Evaluating pointing errors on ergodic capacity of DF relay-assisted FSO communication systems

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***Abstract*—Ergodic capacity of decode-and-forward (DF) relay-assisted free-space optical (FSO) communication systems when line of sight is available is analyzed over gamma-gamma fading channels with pointing errors. Novel closed-form approximate ergodic capacity expression is obtained in terms of the H-Fox function for a 3-way FSO communication system when the α-μ distribution to efficiently approximate the probability density function (PDF) of the sum of gamma-gamma with pointing errors variates is considered. Moreover, we present a novel asymptotic expression at high signal-to-noise ratio (SNR) for the ergodic capacity of DF relay-assisted FSO systems. The main contribution in this work lies in an in-depth analysis about the impact of pointing errors on the ergodic capacity for cooperative FSO systems. In order to maintain the same performance in terms of capacity, it is corroborated that the presence of pointing errors requires an increase in SNR, which is related to the fraction of the collected power at the receive aperture, i.e. A_0. Simulation results are further demonstrated to confirm the accuracy and usefulness of the derived results.**

Keywords*—Free-space optical (FSO), cooperative communications, decode-and-forward (DF), ergodic capacity, asymptotic ergodic capacity analysis, H-Fox function.

I. Introduction

Free-space optical (FSO) systems allow communication at data rates as high as hundreds of gigabits per second for a variety of applications [1]. However, the major impairment in FSO communication systems is the atmospheric turbulence, which produces fluctuations in the irradiance of the transmitted optical beam, as a result of random variations in the refractive index along the link [2]. Additionally, the high directivity of the transmitted beam in FSO systems, or building sway can produce an unsuitable alignment between transmitter and receiver and, hence, a greater deterioration in performance. In the last decade, several works have investigated the adoption of cooperative communications in the context of FSO systems in order to solve these inconveniences [3], [4], which have proved that cooperative communications are quite a efficient technique to satisfy the typical bit error-rate (BER) targets for FSO applications without much increase in hardware. Lately, there have been few studies on ergodic capacity for cooperative FSO systems, which have demonstrated that cooperative communications are also able to increase the channel capacity [5]–[7]. In [5], the end-to-end ergodic capacity of dual-hop FSO system employing amplify-and-forward (AF) relaying is evaluated over gamma-gamma fading channels with pointing errors, by approximating the probability density function (PDF) of the end-to-end signal to noise ratio (SNR), by the α-μ distribution. In [6], [7], the capacity performance of dual-hop subcarrier intensity modulation (SIM)-based FSO system with decode-and-forward (DF) and AF relay is evaluated, respectively, over gamma-gamma fading channels with pointing errors. The derived results are obtained in terms of special function known as generalized bivariate Meijers G-function (GBMGF). Previously, the ergodic capacity was analyzed for FSO communication links in [8]–[11], showing that ergodic capacity can be perfectly applied to FSO links, despite the fact that the atmospheric turbulence channels can be well described as slow fading or block fading channels and, hence, outage capacity becomes a more realistic measure of channel capacity than ergodic capacity in FSO systems.

However, to the best of the author's knowledge, the study of the ergodic capacity for cooperative FSO systems wherein the line of sight is taken into account has not been studied yet. Motivated by this issue, the purpose of this work is to study the ergodic capacity for the bit-detect-and-forward (BDF) cooperative protocol presented in [4], over gamma-gamma fading channels with pointing errors when line of sight is available. As concluded in [4], the BDF cooperative protocol is able to achieve a higher diversity order, strongly dependent not only on the relay location but also on the pointing errors. In this work, a novel approximate closed-form ergodic capacity expression is obtained in terms of the H-Fox function for a 3-way FSO communication system when the irradiance of the transmitted optical beam is susceptible to moderate-to-strong turbulence conditions, following a gamma-gamma distribution of parameters a and b, or pointing error effects, following a misalignment fading model where the effect of beam width, detector size and jitter variance is considered. Here, as proposed in [12], the α-μ distribution is used in order to obtain a closed-form expression for the distribution of the sum of gamma-gamma with pointing errors variates. Unlike [5], here, a numerical observation of the α-μ parameters is included in order to evaluate how these parameters are affected by pointing errors. Moreover, we present an asymptotic expression at high signal-to-noise ratio (SNR) for the ergodic capacity of DF relay-assisted FSO systems, showing a greater and robust capacity when line of sight is available compared to a direct transmission without cooperative communication. The main contribution in this work lies in an in-depth analysis about the impact of pointing errors on the ergodic capacity for cooperative FSO systems. In order to maintain the same performance in terms of capacity, it is corroborated that the presence of pointing errors requires an increase in SNR, which is related to the fraction of the collected power at the receive aperture, i.e. A_0 and, hence, not being dependent of the relay location and atmospheric turbulence conditions.

