On the capacity of MISO FSO systems over gamma-gamma and misalignment fading channels

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Abstract: In this work, the ergodic capacity performance for multiple-input/single-output (MISO) free-space optical (FSO) communications system with equal gain combining (EGC) reception is analyzed over gamma-gamma and misalignment fading channels, which are modeled as statistically independent, but not necessarily identically distributed (i.n.i.d.). Novel and analytical closed-form ergodic capacity expression is obtained in terms of H-Fox function by using the well-known inequality between arithmetic and geometric mean of positive random variables (RV) in order to obtain an approximate closed-form expression of the distribution of the sum of M gamma-gamma with pointing errors variates. In addition, we present an asymptotic ergodic capacity expression at high signal-to-noise ratio (SNR) for the ergodic capacity of MISO FSO systems. It can be concluded that the use of MISO technique can significantly reduce the effect of the atmospheric turbulence as well as pointing errors and, hence, provide significant capacity gain over the direct path link (DL). The impact of pointing errors on the MISO FSO system is also analyzed, which only depends on the number of laser sources and pointing error parameters. Moreover, it can be also concluded that the ergodic capacity performance is dramatically reduced as a consequence of the severity of pointing error effects. Simulation results are further demonstrated to confirm the analytical results.

OCIS codes: (010.1330) Atmospheric turbulence; (060.2605) Free-space optical communication; (060.4510) Optical communications.

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1. Introduction

Free-space optical (FSO) systems have become a very important application area providing communication at data rates as high as hundreds of gigabits per second. FSO links can be used to setup a complete FSO network or as a supplement to conventional radio-frequency (RF) systems and fiber optics. These systems present a wide range of applications such as metropolitan area network (MAN) extension as well as local area network (LAN)-to-LAN connectivity, among other applications [1]. Despite the major advantages of FSO technology, these communication systems are vulnerable to adverse conditions such as molecular absorption, light scattering from suspended particulates, fog, rain and even snow. In addition to this, another adverse effect known as atmospheric scintillation is present here, which produces fluctuations in the irradiance of the received optical beam, severely degrading the link performance [2]. Moreover, thermal expansion, dynamic wind loads as well as weak earthquakes result in the building sway that causes vibration of the transmitter beam leading to a misalignment between transmitter and receiver known as pointing error. There is extensive literature on the study of the performance of FSO communication systems over atmospheric turbulence and misalignment fading channels. For many years researchers have demonstrated that effective fading-mitigation techniques are required in order to satisfy the typical bit error-rate (BER) targets for potential FSO applications. The effect of fading in FSO can be substantially reduced by creating spatial diversity based on multiple-input/multiple-output (MIMO) FSO system with multiple lasers at the transmitter and multiple photodetectors at the receiver [3–5]. The combined effect of atmosphere and misalignment fading has been analyzed in the case of MIMO FSO channels [6–8].

Lately, the study of ergodic capacity for FSO communication links has become an interesting field, which has generated recent research interest. Ergodic capacity, also known as average channel capacity, defines the maximum data rate that can be sent over the channel with asymptotically small error probability, without any delay or complexity constraints [9]. A remarkable variety of works have been reported wherein the ergodic capacity is analyzed over different statistical models to describe the irradiance of FSO links [10–16]. In FSO communications, MIMO systems can be employed to reduce scintillation and therefore improves FSO channel capacity. However, to our knowledge, only a few works have studied the ergodic capacity of MIMO FSO systems [17–21]. In [17], a closed-form expression for the average capacity of MIMO FSO channel with equal gain combining (EGC) reception is obtained over log-normal fading channels without pointing errors. In [18], closed-form expressions for the average capacity of MIMO FSO channels with EGC and maximum ratio combining (MRC) reception are obtained over gamma-gamma fading channels without pointing errors. In [19], the ergodic capacity of MIMO FSO systems is investigated over strong turbulence channels by using a sin-

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gle gamma-gamma approximation [22]. In [20], the effect of MIMO FSO systems with EGC and aperture averaging on the ergodic capacity over gamma-gamma fading channels is studied and compared for different weather conditions. In [21], the ergodic capacity performance of MIMO FSO systems with EGC reception is studied over gamma-gamma fading channels without pointing errors by using α-μ distribution proposed in [23]. However, to the best of the authors' knowledge, the combined effect of turbulence and misalignment fading has not been taken into account on the study of the ergodic capacity for MIMO FSO communication systems.

There remains a need for studying the ergodic capacity of MIMO FSO systems when pointing error effects are taken into account in order to observe how pointing errors impact on the capacity. Hence, motivated by the fact, we focus on the study of the ergodic capacity for multiple-input/single-output (MISO) FSO communication systems with EGC reception over atmospheric turbulence and misalignment fading channels, which are modeled as statistically independent, but not necessarily identically distributed (i.n.i.d.). In this way, a novel and analytical closed-form ergodic capacity expression is obtained when the irradiance of the received optical beam is susceptible to moderate-to-strong turbulence conditions, following a gamma-gamma distribution, and pointing error effects, following a misalignment fading model where the effect of beam width, detector size and jitter variance is considered. This novel closed-form ergodic capacity expression is derived in terms of H-Fox function by using the well-known inequality between arithmetic and geometric mean of positive random variables (RV) in order to obtain an approximate closed-form expression of the distribution of the sum of M gamma-gamma with pointing errors variates. Here, unlike [19,21], wherein the gamma-gamma approximation presented in [22] was used and pointing error effects were not considered, the approximate expression of the distribution of the sum of M gamma-gamma with pointing errors variates only depends on channel parameters and does not depend on any numerical adjustment parameter. In addition, we present an asymptotic ergodic capacity expression at high signal-to-noise ratio (SNR) for the ergodic capacity of MISO FSO systems. It can be concluded that the use of MISO technique can significantly reduce the effect of the atmospheric turbulence as well as pointing errors and, hence, provide significant capacity gain over the direct path link (DL). The impact of pointing errors on the MISO FSO system is also analyzed, which only depends on the number of laser sources and pointing error parameters.

