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Hardening study on the application of the Upper Bound Theorem in indentation processes by means of modules of Triangular Rigid Zones

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ABSTRACT

In previous studies based on indentation processes, the application of the Upper Bound Theorem (UBT) Modular Model by means of Triangular Rigid Zones (TRZ) has being analyzed. These studies were carried out with plane strain conditions and rigid-perfectly plastic material. One of the advantages of working with the modular procedure of the UBT, is the possibility of the introduction of the different parameters that take place in the process. This factors introduction allows the enrichment of the study, bringing the research nearest to the reality of the processes. Therefore, in order to approach the method to a real indentation process, the introduction and study of the material hardening is inevitable. Furthermore, thanks to the new incremental processes that are being developed, the indentation processes are becoming important in the current industry.

Keywords: Indentation; Upper Bound Theorem; Triangular Rigid Zone; Hardening; Modular distribution.

1. Introduction

In previous studies of the application of the Upper Bound Theorem (UBT) by means of modules composed of Triangular Rigid Zones (TRZ), a new model has been developed for its implementation in the indentation process with flat punch. This new model consists of three modules with only two TRZ each. The research was made in conditions of plane strain and rigid-perfectly plastic material [1-3].

Nowadays, one of the principal processes starting to be widely studied is the indentation process. Its implementation in the metalworking industries is been analyzed due to the evolution of new innovative approaches, like the Incremental Forming Processes (IFP). Several authors [4-6] are exploring the different applications of the IFP. Bouffioux et al [7] and Camacho et al [8] also present different studies for the Single Point Incremental Forming (SPIF) in its different variants. The principal advantage of this new processes is the lower forces required and a complete management with CNC machines.

The IFP is considered as an alternative to traditional plastic deformation processes. These new processes appear thus as a possible way to address many of the current industry objective. Conversely, the industrial application is still very restricted [9].

In order to make an approximation of the analytical developed method to the actual deformation process that occurs in indentations processes and make a proper comparison with other methods of current application or experimental testing, the introduction of different parameters is necessary. In this paper, the benefits of the new modular configuration in the application of the UBT are presented by the introduction of the hardening effect to the model and the comparison with another current method of analysis as the Finite Element Method (FEM).

As for the UBT, one of the advantages of the modular implementation is the possibility of the introduction of different parameters that allows enriching the study. In this study case, the indentation process, the introduction of the hardening effect will offer results increasingly closer to the reality of industrial processes. Therefore, the introduction and study of hardening of the material is inevitable.

In Figure 1 can be observed the difference between a rigid-perfectly plastic material (a) and a deformation implemented in a material that suffers from the hardening effect (b).

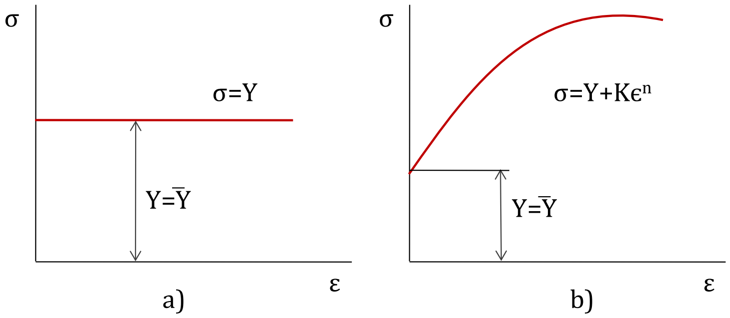


Figure 1. *Stress deformation curves. a) Non hardening and b) Hardening material.*

Always under plane strain consideration, for cold worked metals with yield stress Y, taking in account the hardening effect in the indentation process object of study will be feasible thanks to the introduction of the simplified equation form for the yield curve due to Ludwik [10], represented in Eq.1.

$$\sigma = Y + K \cdot \epsilon^{n} \qquad \text{Eq.1}$$

Where n is the strain-hardening exponent, specific constant of each material and K is the constant stress or resistance coefficient, which depends on the material structure and varies with the processing of it. In Table 1 are some examples of different n and K values for different metals or alloys [11].