II. System and Channel Model

We adopt a three-node cooperative system based on three separate FSO links, as shown in Fig. 1, assuming laser sources intensity-modulated and ideal non-coherent (direct-detection) receivers. The cooperative strategy works in two phases. In the first phase, the source node S sends its own data to the relay node R and the destination node D. In the second phase, the relay node R sends the received data from the source node S in the first phase to the destination node D. In this fashion, the relay node R detects each code bit to "0" or "1" and sends the bit with the new power to the destination node D regardless of these bits are detected correctly or incorrectly. The received electrical signal for each link is given by $Y_m = XI_m + Z_m$, where X is the binary transmitted signal, I_m is the equivalent real-value fading gain (irradiance) through the optical channel, and Z_m is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 = N_0/2$, i.e. $Z_m \sim N(0, N_0/2)$, independent of the on/off state of the received bit. Here, on-off keying (OOK) modulation scheme is used, X is either 0 or $2P_{opt}\sqrt{T_b}$, where P_{opt} is the average transmitted optical power from each node and, T_b is the bit period. The received instantaneous electrical SNR can be written as in [13], as

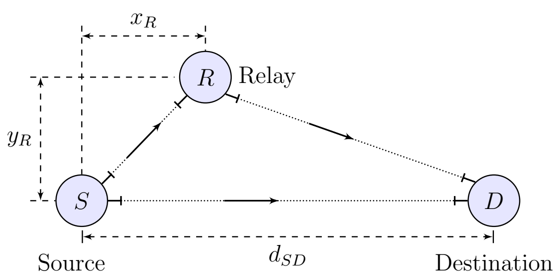


Fig. 1. Block diagram of the considered 3-way FSO system.

$$\gamma = d_E^2 I_m^2 / 2\sigma_m^2 = 4P_{\text{opt}}^2 T_b I_m^2 / N_0 = 4\gamma_0 I_m^2, \tag{1}$$

where d_E is the Euclidean distance and γ_0 represents the received electrical SNR in absence of turbulence when the classical rectangular pulse shape is adopted for OOK formats. The irradiance I_m is considered to be a product of three factors i.e. $I_m = L_m I_m^{(a)} I_m^{(p)}$, where L_m is the deterministic propagation loss, $I_m^{(a)}$ is the attenuation due to atmospheric turbulence and $I_m^{(p)}$ the attenuation due to geometric spread and pointing errors. L_m is determined by the exponential Beers-Lambert law as $L_m = e^{-\Phi d}$, where d is the link distance and Φ is the atmospheric attenuation coefficient. It is given by $\Phi = (3.91/V(km))\,(\lambda(nm)/550)^{-q}$ where V is the visibility in kilometers, λ is the wavelength in nanometers and q is the size distribution of the scattering particles, being $q = 1.3$ for average visibility (6 km $< V <$ 50 km), and $q = 0.16V + 0.34$ for haze visibility (1 km $< V <$ 6 km). To consider a wide range of turbulence conditions, the gamma-gamma turbulence model proposed in [2] is assumed here. Regarding the impact of pointing errors, we use the general model of misalignment fading given in [13], wherein the effect of beam width, detector size and jitter variance is considered. A closed-form expression of the combined PDF of I_m was derived in [14] as

$$f_{I_m}(i) = \frac{\varphi_m^2 i^{-1} G_{1,3}^{3,0}\left(\frac{a_m b_m}{A_0 L_m} i \,\middle|\, \begin{matrix}\varphi_m^2 + 1\\ \varphi_m^2, a_m, b_m\end{matrix}\right)}{\Gamma(a_m)\Gamma(b_m)}, \tag{2}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function [15, eqn. (9.301)]. It must be mentioned that the parameters a and b can be directly linked to physical parameters through the following expressions [16]:

$$a = \left[\exp\left(0.49\sigma_R^2/(1 + 1.11\sigma_R^{12/5})^{7/6}\right) - 1\right]^{-1}, \tag{3a}$$

$$b = \left[\exp\left(0.51\sigma_R^2/(1 + 0.69\sigma_R^{12/5})^{5/6}\right) - 1\right]^{-1}, \tag{3b}$$

