

# Application of the Upper Bound Theorem to indentation processes with tilted punch by means of Modular Model

C. Bermudo<sup>(1,a)</sup>, F. Martín<sup>(1,b)</sup>, L. Sevilla<sup>(1,c)</sup>

<sup>(1)</sup> Department of Manufacturing Engineering, University of Malaga. C/Dr. Ortiz Ramos, s/n E-29071 Malaga. SPAIN.

<sup>(a)</sup>bgambo@uma.es, <sup>(b)</sup>fdmartin@uma.es, <sup>(c)</sup>lsevilla@uma.es.

## ABSTRACT

Nowadays, due to the new deformation processes that are being developed like the Incremental Forming Process (IFP) or the Localized-Incremental Forging Process (LIFP), indentation processes are regaining importance again. In this paper, in order to cover the largest possible number of cases in the study of indentation a tilted punch is considered. Thus, using combinations of flat and tilted punches makes possible the analysis of complex geometries. Present study shows the analysis of the indentation process by the Upper Bound Theorem (UBT) when it is carried out with a tilted punch. A modular model is developed. This modular model is compound by three modules, each with two Triangular Rigid Zones (TRZ), adapting better to the configuration of the punch this way.

**Keywords:** Indentation; Upper Bound Theorem; Triangular Rigid Zone; Tilted punch; Modular distribution.

## 1. Introduction

On the one hand, until today, the indentation processes have been considered as secondary processes due to the deformations obtained. This deformation, often, are localized, small and, especially in the hardening tests, have a superficial effect [1, 2]. However, nowadays, with the recent industry development, indentation is gaining importance. There are several indentation based processes, like the Localized-Incremental Forging Process (LIFP) or the Multiple Indentation Process (MIP), both classified as Bulk Metal Forming Processes, that apply lower forces to obtain the same deformation similarly to other current processes, do not require complex dies and are very flexible in their implementation with CNC machines [3].

On the other hand, the Upper Bound Theorem (UBT) by means of modules of Triangular Rigid Zones (TRZ), developed by F. Martín in 2009 [4] and based on studies carried out by Kudo [5-7] in the 60s, is a method that allows to obtain the necessary deformation effort to achieved the estimated shape in the work-piece deformed, depending of the processes used. This technique is characterized by its easy application, after the mathematical model is generated. Just introducing known parameters of the process, as the measurements of the dies or the depth required, the model will provide the effort that has to be applied to reach such deformation.

Within the present research, the indentation processes are study from the UBT perspective. A new modular model has been developed adapted to these kind of processes. In this paper, an indentation case carried out with a tilted punch is analyzed (**¡Error! No se encuentra el origen de la referencia.**). After the mathematical formulation acquired for the tilted case, different geometries can be study combining the tilted shape with the previous case of flat punch [8, 9] is possible, thus making feasible the resolution of indentation cases with geometrically more complex punches.

## **5. Acknowledgements**

The authors want to thank the University of Malaga – Andalucía Tech, International Campus of Excellence, and the FPU program of the Ministry of Education, Culture and Sport of Spain for its contribution on this paper

## **6. References**

- [1] C. Bernal, A. M. Camacho, J. M. Arenas, and E. M. Rubio. Analytical procedure for geometrical evaluation of flat surfaces formed by multiple indentation processes. *Applied Mechanics and Materials*, 217-219 (2012), pp. 2351-2356.
- [2] J. Chakrabarty, *Theory of plasticity*, Elsevier Science, Oxford (U.K.), 2006.
- [3] A. M. Camacho, M. M. Marín, E. M. Rubio, and M. A. Sebastian. Application of different simulation strategies for the analysis of multi-stroke localised-incremental forming operations. *Materials Science Forum*, 2013 (2012), pp. 19-24.
- [4] F. Martín, *Desarrollo, integración y optimización en el estudio del proceso de forja mediante el teorema del límite superior a través del modelo de bloques rígidos triangulares*, Ingeniería de Fabricación, Universidad de Málaga, Málaga, 2009.
- [5] H. Kudo. An upper-bound approach to plane-strain forging and extrusion-i. *International Journal of Mechanical Sciences*, 1 (1960), pp. 57-83.
- [6] H. Kudo. An upper-bound approach to plane-strain forging and extrusion-ii. *International Journal of Mechanical Sciences*, 1 (1960), pp. 229-252.
- [7] H. Kudo. An upper-bound approach to plane-strain forging and extrusion-iii. *International Journal of Mechanical Sciences*, 1 (1960), pp. 366-368.
- [8] C. Bermudo, F. Martín, and L. Sevilla. Analysis and selection of the modular block distribution in indentation process by the upper bound theorem. *Procedia Engineering*, 63 (2013), pp. 388-396.
- [9] C. Bermudo, F. Martín, and L. Sevilla. Optimización del modelo modular en procesos de indentación mediante el teorema del límite superior. *Anales de Ingeniería Mecánica*, 19 (2014), p. 97.
- [10] I. G. Goryacheva, H. Murthy, and T. N. Farris. Contact problem with partial slip for the inclined punch with rounded edges. *International Journal of Fatigue*, 24 (2002), pp. 1191-1201.
- [11] A. Sackfield, D. Dini, and D. A. Hills. The finite and semi-infinite tilted, flat but rounded punch. *International Journal of Solids and Structures*, 42 (2005), pp. 4988-5009.
- [12] A. Babaei, G. Faraji, M. M. Mashhadi, and M. Hamdi. Repetitive forging (rf) using inclined punches as a new bulk severe plastic deformation method. *Materials Science and Engineering: A*, 558 (2012), pp. 150-157.
- [13] L. Tajul, T. Maeno, and K.-i. Mori. Successive forging of long plate having inclined cross-section. *Procedia Engineering*, 81 (2014), pp. 2361-2366.
- [14] W. Johnson and P. P. B. Mellor, *Engineering plasticity*, Ellis Horwood Limited, Oxford (U.K.), 1983.
- [15] C. Bermudo, F. Martín, and L. Sevilla. Selection of the optimal distribution for the upper bound theorem in indentation processes. *Materials Science Forum*, 797 (2014), pp. 117-122.
- [16] W. Prager and P. G. Hodge, *Theory of perfectly plastic solids*, John Wiley & Sons, Michigan (U.S.A.), 1951.