2. System and channel model

We adopt a MISO FSO system with M transmitters or laser sources and, a single receiver, as shown in Fig. 1. The receiver is assumed to be centered at the origin and consists of optical

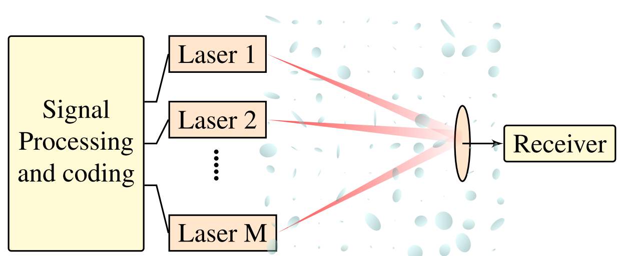

Fig. 1. Block diagram of the considered MISO FSO communications system.

filters and a lens which has the role of collecting and focusing the received beam onto the photodiode. Additionally, the laser sources are separated by a fixed distance so that uncorrelated fading can be considered [24, 25], which will be quantified in this section. The use of

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infrared technologies based on intensity-modulation direct-detection (IM/DD) is also considered. IM/DD systems are commonly used in the terrestrial FSO links due to their simplicity and low cost. The intensity of the emitted light is used for transmitting the information and, the photodetector directly detects changes in the light intensity without the need for a local oscillator. The laser sources and the receiver are physically situated so that all transmitters are simultaneously observed by the receiver. Here, the EGC reception technique is adopted due to its considerable lower implementation complexity even maintaining relevant performance in FSO links [4, 5].

The received electrical signal for each link is given by $Y = XI + Z$, where X represents the optical power supplied by the source, I is the equivalent real-value fading gain (irradiance) through the MISO channel between the source and the receiver, and Z is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 = N_0/2$, i.e. $Z \sim N(0, N_0/2)$, independent of the on/off state of the received bit. The irradiance is considered to be a product of three factors, i.e. $I = L \cdot I_a \cdot I_p$, atmospheric path loss L, atmospheric turbulence I_a, and geometric spread and pointing errors I_p. L is determined by the exponential Beers-Lambert law as $L = e^{-\Phi d}$, where d is the link distance and Φ is the atmospheric attenuation coefficient. It is given by $\Phi = (3.91/V(km))\,(\lambda(nm)/550)^{-q}$ where V is the visibility in kilometers, λ is the wavelength in nanometers and q is the size distribution of the scattering particles, being $q = 1.3$ for average visibility (6 km $< V <$ 50 km), and $q = 0.16V + 0.34$ for haze visibility (1 km $< V <$ 6 km) [26]. Atmospheric turbulence, I_a, causes fluctuations in both the intensity and the phase of the received signal due to variations in the refractive index along the FSO link. To consider a wide range of turbulence conditions, the gamma-gamma turbulence model proposed in [2] is assumed here. Regarding the impact of pointing errors, we use the general model of misalignment fading given in [27] by Farid and Hranilovic, wherein the effect of beam width, detector size and jitter variance is considered. Building sway can be caused by strong winds, thermal expansion of building frame parts, and weak earthquakes [28]. Although the effects of turbulence and pointing are not strictly independent, for smaller jitter values they can be approximated as independent [29]. In addition to this, as in [27], pointing errors are due to building sway and, hence, the correlation time is on the order of a few seconds [30], which is bigger than correlation time of the atmospheric turbulence (10-100 ms). Hence, both atmospheric turbulence and pointing errors can be considered to be independent. A closed-form expression of the combined probability density function (PDF) of I was derived in [30] as

$$f_I(i) = \frac{\varphi^2 i^{-1}}{\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 L} i \,\middle|\, \begin{matrix} \varphi^2+1 \\ \varphi^2, \alpha, \beta \end{matrix}\right), \quad i \geq 0 \tag{1}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function [31, eqn. (9.301)] and $\Gamma(\cdot)$ is the well-known Gamma function. For the sake of simplicity, plane wave propagation is considered in this work since when a Gaussian beam has a relatively large divergence, its statistical properties are close to the case of a point source [32] and, hence, the approximations of either plane or spherical wave can be used. Assuming plane wave propagation, α and β can be directly linked to physical parameters through the following expresions [33]:

$$\alpha = \left[\exp\left(0.49\sigma_R^2/(1+1.11\sigma_R^{12/5})^{7/6}\right) - 1\right]^{-1}, \tag{2a}$$

$$\beta = \left[\exp\left(0.51\sigma_R^2/(1+0.69\sigma_R^{12/5})^{5/6}\right) - 1\right]^{-1}. \tag{2b}$$

where $\sigma_R^2 = 1.23 C_n^2 \kappa^{7/6} d^{11/6}$ is the Rytov variance, which is a measure of optical turbulence strength. Here, $\kappa = 2\pi/\lambda$ is the optical wave number and d is the link distance in meters. C_n^2 is the refractive index structure parameter, which is the most significant parameter that determines the turbulence strength. Clearly, C_n^2 not only depends on the altitude, but also on the

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local conditions such as terrain type, geographic location, cloud cover, and time of day [34]. In addition, C_n^2 is typically within the range 10^{-13}-10^{-17} $m^{-2/3}$ [2]. It must be emphasized that parameters α and β cannot be arbitrarily chosen in FSO applications, being related through the Rytov variance. It can be shown that the relationship $\alpha > \beta$ always holds, and the parameter β is lower bounded above 1 as the Rytov variance approaches ∞ [35]. In relation to the impact of pointing errors [27], assuming a Gaussian spatial intensity profile of beam waist radius, ω_z, on the receiver plane at distance z from the transmitter and a circular receive aperture of radius r, $\varphi = \omega_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver, $\omega_{z_{eq}}^2 = \omega_z^2\sqrt{\pi}\text{erf}(v)/2v\exp(-v^2)$, $v = \sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0 = [\text{erf}(v)]^2$ and $\text{erf}(\cdot)$ is the error function [31, eqn. (8.250)]. It should, however, be commented that independent identical Gaussian distributions for the elevation and the horizontal displacement (sway) are considered, being σ_s^2 the jitter variance at the receiver. It must be also commented that the pointing error model in [27] is not general enough to be directly applied to MISO FSO communications system even when the lasers are pointed at the same detector since pointing displacements will not be exactly on the same transverse plane for all lasers, not being strictly applicable in that case, and therefore, the study of the ergodic capacity over i.n.i.d. gamma-gamma and misalignment fading channels is considered. Nonetheless, this can be considered negligible under the assumption that the source-destination (SD) link distance d_{SD} between transmit lasers and photodetector is several orders of magnitude the spacing between transmitters, which will be checked in the next section.

