



Implementation in dominant strategies of quota rules to choose one candidate[☆]

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ABSTRACT

A group of n experts has to pick one winner from a group of candidates. Each expert has a judgment about who is the best candidate. A social choice rule (SCR) is a function that aggregates experts' judgments to determine the deserving winner. Experts may be biased and not want to reveal their judgments. We state a necessary and sufficient condition for implementing an SCR in dominant strategies. We show that no q -supermajoritarian SCR with $q \leq n - 1$ or q -submajoritarian SCR with $q \geq 2$ satisfies this condition.

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1. Introduction

We study the evaluation problem that arises when a group of n experts has to pick one winner from a group of m candidates. Different experts may have different judgments about who is the best candidate. A social choice rule (SCR) is a function that identifies the deserving winner based on the experts' judgments. However, an expert may be biased, and then he may want to misreport his judgment if, by doing so, some candidate he likes best is selected. To solve this problem, we have to design a mechanism that implements the SCR. Examples of this problem are the selection of the city that will host the Olympic games or the papal conclave to elect the pope.

In this paper, we focus on dominant strategy implementation. First, we show that an SCR is implementable in dominant strategies if and only if the group of experts satisfies an impartiality requirement that we call *impartiality of decisive experts* (IDE). Following to Amorós (2021, 2020a,b), an expert is said to be impartial with respect to two candidates if, whenever he believes that one of them is the best, he prefers that candidate to the other. IDE requires that if the deserving winner changes when a single expert changes his judgment from candidate x to candidate y , the

expert must be impartial with respect to these candidates, and his judgment must coincide with the deserving winner.

An SCR is q -supermajoritarian ($q \geq \lfloor \frac{n}{2} \rfloor + 1$) if, whenever a candidate is judged as best by at least q experts, that candidate is the deserving winner. An SCR is q -submajoritarian ($q \leq \lfloor \frac{n}{2} \rfloor$) if, whenever there is at least one candidate judged as best by at least q experts, the deserving winner is one of these candidates. Amorós (2021, 2020b) showed that, for a q -supermajoritarian or a q -submajoritarian SCR to be implementable in an ordinal equilibrium concept, there must be a minimum number of impartial experts for each pair of candidates.¹ According to these necessary conditions, the most favorable situation is that all experts are impartial with respect to all pairs of candidates. We show that, even in this case, no q -supermajoritarian SCR with $q \leq n - 1$ or q -submajoritarian SCR with $q \geq 2$ is implementable in dominant strategies since none of these SCRs satisfy IDE.

Mackenzie (2020) studied how the pope is elected in the Roman Catholic Church. This problem is a particular case of our model where the cardinals are both the experts and the candidates. Holzman and Moulin (2013) characterized impartial nomination rules when the experts are the candidates themselves and all they care about is whether they win or not. In

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¹ A game-theoretic equilibrium concept is ordinal if it only depends on the ordinal preferences. For example, dominant strategy and Nash equilibria are ordinal equilibrium concepts, but Bayesian equilibrium is not. Amorós (2021) generalized a previous result by Amorós (2020a) for majoritarian SCRs.

this case, each expert is indifferent among everyone else, and then his preferences do not depend on his judgment, which in turn implies that the nomination rule cannot be implemented. Mackenzie (2015) analyzed a stochastic version of the Holzman and Moulin (2013) model. Tamura (2016) established a characterization result in the context of impartial nomination rules that satisfy anonymity, symmetry, and monotonicity.

2. The setting

A group E of $n \geq 2$ experts has to choose one winner from a set C of $m \geq 2$ candidates. Each expert i has an (honest) judgment about who is the best candidate, $J_i \in C$. A social choice rule (SCR) is a function $f : C^n \rightarrow C$ that aggregates the experts' judgments to determine which candidate is socially considered the *deserving winner*.

For each $J \in C^n$ and $x \in C$, let $E_J^x = \{i \in E \mid J_i = x\}$. For each $J \in C^n$ and $q \in \mathbb{N}$, let $C_J^q = \{x \in C : |E_J^x| \geq q\}$.