Table 1. *n and K values for different materials and alloys (UNS designation). Manufacturing Engineering Technology. S. Kalpakjian, S.R. Schimd.*

Material	***K*** [N/mm²]	***n***
A91100	180	0.20
A96061	410	0.05
C26000	900	0.49
C23000	580	0.34
C10100	315	0.54
G43400	530	0.26
G41350	1100	0.14
S30400	1275	0.45

The Ludwik equation is preferred to the Hollomon equation in view of the fact that, unlike the Hollomon Eq.2 it does include the tension until the yield point [12], not just the produced tension by the hardening effort.

$$\sigma = K \cdot \epsilon^{n} \qquad \text{Eq.2}$$

Now, Eq.1 is expressed on the yield strength (Y) and the true or natural strain (ϵ) basis. Working with the analytical model of the UBT, a transformation of these parameters is necessary in order to adapt the equation application to the study of the hardening effect to the case study. Working from the engineering deformation point of view is necessary due to the fact that the UBT divides the deformation sequence in moments of deformation or instants. True and engineering deformations are related as shown in the following equation.

$$\epsilon = \ln(1 + e) \qquad \text{Eq.3}$$

Knowing that:

$$2k = 1.155Y \quad \text{Eq.4}$$

$$k = 0.5775Y = 0.6Y \quad \text{Eq.5}$$

The final equation applied to the modular model for the hardening effect study, is expressed in Eq.6

$$k = k_0 + 0.6 \cdot K \cdot \epsilon^n \quad \text{Eq.6}$$

2. Methodology

In order to establish a widespread system that allows covering the largest number possible of materials by the application of the UBT, a series of simulations for different materials were performed with a Finite Element Method based program (FEM), DEFORM 2D. These simulations were feasible due to the material database of the program used. Thus, working with four different groups of materials is possible: aluminium, steel, titanium and superalloys.

As stated in previous studies [13], the finite configuration is not part of the study field of indentation, resembling more to processes such as deep drawing. Therefore, for the analysis presented in this paper, the infinite configuration will only be considered. The infinite consideration is a study case closer to the current material treatment in the present industry. An application example of a deep indentation is the case of rings manufacture without welding. A deep indentation is made in a material billet. Later, in another stage, the form initial form is finished with the material shearing in order to remove the material indented previously, leaving the initial part hollow. Therefore, the final phase that would correspond to finite configuration is mistaken, in this case, with the shear produced to remove the core of the work-piece.

Corresponding to the modular configuration developed for a flat punch case, to develop a series of hardening models within the current model is necessary. This is required to establish the appropriate model witch will deliver the most according results, in relation to the characteristics of the processed materials.

For the Hardening Model 1 (HM1) (Figure 2), the consideration that all material under the punch presents the hardening effect (ϵ) has been taken into account. Consequently, the equation developed in Ec.6 is applied to the three modules, to both located under the punch as to the exterior module that does not receive a direct deformation from the punch compression. The material situated under the punch will move to the sides, affecting the material considered for the exterior module.

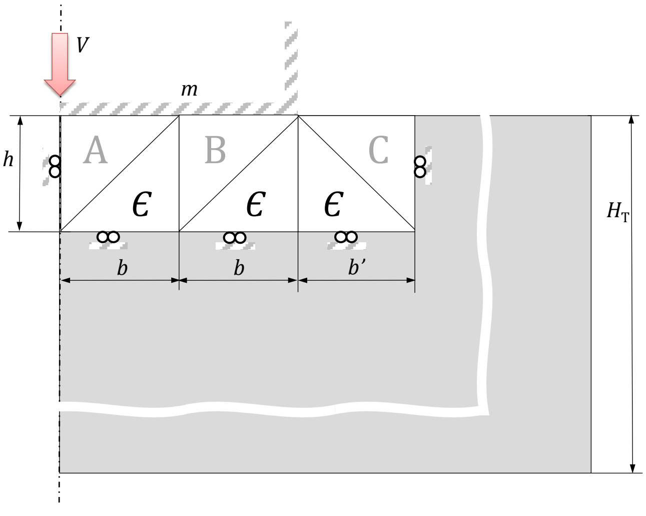


Figure 2. *Hardening Model 1*

Similarly, for the Hardening Model 2 (HM2), the same consideration as for the HM1 is contemplated. The differences is based in the application of the hardening effect because, in this option, the hardening effect is only applied to the modules that are under the direct influence of the punch, modules A and B as seen in Figure 2. In this case raises the hypothesis that the material in the periphery of the punch does not suffer the same deformation as the material contained in the previous modules. Hence, its hardening effect might be negligible.

