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EFFICIENCY OF SECONDARY SCHOOLS IN PORTUGAL: A NOVEL DEA HYBRID APPROACH

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Abstract

This paper is aimed at casting new light on the assessment of secondary schools' performance in Portugal by considering data from the Programme for International Student Assessment 2015. We use the Weighted Russel Directional Distance approach and establish an equivalence model between this Data Envelopment Analysis model and the super-ideal point model, which is then combined with an interactive algorithm. Besides, a procedure to perform robustness analysis has been proposed, allowing decision-makers to understand how sensitive the efficiency of each school to data variation is.

Overall, our findings suggest that the average efficient public school has mean scores across all competences above the scores attained by the average public national school, but below the OECD average. In addition, we have concluded that schools operating with the lowest efficiency scores usually need to make a greater effort to improve results in reading when compared to maths and science competences. Finally, schools more often selected as a reference in terms of best practices and which are classified as robust in terms of efficiency are not necessarily performing with high test results, highlighting the need of seeking new benchmarks more consistent with the preferences of the decision-maker.

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1 INTRODUCTION

Because the resources being assigned to the educational system are scarce and are competing with other public expenses (e.g. health system), there has been an increasing awareness of the importance of the efficiency assessment of the public education sector. In fact, the topic of efficiency in education has received broad coverage in the scientific literature (for a recent review of these studies see Jonhes (2015) and De Witte and López-Torres (2017); specifically on the effects of innovations on education see Haelermans and De Witte (2012) and on allocative efficiency see Haelermans *et al*., (2012)), being at the forefront of the political agenda in many countries.

In this context, the educational production function can be empirically obtained through two different approaches. Generally, stochastic methods are applied to the estimation of the frontier production function either by using regression analysis or by applying other similar approaches. The alternative for establishing the efficiency frontier requires the use of non-parametric methods, such as Data Envelopment Analysis (DEA). This type of methodology can easily tackle multiple inputs and outputs, whilst most stochastic methods require the consideration of a single output. Moreover, the DEA methodology has been broadly accepted and applied to the topic of education (in its distinct variants), representing one of the top fields of assessment with this methodological tool (Emrouznejad *et al*., 2008; Liu *et al*., 2013; Emrouznejad and Yang, 2018[1]). Due to its flexibility, DEA provides help with the identification of possible sources of inefficiency offering public decision-makers (DMs) the chance of studying ways to overcome them.

One of the main advantages of the application of DEA in efficiency assessment is the possibility of finding the benchmarks of inefficient Decision-Making Units (DMUs), providing managers with valuable information regarding the best practices to be followed. These benchmarks are computed through linear programming (LP) models and are obtained just by using the original inputs and outputs. Nevertheless, one of the limitations of traditional DEA models is that they do not typically incorporate the preference structure or value judgments of DMs (Allen *et al.*, 1997). This feature can be particularly relevant in an educational context, since the benchmarks thus computed might not be reachable due to budgetary constraints or they may not reflect the DMs' aspirations (Tavana *et al.*, 2018). In this vein, the existing approaches that

[1] These authors also highlight that the journal of Socio-Economic Planning Sciences has been identified as the first choice for DEA papers with applications in the public sector.

incorporate the DMs' preferences in DEA models do not allow accounting for different priorities and individual expansion and contraction scales regarding the outputs and inputs. However, many real-world problems require the use of non-radial measures of technical efficiency (Aparicio *et al*., 2018). In the context of education, some schools might be interested in improving the scores in specific competences (e.g. mathematics), due to, for example, cultural characteristics and traditions inherent to the country where they are geographically located. In the framework of non-radial measures, Aparicio *et al*. (2018) published the first paper in the literature dedicated to the estimation of technical efficiency in education for OECD countries resorting to this type of approach. However, this latter study uses an oriented model. In this case, as noted by Chen *et al*. (2015), on the one hand, the inefficiency linked to the use of a specific input by a DMU is not necessarily related to the inefficiency regarding the use of another input by the same DMU; on the other hand, a DMU may produce distinct outputs at the same time, but with a different production capacity, and hence the production efficiency for different outputs may also be distinct. Therefore, besides being non-radial (unlike the DEA model used in Tavana *et al*. (2018)), the modelling framework herein considered is also non-oriented (differently from the DEA model employed in Aparicio *et al*. (2018)).

In this context, an equivalence model between the Weighted Russell Directional Distance Model (WRDDM) and the super-ideal point model based on the reference point approach is herein established which is then embedded in an interactive algorithm that allows for the explicit incorporation of the DM's preferences. Additionally, we also suggest a new procedure to perform the robustness analysis of the results obtained either in a worst or best-case scenario, allowing the DMs to understand how sensitive the efficiency of each DMU to data variation is. This tool is mostly relevant to assess how changes in those measures with intra-school variability (i.e. with more than one observation for each school like test scores and the socio-economic status of students), might affect the scores of efficient DMUs.

Finally, since there is a lack of scientific literature that assesses the efficiency of the global population of Portuguese secondary schools by means of DEA and at the same time there are no studies based on the most recent Programme for International Student Assessment (PISA) data, an empirical application of this novel modelling framework is suggested which enables assessing Portuguese secondary education schools' efficiency.

This study is specifically relevant, as indicators of educational results in Portugal have – historically - shown a low performance - relative to the EU countries - in terms of enrolment, progress and completion rates at the level of secondary school. Nevertheless, along the last decade, the Portuguese secondary education system presents a significant improvement on average scores in maths and science. Therefore, we are aimed at understanding what are the main characteristics of efficient schools and to explore the different opportunities of improvement of inefficient schools according to the selection of new benchmarks consistent with distinct DMs' concerns.

The dataset used for the current research is that from PISA 2015 because it contains questions which allow measuring, on a per-school basis, mean scores in maths, reading and science, the mean socio-economic status, the student-teacher ratio and the shortage of educational materials[2], among other contextual factors. Last, but not least, the sample under scrutiny is representative of the whole country.

The remainder of this paper is given as follows. Section 2 delivers a literature review of studies specifically devoted to the appraisal of Portuguese schools. Section 3 provides an overview of the methodological approach followed. Section 4 describes the main premises considered regarding data collection. Section 5 discusses some illustrative results obtained with the proposed methodology. Finally, the main conclusions will be presented in Section 6.

2 LITERATURE REVIEW

Several efforts have been undertaken to appraise Portuguese schools. In Portugal, the General Inspectorate for Education of the Ministry of Education is held responsible for the evaluation of schools, selecting every year the target schools to be visited by inspectors, who are responsible for providing an assessment report of these schools (Portela *et al.*, 2012). However, this type of evaluation is not able to recognize the schools that could be used as benchmarks, making it hard to understand how schools are performing against their peers.

Another evaluation system available for Portuguese schools is also known as Assessment of Secondary Schools, which is conducted by Fundação Manuel Leão. However, this programme does not include schools from the whole country and the schools' involvement is not compulsory. In this sort of evaluation system data are gathered at the students' level, without accounting for other sorts of variables at the teachers' level or the school level.

In spite of the criticism raised by these types of evaluation procedures, a more comprehensive assessment of schools has been postponed due to the lack of data availability.

In this context, in 2010, a platform for secondary schools (Benchmarking of Portuguese Secondary Schools)[3] was settled aimed at helping Portuguese schools with their internal and external evaluation. With this platform several key performance indicators are used to appraise in real-time schools' performance against other schools, also providing the progression of their performance throughout time. The set of indicators constructed within this platform incorporates four major areas: context, resources, results, and processes (for further information regarding this evaluation system see Portela *et al.* (2011)).

[2] The latest version available of PISA is from 2018 but it does not include the index of shortage of educational materials.
[3] http://feg.porto.ucp.pt/besp/

Other examples of studies involving the evaluation of Portuguese schools might be found in Ferrão and Goldstein (2009) who used the multilevel value-added approach and proposed a method to adjust for measurement error to investigate the extent to which this changes school value-added estimates in the particular region of Cova da Beira. The main focus of this study was to find the main explanatory factors for the results obtained by students in maths as well as those responsible for the value-added of schools.

At the aggregated school level, we find the study conducted by Oliveira and Santos (2005) which assesses a set of 42 Portuguese public schools employing a Free Disposal Hull reference technology. The authors concluded that certain characteristics of the municipalities such as where the schools are located, the unemployment rate, the access to health care services, adult education and living infrastructures, were determinants of those schools' efficiency. Similarly, Pereira and Moreira (2007) appraised the efficiency of a big sample of secondary schools (502) but using a parametric approach (stochastic frontier analysis). These authors deemed only one variable of school performance (the average secondary national exams score), being contextualized by school (number of students, teachers, and classes), teacher (average age) and environmental variables (living standard and average years of schooling of the corresponding municipality).

Nevertheless, studies in Portugal that assess the performance of the overall population of Portuguese schools are less prolific, mainly due to data scarcity. Therefore, the Data Envelopment Analysis (DEA) model has essentially been used in small school samples. In this context, Sarrico and Rosa (2009) assessed 51 Portuguese secondary education schools (using data for 2007); this was not a random sample. These authors also contrasted the results obtained from the DEA assessment with the qualitative evaluation followed by the general inspectorate of education and revealed that the results of both assessments were uncorrelated. Later on, Sarrico *et al.* (2010) evaluated 29 self-selected[4] schools in the central region of Portugal with the application of DEA to obtain two measures of efficiency for schools, a 'school's choice model' and a 'school's management model' both differing in the type of inputs used. This work was inspired by the work done in Portela and Camanho (2007) where 22 Portuguese schools were evaluated in terms of efficiency performance also through DEA following two standpoints: society and educational authorities.

More recently, Portela *et al.* (2012) evaluated the efficiency performance of Portuguese secondary schools applying DEA, using as outputs the number of average scores (obtained on national examinations) on leaving school. These scores were contextualized by average scores obtained on entering secondary education by a cohort of students similar to that evaluated while leaving if students remained essentially the same from basic to secondary education in that school.

[4] The schools in the sample chose to answer the electronic questionnaire, thus it is not a random sample.

From the review conducted it can be concluded that studies that evaluate the efficiency of the global population of Portuguese secondary schools through DEA are less abundant and to the best of our knowledge do not consider the recent PISA 2015 data.

3 THE METHODOLOGICAL APPROACH

Numerous approaches have been proposed to incorporate the DMs' preferences in DEA models. With this regard, there are several categories of models: the efficiency score models which use the DMs' preference information to generate more valuable efficiency scores; models which incorporate the DM's preferences through weight restrictions; and the target setting models which contemplate the DMs' preference information to obtain more reasonable targets.

In what concerns the first type of models, the selection of the most preferred solution (MPS), by explicitly locating the DM's most preferred vector of inputs-outputs on the efficient frontier, has been explored through value efficiency analysis in Halme *et al*. (1999), Korhonen *et al*. (2003) and Halme and Korhonen (2000). This approach allows incorporating the DM's preference information in order to obtain the value efficiency scores, being mainly applied to CCR (Charnes *et al*., 1978) and BCC (Banker, 1984) models and as such considers the same expansion and contraction rates of inputs or outputs according to the model's orientation.

Models for estimating restrictions for the DEA weights have been developed by several authors (a review of some of these models following the assumption of constant returns to scale (CRS) is given in Allen *et al*. (1997) and for the general case in Thanassoulis *et al*. (2004, 2008)). Usually this type of approach either involves the use of direct restrictions on the weights, namely absolute weight restrictions (see e.g. Beasley (1990, 1995), Podinovski and Athanassopoulos, 1998), assurance regions of type I and type II (see e.g. Thompson *et al*. (1990), Halkos *et al*. (2015), Degl'Innocenti *et al*. (2017) and Kourtzidis *et al*. (2019)), and restrictions on virtual inputs and outputs[5] (an approach which was first suggested by Wong and Beasley (1990)), or it applies changes to the data set in a way that aims at incorporating value judgments, including the cone ratio approach and the unobserved DMUs approach (this latter approach was developed by Allen and Thanassoulis (2004) and Thanassoulis *et al*. (2012) both for CRS and variable returns to scale (VRS) technologies, respectively).

In the case of target setting models (the type of models employed in this study), one of the most used approaches is Multiobjective Linear Programming (MOLP). Golany (1988) introduced this methodology by asking the DMs to assign a set of input levels as resources and to select the most preferred set of output levels from a set of feasible points on the efficient frontier, leading to the joint use of DEA with MOLP.

