# Teaching improper integrals with CAS


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When teaching how to compute improper integrals such as:

$$\int_0^{\infty} f(t)\,\mathrm{d}t \qquad ; \qquad \int_{-\infty}^{0} f(t)\,\mathrm{d}t \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(t)\,\mathrm{d}t$$

the basic approach to compute such improper integrals is as follows:

$$\begin{aligned}
\int_0^{\infty} f(t)\,\mathrm{d}t &= \lim_{m\to\infty}\int_0^{m} f(t)\,\mathrm{d}t = \lim_{m\to\infty}\big(F(m)-F(0)\big)\\
\int_{-\infty}^{0} f(t)\,\mathrm{d}t &= \lim_{m\to-\infty}\int_m^{0} f(t)\,\mathrm{d}t = \lim_{m\to-\infty}\big(F(0)-F(m)\big)\\
\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t &= \int_{-\infty}^{0} f(t)\,\mathrm{d}t + \int_0^{\infty} f(t)\,\mathrm{d}t \quad \text{or, in case of convergence,}\\
\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t &= \lim_{m\to\infty}\int_{-m}^{m} f(t)\,\mathrm{d}t = \lim_{m\to\infty}\big(F(m)-F(-m)\big)
\end{aligned}$$

(Cauchy principal value)

where $F$ is an antiderivative of $f$.

But, what happens if an antiderivative $F$ for $f$ or the above limits do not exist?

For example, the antiderivatives for the improper integrals:

$$\int_0^{\infty} \frac{\sin(at)}{t}\,\mathrm{d}t \quad ; \quad \int_0^{\infty} \frac{\cos(at)-\cos(bt)}{t}\,\mathrm{d}t \quad \text{or} \quad \int_{-\infty}^{\infty} \frac{\cos(at)}{t^2+1}\,\mathrm{d}t$$

can not be computed. Hence, the above procedures cannot be used for these examples.

In this talk we will show, as an application of advanced calculus subjects, how to compute this kind of improper integrals using a CAS. Laplace and Fourier transforms or Residue Theorem in Complex Analysis are some advance techniques which can be used for this matter.

As an example of use, we will describe the file `ImproperIntegrals.mth`, developed in DERIVE 6, which deals with such computations. This utility file was first introduced at TIME 2014 Conference [1].

Some CAS use different rules for computing integrations. For example RUBI system [2], a **ru**le-**b**ased **i**ntegrator developed by Albert Rich, is a very powerful system for computing integrals using rules. We will be able to develop new rules schemes for some improper integrals using `ImproperIntegrals.mth`. These new rules can extend the types of improper integrals that a CAS can compute.

Finally, we will show some examples that we use with our students which can not be computed with the basic procedures implemented in CAS. Using the utility file `ImproperIntegrals.mth`, these examples will be easily solved.

# References


[1] G. Aguilera, J.L. Galán, M.Á. Galán, Y. Padilla, P. Rodríguez and R. Rodríguez, *Advanced techniques to compute improper integrals using a* CAS, in *Proceedings of the International Conference Technology and its Integration in Mathematics Education* (TIME-2014), Krems, Austria, (2014).

[2] A.D. Rich, *Rule-based Mathematics. Symbolic Integration Rules*, in `http://www.apmaths.uwo.ca/~arich/`.