2.1. Uncorrelated fading

It is well known that diversity techniques are most efficient under the conditions of uncorrelated fading. It is assumed that the spacing between transmitters is sufficiently larger than the coherence/correlation length, l_c, then uncorrelated fading can be considered [24, 25]. In this way, when the refractive index structure parameter C_n^2 is treated as constant, i.e., horizontal FSO link, the plane wave coherence radius ρ_0 is defined as $\rho_0 = 0.79\left(C_n^2\kappa^2 d\right)^{-3/5}$ in both weak and strong turbulence conditions, where d is the link distance and κ is the optical wave number. Under weak turbulence the correlation length of irradiance fluctuations is determined by the Fresnel zone $\sqrt{d/\kappa}$, whereas under strong turbulence the correlation length of irradiance fluctuations is defined by the plane wave coherence radius ρ_0 [19, 36]. The plane wave coherence radius, ρ_0 is smaller than the Fresnel Zone under strong turbulence conditions. Different atmospheric turbulence conditions are adopted here: $C_n^2 = 1.7\times10^{-14}$ $m^{-2/3}$ and $C_n^2 = 8\times10^{-14}$ $m^{-2/3}$, corresponding to moderate and strong turbulence conditions, respectively. A value of wavelength of $\lambda = 1550$ nm is chosen as well as a source-destination link distance of $d_{SD} = 3$ km is considered. Hence, a spacing of 14 mm between the lasers is required under moderate turbulence as well as 5.4 mm under strong turbulence in order to consider uncorrelated fading in this analysis. Both obtained spacing between the lasers at the transmitter are perfectly feasible in this MISO FSO communications system in order to consider that fading is approximately independent of one another. Furthermore, another fundamental optical turbulence parameter, i.e., the isoplanatic angle (θ_0) for adaptive optics, laser propagation in atmosphere, and imaging through turbulence can be defined as angular distance over which the atmospheric turbulence is essentially unchanged [37]. The isoplanatic angle, θ_0, is defined as in [38] for a horizontal FSO link as follows

$$\theta_0 = \left(2.91\kappa^2\int_0^d C_n^2(z)z^{5/3}dz\right)^{-3/5} = 0.94\left(\kappa^2C_n^2d^{8/3}\right)^{-3/5}. \quad (3)$$

Here, a horizontal FSO link is considered as well as the condition of homogeneous turbulence is assumed. Hence, the refractive index structure parameter C_n^2 does not depend on the dis-

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tance. The isoplanatic angle is evaluated both moderate and strong turbulence. Hence, given $\lambda = 1550$ nm, an isoplanatic angle of 5.57 μrad is obtained under moderate turbulence as well as 2.2 μrad under strong turbulence. From the expression in Eq. (3), it can be easily deduced that the isoplanatic angle, θ_0, is related to the plane wave coherence radius ρ_0 by $\rho_0 \approx 0.84 d_{SD}\theta_0$.

3. Ergodic capacity analysis of MISO FSO systems

In this section, the ergodic capacity as a performance measure is evaluated for a MISO FSO communications system over gamma-gamma fading channels with pointing errors. When the EGC reception is used, the statistical channel model can be written as

$$Y = X\frac{1}{M}\sum_{k=1}^{M} I_k + Z, \quad X \in \{0, 2P_{\text{opt}}\}, \quad Z \sim N(0, N_0/2), \tag{4}$$

where I_k represents the equivalent irradiance through the optical channel between the kth transmit aperture and the photodetector and, X is either 0 or $2P_{\text{opt}}$, i.e., on-off keying (OOK) with average power constraint P_{opt}. Here, the division by M in Eq. (4) is considered so as to maintain the average optical power in the air at a constant level of P_{opt}, being transmitted by each laser at source node an average optical power P_{opt}/M. In this way, the total transmit power is the same as in a FSO system with no transmit diversity, i.e. direct path link. The resulting received electrical SNR, γ_{MISO}, can be defined, as in [27], as

$$\gamma_{\text{MISO}} = \frac{1}{2}\frac{\left(2P_{\text{opt}}/M\right)^2}{\sigma_n^2}\left(\sum_{k=1}^{M} I_k\right)^2 = \frac{4P_{\text{opt}}^2 I_T^2}{M^2 N_0} = \frac{4\gamma_0}{M^2} I_T^2, \tag{5}$$

where γ_0 represents the received electrical SNR in absence of turbulence when the classical rectangular pulse shape is adopted for OOK formats, I_T represents the total channel gain and, P_{opt} is the average optical power transmitted. Assuming instantaneous channel side information at the receiver (CSIR), the ergodic capacity corresponding to the considered MISO FSO system in bits/s/Hz is defined as $C_{\text{MISO}} = (B/2\ln(2))\mathbb{E}\left[\ln(1+\frac{4\gamma_0}{M^2}I_T^2)\right]$, with $\mathbb{E}[\cdot]$ denoting expectation. Hence, this ergodic capacity, C_{MISO}, can be obtained by averaging over the PDF of I_T as follows

$$C_{\text{MISO}} = \frac{B}{2\ln(2)}\int_0^\infty \ln\left(1 + \frac{4\gamma_0}{M^2} i^2\right) f_{I_T}(i)di, \tag{6}$$

where B is the channel bandwidth, $\ln(\cdot)$ is the natural logarithm [31, eqn. (1.511)], and $f_{I_T}(i)$ is the PDF corresponding to the sum of M gamma-gamma with pointing errors variates statistically independent but not necessarily identically distributed, i.e., $I_T = I_1 + I_2 + \cdots + I_M$. It should be noted that the factor $1/2$ in Eq. (6) is because the transmitter is assumed to operate in half-duplex mode. It must be also noted that obtaining the PDF of I_T is remarkably tedious and not easily tractable due to the difficulty in finding its statistics. Hence, a lower bound (LB) for this sum can be obtained by using the well-known inequality between arithmetic mean (AM) and geometric mean (GM) given by

$$AM \geq GM, \tag{7}$$

where $AM = (1/M)\sum_{k=1}^{M} I_k$ and $GM = \sqrt[M]{\prod_{k=1}^{M} I_k}$ are the arithmetic and geometric means, respectively. Therefore, a lower bound for the sum of M gamma-gamma with pointing errors variates, i.e. I_T, can be obtained as

$$I_T = \sum_{k=1}^{M} I_k \geq M\sqrt[M]{F \cdot \prod_{k=1}^{M} I_k} = M\sqrt[M]{F \cdot I_{LB}}. \tag{8}$$

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Note that $f_{I_{LB}}(i)$ is mathematically more tractable than $f_{I_T}(i)$ and can be efficiently applied to the analysis of the ergodic capacity of MISO FSO communication systems. From Eq. (8), it can be easily deduced that the mathematical expectation in both sides of inequality takes different values and, hence, a correcting factor F must be added to the inequality in order to maintain the same value in both sides. Taking into account this fact, the correcting factor F is added to Eq. (8) in order to obtain a strict approximation on the ergodic capacity analysis. This correcting factor can be derived from Eq. (8) and can be seen in more detail in appendix. In addition, this correcting factor F only depends on channel parameters. Substituting Eq. (8) into Eq. (6) and, after performing some algebraic manipulations, the ergodic capacity for a MISO FSO system can be accurately approximated as follows

$$C_{\mathrm{MISO}} \simeq \frac{B}{\ln(4)} \int_0^\infty \ln\left(1 + 4\gamma_0 \left(i \cdot F\right)^{\frac{2}{M}}\right) f_{I_{LB}}(i) di. \tag{9}$$