The material of the previous modules receives all the punch push and the exterior module receives only the push of the material that is displaced from the first part. This exterior module is considered to the capacity to absorb the displacement of material without increasing its hardening significantly. So, in relation to the large size of the work-piece, the hardening effect become diluted and could be disregarded.

In the Hardening Model 3 (HM3) case, the hardening effect is only considered in the first module (A). This model is suitable for materials that suffer from a low hardening effect due to deformation, i.e. materials that have a small n, of 0.05 to 0.10 orders. For this reason, the hardening effect will be concentrated in the first module calculation, leaving the rest of the modules without this effect. Figure 3 shows a first approximation of the three developed Hardening Models with an indentation performed for Aluminium A92024 (UNS designation).

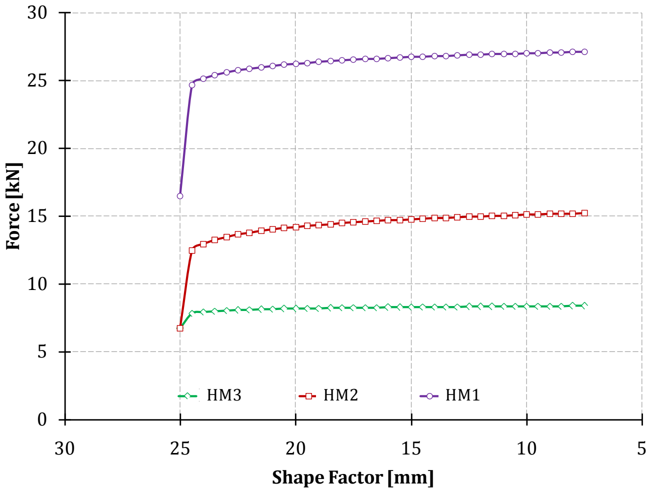


Figure 3. *Comparative between the Hardening Models.*

The hardening exponent (n) is the property related to the metal hardening capacity during the forming process. It is also a good measure of the increase of the material resistance due to plastic deformation. The values for ductile materials at room temperature tend to oscillate between 0.02 and 0.5 [14, 15].

The standard ASTM 646 [16] is applied for the n calculation. It performs the least-squares method to the line $\ln(\sigma_v)=\ln(K)+n\cdot\ln(\varepsilon_v)$, making the following change of variable (Eq.7 and Eq.8).

$$y = \ln(\sigma_V) \quad \text{Eq.7}$$

$$x = \ln(\varepsilon_V) \quad \text{Eq.8}$$

Knowing that the data obtained from the FEM based program database are only from the plastic zone, to obtain the necessary stress-strain data for the deduction of n and K for each simulated materials is possible. A selection of these results is shown in Table 2. As it can be seen, the results obtained are consistent with those shown before.

Table 2. *K and n values for the selection of the materials studied (UNS designation)*

Material	***Y*** [MPa]	***K*** [N/mm^2]	***n***
Aluminium			
A95024	270	366.44	0.12
A95052	140	192.48	0.09
A96062	138	198.54	0.10
Steel			
G10450	640	881.83	0.10
G10080	280	577.67	0.17
S30400	510	1073.84	0.19
Titanium			
R58010	1050	1758.12	0.17
R50250	510	951.46	0.23
R50400	850	1398.28	0.20
Superalloys			
N06600	434.37	1101.03	0.20
G52986	762.62	1124.39	0.12
N02211	337.84	896.05	0.21

3. Results

Thanks to the comparisons performed, to find an action pattern for each selected metal group is possible. In this way, a criterion of classification is set according to the *n* exponent value, shown in Table 2.

