

GOTHIC VAULTING IN SPANISH MILITARY ARCHITECTURE FROM THE 18TH CENTURY

Cinta Lluís-Teruel¹, Josep Lluís i Ginovart¹, Sergio Ramírez González², Antonio Bravo Nieto³

¹ Universitat Internacional de Catalunya cintalluis@uic.es (corresponding author), jlluis@uic.es

² Universidad de Málaga ramirezgonzalezs@hotmail.com

³ Universidad Nacional de Educación a Distancia anieto4@hotmail.com

Abstract. The construction of gunpowder magazines, undertaken by military engineers in Spain throughout the 18th century bears testament to the Enlightenment's construction ethos in the different domains forming the Hispanic Monarchy. In the study on the typological classification that we have performed on 271 projects involving these magazines, 31 designed using gothic vaults have been unearthed. The geometric study into this constructive system unveils that 68.97% of these pointed arches feature centres placed below the impost, suggesting that the angle of incidence on the vertical faces is below 90°, thus sharing this property with catenary vaults. Viewing the appraisals given in *La théorie et la pratique de la coupe de pierres* (1738) by Frézier on the structural coherence of these figures, it can be concluded that this pointed system employed by Hispanic military engineers is a simplified attempt to standardise the gothic vault in line with the catenary model.

Keywords: Catenary, gothic vaults, gunpowder magazines, military engineers, scientific mechanics.

Introduction and purpose

During the Spanish War of Succession (1701-1713), King Philip the Fifth (1683-1746) created the Royal Military Engineering Corps (1711) along with the Academy of Mathematics (1720) to ensure their proper training. Construction design in military architecture brings to light the sizable complexity involved in the defensive strategy of fortified places, in which the gunpowder magazines formed one of the standout elements with the duty to supply the artillery. These *ex novo* constructions complemented or replaced the tradition of placing them in natural cavities or underground (Cassani 1705: 86).

The work of Hispanic military engineers in the 18th century was highlighted in their respective disciplines by geographer Horacio Capel Sáez (Capel 1983) and art historian Alicia Cámara Muñoz (Cámara 1989: 80-106). However, studies on the mechanics knowledge utilized by these military engineers would follow later. These early works concluded that catenary shapes were not introduced in Spain by Gaudí but were already in use by these military engineers (Lluís i Ginovart 2015: e103). Subsequently, at the International Congress on Fortifications and Military Heritage (ICOFORT) 2022 held in Ceuta, the paper titled 'The gothic or ogival vault in the military architecture of the 18th century. The case of gunpowder stores, from Gibralfaro to San Fernando de Figueras' was presented from the perspective of architectural history (Bravo and Ramírez 2022). Several questions were raised in the debate, such as the reason for this typology and why it had gone virtually unnoticed among specialists. At that time, neither the quantity of this type of vaults, their geometric shape, nor their structural behavior was known. The research carried out in the last two years and presented in this study addresses the reason for the use of the medieval pointed vault in the construction of Enlightenment Spain.

The baseline reference for their construction is found in the model devised by Sébastien Le Prestre, Mr Vauban (1633-1707), edited in the work 'Maniere de fortifier selon la méthode de Monsieur de Vauban', by the Abbot Du Fay (f. 1681). Therein, a description is given of a bombproof magazine featuring an exterior enclosure (Du Fay 1681: 170-172), providing a diagram of its constructive elements in the 1693 edition (Du Fay 1693: 191-193). Its sizing was empirical depending on the thrust forwards of the artillery and taking as a benchmark those in place in Landaw (Fig. 1.a).

MODEL POWDER-MAGAZINES SÉBASTIEN LE PRESTRE, MONSIEUR DE VAUBAN (1691)

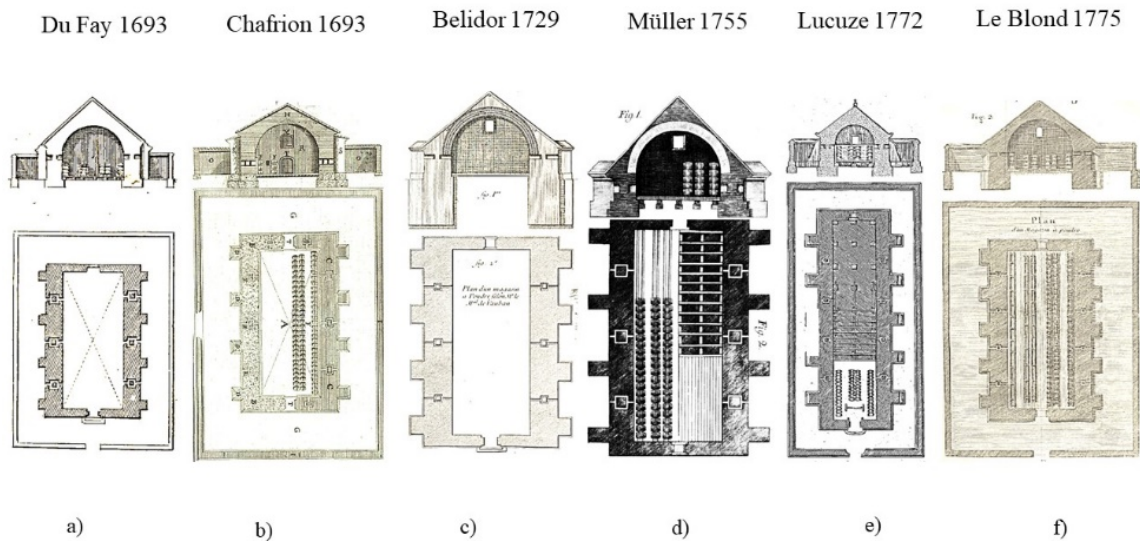


Fig 1 Vauban System gunpowder magazine models 1681. a) Abbé Du Fay, *Maniere de fortifier selon la methode de Monsieur de Vauban*. Image: Du Fay 1693: 193. b) José Chafrion, *Escuela de Palas ó sea Curso Mathematico*. Image: Chafrion 1693: LII, 175. c) Bernard Forest de Belidor, *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile*. Image: Belidor 1729: Book IV, Sheet 26, 64; Sheet 27, figs.1, 2, 68. d) John Müller *A treatise containing the elementary part of fortification, regular and irregular*. Image: Müller 1755: Sheet 17, figs. 1, 2, 222. e) Pedro de Lucuze, *Principios de fortificación que contienen las definiciones de los términos principales de las obras de Plaza, y de Campaña*. Image: Lucuze 1772: Sheet 6, fig. 82. f) Guillaume Le Blond, *Eléments de fortification, contenant la construction raisonnée des ouvrages de la fortification*. Image: Le Blond 1775: sheet 4, figs. 1, 2.

This model was similarly proposed by Sebastián Fernández de Medrano (1646-1705) in ‘El Ingeniero Primera Parte, de la Moderna Architectura Militar’ (Fernández 1687: 238-240). While rectifying the section upon the conclusion of the Luxembourg campaign, it was unable to withstand enemy fire. This extent is given in ‘El Architecto Perfecto en el Arte Militar’ (1700) (Fernández 1700: 233-234). It subsequently became the norm in the ‘Escuela de Palas o sea Curso Matemático’ (Chafrion 1693: 175-177) (Fig. 1.b)

The inventory for the gunpowder magazines project initiated by the Spanish military engineers safeguarded in Hispanic archives has enabled a morphological and typological grading of these constructions that allows for their formal, geographical and chronological sequence to be put in place, thereby selecting 271 projects (1610-1887). Their grading states that 47 of them pertain to actions performed on existing buildings, while the remainder, 82.72%, correspond to the edification of new buildings. From a geographical standpoint, 225 are located on land that comprises Spain in its current state, while 17.28% are found in Cuba, Puerto Rico, Italy, Argentina, Colombia, the United States, North Africa or Trinidad and Tobago.

Duly established as criteria for the grading are namely the archive’s reference, the place in which it is built, the country, date, year, the creator and project title. The construction systems used have been determined and which reflect their formal typology through vaults: barrel (Fig. 2.a), segmented (Fig. 2.b), timbrel (Fig. 2.c), those with special cross-sections (Fig.2.d) and pointed examples. Others used are wood covered structural systems featuring: diaphragm arches, brickwork walls with beams, trusses or log bearing structures. From another stance, within this characterisation of the constructive typology, the difference between a new or refurbished construction, alongside the number of naves these gunpowder magazines were equipped with is duly clarified.

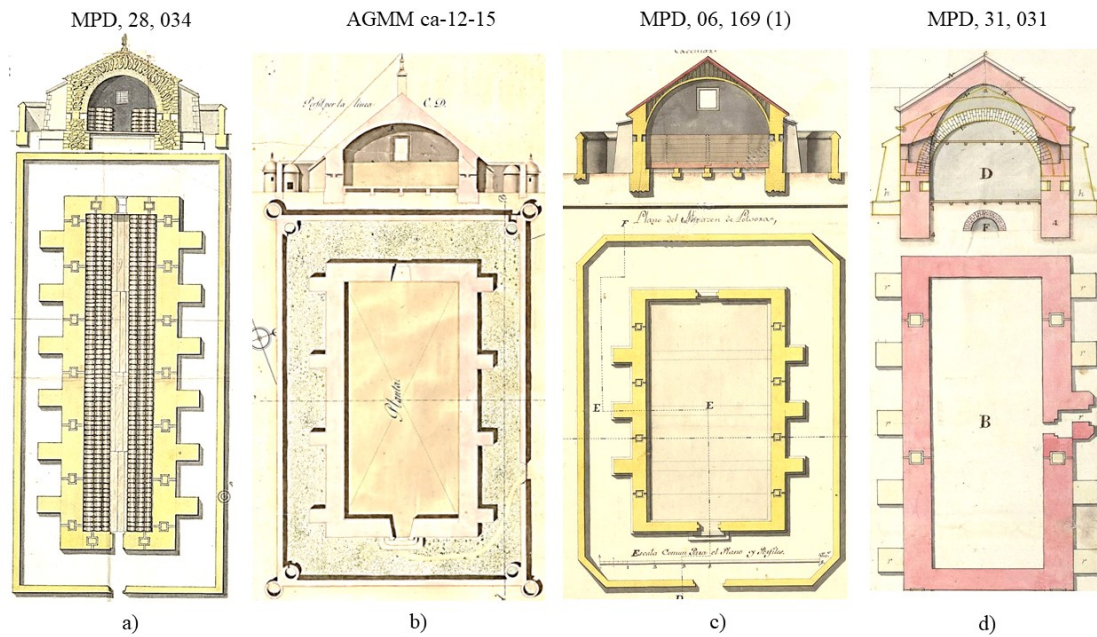


Fig 2 Gunpowder magazine projects featuring vaults as part of Hispanic military engineering from the 18th century. a) Castillo de Santa Cruz de la Motta in San Sebastián (1722), Antonio Álvarez Barba (fl. 1719-1739) (MPD, 28, 034). b) Redrawn, Virgen de Regla in Cádiz (1840), work of Antonio Ruiz Hurtado (fl.1756-1760), (AGMM ca-12-15). c) Benimamet in Valencia (1751), Carlos Beranger (1698-1756), (MPD, 06, 169) pag.1. d) Ciudadela de Pamplona (1718?), Francisco Larrando de Mauleón (1644 -1736), (MPD, 31, 031).

