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Wageningen University and University of Almeria
Dpt.Computer Architecture, Operations Research and Logistics – Computer Architecture and Electronics

ON MULTIMODALITY
OF OBNOXIOUS FACILITY LOCATION MODELS

MSc. THESIS

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Preface

Writing this Master thesis was an interesting and joyful journey. First of all it was a scientific journey, where I learned more than I could ever imagine on forehand. The first person I need to thank for that is my supervisor Dr. Eligius Hendrix. The second supervisor, Dra. Pilar Martinez Ortigosa, I am very grateful for sharing her in dept knowledge on algorithms. I like to thank Dra. Juana Lopez Redondo for her useful comments on my thesis. Dr. Leocadio Gonzalez Casado taught me the basics of programming and Branch and Bound algorithms in the course ´Algoritmos de Optimizacion Global: Estrategias Paralelas´ of the University of Almeria (UAL), he was very patient if I didn't understand everything immediately. Furthermore I want to thank all people of the department of Computers Architecture and Electronics of the UAL, led by Dra. Inmaculada Garcia Fernandez, for giving me a warm welcome and the necessary facilities for writing this thesis. The journey I have made during the research for my thesis was not bounded by the walls of the university. I've got the opportunity to meet many international researchers on location science during the Euro Working Group on Locational Analysis (EWGLA XVII) in Elche, Spain, see (EWGLA, 2008). It helped me to get a better insight in locational problems. The University of Seville invited me to present my research at their University. I am really grateful that they gave me the chance to share ideas about my research. Finally, Wageningen University gave me the education that led to this MSc. thesis, the result of my journey.

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Abstract

Obnoxious single facility location models are models that have the aim to find the best location for an undesired facility. Undesired is usually expressed in relation to the so-called demand points that represent locations hindered by the facility. Because obnoxious facility location models as a rule are multimodal, the standard techniques of convex analysis used for locating desirable facilities in the plane may be trapped in local optima instead of the desired global optimum. It is assumed that having more optima coincides with being harder to solve. In this thesis the multimodality of obnoxious single facility location models is investigated in order to know which models are challenging problems in facility location problems and which are suitable for site selection. Selected for this are the obnoxious facility models that appear to be most important in literature. These are the maximin model, that maximizes the minimum distance from demand point to the obnoxious facility, the maxisum model, that maximizes the sum of distance from the demand points to the facility and the minisum model, that minimizes the sum of damage of the facility to the demand points. All models are measured with the Euclidean distances and some models also with the rectilinear distance metric. Furthermore a suitable algorithm is selected for testing multimodality. Of the tested algorithms in this thesis, Multistart is most appropriate. A small numerical experiment shows that Maximin models have on average the most optima, of which the model locating an obnoxious linesegment has the most. Maximin models have few optima and are thus not very hard to solve. From the Minisum models, the models that have the most optima are models that take wind into account. In general can be said that the generic models have less optima than the weighted versions. Models that are measured with the rectilinear norm do have more solutions than the same models measured with the Euclidean norm. This can be explained for the maximin models in the numerical example because the shape of the norm coincides with a bound of the feasible area, so not all solutions are different optima. The difference found in number of optima of the Maxisum and Minisum can not be explained by this phenomenon.

1.Introduction

1.1 Background of the problem

Since the second half of the last century, recent advances and innovations in technology and industry created many facilities that are needed but may pose a serious danger to the individuals living nearby. These facilities are called obnoxious (or undesired) facilities. Obnoxious is usually expressed in relation to the so-called demand points that represent locations hindered by the facility. Erkut and Neuman (1989) defined an obnoxious facility as one that generates a disservice to the people nearby while producing an intended product or service. Examples of obnoxious facilities are a hazardous waste disposal site or a nuclear plant. In order to find the optimal site for an obnoxious facility existing (friendly) facility location models are adjusted (Carrizosa, 1999). Instead of locating the facility as near as possible, the new models aim for maximizing the minimal distance between the facility and the demand point (Boas Ben-Moshe, 2000).
As a result of adapting existing friendly facility location models, things are more complicated from an algorithmic point of view. Carrizosa states in (Carrizosa, 1999) that because obnoxious models as a rule are multimodal, the standard techniques of convex analysis used for locating desirable facilities in the plane (e.g. Michelot, 1993) may be trapped in local optima instead of the desired global optimum (see Figure 1). Multimodal functions are characterized by having more than one optima, but can either have a single or more than one global optima. In ´Stochastic Global Optimization´ (Zhigljavsky, 2008) an objective function is called multimodal if either there is more than one local minimum or the number of local minima is unknown.

