastic deformation energy is not totally clear, the establishment of accurate relations between FCG rate and thermal variables is not straightforward. Besides, when the plastic zone is very small, which happens with short cracks, and in brittle materials (such as martensitic steels), temperature changes are very small (Palumbo et al., 2017).

(v) Potential energy, π

Usually the plastic dissipation is significantly higher than the elastic potential energy or the energy for surface creation (Quan and Alderliesten, 2022). However, in brittle materials, the elastic energy becomes relevant. So, FCG has also been related to the potential energy, which is the sum of the potential energy due to macroscopic elastic deformation and the potential energy due to external forces:

$$\pi = E_{p,elas} + E_{p,ext} \tag{15}$$

where $E_{p,ext}$ is the potential energy of the external forces. The definition of potential energy must be remembered: it is a scalar quantity equal to the work done by the forces to transport the body to the position where, by convention, it is assumed that there is no potential energy. The energy release rate G, is defined as:

$$G = -\frac{\partial \pi}{\partial A} \tag{16}$$

where A the cracked area. G, was initially proposed by Griffith (Griffith, 1920) to study the fracture of brittle materials. The release of elastic strain energy was the source of energy to create the fracture surfaces and eventually produce some plastic deformation at the crack tip. In small-scale yielding conditions, G is equal to J-integral (Amsterdam et al., 2023).

Barsom (Barsom, 1971) correlated the cyclic strain energy release rate ΔG, with da/dN. The cyclic strain energy release rate was calculated using:

$$\Delta G = \frac{(1 - R^2)}{E} K_{max}^2 \tag{15}$$

Amsterdam *et al.* (Amsterdam et al., 2023) proposed that da/dN has a power law relationship with the cyclic strain energy release rate over the maximum stress intensity factor, i.e. $\Delta G/Kmax$. This is expected to account for the plasticity in metals. The strain energy release rate equation provided a physical explanation for the load ratio effect and is dimensionally correct. This means that the mean stress effect disappears when da/dN is plotted as a function of $\Delta G/Kmax$. Moreover, it has been applied satisfactory to variable amplitude loading cases.

Quan and Alderlestien (Quan and Alderliesten, 2022) developed a detailed study of the relations between da/dN and energy parameters. They found that $dE_{p,elas}/dN$ is not straightforwardly related to da/dN, because there is a significant effect of stress ratio. Even more worrying is the dependence of $dE_{p,elas}/dN$ relatively to the load considered to start the load cycle. The ambiguity of $dE_{p,elas}/dN$ invalidates its use as crack driving parameter for FCG analysis.

Ravi-Chandran (Ravi Chandran, 2017) proposed the correlation of da/dN with the variation of macroscopic elastic energy, ΔC_a, called the net-section strain energy parameter. Moghaddam *et al.* (Rashidi Moghaddam et al., 2018) linked da/dN with the average strain energy density around crack tip, for compact tension (CT), double cantilever beam (DCB) and Brazilian disc (BD) specimens made of AA7075-T6. This energy was calculated using not only the stress intensity factor, but also the first non-singular term of the Williams series expansion (i.e. the T-stress). Pascoe *et al.* (Pascoe et al., 2015, 2014) studied the fatigue disbond growth in adhesively bonded joints, and correlated the disbonding growth rate da/dN with the release of elastic strain energy. Similarly, Yao *et al.* (Yao et al., 2014) studied the Mode I fatigue delamination growth in composites and Amaral *et al.* (Amaral et al., 2017a, 2017b) studied the interlaminar ply delamination growth under mode II and mixed-mode loading. They both related the delamination growth rate da/dN to the elastic strain energy dissipation and demonstrated that the relation between elastic strain energy dissipation per cycle and *da/dN* is almost linear. Chow et al. used ΔG to correlate the FCGR of various metals and polymers (Chow and Lu, 1990; Chow and Woo, 1985). ΔG, is widely used to correlate fatigue crack growth rates of composite materials, because G is easy to calculate and K is ill-defined in an inhomogeneous material even though they are correlated (Armanios et al., 2001).