[5] A virtual input/output is the product of the input/output level and the corresponding DEA weight.

More recently, Yang *et al*. (2009) proposed three equivalence models between the output-oriented dual DEA model and the minimax reference point formulations: the super-ideal point, the ideal point, and the shortest distance models. Like traditional DEA models, these can assess efficiency also providing the trade-off evaluation involved in setting the target values. In the same vein, Wong *et al*. (2009) suggested an equivalence model between DEA and MOLP and revealed how a DEA model can be solved interactively without any prior judgement by using an MOLP formulation.

However, one of the problems faced by the models used by the previous authors is that their major concern is focused on the output increase since they are output-oriented dual DEA models. As a result, the DM can only reflect his/her preferences on the selection of the output values to reach the MPS as the best target unit, having no control at all on the inputs. Also, the DEA model usually suggested to solve this sort of problems is a radial model, only accounting for similar expansion rates of the outputs. In order to partially overcome this limitation, Malekmohammadi *et al*. (2011) suggested a super-ideal model, identical to a target model that enables contemplating identical reduction and expansion scales in total inputs and outputs, respectively, also projecting all inputs and outputs onto the efficient frontier using a single LP problem.

The equivalence between DEA and MOLP models has also been studied in Yang and Xu (2014). These authors demonstrated that minimax reference point models are equivalent to input-oriented dual CCR models if certain conditions are met, proposing an interactive minimax reference point approach. With the same reasoning, Lotfi *et al.* (2010) proposed a novel association between the output-oriented BCC model and the weighted minimax reference point approach. Subsequently, these authors applied the satisfying trade-off method to support the DM in the search for the most preferred reference DMU.

Finally, Tavana *et al*. (2018) established an extended equivalence model between the directional distance function radial non-oriented DEA model and the super-ideal point model. The usefulness of the proposed method is explored by considering the appraisal of the efficiency performance of high schools in the City of Philadelphia. When contrasted to other approaches, this DEA MOLP framework enables the DM to select the variables, as well as their relative intensities, in order to move the DMU closer to the efficient frontier. In this way, the DM can evaluate the strategic trade-offs between the different factors that can reduce the DMU's distance to the efficient frontier. This is an important feature in real-world problems where the DM usually faces budget constraints that preclude him/her from improving all the inefficient factors of a given DMU.

Despite the merits of the comprehensive hybrid DEA MOLP model developed in Tavana *et al*. (2018), this study assumes that the expansion of all outputs and contraction of all inputs is considered to have the same rate. Furthermore, another limitation of this approach is that it does not account for inefficiencies associated with non-zero slacks and it eventually has the problem of miss specifying some evaluated DMUs as efficient units (Chen *et al.*, 2015).

Therefore, we consider the WRDDM further explained in Chen *et al.* (2015), which besides allowing for the technical inefficiency associated with inputs and outputs to be different, also allows for the technical inefficiency among each of the inputs and outputs to be distinct. Additionally, it offers the possibility of comprehending a broad type of factors typically found in the educational context, e.g. discretionary and non-discretionary (for a comprehensive review of the typical inputs and outputs used in education efficiency assessment, please see De Witte and López-Torres (2017)).

3.1. The Weighted Russell Directional Distance Model

We consider the WRDDM formulation suggested in Chen *et al.* (2015), which can easily be adjusted to account for inputs that cannot be varied at the discretion of management (in our case the index of economic, social and cultural status), by considering the one-stage models approach proposed in Banker and Morey (1986), given as follows:

max $\beta_o^R = max\ (w_y(\sum_r \varpi_y^r \alpha_o^r) + w_x(\sum_i \varpi_x^i \zeta_o^i))$

s.t. $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \alpha_o^r g_{yr}$, $r = 1, \ldots, s$,

$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \zeta_o^i g_{xi}$, $i = 1, \ldots, m$,

$\sum_{j=1}^{n} \lambda_j z_{uj} \leq z_{uo}$, $u = 1, \ldots, q$,

$\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$, $j = 1, \ldots, n$, (1)

where the vectors of inputs and outputs of DMU$_o$ are $\boldsymbol{x}_o$ and $\boldsymbol{y}_o$, respectively, and the vector of non-discretionary factors of DMU*o* is given by $\boldsymbol{z}_o$. The parameters α_o^r and ζ_o^i are the individual inefficiency measures for each output and input, respectively, and all variables are nonnegative except for β_o^R. The parameter $\zeta_o^i g_{xi}$ indicates the level by which DMU$_o$ must reduce its *i*-th input to become efficient. Analogously, the parameter $\alpha_o^r g_{yr}$ provides information on the level by which DMU$_o$ must enlarge its *r*-th output in order to become efficient. The coefficients w_y and w_x may be regarded as the given priorities associated with the outputs and inputs, and their sum should be one. Furthermore, the inefficiencies of each related output and input can also have different priorities and $\sum_{r \in O} \varpi_y^r = 1$, $\sum_{i \in I} \varpi_x^i = 1$. In this case, it is necessary that the directional vectors $\boldsymbol{g}_x$ and $\boldsymbol{g}_y$ are measured according to the same measurement units as the original vectors of inputs and outputs, i.e. $= (-\boldsymbol{g}_x, \boldsymbol{g}_y) = (-\boldsymbol{x}^o, \boldsymbol{y}^o)$, in order to add α_o^r and ζ_o^i. Finally, we assume the VRS technology, which implies the imposition of the additional constraint $\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$ $(\forall_j)$.

If the WRDDM inefficiency measure is zero ($\beta_o^R = 0$), then the DMU is fully efficient.

The reference set of the inefficient DMU$_o$ based on (1) can be obtained through problem (2), assuming that α_o^{r*} and ζ_o^{i*} are the optimal solutions to problem (1):

$\max \sum_r s_r^+ + \sum_i s_i^- + \sum_u s_u^-$,

s.t. $\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} + \alpha_o^{r*} g_{yr}$, $r = 1, \ldots, s$,

$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} - \zeta_o^{i*} g_{xi}$, $i = 1, \ldots, m$,

$\sum_{j=1}^{n} \lambda_j z_{uj} + s_u^- = z_{uo}$, $u = 1, \ldots, q$,

$\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$, $j = 1, \ldots, n$,

$s_r^+ \geq 0\ (\forall_r), s_i^- \geq 0\ (\forall_i), s_u^- \geq 0\ (\forall_u)$ (2)

The point of the efficient frontier which can be viewed as a target DMU for the WRDDM - inefficient DMU$_o$ is given by:

$(\hat{\boldsymbol{x}}_o, \hat{\boldsymbol{y}}_o) = (\sum_{j\epsilon E_o} \lambda_j^* \boldsymbol{x}_j, \sum_{j\epsilon E_o} \lambda_j^* \boldsymbol{y}_j, \sum_{j\epsilon E_o} \lambda_j^* \boldsymbol{z}_j)$ (3)

where (s_r^{+*}, s_i^{-*}, s_u^{-*}, λ_j^*) is the optimal solution to (5), α_o^{r*}, $\zeta_o^{i*}$are already given by model (1) and the reference set of the WRDDM-inefficient DMU$_o$ is:

E$_o$ ={ j: $\lambda_j^* > 0$, j=1, ..., n}. (4)

3.2 DEA and MOLP models

Consider the following MOLP problem:

$\max\ (f_1(\boldsymbol{\lambda}), \ldots, f_{s+m}(\boldsymbol{\lambda}))$,

s.t. $\boldsymbol{\lambda} \in \Lambda$ (5)

where $(f_1(\boldsymbol{\lambda}), \ldots, f_{s+m}(\boldsymbol{\lambda})) = (\sum_{j=1}^{n} \lambda_j^* y_{1j}, \ldots, \sum_{j=1}^{n} \lambda_j^* y_{sj}, -\sum_{j=1}^{n} \lambda_j^* x_{1j}, \ldots, -\sum_{j=1}^{n} \lambda_j^* x_{mj})$ and the decision variables $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)^T$ belong to the non-empty feasible space Λ.

Definition 1. A solution $\boldsymbol{\lambda}' \in \Lambda$ is weakly Pareto efficient to problem (5) if and only if there is no other $\boldsymbol{\lambda} \in \Lambda$ such that $f_k(\boldsymbol{\lambda}') < f_k(\boldsymbol{\lambda})$ for all $k = 1, \ldots, s+m$.

Definition 2. A solution $\boldsymbol{\lambda}' \in \Lambda$ is Pareto efficient to problem (5) if and only if there is no other $\boldsymbol{\lambda} \in \Lambda$ such that $f_k(\boldsymbol{\lambda}') \leq f_k(\boldsymbol{\lambda})$ for all $k = 1, \ldots, s+m$ with at least one strict inequality.

A way of expressing preferences about efficient solutions in MOLP is through the use of a *reference point* $\mathbf{f}^{ref} = (f_1^{ref}, \ldots, f_{s+m}^{ref})^T$, which consists of a desirable or a reference value for the objective functions.

Problem (6) can then be used to account for simultaneous modifications of the input and output values of DMUs.

$$
\begin{aligned}
&\min \quad \beta_o \\
&s.t.: \ \tau_k\left(f_k^{ref} - f_k\,(\boldsymbol{\lambda})\right) \leq \ \beta_o, \ \ k = 1, \dots, s+m \qquad (6)\\
&\qquad \boldsymbol{\lambda} \in \Lambda
\end{aligned}
$$

which implies in our case that problem (11) is linear.

In order to require the uniqueness of the solution, an augmentation term can be added to the objective function of problem (6) (Wierzbicki, 1980).

In fact, Ebrahimnejad and Lotfi (2012) established an equivalence model between the general combined oriented radial DEA model and model (6) with the use of the ideal point, i.e. the individual optimal solutions of each objective function, as a reference point, being also known as the super-ideal point model. Later, this modelling approach was also followed in Tavana et al. (2018). In this line of research, in the next section, we establish an equivalence model between the non-oriented non-radial WRDDM and the super-ideal point model.

3.3 The super-ideal equivalent model to the WRDDM

Let the following MOLP problem be given as:

$$
\begin{aligned}
&\max\ (f_r\,(\boldsymbol{\lambda}), r = \ 1, \dots, s, f_i(\boldsymbol{\lambda}), i = \ 1, \dots, m) \\
&\text{s.t. } \boldsymbol{\lambda} \in \Lambda \qquad (7)
\end{aligned}
$$

Considering a similar reasoning to the one given previously, but bearing in mind that unlike in Tavana et al. (2018) we are in the presence of a non-radial model and thus new conditions have to be employed, the next super-ideal point model (8) can be used to generate any efficient solution to MOLP problem (7):

$$
\begin{aligned}
&\min \beta, \\
&\text{s.t. } \tau_{yr}(f_r^{ref} - f_r\,(\boldsymbol{\lambda})) \leq \ \alpha^r\,, r = \ 1, \dots, s, \\
&\tau_{xi}\left(f_i^{ref} - f_i\,(\boldsymbol{\lambda})\right) \leq \ \zeta^i\ , \ i = \ 1, \dots, m, \\
&(w_y(\textstyle\sum_{r\,\epsilon\,O} \varpi_y^r(\alpha^r\,)) + w_x\left(\textstyle\sum_{i\,\epsilon\,I} \varpi_x^i(\zeta^i\,)\right) \leq \beta, \\
&\textstyle\sum_{r\,\epsilon\,O} \varpi_y^r{=}1, \\
&\textstyle\sum_{i\,\epsilon\,I} \varpi_x^i = 1, \\
&w_y + \ w_x = 1, \\
&\boldsymbol{\lambda} \in \Lambda_o \qquad (8)
\end{aligned}
$$

where f_r^{ref}, $r = \ 1, \dots, s$, and f_i^{ref}, $i = \ 1, \dots, m$, are the reference values for each objective function.