This set is noteworthy owing to the chronologically late construction of 31 projects with pointed vaults (1689-1874) (table 1). Similarly referred to as ogival or gothic, this model came into being through the shifting of a pointed arch through its ogival generatrix, shifting along the longitudinal axis of the straight directrix. This research work aims to study pointed vaults, from viewing the vault's geometrical shape, so that its mechanical behaviour can be ascertained (Calvo-López 2011: 5003-533; Pinto-Puerto and Jiménez-Martin 2016: 439-466).

Pointed vaults in the gunpowder magazine projects

Architecture displaying pointed diaphragm or round arches remained in use as a structure for projects involving gunpowder magazines throughout the 17th century. Wooden beams were fitted to the same guide line in the main space, while the arches are placed perpendicular to this, thereby constructing on the nave's generatrix the roof's slope: Fuenterrabía (1684) (MPD, 63, 004) (Fig. 3.a) y La Coruña (1774) (MPD, 28, 027) (Fig. 3.b); these are found with round arches, as is the case of Valencia (1751) (MPD, 06, 169) (Fig. 3.c); Peñíscola (1756) (MPD, 07, 208) and Barcelona (1761), (MPD, 20, 031) (Fig. 3.d). Others take the form of a dome, namely Fuenterrabía (c.1689) (MPD, 09, 101) (Fig. 4.a).

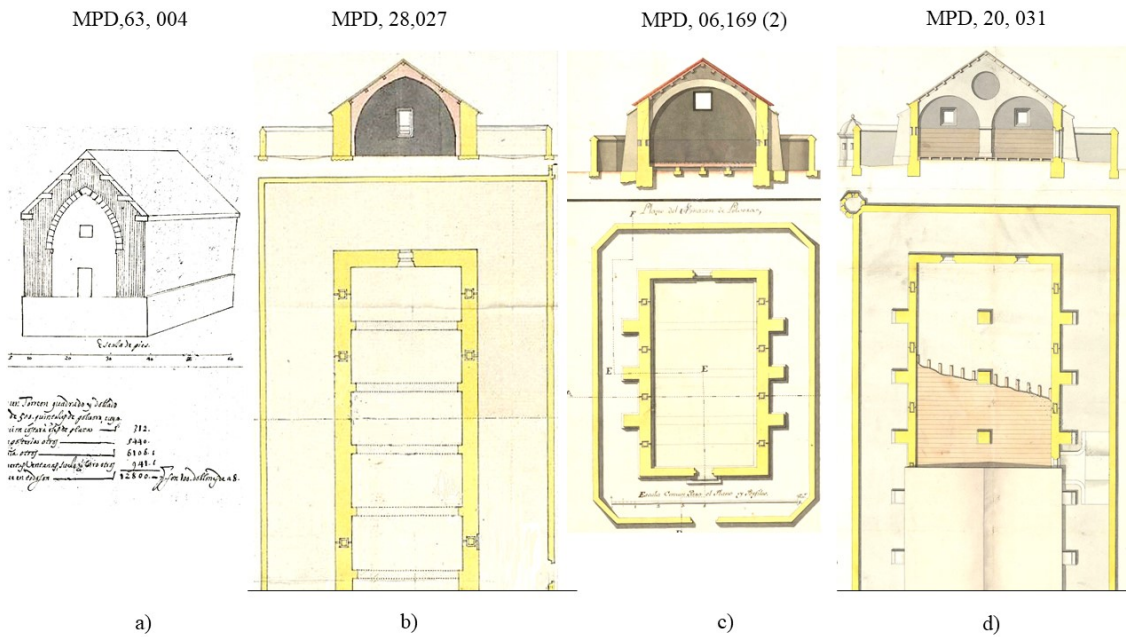


Fig 3 Gunpowder magazine projects using diaphragm arches. a) Fuenterrabía (1684) Octaviano Meni (fl. 1661-1702), (MPD, 63, 004). b) La Coruña (1774), Pedro Martín Cermeño (1722-1792), (MPD, 28, 027). c) Benimamet in Valencia (1751) Carlos Beranger (1698-1756) (MPD, 06, 169), pag.2. d) Barcelona (1761), Pedro Martín Cermeño (1722-1792) (MPD, 20, 031).

Table 1. Gunpowder magazine projects using pointed vaults

Reference	City	Country	Date	Autor
MPD, 09, 101	Fuenterrabía	Spain	1689	Unknown
MPD, 12, 142	Ciudad Rodrigo	Spain	1715	Moreau, Pedro
MPD, 26, 119	Peñíscola	Spain	1720	Montaigu de la Perille, Antonio
AGMM ss-7-17	San Sebastián	Spain	1722	Solis, Lorenzo de
MPD, 64, 005	Fuenterrabía	Spain	1723	Langot, Luis de
CGE Ar.G-T.2-77	Peñíscola	Spain	1723	Unknown
MPD, 59, 046	Gibralfaro	Spain	1724	Unknown
MPD, 59, 047	Gibralfaro	Spain	1725	Unknown
CGE Ar.F-T.2-C1-180.b	San Sebastián	Spain	1727	Subreville, Juan Pedro
AGMM ca-22-20	Cádiz, Isla de León	Spain	1728	Salas, Ignacio
MPD, 08, 239	Gibralfaro	Spain	1728	Desconocido
CGE Ar.F-T.2-C.3-250	Fuenterrabía	Spain	1730	Langot, Luis de
MPD, 19, 193	Ceuta	Spain	1736	Cardoso, Diego
MPD, 07, 180	Ceuta	Spain	1737	Cardoso, Diego
MPD, 19, 194	Ceuta	Spain	1737	Cardoso, Diego
MPD, 01, 018	Gerona	Spain	1738	Fabré José M.
MPD, 07, 181	Ceuta	Spain	1746	Canelas, Thomas
MPD, 27, 094	San Sebastián	Spain	1749	Solis, Lorenzo
AGMM l-2-17	Lerida	Spain	1750	Sertevens, Marcos t'
MPD, 27, 095	San Sebastián	Spain	1750	Martín Zermeño, Juan
MPD, 71, 069	Zaragoza, Torre Torreo	Spain	1750	Unknown
AGMM CUB-17-21	La Habana	Cuba	1760	Couvreu, Alfredo
AGMM col-5-7	Cartagena Indias	Colombia	1769	Arévalo, Antonio
AGMM ge-4-12	Figueras	Spain	1771	Villalonga, Ramón de
MPD, 65, 093	Peñíscola	Spain	1785	Labauume, Carlos
AGMM ca-11-12	Cádiz	Spain	1837	Muñoz, L; Hurtado, M.
AGMM ca-10-6	Cádiz, San Fernando	Spain	1842	Ruiz Hurtado, Antonio
AGMM vi-2-14	Vitoria	Spain	1852	De Yarzaz, Jose Maria
AGMM ca-22-9	Cádiz	Spain	1874	Unknown

The categorization of bombproof pointed vaults accounts for 55.17% of the total. This is evident in Figueras (c. 1771) (AGMM. ge-4-12) and the simple vault in the project undertaken in Gerona (1738) (MPD, 01, 018) (Fig. 4.b). From 1724 onwards, a specific classification was created featuring two naves and a roof with four slopes. This classification is clearly visible in those constructed in Gibralfaro (MPD, 59, 046) (1724) and (MPD, 59, 047) (1725) (Fig. 4.c). It contains a variant with ridges in the shaft of the naves and another fitted perpendicular to those in the center of these spaces. In this way, a total of eight slopes are created in Gibralfaro (MPD, 08, 239) (Fig. 4.d). This can be seen in the projects to erect four magazines in Cadiz (1728) and in Ceuta dating to 1736 (MPD, 19, 193), 1737 (MPD, 07, 180), and 1746 (MPD, 19, 194 and MPD, 07, 181). The subsequent representation of the one located in Cadiz (1874) (AGMM ca-22-9) was most likely built in the first half of the 19th century.

The proposal of this construction method using ogival vaults focused on twenty projects dating from 1715 to 1755. These disappeared in 1785, only to reappear after 1837 upon the review of five projects situated in Cadiz (AGMM ca-10-6, AGMM ca-11-12, AGMM ca-22-9), Vitoria (AGMM vi-2-14), and Colombia (AGMM col-5-7).