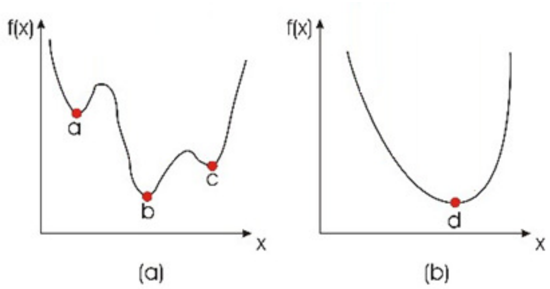

Fig. 1. Optimization problem with nonconvex objective f(x) (a) with local optima point a and c and a global optimum in point b and convex objective (b) with only a global optima, indicated by d (Demeulenaere, 2008).

Many optima as an outcome of the model are undesired because the aim of the model is to find a global optimum, as this is the best location for the obnoxious facility. More optima give a higher chance to get stuck in a local optimum.
The objective of this thesis is to determine which obnoxious single facility location models are suitable for site selection, detecting a globally optimal plan, which means there are no better plans in the rest of the feasible area (Hendrix, 2007).

1.2 Objective

The objective of this thesis is to investigate which obnoxious single facility location models are suitable to determine the optimal location for an obnoxious facility by having a globally optimal plan. Therefore the obnoxious single facility location models are tested on their multimodality since is assumed that models with more optima are harder to solve.

1.3 Research questions

Research questions have been formulated to deal with the problem in a structured way.

1. How can a globally optimal plan be generated from multimodal obnoxious single facility location models?

2. What are challenging problems in obnoxious facility location models?

3. Which obnoxious 1-facility location models are known in literature?
   a) Do these methods have local or global optima as result?
   b) How many local optima do these models have?
   b) Are these models easy to solve?

4. Which algorithms can be used to test obnoxious single facility location models on multimodality?
   a) Are these algorithms effective:
   - Do they find the global optimum?
   - Do they find local optima?
   - Do they have a small variance in the found optimum

   b) Are these algorithms efficient:
   1. Which algorithm needs the least number of function evaluations per run to converge to an optimum?

2. Literature on Obnoxious Facility Location Models

2.1 Literature overview

In facility location problems the aim is to determine the best site for a new facility. The focus of most models is locating a facility in the centre of the so-called demand points in order to be able to serve as many customers as possible. However, there are facilities that are not wanted close to demand points because they have noxious effects. Within facility location science, there is special attention for obnoxious facility location models. These models are designed to find the best location for an obnoxious facility. Erkut and Neuman (1989) composed a list of criteria in order to classify the obnoxious facility location problems.

1. Number of facilities to be located
single facility
multiple facilities (fixed or variable number)

This criterion is whether to locate one undesired facility or more.

2. Feasible region
discrete
continuous
network

The second criterion is on the type of feasible region. The main types are discrete, continuous and network regions. Discrete location models are used when a facility is to be located to a site chosen among a discrete set of predetermined alternative locations. Continuous location models are models where a facility can be located anywhere in the feasible area. Network location models try to site a facility to a node or edge of a graph.

3. Distance measure
Euclidean
rectilinear
network

The third criterion is the distance measure. In order to measure distances between demand points and the facility one of the Minkowski's distances can be used:

Minkowski distance $l_p(x,y) = \left((x_1 - y_1)^p + (x_2 - y_2)^p\right)^{1/p} \qquad 1 \le p \le \infty$

Rectilinear distance:

$p = 1 \qquad l_1(x,y) = \left((x_1 - y_1)^1 + (x_2 - y_2)^1\right)^{1/1} = |x_1 - y_1| + |x_2 - y_2|$

Euclidean distance:

$p = 2 \qquad l_2(x,y) = \left((x_1 - y_1)^2 + (x_2 - y_2)^p\right)^{1/2} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$

The rectilinear distance corresponds to the shortest way of travelling from x to y using only horizontal and vertical movement.
The Euclidean distance is appropriate when movement is allowed in any direction and there are no obstacles to this movement.