Definition. Let $q \in \mathbb{N}$ be such that $\lfloor \frac{n}{2} \rfloor + 1 \leq q \leq n$. An SCR f is q -supermajoritarian if, whenever $J \in C^n$ is such that $|E_J^x| \geq q$ for some $x \in C$, then $f(J) = x$.²

Definition. Let $q \in \mathbb{N}$ be such that $1 \leq q \leq \lfloor \frac{n}{2} \rfloor$. An SCR f is q -submajoritarian if, for every $J \in C^n$ with $C_J^q \neq \emptyset$, we have $f(J) \in C_J^q$.

Experts preferences over candidates may depend on their judgments. Let \mathfrak{R} denote the class of all complete, reflexive, and transitive preference relations over C . A preference function for expert i is a mapping $R_i : C \rightarrow \mathfrak{R}$ that associates with each possible judgment J_i a preference relation $R_i(J_i)$ (the strict part is denoted $P_i(J_i)$).

Let $[C]^2$ denote the collection of pairs of candidates. An expert is *impartial* with respect to a pair of candidates if, whenever he believes that one of the two candidates is the best, he prefers that candidate to the other.

Each expert i is characterized by a set of pairs of candidates with respect to whom the planner knows that i is impartial, $I_i \subset [C]^2$. A preference function $R_i : C \rightarrow \mathfrak{R}$ is *admissible* for i at I_i if, for every $J_i, x, y \in C$ such that $J_i = x$ and $xy \in I_i$, we have $x P_i(J_i) y$. Let $\mathcal{R}(I_i)$ be the class of all preference functions that are admissible for i at I_i .

A jury configuration is a profile $I = (I_i)_{i \in E}$. A profile $R \equiv (R_i)_{i \in E}$ is admissible at I if $R_i \in \mathcal{R}(I_i)$ for every $i \in E$. Let $\mathcal{R}(I)$ denote the set of admissible preference functions at I .

Given a jury configuration I , a state is a profile $(J, R) \in C^n \times \mathcal{R}(I)$. A mechanism is a pair $\Gamma = (M, g)$, where $M \equiv \times_{i \in E} M_i$, M_i is a message space for expert i , and $g : M \rightarrow C$ is an outcome function. Given a mechanism $\Gamma = (M, g)$, $m \in M$ is a *dominant strategy equilibrium* of Γ at state (J, R) if, for every $i \in E$, $\hat{m}_i \in M_i$, and $\hat{m}_{-i} \in M_{-i}$, $g(m_i, \hat{m}_{-i}) R_i(J_i) g(\hat{m}_i, \hat{m}_{-i})$. Let $D(\Gamma, J, R) \subset M$ denote the set of dominant strategy equilibria of Γ at (J, R) . The corresponding candidates selected by the mechanism are denoted $g(D(\Gamma, J, R))$.

Given a jury configuration I , a mechanism $\Gamma = (M, g)$ implements an SCR f in dominant strategies if, for each state $(J, R) \in C^n \times \mathcal{R}(I)$, $g(D(\Gamma, J, R)) = f(J)$.

3. The results

Firstly, we identify a necessary and sufficient condition for an SCR f to be implementable in dominant strategies: If the deserving winner changes when a single expert changes his judgment from J_i to \hat{J}_i , the expert must be impartial with respect to the pair $J_i \hat{J}_i$ and his judgment must coincide with the deserving winner.

Definition. Given a jury configuration I , an SCR f satisfies *impartiality of decisive experts* (IDE) if, for every $i \in E$, $J_i, \hat{J}_i \in C$, and $J_{-i} \in C^{n-1}$ such that $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$, we have $J_i \hat{J}_i \in I_i$ and $f(J_i, J_{-i}) = J_i$.

Theorem 1. Given a jury configuration I , an SCR f is implementable in dominant strategies if and only if it satisfies IDE.