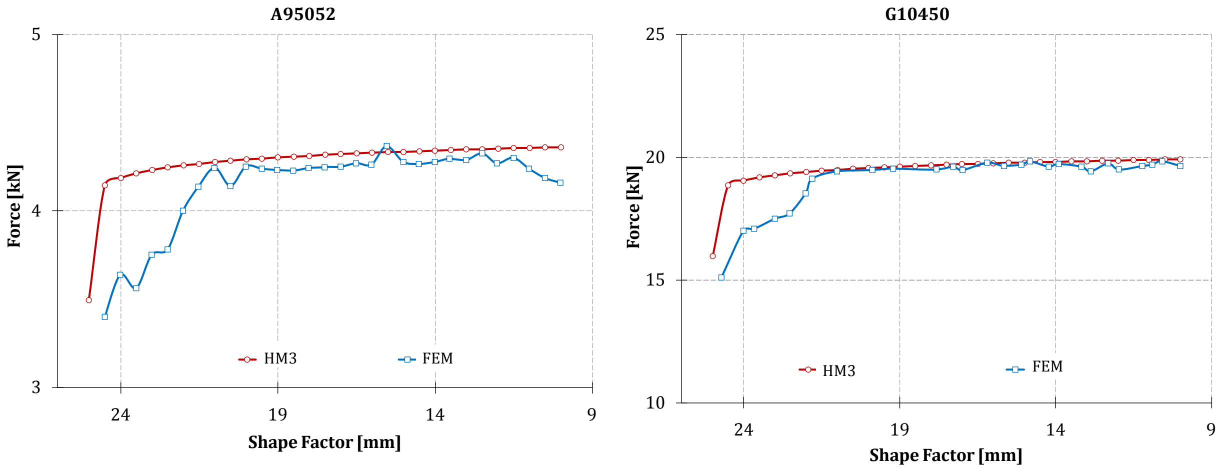
Table 3. *Classification according to the hardening exponent values*

Material	***n***	**Hardening Model**
Aluminium, Steel, Titanium and alloys	$0 \le n \le 0.10$	HM 3
	$n > 0.10$	HM 2
Superalloys	$0 \le n \le 0.12$	HM 3
	$n > 0.12$	HM 1

Following, the results are displayed according of its adjustment level, being HM3 the one that shows the best adjustment.

3.1. HM3 Results

HM3 can be implemented to those materials that do not suffer much from the hardening effect. What is to say, which can go through high deformations without hardening much.



(a) (b)

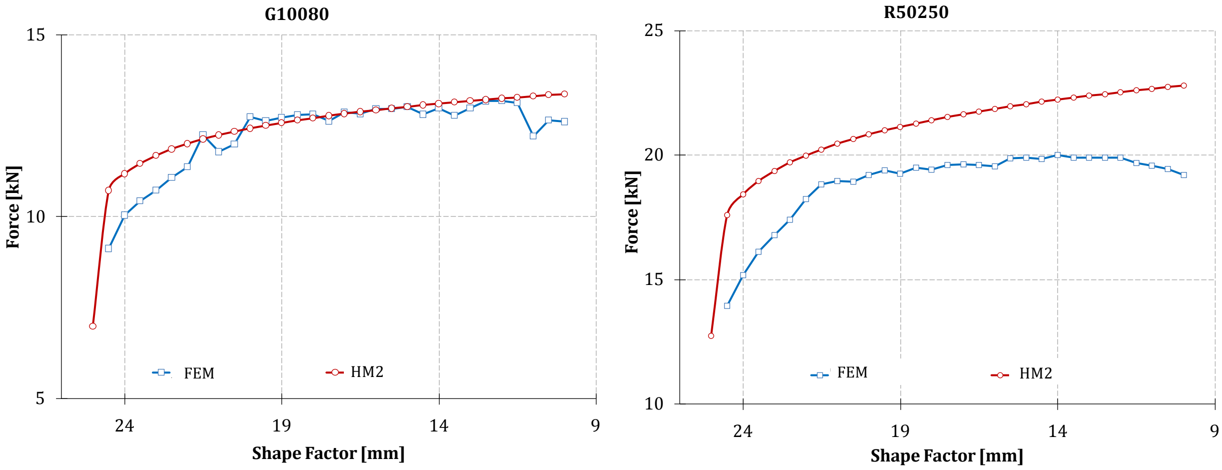
***Figure 4**. Comparatives with HM3.(a) Aluminium A95052(n=0.09) and (b) steel G10450 (n=0.10)*

For the materials selected, this model is applicable to those which $n \leq 0.10$, except for superalloys. This material presents special characteristics so it needs special treatment.

Figure 4 shows a comparative of aluminium A95052 (n=0.09) and steel G10450 (n=0.10) with the Hardening Model 3. The adjustment of the UBT model to the results provided with FEM can be appreciated.

3.2. HM2 Results

Figure 5 presents the comparative between the HM2 and metals that comply with the classification in Table 3. The adjustment is better for low n values.