5. Discussion

In metallic materials FCG is usually due to cyclic plastic deformation at the crack tip. Therefore, da/dN should be correlated with the energy dissipated to change the dislocation structure, dU_p, in a small region immediately ahead of crack tip. In the case of a damage accumulation approach, the crack propagation path should be the reference for calculating the accumulated dU_p. However, da/dN is correlated with other energy parameters, as it was just presented. Therefore, analysing the entire reversed plastic zone is certainly not the perfect solution. This becomes evident in the case of gross plasticity, where the dissipated energy clearly is not all available for FCG. Still, there is some correlation between different energy parameters, which justifies the reasonable correlations observed between da/dN and the different geometric parameters. Besides, the replacement of ΔK by energy parameters is a step towards a better understanding of FCG, but the relationships da/dN versus energy parameters continue to be phenomenological. The discussion will now continue, organized in the form of a SWOT analysis (Strengths, Weaknesses, Opportunities and Threats).

(i) Strengths

Some general aspects of the methodology used to numerically calculate da/dN from energy-based parameters are well-established. The node release strategy is interesting and the simulation models must include the contact of crack flanks, i.e. crack closure. This way, the crack closure phenomenon is included in a natural way in the energy-based parameters. Amsterdam *at al.* (Amsterdam et al., 2023) showed that using $\Delta G/Kmax$ as an energy parameter, the mean stress effect is included without the need of crack closure concept. Thus, the definition of crack closure parameter, which is not in any way consensual (van Kuijk et al., 2021), ceases to be a problem.

Some variations can be considered, which give some flexibility to the researcher. For example, the number of load cycles between load releases may be fixed or variable. Besides, the researcher can choose between a local damage approach or a cumulative damage approach. In the first case, it is assumed that FCG is due to the localized damage occurring at the crack tip, as proposed by Laird's model (Laird and Smith, 1963). In this case, the correlation of da/dN must be done with the energy dissipated in a small region near crack tip. If a cumulative damage model is used, a progressive degradation of the material is assumed from the moment the point under study enters the process zone (Nittur et al., 2014). Nittur *et al.* (Nittur et al., 2014) calculated the accumulated energy in the reversed plastic zone, while Zheng *et al.* (Zheng et al., 2013) considered the accumulation of damage ahead of the critical point. A node release strategy is usually used, but an alternative strategy based on the elimination of elements was also proposed (Nečemer et al., 2022).

Another strength of energy-based approaches is that they can be applied to study the failure of different materials, apart from metallic ones. In other words, the energy concepts are universal, and a transference of knowledge may occur between researchers from different fields.

(ii) Weaknesses

The competence of the energy-based approaches to study FCG has been proved by different authors. However, a broad implementation of this approach has not occurred. In fact, there are analytical expressions of *K* for different cracked geometries, namely the standard ones, and *K* can be obtained using commercial FE software, which facilitates the use of K-based approaches. Even the ability to measure crack closure level is usually available in research laboratories using extensometers or DIC. On the other hand, the calculation of dissipated energy requires complex numerical models, involving the correct modelling of material behaviour and crack propagation. The experimental determination of thermal energies also requires specific equipment and knowledge. The wide implementation of energy-based approaches requires the development of user user-friendly tools, to facilitate the work of researchers.

The energy-based approaches must evolve to include additional damage mechanisms besides cyclic plastic deformation. In fact, a single mechanism cannot be expected to be responsible for a phenomenon which greatly depends on material, environmental conditions and loading, and which spans over seven orders of magnitude in terms of da/dN. Potential mechanisms are the environmental damage, brittle failure and growth and coalescence of microvoids. The relevance of the crack propagation mechanism was pointed out by Ranganathan *et al.* (Ranganathan et al., 2008). Sgambitterra *et al.* (Sgambitterra et al., 2023) plotted da/dN versus dissipated energy for a shape memory alloy. A linear relation was found in log–log scales, but a change of slope is observed for the highest values of energy, which may indicate a change of mechanism. Zheng *et al.* (Zheng et al., 2014), brought attention to a limitation in the proposed model. Specifically, the model solely accounts for energy dissipation from reversed plasticity, which means it cannot accurately predict crack propagation during stage III, commonly referred to as the fast fracture stage. This stage often involves a mix of fatigue failure and monotonic fracture. Cvijovic *et al.* (Cvijović et al., 2008) studied 7000 aluminium alloys with different contents of iron. They found that the large Fe-rich intermetallic particles (identified as $(Cu,Fe,Mn)Al_3$ and Al_7Cu_2Fe particles increase da/dN (Cvijović et al., 2007). These particles are brittle, therefore easily fracture without absorbing much energy. Owing to their spheroidized and small morphology, the particles identified as the η-$Mg(Zn,Cu,Al)_2$ or S-$CuMgAl_2$ exhibit a lower propensity for fracture. Consequently, the generation of new crack surfaces along elongated trajectories necessitates a greater energy expended. Lesiuk *et al.* (Lesiuk et al., 2017) proposed a static component of the energy corresponding to the maximal value of loading, however no link was established with a different damage mechanism. Li *et al.* (Li et al., 2023) proposed that the energy consumed during FCG is linked to the blunting-resharpening (BRS) mechanism, to the microscale void coalescence (MVC) and to a BRS-MVC equilibrium mechanism. Also, the inclusion of other damage mechanisms into the energetic analysis may be problematic. For example, how is the energy associated with environmental damage calculated?