Let

$$f_r\,(\boldsymbol{\lambda}) = \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} - y_{ro}, r = \ 1, \dots, s, \qquad (9)$$

and in a similar way:

$$f_i\,(\boldsymbol{\lambda}) = x_{io} - \textstyle\sum_{j=1}^{n} \lambda_j x_{ij}\,, i = \ 1, \dots, m. \qquad (10)$$

Consider that

$\Lambda_o = \Lambda = \{ \boldsymbol{\lambda}: \sum_{j=1}^{n} \lambda_j z_{uj} \leq z_{uo}, u = 1, ..., q, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \ (\forall_j)\}$. (11)

The maximum feasible variation values of the r^{th} outputs for DMU$_o$ are $\widehat{f_{ro}} = f_r(\boldsymbol{\lambda}^*)$, for $r = 1, \dots, s$, where $\boldsymbol{\lambda}^*$ can be computed by solving the following problems:

$$\widehat{f_{ro}} = \max_{\boldsymbol{\lambda} \in \Lambda_o} f_{ro}(\boldsymbol{\lambda}), r = 1, \dots, s, \quad (12)$$

Analogously, the maximum feasible variation values of the i^{th} inputs for DMU$_o$ are $\widehat{f_{io}} = f_i(\boldsymbol{\lambda}^*)$, $i = 1, \dots, m$, where $\boldsymbol{\lambda}^*$ can be computed by solving the following problems:

$$\widehat{f_{io}} = \max_{\boldsymbol{\lambda} \in \Lambda_o} f_{io}(\boldsymbol{\lambda}), i = 1, \dots, m. \quad (13)$$

The equivalence between the general DEA Model (1) and problem (8) can be settled by the following theorem (see the proof in Appendix).

Theorem 1. Let $g_{yr} > 0$ $(\forall r)$, $g_{xi} > 0$ $(\forall i)$. The general combined-oriented DEA Model (1) can be equivalently transformed into the super-ideal point Model (8) using Equations (9)-(13) and the next definitions:

$$\tau_{yr} = \frac{1}{g_{yr}}, r = 1, \dots, s, \quad (14)$$

$$\tau_{xi} = \frac{1}{g_{xi}}, i = 1, \dots, m, \quad (15)$$

$$f_r^{ref} = \frac{F_r^{max}}{\tau_{yr}}, r = 1, \dots, s, \quad (16)$$

$$f_i^{ref} = \frac{F_i^{max}}{\tau_{xi}}, i = 1, \dots, m, \quad (17)$$

$$F_r^{max} = \{\tau_{yr} \widehat{f_{ro}}\} = \{\frac{\widehat{f_{ro}}}{g_{yr}}\}, r = 1, \dots, s \quad (18)$$

$$F_i^{max} = \{\tau_{xi} \widehat{f_{io}}\} = \{\frac{\widehat{f_{io}}}{g_{xi}}\}, i = 1, \dots, m \quad (19)$$

$$\alpha^r = F_r^{max} - \alpha_o^r, r = 1, \dots, s, \quad (20)$$

$$\zeta^i = F_i^{max} - \zeta_o^i, i = 1, \dots, m, \quad (21)$$

$$\beta = ((w_y(\textstyle\sum_{r \in O} \varpi_y^r F_r^{max}) + w_x(\sum_{i \in I} \varpi_x^i F_i^{max})) - \beta_o^R. \quad (22)$$

3.4. The interactive approach to obtain solutions to the hybrid DEA MOLP model

In this section we describe an interactive approach inspired in Tavana et al. (2018) to obtain solutions to the hybrid DEA MOLP model previously considered. Nevertheless, some differences might be found in the approach herein followed when contrasted to the one proposed in Tavana et al. (2018): we use a non-radial model which allows the DMs to assign different weights to the inputs and outputs according to his/her preferences, thus allowing the examination of different weight profiles in the search of new reference targets. Furthermore, the search of new targets used in the search of the new reference DMUs is facilitated by the information given by the robustness approach proposed in Section 3.5.

Step 1. Solve problem (1) to identify the efficient and inefficient DMUs using a selected combination of weights for each input and output and remaining with this same combination for all the steps of the algorithm. Additionally, use the same weight profiles for all DMUs in order to guarantee the ordering of the DMUs based on the resulting efficiency scores (Ruiz and Sirvent, 2016). Solve the super-efficiency model in order to rank the efficient DMUs:

$\max \beta_o^R = max\ (w_y(\sum_r \varpi_y^r \alpha_o^r) + w_x(\sum_i \varpi_x^i \zeta_o^i)),$

s.t. $\sum_{j\neq o} \lambda_j y_{rj} \geq y_{ro} + \alpha_o^r g_{yr}$, $r = 1, \ldots, s$,

$\sum_{j\neq o} \lambda_j x_{ij} \leq x_{io} - \zeta_o^i g_{xi}$, $i = 1,\ldots, m$,

$\sum_{j\neq o} \lambda_j z_{uj} \leq z_{uo}$, $u = 1, \ldots, q$,

$\sum_{j\neq o} \lambda_j = 1$, $\lambda_j \geq 0, j = 1, \ldots, n$. (23)

Step 2. For each problem given in Step 1, solve problem (2) for each inefficient DMU and obtain the corresponding DMU target set. If the DM is satisfied with the previous target set, then the solution process ends. Otherwise, go to Step 3.

Step 3. Set $h = 1$. Obtain the values of f_r^{ref}, $r = 1, \ldots, s$, f_i^{ref}, $i = 1, \ldots, m$, through expressions (9) to (14) and (16) to (19).

Step 4. Compute $f_r(\boldsymbol{\lambda}^*)$, $r = 1, \ldots, s$, $f_i(\boldsymbol{\lambda}^*)$, $i = 1, \ldots, m$, where $\boldsymbol{\lambda}^*$ is the optimal solution to each model obtained with formulation (2) and expressions (9) and (10).

Step 5. Consider the information provided by the robustness assessment given in Section 5.2 and assign for each of the main components of the target unit under evaluation to the following three categories: the ones that require further improvement (I^h), the ones that are to be maintained (M^h) and the ones that must be relaxed (R^h). For each $r, i \in I^h$, specify how much the DM wants to improve the corresponding output Δf_r^h or input Δf_i^h and, for each $r, i \in R^h$, specify how much the DM allows to worsen the corresponding output Δf_r^h or input Δf_i^h. This interaction is inspired in a version of the STOM method and in Miettinen and Mäkelä (2006).

Step 6. Update the new reference values given as:

$q_r^h = f_r(\boldsymbol{\lambda}^*) + \Delta f_r^h$ for each $r \epsilon I^h$ and $r = 1, \ldots, s$,

$q_i^h = f_i(\boldsymbol{\lambda}^*) - \Delta f_i^h$ for each $i \epsilon I^h$ and $i = 1, \ldots, m$,

$q_r^h = f_r(\boldsymbol{\lambda}^*) - \Delta f_r^h$ for each $r \epsilon R^h$ and $r = 1, \ldots, s$,

$q_i^h = f_i(\boldsymbol{\lambda}^*) + \Delta f_i^h$ for each $i \epsilon R^h$ and $i = 1, \ldots, m$,

$q_r^h = f_r(\boldsymbol{\lambda}^*)$ for each $r \epsilon M^h$ and $r = 1, \ldots, s$,

$q_i^h = f_i(\boldsymbol{\lambda}^*)$ for each $i \epsilon M^h$ and $i = 1, \ldots, m$.

Step 7. According to the new reference points given in Step 6 compute the new weights as follows:

$\tau_{yr}=\frac{1}{f_r^{ref}-q_r^h}$, $r = 1, \dots, s$,

$\tau_{xi}=\frac{1}{f_i^{ref}-q_i^h}$, $i = 1, \dots, m$.

Step 8. Solve the super-ideal problem (8) with the weights computed in Step 7 and obtain the optimal values $\alpha^{r*}$and ζ^{i*}.

Step 9. Let:

$\alpha_o^{r*} = F_r^{max} - \alpha^{r*}, r = 1, \dots, s,$

$\zeta_o^{i*} = F_i^{max} - \zeta^{i*}, i = 1, \dots, m.$

Step 10. Solve problem (2) and obtain the new reference target units of the DMU under assessment. If the DM accepts the new reference target unit as the MPS, then stop. Otherwise, let h:= h+1. Go to Step 4 or to Step 1 if new weight profiles for the inputs and outputs should be explored (accounting for the same weight profiles for all DMUs).

3.5. A proposal for the robustness assessment of efficient DMUs

The sensitivity analysis of perturbations in data and the robustness of the efficiency score perturbations, based on super-efficiency DEA approaches, has been a topic of interest within the different DEA models (Zhu, 1996, 2001, 2003). In the context of education, this tool is particularly important to evaluate how those measures with intra-school variability (e.g. how changes in maths, reading and science scores and in the socio-economic status of students) might affect the scores of efficient DMUs.

In order to deal with uncertainty, we consider that the perturbations in the value of each factor are within an interval range. This interval is obtained by applying a common tolerance δ to all factors, such that $x_{ij}^L = x_{ij}(1-\delta) \leq x_{ij} \leq x_{ij}(1+\delta) = x_{ij}^U$, $z_{ij}^L = z_{ij}(1-\delta) \leq z_{ij} \leq z_{ij}(1+\delta) = z_{ij}^U$ and $y_{ij}^L = y_{ij}(1-\delta) \leq y_{ij} \leq y_{ij}(1+\delta) = y_{ij}^U$.

Let the inputs (discretionary and non-discretionary) and outputs of DEA model (4) be given within a positive interval range of variation, i.e. $[x_{ij}^L, x_{ij}^U]$, $[z_{ij}^L, z_{ij}^U]$ and $[y_{ij}^L, y_{ij}^U]$. On the other hand, the directional vectors corresponding to the original interval valued (discretionary) inputs and outputs are given as $[g_{xi}^L, g_{xi}^U]$ and $[g_{yr}^L, g_{yr}^U]$, respectively. Additionally, we set the same weight profiles for all DMUs.

Furthermore, we simultaneously contemplate data perturbations for other DMUs according to a worst-case scenario and to a best-case scenario. While the former assumes increased outputs and decreased inputs for all other DMUs (i.e. the efficiency of DMU$_o$ declines and the efficiency of all the other DMUs improve), the latter supposes the reverse situation.

The upper bound, $(1-\beta_o^{LR})$, of the interval efficiency, $[(1-\beta_o^{UR}), (1-\beta_o^{LR})]$, for DMU$_o$ is obtained by solving the following LP problem, which corresponds to an optimistic scenario of coefficients:

$\max \beta_o^{LR} = max\ (w_y(\sum_{r\,\epsilon\,O} \varpi_y^r \alpha_o^r) + w_x(\sum_{i\,\epsilon\,I} \varpi_x^i \zeta_o^i))$

s.t. $\sum_{j\neq o} \lambda_j y_{rj}^L \geq y_{ro}^U + \alpha_o^r g_{yr}^U,\ r = 1, \ldots, s,$

$\sum_{j\neq o} \lambda_j x_{ij}^U \leq x_{io}^L - \zeta_o^i g_{xi}^L,\ i = 1, \ldots, m,$

$\sum_{j\neq o} \lambda_j z_{uj}^U \leq z_{uo}^L,\ u = 1, \ldots, q,$

$\sum_{j\neq o} \lambda_j = 1,\ \lambda_j \geq 0, j=1, \ldots, n.$ (24)

The lower bound, $(1-\beta_o^{UR})$, of the interval efficiency, $[(1-\beta_o^{UR}), (1-\beta_o^{LR})]$, for DMU$_o$ is obtained by solving the following LP problem, which corresponds to a pessimistic scenario of coefficients:

$\max \beta_o^{UR} = max\ (w_y(\sum_{r\,\epsilon\,O} \varpi_y^r \alpha_o^r) + w_x(\sum_{i\,\epsilon\,I} \varpi_x^i \zeta_o^i))$

s.t. $\sum_{j\neq o} \lambda_j y_{rj}^U \geq y_{ro}^L + \alpha_o^r g_{yr}^L,\ r = 1, \ldots, s,$

$\sum_{j\neq o} \lambda_j x_{ij}^L \leq x_{io}^U - \zeta_o^i g_{xi}^U,\ i = 1, \ldots, m,$

$\sum_{j\neq o} \lambda_j z_{uj}^L \leq z_{uo}^U,\ u = 1, \ldots, q,$

$\sum_{j\neq o} \lambda_j = 1,\ \lambda_j \geq 0, j=1, \ldots, n.$ (25)

While considering (24) and (25), it becomes clear that $1-\beta_o^{UR} \leq 1-\beta_o^{LR}$.

According to the previous efficiency score intervals, the DMUs can be classified into three subsets as follows: $E^{++} = \{j \in J : (1-\beta_o^{UR}) \geq 1\}$, $E^{+} = \{j \in J: (1-\beta_o^{UR}) < 1$ and $(1-\beta_o^{LR}) \geq 1\}$ and $E^{--} = \{j \in J: (1-\beta_o^{LR}) < 1\}$, where J is the index set of DMUs ($j=1, \ldots, n$).

