Removing redundancy for attribute implications in data with grades

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Extended abstract

Reasoning with if-then rules –in particular, with those taking from of implications between conjunctions of attributes– is crucial in many disciplines ranging from theoretical computer science to applications. One of the most important problems regarding the rules is to remove redundancies in order to obtain equivalent implicational sets with lower size. This problem, which has been widely studied in the classical setting, is partially addressed in [4] for the case of attribute implications in data with grades. In this work, we tackle this problem using the so-called *Fuzzy Attribute Simplification Logic*, FASL, which has been introduced in [1]. This logic leads to an automatic reasoning method for implications in data with grades.

FASL manages formulas $A \Rightarrow B$ where A and B are fuzzy sets over an alphabet Ω whose elements are named attributes. Interpretations are fuzzy formal contexts and, informally, an implication such as $\{a, {}^{0.5}/b\} \Rightarrow \{{}^{0.9}/c\}$ means every object that has attribute a to degree 1 (i.e. fully possesses a), and attribute b to degree 0.5, has attribute c to degree at least 0.9.

Specifically, truthfulness structures in FASL are tuples $\langle L, \vee, \wedge, \otimes, \rightarrow, \smallsetminus, ^*, 0, 1\rangle$ where $\langle L, \vee, \wedge, \otimes, \rightarrow, 0, 1\rangle$ is a complete residuated lattice, * is an hedge (a "very true" function [2]) and $\smallsetminus$ is a binary operation satisfying the following adjointness property: $a \smallsetminus b \leq c$ if and only if $a \leq b \vee c$ for all $a, b, c \in L$. As a consequence, $a \smallsetminus b = \bigwedge\{c \in L \,|\, a \leq b \vee c\}$. In particular, when the lattice is linearly ordered, we have

$$a \smallsetminus b = \begin{cases} a, & \text{if } a > b, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

These operations are pointwise extended to fuzzy sets.

For a fuzzy formal context $\mathbb{K} = \langle X, Y, I\rangle$ such that $Y = \Omega$, the degree in which $A \Rightarrow B$ is true in $\mathbb{K}$ is defined as follows:

$$\|A \Rightarrow B\|_{\mathbb{K}} = \bigwedge_{x \in X} \Big(\big(\bigwedge_{y \in Y} (A(y) \rightarrow I(x,y))\big)^* \rightarrow \bigwedge_{y \in Y} (B(y) \rightarrow I(x,y)) \Big) \tag{2}$$

The axiomatic system in FASL is defined as follows: for all $A, B, C, D \in L^{\Omega}$ and $c \in L$,

[Ax] infer $A \cup B \Rightarrow A$ (*Axiom*)

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[Mul] from $A \Rightarrow B$ infer $c^*{\otimes}A \Rightarrow c^*{\otimes}B$ (*Multiplication*)

[Sim] from $A \Rightarrow B$ and $C \Rightarrow D$ infer $A \cup (C \smallsetminus B) \Rightarrow D$ (*Simplification*)

The soundness and completeness are ensured when we assume that both L and Ω are finite. In addition, for any $A,B,C,D \in L^Y$, the following equivalences hold true:

(DeEq) $\{A \Rightarrow B\} \equiv \{A \Rightarrow B \smallsetminus A\}$;

(UnEq) $\{A \Rightarrow B, A \Rightarrow C\} \equiv \{A \Rightarrow B \cup C\}$;

(SiEq) If $A \subseteq C$ then $\{A \Rightarrow B, C \Rightarrow D\} \equiv \{A \Rightarrow B, A \cup (C \smallsetminus B) \Rightarrow D \smallsetminus B\}$.

In [4], V. Vychodil consider *globalization* as hedge and provides a polynomial algorithm for computing a non-redundant equivalent set for a given implicational set. An implicational set T is said to be non-redundant if no implications in T can be inferred from others implications. The main goal here is in the same direction, but considering an arbitrary hedge and using FASL. As a first stage in this line, we propose the use of **(DeEq)**, **(UnEq)** and **(SiEq)** for removing redundant information in an implicational set. Specifically, the method here proposed crosses the sets of implications (quadratic complexity) taking pairs of implications and applies the above mentioned equivalences.

For instance, consider $\langle\{0,0.5,1\},\vee,\wedge,\otimes,\rightarrow,\smallsetminus,^*,0,1\rangle$ being the three-element equidistant subchain of the standard Łukasiewicz algebra with * being the identity and $\smallsetminus$ as described in (1). For the implicational set

$$T_1 = \big\{\{{}^{0.5}/x, y, {}^{0.5}/z\} \Rightarrow \{x, {}^{0.5}/y, {}^{0.5}/z\}, \{{}^{0.5}/y\} \Rightarrow \{{}^{0.5}/x, {}^{0.5}/y, z\},$$
$$\{{}^{0.5}/x, {}^{0.5}/y\} \Rightarrow \{y,z\}, \{x,z\} \Rightarrow \{{}^{0.5}/x, y, {}^{0.5}/z\}, \{z\} \Rightarrow \{{}^{0.5}/x\}\big\}$$

in [4], the author obtains

$$T_2 = \big\{\{{}^{0.5}/y\} \Rightarrow \{x,y,z\}, \{z\} \Rightarrow \{{}^{0.5}/x\}\{x,z\} \Rightarrow \{{}^{0.5}/x, y, {}^{0.5}/z\}\big\}$$

However, our method, which has been implemented in PROLOG, renders the following equivalent set:

$$T_3 = \big\{\{{}^{0.5}/x\} \Rightarrow \{x\}, \{{}^{0.5}/y\} \Rightarrow \{{}^{0.5}/x, y, z\}, \{z\} \Rightarrow \{y\}\big\}$$

References

[1] Radim Belohlavek, Pablo Cordero, Manuel Enciso, Ángel Mora, Vilem Vychodil: *Automated prover for attribute dependencies in data with grades.* International Journal of Approximate Reasoning. Vol. 70 (2016) 51–67.

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[3] Radim Belohlavek, Vilem Vychodil. Attribute implications in a fuzzy setting. Lecture Notes in Computer Science. Vol. 3874, 45–60. Vilem Vychodil. On minimal sets of graded attribute implications, Information Sciences. Vol. 294, 478–488.

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