4. Objective
maximin
maxisum
minisum
single objective
multiobjective

The fourth criterion considers the objective function. Erkut and Neuman (1989) distinguish two important objectives: the maximin and the maxisum objective. In this thesis models with the minisum objective are investigated, therefore the minisum objective is describes as well.

Maximin
The maximin objective maximizes the distance of the facility from the nearest demand point see a) in Figure 2.
The 1-maximin objective is equivalent to the geometrical problem of finding the largest empty circle that does not include any given demand point (Cappanera, 2003). This objective provides the highest protection on the demand point that is most influenced by the undesirable effects of the facility.

Maxisum
The maxisum objective considers the effect of the facility to all demand points by maximizing the sum of the distances between the obnoxious facility to the demand points see b) in Figure 2.

Minisum
The aim of Minisum models is to find an optimal location for an obnoxious facility where the damage to all demand points is minimized, see c) in Figure 2.

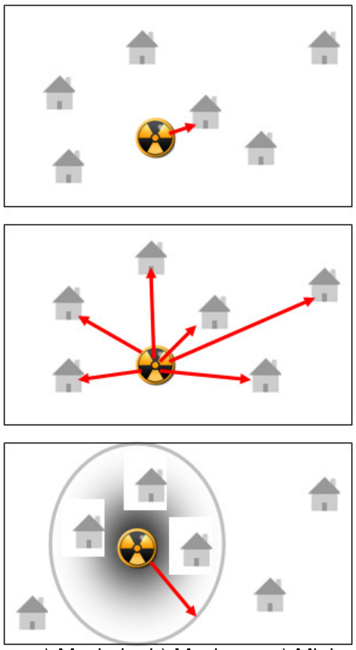

Fig. 2. From the top to the bottom: a) Maximin, b) Maxisum, c) Minisum

Single objective models are models with only the goal to minimize the obnoxious effect on individuals, while multi-objective models consider more than one objective. An example of a multi objective model is a semi obnoxious model, which has for instance the aim to maximize distance and to minimize costs at the same time.

The focus of this thesis is on single obnoxious facility location models in a continuous environment in which distances are measured either with Euclidean or rectilinear metrics. Therefore the literature overview is concentrated on models with the same criteria.

2.1 Literature search

In order to find models on the optimal location for a single obnoxious facility, the search machine of Scopus was used on the 3rd of November 2008. The database chosen is Scopus because it contains the largest number of articles with the keyword ¨obnoxious facility model¨ and ¨undesirable facility model¨ of the databases that are offered by the library of WUR. In Scopus a basic search was conducted as follows:

I.

Search for: obnoxious facility model **in** Article Title, Abstract, Keywords
Date Range
Published All years to Present **Document Type** All
Results: 42

Because an obnoxious facility is by some authors called an undesirable facility, a second search is done in Scopus.

II.

Search for: undesirable facility model **in** Article Title, Abstract, Keywords
Date Range
Published All years to Present **Document Type** All
Results: 106

To obtain only the relevant articles, in **Subject Area** the results of this second search are limited to:

- Decision Sciences	(28 articles)
- Mathematics	(18 articles)
- Business, Management and Accounting	(6 articles)
- Computer Science	(5 articles)
- Economics, Econometrics and Finance	(2 articles)

This limitation leads to 38 results for the second search. Some articles are tagged in more than one subject area.

The hits of the first and second search can be added up. So the total number of articles is 80 (see Appendix A). Ten articles are found in both searches, they overlap the searches, which results in 70 articles. Of these 70 articles, 25 articles can not be used for this thesis because they:

- are not available by Wageningen University library
- are not available on-line
- do not contain a model
- are still in press

Some of the articles that are not considered further do have an abstract that is available, but since that is the only information, they are deleted from the list anyway.

To conclude, 45 articles are used for this thesis.