Let the efficient DMUs be classified into E^{++} (strongly efficient), E^{+} (potentially efficient) and E^{--} (strongly inefficient).

In this context, a DMU is said to be robust to changes in its factors if it remains efficient (or inefficient). In such a case, the DMU can be stated as robustly efficient (or robustly inefficient) for the tolerance considered.

4 DATA AND VARIABLES

There has been a prolific number of studies with the application of DEA to educational efficiency, specifically resorting to data gathered from PISA. In this context, two different streams of research are usually followed. The first one considers countries as units of analysis (see e.g. Afonso and Aubyn, 2006; Giménez *et al.*, 2007; Thieme *et al.*, 2012; Agasisti, 2014; Bogetoft *et al.*, 2015); whereas the second one is more focused on schools as the centre of evaluation (see e.g. Agasisti, 2013; Agasisti and Zoido, 2018; Aparicio *et al.*, 2018; Aparicio *et al.*, 2019). Our study is framed in this latter stream of research and it uses data from PISA 2015. This programme, which began in the year 2000, assesses 15-year-old students' competences in reading, maths, and science every three years. With the objective of making easier international comparisons, students are chosen by their age (those who have between 15 years and three months and 16 years and two months at the beginning of the assessment) and not by the grade they are coursing. This age corresponds to the tenth grade (unless the student has repeated at least one course during his/her academic track).

PISA data are collected by the use of a two-stage stratified sample design, in which a minimum number of schools are randomly selected in each country and, within them, 42 random students (OCED, 2016). We focus on PISA 2015 Portuguese data[6], which contains information about a total of 202 public schools[7].

Despite the difficulties inherent to the quantification of the level of education apprehended by students, there is a wide unanimity in scientific literature on considering the outcomes from standardized tests as educational outputs (e.g. Afonso and Aubyn, 2006; Thieme *et al*. (2012); Agasisti and Zoido (2018); De Witte and López-Torres, 2017; Tavana *et al*., 2018; Aparicio *et al*., 2018; 2019). Therefore, we have selected as outputs the mean scores of students belonging to the same school in the three most important competencies (mathematics (*o_meanmaths*), reading (*o_meanread*) and science (*o_meansci*)). It is worth mentioning that despite these scores show correlation among them, we have opted to consider all three of them in order to follow the protocol to avoid one of the pitfalls of DEA as suggested in Dyson *et al*. (2001). In what regards the selection of inputs, the use of the PISA dataset presents additional challenges, because of the prolific number of indicators available. Therefore, following the same approach considered in the studies conducted by Aparicio *et al*. (2018; 2019) we have based our choice on the following premises: input factors must obey to the property of monotonicity, i.e. ceteris paribus, this means that a higher level of inputs is required to obtain a higher level of outputs. Hence, inputs should have a positive correlation regarding outputs. In addition, inputs must correspond to effective measures of educational resources used in the learning process.

Since students can be viewed as the “raw material” to be transformed through the learning process, we have used the student-teacher ratio in our analysis. This indicator compares the number of students (in full-time equivalent) to the number of teachers (in full-time equivalent) at a certain level of education and in similar types of institutions (OECD, 2015). Several studies have used this indicator in the assessment of efficiency in education (see e.g. Agasisti, 2014; Tavana *et al*., 2018). Nevertheless, in order to obtain a positive relationship between this indicator and the amount of resources invested, we have transformed it into its inverse, i.e. the teacher-

[6] Secondary education in Portugal encompasses 3 years with students starting at this stage and ending it with 18 years (for further information on this topic see e.g. Portela and Camanho (2010)).
[7] We remove private and semi-private schools because the way to hire teachers is different, which deserve a separate analysis and, also, because the proportion of students in private and semi-private schools in Portugal is relative low (17% in 2015), as compared to the rest of EU countries.

student ratio (*i_tsratio*). A similar procedure can be found in Agasisti (2013), Agasisti and Zoido (2018) and Aparicio *et al*. (2018; 2019).

Another important indicator, which has been used in scientific literature as a proxy to the quality of resources available, is the index of shortage of educational material (see e.g. Aparicio *et al*. (2018; 2019)) which includes data regarding several resources (e.g. textbooks, library, laboratory material and information technology equipment). Higher values reflect a greater shortage of educational material (OECD, 2015). Therefore, we have considered its symmetrical values to obtain an index of quality of schools' educational resources (*i_qser*). In this latter case, positive values reflect principals' opinions that a shortage of educational resources hampers learning to a reduced extent than the OECD average, and negative values indicate that school principals believe that this shortage hampers learning to a larger extent (OECD, 2015).

It has been commonly acknowledged that socio-economic divergencies between students bear a significant effect on student's learning outcomes (OECD, 2015). Therefore, the performance of schools might be impacted by the socio-economic status of the intake of pupils (Dyson *et al*., 2001). This problem can be tackled through the inclusion of environmental variables that allow measuring the social and cultural status of students in the school. Therefore, recent publications have incorporated the index of economic, social and cultural status (*i_ESCS*), created by PISA, in the assessment of schools' efficiency (e.g. Giménez *et al*., 2007; Thieme *et al*., 2012; Agasisti and Zoido (2018); Aparicio *et al*. (2018; 2019)). This index was derived from three variables related to family background: the highest level of parents' education and occupation status and home possessions. Since this input is not under the control of the DM (e.g. the school's principal) it will be considered as an uncontrollable exogenous factor.

Since the indexes of quality of schools' educational resources and ESCS might have negative values, we have transformed the problem by following a procedure based on the one proposed by Tone (2019) in the context of the slacks based measure (SBM). We have followed this procedure because, as demonstrated by Chen et al. (2015), when the directional vectors for inputs and outputs are chosen to be equal to the actual input and output vectors (which is our case), the WRDDM can equivalently be transformed into a corresponding weighted SBM yielding the same information on performance. Furthermore, the advantage of using Tone's procedure for handling negative data is the following (Tone, 2019): it holds important properties of consistency and units invariance; it avoids division by zero; it also allows obtaining projections onto the efficient frontier.

As can be seen in Table 1 the average scores on the PISA mathematics, reading and science literacy scale obtained by 15-year-old students in public secondary schools in Portugal are below the OECD average in 2015 (OCED, 2016). Furthermore, the teacher-student ratio in Portugal is above the OECD average while the index of quality of schools' educational resources and the index of economic, social and cultural status (ESCS) are lower in Portugal.

Table 1. Descriptive statistics for Portuguese public schools

	o_meanmaths	*o_meanread*	*o_meansci*	*i_ESCS*	*i_tsratio*	*i_qser*
Mean	470.37 (490)	476.71 (493)	480.10 (493)	-0.64 (-0.04)	0.11 (0.08)	-0.21 (0.00)
S.D.	55.25	57.02	54.04	0.62	0.06	1.04
Minimum	355.79	351.30	365.37	-2.04	0.02	-3.61
Maximum	581.83	574.95	584.86	1.05	0.50	1.25
Count	202	202	202	202	202	202

Note: S.D. stands for "Standard deviation". Between brackets it is possible to see the average OECD values.
Source: Authors' own calculations.

5 DISCUSSION OF RESULTS[8]

As shown in Table 2, there are 25 public efficient schools in Portugal (efficiency score ≥ 1). The mean efficiency score of the whole sample is 3.22, though the mean values across schools differ significantly, varying between an efficiency level of 0.46 to 497.7 (see Table A1 of the appendix). If we contrast those scores with the average test results attained in maths, reading, and science for each school, it is possible to conclude that there is not a consistent association between these results and efficiency scores. For example, DMU 186 has the highest (aggregate) efficiency level although its test results are rather modest. Something analogous also happens for other schools (see DMUs 52, 60, 75, 84, 116, 124, 132, 157 and 159), which are also positioned in a high place in the rank of efficient schools in spite of having poor results in either competence (see Table A1 of the appendix). Contrastingly, some schools with quite good outcomes in maths, reading, and science are in the last positions in terms of efficiency (e.g. DMUs 44, 53, 98, 120, 161 e 169), suggesting that these schools are not adequately using their resources (see Table A1 of the appendix). Despite this fact, Figure 1 shows a positive relationship of maths, reading, and science scores with efficiency[9] (0.3, 0.25 and 0.26, respectively) and a negative correlation between the teacher-student ratio, the index of ESCS and the index of quality of educational resources and efficiency (-0.30, -0.13 and -0.25, respectively).

From the analysis of Tables 2 and 3, it is possible to conclude that despite efficient schools present higher average scores than inefficient schools across all competences considered, these also have a wide variability of scores when contrasted to inefficient schools. Additionally, efficient public schools have average scores in all competences above the average national scores but below the OECD average.

Inefficient schools have scores across all competences quite similar to those obtained for the national average PISA scores. The factor that stands out for these schools is the quality of

[8] The software used was Opensolver 2.9.0 available from https://opensolver.org/ and we have created an Excel-Visual Basic based application that uses OpenSolver as backend to model and solve our DEA problems.

[9] After removing school 186 from the sample which is an outlier in terms of the aggregate efficiency level obtained (it is worth mentioning that super-efficiency has also been used in the literature to find outliers).

school educational resources that is slightly higher than the national average, with both the average index of ESCS and the teacher-student ratio attaining values near the national average.

Overall, these outcomes suggest that, regardless of the significant variation across schools, the average Portuguese public school is closer to the average inefficient school than to the average efficient school.

Finally, the teacher-student ratio reaches similar average values both for efficient and inefficient schools, but there is a wide variability of this indicator in both types of schools.

Table 2. Descriptive statistics regarding efficient Portuguese public schools

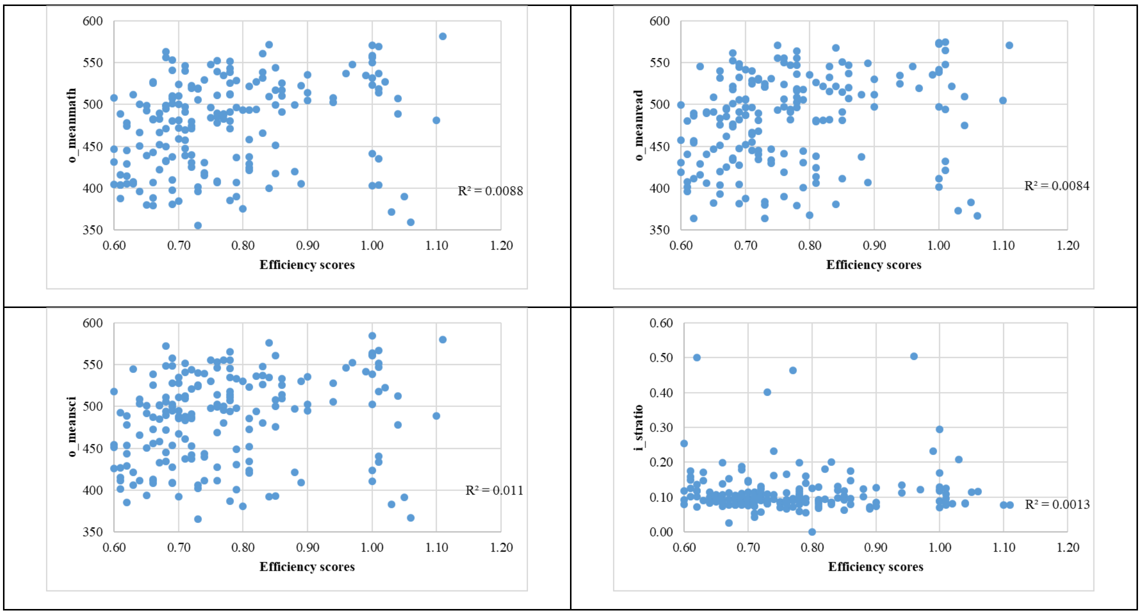
	o_meanmaths	*o_meanread*	*o_meansci*	*i_ESCS*	*i_tsratio*	*i_qser*
Mean	482.34 (490)	484.13(493)	488.41(493)	-0.88 (-0.04)	0.11(0.08)	-1.10 (0.00)
S.D.	70.96	73.52	69.76	0.80	0.07	1.64
Minimum	359.33	366.60	367.32	-2.04	0.02	-3.61
Maximum	581.83	574.95	584.86	1.05	0.40	0.85
Count	25	25	25	25	25	25

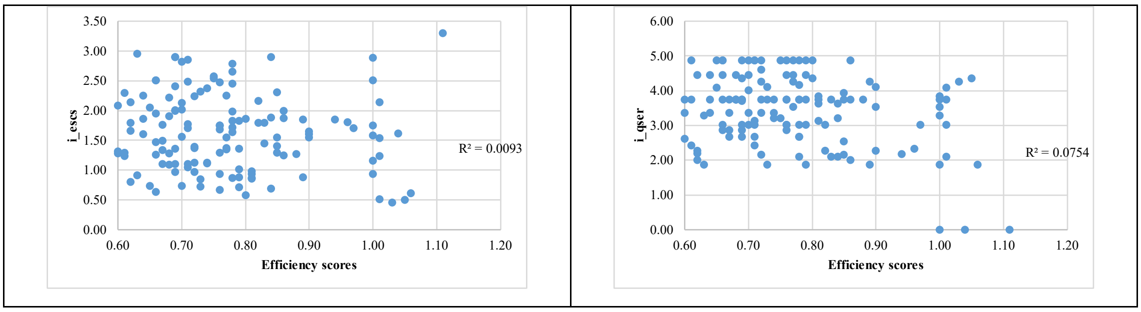
Note: S.D. stands for "Standard deviation". Between brackets it is possible to see the average OECD values.