The PDF $f_{I_{LB}}(i)$ can be derived in closed-form via inverse Mellin transform, which is an essential tool in studying the distributions of products and quotients of independent random variables. The importance of the Mellin transform in probability theory lies in the fact that if I_K for $k = \{1, 2, \ldots, M\}$ are M independent random variables, then the Mellin transform of their product is equal to the product of the Mellin transforms of I_k for $k = \{1, 2, \ldots, M\}$ [39]. Hence, the closed-form solution for the PDF corresponding to I_{LB} can be expressed in terms of the Meijer G-function by employing the definition of the Mellin transform [31, eqn. (17.41)] as follows

$$f_{I_{LB}}(i) = \frac{i^{-1} \prod_{k=1}^M \varphi_k^2 G_{M,3M}^{3M,0}\left(\prod_{k=1}^M \frac{\alpha_k \beta_k}{A_k L_k} i \,\middle|\, \begin{matrix} \varphi_1^2 + 1, \ldots, \varphi_M^2 + 1 \\ \varphi_1^2, \alpha_1, \beta, \ldots, \varphi_M^2, \alpha_M, \beta_M \end{matrix}\right)}{\prod_{k=1}^M \Gamma(\alpha_k)\Gamma(\beta_k)}. \tag{10}$$

The integral in Eq. (9) can be solved using [40, eqn. (8.4.6.5)] in order to express the natural logarithm in terms of the Meijer's G-function as $\ln(1+z) = G_{2,2}^{1,2}\left(z\big|_{1,0}^{1,1}\right)$ and, then, using [41, eqn. (07.34.21.0012.01)], we can obtain the approximate closed-form solution for the ergodic capacity corresponding to this MISO FSO communications system, C_{MISO}, as follows

$$C_{\mathrm{MISO}} \simeq \frac{B \prod_{k=1}^M \varphi_k^2 H_{2+3M,2+M}^{1,2+3M}\left(4\gamma_0 \sqrt[M]{F} \left(\prod_{k=1}^M \frac{A_k L_k}{\alpha_k \beta_k}\right)^{\frac{2}{M}} \,\middle|\, \begin{matrix} (1,1), (1,1), \xi_1 \\ (1,1), \xi_2, (0,1) \end{matrix}\right)}{\ln(4) \prod_{k=1}^M \Gamma(\alpha_k)\Gamma(\beta_k)}, \tag{11}$$

where $\xi_1 = \{(1-\varphi_1^2, \frac{2}{M}), (1-\alpha_1, \frac{2}{M}), (1-\beta_1, \frac{2}{M}), \ldots, (1-\varphi_M^2, \frac{2}{M}), (1-\alpha_M, \frac{2}{M}), (1-\beta_M, \frac{2}{M})\}$, $\xi_2 = \{(-\varphi_1^2, \frac{2}{M}), \ldots, (-\varphi_M^2, \frac{2}{M})\}$ and, $H_{p,q}^{m,n}[\cdot]$ is the H-Fox function [42, eqn. (1.1)]. A computer program in Mathematica for the efficient implementation of the H-Fox function is given in [43, appendix A]. Notwithstanding, obtained results via H-Fox function will be checked through Monte Carlo simulation in following sections. At this point, it should be mentioned that an asymptotic ergodic expression can be derived from Eq. (11) by using the corresponding series expansion of the H-Fox function [42, Chapter 1]. However, this ergodic capacity analysis might not result in a closed-form expression and, hence, we cannot always obtain an asymptotic expression from its corresponding closed-form expression. Within this context, from Eq. (9) and knowing that $I_T = I_1 + I_2 + \cdots + I_M$, an asymptotic expression at high SNR can be readily and accurately lower-bounded due to the fact that $\ln(1+z) \approx \ln(z)$ when $z \to \infty$ as follows

$$C_{\mathrm{MISO}}^H \doteq \frac{B \ln(4\gamma_0)}{\ln(4)} + \frac{B \ln(F)}{M \ln(2)} + \frac{B}{M \ln(2)} \sum_{k=1}^M \int_0^\infty \ln(i_k) f_{I_k}(i_k) di_k. \tag{12}$$

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To evaluate the integral in Eq. (12), the natural logarithm is expressed as a subtraction of two Meijer's G-function by adding redundancy as

$$\ln(z) = \frac{z\ln(z)}{z-1} - \frac{\ln(z)}{z-1}. \tag{13}$$

In addition, we can use the fact that the natural logarithm is related to the Meijer's G-function by $\frac{\ln(z)}{z-1} = G_{2,2}^{2,2}\left(z\middle|_{0,0}^{0,0}\right)$ [41, eqn. (01.04.26.0004.01)], therefore, the expression in Eq. (12) can be re-written as

$$\begin{aligned} C_{\mathrm{MISO}}^{H} &\doteq \frac{B\ln(4\gamma_0)}{\ln(4)} + \frac{B\ln(F)}{M\ln(2)} \\ &+ \frac{B}{M\ln(2)}\left(\sum_{k=1}^{M}\int_0^\infty i_k G_{2,2}^{2,2}\left(i_k\middle|_{0,0}^{0,0}\right) f_{I_k}(i_k)di_k - \sum_{k=1}^{M}\int_0^\infty G_{2,2}^{2,2}\left(i_k\middle|_{0,0}^{0,0}\right) f_{I_k}(i_k)di_k\right). \end{aligned} \tag{14}$$

Both integrals in Eq. (14) can be solved using [40, eqn. (2.24.1.2)]. Hence, the asymptotic closed-form solution for the ergodic capacity corresponding to this MISO FSO communications system at high SNR, C_{MISO}^{H}, can be accurately estimated as

$$\begin{aligned} C_{\mathrm{MISO}}^{H} &\doteq \frac{B\ln(4\gamma_0)}{\ln(4)} + \frac{B\ln(F)}{M\ln(2)} + \frac{B}{M\ln(2)}\sum_{k=1}^{M}\frac{\varphi_k^2}{\Gamma(\alpha_k)\Gamma(\beta_k)} \\ &\times\left(G_{3,5}^{5,2}\left(\frac{\alpha_k\beta_k}{A_kL_k}\middle|\begin{matrix}0,0,\varphi_k^2+1\\ \varphi_k^2,\alpha_k,\beta_k,0,0\end{matrix}\right) - G_{3,5}^{5,2}\left(\frac{\alpha_k\beta_k}{A_kL_k}\middle|\begin{matrix}1,1,\varphi_k^2+1\\ \varphi_k^2,\alpha_k,\beta_k,1,1\end{matrix}\right)\right). \end{aligned} \tag{15}$$

As can be seen in Eq. (15), the subtraction of two Meijer's G-function is independent of the SNR, γ_0, resulting in a positive value, which depends on the MISO FSO channel. In order to simplify the expression in Eq. (15) and observe how the atmospheric turbulence and pointing errors impact on the capacity of MISO FSO communication systems, the series asymptotic expansion corresponding to the Meijer's G-function [41, eqn. (07.34.06.0002.01)] is used in Eq. (15). Therefore, the following asymptotic expression can be derived as follows

$$C_{\mathrm{MISO}}^{H} \doteq \frac{B\ln(4\gamma_0)}{\ln(4)} + \frac{B\ln(F)}{M\ln(2)} + \frac{B}{M\ln(2)}\sum_{k=1}^{M}\psi(\alpha_k)+\psi(\beta_k) - \frac{1}{\varphi_k^2} - \ln\left(\frac{\alpha_k\beta_k}{A_kL_k}\right), \tag{16}$$

wherein $\psi(\cdot)$ is the psi (digamma) function [31, eqn. (8.360.1)]. It can be corroborated that only two first terms have influence on the series expansion and, hence, the rest of the summation terms corresponding to this series expansion have been canceled as a result of the subtraction of two Meijer's G-function in Eq. (15).