Both the results for ¨obnoxious facility model¨ and ¨undesirable facility model¨ are scanned on the following criteria (see Appendix B):

1. Obnoxious and undesirable facility
2. Single facility
3. Single objective
4. Euclidean and rectilinear distance metric
5. In continuous space

Furthermore the focus is on three models, the Maximin, Maxisum and Minisum model.

The result leads to a collection of 12 articles, of which 3 contained two different models:

- 8 articles containing Maximin models
- 2 articles that contain Maxisum models
- 5 articles on Minisum models

In total 15 models are investigated on their multimodality.

By searching on the words ´obnoxious facility model´ and ´undesirable facility model´ in Scopus not all existing literature on obnoxious facility location models is found. The aim of this thesis is not to give a broad literature overview, but to find models that are commonly used, and test them on their multimodality. The search, how it is conducted for this thesis, is assumed to be sufficient to find all commonly used obnoxious single facility location models.

3. Obnoxious Single Facility Location Models

In this chapter the obnoxious single facility models that are selected from literature are presented (see Table 3). Before the description of the models, in the first section an overview is given of the notation of the indices, variables and data used to describe the models.

Table 3. Overview of selected obnoxious single facility location models

Model	Index	Title
Maximin	0	Generic model
	1	Obnoxious linesegment
	2	Polyhedral forbidden regions
	3	Mixed Integer Programming
	4	Circular and rectangular forbidden regions
	5,7,8	Generic model with weights
	6	Minimize maximum damage
Maxisum	0	Generic model
	1,2	Generic model with weights
Minisum	0	Generic model
	1,3	Wind dispersion (wind direction)
	2	Wind dispersion (wind velocity)
	4	Global repulsion
	5	Generic model with weights

3.1 Notation

The following notation will be used throughout this chapter:

<u>Indices</u>

i	index of demand points	
j	index of protected areas	
k	index of circular protected areas	
l	index of dimensions	l =1, 2
m	index of rectangular protected areas	
n	index of wind directions	n =N, S, W, E
o	index of wind velocities	

<u>Data</u>

P_i	location of demand point i
S	feasible area, a closed subset of R^2
T	feasible region minus the protected areas
U	feasible region minus the protected areas E_i and the forbidden regions F_j
l	length of linesegment $\underline{x}$, l = 0.25
E_j	polyhedron j, the protected area around a demand point
E_i	protected area around demand point i
P_{il}	dimension l of location vector P_i
M	a sufficiently large number, the size of feasible area S
C_k	location of circular protected area k
R_m	location of rectangular protected area m
w_i	positive weight associated to demand point i
L_i	distance decay of demand point i, $L_i > 0$
G	damage radius
n_n	frequency of occurrence of wind direction n
n_o	frequency of occurrence of wind velocity o
O	pollution dispersion rate
π	3.1415…
μ_n	mean wind velocity of wind direction n

u_o wind velocity o (in knots)
h stack height (15 m)
F_j forbidden region j
α_i start repulsion value
β_i repulsion decay rate

Variables
z_a minimum distance over demand points i to the location of obnoxious facility x
z_b minimum distance over demand points i to the location of obnoxious linesegment $\underline{x}$
z_c minimum distance over protected areas i to the location of obnoxious facility x
z_d sum of distance over demand points i to the location of obnoxious facility x
z_e sum of damage from obnoxious facility x to demand points i
z_f sum of pollution damage from obnoxious facility x to demand points i
z_g sum of repulsion from demand points i to the location of obnoxious facility x
x location of the obnoxious facility
$\underline{x}$ location of the obnoxious linesegment, consisting of x_0 and x_E, where x_0 is the origin and x_E is the end of the linesegment
d_i distance from x to P_i
d_{il}^- surplus variable to express $x_l < P_{il}$
d_{il}^+ surplus variable to express $x_l > P_{il}$
t_{il} binary variable
q damage from obnoxious facility x to the most affected demand point
f_i damage to demand point i, binary variable
p_i (1st coordinate of P_i) – (1st coordinate of x)
y_i (2nd coordinate of P_i) – (2nd coordinate of x)

3.2 Maximin Models

A maximin model is defined as a model that has the aim to find a location for an obnoxious facility in a geographical region, for which the minimum distance from the obnoxious facility to the closest demand point is maximized. A generic single facility maximin location problem is given by model 0.