where $\sigma_R^2 = 1.23C_n^2\kappa^{7/6}d^{11/6}$ is the Rytov variance, which is a measure of optical turbulence strength. Here, $\kappa = 2\pi/\lambda$ is the optical wave number and d is the link distance in meters. C_n^2 stands for the altitude-dependent index of the refractive structure parameter and varies from 10^{-13} $m^{-2/3}$ for strong turbulence to 10^{-17} $m^{-2/3}$ for weak turbulence [2]. It must be emphasized that parameters a and b cannot be arbitrarily chosen in FSO applications, being related through the Rytov variance. In relation to the impact of pointing errors [13], assuming a Gaussian spatial intensity profile of beam waist radius, ω_z, on the receiver plane at distance z from the transmitter and a circular receive aperture of radius r, $\varphi = \omega_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver, $\omega_{z_{eq}}^2 = \omega_z^2\sqrt{\pi}\text{erf}(v)/2v\exp(-v^2)$, $v = \sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0 = [\text{erf}(v)]^2$ and $\text{erf}(\cdot)$ is the error function [15, eqn. (8.250)]. In the following section, the fading coefficient I_m for the paths S-D, S-R and R-D is indicated by I_{SD}, I_{SR} and I_{RD}, respectively. Here, we assume that all coefficients are statistically independent.

III. Ergodic capacity analysis

In this section, firstly, we analyze the ergodic capacity of cooperative FSO system under study. Here, two cases can be considered to evaluate the ergodic capacity corresponding to the BDF relaying scheme, depending on the fact that the bit from the relay S-R-D is detected correctly or incorrectly. Assuming a statistical channel model as follows

$$Y_{\text{DF}} = \frac{1}{2}XI_{SD} + X^* I_{RD} + Z_{SD} + Z_{RD}, \tag{4}$$

being $X \in \{0, d_E\}$ and $Z_{SD}, Z_{RD} \sim N(0, N_0/2)$. X^* represents the random variable corresponding to the information detected at node R and, hence, $X^* = X$ when the bit has been detected correctly at node R and $X^* = d_E - X$ when the bit has been detected incorrectly. The division by 2 in Eq. (4) is considered so as to maintain the average optical power in the air at a constant level of P_{opt}, being transmitted by each node an average optical power P_{opt}. In this manner, the source node transmits by each laser an average optical power $P_{\text{opt}}/2$ as well as the relay node transmits an average optical power P_{opt} because only one laser is available. Hence, the ergodic capacity corresponding to the BDF cooperative protocol is given by

$$C_{\text{DF}} = C_0 \cdot (1 - P_b^{SR}) + C_1 \cdot P_b^{SR} = C_0 + (C_1 - C_0) \cdot P_b^{SR}, \tag{5}$$

where P_b^{SR} denotes the BER corresponding to the S-R link and, C_0 and C_1 are the ergodic capacity when the bit is correctly and incorrectly detected at node R, respectively. The resulting received electrical SNR when $X^* = X$, can be defined as $\gamma_{\rm DF}^0 = \frac{\gamma_0}{2}(I_{SD}+2I_{RD})^2$ and, when $X^* = d_E - X$, the resulting received electrical SNR can be defined as $\gamma_{\rm DF}^1 = \frac{\gamma_0}{2}(I_{SD}-2I_{RD})^2$ [4]. It must be noted that the ergodic capacity corresponding to BDF relaying scheme in Eq. (5) can be accurately approximated as follows $C_{\rm DF} \approx C_0$ as SNR increases, since the term P_b^{SR} tends to zero as SNR increases. This approximation has been numerically corroborated by Monte Carlo simulation and, it will be checked in following sections. Hence, the ergodic capacity of BDF cooperative protocol can be written as

$$C_{\rm DF} \approx \frac{B}{2\ln(2)}\int_0^\infty \ln\left(1+\frac{\gamma_0}{2}i^2\right) f_{I_T}(i)di, \qquad (6)$$

where B is the channel bandwidth, $\ln(\cdot)$ is the natural logarithm [15, eqn. (1.511)], and $I_T = I_{SD}+2I_{RD}$. It should be noted that the factor $1/2$ in Eq. (16) is because the source node S is assumed to operate in half-duplex mode. It should be also mentioned that obtaining the corresponding PDF of I_T is remarkably tedious and complicated. In order to solve the integral in Eq. (6), we approximate the PDF $f_{I_T}(i)$ by the α-μ PDF as proposed in [12]

$$f_{I_T}(i) \approx \frac{\alpha\mu^\mu i^{\alpha\mu-1}}{\hat{i}^{\alpha\mu}\Gamma(\mu)}\exp\left(-\mu\frac{i^\alpha}{\hat{i}^\alpha}\right). \qquad (7)$$