Table 3. Descriptive statistics regarding inefficient Portuguese public schools

	o_meanmaths	*o_meanread*	*o_meansci*	*i_ESCS*	*i_tsratio*	*i_qser*
Mean	468.68 (470)	475.66 (477)	478.92 (480)	-0.61 (-0.64)	0.11 (0.11)	-0.08 (-0.21)
S.D.	52.69	54.47	51.57	0.58	0.06	0.86
Minimum	355.79	351.30	365.37	-1.67	0.05	-1.74
Maximum	571.46	570.84	576.41	0.91	0.50	1.25
Count	177	177	177	177	177	177

Note: S.D. stands for "Standard deviation". Between brackets it is possible to see the average national values.

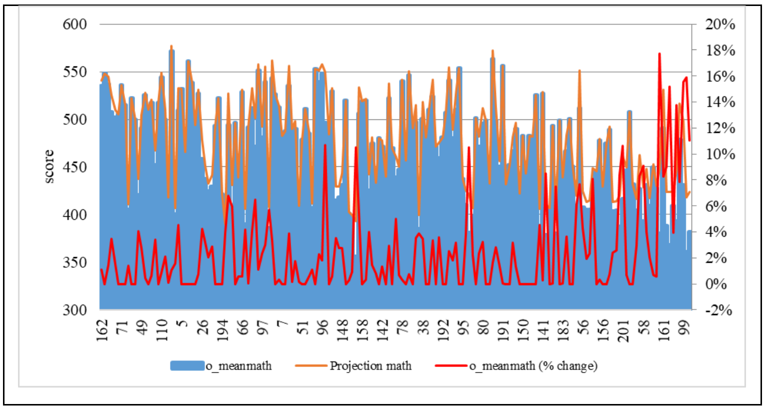


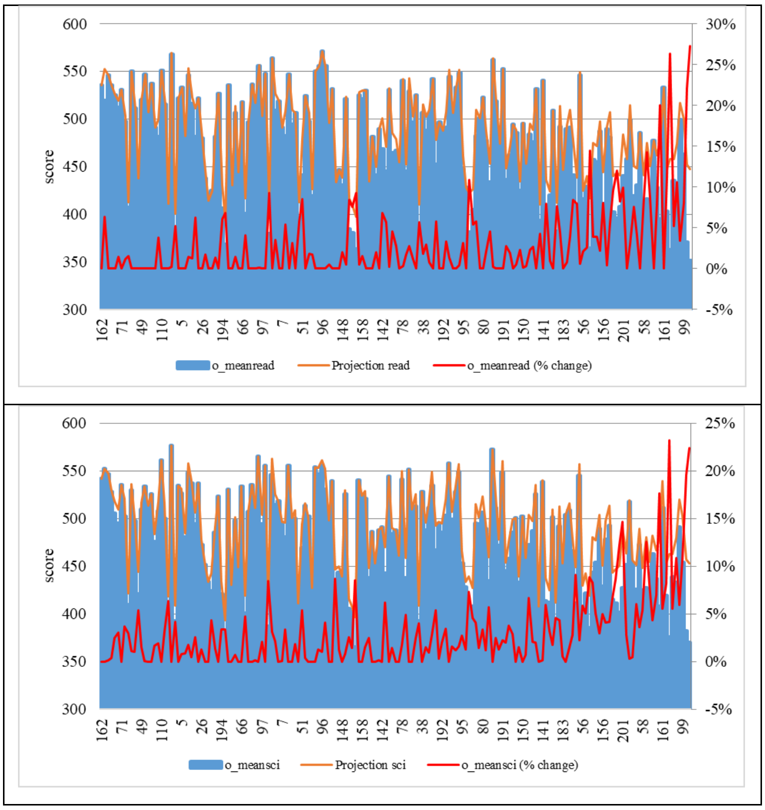


Source: Authors’ own calculations.

Figure 1. Efficiency score vs outputs/inputs.

Figures 2 and 3 illustrate the projections initially computed for the inputs and outputs that could turn inefficient schools into efficient, by decreasing order of efficiency. If we explore the initial projections obtained for each output calculated with the WRDDM, it is possible to observe that there are divergences between maths, reading and science projection values that are not perceivable when only average values are analysed. Hence, for instance, schools operating with the lowest efficiency scores require substantially higher adjustments in the results obtained (in increasing order of required percentage change) in reading, science, and maths, suggesting that they need to make a greater effort to improve results in reading when compared to the other two competences. In fact, this divergence is about 9%, 5%, and 4% if we compare the maximum adjustments required between reading and maths, science and maths and reading and science, respectively. These conclusions are consistent with the ones obtained in a recent study that suggests that most schools in OECD countries are usually less efficient in reading than in mathematics (Aparicio *et al*., 2018). Another issue that distinguishes efficient schools from the others is that these manage to reach higher PISA scores even with lower than average national and OECD indexes of ESCS and quality of school educational resources.

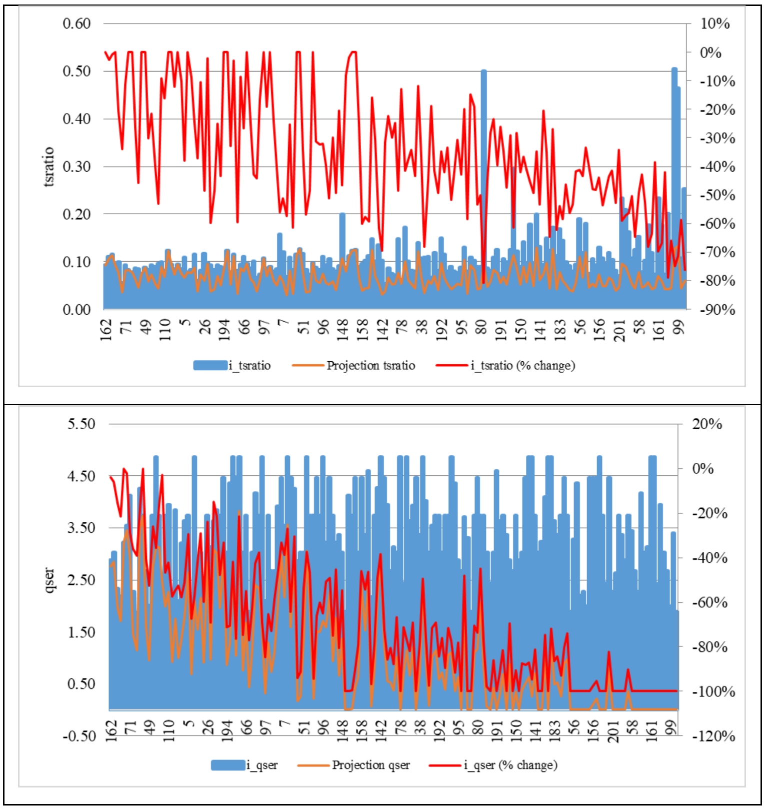




Source: Authors' own calculations.

Figure 2. Real values vs projections (outputs)

Analogously, if we observe the initial projections obtained for each input, it is possible to perceive discrepancies between the teacher-student ratio and the quality of school educational resources projection values across schools. Then again, schools operating with the lowest efficiency scores usually require significantly higher adjustments in both inputs. Another interesting evidence refers to the fact that a higher teacher-student ratio does not necessarily mean higher results in the different competences. This is particularly true for the schools placed on the first quintile of inefficient schools. Specifically, DMUs 99 and 122 have the highest teacher-student ratios of the sample (0.46 and 0.50, respectively) with rather poor results in maths, reading and science. Nevertheless, other schools follow a similar behaviour (e.g. DMUs 4, 35, 55, 56, 58, 68, 81, 101, 114, 115, 139 and 178 - see Table A1 of the appendix). Our findings might indicate that the investment that has been made in Portugal in more teaching resources in disadvantaged schools, through smaller classes and more teaching hours, did not allow attaining the desired outcomes. This can be related to the fact that teachers in the most disadvantaged schools are usually less qualified or experienced than those in the most advantaged schools (OECD, 2015). Therefore, as acknowledged in PISA, Portugal should foster policies to improve teacher quality – e.g. by raising salaries to attract good candidates and retaining effective teachers – even if the trade-off is larger classes (OECD, 2015).



Source: Authors’ own calculations.

Figure 3. Real values vs projections (inputs)

To see if the schools elected as benchmarks are robust in face of changes in its intra-school variability factors (i.e. maths, reading and science scores and the index of ESCS), we have considered data perturbations of these factors of 5%, 10% and 20%[10], respectively.

The top five schools which are more often seen as benchmarks according to the non-oriented WRDDM are in decreasing order of nomination DMUs 132, 16, 33, 174, 173 (135, 111, 109, 82 and 74 times, respectively) – see Table 4. Out of these, only two are classified as strongly efficient, assuming data perturbations of 5% and 10%, i.e. DMUs 132 and 16, whereas DMU 174 is only classified as strongly efficient, considering data perturbations of 5% – see Table 4.

Despite being classified as a robust DMU in terms of (aggregate) efficiency, DMU 132 presents poor results across all competences but requires a low level of resources to produce them (see Table A1 of the appendix). In the case of DMU 16, a very good performance is reached both in terms of results in the PISA scores and resource utilization. DMU 174 has the lowest teacher-student ratio within the sample of schools, particularly excelling in reading scores (which are higher than the OECD average). Finally, DMU 173 does not have the highest PISA results, but it manages to use fewer resources.

[10] From this point forward, whenever we refer to data perturbations, we only mean data perturbations on intra-school variability factors.

From the analysis of these schools, it is also possible to identify some trade-offs. For example, DMU 16 has the same index of quality of school educational resources of DMU132, but a higher teacher-student ratio, leading to a better performance of this latter DMU across all competences. Similar conclusions can be drawn if we consider DMU 33 which has a higher level of resources when contrasted to DMU 132 but attains better results. A trade-off might also exist between the index of quality of school educational resources and the teacher-student ratio – see e.g. DMUs 174 and 173.

Table 4. Specific characteristics of efficient Portuguese public schools

DMU	Nº. Ref. (N.O)	Nº. Ref. (O)	Nº. Ref. (I)	Efficiency score	Classif. (5%)	Classif. (10%)	Classif. (20%)
132	135	3	129	1.29	E++	E++	E+
16	111	53	100	1.11	E++	E++	E+
33	109	54	37	1.01	E+	E+	E+
174	82	35	61	1.10	E++	E+	E+
173	74	33	56	1.04	E+	E+	E+
74	47	13	24	1.04	E+	E+	E+
159	45	1	25	1.01	E+	E+	E+
190	45	102	19	1.01	E+	E+	E+
186	44	4	29	497.70[(a)]	E++	E++	E+
75	21	1	15	1.35	E++	E++	E+
180	15	80	8	1.02	E+	E+	E+
2	14	0	18	1.00	E+	E+	E+
64	10	0	1	1.01	E+	E+	E+
47	7	66	7	1.01	E+	E+	E+
157	7	46	0	1.00	E++	E++	E+
124	6	7	1	1.05	E+	E+	E+
27	5	32	9	1.00	E+	E+	E+
52	3	0	8	1.06	E+	E+	E+
60	2	0	20	1.00	E+	E+	E+
109	2	21	1	1.00	E+	E+	E+
65	1	0	0	1.00	E+	E+	E+
84	1	0	0	1.01	E+	E+	E+
116	1	0	1	1.03	E+	E+	E+
136	1	0	0	1.00	E+	E+	E+
184	1	1	0	1.00	E+	E+	E+

Note: Nº ref. is the number of times that the DMU is selected as a benchmark in the WRDDM.
NO – non-oriented; O – output oriented; I – input oriented; (a) this DMU is an outlier in terms of the aggregate efficiency level obtained.
Classif. 20%, Classif. 10% and Classif. 5% correspond to the classification of DMUs when all the factors are assumed to be varying simultaneously within an interval range of 20%, 10% and 5%, respectively.
Source: Authors' own calculations.

