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Coordinated and decentralized trading of flexibility products in Inter-DSO Local Electricity Markets via ADMM

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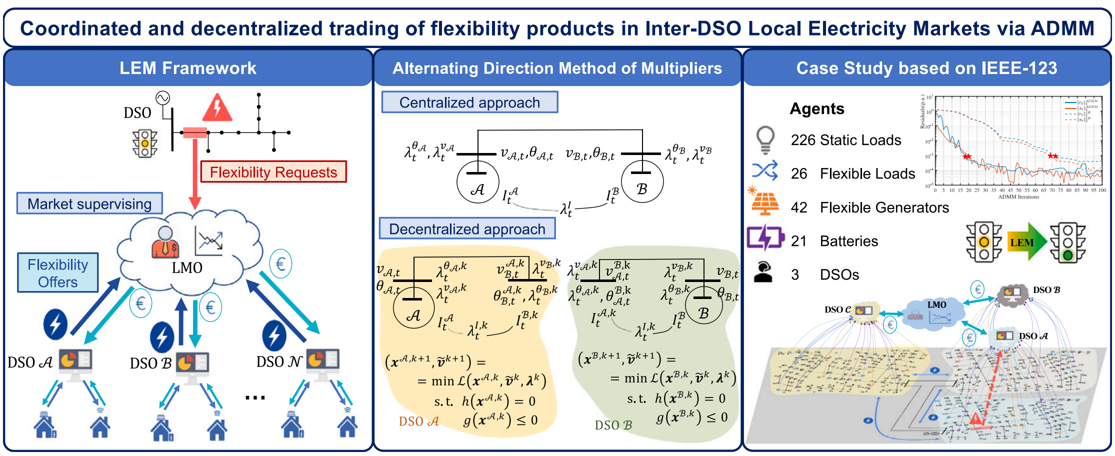
GRAPHICAL ABSTRACT



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ABSTRACT

Local Electricity Markets (LEMs) arise as new layers of current market designs, enabling local trading of flexibility products. In many cases, LEMs encompass several networks, involving different Distribution System Operators (DSOs). This setting raises intrinsic concerns regarding the information privacy of the involved DSOs while trying to achieve system-wide efficiency. In this paper, a market design for the trading of flexibility products in inter-DSO LEMs is proposed where the DSOs are allowed to trade among the areas under the coordination of a Local Market Operator (LMO). A coordinated and decentralized approach based on the Alternating Direction Multiplier Method is proposed. The novel aspect of this work is that each DSO self-schedules its own assets in response to market signals (optimal dual variables) to fulfil the flexibility requests from itself or from other DSOs. An illustrative case study based on the IEEE 123 bus test systems is used for testing the proposed framework.

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Nomenclature

Parameters are in upper case letter and variables in lower case letter. $|\Omega|$ denotes the cardinality of set Ω.

Acronyms

ADMM	Alternating Direction Method of Multipliers
BESS	Battery Energy Storage System
DSO	Distribution System Operator
FG	Flexible Generator
FL	Flexible Load
LEM	Local Electricity Market
LMO	Local Market Operator
LR	Lagrangian Relaxation
SL	Static Load
SOC	State of Charge

Indices and sets

$l,\ \Omega_l$	Index and set for SLs, $l \in \Omega_l$.
$f,\ \Omega_f$	Index and set for FLs, $f \in \Omega_f$.
$g,\ \Omega_g$	Index and set for FGs, $g \in \Omega_g$.
$s,\ \Omega_s$	Index and set for BESSs, $s \in \Omega_s$.
$t,\ \Omega_t$	Index and set for time periods, $t \in \Omega_t$.
$i,\ \Omega_i$	Index and set for buses, $i, j \in \Omega_i$.
$(i,j),\ \Omega_b$	Index and set for branches, $(i,j) \in \Omega_b$.
$e,\ \Omega_e$	Index and set for scenarios, $e \in \Omega_e$.
$p,\ \Omega_p$	Index and set for DSOs.
Λ^p	Set for interconnecting buses of the DSO p.
k	Iteration counter.

Parameters

$B_{i,j}$	Susceptance of the line i,j (S).
$G_{i,j}$	Conductance of the line i,j (S).
Δt	Time interval duration (h).
$S^u_{f,t},\ S^d_{f,t}$	Price of the upward u and downward d energy flexibility product provided by FL f at period t (€/kWh).
$S^u_{g,t},\ S^d_{g,t}$	Price of the upward u and downward d energy flexibility product provided by FG g at period t (€/kWh).
$S^u_{s,t},\ S^d_{s,t}$	Price of the upward u and downward d energy flexibility product provided by BESS s at period t (€/kWh).
S_t	Price of energy in the wholesale market at period t (€/kWh).
$\eta^C_s,\ \eta^D_s$	Charging C and discharging D efficiencies for the BESS s (p.u.).
$P^{sch}_{l,t},\ P^{sch}_{f,t}$	Scheduled sch demand for SL l and FL f at period t, respectively (kW).
$P^{sch}_{g,t}$	Scheduled sch generation for FG g at period t (kW).
$P^{am}_{l,t}$	Power demand after market am clearing for SL l at period t (kW).
$Q^{sch}_{l,t}$	Scheduled sch reactive demand for SL l at period t (kVAr).
$Q^{sch}_{g,t}$	Scheduled sch reactive generation for FG g at period t (kVAr).
$\overline{P}_f,\ \underline{P}_f$	Upper and lower demand bounds for FL f at period t (kW).
$\overline{P}_g,\ \underline{P}_g$	Upper and lower generation bounds for FG g at period t (kW).
P^{conv}_s	BESS converter power rating $conv$ for BESS s (kW).
$\overline{SOC}_s,\ \underline{SOC}_s$	Upper and lower bounds for SOC of BESS s (kWh).
$SOC0_s$	Initial SOC for BESS s (kWh).
$\overline{S}_{i,j}$	Thermal limit of the line i,j (kVAr).
γ	Penalty factor (p.u.).
ε	Convergence tolerance (p.u.).

Variables

$\omega^u_{f,t},\ \omega^d_{f,t}$	Upward u and downward d flexibility product from FL f at period t (kWh).
$\omega^u_{g,t},\ \omega^d_{g,t}$	Upward u and downward d flexibility product from FG g at period t (kWh).
$\omega^u_{s,t},\ \omega^d_{s,t}$	Upward u and downward d flexibility product from BESS s at period t (kWh).
$p^{am}_{f,t},\ p^{am}_{s,t}$	Power demand of FL f and BESS s, in period t after market clearing am, respectively (kW).
$p^{am}_{g,t}$	Power generation of FG g at period t after market am clearing (kW).
$soc_{s,t}$	State of charge of BESS s at period t (kWh).
$\theta_{i,t}$	Voltage phase angle in bus i at period t (rad).
$v_{i,t}$	Voltage magnitude in bus i at period t (pu).
$p_{i,j,t}, q_{i,j,t}$	Active and reactive power flow between buses i and j at period t (kW, kVAr).
$p_{i,t}$	Active power exchanged at the point of common coupling i at period t (kW).
$q_{i,t}$	Reactive power exchanged at the point of common coupling i at period t (kVAr).
$c_{f,t},\ c_{g,t},\ c_{s,t}$	Cost of the flexibility products traded by FL f, FG g and BESS s, respectively, at period t (€).
$\tilde{m}^{\theta,k}_t$	Complicating constraint associated with the voltage phase angle at the interconnection θ at period t in iteration k (rad).
$\tilde{m}^{I,k}_t$	Complicating constraint associated with balance I at period t in iteration k (kW).
x^p	General variable x of DSO p.
$\lambda^{\theta,k}_t$	Lagrange multiplier associated with the voltage phase angle θ constraint at the interconnection in iteration k and period t (€/rad).
$\lambda^{v,k}_t$	Lagrange multiplier associated with the voltage magnitude v constraint at the interconnection in iteration k and period t (€/rad).
$\lambda^{I,k}_t$	Lagrange multiplier associated with balance constraint I in iteration k and period t (€/kWh).

1. Introduction

1.1. Motivation

Renewable energy sources operating at distribution grids are becoming increasingly common in modern smart grids. In this context, LEMs are being deployed as new layers of the traditional electricity markets [1], providing new business models for the renewable agents that in many cases did not have opportunities to participate in the conventional wholesale markets. Promoting LEMs among distribution networks is a strategic focus of global energy policies. Some examples can be found in the EU-funded pilot projects FLEXITRANSTORE [2] InterFlex [3], EMPOWER [4], and DREAM-GO [5].

The behaviour of market agents can lead to distribution line congestion [6], voltage magnitude deviations [7], or even power imbalances [8]. The adoption of LEMs allows the resolution of the distribution operational constraints through local agents connected to the same grid without involving upstream assets.

Trading of energy among LEMs to provide flexibility is an important challenge for modern distribution networks [9]. Interoperability of these markets offers a route to overcome this barrier [10] and motivates the study of inter-DSO local market structures. This setting raises intrinsic concerns regarding the information privacy of the involved -DSO in the effort to achieve system-wide efficiency. Therefore, it is necessary to develop market designs for trading the flexibility products in inter-DSO LEMs where -DSOs are allowed to trade among areas.

1.2. Literature review

Several market structures have been studied for LEMs, including different combinations of agents. References [11] use the Aggregator as the central entity of the market, offering flexibility services to the DSO. Other approaches assume that the DSO is responsible for the energy balance of the market as well as for the grid operation [12–14]. LEMs can also be cleared on an auction basis; in this type of clearing, an auctioneer is responsible for the matching process between the asks and bids [15]. In Ref. [16], a transactive energy operator operates aggregated fleets of electric vehicles while providing peak-shaving services to the DSO. Traditional market structures have also been considered by [17] where energy from hydroelectric systems is traded in decentralized manner following a hierarchical structure. Additionally, another branch of research which considers LEMs without governance, i.e. with no market operator. A bi-level peer-to-peer-to-grid market is proposed in [18], where the market is fully distributed among the participating agents. [19] investigates a methodology where the market is jointly hosted by community managers and peers. Blockchain technology was used in [20] to host a market for energy trading among households. However, this feature may have profound implications for the transparency of the market. This work considers the DSO as the entity that receives offers from flexible agents within its network, but does not act as the market operator. The LEM is hosted by an independent Local Market Operator (LMO) to promote transparency, as in [21].

In many cases, LEMs encompass several geographically restricted jurisdictions, such as energy communities, neighbourhoods, districts, or towns [4]. Thus, several DSOs can be involved in the participation of LEMs. This type of participation in pioneering flexibility market projects has been discussed in [22]. [23], the relevance of this type of coordination for congestion and imbalance management is assessed when organizing flexibility markets. DSOs cooperation is also highly important for flexibility providers operating assets across borders [24]. However, currently there is a gap in the knowledge about how such cross-border coordination can be facilitated. Only [25, 26] proposed a methodology to coordinate flexible power units using converted-coupled units that consider costs and potential flexibility procurement.

inter-DSO coordination raises considerable concerns regarding the information privacy of the involved agents while trying to achieve system-wide cost-efficient solutions [27]. Privacy can be preserved using decentralized market protocols, e.g. peer-to-peer [18], agent-centric protocols [28], or hybrid approaches [29]. Nevertheless, little research has been conducted on the protection of privacy when several DSOs are cooperating in local market structures. The coordination of these DSOs while preserving confidential participant and network data is a noteworthy feature, which is one of the contributions of this paper. This paper offers a decentralized and coordinated market-clearing procedure for flexibility trading in inter-DSO LEMs, achieving cost-effective system-wide solutions without compromising the market agents' data privacy.