For the sake of simplicity, the numerical results have been computed for independent and identically distributed (i.i.d.) gamma-gamma and misalignment fading channels. As commented in Section 2, due to the fact that the distance between transmit lasers and photodetector is several orders of magnitude the spacing between transmitters, the MISO FSO channel can be considered as i.i.d, obtaining the similar results as in i.n.i.d. fading channels. At this point, it should, however, be noted that obtained expressions in Eqs. (11) and (16) are for i.n.i.d. fading channels and subscripts are retained in order to present more general expressions, which are totally valid for i.i.d. fading channels. The results corresponding to this ergodic capacity analysis are depicted in Fig. 2 when different values of $M = \{2,4,8\}$ are considered for a source-destination link distance of $d_{SD} = 3$ km. Additionally, we also include the performance analysis for the direct path link in order to establish the baseline performance. Different weather conditions are adopted: haze visibility of 4 km with $C_n^2 = 1.7\times10^{-14}\ m^{-2/3}$ and clear visibility of 16

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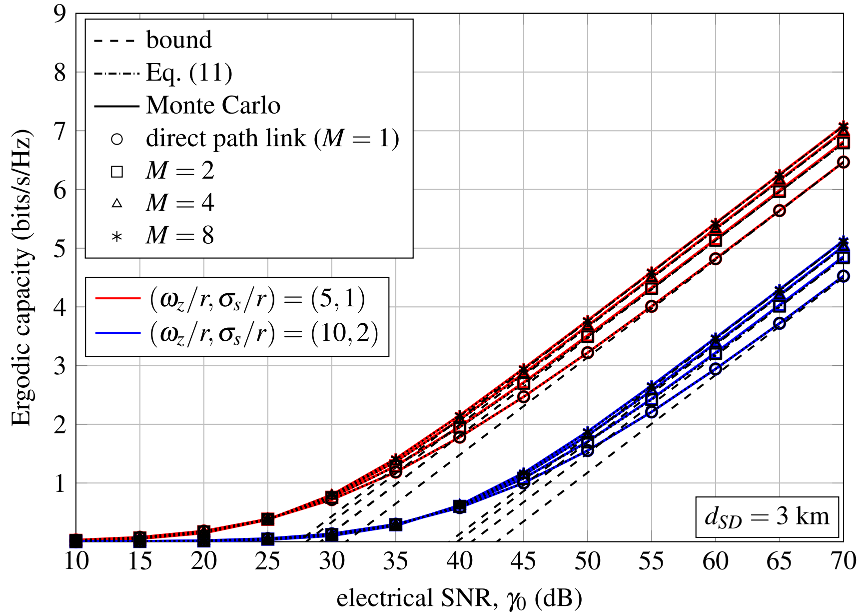
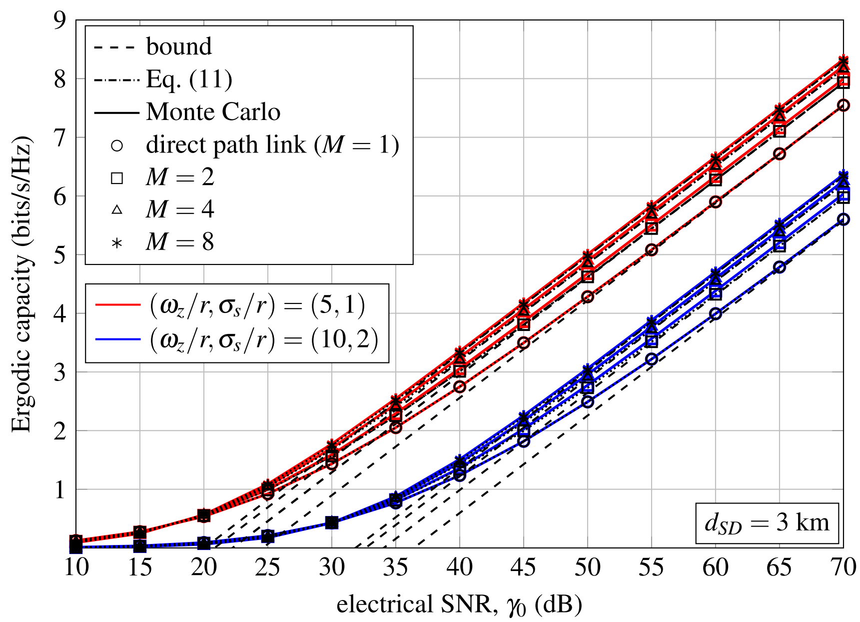


(a) Moderate turbulence. $C_n^2 = 1.7 \times 10^{-14}\ m^{-2/3}$.



(b) Strong turbulence. $C_n^2 = 8 \times 10^{-14}\ m^{-2/3}$.

Fig. 2. Ergodic capacity for a source-destination link distance of d_{SD}= 3 km when different weather conditions and values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r)$ =(5,1) and $(\omega_z/r, \sigma_s/r)$=(10,2) are assumed.

km with $C_n^2 = 8\times10^{-14}\ m^{-2/3}$, corresponding to moderate and strong turbulence conditions, respectively. Here, parameters α and β are calculated from Eq. (2). A value of wavelength of $\lambda = 1550$ nm is chosen. Pointing errors are present here assuming values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (5,1)$ and $(\omega_z/r, \sigma_s/r) = (10,2)$. Monte Carlo simulation results are furthermore included as a reference, which are obtained from Eq. (6), confirming the usefulness and accuracy of the proposed approximation given in Eq. (11). In addition, this figure shows the high accuracy of the asymptotic results based on the logarithm approximation given in Eq. (16) at high SNR. The exact closed-form solution for the ergodic capacity corresponding to the direct path link as well as the asymptotic ergodic capacity expression can be obtained from Eqs. (11) and (16), respectively when the parameter M is set to 1. It is noteworthy to mention that the obtained results provide an excellent match between the analytical and the respective simulated results, which verify the high accuracy of the proposed approximation. In addition to this, it must be highlighted that the analytical ergodic capacity given in Eq. (11) is very precise in the entire SNR regime, i.e. from low to high SNR.