The α-μ distribution is characterized by the α and μ parameters as well as the α-root mean value $\hat{i}$ of the random variable I_T. The use of this approximate PDF is suitable in order to study the ergodic capacity of cooperative FSO systems due to the fact that this PDF contains information regarding the mean, the variance, and the fourth moment of I_T. These parameters are obtained as the solution of the system of transcendental equations derived in [12, eqn. (24) and (25)]. The required solution for the system of transcendental equations has been numerically solved in an efficient manner. The parameter $\hat{i}$ can be obtained as $\hat{i} = \mu^{1/\alpha}\Gamma(\mu)\mathbb{E}[I_T]/\Gamma(\mu+1/\alpha)$, where $\mathbb{E}[\cdot]$ denotes the expectation operator. Therefore, the nth moment of I_T can be determined as follows

$$\mathbb{E}[I_T^n] = \int_0^\infty\int_0^\infty (i_1+2i_2)^n f_{I_{SD}}(i_1)f_{I_{RD}}(i_2)di_1di_2. \qquad (8)$$

According to the binomial theorem, it is possible to expand the power $(i_1+2i_2)^n$ into a sum and, after performing some straightforward manipulations in Eq. (8), we can express $\mathbb{E}[I_T^n]$ as

$$\begin{aligned}\mathbb{E}[I_T^n] = &\sum_{k=0}^n \frac{n!2^k}{k!(n-k)!}\\ &\times\int_0^\infty i_1^{n-k}f_{I_{SD}}(i_1)di_1\cdot\int_0^\infty i_2^k f_{I_{RD}}(i_2)di_2.\end{aligned} \qquad (9)$$

Both integrals in Eq. (9) can be solved with the help of [17, eqn. (2.24.2.1)] and, then, performing some algebraic manipulations, the corresponding closed-form solution for the nth moment of I_T can be written as

$$\begin{aligned}\mathbb{E}[I_T^n] = &\varphi_{SD}^2\varphi_{RD}^2\sum_{k=0}^n\frac{n!2^k}{k!(n-k)!}\\ &\times\left(\frac{a_{SD}b_{SD}}{A_{SD}L_{SD}}\right)^{k-n}\frac{\Gamma(n-k+a_{SD})\Gamma(n-k+b_{SD})}{\Gamma(a_{SD})\Gamma(b_{SD})(n-k+\varphi_{SD}^2)}\\ &\times\left(\frac{a_{RD}b_{RD}}{A_{RD}L_{RD}}\right)^{-k}\frac{\Gamma(k+a_{RD})\Gamma(k+b_{RD})}{\Gamma(a_{RD})\Gamma(b_{RD})(k+\varphi_{RD}^2)}.\end{aligned} \qquad (10)$$

For $n=1$, we can obtain the mean of I_T, i.e. $\mathbb{E}[I_T]$. Substituting Eq. (7) into Eq. (6) and, after performing the change of variables $i' = i^\alpha/\hat{i}^\alpha$, the ergodic capacity for BDF relaying scheme can be accurately approximated as follows

$$\begin{aligned}C_{\rm DF} \approx &\frac{B\mu^\mu}{2\ln(2)\Gamma(\mu)}\\ &\times\int_0^\infty \ln\left(1+\frac{\hat{i}\gamma_0}{2}i'^{2/\alpha}\right)\frac{i'^\mu}{i'}\exp(-\mu i')di'.\end{aligned} \qquad (11)$$

The integral in Eq. (11) can be solved using [17, eqn. (8.4.6.5)] and [17, eqn. (8.4.3.1)] in order to express the natural logarithm in terms of the Meijer's G-function as $\ln(1+x) = G_{2,2}^{1,2}\left(x\middle|_{1,0}^{1,1}\right)$ and, the exponential in terms of the Meijer's G-function as $e^x = G_{0,1}^{1,0}\left(-x\middle|_0^-\right)$, respectively. Afterwards using [18, eqn. (07.34.21.0012.01)], we can obtain the approximate closed-form solution for the ergodic capacity corresponding to the BDF cooperative protocol, $C_{\rm DF}$, as follows

$$\begin{aligned}C_{\rm DF} \approx &\frac{B}{2\ln(2)\Gamma(\mu)}\\ &\times H_{1,3}^{3,2}\left(\frac{\gamma_0\Gamma(\mu)^2\mathbb{E}[I_T]^2}{2\Gamma(\mu+1/\alpha)^2}\middle|\begin{matrix}(1,1),(1,1),(1-\mu,2/\alpha)\\(1,1),(0,1)\end{matrix}\right),\end{aligned} \qquad (12)$$

where $H_{p,q}^{m,n}[\cdot]$ is the H-Fox function [17, eqn. (8.3.1)]. A computer program in Mathematica for the efficient implementation of the H-Fox function is given in [19, appendix A]. An asymptotic expression for the ergodic capacity corresponding to the BDF relaying scheme at high SNR can be readily and accurately lower-bounded as in [20, Eqs. (8) and (9)] as follows