Coordinated and decentralized solutions have been previously proposed. Coordination is attained by multi-bilateral trades between agents in peer-to-peer markets. The electrical distance of the peers in a distribution network drives the preference lists for coordinating a market solution in a peer-to-peer methodology [13]. [20] used an ant-colony optimization algorithm to coordinate the solution among peers. Nevertheless, since the algorithm is based on heuristics, this type of coordination may yield several solutions for the same input data. Consensus is used in [29] to coordinate a hybrid method for community formation and energy trading among peers, but this method may suffer from convergence difficulties when seeking to coordinate system-wide market properties. Grid usage prices are defined by the DSO in [30] to enforce network constraints in a peer-to-peer market. However, this approach does not guarantee the optimal solution of the market, hindering its efficient operation. Following an alternative research direction, several studies coordinate the solution of the market based on market signals. For example, [27] used only the variables associated with the energy transfer among peers. Other researchers also included market signals associated with the grid variables [27]. Following this approach, this paper proposes a coordination methodology based on market signals that can be applied to inter-DSO LEM and ensures that an optimal solution will be achieved.

In the above-discussed works, several solution approaches have been proposed. In particular, Lagrangian relaxation has been applied to decentralize market-clearing procedures in distribution grids [17,28]. Nevertheless, the ADMM algorithm [31] improves the Lagrangian relaxation decentralization process due to its convergence properties. Multiples methodologies have been proposed to decentralize the market-clearing using this algorithm atthe distribution level [18,30]. However, little research has been performed regarding the coordination among the physically adjacent DSOs operating in the same LEMs.

A summary of the above literature review is shown in Table 1. Additionally, the limitations of the current approaches are described as follows:

- L1. Coordination among DSOs when trading flexibility products for congestion and imbalance management has not been sufficiently addressed.
- L2. Most of the current market-clearing methodologies have limited privacy protection of the participating users, and the decentralized methodologies that do provide such protection may obtain to suboptimal solutions.
- L3. Little attention has been paid to coordination methodologies for the trading of flexibility products based on market signals while ensuring optimal market solution.

1.3. Contributions

The main contribution of this paper is a decentralized and coordinated approach for flexibility products trading in inter-DSO LEMs. The specific contributions of this paper are as follows:

- C1. An operational framework that can handle flexibility needs an inter-DSO LEM environment while preserving market privacy information.

Table 1
Summary of the literature review.

	DSO–DSO coord.	Type of products	Decentr. market	Market host	Privacy preserv.	Modelling approach	Opt. granted	Grid	Math. model
[4]	No	Flexibility	No	Aggregator	No	Optimization	Yes	None	MILP
[11]	No	Capacity	No	LMO	No	Optimization	Yes	Linear DistFlow	LP
[12]	No	Flexibility	No	DSO	No	Optimization	Yes	AC	LP
[13]	No	Energy	Yes	None	Yes	Peer-to-peer	No	None	–
[14]	No	Energy	No	DSO	No	Optimization	Yes	None	MILP
[15]	No	Energy	Yes	Auctioneer	Yes	Auction model	No	DC	–
[16]	No	Flexibility	No	Transactive Energy Operator	No	Optimization	Yes	DC	LP
[17]	No	Energy	Yes	Market operator	Yes	Lagrangian Relaxation	Yes	DC	MILP
[18]	No	Energy	Yes	None	Yes	Peer-to-peer	No	DistFlow	LP
[19]	No	Energy	Yes	None	Yes	Peer-to-peer	No	DC	QCP
[20]	No	Energy	Yes	None	Yes	Peer-to-peer	No	None	MINLP
[21]	No	Energy, Reserve	No	LMO	No	Optimization	Yes	None	MILP
[26]	Yes	Flexibility	No	–	No	Converter-coupled units	Yes	None	MILP
[27]	No	Energy	Yes	None	Yes	ADMM	Yes	DC	QCP
[28]	No	Energy	Yes	LMO	Yes	ADMM	Yes	None	QCP
[29]	No	Energy	Yes	None	Yes	Hybrid (Opt. and P2P)	No	PTDF	MILP
[30]	No	Energy	Yes	None	Yes	Peer-to-peer	No	DistFlow	MINLP
This paper	Yes	Flexibility	Yes	LMO	Yes	ADMM	Yes	LinearL AC	QCP

C2. A decentralized solution approach based on the ADMM where optimal dual variables associated with the voltage magnitude and phase angle at the interconnecting DSOs nodes and system-wide imbalance equations, are the signals that properly drive the coordination mechanism to achieve system-wide efficiency.

1.4. Paper organization

The remainder of this paper is organized as follows. Section 2 defines and formulates the flexibility product-trading framework for the LEM considering the distribution network constraints. Further, Section 3 provides a decentralized solution approach to the inter-DSO LEM clearing problem. Section 4 illustrates the proposed approach using the IEEE 123 bus test system. Finally, Section 5 states the main conclusions of this work.

2. Local energy market-clearing problem

This section describes the elements of the proposed LEM model. The market consists of several DSOs, demanding flexibility and different agents offering their products. The market mitigates the congestion and corrects balance deviations using a linear model of the distribution network. Moreover, the LEM is designed to be compatible with upward market-clearing solutions, such as those from wholesale markets. Power exchanged with upstream grid remains constant after the LEM market-clearing procedure. The centralized market clearing is based on [32–34].

2.1. LEM Architecture

Fig. 1 presents the interactions among the different actors involved in the operation of LEMs. The LMO runs the market clearing for flexibility products, receiving asks and bids from DSOs. Each DSO manages the bids from the agents connected to its respective distribution network. In this paper, we assume that three different agents provide flexibility, namely FLs, FGs, and BESSs. It is also assumed that a unique flexibility product is traded regardless of its origin.

More detailed information regarding the interactions between different agents is presented in Fig. 2 using a sequence diagram, where a visual representation of the communication protocols is shown. LEM is cleared on a near real-time basis, with a time frame of 15 min, assuming forecast values for wholesale market prices and demands.

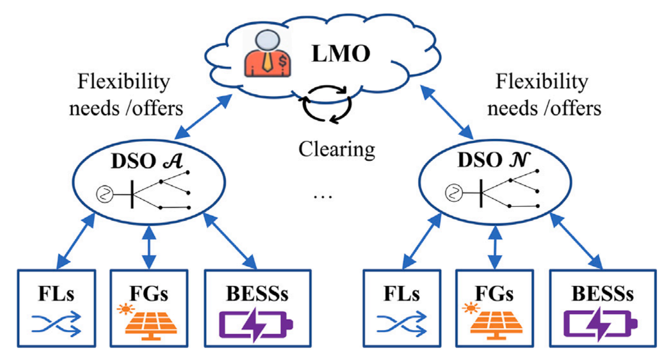

Fig. 1. General diagram of the relationships between the stakeholders of the LEM formulation.

This market is cleared whenever an event occurs in the operation of the distribution network. The market-clearing results in the optimal scheduling of flexible assets for next 24 h. If new congestions or imbalances appear as a result of forecast inaccuracies, the market clearing will be cleared again while incorporating the most updated information available.

The clearing process is as follows. First, DSOs predict their flexibility needs [35] and, if required, they ask the LMO to organize a LEM for providing flexibility. Each agent submits their flexibility availability to its respective DSO that interacts with the LMO submitting their bids. After collecting asks and bids, an iterative market-clearing protocol starts. Each DSO schedules the resources of its distribution network based on the costs and dual variables set by the LMO. After each iteration, coupling variables at the interface are shared with the LMO that updates the market signals. This process is repeated until convergence. Then, LMO sends accepted requests and matched bids to DSOs that finally inform the participating agents. As a result, agents schedule their internal control signals to deliver the flexibility products under surveillance of the LMO and DSO. Finally, the DSO leaves a delivery note of the products traded to the agents.

So far, it has been assumed that the optimal scheduling of the flexible resources is performed by the DSO. Nevertheless, the above-described protocol is compatible with the participation of flexibility aggregators and private entities. The only difference in the protocol in

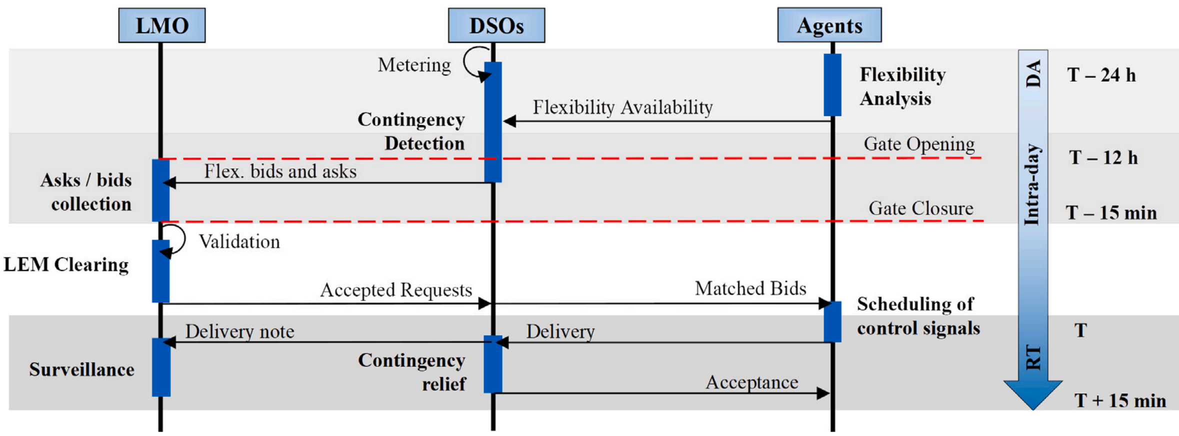

Fig. 2. Sequence diagram of the LEM representing the interactions between the agents in the proposed framework [32–34].

this case would be that the control signals will be determined by either the aggregator or the private owner.

2.2. Problem formulation

The LEM market-clearing problem is formulated in terms of the flexibility products offered, the market restrictions, the distribution network model and the objective function.

2.2.1. Flexibility product definition

In this section, the flexibility products offered by FLs, FGs and BESSs are defined. It is assumed that the energy traded by those flexible units will be collected in the wholesale market. Thus, the price of the energy should be considered in the costs functions to determine the real cost of obtaining flexibility. Then, the baseline of the flexible units or scheduled power arises from their participation in the wholesale market. For simplicity, this baseline is considered to be constant.