Proof. First, we prove the necessity part. Given a jury configuration I , let f be an SCR implementable in dominant strategies. Suppose by contradiction that f does not satisfies IDE. Then, for some $i \in E$, $J_i, \hat{J}_i \in C$, and $J_{-i} \in C^{n-1}$ such that $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$, we have either (i) $f(J_i, J_{-i}) \neq J_i$ or (ii) $f(J_i, J_{-i}) = J_i$ and $J_i \hat{J}_i \notin I_i$. Then, there exists $R_i \in \mathcal{R}(I_i)$ such that either $f(J_i, J_{-i}) P_i(J_i) f(J_i, J_{-i})$ or $f(J_i, J_{-i}) P_i(\hat{J}_i) f(\hat{J}_i, J_{-i})$. To see this, note that, if $f(J_i, J_{-i}) \neq J_i$, by definition of admissible preference function, there exists $R_i \in \mathcal{R}(I_i)$ such that $f(\hat{J}_i, J_{-i}) P_i(J_i) f(J_i, J_{-i})$. The same happens if $f(J_i, J_{-i}) = J_i$, $J_i \hat{J}_i \notin I_i$, and $f(\hat{J}_i, J_{-i}) = \hat{J}_i$. Finally, if $f(J_i, J_{-i}) = J_i$, $J_i \hat{J}_i \notin I_i$, and $f(\hat{J}_i, J_{-i}) \neq \hat{J}_i$, by definition of admissible preference function, there exists $R_i \in \mathcal{R}(I_i)$ such that $f(J_i, J_{-i}) P_i(\hat{J}_i) f(\hat{J}_i, J_{-i})$. Let $\Gamma = (M, g)$ be a mechanism implementing f in dominant strategies. Let $\hat{R}_i \in \mathcal{R}(I_i)$ and $R_{-i} \in \mathcal{R}(I_{-i})$. Let $J = (J_i, J_{-i})$, $\hat{J} = (\hat{J}_i, J_{-i})$, $R = (R_i, R_{-i})$, and $\hat{R} = (\hat{R}_i, R_{-i})$. Because Γ implements f in dominant strategies, there exists $m^*(R(J)) = (m_i^*(R_i(J_i)), m_{-i}^*(R_{-i}(J_{-i}))) \in D(\Gamma, J, R)$ such that $g(m^*(R(J))) = f(J)$. Similarly, there exists $m^*(\hat{R}(\hat{J})) = (m_i^*(\hat{R}_i(\hat{J}_i)), m_{-i}^*(R_{-i}(J_{-i}))) \in D(\Gamma, \hat{J}, \hat{R})$ such that $g(m^*(\hat{R}(\hat{J}))) = f(\hat{J})$. If $f(\hat{J}_i, J_{-i}) P_i(J_i) f(J_i, J_{-i})$, we have $g(m_i^*(\hat{R}_i(\hat{J}_i)), m_{-i}^*(R_{-i}(J_{-i}))) P_i(J_i) g(m_i^*(R_i(J_i)), m_{-i}^*(R_{-i}(J_{-i})))$, which contradicts that $m_i^*(R_i(J_i))$ is a dominant strategy for expert i at state (J, R) . If $f(J_i, J_{-i}) P_i(\hat{J}_i) f(\hat{J}_i, J_{-i})$, then $g(m_i^*(R_i(J_i)), m_{-i}^*(R_{-i}(J_{-i}))) P_i(\hat{J}_i) g(m_i^*(\hat{R}_i(\hat{J}_i)), m_{-i}^*(R_{-i}(J_{-i})))$ which contradicts that $m_i^*(\hat{R}_i(\hat{J}_i))$ is a dominant strategy for expert i at state (\hat{J}, \hat{R}) .