3.2.1 Model 0. Generic model

Objective function

$$\max z_a \qquad (1)$$

Subject to

$$z_a \leq d(P_i, x) \qquad \textit{for all } i \qquad (1.1)$$

Where $d(P_i, x)$ is the Euclidean or rectilinear distance from demand point i to obnoxious facility x

$$x \in S \qquad (1.2)$$

Result of the literature research
In the literature research eight maximin models have been found. These models correspond to the following numbers (in Appendix A):
Search I: Obnoxious facility model:
Model 1 (article nr. 16: Barcia et al. 2003)
Model 2 (article nr. 31: Fernandez et al. 1997)

Search II: Undesirable facility model:
Model 3 (article nr. 1: Nadirler et al. 2007)
Model 4 (article nr. 6: Caceres et al. 2007)
Model 5 (article nr. 8: Saameno et al. 2006)
Model 6 (article nr. 23: Carrizosa et al. 1998)
Model 7 (article nr. 29: Erkut et al. 1989)
Model 8 (article nr. 32: Melachrinoudis, 1985)
In the following sections these models are discussed.

3.2.2 Model 1. Obnoxious linesegment

This paper is considering an obnoxious facility that is not a point but a line segment. A location for an undesirable anchored segment of fixed length has to be found. An example for an application is the transportation of hazardous materials from a fixed site across a linear path with a bounded length.

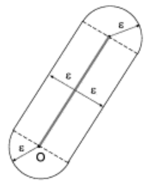

Fig. 3.1. Obnoxious linesegment where ε is the damage radius of the obnoxious linesegment (Barcia, 2003).

In Figure 3.1, the distance of size ε from the obnoxious linesegment $\underline{x}$ to a demand point i, if the demand point is on the hippodrome, is shown.

Objective function

$$\max z_b \qquad (1.3)$$

Subject to

$$z_b \leq d(P_i, \underline{x}) \qquad \textit{for all } i \qquad (1.4)$$

Where $d(P_i, \underline{x})$ is the Euclidean minimum distance between demand point i and a point of linesegment $\underline{x}$

$$\underline{x} = \left\{ x \in R^2 \,/\, x = \lambda \begin{pmatrix} x_{01} \\ x_{02} \end{pmatrix} + (1-\lambda) \cdot \begin{pmatrix} x_{E1} \\ x_{E2} \end{pmatrix} \right\} \quad 0 \leq \lambda \leq 1 \qquad (1.5)$$

$$\sqrt{((x_{E1} - x_{01})^2 + (x_{E2} - x_{02})^2)} = l \qquad (1.6)$$

$$\underline{x} \in S \qquad (1.7)$$

3.2.3 Model 2. Polyhedral forbidden regions

The problem deals with locating a point in a given convex polyhedron which maximizes the minimum Euclidean distance from a given set of convex polyhedra representing protected areas around population points (Figure 3.2). The problem is to locate a single undesirable facility in the permissible region so as to maximize its Euclidean distance from the nearest polygonal forbidden region. This means that the facility should be located as far away as possible from the protection area around the nearest demand point.
In the test case there is no information on size of protection areas around demand points. Therefore weights are given to the demand points, depending on the size of the areas around them. If an area around a demand point is large, a high weight is given (e.g. 5). The other way around, a low weight is given (e.g. 1). These weights are assigned to generated protection areas. The aim of this is to locate the facility as far away as possible from the closest border of the protection area.

Objective function

$$\max z_c \tag{1.8}$$

Subject to

$$z_c \le d(E_j, x) \qquad \textit{for all j} \tag{1.9}$$

Where $d(E_j, x)$ is the minimum Euclidean distance from facility location x to a point of protected area E_j

$$x \in T \tag{1.10}$$

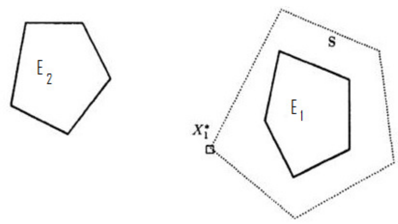

Fig. 3.2. X_1^* is a global maximum although it is not equidistant to E_1 and E_2 (Fernandez, 1997).