FLs can offer the flexibility product in two directions, either increasing or decreasing their scheduled consumption $P^{sch}_{f,t}$, because they are assumed to modify their demand within a given range. The associated costs for the provision of flexibility products are presented in (1). The power demand of the FL f after the LEM clearing, at period t, $p^{am}_{f,t}$, is bounded by $\overline{P}_f$, $\underline{P}_f$ and is defined in (2).

$$c_{f,t} = (S_t - S^u_{f,t})\omega^u_{f,t} + (S^d_{f,t} - S_t)\omega^d_{f,t} \quad \forall f \in \Omega_f, \forall t \in \Omega_t \tag{1}$$

$$p^{am}_{f,t} = P^{sch}_{f,t} + \frac{1}{\Delta t}(\omega^u_{f,t} - \omega^d_{f,t}) \quad \forall f \in \Omega_f, \forall t \in \Omega_t \tag{2}$$

To obtain profits, the upward flexibility product, $\omega^u_{f,t}$, is offered at a price, $S^u_{f,t}$, that is lower than the wholesale market price S_t. On the other hand, for the same reason the downward flexibility product, $\omega^d_{f,t}$, is offered at a price, $S^d_{f,t}$, that is higher than the previously settled wholesale market price.

FGs are assumed to be controllable in both directions (e.g. co-generation plants or PV with storage), providing upward and downward flexibility. Considering $P^{sch}_{g,t}$ as the scheduled power generation of FG g, at time t, the asset can provide upward, $\omega^u_{g,t}$, and downward, $\omega^d_{g,t}$, flexibility products at prices $S^u_{g,t} > S_t$ and $S^d_{g,t} < S_t$, respectively. Thus, the output power after market clearing must satisfy $\underline{P}_{g,t} \leq p^{am}_{g,t} \leq \overline{P}_{g,t}$ for all time periods $t \in \Omega_t$. Then, the cost of the flexibility product is defined in (3) and the power generation output after LEM clearing in (4).

$$c_{g,t} = (S^u_{g,t} - S_t)\omega^u_{g,t} + (S_t - S^d_{g,t})\omega^d_{g,t} \quad \forall g \in \Omega_g, \forall t \in \Omega_t \tag{3}$$

$$p^{am}_{g,t} = P^{sch}_{g,t} + \frac{1}{\Delta t}(\omega^u_{g,t} - \omega^d_{g,t}) \quad \forall g \in \Omega_g, \forall t \in \Omega_t \tag{4}$$

Lastly, BESSs constraints are described by (5)-(8). These agents can shift generation or consumption from one period to another. In this case, no baseline is considered for clarity. The agents are assumed to only participate in the proposed market clearing. BESSs provide both upward, $\omega^u_{s,t}$, and downward, $\omega^d_{s,t}$, flexibility, offered at the prices $S^u_{s,t} < S_t$ and $S^d_{s,t} > S_t$, respectively. The costs for BESS products are given by

$$c_{s,t} = (S_t - S^u_{s,t})\omega^u_{s,t} + (S^d_{s,t} - S_t)\omega^d_{s,t} \quad \forall s \in \Omega_s, \forall t \in \Omega_t \tag{5}$$

Operation limits for the BESSs are set by the battery converter rating P^{conv}_s and the State of Charge (SOC) bounds. Let η^C_s and η^D_s be the charging and discharging efficiencies, then equation (6) sets the SOC trajectories after LEM clearing while (7) establishes the SOC bounds. For realistic modelling, it is considered that $soc_{s,0} = soc_{s,|\Omega_t|} = SOC0_s$. The power of the BESS s after market clearing is computed in (8) considering that if $p^{am}_{s,t} \leq 0$, BESS is being discharged, else if $p^{am}_{s,t} \geq 0$, BESS is charged. This power is limited by the converter rating P^{conv}_s, setting the upper and lower bounds for $p^{am}_{s,t}$ according to (9).

$$soc_{s,t} = soc_{s,t-1} + \eta^C_s \omega^u_{s,t} - \frac{\omega^d_{s,t}}{\eta^D_s} \quad \forall s \in \Omega_s, \forall t \in \Omega_t \tag{6}$$

$$\underline{SOC}_s \leq soc_{s,t} \leq \overline{SOC}_s \quad \forall s \in \Omega_s, \forall t \in \Omega_t \tag{7}$$

$$p^{am}_{s,t} = \frac{1}{\Delta t}(\omega^u_{s,t} - \omega^d_{s,t}) \quad \forall s \in \Omega_s, \forall t \in \Omega_t \tag{8}$$

$$-P^{conv}_s \leq p^{am}_{s,t} \leq P^{conv}_s \quad \forall s \in \Omega_s, \forall t \in \Omega_t \tag{9}$$

2.2.2. LEM Balancing Constraint

LEMs are organized to take advantage of local assets to fulfil local flexibility needs. The power exchanged with the upstream network $p_{i,t}$ does not change after market operation, alleviating congestion by a physical translation of the consumption from the high-load areas to other areas where thermal restrictions are not activated. Thus, the balance for all flexible upwards and downwards products must be zero as described by (10). This equation guarantees that the clearing results do not affect to the upstream networks, ensuring the compatibility of the proposed framework with the current market structures. Associated with this equation, the dual variable λ^I_t represents the marginal cost of the traded flexibility products.

$$\sum_{l \in \Omega_l}(P^{sch}_{l,t} - p^{am}_{l,t}) + \sum_{f \in \Omega_f}(P^{sch}_{f,t} - p^{am}_{f,t}) - \sum_{g \in \Omega_g}(P^{sch}_{g,t} - p^{am}_{g,t}) - \sum_{s \in \Omega_s} p^{am}_{s,t} = 0 \quad \forall t \in \Omega_t \quad : \lambda^I_t \tag{10}$$

2.2.3. Network model

We consider a linear version of the AC network model to characterize the power balance and thermal limits [36]. Linear models of the grid are widely used in market studies [16,19,37] because such models eliminate the duality gaps that appear when using non-linear

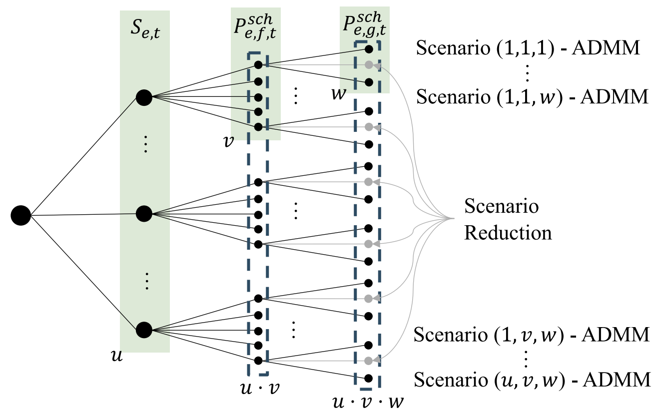

Fig. 3. Scenario tree of the stochastic version of the Inter-DSO LEM with scenario reduction.

definitions. Thus, a robust economic interpretation of the dual variables can be provided. The power balance for all buses is given by (11) and (12). Active and reactive power flows are computed using (13) and (14). Eq. (15) sets the power flow limits $\overline{S}_{i,j}$.

$$\sum_{j\in\Omega_i}\left(G_{i,j}v_{j,t}-B_{i,j}\theta_{j,t}\right)=p_{i,t}+p^{am}_{g,t}-P^{am}_{l,t}-p^{am}_{f,t}-p^{am}_{s,t}\quad \forall i\in\Omega_i,\forall t\in\Omega_t \tag{11}$$

$$-\sum_{j\in\Omega_i}\left(B_{i,j}v_{j,t}+G_{i,j}\theta_{j,t}\right)=q_{i,t}+Q^{am}_{g,t}-Q^{am}_{l,t}\quad \forall i\in\Omega_i,\forall t\in\Omega_t \tag{12}$$

$$p_{i,j,t}=G_{i,j}(v_{i,t}-v_{j,t})-B_{i,j}(\theta_{i,t}-\theta_{j,t})\quad \forall(i,j)\in\Omega_b,\forall t\in\Omega_t \tag{13}$$

$$q_{i,j,t}=B_{i,j}(v_{j,t}-v_{i,t})+G_{i,j}(\theta_{j,t}-\theta_{i,t})\quad \forall(i,j)\in\Omega_b,\forall t\in\Omega_t \tag{14}$$

$$p^2_{i,j,t}+q^2_{i,j,t}\le\overline{S}^2_{i,j}\quad \forall(i,j)\in\Omega_b,\forall t\in\Omega_t \tag{15}$$

2.2.4. Objective function

The LEM clearing problem objective function minimizes the total cost for the products offered by FLs, FGs and BESSs.

$$\min\sum_{t\in\Omega_t}\left[\sum_{f\in\Omega_f}c_{f,t}+\sum_{g\in\Omega_g}c_{g,t}+\sum_{s\in\Omega_s}c_{s,t}\right] \tag{16}$$

2.3. Stochastic formulation

In this section, the stochasticity of the generation, the loads and the wholesale market prices is discussed. Loads and prices in distribution networks have an nondeterministic nature, which may influence the solution of the problem. Nevertheless, although DSOs have powerful prediction tools which are able to predict these variables, they may introduce an error in the parameters of the problem.

To characterize this behaviour, we assume that the probability density distribution of the forecast error of the loads and the prices is a normal distribution as in [38] with a standard deviation of 25% and mean value equal to the forecasted values. A scenario tree based approach is used to incorporate the uncertainty into the model as Fig. 3 shows. In order to avoid computational burden, a scenario reduction technique is used to include the most representative scenarios in the stochastic formulation [39]. Each scenario is simulated using the proposed ADMM algorithm.

Let $\hat{X}$ be the mean of the parameter X and $\Delta\tilde{e}^X_e$ be the forecast error in the scenario e for the parameter X. The scheduled demand $P^{sch}_{e,f,t}$, the scheduled generation $P^{sch}_{e,g,t}$, and the wholesale market price $S_{e,t}$ in the scenario e are described as follows

$$P^{sch}_{e,f,t}=\hat{P}^{sch}_{f,t}+\Delta\tilde{e}^f_e\quad \forall e\in\Omega_e,\forall f\in\Omega_f,\forall t\in\Omega_t \tag{17}$$

$$P^{sch}_{e,g,t}=\hat{P}^{sch}_{g,t}+\Delta\tilde{e}^g_e\quad \forall e\in\Omega_e,\forall g\in\Omega_g,\forall t\in\Omega_t \tag{18}$$

$$S_{e,t}=\hat{S}_{e,t}+\Delta\tilde{e}^S_e\quad \forall e\in\Omega_e,\forall t\in\Omega_t \tag{19}$$

The expected value of the cost is minimized in the objective function for all scenario e in the set of possible realizations Ω_e. Let $\mathcal{P}_e$ be the probability of the scenario e, then, the market formulation stands as follows,

$$\min\sum_{e\in\Omega_e}\mathcal{P}_e\left[\sum_{t\in\Omega_t}\left[\sum_{f\in\Omega_f}c_{e,f,t}+\sum_{g\in\Omega_g}c_{e,g,t}+\sum_{s\in\Omega_s}c_{e,s,t}\right]\right] \tag{20}$$
$$\text{s.t. (1)–(15)}\quad \forall e\in\Omega_e$$

The output variables of (20) are now variables with uncertainty with a determined probability density function.