$$C_{\rm DF}^H \doteq \frac{B}{2\ln(2)}\frac{\partial\mathbb{E}[(\frac{\gamma_0}{2}I_T^2)^n]}{\partial n}\bigg|_{n=0}, \qquad (14)$$

where $\mathbb{E}[(\frac{\gamma_0}{2}I_T^2)^n]$ denotes the nth moment of instantaneous electrical SNR, $\gamma_{\rm DF}^0$. Hence, the ergodic capacity corresponding to the BDF cooperative protocol at high SNR, $C_{\rm DF}^H$, can be asymptotically expressed as follows

$$\begin{aligned}C_{\rm DF}^H \doteq &\frac{B\ln(\gamma_0/2)}{2\ln(2)}\\ &+\frac{B}{\ln(2)}\left(\ln\left(\frac{\Gamma(\mu)\mathbb{E}[I_T]}{\Gamma(\mu+1/\alpha)}\right)+\frac{\psi(\mu)}{\alpha}\right),\end{aligned} \qquad (15)$$

where $\psi(\cdot)$ is the psi (digamma) function [21, eqn. (6.3.1)]. Next, we study the ergodic capacity corresponding to the direct transmission (DT) without cooperative communication in order to establish the baseline performance. This closed-form expression was obtained in [9] and, it is here reproduced for convenience. Assuming channel side information at the receiver, the ergodic capacity, C_{DT}, can be obtained as

$$C_{\mathrm{DT}} = \frac{B}{2\ln(2)} \int_0^\infty \ln\left(1 + 4\gamma_0 i^2\right) f_{I_{SD}}(i) di, \qquad (16)$$

The integral in Eq. (16) can be solved using [17, eqn. (8.4.6.5)] in order to express the natural logarithm in terms of the Meijer's G-function as in Eq. (11), and afterwards using [17, eqn. (2.24.1.1)]. Hence, the closed-form solution for the ergodic capacity corresponding to the direct transmission can be seen in Eq. (17) at the top of the next page. An asymptotic expression for the ergodic capacity corresponding to the direct transmission at high SNR can be obtained as in Eq. (14) as follows

$$C_{\mathrm{DT}}^{H} \doteq \frac{B}{2\ln(2)} \frac{\partial \mathbb{E}[(4\gamma_0 I_{SD}^2)^n]}{\partial n}\bigg|_{n=0}, \qquad (18)$$

where $\mathbb{E}[(4\gamma_0 I_{SD}^2)^n]$ denotes the nth moment of instantaneous electrical SNR. Performing some algebraic manipulations in Eq. (18), the asymptotic closed-form solution for the ergodic capacity corresponding to the direct transmission at high SNR, C_{DT}^{H}, can be accurately estimated as

$$\begin{aligned} C_{\mathrm{DT}}^{H} &\doteq \frac{B\ln(4\gamma_0)}{2\ln(2)} + \frac{B}{\ln(2)} \\ &\times \left(\psi(a_{SD}) + \psi(b_{SD}) - \frac{1}{\varphi_{SD}^2} - \ln\left(\frac{a_{SD} b_{SD}}{A_{SD} L_{SD}}\right)\right). \end{aligned} \qquad (19)$$

For the better understanding of the study of the ergodic capacity in cooperative FSO systems when line of sight is taken into account, the ergodic capacity is depicted in Fig. 2 for a source-destination link distance of $d_{SD} = 3$ km when different relay locations are considered. Different weather conditions are adopted: haze visibility of 4 km with $C_n^2 = 1.7\times10^{-14}\ m^{-2/3}$ and clear visibility of 16 km with $C_n^2 = 8\times10^{-14}\ m^{-2/3}$, corresponding to moderate and strong turbulence, respectively. Here, a and b are calculated from Eq. (3) and, a value of $\lambda = 1550$ nm is assumed. Pointing errors are here present assuming values of normalized beam width of $\omega_z/r = \{5, 10\}$ and a value of normalized jitter of $\sigma_s/r = 1$ for each link. It is clear to observe that the obtained ergodic capacity in Eq. (12) is very accurate in the entire SNR regime, i.e. from low to high SNR. A relevant improvement in terms of the capacity has been achieved under different turbulence conditions and pointing error effects. As previously mentioned, the ergodic capacity for direct transmission is also included as a benchmark in order to establish the baseline performance. As expected, the ergodic capacity of the considered cooperative FSO system is strongly dependent not only on the relay location but also on the pointing error effects. Monte Carlo simulations results are also included as a reference (Eq. (5) for BDF cooperative protocol and, Eq. (16) for direct transmission), confirming the accuracy of the proposed α-μ approximation, and usefulness of the derived results. There is quite a match between simulated and analytical results as well as between simulated and asymptotic results at high SNR. This analysis can be extended in order to obtain a point where the asymptotic ergodic capacity at high SNR intersects with the γ_0-axis. This point can be understood as a SNR threshold, i.e. $\gamma_{\mathrm{DF}}^{th}$, in which the ergodic capacity is significantly increased. From Eq. (15), it is easy to derive the corresponding expression of $\gamma_{\mathrm{DF}}^{th}$ in terms of the α-μ parameters and, it is given by