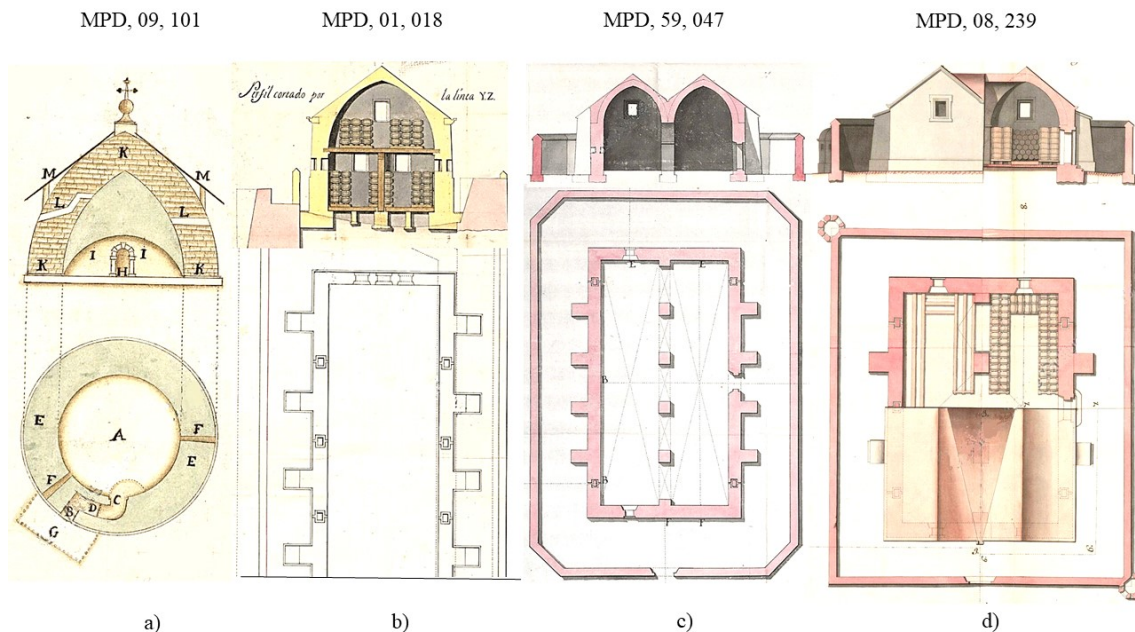


Fig 4 Gunpowder magazine projects featuring pointed vaults. a) Fuenterrabía (c.1689), unknown, (MPD, 09, 101). b) Gerona (1738) Miguel Marín Truq (+ 1764), (MPD, 01, 018). c) Gibralfaro in Malaga (1725), unknown, (MPD, 59, 047). d) Gibralfaro in Malaga (1728) Ignacio Sala (1686-1754), (MPD, 08, 239).

The geometry of an ogival vault

In the codex of Villard de Honnecourt (c.1175-c.1240) (ms. fr. 190093) the following term appears; 'tiirc o quint point' (fol. 20v), 'Par chu fait om trois manires d'ars à compas ovrir one fois' (fol. 21r). In this scenario, the ogive is determined through its span (l), setting the swek or centre of curvature of the voussoirs (t), thereby deducing the rise (f), and accordingly, the keystone (Bechmann 1993, 212-230). Likewise, the *quinto acuto* used by Giovanni di Gherardo da Prato (c.1360- c.1446) in the section of the Dome at Santa Maria del Fiore (1426) (Saalman 1959, 11-20). These typologies featuring the $A_{(l/n_i, l/n_{i-1})}$ design, divide the arch's span into n_i parts (l/n_i), placing the ogive's centre in (l/n_{i-1}) (Fig. 5.a). A further option would be to set the span and keystone as to deduce the swek, in such a way that *the principles leading proportion in the tradition are based on the Timaeus* (c. 360 BC) of Plato (c. 424 - c. 347 BC), with the ratios (2:3) sesquialteran, (4:3) sesquitercian, (9/8) sesquioctavan (Tim. 35B-36B) (Cornford 1937: 66 -71), or the notes deduced from the foregoing; octave (1:2), fifth (2:3), fourth (3:4) or tonal (8:9). Upon ascertaining the span (l) and its rise (f), its swek is determined $A_{(l/f)}$. (Fig. 5.b). Pointed arches always have a deduced geometric parameter, in which the centre is situated in an interval $[l/2, l]$ while its rise will be between $[f/2, \frac{\sqrt{3}}{2}f]$, which is defined as round arch arco, and the one at *plein cintre* inscribed in an equilateral triangle next to its radius.

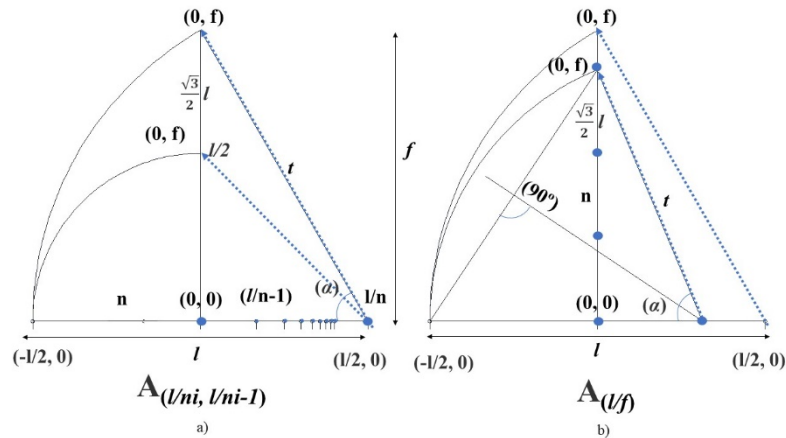


Fig 5 Determination of the point and sweep of the ogival arch, (l) span (f) rise (n_i) parts; a) Methodology traced arc from span (l/n_i) . b) Methodology traced arc from rise $A_{(l/f)}$.

Military engineers accessed scientific holdings of the Library of the Academy of Mathematics (*Biblioteca de la Academia de Matemáticas*) (Segovia 2004, 77-92), with its own allocation of funds (Galland-Seguela 2004: 157-180), and where Vicente Tomás Tosca (1651-1723) was renowned as one of its most influential authors. In the ‘Compendio matemático’ (1712), it is noted that the round arch, the pointed arch, and the elliptical arch will eventually collapse in their upper third if the keystone is not adequately reinforced. Sufficient arches are properly defined, while the author also states that the ogival arch is not aesthetically pleasing and would collapse in its upper third if the keystone is not sufficiently strengthened. However, fewer abutments were needed than for other arches (Tosca 1712: 104-105) (Tosca 1712, imprint 8, Volume 5, fig. 15, 108) (Fig. 6.a), and almost goes unnoticed when using an upper pointed arch and a further round arch placed below (Tosca 1712: 118).

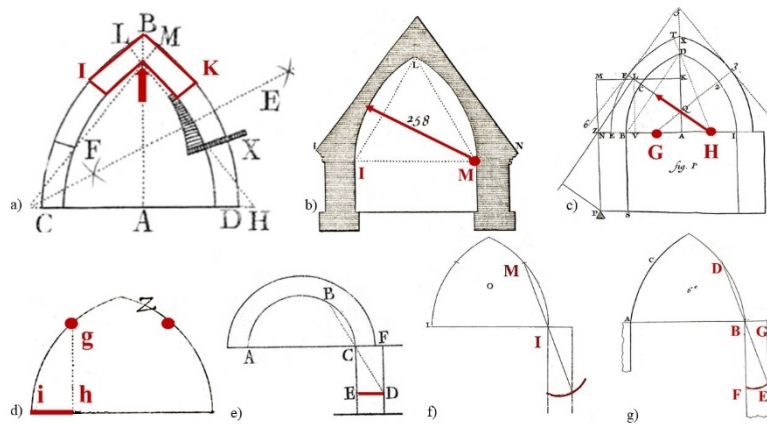


Fig 6 Designs for ogival arches and sizing of their abutments. a) Tomàs Vicent Tosca Mascó, *Compendio mathematico. Tomo V* (Tosca 1712: imprint 8, Volume 5, fig. 15, 108). b) Bernard Forest de Belidor, *Nouveau cours de Mathématique* (1725.), (Belidor 1725: Sheet 18, fig. 258, 336). c) Bernard Forest de Belidor *La science des ingénieurs* 1729, (Belidor 1729: liv. II, Sheet 6, fig1, 64). d) Ginés Martínez de Aranda (Ms. 457: fol. 6). f) François Derand, *L'architecture des voûtes* 1643. (Derand 1643: Cap IV, fig. O, 11). e) (Tosca 1712: imprint 9, Volume 5, fig. 22, 138). g) (Belidor 1729: Book. II, Sheet 6, fig. 1, 64).

Bernard Forest de Belidor (1698-1761) outlines in the work ‘Nouveau cours de Mathématique à l’usage de l’Artillerie’ (1725) the section describing the Vauban model while expanding on the categorisation of the segmental and pointed elements represented in *plein cintre* (Belidor 1725: Sheet 18, fig. 258, 336) (Fig. 6.b), thus determining the abutments or stirrups used with these categories (Belidor 1725: Sheet 31, 490-498). In ‘La science des ingénieurs’ the pointed arch is traced from the span and rise outwards, upon deducing the sweep (Belidor 1729: liv. II, Sheet 6, fig1, 64) (Fig. 6.c), then introducing the elliptical and structural beam, thereby proving that the catenary is the most suitable curvature so that all of its parts are properly balanced (Belidor 1729: Book II, Sheet. 6-7, 64-65, Fig. 8).

Formal knowledge of Spanish military engineers

Phillip the Second (1527-1598) created the *Academia Real Mathematica* in Madrid (1583) under the guidance of Juan de Herrera (1530-1597), advising the study of *Mechanica* (Herrera 1584: page. 10r-11r), and in which one of its professors, Cristóbal de Rojas (1555-1614), published the rule of thirds used to size the arches' abutments. Detail is given as to how this can be reduced whenever a heavy load is exerted (Rojas 1598: III, fol. 97 r), coinciding chronologically with the manuscript entitled *Cerramientos y trazas de montea* (c. 1598- 1608) published by the architect Ginés Martínez de Aranda (1556-1620) (Ms. 457: fol. 5-6) (Fig. 6.d). This rule was also adhered to in the bibliographical repository provided to the engineers (Lluis i Ginovart 2015: e103). Among these sources, we can list François Derand (1588-1644), who wrote 'L'architecture des voûtes ou l'art des traits et coupe des voûtes' (Derand 1643: Cap IV, fig. O, fol.11) (Fig. 6.f) and François Blondel (1616-1686) with *Cours d'architecture* (Blondel 1675: IV Partie, 419). Additionally, Tosca (Tosca 1712: imprint 9, Volume 5, fig. 22, 138) (Fig. 6.e) and Bélidor (Belidor 1729: Book II, Sheet 6, fig. 1, 64) (Fig. 6.g) detailed the categorization of gunpowder magazines (Belidor 1729: Book IV, Sheet 26, 64; Sheet 27, 68), in which the Vauban model (Fig.1.c) was also included. It is remarked that the pointed vaults tend to be weaker than the barrel model, as could be ascertained from the fort at Tornay (Belidor 1729: Book IV, 62-63), and sizing to render them bombproof must be calculated experimentally through the application of the laws of mechanics (Belidor 1729: 1729, Book IV, 65-66). The above is completed through 'Traité des Ponts' (1716), de Hubert Gautier (1660-1737), in its second edition published in 1723, with the time-served phrase: 'Ut pondera libra, sic aedificia architectura' (Gautier 1723: frontispiece), in such a way that the round arch tips the equilibrium away from the pointed arch (Huerta 1996: 18-29).