Another problem is the dimensional differences involved in relations between da/dN and energy parameters. In fact, since da/dN and the energy parameters have different units, the constants appearing in the relations must have units to maintain dimensional coherence. Therefore, any change in the units of da/dN of energy parameters must be accompanied by a change in the values of the constants. This problem was also identified in da/dN-ΔK relations, and solved with da/dN versus plastic CTOD relations. This means that non-dimensional relations must be searched.

(iii) Opportunities

The link with low-cycle fatigue (LCF) properties provides a great opportunity to increase the relevance and applicability of energy-based approaches. Fig. 4 shows a stress–strain loop with the indication of the plastic deformation energy density, Δw_p. The area of the hysteresis loop, i.e. Δw_p, is related to the total energy dissipation and hence to plastic deformation and heat generation. Lefebvre and Ellyin (Lefebvre and Ellyin, 1984) found that each material has a limit of energy that it can absorb and when this limit is exceeded, cracks appear that could lead to the material's rupture. In 1997, Ellyin (Ellyin, 1996) related the plastic strain energy density to the number of cycles until failure:

$$\Delta w_p = k_p (2N_f)^{\alpha_p} \tag{14}$$

where k_p and α_p are obtained from experimental testing. A similar approach relates the total energy, which is the sum of elastic energy and plastic energy, with fatigue life:

$$\Delta w_t = \Delta w_e + k(2N_f)^{\alpha} \tag{15}$$

Since this approach does not consider the effect of mean tension, Golos and Ellyin (Golos and Ellyin, 1988, 1987) proposed a new formulation to take this effect into account. This formulation considers the algebraic sum of the positive elastic strain energy density, Δw_e^{+},

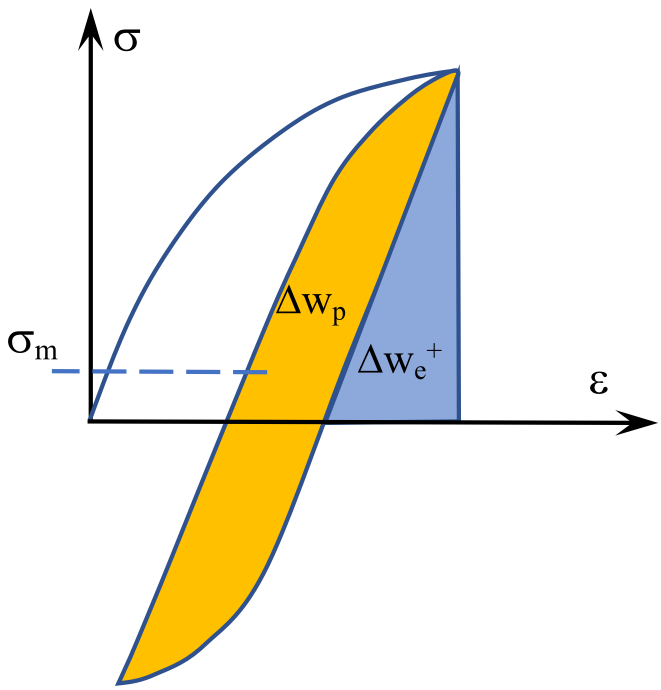

Fig. 4. Stress–strain loop with the indication of dissipated energy and positive elastic energy.

with the plastic strain energy density, Δw_p (Fig. 4). The accumulated strain energy density, W, can be calculated by summing the strain energy dissipated per cycle throughout the test:

$$W = \int_1^{N_f} \Delta w dN \tag{16}$$

The cumulative (hysteresis) energy absorbed to 'failure', W_c, was found to be constant and independent of strain range during LCF tests at constant total strain range (Skelton, 1991). On the other hand, Halford (Halford, 1966) observed an increase of W_c with the number of cycles to failure. When block-loading conditions are applied, the outcome remains unaffected by the order of the cycle (Kliman and Bílý, 1984; Skelton et al., 1996).