Finally, Table 5 depicts the correlation between the number of times each efficient DMU is selected as a benchmark and the type of orientation considered, suggesting that the number of times each efficient school is viewed as a benchmark according to the output-oriented WRDDM is more correlated to the scores obtained in each competence (disregarding the values reached by their corresponding inputs), whereas the number of times each efficient school is viewed as a benchmark with the input-oriented WRDDM specification is more correlated to the discretionary inputs (neglecting the values attained by their corresponding outputs), thus revealing one of the advantages of using the non-oriented WRDDM specification which simultaneously considers inputs and outputs.

Table 5. Correlation between the number of times a school is nominated as a reference and the values of outputs/inputs

	Nº. Ref. (NO)	*Nº. Ref. (O)*	*Nº. Ref. (I)*
o_meanmaths	0.08	0.48	0.01
o_meanread	0.09	0.39	0.06
o_meansci	0.11	0.43	0.06
i_tsratio	-0.32	0.07	-0.37
i_qser	-0.48	0.07	-0.53

Note: Nº ref. is the number of times that the DMU is selected as a benchmark in the WRDDM.

NO – non-oriented; O – output-oriented; I – input-oriented.

Source: Authors' own calculations.

Since the WRDDM does not explicitly address the DM's preferences and aspirations in the selection of the benchmark DMUs, in the next section of the paper we illustrate how the algorithm proposed can provide a valuable tool in the design of educational policies.

5.1. Application of the algorithm

Step 1. We have started by solving problem (1) and (23) to identify the efficient and inefficient DMUs, assuming that all the outputs have the same weights and also by assigning the same importance to inputs and outputs. The directional vectors have been defined as the original inputs and outputs. The efficient DMUs thus obtained have an efficiency score ($1 - \beta_o^R$) lower than one if they are inefficient.

Step 2. For each problem given in Step 1, we have solved problem (23) for each inefficient DMU and we have obtained the corresponding DMU target set.

Step 3. We have obtained the values of f_r^{ref}, $r = 1, 2, 3$, and f_i^{ref}, $i = 1, 2, 3$. We have set $h = 1$. In our case, the reference values for all DMUs were obtained from the ideal point values which are provided in Table 6.

Table 6. Ideal point values for each input/output

o_meanmaths	*o_meanread*	*o_meansci*	*i_tsratio*	*i_qser*
581.83	574.95	584.86	0.02	0.00

Source: Authors' own calculations.

Step 4. We have computed $f_r(\boldsymbol{\lambda}^*)$, $r = 1, 2, 3, f_i(\boldsymbol{\lambda}^*)$, $i = 1, 2, 3$, where $\boldsymbol{\lambda}^*$ is the optimal solution to each model obtained with formulation (2), thus providing the initial projections computed for the outputs and inputs of inefficient schools.

Step 5. In order to specify how much a hypothetical DM wants to improve/worsen the corresponding output or input, we have further investigated the characteristics of both efficient and inefficient schools.

For illustrative purposes (and for space reasons), the potentialities offered by the algorithm will only focus on specific schools located on the first quintile of inefficient schools, i.e. with $0.46 \leq (1 - \beta_o^R) \leq 0.66$. These schools (which are those requiring the largest adjustments both for the inputs and outputs in order to become efficient) have average poor results across all competences and do not have a suitable use of their resources - see Table 7.

Table 7. Descriptive statistics of Portuguese public schools with $0.46 \leq (1 - \beta_o^R) \leq 0.66$

	o_meanmaths	*o_meanread*	*o_meansci*	*i_ESCS*	*i_tsratio*	*i_qser*
Mean	434.80 (470)	444.19 (477)	448.35 (480)	-0.64 (-0.64)	0.15 (0.11)	-0.13 (-0.21)
S.D.	45.04	48.81	44.00	0.08	0.08	0.94
Minimum	361.09	351.30	370.15	0.07	0.07	-1.73
Maximum	526.95	545.59	544.83	0.50	0.50	1.25
Count	50	50	50	50	50	50

Note: S.D. stands for “Standard deviation”. Between brackets it is possible to see the average national values.
Source: Authors’ own calculations.

Step 6. In order to conduct the interactive methodology proposed in this study and to explore its potentialities, a hypothetical DM has been considered (the schools’ principal and other education policy-makers are the target DMs in this sort of approach) like in other studies performed in an educational context (e.g. Tavana *et al*., (2018)) or in other contexts (e.g. Yang *et al*. (2009)).

In our analysis, we have considered schools located in the first quintile of inefficient schools with distinctive features. Specifically, we have selected the following two DMUs for illustrative purposes:

1) DMU 22 which excels in science, with a mean score in this competence of 503 (above the mean OECD scores), but poor mean scores (466) in maths (below the mean scores attained for inefficient schools) and mean scores in reading at the level of efficient schools (489). The teacher-student ratio in this school (0.14) is above the mean value reached for the public average school, whereas the opposite situation occurs with the index of the quality of school educational resources (-0.25). The first projections obtained for this DMU in order to reach efficiency suggest the consideration of the following benchmarks: DMUs 16, 33 and 132 with λ_{16}=0.25, λ_{33}=0.25 and λ_{132}=0.5, leading to an increase of the mean maths and reading scores of 483.26 and 493.08, respectively, maintaining the mean science scores and reducing both the teacher-student ratio and the index of quality of school resources to 0.06 and -3.05, respectively. Let us consider that the DM is not yet satisfied with the maths scores

projected which are still below the mean OECD scores. Additionally, after the information obtained in the previous step, the DM is made aware that, despite being strongly efficient, DMU 132 presents poor results across all competences (and thus it does not satisfy him/her as a reference of best practices). Furthermore, the DM also knows that DMU 16 is more robust regarding the data perturbations considered and at the same time presents the highest maths scores of the sample of schools. Hence, we use a new reference target for this DMU consistent with the values reached for the outputs and inputs of DMU 16. By doing this, the DM is considering the hypothesis of increasing the scores across all competences, at the expense of investing more in the quality of educational resources, keeping the same projected teacher-student ratio.

2) DMU 175 which presents poor results across all competences, with mean scores of 368, 362 and 375 in maths, reading, and science, respectively. Nevertheless, the teacher-student ratio in this school (0.07) and the index of the quality of school educational resources (-1.19) are quite reduced, being below the mean value reached for the public average school. The first projections computed for this DMU in order to reach efficiency indicate the use of the following DMUs as a reference in terms of best practices: DMUs 16 and 132 with λ_{16}=0.03 and λ_{132}=0.97, leading to an initial projection of the mean maths, reading and science scores of 424, 458 and 463, respectively, while reducing both the teacher-student ratio and the quality of school resources to 0.04 and -3.61, respectively. Let us consider that the DM is not yet satisfied with the scores projected across all competences which are below the scores obtained by the average public schools. Additionally, following the assumptions previously made that DMU 132 presents poor results across all competences (and thus it does not satisfy him/her as a reference of best practices), we apply once more as a reference target the values reached for the outputs and inputs of DMU 16. By doing this, the DM is considering the hypothesis of increasing the scores across all competences, at the expense of investing more in the quality of educational resources and teachers per student.

Step 7. According to the preferences given in Step 6, the new reference values were updated.

Steps 8. We have solved the super-ideal problem (13) with the new reference values and obtained the optimal values α^{1*}, α^{2*}, α^{3*} and ζ^{1*}.

Step 9. We have computed the optimal values α_o^{1*}, α_o^{2*}, $\alpha_o^{3*}$and ζ_o^{1*}.

Step 10. We have solved problem (5) and obtained the new reference target units of inefficient DMUs.

In what concerns DMU 22, the use of a new reference target leads to computing the following benchmarks: DMUs 16 and 74 with λ_{16}=0.15, λ_{74}=0.85, leading to an increase of the mean maths, reading and science scores of 518.48, 518.43 and 522.40, respectively, but requiring the investment in more teachers per student (0.13) and a lower investment in the quality of school resources whose index decreases to -3.61, when contrasted to the previous projections. It is worth mentioning that despite PISA scores are now much higher than the original mean values, the projected values computed for the use of resources are now closer to the original values of this DMU (and might be considered as feasible in budgetary terms by the DM).

If we analyse the case of DMU 175, the use of a new reference target allows reaching the following benchmarks: DMUs 16, 132 and 174 with λ_{16}=0.16, λ_{132}=0.15 and λ_{174}=0.69, leading to an increased projection of the mean maths, reading and science scores of 487.53, 507.49 and 498.73, respectively, but requiring less investment in teachers per student (0.03) and a higher investment in the quality of school resources whose index increases to -0.67, when contrasted to the previous projections.

If we assume that the DM accepts the new reference target units as the MPS, the procedure stops. Otherwise, other solutions can be sought according to the DM's preferences just by letting $h := h+1$ and going to Step 4 or to Step 1 (if the purpose is to assign different weights to inputs and/or outputs, always assuming the same weight profiles for the different DMUs).

With this regard, we would like to stress that interactive methods encompass a sequence of computation and dialogue phases and that, in general, it is not usual to fix a maximum number of iterations for an interactive algorithm. The DM is the one that makes the decision to stop when (s)he perceives that (s)he can no longer improve the solution (s)he obtains once (s)he learns which solutions can be reached.

6 CONCLUSIONS

This paper provides an additional understanding of the main factors that influence the efficiency of Portuguese public secondary schools using data from PISA 2015. To the best of our knowledge, this is the first work that proposes the use of the non-oriented non-radial WRDDM in an educational context. This approach allows overcoming the limitations typically found in the literature regarding the use of oriented radial models in efficiency assessment in education since it enables accounting for different priorities and individual expansion and contraction scales for the factors herein viewed as inputs and outputs. In addition, this methodological approach can easily encompass discretionary and nondiscretionary factors typically used in the education context. An equivalence model between the WRDDM and the super-ideal point model has also been established, being combined with an interactive algorithm. With the interactive method developed to search for the MPS (i.e. the most preferred benchmarks), DMs are capable of handling input and output factors according to their preferences, i.e. the target inputs and outputs can be increased, maintained or reduced, depending on what they believe it is reasonable and realistic for them. This can be particularly relevant if educational policy-makers (or school managers) are willing to hire more experienced teachers and/ or are willing to invest more in educational resources in those schools with poor performance across distinct competences, thus allowing to explore the trade-offs between these factors. For example, for those schools with sufficiently higher PISA scores, but which remain inefficient, different possibilities can be explored, namely the reduction of the input factors that have a higher budgetary impact.

We also suggest a new procedure to perform the robustness assessment of the results obtained both considering a worst and best-case scenario (i.e. assuming increased outputs and decreased inputs for all other DMUs and the reverse situation, respectively), enabling DMs to be aware of how sensitive efficiency classification to data variation is. This tool can be of interest to evaluate how those measures with intra-school variability (e.g. how changes in PISA test scores and the ESCS of students), might have an impact on the scores of efficient DMUs.

Our analysis has been conducted by contemplating inputs and outputs found in other publications in a similar context. Therefore, we have used as inputs the teacher-student ratio and an index of the quality of school educational resources (as discretionary factors) and the index of ESCS (as a non-discretionary factor), whereas as outputs we have used the PISA test scores across three main competences, i.e. maths, reading and science scores. Our results show great variability in terms of efficiency scores across the 202 schools under scrutiny, without a consistent association between the results obtained in the PISA test scores and the efficiency scores computed

Overall, according to our findings, the average efficient Portuguese public school has mean scores across all competences above the mean scores reached by the average public national school, but still below the OECD average. In addition, rather than being closer to the average efficient school, the profile of the average Portuguese public school is quite near the average inefficient school. In general, the schools which operate with the lowest efficiency scores need to perform a greater effort to progress in terms of the results in reading when contrasted to the other two competences. Another interesting evidence obtained refers to the fact that a higher teacher-student ratio in these schools does not necessarily entail higher outcomes in the test scores. Therefore, rather than increasing the teacher-student ration in these schools, policy-makers should promote policies to improve teacher quality even if this must be compensated with larger classes.