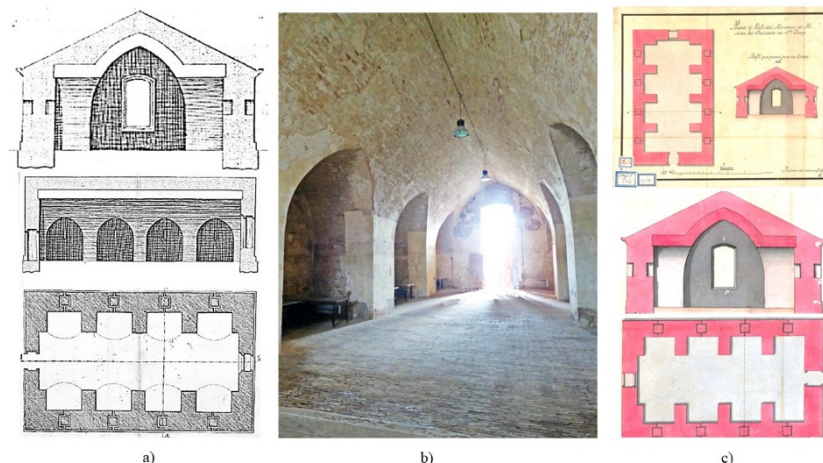


Fig 7 System using pointed barrel vault with lunettes. a) Miguel Sánchez Taramas, *Tratado de fortificación, ó Arte de construir los edificios militares, y civiles, escrito en inglés por Juan Muller* (1769) (Sánchez 1769, Sheet 2.A, 372 bis). b) Image of San Fernando in Figueras (authors), designed in (1753) by Juan Martín Zermeno (1700- 1773) and executed by Pedro Martín Zermeno (1721-1792). c) Project redrawn (c.1771) by Ramón de Villalonga (1748 - c.1796), AGGM Ge-4-12).

Belidor's work was translated by John Müller (1699-1784) and outlined the sizing of the ogival section (Müller 1755: Sheet 2, fig. 5-6, 42). He discusses in 'Of Powder-Magazines' (Muller 1755: 215-222) the Vauban model (Müller 1755: Sheet 17, fig.1,2,3,4, 222) (Fig.1. d), coinciding with the view that the vault performed best with a barrel vault over a pointed one. In turn, John Müller's text was translated by Miguel Sánchez Taramas (1733-1799) to be used by the alumni of the School of Mathematics. In the chapter dealing with the arches' abutment, he adheres to Müller's views (1755), while in its 'Sección XIX De los Almacenes de Polvora' (Sánchez 1769: 357-370) fitted with dual barrel vaults with major weighting on the roofed elements, these are replaced by pointed one affording greater capacity. Furthermore, the success of the Vauban (1681) categorisation is explained in Landaw (Sánchez 1769: Sheet 17, figs.1-4) along with the shortfall of pointed vaults in Tornay. In Note 27, a pointed barrel vault is described, featuring lunettes and stirrups, while making reference to the keystone this has (Sánchez 1769: Sheet 2.A, 372 bis) (Fig. 7.a). The account is taken as an experiment in the project for San Fernando de Figueras (1753) de Juan Martín Zermeno (1700- 1773), undertaken by Pedro Martín Zermeno (1721-1792) (Fig. 7.b), and drafted by Ramón de Villalonga (1748 - c.1796) (AGGM Ge-4-12) (c.1771) (Fig. 7.c). Despite this, Pedro Lucuze (1692-1779) in the *Principios de fortificación* (1772), edited for usage within the Academy, continues to foster the barrel model devised by Vauban (Lucuze 1772: Sheet 6. Fig. 82, 85-90) (fig. 1.e).

Pointed vault versus the catenary model in military architecture

The influence exerted over the construction of gunpowder magazines seen in the Vauban model is decisive in the building of semi-cylindrical or barrel vaults, to the extent that they account for 62.31% compared to pointed vaults totalling 26.92% and the remainder, 10.77%, corresponding to segmented vaults. Within the inventory performed, 129 magazines feature vaults affected by the forces of gravity, while 58.91% are bombproof and have been sized experimentally, thereto applying the laws of mechanics with regard to withstanding artillery impact (Belidor 1729: Book IV, 65-66). Owing to this evolution, bombproof construction techniques entered a period of decline in the mid 17th century, as can be noted in the similitude of Lorenzo de Solís's dimensions for the magazine, the troops' barracks and storehouses for perishable goods in Cartagena de Indias 1754 (CGE: Ar. J-T.7-C.2-49) (Fig. 8.a), thus corroborated through its construction by Antonio Arévalo in 1769 (AGMM col-5-7) (Fig. 8.b)

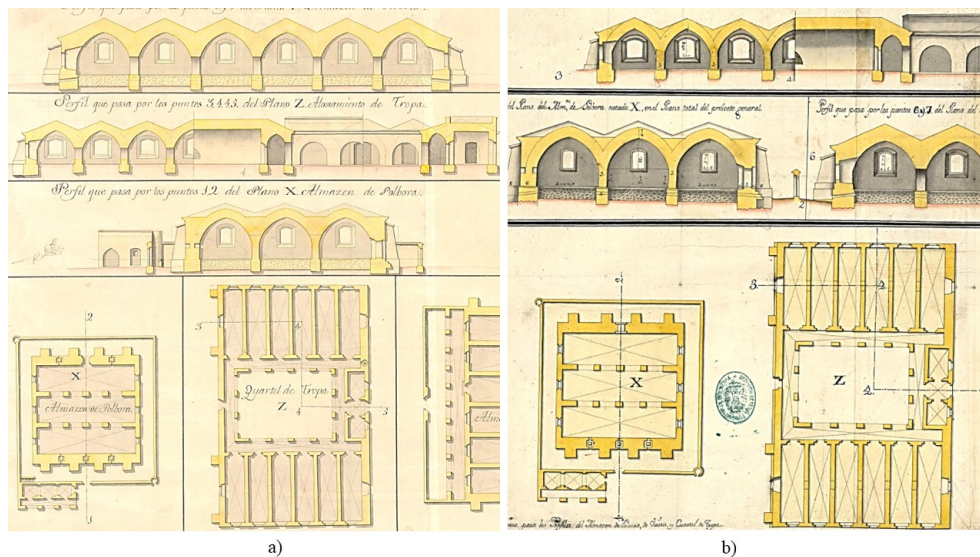


Fig 8 Layout of constructions in the curtain walling in Santa Catalina de Cartagena de Indias. a) Project for a Gunpowder store, troop barracks, supply depot, supply warehouse gunpowder store, troop barracks, supply store (1754), Lorenzo de Solís (+1761), (CGE, Ar. J-T.7-C.2-49). b) Redrawing of the construction made (1769) by Antonio Arévalo (1715-1800), (AGMM col-5-7).

In order to size the abutment, the maximum angle of sweep is defined (α) as the one formed between the impost and the arch's centre with its keystone, placed at an interval $[0, \alpha]$ (Fig. 4.b). From this, it is proven that the length of the ogive is calculated:

$$L = \frac{2\pi \left(\frac{L}{2}\right)\alpha}{360} \dots [1]$$

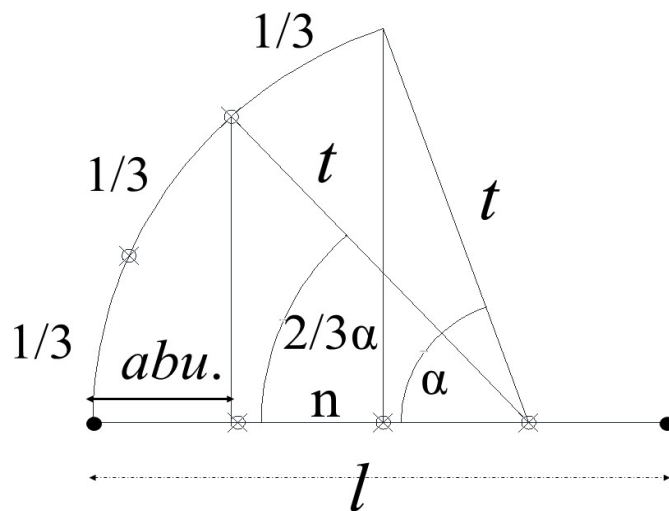
The value of the abutment following the rule of thirds for an ogive is calculated at a point $(2/3 \alpha)$, whereby:

$$abu = t - t \cos (2/3 \alpha) \dots [2]$$

Taking into account an interval for a tautness value $\alpha = [90^\circ - 60^\circ]$, of a round arch placed at *plein cintre*, its abutment would then be $[1,069 - 1,000]$ (table 2). In a vault or dome measuring 20 *toises* (6,487m), the measurement difference amounts to (0.104 m), with a value between both of 6,215%.

Table 2. Abutment of a pointed arch depending on its span

l	$l/2$	n	$n-1$	t	$1/2 l-l/n$	$\cos \alpha^\circ$	α rad.	α°	$2/3 \alpha^\circ$	\cos	abutment	$l/abu.$
1	0.5	2	1	0.500	0.000	1.000	0.000	90.000	60.000	0.500	0.250	4.000
1	0.5	3	2	0.667	0.167	0.969	0.250	75.522	50.348	0.638	0.241	4.145
1	0.5	4	3	0.750	0.250	0.945	0.333	70.529	47.019	0.682	0.239	4.190
1	0.5	5	4	0.800	0.300	0.931	0.375	67.976	45.317	0.703	0.237	4.211
1	0.5	6	5	0.833	0.333	0.921	0.400	66.422	44.281	0.716	0.237	4.224
1	0.5	7	6	0.857	0.357	0.914	0.417	65.376	43.584	0.724	0.236	4.233
1	0.5	8	7	0.875	0.375	0.910	0.429	64.623	43.082	0.730	0.236	4.239
1	0.5	9	8	0.889	0.389	0.906	0.438	64.056	42.704	0.735	0.236	4.243
1	0.5	10	9	0.900	0.400	0.903	0.444	63.612	42.408	0.738	0.235	4.247
1	0.5	11	10	0.909	0.409	0.900	0.450	63.256	42.171	0.741	0.235	4.249
1	0.5	12	11	0.917	0.417	0.898	0.455	62.964	41.976	0.743	0.235	4.252
1	0.5	1	1	1.000	-0.500	0.878	0.500	60.000	40.000	0.766	0.234	4.274



The catenary model was introduced into the Academy of Mathematics by Belidor (Belidor 1729: liv. II, Sheet 6, fig. 7-8, 64) (Fig. 9.a), plus likewise through the stereotomy of Amédée François Frézier (1682-1773). This work was adhered to in the Academies (Calvo-López 2007: 167-176), as well as the Artillery of Segovia (Navascués 1996: 17-44) along with the Marine Guards in Cadiz (García 2015: 123-154). It is defined as *Chaînesse* and puts forward the argument that it is a suitable curve to balance the voussoirs or segments, although this does not endow it with beauty as it entails a flexion to the abutment, suggesting that the formwork is more similar to that of the pointed arch (Frézier 1738: 97-98, Sheet 33, fig. 50, 116) (Fig. 9.c).