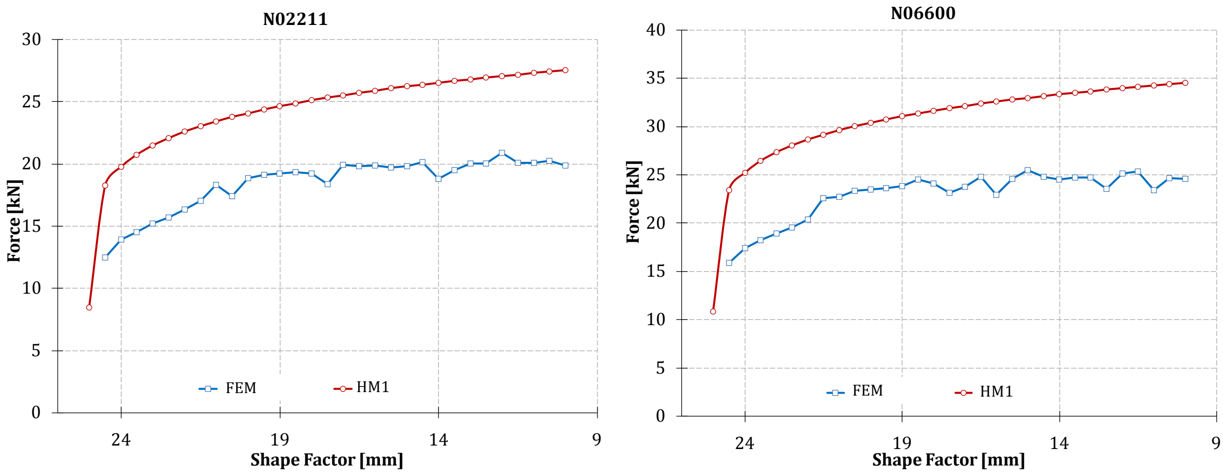


(a) (b)

***Figure 5.** Comparatives with HM2. (a) Steel G10080 (n=0.17) and (b) Titanium R50250 (n=0.23)*

3.3. HM1 Results

HM1 is only applicable to those materials which present a high hardening effect. That is to say, that the materials hardens with low deformations. In this case study, HM1 is feasible for Superalloys. Figure 6 presents the comparative with N02211 and N06600. Due to the nature of this type of material, the behaviour of the analysis results differ from the other examples exposed. Although these results present more remote values compared with the previous ones, are still within the allowed range. However, these types of materials will be less used in indentation processes.



(a) *(b)*

Figure 6. *Comparatives with HM1.(a) N02211 (n=0.21) and (b) N06600 (n=0.20)*

4. Conclusions

The implementation of the hardening effect to the new modular configuration of the Upper Bound Method is introduced in the present research. Is has been necessary to develop three Hardening Models to have a proper adjustment to the materials considered for the study. These Hardening models depend on the hardening exponent *n*, which is the metal property related the metal hardening capacity during the forming process. Is a variable directly related with the hardening so, the introduction of *n* in the mathematical development of the new modular Upper Bound Method, faithfully reflects the hardening process in the results obtained.

For the metals taken into account in the case studies, we found that superalloys need a special classification. This special treatment is not unusual since superalloys do not typically follow the normal behaviour of the other metals. So the need to adjust the *n* classification to this type of metal is considered normal.

We show through our analysis that this model result consistent with the solutions that the numerical method provides. The resolution obtained with FEM have been compared with the one procured by the new Upper Bound Method configuration.

For *n* values from 0 to 0.10, the model evolution is similar to the simulations and the effort values are close, with a difference of 3 or 5%. The Hardening Model 3 is the model that adapts better to the metals chosen. For *n* values above 0.10 the model show efforts with a higher difference with the effort provided with FEM. However, this disparity is contain and is usually not a 15% superior, applying the Hardening Model 2

Also, for superalloys, the suitable model is the Hardenind Model 1, that presents the hardening effect in all the modules of the modular model. On the one hand, for the superalloys considered, the Hardening Model present more remote values compared with the previous materials but the results are still within the allowed range. On the other hand, these types of materials are not usually used in indentation processes.

So, in this paper, the application of the new modular Upper Bound Method model to an indentation process considering with non rigid-perfectly plastic materials is presented, showing the suitability of the model and the accurate results obtained.

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6. References

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