From previous results, this ergodic asymptotic capacity analysis can be extended in order to obtain a point where the asymptotic ergodic capacity at high SNR intersects with the γ_0-axis. This point can be understood as a SNR threshold, i.e. $\gamma_{\text{MISO}}^{th}$, in which the ergodic capacity corresponding to this MISO FSO communications system is significantly increased. From Eq. (16) the corresponding expression of $\gamma_{\text{MISO}}^{th}$ in terms of the MISO FSO channel can be derived as

$$\gamma_{\text{MISO}}^{th}[dB] = \frac{-20}{\ln(10)}\left(\frac{\ln(F)}{M} + \ln(2) + \frac{1}{M}\sum_{k=1}^{M}\psi(\alpha_k) + \psi(\beta_k) - \ln\left(\frac{\alpha_k\beta_k}{A_kL_k}\right) - \frac{1}{\varphi_k^2}\right). \quad (17)$$

Similar to Eq. (17), we can also obtain the corresponding SNR threshold corresponding to the direct path link when the parameter M is set to 1, γ_{DL}^{th}, as follows

$$\gamma_{\text{DL}}^{th}[dB] = \frac{-20}{\ln(10)}\left(\ln(2) + \psi(\alpha_{\text{DL}}) + \psi(\beta_{\text{DL}}) - \ln\left(\frac{\alpha_{\text{DL}}\beta_{\text{DL}}}{A_{\text{DL}}L_{\text{DL}}}\right) - \frac{1}{\varphi_{\text{DL}}^2}\right), \quad (18)$$

where α_{DL}, β_{DL}, φ_{DL}^2, A_{DL} and L_{DL} are parameters corresponding to the direct path link. Next, it can be easily deduced from asymptotic analysis at high SNR that the shift of the ergodic capacity versus SNR is more relevant than the slope of the curve in SNR compared to other performance metrics such as BER and outage probability. This shift can be interpreted as an improvement on ergodic capacity in order to maintain the same performance in terms of capacity with less SNR. From Eqs. (17) and (18), we can obtain this improvement or gain corresponding to this MISO FSO communications system for i.i.d. fading channels, i.e. $G_{\text{MISO}}[dB]$, as follows

$$G_{\text{MISO}}[dB] = \gamma_{\text{DL}}^{th}[dB] - \gamma_{\text{MISO}}^{th}[dB] = \frac{20\ln(F)}{M\ln(10)}. \quad (19)$$

The expression in Eq. (19) has been derived for independent and identically distributed (i.i.d.) fading channels in order to observe the impact of the number of lasers on the MISO FSO system. For the better understanding of the impact of the MISO FSO system under study, the gain in Eq. (19) as a function of the source-destination link distance is depicted in Fig. 3 when a value of normalized beam width $\omega_z/r = 7$ and values of normalized jitter of $\sigma_s/r = \{1,3,4\}$ are considered. Different weather conditions and different values of $M = \{2,6\}$ are also assumed. As expected, the ergodic capacity corresponding to the MISO FSO system is strongly dependent not only on the number of laser sources, but also on the severity of pointing errors. This capacity is significantly increased as the parameter M increases. Furthermore, a greater gain can be achieved as the value of normalized jitter increases, showing an excellent robustness against the pointing error as well as atmospheric turbulence. It must be also commented

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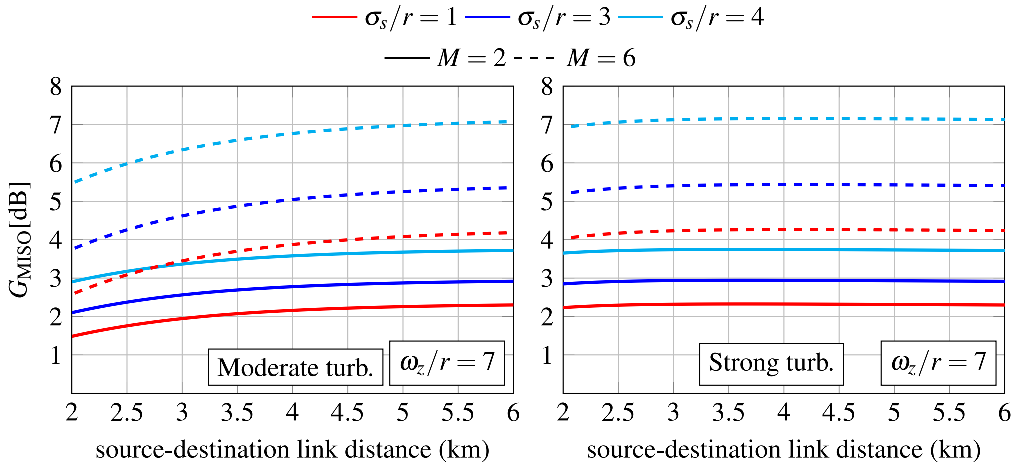

Fig. 3. Gain for a source-destination link distance of $d_{SD} = 3$ km when different weather conditions and a value of normalized beam width of $\omega_z/r = 7$ and values of normalized jitter of $\sigma_s/r = \{1,3,4\}$ are assumed.

that the gain is less vulnerable as the source-destination link distance increases under strong turbulence conditions. These results show a relevant impact of pointing errors on the capacity of the MISO FSO communications system. Taking into account this fact and, knowing that the impact of pointing errors in our analysis can be suppressed by assuming $A_0 \to 1$ and $\varphi^2 \to \infty$ [27], the corresponding ergodic capacity expression is derived from Eq. (11). This expression can be easily obtained from the definition of the H-Fox function [42, eqn. (1.1)] by using the following property $z\Gamma(z) = \Gamma(z+1)$ given in [41, eqn. (06.05.17.0002.01)] as

$$\prod_{k=1}^{M} \lim_{\varphi_k^2 \to +\infty} \frac{\Gamma\left(\varphi_k^2+1\right)\Gamma\left(\varphi_k^2-\frac{2s}{M}\right)}{\Gamma\left(\varphi_k^2\right)\Gamma\left(1+\varphi_k^2-\frac{2s}{M}\right)} = \prod_{k=1}^{M} \lim_{\varphi_k^2 \to +\infty} \frac{M\varphi_k^2}{M\varphi_k^2-2s} = 1. \quad (20)$$