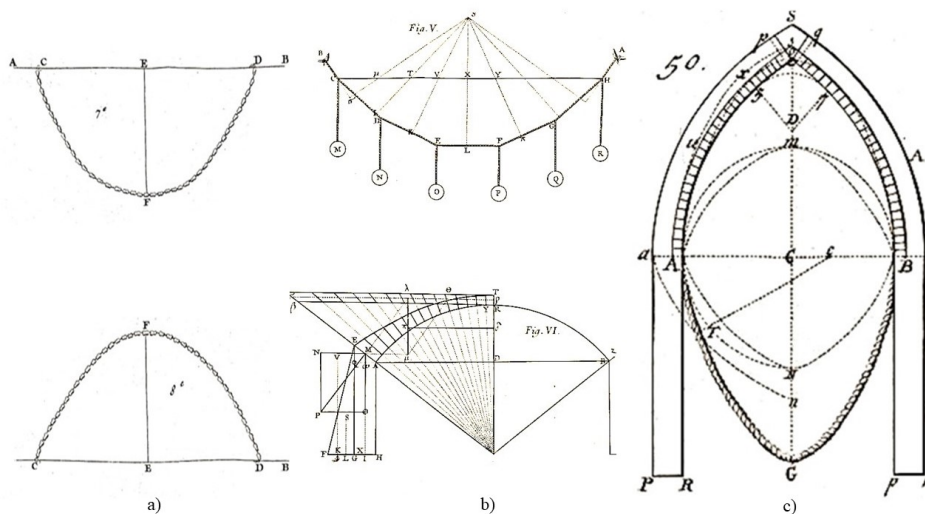


Fig 9 The catenary model in architectural treatises of the French Benightment. a) Bernard Forest de Belidor, *La science des ingénieurs* (1729) (Belidor 1729: liv. II, Sheet 6, fig. 7-8, 64). b) Pierre Couplet *De la poussée des voûtes* (1729) (Couplet 1729: Sheet 6, fig. 5, 116). c) Amédée François Frézier, *La théorie et la pratique de la coupe de pierres et des bois pour la construction des voûtes* (1738) (Frézier 1738: Sheet 33, fig. 50, 116).

Frézier refers to ‘De la poussée des voûtes’ (1729) by Pierre Couplet (c.1670-1743), stating the *chaînette* as the standout form for the construction of vaults (Couplet 1729: 95-97, Sheet 6, fig. 5, 116) (Fig. 9.b). At the same time, he deemed the catenary the ideal shape for an arch of constant thickness upon comparing it with the round and pointed arches (Frézier 1738: 97-98, Sheet 33, fig. 50, 116), drawing it within the thickness of the ogival arch (Huerta 2004: 334). Included in the chapter entitled *Des courbes composées* he makes appreciations akin to those of Tosca, grading them as a *tiers point* and elliptical. Gothic vaults, defined as eyesores, boast the advantage of facilitating the construction process with very small and unshaped voussoirs. These were less costly to produce and required less counterbalance (Frézier 1738: 102-104). He states that they fell into disuse due to the fact that projects struck the parabolic line and could reach the upper surface perpendicular to its tangent, thus exerting greater stress on the keystone, unlike the case of round variants (Frézier 1738: 113-114). Guillaume Le Blond (1704-1781) published Vauban’s typology in his work *Elémens de Fortification* (Le Blond 1739: Sheet 4, fig 6, 46-48) and its subsequent editions (Le Blond 1775: Sheet 4, fig 1-4, 48-50) (Fig. 1.f).

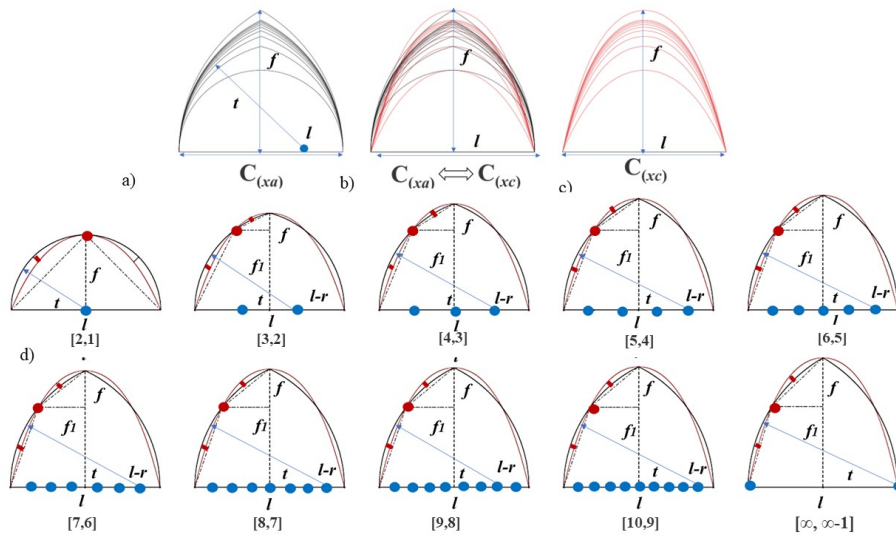


Fig 10 Comparison between pointed and catenary vaults. Comparing both systems as a whole (Fig. 10.b), pointed vaults $C_{(xa)}$ (Fig 10.a) with a span of an arch dividing n_i parts (l/n_i), in the centre of the ogive is situated as (l/n_{i-1}) (Fig. 5.a), in the interval $[-l/2, 0]$ and whose equation to obtain a circumference gives:

$$f(x_a) = \sqrt{r^2 - x^2} \dots [3]$$

Turning to catenary arch forms, $C_{(xc)}$ (Fig. 10.c) which feature the same span and rise in the interval $[-l/2, 0]$ whose equation would be:

$$f(x_c) = a \cosh(x/a) \dots [4]$$

Comparing them as a whole $C_{(xa)} \leftrightarrow C_{(xc)}$ inasmuch as the length of the semi arches, it is seen that:

$$L f(x_a) = \int_{-l/2}^0 \sqrt{r^2 - x^2} dx \dots [5]$$

$$L f(x_c) = \int_{-l/2}^0 a \cosh(x/a) dx \dots [6]$$

Along with their interior surface:

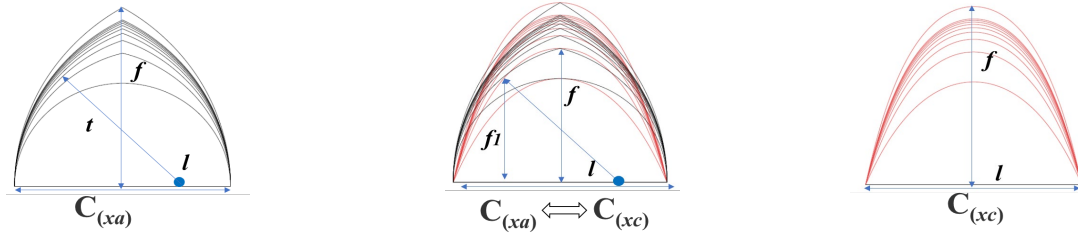
$$S f(x_a) = 2 \iint_{-l/2}^0 \sqrt{r^2 - x^2} dx dy \dots [7]$$

$$S f(x_c) = \iint_{-l/2}^0 a \cosh(x/a) dx dy \dots [8]$$

The calculation shows us that the most coherent values in terms of lengths and surfaces between catenaries and arches with the same span and rise (Fig. 10.d), are those for $A_{(l/n_i, l/n_{i-1})}$, the arches $A_{(4,3)}$ y $A_{(5,4)}$, given in table 3.

Table 3. Geometric relationship between the ogive and catenary

Prop. $l \neq t$	l (m)	t (m)	$l-t$ (m)	f (m)	$l \neq t$	lon. arc. (m)	lon. cat. (m)	dif. lon. (m)	sup. arc.	sup. cat.	dif. sup.	f_l (m)	$f \cdot f_l$ (m)	dif. cat- arc (m)	dif. arc- cat (m)
2-1	1.000	0.500	0.500	0.500	0.200	1.571	1.496	0.075	3.927	3.466	0.461	0.500	0.000	0.048	0.048
3-2	1.000	0.736	0.264	0.646	0.155	1.757	1.736	0.216	4.782	4.536	0.246	0.532	0.114	0.014	0.037
4-3	1.000	0.779	0.221	0.707	0.141	1.846	1.842	0.042	5.156	4.996	0.161	0.532	0.175	0.021	0.033
5-4	1.000	0.797	0.203	0.742	0.135	1.898	1.903	-0.047	5.368	5.255	0.113	0.532	0.209	0.025	0.031
6-5	1.000	0.807	0.193	0.764	0.131	1.932	1.942	-0.098	5.505	5.421	0.083	0.533	0.231	0.027	0.030
7-6	1.000	0.814	0.186	0.779	0.128	1.956	1.969	-0.133	5.600	5.538	0.062	0.534	0.246	0.029	0.030
8-7	1.000	0.818	0.182	0.791	0.127	1.974	1.990	-0.158	5.671	5.624	0.047	0.534	0.257	0.029	0.030
9-8	1.000	0.821	0.179	0.799	0.125	1.988	2.005	-0.175	5.725	5.690	0.035	0.534	0.265	0.031	0.029
10-9	1.000	0.824	0.176	0.806	0.124	1.998	2.018	-0.192	5.768	5.742	0.026	0.535	0.272	0.032	0.028
1-1	1.000	0.843	0.157	0.867	0.115	2.094	2.125	-0.310	6.142	6.196	-0.054	0.538	0.329	0.037	0.026



Practical geometry employed by military engineers of the Enlightenment

Upon analysing 31 pointed vaults included in the aforesaid inventory of gunpowder magazines, we have been able to determine accurately, through the lines of the points on a compass, the centres of the arch's outline: nine have the point of the arch on the impost line (Fig. 11) and twenty below this (Fig. 12), while it has not been possible to ascertain that of a further three. The findings metrologically determine the span and rise of arch that generates the structure, thus deducing the vault's tautness (table 4).