Now we prove the sufficient part. Given a jury configuration I , let f be an SCR that satisfies IDE. Then, for every $i \in E$, $J_i, \hat{J}_i \in C$, and $J_{-i} \in C^{n-1}$ with $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$, we have $J_i \hat{J}_i \in I_i$, $f(J_i, J_{-i}) = J_i$, and $f(\hat{J}_i, J_{-i}) = \hat{J}_i$ and hence, by definition of admissible preference function, we have $f(J_i, J_{-i}) P_i(J_i) f(\hat{J}_i, J_{-i})$ for every $R_i \in \mathcal{R}(I_i)$. Then, the direct mechanism implements f in dominant strategies. To see this, note that, for every state (J, R) and expert i , either (1) $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$ for some $\hat{J}_i \in C$ and $\hat{J}_{-i} \in C^{n-1}$, and then J_i is the only weakly dominant strategy for expert i , or (2) $f(J_i, J_{-i}) = f(\hat{J}_i, J_{-i})$ for every $\hat{J}_i \in C$ and $\hat{J}_{-i} \in C^{n-1}$, and then every $\hat{J}_i \in C$ is a weakly dominant strategy for expert i . Therefore, for every state (J, R) , there is a dominant strategy equilibrium $J^* \in \Pi^n$ such that, for each $i \in E$, either (i) $J_i^* = J_i$, or (ii) $J_i^* \neq J_i$ and $f(J_i, \hat{J}_{-i}) = f(J_i^*, \hat{J}_{-i})$ for every $\hat{J}_{-i} \in C^{n-1}$. Suppose $J^* \neq J$. By (ii), $f(J_1^*, J_2^*, \dots, J_n^*) = f(J_1, J_2^*, \dots, J_n^*) = f(J_1, J_2, \dots, J_n^*) = \dots = f(J_1, J_2, \dots, J_n)$. Hence, every dominant strategy equilibrium of the direct mechanism at state (J, R) results in $f(J)$. ■

Secondly, we show that, if $m \geq 3$ and $n \geq 3$, no q -supermajoritarian SCR with $q \neq n$ or q -submajoritarian SCR with $q \neq 1$ is implementable in dominant strategies, since none of these SCRs satisfy IDE, regardless of the jury configuration.

² For each real number $\alpha \in \mathbb{R}$, $\lfloor \alpha \rfloor = \max\{\beta \in \mathbb{Z} \mid \beta \leq \alpha\}$.

Theorem 2. Suppose $m \geq 3$ and $n \geq 3$. If an SCR f is q -supermajoritarian for some $q \leq n - 1$ or q -submajoritarian for some $q \geq 2$, it is not implementable in dominant strategies, regardless of the jury configuration.

Proof. Given any jury configuration I , let f be an SCR implementable in dominant strategies. By Theorem 1, f satisfies IDE. First, suppose by contradiction that f is $(n - 1)$ -supermajoritarian.

Claim 1. Let $J \in C^n$, $x, y, z \in C$, and $i, j \in E$ be such that $J_i = x$, $J_j = y$, and $J_k = z$ for every $k \in E \setminus \{i, j\}$. Then $f(J) = z$.

Suppose first that $f(J) = x$. Let $\hat{J}_i = z$. Because f is $(n - 1)$ -supermajoritarian, $f(\hat{J}_i, J_{-i}) = z$. Then, $f(J_j, J_{-j}) \neq f(\hat{J}_i, J_{-i})$ and $f(J_j, J_{-j}) = x \neq J_j$, which contradicts IDE. Suppose now that $f(J) \notin \{x, z\}$. Let $\hat{J}_i = z$. Because f is $(n - 1)$ -supermajoritarian, $f(\hat{J}_i, J_{-i}) = z$. Then, $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$ and $f(J_i, J_{-i}) \neq x = J_i$, which contradicts IDE.

Claim 2. $n \geq 4$.

Suppose $n = 3$. Let $J \in C^n$ be such that, for some $x, y, z \in C$, $J_1 = x$, $J_2 = y$, and $J_3 = z$. By Claim 1, $f(J) = z$. Using the same argument, $f(J) = x$ and $f(J) = y$, a contradiction.