3.2.4 Model 3. Mixed Integer Programming

In this paper a 1-maximin problem with rectilinear distances is studied. A single undesirable facility in a continuous planar region is located while considering the interaction between the facility and existing demand points. The 1-maximin problem has been formulated as an MIP model in the literature (Sayin, 2000). New bounding schemes are suggested to increase the solution efficiency. It is an adjustment on Sayins model to make it easier to solve.

Objective function

$$\max z_a \tag{1.11}$$

Subject to

$$z_a \le d_i \qquad \textit{for all i} \tag{1.12}$$

$$d_i = d_{i1}^+ + d_{i1}^- + d_{i2}^+ + d_{i2}^- \qquad \textit{for all i} \tag{1.13}$$

$$d_{il}^+ - d_{il}^- = x_l - P_{il} \qquad \textit{for all i, for all l} \tag{1.14}$$

$$d_{il}^+ \le M t_{il} \qquad \textit{for all i, for all l} \tag{1.15}$$

$$d_{il}^- \le M(1 - t_{il}) \qquad \textit{for all i, for all l} \tag{1.16}$$

$$d_{il}^+, d_{il}^- \ge 0 \qquad \textit{for all i} \tag{1.17}$$

$$t_{il} \in \{0,1\} \qquad \textit{for all i, for all l} \tag{1.18}$$

$$x \in S \tag{1.19}$$

Sayin tries to linearize the problem by using a mixed integer mathematical model in which the rectilinear distance is calculated by a set of constraints controlled by binary variables.

3.2.5 Model 4. Circular and rectangular forbidden regions

All demand points are surrounded by a protection area. The aim is to locate the facility as far as possible from the closest border, while remaining in the feasible area. The shape of the border can have the shape of a rectangle or of a circle (see Figure 3.3). The sides of the rectangle should be parallel to the axes. A set of points and line segments should be considered in order to find the optimal location for the facility.

Objective function

$$\max z_c \qquad (1.20)$$

Subject to

$$z_c \leq d_k(C_k, x) \qquad \textit{for all } k \qquad (1.21)$$

$$z_c \leq d_m(R_m, x) \qquad \textit{for all } m \qquad (1.22)$$

Where $d_k(C_k, x)$ is the Euclidean distance from obnoxious facility x to the circular protected area k and $d_m(R_m, x)$ is the Euclidean distance from obnoxious facility x to the rectangular protected area m.

$$x \in T \qquad (1.23)$$

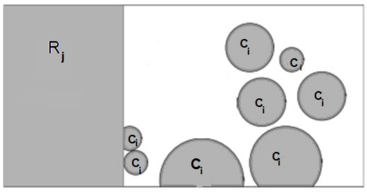

Fig. 3.3. A situation with a rectilinear protected area R_m and circular protected areas C_k (Carceres, 2007).

3.2.6 Model 5, 7, 8. Generic model with weights

Model 5.

This paper describes a method to determine a finite set in which an optimal solution is located for a general Euclidean problem of locating an undesirable facility in a polygonal region. It can be determined in polynomial time. The general problem that is proposed leads to several well known problems, such as the maximin problem.

Model 7.

This paper contains a survey of the maximization location models in the Operations Research literature. One of the models is the maximin model. In this article both Euclidian and rectilinear distances are suggested as distance metric.

Model 8.

The problem in this paper is formally defined to be the selection of a location within the convex region that maximizes the minimum weighted Euclidean distances with respect to all existing facilities (Figure 3.4).

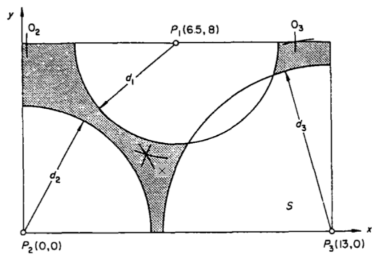

Fig. 3.4. An example with 3 weighted demand points and a possible location in between them (Melachrinoudis, 1985).