2.4. Inter-DSO Local Energy Markets

Considering that LEMs are composed of a set of $|\Omega_p|$ DSOs, interconnected by tie-lines, the objective function is re-written as the cost minimization of the flexibility products offered by each independent DSO as given by (21).

$$\min\sum_{p\in\Omega_p}\sum_{t\in\Omega_t}\left[\sum_{f\in\Omega_f}c^p_{f,t}+\sum_{g\in\Omega_g}c^p_{g,t}+\sum_{s\in\Omega_s}c^p_{s,t}\right] \tag{21}$$

The balancing constraint (10) is then re-defined as follows,

$$\sum_{p\in\Omega_p}I^p_t=0\quad \forall t\in\Omega_t\ :\lambda^I_t \tag{22}$$

$$I^p_t=\sum_{l\in\Omega^p_l}(P^{sch}_{l,t}-P^{am}_{l,t})+\sum_{f\in\Omega^p_g}(P^{sch}_{f,t}-p^{am}_{f,t})-\sum_{g\in\Omega^p_g}(P^{sch}_{g,t}-p^{am}_{g,t})-\sum_{s\in\Omega^p_s}p^{am}_{s,t}\quad \forall t\in\Omega_t,\forall p\in\Omega_p \tag{23}$$

where I^p_t is the imbalance of the DSO p at period t as defined in (23). It is important to note that I^p_t is computed only considering the assets of DSO p. Node balance equations are also affected by this inter-DSO setting. Let Ω^p_i be the set of nodes belonging to the DSO p, and Λ_p be the set of interconnecting nodes. Then, the node balance is given by

$$\begin{aligned}&\sum_{j\in\Omega^p_i}\left(G_{i,j}v_{j,t}-B_{i,j}\theta_{j,t}\right)+\sum_{j\in\Lambda^p}\left(G_{i,j}v_{j,t}-B_{i,j}\theta_{j,t}\right)=p_{i,t}\\&-\sum_{j\in\Omega^p_i}\left(B_{i,j}v_{j,t}+G_{i,j}\theta_{j,t}\right)-\sum_{j\in\Lambda^p}\left(B_{i,j}v_{j,t}+G_{i,j}\theta_{j,t}\right)=q_{i,t}\\&\forall i\in\Omega^p_i,\forall p\in\Omega_p,\forall t\in\Omega_t\end{aligned} \tag{24}$$

The inter-DSO LEM clearing problem can be formulated in compact form. Let $\mathbf{x}^p$ be the vector of the variables of the DSO p, $\tilde{\mathbf{v}}$ be the vector of the complicating variables, i.e. variables that belongs to different DSOs, λ be the vector of all Lagrange multipliers at complicating constraints. The inter-DSO LEM is re-written in (25) considering the cost function $f^p(\mathbf{x}^p,\tilde{\mathbf{v}})$, subject to the DSO constraints $h(\mathbf{x}^p)=0$, $g(\mathbf{x}^p\le 0)$ and the complicating constraints $\tilde{m}(\mathbf{x}^{\mathcal{A}},\mathbf{x}^{\mathcal{B}},\dots,\mathbf{x}^{|\Omega_p|},\tilde{\mathbf{v}})=0$.

$$\begin{aligned}&\min\sum_{p\in\Omega_p}f^p(\mathbf{x}^p,\tilde{\mathbf{v}})\\&\text{subject to}\quad h(\mathbf{x}^p)=0,\quad g(\mathbf{x}^p)\le 0\\&\qquad\tilde{m}(\mathbf{x}^{\mathcal{A}},\mathbf{x}^{\mathcal{B}},\dots,\mathbf{x}^{|\Omega_p|},\tilde{\mathbf{v}})=0\ :\lambda\end{aligned} \tag{25}$$

3. Solution approach to Inter-DSO LEMs

In this section, the solution approach to the inter-DSO LEM clearing problem based on the ADMM is described. The decentralized coordination mechanism among the areas is activated when there is a request for flexibility. The market-clearing problem is iteratively solved where information is exchanged between the LMO and the involved DSOs. In this setting, information exchanges are reduced, attaining the same centralized solution sharing a reduced quantity of information.

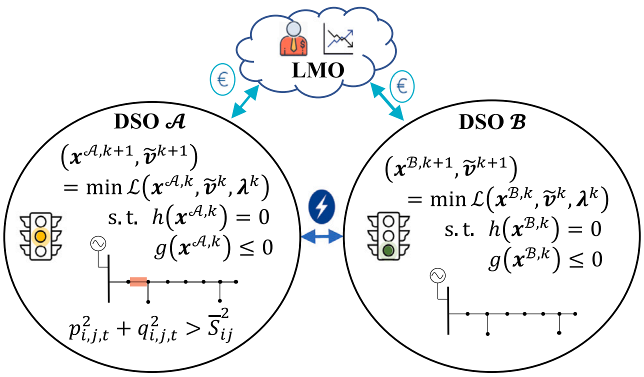

Fig. 4. Coordination procedure among the agents in inter-DSO LEM settings. Dark and light blue arrows represents flexibility requests and economical signals, respectively.

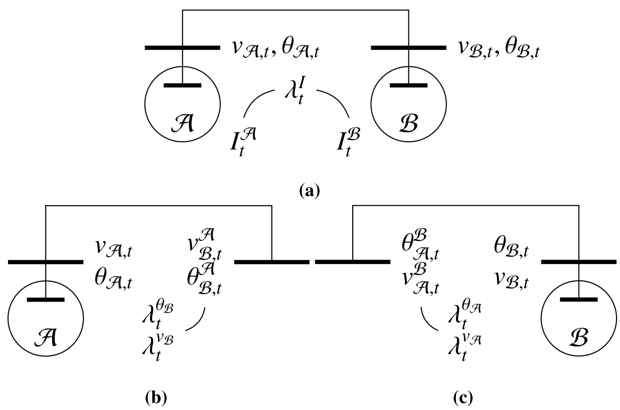

Fig. 5. Representation of the coordination procedure between two different areas connected by a tie-line.

3.1. Coordination scheme

The coordination scheme is presented in Fig. 4, where the LMO acts as coordinator of the LEM clearing solution with the information provided by DSOs. Imbalance information is exchanged among DSOs to fulfil the flexibility requests without involving the upstream assets. Additionally, economic signals are also exchanged representing their offers to satisfy these flexibility requests.

The objective of this approach is to find the overall optimal solution in a decentralized and coordinated manner. To achieve this goal, the coupling constraints (22) and (24) are coordinated. Information exchanges include the primal and dual variables at the interconnecting nodes ($v_{i,t}, \lambda_t^v, \theta_{i,t}, \lambda_t^\theta$) and flexibility products (I_t^p, λ_t^I). The first and second magnitudes are widely used in the literature as a method for coordination [30,40]. Furthermore, this inter-DSO LEM algorithm also considers area imbalances in the coordination procedure, using the Lagrange multiplier λ_t^I as the marginal cost for flexibility provision.

3.1.1. Coordination through the voltage magnitude and phase angle at interconnection

For readability, coordination is explained considering a LEM with two interconnected DSOs as shown in Fig. 5(a). In this case, coordination is achieved by duplicating the DSO's node voltage magnitude and voltage phase angle $v_{\mathcal{A},t} = v_{\mathcal{A},t}^{\mathcal{B}}$, $\theta_{\mathcal{A},t} = \theta_{\mathcal{A},t}^{\mathcal{B}}$ and $v_{\mathcal{B},t} = v_{\mathcal{B},t}^{\mathcal{B}}$, $\theta_{\mathcal{B},t} = \theta_{\mathcal{B},t}^{\mathcal{A}}$, at the interconnection as depicted in Figs. 5(b) and 5(c).

Those internal variables replace the complicating variables $v_{j,t}$, $\theta_{j,t}$ in (24) for all $j \in \Lambda_p$, making the problem separable. Duplicated variables are coordinated using (26) and (27), ensuring that the node voltage obtained in DSO $\mathcal{B}$ in iteration k, $v_{\mathcal{B},t}^k$, $\theta_{\mathcal{B},t}^k$, is equal to the duplicated variables $v_{\mathcal{B},t}^{\mathcal{A}}$, $\theta_{\mathcal{B},t}^{\mathcal{A}}$ of DSO $\mathcal{A}$.

$$v_{\mathcal{B},t}^{\mathcal{A}} - v_{\mathcal{B},t}^{k} = 0 \;:\; \lambda_t^{v_{\mathcal{B}},k} \tag{26}$$

$$\theta_{\mathcal{B},t}^{\mathcal{A}} - \theta_{\mathcal{B},t}^{k} = 0 \;:\; \lambda_t^{\theta_{\mathcal{B}},k} \tag{27}$$

3.1.2. Coordination through the overall balance of the grid

The overall balance for all DSOs must be met. Following Fig. 5, the sum of the imbalances from DSO $\mathcal{A}$ and $\mathcal{B}$ must be equal to zero,

$$I_t^{\mathcal{A}} + I_t^{\mathcal{B},k} = 0 \quad \forall t \in \Omega_t \quad : \lambda_t^{I,k} \tag{28}$$

Through iterative exchange of information associated with (26) – (28), the centralized solution of the problem described by (21) can be obtained without compromising the information privacy of the different DSOs and without solving a large economic dispatch.