$$\gamma_{\mathrm{DF}}^{th}[dB] = \frac{20}{\ln(10)} \left(\ln\left(\frac{\sqrt{2}\Gamma\left(\mu + 1/\alpha\right)}{\mathbb{E}[I_T]\Gamma(\mu)}\right) - \frac{\psi(\mu)}{\alpha}\right). \qquad (20)$$

Similar to Eq. (20), we can obtain the corresponding SNR threshold, i.e. $\gamma_{\mathrm{DT}}^{th}$, for the direct transmission without cooperative communication and, it is given by

$$\begin{aligned} \gamma_{\mathrm{DT}}^{th}[dB] &= -\frac{20}{\ln(10)} \left(\ln(2) - \ln\left(\frac{a_{SD} b_{SD}}{A_{SD} L_{SD}}\right)\right) \\ &- \frac{20}{\ln(10)} \left(\psi(a_{SD}) + \psi(b_{SD}) - \frac{1}{\varphi_{SD}^2}\right). \end{aligned} \qquad (21)$$

It can be observed from this asymptotic analysis at high SNR that the shift of the ergodic capacity versus SNR is more relevant than the slope of the curve in SNR compared to other performance metric such as BER and outage probability. This shift can be interpreted as an improvement on ergodic capacity. From Eqs. (20) and (21), we can obtain this improvement or gain, i.e. $G[dB]$, as $G[dB] = \gamma_{\mathrm{DT}}^{th}[dB] - \gamma_{\mathrm{DF}}^{th}[dB]$. It can be seen in Fig. 2 gain values of 3.21 and 3.18 dB for moderate turbulence as well as gain values of 3.34 and 3.31 dB for strong turbulence, when values of normalized

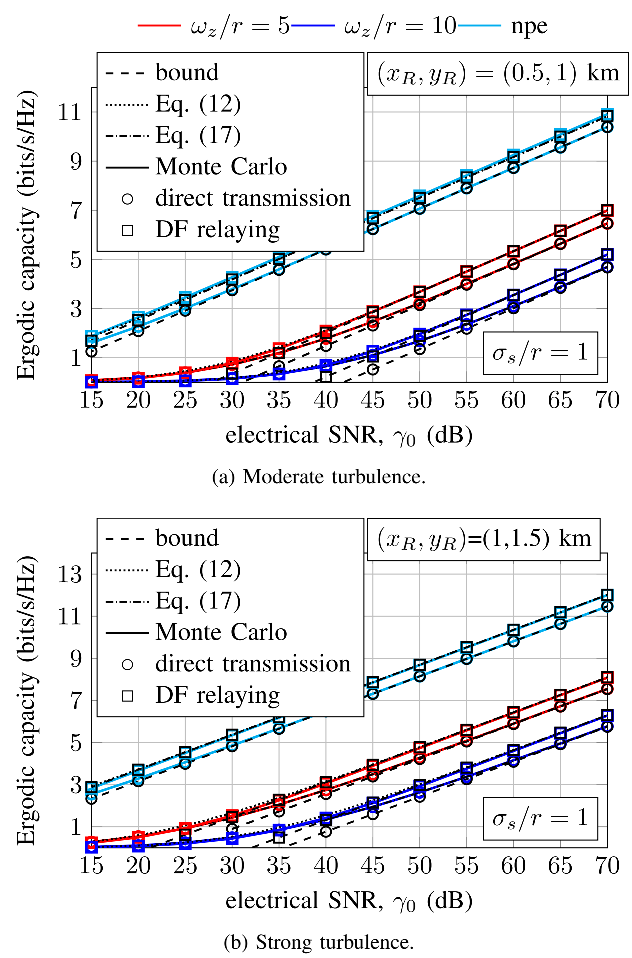


Fig. 2. Ergodic capacity of BDF relaying scheme for a source-destination link distance of $d_{SD} = 3$ km when different weather conditions are assumed.