Claim 3. Let $J \in C^n$, $x, y \in C$, and $i, j \in E$ be such that $J_i = J_j = x$ and $J_k = y$ for every $k \in E \setminus \{i, j\}$. Then $f(J) = y$.

Suppose $f(J) \neq y$. Given any $z \in C \setminus \{x, y\}$, let $\hat{J}_i = z$. By Claim 1, $f(\hat{J}_i, J_{-i}) = y$. Then, $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$ and $f(J_i, J_{-i}) = y \neq \hat{J}_i$ which contradicts IDE.

Claim 4. $n \geq 5$.

Suppose $n = 4$. Let $J \in C^n$ be such that, for some $x, y \in C$, we have $J_1 = J_2 = x$ and $J_3 = J_4 = y$. By Claim 3, $f(J) = y$. Using the same argument, $f(J) = x$, a contradiction.

Claim 5. Let $J \in C^n$, $x, y, z \in C$, and $i, j, k \in E$ be such that $J_i = x$, $J_j = J_k = y$, and $J_l = z$ for every $l \in E \setminus \{i, j, k\}$. Then $f(J) = z$.

Suppose first that $f(J) = y$. Let $\hat{J}_i = z$. By Claim 3, $f(\hat{J}_i, J_{-i}) = z$. Then, $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$ and $f(J_i, J_{-i}) = y \neq \hat{J}_i$ which contradicts IDE. Suppose now that $f(J) \notin \{y, z\}$. Let $\hat{J}_j = z$. By Claim 1, $f(\hat{J}_j, J_{-j}) = z$. Then, $f(J_j, J_{-j}) \neq f(\hat{J}_j, J_{-j})$ and $f(J_j, J_{-j}) \neq y = J_j$ which contradicts IDE.

Claim 6. $n \geq 6$.

Suppose $n = 5$. Let $J \in C^n$ be such that, for some $x, y, z \in C$, we have $J_1 = x$, $J_2 = J_3 = y$, and $J_4 = J_5 = z$. By Claim 5, $f(J) = z$. Using the same argument, $f(J) = y$, a contradiction.

Claim 7. Let $J \in C^n$, $x, y \in C$, and $i, j, k \in E$ be such that $J_i = J_j = J_k = x$, and $J_l = y$ for every $l \in E \setminus \{i, j, k\}$. Then $f(J) = y$.

Suppose $f(J) \neq y$. Given any $z \in C \setminus \{x, y\}$, let $\hat{J}_i = z$. By Claim 5, $f(\hat{J}_i, J_{-i}) = y$. Then, $f(J_i, J_{-i}) \neq f(\hat{J}_i, J_{-i})$ and $f(J_i, J_{-i}) = y \neq \hat{J}_i$ which contradicts IDE.

Claim 8. $n \geq 7$.

Suppose $n = 6$. Let $J \in C^n$ be such that, for some $x, y \in C$, we have $J_1 = J_2 = J_3 = x$ and $J_4 = J_5 = J_6 = y$. By Claim 7, $f(J) = y$. Using the same argument, $f(J) = x$, a contradiction.

This argument can be used recursively to show that $n \geq 8$, $n \geq 9$, etc. Therefore, there is no n such that f is $(n - 1)$ -supermajoritarian. Finally, notice that (1) for every $q \leq n - 1$, q -supermajoritarianism implies $(q + 1)$ -supermajoritarianism and (2) for every $q \leq \lfloor \frac{n}{2} \rfloor$, q -submajoritarianism implies $(n - q + 1)$ -supermajoritarianism. ■

4. Concluding comments

We have studied the problem of implementing a social choice rule that aggregates experts' judgments to determine the deserving winner of a competition. We have stated a necessary and sufficient condition on the impartiality of the experts for the case of dominant strategy implementation. We have shown that no q -supermajoritarian rule with $q \neq n$ or q -submajoritarian rule with $q \neq 1$ satisfies this condition. The question of whether any SCR like that is implementable in equilibrium concepts other than dominant strategies is still open.

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