The link of energy parameters with other non-linear parameters is also interesting, providing a better understanding of crack tip phenomena. In fact, the different non-linear crack tip parameters are expected to be faces of the same coin. Antunes *et al.* (Antunes et al., 2015) have established a definitive association between the energy dissipated in the vicinity of the crack tip and the plastic strain range. Specifically, the research found that the energy dissipated is directly proportional to $\Delta\varepsilon_p^{1.5}$, measured at the Gauss point located immediately ahead of the last crack tip position. This relation was found to be independent of the size of crack tip elements and of the presence of crack closure.

The study of FCG under mixed-mode loading using energy parameters is an opportunity for a better understanding of this loading situation.

(iv) Threats

The energy effectively responsible for crack propagation is just a fraction of the energy dissipated in the cyclic plastic zone. According to the Taylor–Quinney coefficient, only about 10 % of the dissipated energy is stored internally in the form of dislocations. This percentage is not constant, changing with plastic strain and plastic strain rate. Besides, only a part of the dislocations is acting at the crack tip and is responsible for crack increment. The FCG rate should be correlated with this small portion of the dissipated energy, but the accurate measurement of this energy is very difficult.

A second relevant problem is the application of these approaches to 3D crack fronts. In fact, all previous studies, numerical and experimental, are 2D. The application to corner cracks or surface cracks, requires the calculation of the energy parameters along the crack front, i.e. the ability to isolate energy parameters to each node considered along the crack front. In studies with CTOD, the finite element mesh is defined perpendicularly at each crack front node (Escalero et al., 2023). Anyway, if the dissipated energy is calculated over the reversed plastic zone, the association of a value with a specific node is not easy, since some cross-influence may be expected. The solution may be the use of the Gauss points closest to the crack front nodes, avoiding the calculation over a region. However, this may have problems of errors of FE solutions, as already pointed out. In the experimental studies, the problem is even more dramatic, since the measurement of temperature is made only at the surface.

6. Final remarks

The current works describes some of the most relevant works for FCG prediction with energy parameters, considering analytical, numerical and experimental perspectives. The success in the use of energy parameters has been described in detail. The effect of ΔK (Nittur et al., 2014) and overloads (Smith, 2011), for example, has been successfully predicted with energy parameters.

There is a reasonable consensus about some points of the numerical simulation of FCG based on energy parameters. A node release strategy is recommended and the models must include the simulation of the contact of crack flanks, i.e. crack closure. The inclusion of the contact of crack flanks is particularly relevant in regime I and for low stress ratio values. Besides, the numerical models must have a realistic number of load cycles between node releases for a better simulation of plasticity-induced crack closure. The accurate simulation of material's elastic–-plastic behaviour is fundamental.

Considering that the damage is localized at the crack tip, it is not logical to use the dissipated energy on all reversed plastic zone. A distinction is here made between the region very close to the crack tip and the remaining region of reversed plastic zone. Assuming an approach based on damage accumulation, only the line of points ahead of the crack tip, along the crack path, should be studied. This is also advantageous since the reversed plastic zone is small and with complex shape and size. However, a comparative study of the different energy parameters applied to experimental results would be very interesting and is addressed for future work.

Additionally, there are issues requiring further studies. The relationship between da/dN and the plastic work dissipated per cycle must be studied, in order to check if it is linear in natural scales or in log–log scales. The relation between the dissipated energy quantified at the first Gauss point and at a critical point placed at some distance ahead of the crack tip must also be clarified. The relevance of FEM errors in this context must be studied. The relation between the dissipated energy over the reversed plastic zone and the energy at a critical point is also an interesting issue. The rise of temperature on crack tip region and the impact of this on FCG behaviour must be studied. Finally, the effect of crack closure on the relation between dissipated energy and ΔK is not known.

CRediT authorship contribution statement

F.V. Antunes: Writing – review & editing, Writing – original draft, Visualization, Supervision, Project administration, Investigation, Funding acquisition, Data curation, Conceptualization. **E.R. Sérgio:** Writing – review & editing, Writing – original draft, Resources, Methodology, Investigation, Formal analysis, Data curation. **P.M. Cerezo:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Data curation. **P. Lopez-Crespo:** Writing – review & editing, Supervision, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **D.M. Neto:** Writing – review &

editing, Writing – original draft, Resources, Methodology, Investigation, Formal analysis, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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