The models of articles 5, 7 and 8 have the same model formulation. The Maximin model in these articles is formulated as the generic maximin model, model 0, only weights are attached to the demand points. Constraint 1.1 of model 0 is replaced by constraint 1.24:

$$z_a \leq w_i d(P_i, x) \qquad \textit{for all } i \qquad (1.24)$$

3.2.7 Model 6. Minimize maximum damage

Suppose that an undesirable facility is to be located at some point x within a region S. The facility will affect the existing population. In this paper a model is described which seeks location x for which the highest damage to any of the demand points P_i is minimized.

Objective function

$$\min q \qquad (1.25)$$

Subject to

$$q \geq \frac{w_i}{d(P_i, x)^{L_i}} \qquad \textit{for all } i \qquad (1.26)$$

Where $d(P_i, x)$ is the Euclidean distance between P_i and x

$$x \in S \qquad (1.27)$$

3.3 Maxisum Models

Introduction
The maxisum criterion (or maxian or antimedian) attempts to maximize the sum of the distances from the undesirable (obnoxious) facility to the population centres. The optimal facility location will always be in the boundary of the feasible region (Saameño, 2006). This means that there is a limited number of optimal solutions. A generic single facility maxisum location problem is given by model 0.

3.3.1 Model 0. Generic model

Objective function

$$\max z_d \tag{2}$$

Subject to

$$z_d = \sum_{i=1}^{n} d(P_i, x) \tag{2.1}$$

Where $d(P_i, x)$ is the Euclidean or rectilinear distance from the obnoxious facility x to the demand point i.

$$x \in S \tag{2.2}$$

Result of the literature research
In the literature research two Maxisum models are found in articles that correspond with the number between brackets (see Appendix A for article titles).
Maxisum models that satisfy all criteria:
Search II: Undesirable facility model:
Model 1 (article nr. 8: Saameno et al. 2006)
Model 2 (article nr. 29: Erkut et al. 1989)
In the following sections these models are discussed.

3.3.2 Model 1, 2. Generic model with weights

Model 1.
The general problem that is proposed in this article leads to several well known problems, such as the maxisum problem. The Maxisum model given in this article is the same as model 2 (Erkut et al.,1989).

Model 2.
Erkut et al. express the generic single facility maximum problem an the feasible region S. They give Euclidean as well as rectilinear versions of this problem.

The model is defined as the generic model, only weights are attached to the demand points. Constraint (2.1) of model 0 is replaced by constraint (2.3).

$$z_d = \sum_{i=1}^{n} w_i d(P_i, x) \tag{2.3}$$

The authors state that there is little literature on 1-maxisum problems. In search for Maxisum problems for this thesis the same can be concluded.

3.4 Minisum Models

The objective of a Minisum model is to locate an obnoxious facility such that it gives minimum damage to all demand points. A generic single facility Minisum model is given by model 0.

3.4.1 Model 0. Generic model

Objective function

$$\min z_e \tag{3}$$

Subject to

$$z_e = \sum_{i=1}^{n} f_i \cdot \frac{1}{d(P_i, x)^{L_i}} \tag{3.1}$$

Where $d(P_i, x)$ is the Euclidean or rectilinear distance from obnoxious facility x to demand point i.

$$f_i = \begin{cases} 0 & if \quad d(P_i, x) > G \\ 1 & if \quad d(P_i, x) \le G \end{cases} \qquad for\ all\ i \tag{3.2}$$

$$x \in S \tag{3.3}$$

Result of the literature research

Five Minisum models are found in articles corresponding to the numbers between brackets (see Appendix A for article titles).
Minisum models that satisfy all criteria:
Search I: Obnoxious facility model:
Model 1 (article nr. 34: Karkazis et al. 1992)
Model 2 (article nr. 35: Karkazis et al. 1991)
Model 3 (article nr. 36: Karkazis et al. 1992)

Search II: Undesirable facility model:
Model 4 (article nr. 20: Fernandez et al. 2000)
Model 5 (article nr. 23: Carrizosa et a. 1998)
In the following sections these models are discussed.

3.4.2 Model 1, 3 Wind dispersion (wind direction)

Model 1.