Hence, the approximate closed-form solution for the ergodic capacity corresponding to this MISO FSO communications system, C_{MISO}^{npe}, as follows

$$C_{\mathrm{MISO}}^{npe} \simeq \frac{B H_{2+2M,2}^{1,2+2M}\left(4\gamma_0 \sqrt[M]{F^{npe}}\left(\prod_{k=1}^{M}\frac{L_k}{\alpha_k\beta_k}\right)^{\frac{2}{M}} \middle| \begin{matrix}(1,1),(1,1),\xi_3 \\ (1,1),(0,1)\end{matrix}\right)}{\ln(4)\prod_{k=1}^{M}\Gamma(\alpha_k)\Gamma(\beta_k)}, \quad (21)$$

where $\xi_3 = \{(1-\alpha_1,\frac{2}{M}),(1-\beta_1,\frac{2}{M}),\ldots,(1-\alpha_M,\frac{2}{M}),(1-\beta_M,\frac{2}{M})\}$ and F^{npe} is the corresponding correcting factor when the pointing error effects are suppressed. In this way, we can obtain the corresponding asymptotic ergodic capacity of the MISO FSO communications system when no pointing errors are present from Eq. (16) as follows

$$C_{\mathrm{MISO}}^{H^{npe}} \doteq \frac{B\ln(4\gamma_0)}{\ln(4)} + \frac{B\ln(F^{npe})}{M\ln(2)} + \frac{B}{M\ln(2)}\sum_{k=1}^{M}\psi(\alpha_k)+\psi(\beta_k)-\ln\left(\frac{\alpha_k\beta_k}{L_k}\right). \quad (22)$$

Similar to Eq. (17), we can derive the corresponding SNR threshold corresponding to the MISO FSO system when no pointing errors are present, $\gamma_{\mathrm{MISO}}^{th_{npe}}$, as follows

$$\gamma_{\mathrm{MISO}}^{th_{npe}}[dB] = \frac{-20}{\ln(10)}\left(\frac{\ln(F^{npe})}{M} + \ln(2) + \frac{1}{M}\sum_{k=1}^{M}\psi(\alpha_k)+\psi(\beta_k)-\ln\left(\frac{\alpha_k\beta_k}{L_k}\right)\right). \quad (23)$$

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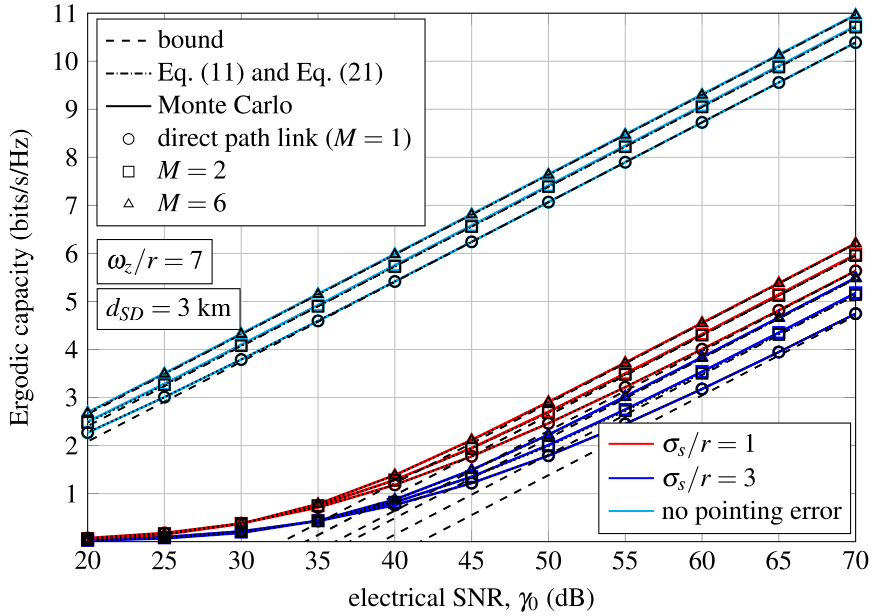
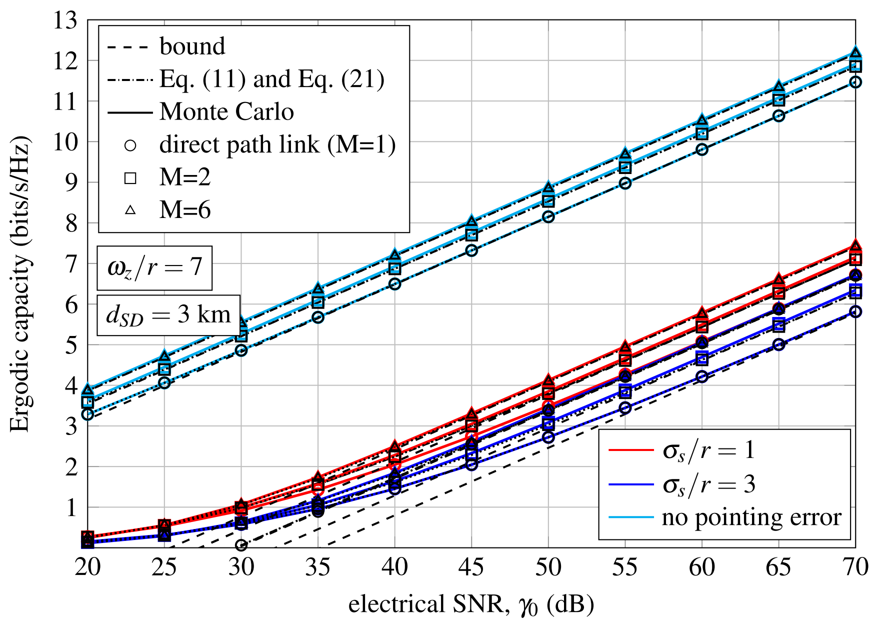


(a) Moderate turbulence. $C_n^2 = 1.7 \times 10^{-14}\ m^{-2/3}$.



(b) Strong turbulence. $C_n^2 = 8 \times 10^{-14}\ m^{-2/3}$.

Fig. 4. Ergodic capacity for a source-destination link distance of $d_{SD} = 3$ km when different weather conditions and a value of normalized beam width of $\omega_z/r = 7$ and values of normalized jitter of $\sigma_s/r = \{1,3\}$ are assumed.