3.2. DSO sub-problem

In this section, the DSO sub-problem is defined. At each iteration k, DSO information is shared with the LMO to coordinate the market-clearing solution. DSO $\mathcal{A}$ solves its sub-problem assuming known complicating variables of DSO $\mathcal{B}$, and complicating constraints (26)–(28) are relaxed and included in the objective function. For privacy reasons, the DSO imbalance $I_t^{\mathcal{B},k}$ is computed inside the jurisdiction of DSO $\mathcal{B}$ and then, is shared.

$$\tilde{m}_t^{v,k,\mathcal{A}} = v_{\mathcal{B},t}^{\mathcal{A}} - v_{\mathcal{B},t}^{k} \quad \forall t \in \Omega_t \tag{29}$$

$$\tilde{m}_t^{\theta,k,\mathcal{A}} = \theta_{\mathcal{B},t}^{\mathcal{A}} - \theta_{\mathcal{B},t}^{k} \quad \forall t \in \Omega_t \tag{30}$$

$$\tilde{m}_t^{I,k,\mathcal{A}} = I_t^{\mathcal{A}} + I_t^{\mathcal{B},k} \quad \forall t \in \Omega_t \tag{31}$$

The DSO problem is described by (32). Its objective function is composed of three terms. The first term involves the sum of flexible agent costs of the DSO $\mathcal{A}$. The second term relaxes the complicating constraints $\tilde{m}_t^{v,k,\mathcal{A}}$, $\tilde{m}_t^{\theta,\mathcal{A},k}$, $\tilde{m}_t^{I,\mathcal{A},k}$ multiplied by their associated dual variables $\lambda_t^{v,k}$, $\lambda_t^{\theta,k}$, $\lambda_t^{I,k}$. Lastly, a quadratic term of the sum of coupling constraints multiplied by the penalty factor γ is included. The last term is specific to the ADMM algorithm, enhancing the speed of the convergence to the optimal solution [31]. This DSO problem is subject to restrictions (1)–(15) for all assets connected to the network of the DSO.

$$\begin{aligned}\min \textstyle\sum_{t\in\Omega_t} &\left[\textstyle\sum_{f\in\Omega_f^{\mathcal{A}}} c_{f,t}^{\mathcal{A}} + \sum_{g\in\Omega_g^{\mathcal{A}}} c_{g,t}^{\mathcal{A}} + \sum_{s\in\Omega_s^{\mathcal{A}}} c_{s,t}^{\mathcal{A}} \right] + \textstyle\sum_{t\in\Omega_t} \left[\lambda_t^{v,k} \tilde{m}_t^{v,\mathcal{A},k} \right.\\ &\left. + \lambda_t^{\theta,k} \tilde{m}_t^{\theta,\mathcal{A},k} + \lambda_t^{I,k} \tilde{m}_t^{I,\mathcal{A},k} \right] + \frac{\gamma}{2} \left[\textstyle\sum_{t\in\Omega_t} \left(\tilde{m}_t^{I,\mathcal{A},k} \right)^2 \right.\\ &\left. + \textstyle\sum_{t\in\Omega_t} \left(\tilde{m}_t^{v,\mathcal{A},k} \right)^2 + \sum_{t\in\Omega_t} \left(\tilde{m}_t^{\theta,\mathcal{A},k} \right)^2 \right]\end{aligned} \tag{32}$$

subject to (1), (2) $\forall f \in \Omega_f^{\mathcal{A}}$
(3), (4) $\forall g \in \Omega_g^{\mathcal{A}}$
(5)–(8) $\forall s \in \Omega_s^{\mathcal{A}}$
(11)–(15) $\forall i,j \in \Omega_i^{\mathcal{A}}$

3.3. Lagrange's multipliers update

The LMO updates the Lagrange multipliers once all DSOs have exchanged the information resulting from the solution of (32).

$$\lambda_t^{v,k+1} = \lambda_t^{v,k} + \gamma\, \tilde{m}_t^{v,k} \quad \forall t \in \Omega_t \tag{33}$$

$$\lambda_t^{\theta,k+1} = \lambda_t^{\theta,k} + \gamma\, \tilde{m}_t^{\theta,k} \quad \forall t \in \Omega_t \tag{34}$$

$$\lambda_t^{I,k+1} = \lambda_t^{I,k} + \gamma\, \tilde{m}_t^{I,k} \quad \forall t \in \Omega_t \tag{35}$$

3.4. Convergence criterion: Primal and dual residuals

Primal residual $\|r^k\|_2$ is defined as $\left\|\tilde{m}(\mathbf{x}^{\mathcal{A}}, \mathbf{x}^{\mathcal{B}}, \dots, \mathbf{x}^{|\Omega_p|}, \tilde{\mathbf{v}})\right\|_2$ while dual residual, $\|s^k\|_2$, is defined as $\gamma \left\|\tilde{\mathbf{v}}^{k+1} - \tilde{\mathbf{v}}^k\right\|_2$. In our case, they can be written as

$$\left\|r^k\right\|_2 = \left[\sum_{t\in\Omega_t}\left(\tilde{m}_t^{v,k}\right)^2 + \sum_{t\in\Omega_t}\left(\tilde{m}_t^{\theta,k}\right)^2 + \sum_{t\in\Omega_t}\left(\tilde{m}_t^{I,k}\right)^2\right]^{1/2} \tag{36}$$

$$\begin{aligned}\left\|s^k\right\|_2 = \quad \gamma \Bigg[& \textstyle\sum_{t\in\Omega_t}\left(v_{\mathcal{B},t}^{\mathcal{A},k+1} - v_{\mathcal{B},t}^{\mathcal{A},k}\right)^2 + \sum_{t\in\Omega_t}\left(v_{\mathcal{A},t}^{\mathcal{B},k+1} - v_{\mathcal{A},t}^{\mathcal{B},k}\right)^2 \\ & + \textstyle\sum_{t\in\Omega_t}\left(\theta_{\mathcal{B},t}^{\mathcal{A},k+1} - \theta_{\mathcal{B},t}^{\mathcal{A},k}\right)^2 + \sum_{t\in\Omega_t}\left(\theta_{\mathcal{A},t}^{\mathcal{B},k+1} - \theta_{\mathcal{A},t}^{\mathcal{B},k}\right)^2 \\ & + \textstyle\sum_{t\in\Omega_t}\left(I_t^{\mathcal{A},k+1} - I_t^{\mathcal{A},k}\right)^2 + \sum_{t\in\Omega_t}\left(I_t^{\mathcal{B},k+1} - I_t^{\mathcal{B},k}\right)^2 \Bigg]^{1/2}\end{aligned} \tag{37}$$

The convergence criterion is selected as $\max\left\{\|r^k\|_2, \|s^k\|_2\right\} \leq \varepsilon$. For real case applications, this convergence criterion is set to $\varepsilon = 10^{-3}$ [41]. This accuracy is sufficient to obtain satisfactory solutions without incurring a large computational cost.

3.5. Solution algorithm for inter-DSO LEM

The proposed solution approach is based on the ADMM decomposition algorithm [31]. Using this procedure, the inter-DSO optimization problem presented in (32) is solved in a coordinated and decentralized manner. This process allows LMOs to solve the market while having only partial access to the information. Thus, the principle of privacy among the agents is maintained [27].

The convergence is checked by calculating norm-2 of primal $\|r^k\|_2$ and dual residual $\|s^k\|_2$. Complicating constraints are relaxed and the objective function $f^p(\mathbf{x}^p, \tilde{\mathbf{v}})$ is replaced by its Lagrangian function $\mathcal{L}(\mathbf{x}^{p,k}, \tilde{\mathbf{v}}^k, \lambda^k)$ at iteration k defined in (32). The ADMM procedure for the LEM depicted in (25) is presented in algorithm 1.

Algorithm 1 ADMM Inter-DSO LEM clearing procedure.

```
1: ṽ^k, λ^k ← 0
2: procedure ADMM Loop
3:     while max{‖r^k‖_2, ‖s^k‖_2} ≥ ε do
4:         for p ∈ Ω_p do
5:             min{L(x^{p,k}, ṽ^k, λ^k) : h(x^p) = 0, g(x^p) ≤ 0}.
6:             update ṽ^k.
7:             distribute ṽ^k and x^{p,k} among DSOs.
8:         end for
9:         update λ^{k+1} = λ^k + γ m̃(x^A, x^B, ..., x^{|Ω_p|}, ṽ)
10:        compute ‖r^k‖_2, ‖s^k‖_2
11:    end while
12: end procedure
```

4. Case study

This section presents a case study to illustrate the proposed approach. The case study builds on three IEEE 123 bus test systems interconnected by tie-lines, as shown in Fig. 6. Each network is managed by an independent DSO that only exchanges the information of the interconnecting nodes and internal imbalances with the LMO. The LMO iteratively sends economic signals to each DSO for market coordination.

The total static load of the system is 8 MW shared among 229 Static Loads (SLs). Load profiles were synthetically generated considering three different types of end-users, namely residential, residential with PV and industrial clients. Regarding flexible assets, there are a total of 26 FLs, 42 FGs and 21 BESSs.

4.1. Operation limits of assets

We assume FLs can offer up to 20% of their average demand [42]. Therefore, upper and lower demand limits are given by $0.8P_{f,t}^{sch} \leq p_{f,t}^{am} \leq 1.2P_{f,t}^{sch}$. For FGs, only PV systems are considered for this case study. The upper bound is set by the scheduled power output $P_{g,t}^{sch}$, while the lower limit is 0 resulting in $0 \leq p_{g,t}^{am} \leq P_{g,t}^{sch}$. Additionally, charging and discharging efficiencies η_s^C, η_s^D are assumed to be constant and equal to 90% for all BESSs. For an effective provision of flexibility, $SOC0_s$ is set to 50%. Lastly, bounds for BESS s are set to $\underline{SOC}_s = 5\%$ and $\overline{SOC}_s = 95\%$.

4.2. Offers for flexibility products

Offers for flexibility products are randomly generated following Spanish balancing market average prices (5.261 €/MWh for upward products and 13.192 €/MWh for downward products). Prices for FGs are set considering that $S_{g,t}^d < S_t$, while for FLs and BESSs, $S_{f,t}^u < S_t$, $S_{s,t}^u < S_t$ and $S_{f,t}^d > S_t$, $S_{s,t}^d > S_t$. The upward flexibility product prices for FLs and BESSs are in the interval $[S_t - 5.261, S_t)$ €/MWh, while the downward flexibility product prices are in the interval $(S_t, S_t + 13.192]$ €/MWh. The downward flexibility product prices for FGs are in the interval $[S_t - 5.261, S_t)$ €/MWh.

4.3. Flexibility Trading at Inter-DSO LEMs: Centralized approach

To illustrate the trading of flexibility in inter-DSO LEMs, we first start with the centralized approach. Given the input data described above, congestion appears at the distribution line 13 – 152 belonging to DSO $\mathcal{A}$. The thermal limit for this line is 1,750 kVA which is exceeded by the end of the day (blue line) as shown in Fig. 7. We assume that the DSO requests a volume of flexibility equal to the shaded area of the figure. Then, a LEM is organized with a time period granularity of 15 minutes for the operation horizon.

We first consider a solution to the LEM where only the agents of the DSO $\mathcal{A}$ (where the congestion is located) are involved in the flexibility provision and no DSO power exchange is allowed. The solution for this market is given in Fig. 8(a), where the power flow through the previously congested line is shown (blue line) along with the traded flexibility (yellow bars). Not only is the power flow modified in the periods with congestion, but also in the previous time slots. This is due to the preparatory operations carried out to alleviate the congestion at the minimum cost considering the whole operation horizon.

Nevertheless, this cost can be further reduced if the rest of the interconnected DSOs are considered in the provision of flexibility through an inter-DSO LEM. The market solution is modified as shown in Fig. 8(b). In this case, the costs of flexibility procurement are reduced. The cost savings are due to the participation of more competitive assets from the neighbouring DSOs.

Fig. 9 shows the distribution of the per unit total costs, considering the costs for DSO $\mathcal{A}$ as the basis. Most of the traded flexibility is concentrated in the periods with congestion (19:00 to 23:00), whereas the flexibility trading in other periods is due to preparatory operations. Since, in this case study, BESSs are the most competitive agents for flexibility products, they are responsible for the major part of the trading (yellow bars in Fig. 9).