Table 4. Measurements of pointed vaults

Graphical reference	Project reference	Toises (feet)				Varas (palms)				Tip.
		l	f	f_1	f_2	l	f	f_1	f_2	
	1 MPD, 09, 101	22	19	19	0					A
	2 MPD, 12, 142	28	20	20	0					A
	3 MPD, 26, 119	20	13	10	3					B
	4 AGMM ss-7-17	-	-	-	-	32	21	10	11	B
	5 MPD, 64, 005	11	10	8	2					B
	6 CGE Ar.G-T.2-77	21	16	11	5					B
	7 MPD, 59, 046	14	9	9	0					A
	8 MPD, 59, 047	14	9	9	0					A
	9 CGE Ar.F-T.2-C1-180.b	19	15	14	1					B
	10 AGMM ca-22-20	-	-	-	-					-
	11 MPD, 08, 239	14	9	8,5	0,5					B
	12 CGE Ar.F-T.2-C.3-250	11	9	7,5	1,5					B
	13 MPD, 19, 193	14	9	8	1					B
	14 MPD, 07, 180	-	-	-	-					-
	15 MPD, 19, 194	15	9	8,75	0,25					B
16 MPD, 01, 018	19	15	15	0					B	
17 MPD, 07, 181	14	9	8,5	0,5					B	
18 MPD, 27, 094	20	16	9	7					B	
19 AGMM l-2-17 A	24	15	15	0					B	
20 AGMM l-2-17 B	24	14	12,5	1,5					B	
21 MPD, 27, 095	21	15	10	5					B	
22 MPD, 71, 069	-	-	-	-	7	6	6	0	A	
23 AGMM CUB-17-21	10	8	8	0					A	
24 AGMM ge-4-12	-	-	-	-	33	15,5	15,5	0	A	
25 AGMM col-5-7	24	17	12	5					-	
26 MPD, 65, 093	-	-	-	-					-	
27 AGMM ca-11-12	-	-	-	-	18	10	10	0	A	
28 AGMM ca-10-6	14	8	5	3					B	
29 AGMM vi-2-14 A	19	13	12	1					B	
30 AGMM vi-2-14 B	11	8	5	3					B	
31 AGMM ca-22-9	10	9	9	0					B	

The maximum dimensions (l, f) in which *toises* are used are (28, 20 To.) (6,580, 4,700 m) found in Ciudad Rodrigo (1715) (MPD, 12, 142) con ($l/f=1,400$), and in palms in San Sebastián (1722) (AGMM ss-7-17) (32, 21 Va.) (6,688, 4,389 m) y ($l/f=1,524$), thereby giving the average of (4,138x2,447 m) and ($l/f=1,691$).

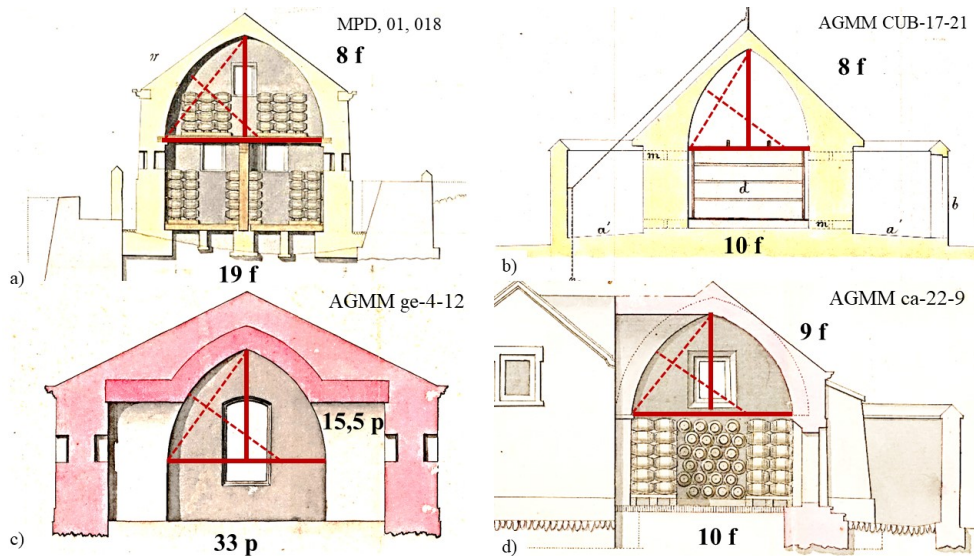


Fig 11 Pointed vaults placed on the centre of the impost line. a) Fuerte de Capuchinos in Gerona (1738), José Fabrè (+ 1785), (MPD, 01, 018). b) Barrio del Vedado in La Habana (Cuba) (1853), José Sacramento de León, (AGMM CUB-17-21). c) Figueras in Gerona (c.1780). Ramon de Villalonga (1748-1796), (AGMM ge-4-12). d) Isla de León in San Fernando, Cádiz (1728), Ignacio Sala Garrigó (1686-1755), (AGMM ca-22-9).

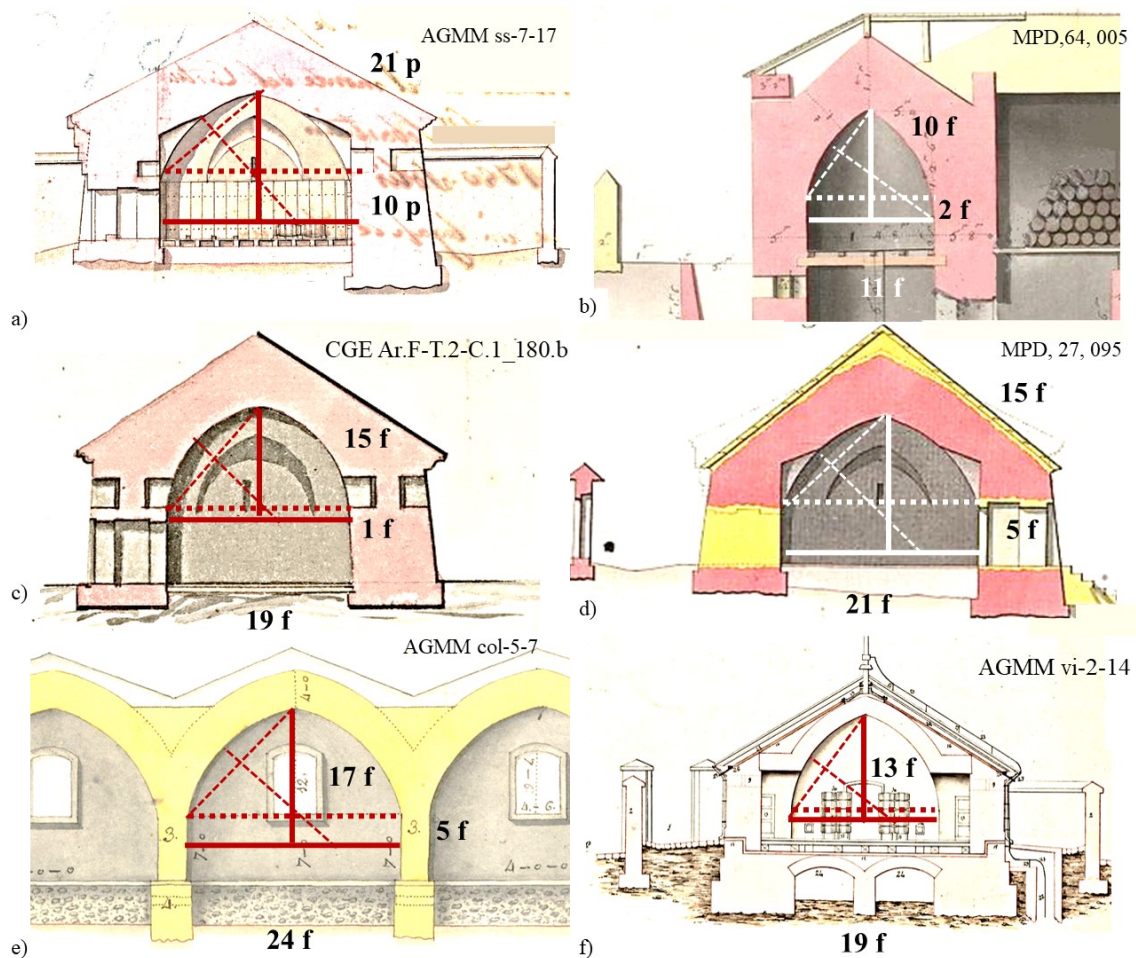


Fig 12 Pointed vaults with the centre below the impost line. a) Monte del Castillo de la Mota in San Sebastián (1728), Ignacio Sala Garrigó (1686-1755), (AGMM ss-7-17). b) Cubo de la Magdalena in Fuenterrabía (1723), Luis de Langot (fl. 1710-1730), (MPD, 64, 005). c) San Sebastián (1727) Juan Pedro Subreville (1705, 1736), (CGE Ar.F-T.2-C.1_180.b). d) Castillo de la Mota in San Sebastián (1750), Juan Martín Zermeño (1700-1773), (MPD, 27, 095). e) Cartagena de Indias in Colombia (1769), Antonio de Arévalo (1715-1800), (AGMM col-5-7). f) Vitoria (1852), Juan Ysla Fernández y Falqués (1796-1860), (AGMM vi-2-14).

Within the listing as vaults with special sections (T-4), six projects are found (1718-1736), with the one by Juan Pedro Subreville (fl. 1718-1739) in San Sebastián (1727) (CGE. Ar. F-T.2-C1_180.a) being particularly noteworthy, as therein the auxiliary outline for their draughting is clearly visible (Fig. 13.b). The main vault has been drawn with two pointed arches (a_1 , a_2) both placing the centres above the impost line, thus creating a horseshoe shaped dome that starts half a foot below the centres. The geometric slope upon the impost forms an angle in excess of 90° . Furthermore, it is noted that it does not contain a horizontal tangent in the keystone, although the measurements showing constructive divergence are negligible. A round arch is drawn upon the impost (a_3) measuring 20 feet in diameter, which would have measurements somewhat less than the arches' span (a_1 , a_2) (Fig. 13.b). In the vaults with pointed lunettes, the round arch (a_6) is placed above the impost, while the arches' centres (a_4 , a_5) are below this, thus forming an angle inferior to 90° (Fig. 13.c).