Finally, the schools more frequently elected as a reference in terms of best practices and which are classified as robust in terms of efficiency do not necessarily perform better in terms of test results, highlighting the advantage of the approach herein proposed that enables seeking new benchmarks more consistent with the preferences of the DM.

Despite the methodological framework herein developed addresses the need to incorporate the preferences of the DM in an educational context, one of the limitations of this work is that it uses a hypothetical DM and not an actual one. Nevertheless, the merit of the proposed approach has been clearly shown with two concrete examples. The choice of the new targets used in the search of the new benchmarks is facilitated by the information provided by the robustness assessment. In this context, it is worth mentioning that this sort of robustness evaluation can further be explored in the comparison between schools from different countries. Future research directions should include the possibility of considering other combined interactive approaches, besides the STOM approach herein contemplated, such as, for example, the STEM method (Benayoun *et al*., 1971), the Tchebycheff method (Steuer and Choo, 1983), the G-D-F method (Geoffrion *et al*., 1972) or the Wierzbicki method (Wierzbicki, 1980).

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APPENDIX

Proof of Theorem 1

Using Equations (9)-(15), the DEA Model (1) can be rewritten as:

$\max \beta_o^R = \max\,(w_y(\sum_r \varpi_y^r \alpha_o^r) + w_x(\sum_i \varpi_x^i \zeta_o^i))),$

s.t. $\alpha_o^r \frac{1}{\tau_{yr}} - f_r\,(\boldsymbol{\lambda}) \leq 0\,,\, r = 1, \ldots, s,$

$\zeta_o^i \frac{1}{\tau_{xi}} - f_i\,(\boldsymbol{\lambda}) \leq 0\,,\, i = 1, \ldots, m,$

$\boldsymbol{\lambda} \in \Lambda_o$ (1A)

From (20) the first set of constraints of problem (1A) for $r = 1, \ldots, s$ can be equivalently transformed into

$\alpha_o^r \frac{1}{\tau_{yr}} - f_r\,(\boldsymbol{\lambda}) \leq 0 \Leftrightarrow \alpha_o^r \frac{1}{\tau_{yr}} \leq f_r\,(\boldsymbol{\lambda}) \Leftrightarrow -\tau_{yr} f_r\,(\boldsymbol{\lambda}) \leq -\alpha_o^r \Leftrightarrow$

$F_r^{max} - \tau_{yr} f_r\,(\boldsymbol{\lambda}) \leq F_r^{max} - \alpha_o^r \Leftrightarrow \tau_{yr}\,(\frac{F_r^{max}}{\tau_{yr}} - f_r\,(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow$

$\tau_{yr}\,(f_r^{ref} - f_r\,(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow \tau_{yr}\,(f_r^{ref} - f_r\,(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow$

$\tau_{yr}\,(f_r^{ref} - f_r\,(\boldsymbol{\lambda})) \leq \alpha^r$. (2A)

Analogously from (21), the second set of constraints of problem (1A) can be equivalently transformed into:

$\tau_{xi}\,(f_r^{ref} - f_i(\boldsymbol{\lambda})) \leq F_i^{max} - \zeta_o^i \Leftrightarrow \tau_{xi}\,(f_r^{ref} - f_i(\boldsymbol{\lambda})) \leq \zeta^i\,,\, i = 1, \ldots, m.$ (3A)

Additionally, the objective function of model (1A) becomes:

$\max\ \beta_o^R = \min\,(-\ \beta_o^R) = \min \beta.$ (4A)

Since $\widehat{f_{ro}} = f_{ro}(\boldsymbol{\lambda}^*),\, r = 1, \ldots, s$, expression (4A) implies that for any $\boldsymbol{\lambda} \in \Lambda_o$

$f_r^{ref} = \frac{F_r^{max}}{\tau_{yr}} \geq \frac{\tau_{yr}\ \widehat{f_{ro}}}{\tau_{yr}} = \widehat{f_{ro}} = \max_{\boldsymbol{\lambda} \in \Lambda_o} f_{ro}(\boldsymbol{\lambda}),\, r = 1, \ldots, s.$ (5A)

Similarly, from (16) and (17) it is obtained, respectively,

$f_i^{ref} = \frac{F_i^{max}}{\tau_{xi}} \geq \frac{\tau_{xi}\ \widehat{f_{io}}}{\tau_{xi}} = \widehat{f_{io}} = \max_{\boldsymbol{\lambda} \in \Lambda_o} f_{io}(\boldsymbol{\lambda}),\, i = 1, \ldots, m.$ (6A)

Equations (5A) and (6A) imply for any $\boldsymbol{\lambda} \in \Lambda_o$

$f_r^{ref} - f_r\,(\boldsymbol{\lambda}) \geq 0,\, r = 1, \ldots, s,$ (7A)

$f_i^{ref} - f_i\,(\boldsymbol{\lambda}) \geq 0,\, i = 1, \ldots, m.$ (8A)

Hence, from (22) it is verified for any $\boldsymbol{\lambda} \in \Lambda_o$ that

$\beta = (w_y(\sum_{r \in O} \varpi_y^r F_r^{max}) + w_x(\sum_{i \in I} \varpi_x^i F_i^{max}) -\ \beta_o^R.$

Moreover, since

$\beta_o^R = w_y(\sum_{r \in O} \varpi_y^r(\alpha_o^r)) + w_x\big(\sum_{i \in I} \varpi_x^i(\zeta_o^i)\big),$

from (18) and (19) it is known that

$\beta = (w_y(\sum_{r \in O} \varpi_y^r F_r^{max}) + w_x(\sum_{i \in I} \varpi_x^i F_i^{max})) - \beta_o^R \Leftrightarrow \beta = (w_y(\sum_{r \in O} \varpi_{yg}^r (F_r^{max} - \alpha_o^r)) + w_x(\sum_{i \in I} \varpi_x^i (F_i^{max} - \zeta_o^i))) = (w_y(\sum_{r \in O} \varpi_y^r (\alpha^r)) + w_x(\sum_{i \in I} \varpi_x^i (\zeta^i)))$ (9A)

With the foregoing in mind, it is proven that the general combined-oriented DEA Model (1) can be equivalently transformed into the super-ideal point Model (8).

Table 1A – Original inputs and outputs vs efficiency scores

DMU (nº)	School Identification	o_meanmaths	o_meanread	o_meansci	i_tsratio	i_escs	i_qser	Efficiency Score
1	62000002	375.47	367.92	380.52	0.12	-1.67	0.74	0.80
2	62000004	556.44	572.61	560.85	0.08	0.26	-1.73	1.00
3	62000005	452.12	461.95	458.32	0.10	-0.75	0.13	0.67
4	62000006	408.90	435.61	438.66	0.20	-0.72	0.33	0.53
5	62000009	531.54	532.83	526.77	0.11	-0.46	0.13	0.83
6	62000010	380.28	395.19	408.03	0.07	-0.46	-0.48	0.56
7	62000011	482.58	490.93	495.95	0.12	-0.90	-0.07	0.77
8	62000012	571.46	568.19	576.41	0.08	0.65	-1.51	0.84
9	62000014	406.65	430.67	430.38	0.11	-1.53	0.85	0.79
10	62000015	469.55	463.99	483.56	0.07	-0.47	-0.93	0.71
11	62000017	442.77	451.42	455.87	0.14	-0.99	-0.74	0.66
12	62000018	496.16	506.04	498.45	0.08	-0.89	1.25	0.79
13	62000022	519.63	520.99	525.33	0.09	0.07	-1.73	0.73
14	62000024	439.12	466.37	487.94	0.07	-0.55	-0.48	0.71
15	62000025	404.92	406.75	409.34	0.08	-1.37	-1.74	0.89
16	62000027	581.83	571.03	579.80	0.06	1.05	-3.61	1.11
17	62000028	519.70	529.87	521.19	0.10	0.00	-0.25	0.72
18	62000029	451.64	490.98	475.59	0.09	-0.96	-1.07	0.85
19	62000030	494.29	481.15	494.22	0.06	-0.46	-0.59	0.82
20	62000031	491.31	496.33	506.96	0.09	-0.53	-0.59	0.78
21	62000032	507.09	491.05	502.92	0.11	-0.24	-0.59	0.69
22	62000034	466.30	489.36	503.59	0.14	-0.39	-0.25	0.64
23	62000035	384.83	387.86	392.58	0.14	-1.52	0.85	0.70
24	62000036	482.81	483.70	472.59	0.10	-0.30	-0.59	0.66
25	62000037	487.31	482.14	480.23	0.07	-0.88	1.25	0.77
26	62000038	458.57	479.58	472.52	0.12	-1.27	0.13	0.81
27	62000039	531.68	541.95	561.80	0.08	-0.49	0.23	1.00
28	62000040	401.50	384.13	405.68	0.11	-1.41	0.51	0.73
29	62000043	466.29	494.62	485.09	0.13	-0.49	-0.25	0.67
30	62000044	489.07	480.16	492.75	0.10	0.05	1.25	0.61
31	62000045	426.93	485.76	472.57	0.13	-0.38	0.13	0.59
32	62000046	493.82	535.74	530.21	0.08	-0.38	1.25	0.80
33	62000047	514.17	494.17	517.75	0.10	-1.02	-1.52	1.01
34	62000048	511.69	536.08	535.18	0.10	-0.27	0.56	0.78
35	62000049	405.21	364.06	385.04	0.18	-1.45	-1.42	0.62
36	62000050	526.99	521.80	536.73	0.08	-0.09	-1.35	0.82
37	62000052	489.60	506.62	499.67	0.11	-0.56	-0.74	0.76
38	62000053	500.72	506.18	528.34	0.11	-0.12	-0.25	0.70
39	62000054	425.07	434.53	437.56	0.11	-1.27	0.99	0.72
40	62000055	408.41	389.83	411.45	0.13	-1.58	-0.59	0.76
41	62000056	505.04	511.76	494.84	0.05	-0.70	-0.07	0.90
42	62000057	436.77	444.37	448.85	0.09	-1.23	-0.59	0.79
43	62000058	447.36	454.98	461.19	0.15	-1.15	-0.59	0.71
44	62000059	511.91	545.59	544.83	0.09	0.70	-1.73	0.63
45	62000060	480.87	487.63	488.13	0.11	-0.68	1.25	0.70
46	62000061	529.74	531.32	539.11	0.08	0.12	0.13	0.74
47	62000062	537.03	547.96	546.77	0.11	-0.71	-0.59	1.01
48	62000063	474.52	481.51	486.09	0.15	-0.87	-1.44	0.72
49	62000066	490.91	520.05	509.28	0.09	-0.38	-1.61	0.86
50	62000067	471.16	516.81	510.92	0.07	-0.42	0.13	0.78
51	62000068	478.38	441.05	469.03	0.12	-0.96	-0.59	0.76
52	62000069	359.33	366.60	367.32	0.09	-1.64	-1.73	1.06
53	62000070	492.99	508.71	501.31	0.12	-0.20	0.48	0.65
54	62000071	430.24	440.83	452.83	0.10	-1.12	0.13	0.72
55	62000072	380.01	382.23	394.15	0.17	-1.52	1.25	0.65
56	62000073	407.51	415.74	405.93	0.19	-1.34	-0.32	0.63
57	62000074	541.05	544.73	558.04	0.08	0.65	0.13	0.69