$$C_{\text{DT}} = \frac{B\varphi_{SD}^2 2^{a_{SD}+b_{SD}-4}}{\pi \ln(2)\Gamma(a_{SD})\Gamma(b_{SD})} G_{8,4}^{1,8}\left(\left(\frac{8A_{SD}L_{SD}}{a_{SD}b_{SD}}\right)^2\gamma_0 \middle| \begin{matrix} 1,1,\frac{1-a_{SD}}{2},\frac{2-a_{SD}}{2},\frac{1-b_{SD}}{2},\frac{2-b_{SD}}{2},\frac{1-\varphi_{SD}^2}{2},\frac{2-\varphi_{SD}^2}{2} \\ 1,0,-\frac{\varphi_{SD}^2}{2},\frac{1-\varphi_{SD}^2}{2}\end{matrix}\right). \tag{17}$$

beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (5, 1)$ and $(\omega_z/r, \sigma_s/r) = (10, 1)$ are considered, respectively. Now, the error-rate performance analysis in [4] is taken into account, in which was demonstrated that the diversity order gain does not depend on pointing errors when the relation $\varphi^2 > \beta$ is satisfied. Under this desirable scenario, the effect of misalignment on the ergodic capacity in this cooperative FSO system is analyzed. Knowing that the impact of pointing errors in our analysis can be suppressed by assuming $A_0 \to 1$ and $\varphi^2 \to \infty$ [13], the corresponding gain disadvantage, $D_{pe}^{\text{DF}}[dB]$, relative to this 3-way cooperative FSO system without misalignment fading can be derived from Eq. (20) as

$$D_{pe}^{\text{DF}}[dB] = \gamma_{\text{DF}}^{th}[dB] - \gamma_{\text{DF}_{npe}}^{th}[dB]. \tag{22}$$

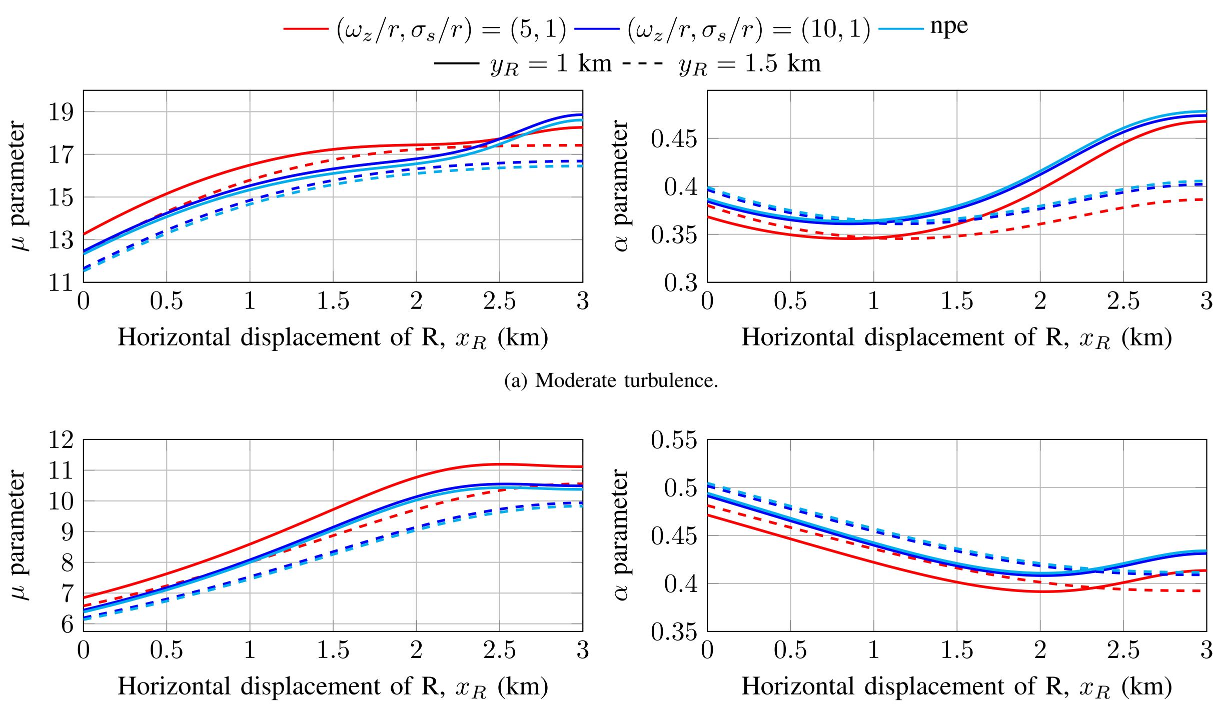
At this point, it must be noted that the parameters α and μ tend to remain at a constant level as the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver increases, i.e. $\varphi^2 \to \infty$. This conclusion has been carefully checked by numerical observation and, hence, the expression in Eq. (22) can be approximated as