The flexibility costs allocated through the periods from 18:00 to 00:00 are shown in Fig. 10. Fig. 10(a) displays the case where only the assets of the DSO $\mathcal{A}$ are included, while Fig. 10(b) presents the flexibility cost distribution when assets from all DSOs participate in the market. The total volume of the traded flexibility products is the same in both cases. In the second case, 11.39% of the traded flexibility is imported from DSOs $\mathcal{B}$ and $\mathcal{C}$, resulting in a cost reduction of 16.95%.

Finally, Fig. 11 shows the Lagrange multiplier (λ_t^I) associated with the balance constraint (28). Dual variable λ_t^I is $(S_t - S_t^u)$ of the marginal asset if the demand for flexibility is in the upward direction, or with $(S_t^d - S_t)$ of the marginal asset if the demand for flexibility is in the downward direction.

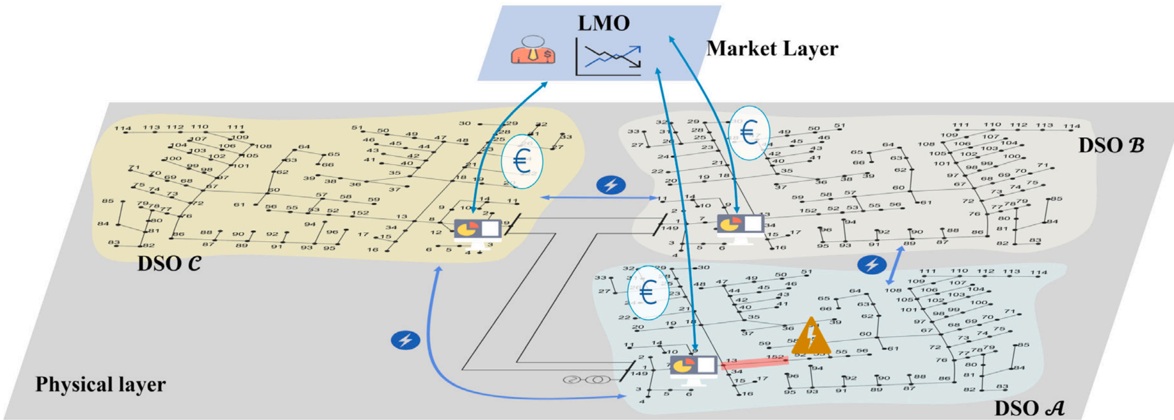

Fig. 6. Illustration of the use case based on the IEEE 123 radial distribution system.

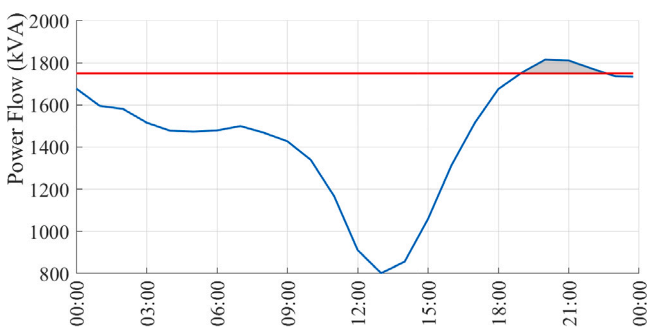

Fig. 7. Power flow in line 13–152 of the DSO $\mathcal{A}$ before LEM clearing (blue line), thermal limit (red line) and flexibility needs (shaded area).

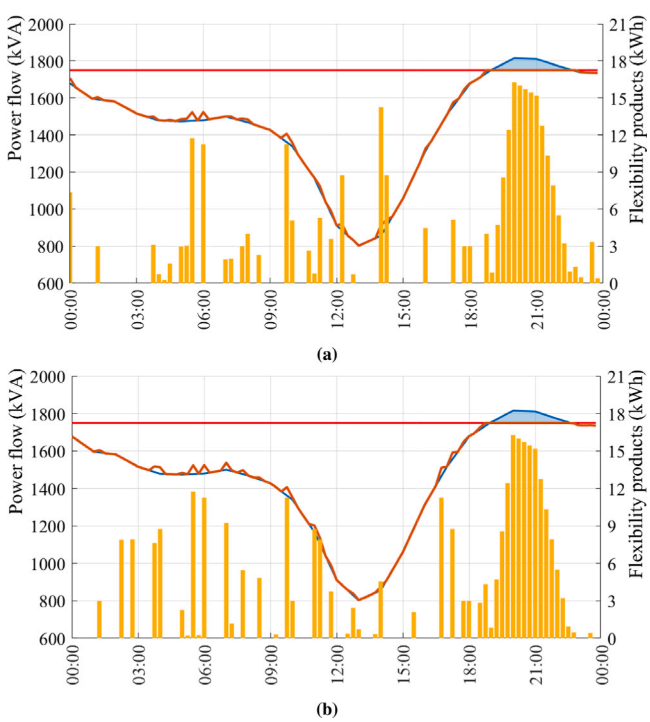

Fig. 8. Power flow in line 13–152 belonging to DSO $\mathcal{A}$ after LEM clearing (blue line), thermal limit (red line) and flexibility products (bars) exchanged in the market. The solution considering only the assets from DSO $\mathcal{A}$ is presented in (a) while in (b) shows the solution considering all areas..

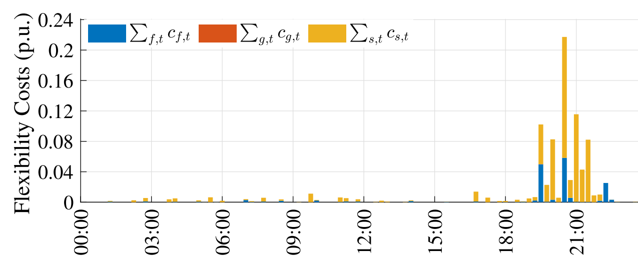

Fig. 9. Distribution of the costs for the periods with market trading. Blue, orange and yellow lines represents the costs of the flexibility products of FLs, FGs and BESSs, respectively.

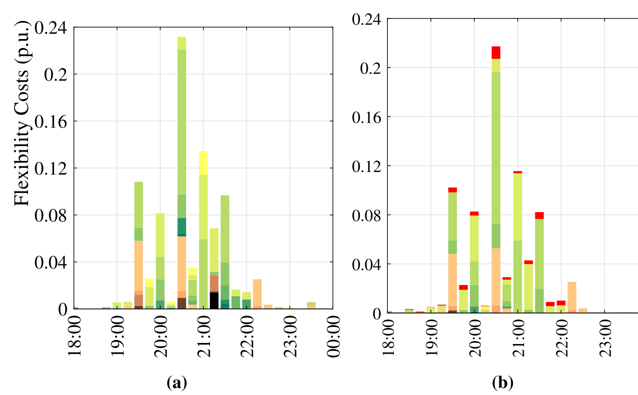

Fig. 10. Distribution of the costs for the periods with congestion considering (a) only flexibility assets from DSO $\mathcal{A}$ and (b) flexibility assets from all areas. Blue, orange, green and red represent the FLs, FGs, BESSs, and DSO $\mathcal{B}$ and $\mathcal{C}$ assets, respectively, included in case (b).

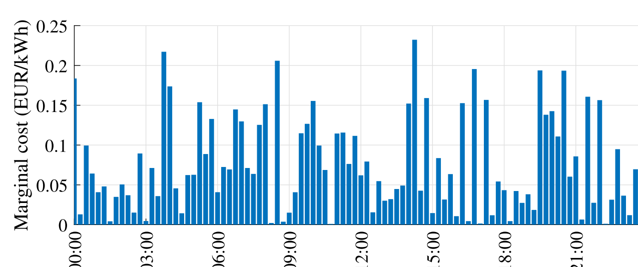

Fig. 11. Representation of the marginal cost associated with the overall imbalance of the grid λ_t^I.

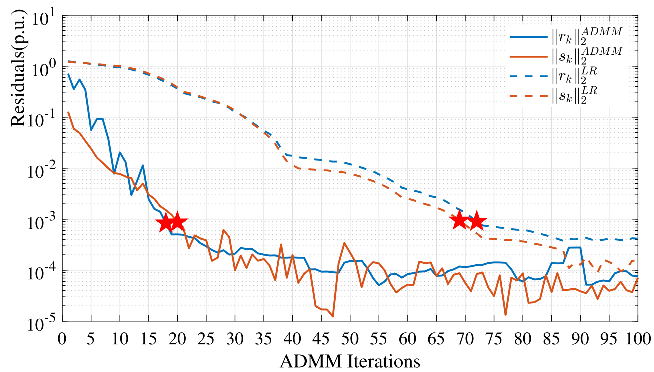

Fig. 12. Evolution of the primal (blue) and dual (orange) residuals for the case study of the LEM using ADMM (solid line) and LR algorithm (dashed line).

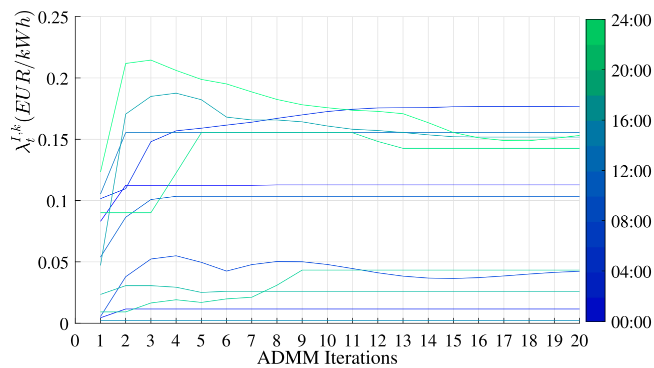

Fig. 13. Evolution of the dual variable λ_t^I in the case study during the iterations of the ADMM algorithm.

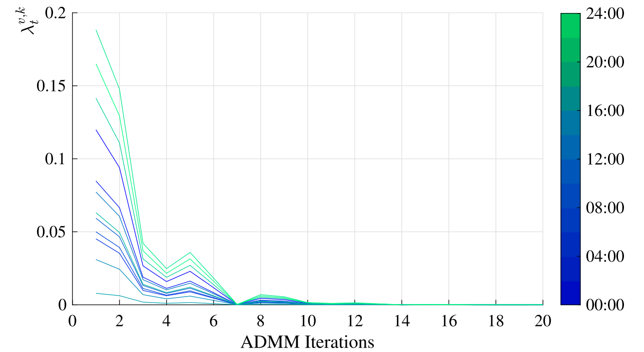

Fig. 14. Evolution of the dual variable $\lambda_t^{v,k}$ associated with voltage magnitude $v_{3149,t}$ in the case study during the iterations of the ADMM algorithm.

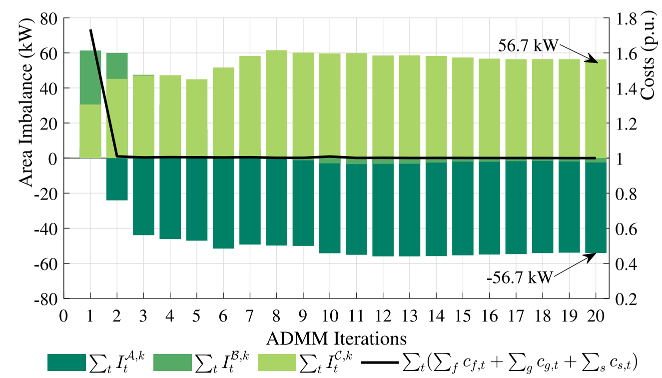

Fig. 15. Evolution of the total costs (green line) and the imbalance variables in the case study during the iterations of the ADMM algorithm (bars).