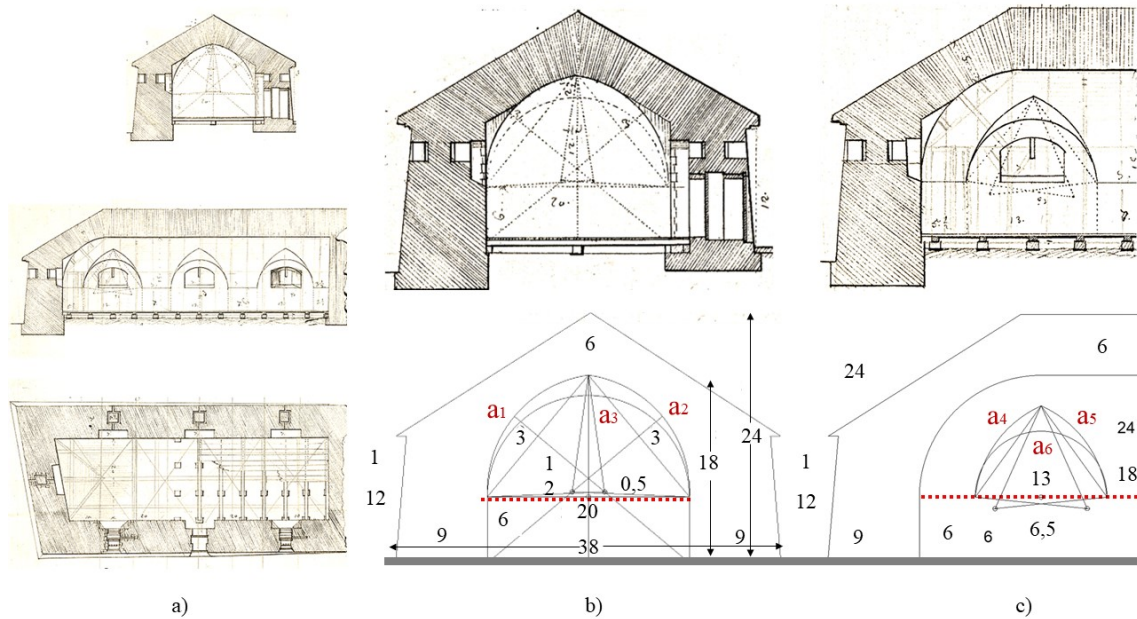


Fig 13 Metrological simulation of vaults featuring special sections. a) Project in San Sebastián (1727), Juan Pedro Subreville (fl. 1718-1739) (CGE. Ar. F-T.2-C1_180.a). b) Cross section and its metric notes. c) Longitudinal section and its metric notes.

Four projects were unearthed representing ovals and whose centres had been offset to below the impost, containing an angle at the start of the dome of less than 90° . One in Barcelona (1731) (MPD, 07, 057) (Fig. 14.a), two in Tortosa (1733) (AGMM t-18-16), (MPD, 13, 035) (Fig. 14.b) and another in La Coruña (1736) (MPD, 17, 057) (Fig. 14.c). This geometric layout, given its horizontal tangent in the keystone, has led to the hypothesis that representation through ovals (Tosca 1707: 292-295) may be interpreted as a catenary (Lluís i Ginovart: et al. 2017a, 85-99).

Knowledge on catenaries in Spain may also have come from the hands of Irish and Scottish emigrants. Upon Phillip the Fifth taking the throne in 1700, the military status and rank was preserved for catholic military families choosing to emigrate from these lands under his protection, with these being versed in 'A description of helioscopes, and some other instruments' (1676) by Robert Hooke (1635-1703). Against this backdrop, chronologically coinciding with the engineers' projects, descendants of the O'Connor family who settled in Benicarló in 1749 built a wine cellar for the vintner Carlón in 1757, in which a catenary arch was used (Lluís i Ginovart, et al. 2017b: 703-716).

With regard to the analysis of the 28 pointed vaults, seven of these are within the reference range stated ($l:f$) [1,5238, 1,556] giving an average of (l,f) (4,138x2,447 m), remaining somewhat close to the geometry ($l:f$) of [3:2]. The average difference between both curves would be found in the upper part of the ogive's intrados in relation to the catenary with a maximum distance of 0.057m. In the lower section, the ogive would be inserted into the catenary's intrados, at a maximum distance of 0.154 m. The difference in drawing an ogive formwork through a traditional style arch and another using a chain for the catenary, while the maximum distortion of the end measurements is somewhat slight. The formwork's length on the ogival arch is 0.089 m, greater than the catenary model, whereas the section of the pointed vault is $1,019 \text{ m}^2$ greater than that on the catenary model.

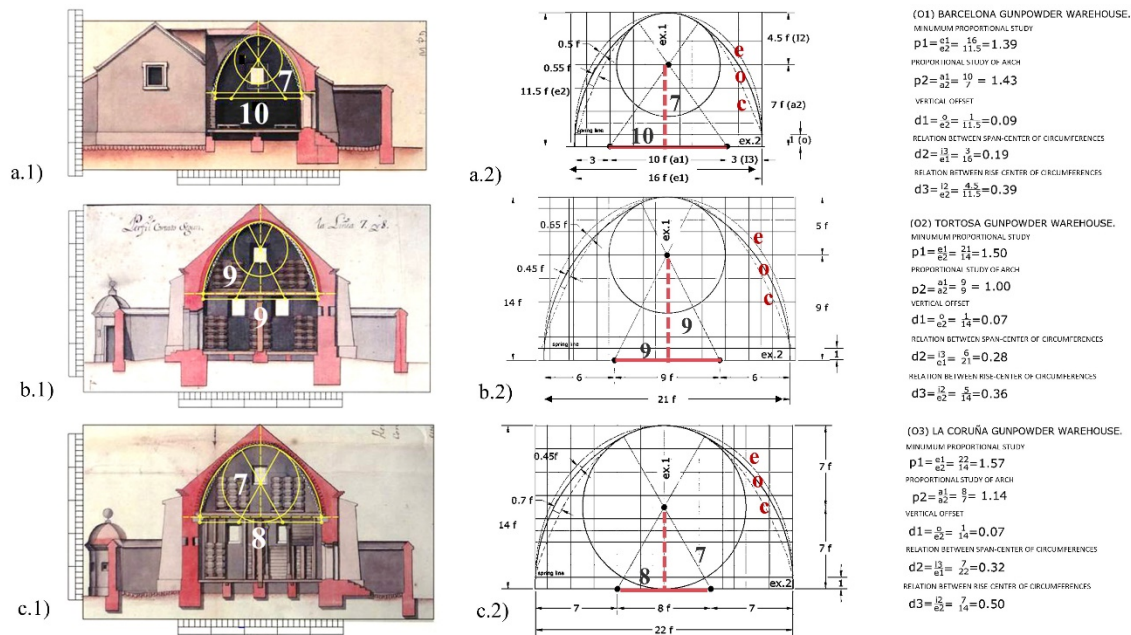


Fig 14 Representation of vaults using ovals, (c) catenary (e) ellipsis, (o) oval. a.1) Barcelona (1731) attributed to Miguel Marín Truq (+1764). a.2) curve metrology simulation (c), (e), (o) (MPD, 07, 057). b.1) Tortosa, (1733), Miguel Marín Truq (+1764). b.2) curve metrology simulation (c), (e), (o), (MPD, 13, 035). c.1) La Coruña (1736) Juan de Laferrière y Valentín. c.2) curve metrology simulation (c), (e), (o) (MPD, 17, 057).

Conclusion

The treatise taught at the Academy of Mathematics in Barcelona incorporates certain metrological principles, both in defensive practices and in auxiliary military constructions. Consequently, among the 31 projects analyzed featuring ogival shapes, their geometry is determined by fixing the span and the rise, denoted as $A(l/f)$. This circumstance necessitates defining the arch's center with a square, a design element inherited from the medieval period and retained into the late Hispanic Gothic era.

The research presents a geometric reinterpretation of the medieval pointed arch by these engineers, as 68.97% of the examined arches have their center positioned below the impost. This feature implies that the angle of incidence on the vertical face is less than 90° , a characteristic shared with the catenary arch and initially elucidated in Belidor's 'La science des ingénieurs' (1729). Additionally, the Academy studied the compatibility and structural similarities between the catenary and Gothic vaulting in Frézier's 'La théorie et la pratique de la coupe de pierres' (1738). Given the decline of bomb-proof vaults in powder magazines and the compatibility known by Frézier, it's demonstrated that this theory is perfectly applicable, with the pointed arch also being attributed with properties of economy, lightness, and reduced counterbalance.

One can speculate that due to the minimal metric discrepancy between the pointed arch and the catenary, the pointed system employed by military engineers, with its center positioned below the impost, represents a simplified attempt at mimicking the structural form of the catenary. Furthermore, in terms of geometry, the intricate depiction of the catenary's graphic representation is simplified with a pointed arc, a form that can be drawn both graphically and constructively using traditional instruments such as string, compass, and square. Scientifically, it's evidenced that 11.44% of the analyzed powder magazine projects aim to adopt the structural innovation of the Enlightenment's catenary, albeit constructed with medieval geometry, thus affirming Frézier's principles.

Glossary

$A(l/m_i, l/m_i-1)$ arc with its center traced from its span

$A(l/f)$ arc with its center traced from its span and rise

Abu abutment

(AGMM) Archivo General Militar Madrid

(AGMS) Archivo General Militar de Segovia

(AGS) Archivo General de Simancas

(AIMML) Archivo Intermedio Militar de Melilla

(BN) Biblioteca Nacional de Madrid

(c) catenary
 $C_{(x_a)}$ system pointed vaults
 $C_{(x_c)}$ system catenary
 (CGE) Archivo Cartográfico de Estudios Geográficos del Centro Geográfico del Ejército
 (f) rise
 $f(x_a)$ mathematical function of the arch
 $f(x_c)$ catenary mathematical function
 (e) ellipsis
 L length of the ogive
 $L f(x_a)$ length of arch
 $L f(x_c)$ length of catenary
 (l), span
 MPD: Mapas, Planos y Dibujos del Archivo General de Simancas.
 (n) parts
 (o) oval
 $S f(x_a)$ arch surface
 $S f(x_c)$ catenary surface
 (t) swek
 (α) angle of swek

Footnotes

¹ Archivo General de Simancas (AGS), Archivo Cartográfico de Estudios Geográficos del Centro Geográfico del Ejército (CGE), Archivo General Militar Madrid (AGMM), Biblioteca Nacional de Madrid (BN), Archivo General Militar de Segovia (AGMS) y Archivo Intermedio Militar de Melilla (AIMML).