$$D_{pe}^{\text{DF}}[dB] \approx \frac{20}{\ln(10)} \ln\left(\frac{\mathbb{E}\left[I_T^{npe}\right]}{\mathbb{E}\left[I_T\right]}\right), \tag{23}$$

where $\mathbb{E}[I_T^{npe}] = L_{SD} + 2L_{RD}$. In order to validate the previous statement, the parameters α and μ are illustrated in Fig. 3 as a function of the horizontal displacement of the relay node R when different relay locations $y_R = \{1, 1.5\}$ km are assumed. The depicted curves in Fig. 3 have been obtained by using a numerical approach due to the fact that finding the relation between α-μ and a-b parameters can be time-consuming and are technically difficult to perform. Note that the obtained results in Fig. 3 both α and μ when pointing errors are suppressed can be considered negligible as the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver increases. Therefore, the expression in Eq. (23) can be simplified under the assumption that all links are affected by the same values of normalized beam width and normalized jitter as follows

$$D_{pe}^{\text{DF}}[dB] \approx \frac{20}{\ln(10)} \ln\left(\frac{1+\varphi^2}{A_0\varphi^2}\right). \tag{24}$$

The expression in Eq. (24) is the gain disadvantage corresponding to the direct transmission and, hence, this can be only used when the same values of normalized beam width and normalized jitter are assumed for each link. Moreover, it can be easily deduced that this gain disadvantage depends neither on the relay location nor on the atmospheric turbulence conditions as φ^2 increases. The expression in Eq. (24) can be simplified even further as $\varphi^2 \to \infty$, obtaining an expression only dependent on the value of normalized beam width, i.e.



(a) Moderate turbulence.

(b) Strong turbulence.

Fig. 3. α-μ parameters as a function of the relay location for a source-destination link distance of $d_{SD} = 3$ km when different weather conditions are assumed.

A_0. Hence, the gain disadvantage corresponding to the BDF cooperative protocol can be accurately approximated by

$$D_{pe}^{\mathrm{DF}}[dB] \approx -\frac{20\ln(A_0)}{\ln(10)}. \quad (25)$$

The gain disadvantage, $D_{pe}^{\mathrm{DF}}[dB]$, is depicted in Fig. 4 as a function of the ratio between ω_z and σ_s. It can be observed that there is a perfect match between the exact results and obtained results for greater values of ω_z/σ_s than 7 by using the approximate expression for the gain disadvantage of BDF relaying. According to the expression in Eq. (25), it can be seen in Fig. 2 gain disadvantage of 22.3 and 34.07 dB when values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (5, 1)$ and $(\omega_z/r, \sigma_s/r) = (10, 1)$ are considered, respectively.

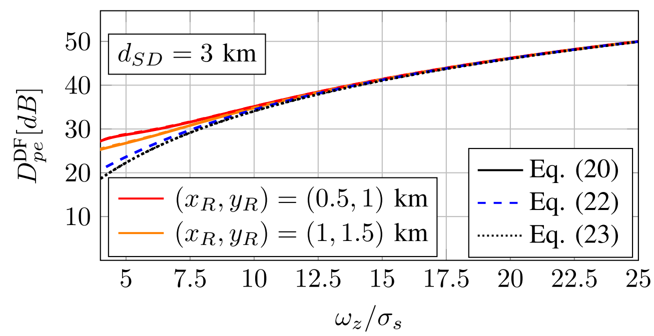


Fig. 4. Gain disadvantage, $D_{pe}^{\mathrm{DF}}[dB]$ for a source-destination link distance of $d_{SD} = 3$ km.

IV. Conclusion

The ergodic capacity of BDF cooperative protocol is analyzed over gamma-gamma fading channels with pointing errors when line of sight is available. Novel closed-form approximate ergodic capacity expression is obtained in terms of the H-Fox function for a 3-way FSO communication system when the α-μ distribution to efficiently approximate the PDF of the sum of gamma-gamma with pointing errors variates is considered. Simple asymptotic expression at high SNR for the ergodic capacity of BDF cooperative protocol is obtained providing a perfect match between simulated and analytical results. It can be concluded that cooperative protocols such as BDF relaying are able to achieve a greater ergodic capacity than a direct transmission without cooperative communication. In addition, it is demonstrated that the ergodic capacity is strongly dependent on the relay location as well as pointing error effects. Apart from that, from the asymptotic ergodic capacity analysis can be concluded that the shift of the ergodic capacity versus SNR is more relevant than the slope of the curve in SNR compared to other performance metric such as BER and outage probability. Finally, the impact of the pointing errors on the ergodic capacity is deeply analyzed, which corroborates that the presence of pointing errors requires an increase in SNR in order to maintain the same performance in terms of capacity. This increase is related to the parameter A_0.

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