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Obtained conclusions in Fig. 2 are contrasted in Fig. 4, where the effect of misalignment on the ergodic capacity of this MISO FSO communications system is evaluated when different values of $M = \{2,6\}$ are considered for a source-destination link distance of $d_{SD} = 3$ km. Here, the performance analysis for the direct path link is also included as well as the obtained result when pointing errors are not considered. Pointing errors are present here assuming a value of normalized beam width of $\omega_z/r = 7$ and values of normalized jitter of $\sigma_s/r = \{1,3\}$. On the one hand, it can be observed that the ergodic capacity corresponding to this MISO FSO system is dramatically decreased as the normalized jitter increases. On the other hand, it can be also observed that when the ratio between normalized beam width and normalized jitter is increased, the ergodic capacity is also increased. Finally, from Eqs. (17) and (23), the impact of the pointing error effects translates into a gain disadvantage, $D_{pe}[dB]$, relative to this MISO FSO system without misalignment fading given by

$$D_{pe}[dB] \triangleq \gamma_{\mathrm{MISO}}^{th}[dB] - \gamma_{\mathrm{MISO}}^{th_{npe}}[dB] = \frac{20}{M\ln(10)} \left(\ln\left(\frac{F^{npe}}{F}\right) + \sum_{k=1}^{M} \frac{1}{\varphi_k^2} - \ln(A_k) \right). \quad (24)$$

The expression in Eq. (24) can be simplified for i.i.d. fading channels as

$$D_{pe}[dB] \triangleq \frac{20}{\ln(10)} \left(M\ln\left(\frac{M\varphi^2}{M\varphi^2+1}\right) + \ln\left(\frac{\varphi^2+1}{\varphi^2}\right) + \frac{1}{\varphi^2} - \ln(A_0) \right). \quad (25)$$

The derived expression in Eq. (25) only depends on the number of laser sources and pointing error parameters. According to the expression in Eq. (25), it can be seen in Fig. 4 that a gain disadvantage of 28.65 dB for $M = \{2,6\}$ is achieved when values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (7,1)$ are assumed for moderate turbulence and, gains disadvantage of 33.6 and 33.02 dB are achieved for $M = 2$ and $M = 6$ respectively, when values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (7,3)$ are assumed for strong turbulence.

4. Conclusions

In this work, the ergodic capacity performance for MISO FSO communication systems with EGC reception is analyzed over i.n.i.d. gamma-gamma and misalignment fading channels. Novel and analytical closed-form ergodic capacity expression is obtained in terms of H-Fox function by using the well-known inequality between arithmetic and geometric mean of positive random variables in order to obtain an approximate closed-form expression of the distribution of the sum of M gamma-gamma with pointing errors variates. Obtained results have confirmed the high accuracy of the proposed approximation in the entire SNR regime based on the well-known inequality between arithmetic and geometric mean. From the asymptotic ergodic capacity analysis is easily deduced that the shift of the ergodic capacity versus SNR is here more relevant than the slope of the curve in SNR compared to other performance metric such as BER and outage probability. This shift is interpreted as an improvement on ergodic capacity in order to maintain the same performance in terms of capacity with less SNR, which depends on the number of laser sources and channel parameters. It can be concluded that the use of MISO technique can significantly increase the capacity gain over the direct path link. The impact of pointing errors on the MISO FSO system is also analyzed, which only depends on the number of laser sources and pointing error parameters. Moreover, it can be also concluded that the ergodic capacity performance is dramatically reduced as a consequence of the severity of pointing error effects. From the relevant results derived here, researching the impact of channel correlation on ergodic capacity of MISO FSO systems as well as studying a MIMO FSO system with non zero boresight are interesting topics for future research in order to complement the study in this work.

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Appendix

In this appendix correcting factors F and F^{npe} are obtained from Eq. (8) when both averages hold and, hence, it can be expressed as follows $\mathbb{E}[I_T] = M\sqrt[M]{F} \cdot \mathbb{E}\left[\sqrt[M]{I_{LB}}\right]$. It should be noted that parameters F and F^{npe} not only depend on the parameter M, i.e. number of lasers, but also on the atmospheric turbulence as well as pointing errors. Hence, we can express the correcting factor, F, as follows

$$F = \frac{\mathbb{E}[I_T]^M}{M^M \cdot \mathbb{E}\left[\sqrt[M]{I_{LB}}\right]^M}. \tag{26}$$

Firstly, before evaluating the parameter F, we obtain the mean of I_T as $\mathbb{E}[I_T] = \sum_{k=1}^{M} \mathbb{E}[I_k]$ since the variates I_k for $k = \{1, \ldots, M\}$ are statistically independent. Therefore, the mean of a generic random variable I_k is given by

$$\mathbb{E}[I_k] = \int_0^\infty i f_{I_k}(i) di. \tag{27}$$

The integral in Eq. (27) can be evaluated with the help of [40, eqn. (2.24.2.1)] and, hence, $\mathbb{E}[I_T]$ can be expressed as follows

$$\mathbb{E}[I_T] = \sum_{k=1}^{M} \frac{A_k L_k \varphi_k^2}{1+\varphi_k^2}. \tag{28}$$

Secondly, we obtain the expectation of positive Mth root of I_{LB}, i.e. $\mathbb{E}\left[\sqrt[M]{I_{LB}}\right]$, as $\mathbb{E}\left[\sqrt[M]{I_{LB}}\right] = \prod_{k=1}^{M} \mathbb{E}\left[\sqrt[M]{I_k}\right]$. Therefore, the expectation of positive Mth root of I_k is given by

$$\mathbb{E}\left[\sqrt[M]{I_k}\right] = \int_0^\infty i^{1/M} f_{I_k}(i) di. \tag{29}$$

In order to evaluate the integral in Eq. (29), we can use [40, eqn. (2.24.2.1)] as in Eq. (28) and, hence, the expectation of positive Mth root of I_{LB} is given by is given by

$$\mathbb{E}\left[\sqrt[M]{I_{LB}}\right] = \prod_{k=1}^{M} \frac{M\varphi_k^2 \Gamma\left(\alpha_k + \frac{1}{M}\right)\Gamma\left(\beta_k + \frac{1}{M}\right)}{\Gamma(\alpha_k)\Gamma(\beta_k)\left(M\varphi_k^2+1\right)} \left(\frac{\alpha_k \beta_k}{A_k L_k}\right)^{-1/M}. \tag{30}$$

The corresponding correcting factor when no pointing errors are present, F^{npe}, is obtained by assuming $A_0 \to 1$ and $\varphi^2 \to \infty$ in Eqs. (28) and (30). Hence, $\mathbb{E}[I_T]^{npe}$ can be written as

$$\mathbb{E}[I_T]^{npe} = \sum_{k=1}^{M} L_k, \tag{31}$$

and, $\mathbb{E}\left[\sqrt[M]{I_{LB}}\right]^{npe}$ is given by

$$\mathbb{E}\left[\sqrt[M]{I_{LB}}\right]^{npe} = \prod_{k=1}^{M} \frac{\Gamma\left(\alpha_k + \frac{1}{M}\right)\Gamma\left(\beta_k + \frac{1}{M}\right)}{\Gamma(\alpha_k)\Gamma(\beta_k)} \left(\frac{\alpha_k \beta_k}{L_k}\right)^{-1/M}. \tag{32}$$

Finally, correcting factors F and F^{npe} are easily derived substituting Eq. (28) and Eq. (30) into Eq. (26) and, Eq. (31) and Eq. (32) into Eq. (26), respectively.

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