There is no conflict of interest.

I+D+i "Cartografías de la ciudad en la Edad Moderna: relatos, imágenes, representaciones" (PID2020-113380GB-I00 / AEI / 10.13039/ 501100011033) funded by the Spanish State Research Agency (Ministry of Science and Innovation).

References

- Bechmann, Ronald. 1993. *Villard de Honnecourt. La pensée technique au XIIIe siècle et sa communication, Nouvelle édition revue et augmentée*. Paris: Picard Éditeur.
- Belidor, Bernard Forest de. 1725. *Nouveau cours de Mathématique à l'usage de l'Artillerie et du Génie, où l'on applique les parties les plus utiles de cette science à la théorie et à la pratique de différents sujets qui peuvent avoir rapport à la guerre*. Paris: Chez Charles-Antoine Jombert.
- Belidor, Bernard Forest de. 1729. *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile*. Paris: Chez Claude Jombert.
- Blondel, François. 1675. *Cours d'architecture enseigné dans l'Académie Royale d'Architecture*. Paris: imprimerie Lambert Roulland.
- Bravo, A. and S. Ramírez. 2022. The gothic or ogival vault in the military architecture of the 18th century. The case of gunpowder stores, from Gibralfaro to San Fernando de Figueras. International Congress on Fortifications and Military Heritage ICOFORT 2022, Ceuta (Spain). Unpublished.
- Calvo-López, José. 2007. Piezas singulares de cantería en la ingeniería y la arquitectura militar de Cartagena en el siglo XVIII. In *Actas del Quinto Congreso Nacional de Historia de la Construcción, Burgos, 7-9 junio 2007*, Arenillas, M. Segura, C. Bueno, F. Huerta, S. (eds.) Madrid: I. Juan de Herrera, SEDHC, CICCIP, CEHOPU, 167-176.
- Calvo-López, José. 2011. From Mediaeval Stonecutting to Projective Geometry. *Nexus Network Journal* 13: 503-533. <https://doi.org/10.1007/s00004-011-0081-5>.
- Cámara, A. 1989. Città e difesa dei regni peninsulari nella Spagna Imperiale (secoli XVI-XVII). In *La città e le mura*, De seta, C.; Le Goff, J. (eds.). Roma-Bari: Laterza: 80-106.
- Capel, H. 1983. *Los Ingenieros militares en España, siglo XVIII: repertorio biográfico e inventario de su labor científica y espacial*. Barcelona: Edicions Universitat de Barcelona.
- Cassani, José. 1705. *Escuela Militar de Fortificación ofensiva y defensiva. Arte de fuegos y de esquadronar*. Madrid: Antonio Gonçalvo de Reyes.
- Cornford, Francis MacDonald. 1937. *Plato's cosmology. The Timaeus of Plato*. London: Kegan

- Paul, Trench, Trubner & Co.
- Couplet, Pierre. 1729. *De la poussée des voûtes. Mémoires de l'Académie Royale des Sciences de Paris*. Paris: Imprimerie Royale.
- Chafrion, José. 1693. *Escuela de Palas à sea Curso Mathematico*. Tomo I. Milan: Empreinte Real, by Marcos Antonio Pandulpho Malatesta.
- Derand, François. 1643. *L'architecture des voûtes ou l'art des traits et coupe des voûtes*. Paris: Sebastian Cramoisy.
- Du Fay, Abbé. 1681. *Maniere de fortifier selon la methode de Monsieur de Vauban... Par monsieur l'abbé Du Fay*. Paris: Veuve Jean-Baptiste Coignard & Jean-Baptiste II Coignard.
- Du Fay, Abbé. 1693. *Maniere de fortifier selon la methode de Monsieur de Vauban... Par monsieur l'abbé Du Fay... Seconde edition augmentée*. Paris: Veuve Jean-Baptiste Coignard & Jean-Baptiste II Coignard.
- Fernández, Sebastián. 1687. *El Ingeniero Primera Parte, de la Moderna Architectura Militar. Dividido en dos Tomos... Por el Capitan Don Sebastian Fernnandez de Medrano, Maestro de Mathematicas de la Academia Militar de los Estados de Flandes*. Brussels: Casa de Lamberto Marchant.
- Fernández, Sebastián. 1700. *El Architecto Perfecto en el Arte Militar dividido en cinco Libros... El General de Batalla Don Sebastian Fernnandez de Medrano, Director de la Academia Real y Militar del Exercito de los Payses Baxos*. Brusselas: Casa de Lamberto Marchant.
- Frézier, Amédée François. 1738. *La théorie et la pratique de la coupe de pierres et des bois pour la construction des voûtes et autres parties des bâtiments civils et militaires, ou traité de stéréotomie à l'usage de l'architecture. Tomo II*. Strasbourg/Paris: Jean Daniel Doulsseker/Charles-Antoine Jombert.
- Galland, Martine. 2004. Las condiciones materiales de la vida privada de los ingenieros militares en España durante el siglo XVIII. *Geo Crítica / Scripta Nova. Revista electrónica de geografía y ciencias sociales*, 8, 157-180.
- García, Manuel. 2015. Las bibliotecas de las academias de guardias marinas en el siglo XVIII. In: *Jorge Juan Santacilia en la España de la Ilustración*, Alberola-Romá; Mas; Die (coord.). Alicante: Universidad de Alicante, 123-154.
- Gautier, Henri. 1723. *Traité des ponts, ou il est parlé de ceux des romains & ceux des moderns. Augmenté d'une Dissertationsur les Culées, Piles, Voussoirs, et Poussées des Ponts* (pp. 335-412). Paris: Chez André Cailleau.
- Herrera, Juan. 1584. *Institucion de la Academia Real Mathematica en Castellano, que la Majestad del Rey Don Phelippe. II./ N. S. mando fundar en su Corte. Dirigida a la C.R.M. del Rey Don Phelippe nuestro Señor*. Madrid: En casa de Guillermo Droy impressor de libros.
- Huerta, Santiago. 1996. La teoría del arco de fábrica: desarrollo histórico. *Obra Pública*. 38, 18-29.
- Huerta, Santiago. 2004. *Arcos, bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica*. Madrid: Instituto Juan de Herrera. Escuela Técnica Superior de Arquitectura Madrid.
- Le Blond, Guillaume. 1739. *Elémens de Fortification dédiés à son Altesse Monseigneur le Prince Charles de Lorraine, Grand Ecuyer de France*. Paris: Chez Charles-Antoine Jombert.
- Le Blond, Guillaume. 1775. *Eléments de fortification, contenant la construction raisonnée des ouvrages de la fortification*. Paris: Chez Charles-Antoine Jombert.
- Lucuze, Pedro de. 1772. *Principios de fortificación que contienen las definiciones de los términos principales de las obras de Plaza, y de Campaña*. Barcelona: Tomás Piferrer.
- Lluis i Ginovart, Josep. 2015. La mecánica ilustrada en los ingenieros militares españoles. El proyecto de los almacenes de pólvora (1715-1798). *Informes de la Construcción* 67: 539, e103. <https://doi.org/10.3989/ic.14.109>
- Lluis i Ginovart, Josep. et al. 2017 a. Layout of Catenary Arches in the Spanish Enlightenment and Modernism. *Nexus Network Journal* 19: 85-99. <https://doi.org/10.1007/s00004-016-0313-9>
- Lluis i Ginovart, Josep. et al. 2017 b. Hooke's Chain Theory and the Construction of Catenary Arches in Spain. *International Journal of Architectural Heritage Conservation, Analysis, and Restoration* 11.5: 703-716. <https://doi.org/10.1080/15583058.2017.1290851>
- Ms. 457 (c. 1598- 1608). Biblioteca Central Militar, Madrid, Ing. 6-12-14. <https://bibliotecavirtual.defensa.gob.es/BVMDefensa/i18n/consulta/registro.cmd?id=4530>
- Muller, John. 1755. *A treatise containing the elementary part of fortification, regular and irregular ... For the use of the Royal academy of artillery at Woolwich. Illustrated with thirty-four copper plates*. London: Printed for A. Millar.
- Navascués, Pedro. 1996. *Tratados de arquitectura y fortificación en la antigua biblioteca del Alcázar*. In 'XII Día del Alcázar', 7 de Julio de 1995, Segovia, pp. 17-44.

- Pinto-Puerto, Francisco and Alfonso Jiménez-Martín. 2016. Geometric Working Drawing of a Gothic Tierceron Vault in Seville Cathedral. *Nexus Network Journal* 18: 439-466. <https://doi.org/10.1007/s00004-015-0271-7>
- Rojas, Cristóbal de. 1598. *Teorica y practica de fortificacion, conforme las medidas y defensas destos tiempos, repartida en tres partes*. Madrid: Luis Sanchez.
- Saalman, Howard. 1959. Giovanni di Gherardo da Prato's Designs concerning the Cupola of Santa Maria del Fiore in Florence. *Journal of the Society of Architectural Historians* 28: 11-20. <https://doi.org/10.2307/987890>
- Sánchez, Miguel. 1769. *Tratado de fortificación, ó Arte de construir los edificios militares, y civiles, escrito en inglés por Juan Muller; traducido en castellano, dividido en dos tomos, y aumentado con notas, adiciones y 22 láminas finas sobre 26, que ilustran al original, por Miguel Sánchez Taramas; tomo primero*. Barcelona: Thomas Piferrer.
- Segovia, Francisco. 2004. Els fons bibliogràfics de l'Acadèmia de Matemàtiques. En: *L'Acadèmia de Matemàtiques de Barcelona. El llegat dels enginyers militars*. Muñoz, J. M. (cor). Madrid: Ministerio de Defensa.
- Tosca, Tomás Vicente. 1707. *Compendio mathematico. Tomo I: Que comprehende Geometria elementar, arithmetica inferior, geometria practica*. Valencia: Antonio Bordazar.
- Tosca, Tomás Vicente. 1712. *Compendio mathematico. Tomo V, Que comprehende Arquitectura civil, montea, y canteria, arquitectura militar, pirotechnia, y artillería*. Valencia: Antonio Bordazar.