4.4. Flexibility Trading at inter-DSO LEMs: Decentralized and Coordinated approach

The benefits of considering the interaction among the neighbouring DSOs are presented in Section 4.3., where the total costs for solving the congestion in DSO $\mathcal{A}$ are reduced when assets from areas $\mathcal{B}$ and $\mathcal{C}$ are considered. Nevertheless, although profitable, this approach may raise privacy concerns. The proposed ADMM approach addresses this issue.

ADMM algorithm, as explained in Section 3, is implemented sharing variables $v_t^k, \lambda_t^{v,k}, \theta_t^k, \lambda_t^{\theta,k}, I_t^{p,k}, \lambda_t^{I,k}$ among DSOs and LMO, at each iteration k. The solution using this approach is found to be the same as that obtained in the centralized approach. Considering a penalty factor of $\gamma = 10^{-5}$ and $\varepsilon = 10^{-3}$, convergence is reached after 20 iterations. Moreover, both primal and dual residuals tend to decrease, as presented in Fig. 12.

The presented results obtained with the ADMM algorithm are compared to the LR algorithm in Fig. 12 [17]. Convergence is reached after 73 iterations of the LR algorithm. Additionally, both primal and dual residual curves are steeper for the ADMM algorithm. This feature enables this method to not only to reach convergence before LR, but also to obtain an order of magnitude higher level of precision after 100 iterations.

Fig. 13 presents the evolution of the marginal costs associated with the overall imbalance restriction for each time period t. For simplicity, only 12 Lagrange multipliers are shown.

The evolution of the dual variable $\lambda_t^{v,k}$ associated with the voltage magnitude $v_{3149,t}$ is depicted in Fig. 14. The values of the multipliers tends to zero.

Evolution of the total costs and imbalance among DSOs is shown in Fig. 15, attaining the centralized solution. These iterations represent the interactions among DSOs and LMO prior to reaching convergence. During the initial iterations, DSOs $\mathcal{B}$ and $\mathcal{C}$ respond to the LMO market signals with upward imbalance. Then, after some iterations, DSO $\mathcal{A}$ trades downward flexibility. Finally, the market is cleared, and complicating constraint (28) is satisfied when the upward and downward imbalances reach convergence.

Comparing the centralized and decentralized solutions in terms of their hourly costs and the dual variable $\lambda_t^{I,k}$, the maximum differences are $1.17 \cdot 10^{-4}$ and $1.42 \cdot 10^{-4}$, respectively.

4.5. Impact of the uncertainty in the Inter-DSO LEM clearing

In this section the impacts of the uncertainty of the forecast errors in the local market-clearing solution is analysed. The number of scenarios included in the formulation is 125 for random parameters $S_{e,t}$, $P_{e,f,t}^{sch}$, and $P_{e,g,t}^{sch}$, after a scenario reduction technique is used.

The results of the deterministic and the stochastic simulations are compared in Fig. 16, where the deterministic costs are compared with the expected costs of the stochastic formulation. The expected costs in the stochastic version are a 6% greater than those obtained in the deterministic version. The flexibility products quantity also increases to 4.79% and it also increases flexibility mean price also to 2.51%. Nevertheless, these variables show a similar temporal distribution as Fig. 16 shows.

The stochastic version of the proposed formulation return values for the variables in each scenario. Thus, the probability density distributions of the flexibility magnitudes can be rebuilt. Fig. 17 represents the probability density functions for the costs, products quantity and prices of the market for different time periods. As figure shows, there are some time periods where it is expected that the costs, the products quantity and the price are close to zero, i.e., when the grid is not congested. However, the time periods around 20:00, shows a displaced distribution

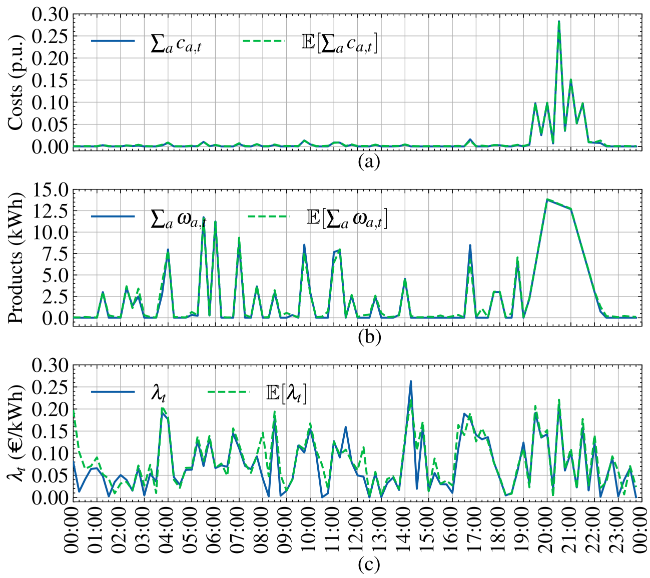

Fig. 16. Comparison of the deterministic solution (blue) versus stochastic solution (green), regarding the market costs (a), the quantity of the products (b) and the marginal price of the flexibility (c).

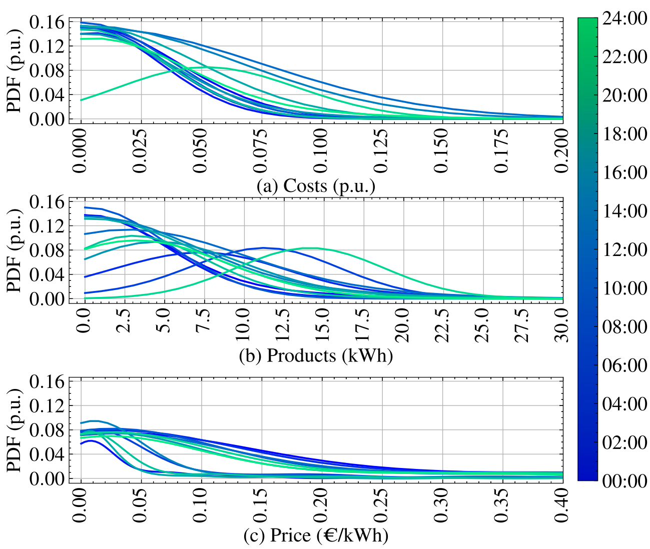

Fig. 17. Probability density functions of the costs of the market solution (a), the quantity of the products (b) and the marginal price of the flexibility (c) for different time periods.

function, which indicates that it is expected that flexibility is traded during these periods to solve the congestion.

4.6. Computational and privacy issues

The decentralized inter-DSO LEM clearing problem can be cast as a quadratic optimization problem with a set of linear constraints. This problem is solved using CPLEX in the GAMS software [43]. All simulations are performed on a personal computer with a quad-core Intel i7–4720 HQ 2.60 GHz with 16 GB of RAM. It takes approximately one second per iteration to solve the market clearing, using the ADMM algorithm, for a total time of 21.397 seconds. The total time spent using LR for the market clearing was 37.954 s. The stochastic version of the problem spent around 29.245 s per scenario to solve the market clearing. Thus, although the time spent per iteration is lower for LR, the high number of iterations needed to obtain the same level of precision makes this algorithm less favourable. The results for both algorithms are compared in Table 2.

Table 2
Comparison of the performance of the ADMM, LR and stochastic algorithms for solving the decentralized version of the LEM.

	ADMM	LR	SCH
Number of its.	20	73	25
Seconds per it.	1.0699	0.5199	1.1698
Computational time	21.397	37.954	3655.63
Model type	QCP	LP	LP
Solver	CPLEX	CPLEX	CPLEX
Precision 100 its.	$\sim 10^{-4}$	$\sim 10^{-3}$	$\sim 10^{-4}$

The performed simulations proved that the proposed market-clearing procedure can be solved within the market time frame. In the case of an increase in the number of participating DSOs or an increase in the complexity of the flexible assets, a parallelized version of the ADMM can be used [44] to streamline the algorithm. In the decentralized setting, the number of equations and variables are 231,651 and 216,963, respectively. Only 16 variables are exchanged among DSOs and LMO, accounting for 0.00690694% of all problem variables. Privacy is preserved because the shared information does not contain any nominative data of the participants.

5. Conclusions

This paper presented a coordinated and decentralized methodology for flexibility products trading in inter-DSO LEMs. Inter-DSO market-clearing problems can be solved using ADMM-based algorithms while preserving the privacy of the information associated with each DSO.

Optimal dual variables associated with the node voltage magnitude and phase angle at the tie-lines and overall imbalance equation, are the signals that properly drive the coordination mechanism to achieve system-wide efficiency. Coordinated self-operation of DSOs, based on minimizing the provision costs of the flexibility product yields the same electrical and economic operating points as of those where DSOs are centrally operated. This result is achieved at the cost of strong and iterative coordination among the involved DSOs.

A case study based on the IEEE 123 bus system demonstrated the feasibility of this approach, achieving savings of 16.95% compared to that of the non-coordinated solutions. The coordination mechanism requires a reduced number of interactions among DSOs and LMO to achieve convergence for a total of 20 iterations in the case study. The stochastic version of the problem shows less than 6% of deviation from the deterministic simulations, which demonstrate the robustness of the proposed approach.

CRediT authorship contribution statement

José A. Aguado: Conceptualization, Investigation, Methodology, Writing – review & editing, Validation. **Ángel Paredes:** Investigation, Methodology, Software, Data analysis, Writing – original draft, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Datasets related to this article can be found at doi.org/10.17632/6ht44tmvtf.1 an open-source online data repository hosted at Mendeley Data [45].