DMU (nº)	School Identification	o_meanmaths	o_meanread	o_meansci	i_tsratio	i_escs	i_qser	Efficiency Score
58	62000075	392.82	406.10	425.23	0.15	-1.05	-0.17	0.59
59	62000076	489.79	485.60	500.90	0.29	-0.91	-0.74	0.67
60	62000077	403.36	411.09	410.91	0.10	-1.31	-3.61	1.00
61	62000078	480.43	506.02	499.24	0.09	-0.25	0.13	0.69
62	62000079	470.98	500.97	494.55	0.09	-0.34	0.13	0.68
63	62000083	535.20	546.97	555.73	0.11	0.01	0.85	0.77
64	62000084	518.84	574.95	552.09	0.07	-0.11	0.48	1.01
65	62000085	550.59	539.06	538.41	0.10	-0.66	0.13	1.00
66	62000087	528.40	517.85	533.64	0.11	-0.42	0.13	0.79
67	62000088	510.07	532.67	528.16	0.08	0.16	1.25	0.69
68	62000089	446.46	457.80	451.28	0.23	-0.96	-0.99	0.60
69	62000090	414.87	433.87	411.05	0.08	-1.13	-0.39	0.74
70	62000093	551.51	555.44	565.67	0.07	0.40	1.25	0.78
71	62000094	535.86	530.40	535.61	0.09	-0.64	0.51	0.90
72	62000095	395.83	380.22	402.05	0.12	-1.53	0.13	0.73
73	62000096	499.39	511.54	497.44	0.08	-0.98	0.13	0.88
74	62000097	507.42	509.24	512.37	0.14	-0.64	-3.61	1.04
75	62000099	422.96	397.49	423.24	0.03	-1.43	-0.14	1.35
76	62000100	422.43	424.90	432.95	0.12	-1.15	-0.93	0.67
77	62000101	547.79	519.38	552.74	0.11	-0.54	-0.59	0.97
78	62000102	540.24	540.38	540.74	0.06	0.60	1.25	0.71
79	62000103	517.41	481.44	507.83	0.10	-0.85	0.13	0.85
80	62000104	495.38	522.18	506.08	0.09	-0.03	0.85	0.68
81	62000106	388.26	402.95	419.37	0.18	-0.91	1.25	0.54
82	62000107	525.82	547.15	533.40	0.08	-0.26	0.13	0.86
83	62000108	511.15	524.02	513.96	0.08	-0.50	1.25	0.76
84	62000109	403.47	421.04	433.64	0.17	-1.73	0.13	1.01
85	62000110	484.88	497.23	502.37	0.07	-0.49	0.13	0.76
86	62000111	414.85	457.04	443.66	0.07	-0.45	-1.35	0.62
87	62000112	457.24	444.98	437.65	0.17	-1.20	-1.18	0.71
88	62000113	449.86	477.35	462.57	0.18	-0.51	0.56	0.57
89	62000114	403.39	401.77	415.50	0.12	-1.01	0.13	0.61
90	62000115	445.44	454.44	453.95	0.10	-0.58	-1.61	0.62
91	62000117	498.79	476.95	489.37	0.50	-1.15	0.13	0.68
92	62000118	477.77	486.62	489.10	0.09	-0.10	0.85	0.62
93	62000119	482.20	476.01	487.29	0.18	-0.78	0.85	0.66
94	62000120	553.61	548.80	548.80	0.08	0.65	0.74	0.69
95	62000121	437.78	447.16	453.36	0.09	-0.88	-0.74	0.69
96	62000124	548.07	570.84	555.96	0.11	0.32	1.25	0.75
97	62000125	480.79	502.76	494.11	0.11	-0.61	-1.51	0.78
98	62000126	525.62	531.44	525.35	0.10	0.26	1.25	0.66
99	62000128	431.21	462.39	454.18	0.46	-0.71	-1.61	0.51
100	62000131	489.20	493.96	500.36	0.08	-0.70	0.64	0.77
101	62000132	388.59	392.95	408.23	0.20	-1.61	1.25	0.66
102	62000133	538.96	547.49	555.33	0.09	0.20	0.13	0.78
103	62000134	407.16	419.39	427.14	0.10	-1.31	0.13	0.76
104	62000135	510.45	505.86	511.06	0.11	-0.23	0.40	0.70
105	62000137	409.82	427.79	427.89	0.13	-1.28	-0.93	0.69
106	62000138	437.66	438.06	451.70	0.10	-1.31	-0.48	0.81
107	62000139	400.27	380.72	392.22	0.09	-1.56	-0.41	0.84
108	62000140	385.73	379.23	386.92	0.09	-1.38	-0.93	0.78
109	62000141	570.99	573.03	584.86	0.09	0.64	-0.32	1.00
110	62000142	544.22	551.23	561.02	0.08	0.06	0.33	0.85
111	62000143	523.92	542.03	534.62	0.07	0.58	-0.59	0.70
112	62000144	522.42	549.69	530.08	0.09	-0.40	0.64	0.89
113	62000146	466.03	482.23	480.48	0.08	-0.80	-1.51	0.83
114	62000147	381.71	351.30	370.15	0.25	-0.92	-1.73	0.46
115	62000149	508.03	499.41	518.03	0.21	-0.16	0.13	0.60
116	62000150	371.21	373.26	382.74	0.13	-1.79	0.65	1.03
117	62000152	393.03	428.82	432.73	0.13	-0.47	-0.59	0.53
118	62000153	509.37	521.98	534.60	0.08	-0.37	0.03	0.84
119	62000154	514.56	497.59	502.50	0.08	-0.60	-1.33	0.90
120	62000155	500.50	490.85	508.52	0.10	0.00	0.13	0.64
121	62000156	499.70	515.13	500.19	0.12	-0.70	-1.44	0.85
122	62000157	478.65	499.32	491.32	0.50	-0.44	-0.93	0.53
123	62000158	536.92	545.87	546.58	0.11	-0.44	-1.28	0.96
124	62000159	389.84	382.68	391.50	0.10	-1.75	0.74	1.05
125	62000160	432.77	433.50	434.67	0.09	-0.96	0.13	0.68
126	62000162	424.28	413.84	434.10	0.09	-1.39	0.03	0.81
127	62000164	429.40	424.76	423.96	0.08	-1.38	0.23	0.81

DMU (nº)	School Identification	o_meanmaths	o_meanread	o_meansci	i_tsratio	i_escs	i_qser	Efficiency Score
128	62000165	496.64	555.38	530.47	0.09	0.29	-0.39	0.75
129	62000166	479.86	489.31	487.63	0.13	-0.88	0.65	0.72
130	62000167	508.06	535.17	528.10	0.09	-0.40	-1.43	0.94
131	62000168	381.08	381.17	409.18	0.08	-1.14	0.09	0.69
132	62000169	419.02	453.91	458.77	0.04	-0.79	-3.61	1.29
133	62000170	546.52	528.62	551.42	0.10	0.24	1.25	0.71
134	62000171	450.60	440.74	465.92	0.09	-0.64	0.85	0.64
135	62000172	552.79	550.16	553.66	0.09	0.22	0.85	0.76
136	62000173	523.36	497.40	502.85	0.16	-1.09	-0.07	1.00
137	62000174	389.83	400.32	400.54	0.10	-1.37	-1.73	0.79
138	62000175	510.14	507.01	514.02	0.09	-1.00	1.25	0.86
139	62000176	431.53	418.83	454.09	0.16	-0.93	-0.25	0.60
140	62000177	418.02	446.75	443.79	0.09	-1.13	-0.25	0.74
141	62000178	526.95	539.97	539.04	0.13	0.26	0.13	0.66
142	62000179	471.12	468.53	491.08	0.10	-0.85	1.25	0.72
143	62000180	404.07	395.85	411.48	0.10	-0.96	-1.18	0.61
144	62000181	543.53	563.68	545.84	0.08	0.54	-0.93	0.78
145	62000182	560.79	545.74	548.15	0.08	0.02	1.25	0.83
146	62000183	471.70	452.14	467.22	0.12	-0.82	-0.07	0.70
147	62000184	417.94	411.16	393.18	0.09	-1.56	0.23	0.85
148	62000185	430.88	430.05	439.68	0.20	-1.51	-0.59	0.74
149	62000186	378.92	403.59	413.30	0.09	-1.09	-0.59	0.66
150	62000187	482.50	495.45	502.14	0.09	-0.24	0.13	0.67
151	62000188	355.79	363.54	365.37	0.12	-1.64	0.85	0.73
152	62000189	525.89	508.79	514.41	0.08	-0.25	0.30	0.78
153	62000191	397.93	404.20	408.10	0.11	-1.25	-0.30	0.69
154	62000192	506.10	525.18	540.38	0.09	0.11	-0.59	0.73
155	62000193	405.03	429.15	421.50	0.12	-1.02	0.74	0.63
156	62000194	429.06	411.05	429.19	0.13	-1.02	0.85	0.62
157	62000195	441.11	401.50	423.98	0.40	-2.04	-1.91	1.00
158	62000196	519.15	521.32	524.11	0.10	-0.24	0.85	0.73
159	62000197	434.93	432.31	440.61	0.10	-1.23	-3.61	1.01
160	62000198	420.23	437.66	421.12	0.08	-1.33	-0.93	0.88
161	62000200	490.81	533.03	511.32	0.23	0.31	1.25	0.55
162	62000201	534.98	535.74	541.60	0.09	-0.61	-0.74	0.99
163	62000202	539.11	555.92	546.64	0.08	0.33	0.13	0.76
164	62000204	361.09	370.58	382.26	0.11	-0.81	-0.22	0.51
165	62000205	431.07	415.59	427.06	0.08	-0.45	-0.93	0.59
166	62000206	563.45	562.05	572.52	0.08	0.91	-0.59	0.68
167	62000207	427.63	428.21	436.60	0.12	-0.66	-0.59	0.57
168	62000208	404.45	430.44	426.06	0.11	-0.89	-0.74	0.60
169	62000209	498.86	491.54	491.79	0.11	-0.22	1.25	0.65
170	62000211	502.43	524.91	505.64	0.10	-0.88	-0.39	0.94
171	62000212	439.85	445.33	442.33	0.05	-0.85	0.85	0.72
172	62000213	491.09	496.40	509.34	0.08	-0.24	0.03	0.71
173	62000214	488.61	474.96	478.36	0.08	-1.42	-0.74	1.04
174	62000215	480.93	504.84	488.94	0.02	-1.12	0.64	1.10
175	62000216	368.14	362.25	375.35	0.07	-0.73	-1.19	0.54
176	62000217	493.53	481.37	485.56	0.09	-0.92	0.13	0.81
177	62000218	522.02	526.36	523.20	0.09	-0.51	0.85	0.81
178	62000219	406.50	420.11	411.61	0.15	-1.28	-0.38	0.66
179	62000220	459.22	496.47	495.09	0.08	-0.42	0.13	0.70
180	62000221	527.01	522.05	522.82	0.08	-1.12	0.85	1.02
181	62000222	498.79	518.35	511.48	0.11	-0.04	-1.18	0.68
182	62000223	501.17	482.17	494.93	0.10	-0.18	-1.73	0.69
183	62000225	438.81	446.75	450.53	0.17	-1.12	0.03	0.65
184	62000226	558.56	573.62	563.69	0.09	0.01	0.13	1.00
185	62000227	395.91	405.79	411.33	0.08	-0.97	0.13	0.64
187	62000229	474.53	489.73	478.06	0.11	-0.26	0.85	0.62
188	62000230	409.07	414.29	420.62	0.10	-0.84	-1.33	0.59
189	62000231	450.07	473.77	471.78	0.13	-0.83	-0.59	0.68
190	62000232	569.62	564.73	567.11	0.10	-0.18	-1.16	1.01
191	62000233	556.16	552.49	548.61	0.08	0.76	0.99	0.68
192	62000234	480.94	487.55	485.44	0.15	-0.81	0.13	0.70
193	62000235	387.51	407.59	401.59	0.08	-0.93	0.85	0.61
194	62000236	421.73	406.10	420.62	0.08	-1.30	-0.59	0.81
195	62000237	432.20	436.00	445.02	0.10	-1.02	0.03	0.68
196	62000238	497.20	524.65	512.58	0.08	0.00	0.31	0.71
197	62000239	517.52	536.89	525.70	0.09	-0.49	0.13	0.86
198	62000242	522.49	530.90	543.96	0.09	0.21	0.33	0.72

DMU (nº)	School Identification	o_meanmaths	o_meanread	o_meansci	i_tsratio	i_escs	i_qser	Efficiency Score
199	62000243	538.46	515.96	537.39	0.12	-0.36	-0.93	0.83
200	62000246	512.37	518.56	518.10	0.16	-0.81	0.85	0.78
201	62000247	416.34	440.31	427.13	0.07	-0.32	-1.33	0.61
202	62000248	484.52	493.35	497.95	0.10	-0.81	0.85	0.75

Source: Authors' own calculations.