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References

[1] Valarezo O, Gómez T, Chaves-Avila JP, Lind L, Correa M, Ulrich Ziegler D, et al. Analysis of new flexibility market models in Europe. Energies 2021;14(12):3521. http://dx.doi.org/10.3390/en14123521.
[2] Fotis G, Dikeakos C, Zafeiropoulos E, Pappas S, Vita V. Scalability and replicability for smart grid innovation projects and the improvement of renewable energy sources exploitation: The FLEXITRANSTORE case. Energies 2022;15(13). http://dx.doi.org/10.3390/en15134519.
[3] Schütz T, Hu X, Fuchs M, Müller D. Optimal design of decentralized energy conversion systems for smart microgrids using decomposition methods. Energy 2018;156:250–63. http://dx.doi.org/10.1016/j.energy.2018.05.050.
[4] Olivella-Rosell P, Bullich-Massagué E, Aragüés-Peñalba M, Sumper A, Ottesen SØ, Vidal-Clos JA, et al. Optimization problem for meeting distribution system operator requests in local flexibility markets with distributed energy resources. Appl Energy 2018;210(May 2017):881–95. http://dx.doi.org/10.1016/j.apenergy.2017.08.136.
[5] Lezama F, Soares J, Hernandez-Leal P, Kaisers M, Pinto T, Vale Z. Local energy markets: Paving the path toward fully transactive energy systems. IEEE Trans Power Syst 2019;34(5):4081–8. http://dx.doi.org/10.1109/TPWRS.2018.2833959.
[6] Horesh N, Quinn C, Wang H, Zane R, Ferry M, Tong S, et al. Driving to the future of energy storage: Techno-economic analysis of a novel method to recondition second life electric vehicle batteries. Appl Energy 2021;295:117007. http://dx.doi.org/10.1016/j.apenergy.2021.117007.
[7] Zhang C, Xu Y, Dong ZY, Zhang R. Multi-objective adaptive robust voltage/VAR control for high-PV penetrated distribution networks. IEEE Trans Smart Grid 2020;11(6):5288–300. http://dx.doi.org/10.1109/TSG.2020.3000726.
[8] Zhou S, Zou F, Wu Z, Gu W, Hong Q, Booth C. A smart community energy management scheme considering user dominated demand side response and P2P trading. Int J Electr Power Energy Syst 2020;114(December 2018):105378. http://dx.doi.org/10.1016/j.ijepes.2019.105378.
[9] Heinrich C, Ziras C, Syrri AL, Bindner HW. EcoGrid 2.0: A large-scale field trial of a local flexibility market. Appl Energy 2020;261:114399. http://dx.doi.org/10.1016/j.apenergy.2019.114399.
[10] Pressmair G, Kapassa E, Casado-Mansilla D, Borges CE, Themistocleous M. Overcoming barriers for the adoption of local energy and flexibility markets: A user-centric and hybrid model. J Clean Prod 2021;317(July):128323. http://dx.doi.org/10.1016/j.jclepro.2021.128323.
[11] Khajeh H, Firoozi H, Hesamzadeh MR, Laaksonen H, Shafie-Khah M. A local capacity market providing local and system-wide flexibility services. IEEE Access 2021;9:52336–51. http://dx.doi.org/10.1109/ACCESS.2021.3069949.
[12] Esmat A, Usaola J, Moreno M. A decentralized local flexibility market considering the uncertainty of demand. Energies 2018;11(8):2078. http://dx.doi.org/10.3390/en11082078.
[13] Guerrero J, Sok B, Chapman AC, Verbič G. Electrical-distance driven peer-to-peer energy trading in a low-voltage network. Appl Energy 2021;287:116598. http://dx.doi.org/10.1016/j.apenergy.2021.116598.
[14] Zakernezhad H, Setayesh Nazar M, Shafie-khah M, Catalão JP. Optimal scheduling of an active distribution system considering distributed energy resources, demand response aggregators and electrical energy storage. Appl Energy 2022;314(February). http://dx.doi.org/10.1016/j.apenergy.2022.118865.
[15] Lange S, Sokolowski P, Yu X. An efficient, open-bid procurement auction for small-scale electricity markets. Appl Energy 2022;314(February):118867. http://dx.doi.org/10.1016/j.apenergy.2022.118867.
[16] Masood A, Hu J, Xin A, Sayed AR, Yang G. Transactive energy for aggregated electric vehicles to reduce system peak load considering network constraints. IEEE Access 2020;8:31519–29. http://dx.doi.org/10.1109/ACCESS.2020.2973284.
[17] Zhong Z, Fan N, Wu L. A hybrid robust-stochastic optimization approach for day-ahead scheduling of cascaded hydroelectric system in restructured electricity market. European J Oper Res 2022. http://dx.doi.org/10.1016/j.ejor.2022.06.061.
[18] Ullah MH, Park JD. DLMP integrated P2P2G energy trading in distribution-level grid-interactive transactive energy systems. Appl Energy 2022;312(September 2021):118592. http://dx.doi.org/10.1016/j.apenergy.2022.118592.
[19] Guo Z, Pinson P, Chen S, Yang Q, Yang Z. Chance-constrained peer-to-peer joint energy and reserve market considering renewable generation uncertainty. IEEE Trans Smart Grid 2021;12(1):798–809. http://dx.doi.org/10.1109/TSG.2020.3019603.
[20] Esmat A, de Vos M, Ghiassi-Farrokhfal Y, Palensky P, Epema D. A novel decentralized platform for peer-to-peer energy trading market with blockchain technology. Appl Energy 2021;282(PA):116123. http://dx.doi.org/10.1016/j.apenergy.2020.116123.
[21] Sheikhahmadi P, Bahramara S, Mazza A, Chicco G, Shafie-Khah M, Catalao JP. Multi-microgrids operation with interruptible loads in local energy and reserve markets. IEEE Syst J 2022;1–12. http://dx.doi.org/10.1109/JSYST.2022.3177637.
[22] Schittekatte T, Meeus L. Flexibility markets: Q&A with project pioneers. Util Policy 2020;63(August 2019):101017. http://dx.doi.org/10.1016/j.jup.2020.101017.
[23] Hadush SY, Meeus L. DSO-TSO cooperation issues and solutions for distribution grid congestion management. Energy Policy 2018;120(June):610–21. http://dx.doi.org/10.1016/j.enpol.2018.05.065.
[24] Stanley R, Johnston J, Sioshansi F. Platforms to support nonwire alternatives and DSO flexibility trading. Consum. prosumer, prosumager how serv. innov. will disrupt util. bus. model, Elsevier Inc.; 2019, p. 111–26. http://dx.doi.org/10.1016/B978-0-12-816835-6.00006-1.
[25] Sarstedt M, Kluß L, Gerster J, Meldau T, Hofmann L. Survey and comparison of optimization-based aggregation methods for the determination of the flexibility potentials at vertical system interconnections. Energies 2021;14(3). http://dx.doi.org/10.3390/en14030687.
[26] Sarstedt M, Hofmann L. Monetarization of the Feasible Operation Region of active distribution grids based on a cost-optimal flexibility disaggregation. IEEE Access 2022;10:5402–15. http://dx.doi.org/10.1109/ACCESS.2022.3140871.
[27] Zhang R, Yan K, Li G, Jiang T, Li X, Chen H. Privacy-preserving decentralized power system economic dispatch considering carbon capture power plants and carbon emission trading scheme via over-relaxed ADMM. Int J Electr Power Energy Syst 2020;121(February):106094. http://dx.doi.org/10.1016/j.ijepes.2020.106094.
[28] He Y, Chen Q, Yang J, Cai Y, Wang X. A multi-block ADMM based approach for distribution market clearing with distribution locational marginal price. Int J Electr Power Energy Syst 2021;128(January):106635. http://dx.doi.org/10.1016/j.ijepes.2020.106635.
[29] Ma L, Wang L, Liu Z. Multi-level trading community formation and hybrid trading network construction in local energy market. Appl Energy 2021;285(December 2020). http://dx.doi.org/10.1016/j.apenergy.2020.116399.
[30] Zhang K, Troitzsch S, Hanif S, Hamacher T. Coordinated market design for peer-to-peer energy trade and ancillary services in distribution grids. IEEE Trans Smart Grid 2020;11(4):2929–41. http://dx.doi.org/10.1109/TSG.2020.2966216.
[31] Boyd S, Parikh N, Chu E, Peleato B, Eckstein J. Distributed optimization and statistical learning via the alternating direction method of multipliers. Found Trends Mach Learn 2010;3(1):1–122. http://dx.doi.org/10.1561/2200000016.
[32] Paredes A, Aguado JA. Capacity and energy local flexibility markets for imbalance and congestion management. In: 2021 IEEE international smart cities conference. IEEE; 2021, p. 1–7. http://dx.doi.org/10.1109/ISC253183.2021.9562971.
[33] Olivella-Rosell P, Lloret-Gallego P, Munné-Collado Í, Villafafila-Robles R, Sumper A, Ottessen S, et al. Local flexibility market design for aggregators providing multiple flexibility services at distribution network level. Energies 2018;11(4):822. http://dx.doi.org/10.3390/en11040822.
[34] Esmat A, Usaola J, Moreno MÁ. Distribution-level flexibility market for congestion management. Energies 2018;11(5). http://dx.doi.org/10.3390/en11051056.
[35] Leiva J, Aguado JA, Paredes Á, Arboleya P. Data-driven flexibility prediction in low voltage power networks. Int J Electr Power Energy Syst 2020;123:106242. http://dx.doi.org/10.1016/j.ijepes.2020.106242.
[36] Yang J, Zhang N, Kang C, Xia Q. A state-independent linear power flow model with accurate estimation of voltage magnitude. IEEE Trans Power Syst 2017;32(5):3607–17. http://dx.doi.org/10.1109/TPWRS.2016.2638923.
[37] Qin J, Rajagopal R, Varaiya P. Flexible market for smart grid: Coordinated trading of contingent contracts. IEEE Trans Control Netw Syst 2018;5(4):1657–67. http://dx.doi.org/10.1109/TCNS.2017.2746347, arXiv:1701.06253.
[38] MansourLakouraj M, Shahabi M, Shafie-khah M, Catalão JP. Optimal market-based operation of microgrid with the integration of wind turbines, energy storage system and demand response resources. Energy 2022;239:122156. http://dx.doi.org/10.1016/j.energy.2021.122156, URL https://linkinghub.elsevier.com/retrieve/pii/S036054422102404X.
[39] Morales J, Pineda S, Conejo A, Carrion M. Scenario reduction for futures market trading in electricity markets. IEEE Trans Power Syst 2009;24(2):878–88. http://dx.doi.org/10.1109/TPWRS.2009.2016072, URL http://ieeexplore.ieee.org/document/4801580/.
[40] Conejo A, Aguado J. Multi-area coordinated decentralized DC optimal power flow. IEEE Trans Power Syst 1998;13(4):1272–8. http://dx.doi.org/10.1109/59.736264.
[41] Hu B, Sun Y, Shao C, Shahidehpour M, Ding Y, Niu T, et al. A decentralized market framework for procurement of operating reserves from district energy systems. IEEE Trans Sustain Energy 2021;12(3):1629–39. http://dx.doi.org/10.1109/TSTE.2021.3059768.

[42] Hasan KN, Wang M, Milanovic JV. A survey on demand side management potential in south-east Europe to support transmission network flexibility. IEEE; 2018, http://dx.doi.org/10.1109/ISGTEurope.2018.8571693.
[43] Corporation GD. General algebraic modeling system (GAMS) release 24.1.3 r41464. 2013, Washington, DC, USA.
[44] Tsaousoglou G, Soumplis P, Efthymiopoulos N, Steriotis K, Kretsis A, Makris P, et al. Demand response as a service: Clearing multiple distribution-level markets. IEEE Trans Cloud Comput 2022;10(1):82–96. http://dx.doi.org/10.1109/TCC.2021.3117598.
[45] Paredes A, Aguado JA. Local flexibility market dataset. 2022, http://dx.doi.org/10.17632/6ht44tmvtf.1.