This article describes a minisum problem, with wind dispersion in Euclidean distances, see Figure 3.5. With regard to the location of the facility it is reasonable to search for a point minimizing the total pollution load, z_f, over all demand points and all wind velocities. The Minisum model given in this article is based on model 3 and therefore only model 3 is described below.

Model 3.

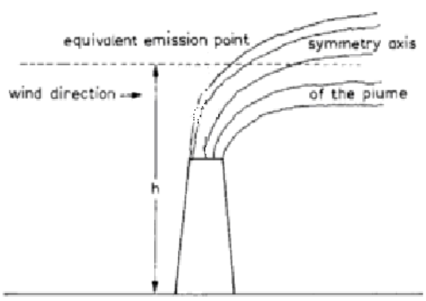

Fig. 3.5. The pollutant dispersion model

Objective function

$$\min z_f \qquad (3.4)$$

Subject to

$$z_f = \sum_i \sum_n n_n w_i f_n(P_i, x) \qquad (3.5)$$

$$f_n(P_i, x) = \frac{O}{\pi \mu_n q(p_i) V(p_i)} \cdot \exp(-0.5((\frac{y_i}{q(p_i)})^2 + (\frac{h}{V(p_i)})^2)) \quad \text{for all } n \qquad (3.6)$$

Where f_n is the pollution dispersion function for wind direction n

$$q(p_i) = 0.0804 p_i^{0.8929} \qquad (3.7)$$

and q is the horizontal diffusion parameter for 100 m ≤ p ≤ 120000 m.

$$V(p_i) = 8.3913 + -1.069 p_i^{1/2} + 19.445 p_i^{1/3} - 27.3357 p_i^{1/4} \qquad (3.8)$$

V is the vertical diffusion parameter.

$$x \in S \qquad (3.9)$$

For a certain wind direction n the damage to all demand points is calculated for obnoxious facility x. The goal of the model is to minimize the sum of damage of all these wind directions. The blue shape in Figure 3.6 shows the influence area from the obnoxious facility located in the middle of the circle, if there is a strong wind velocity from the Nord (N) East (O), with a high frequency of occurrence.

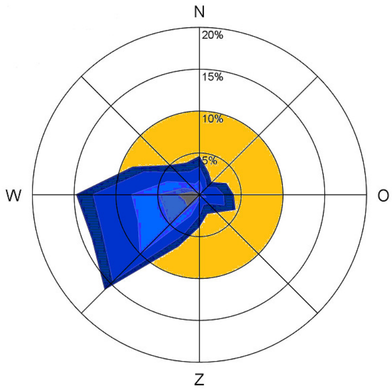

Fig. 3.6. The blue shape is an example of an influence area of an obnoxious facility that is located with model 1 and 3. The space within the orange circle is a possible influence are of an obnoxious facility located with model 2.

3.4.3 Model 2. Wind dispersion (wind velocity)

The model in this article is called the wind model. It has similarities to model 1, 3 but there are some differences. Index o in this model is the index of wind velocity instead of the index of wind directions. Constraint (3.6) of model 1,3 is replaced by constraint (3.10).

$$f_o(P_i, x) = \frac{O}{\pi u_o q(p_i) V(p_i)} \cdot \exp(-0.5((\frac{y_i}{q(p_i)})^2 + (\frac{h}{V(p_i)})^2)) \quad \text{for all } o \qquad (3.10)$$

The difference between this model and model in article 1 and article 3 is that the ´mean wind velocity μ_n´, which is the average wind velocity of wind direction n, is replaced by ´wind velocity u_o´, which stands for the o^{th} wind velocity and where the wind velocity is independent of the direction of the wind.

3.4.4 Model 4. Global repulsion

The objective of the model presented is to minimize the global repulsion of the inhabitants of the geographical region where the facility has to be located as they feel it. A non-linear decay function is proposed to measure the different degrees of repulsion that the inhabitants may feel depending on the kind of facility to be located or the special characteristics of the inhabitants of a city. Furthermore, environmental concerns are taking into account through the definition of protected areas where the location is not allowed. Around each city there is also a forbidden region to avoid the location of the polluting facility too close to it (see Figure 3.7).

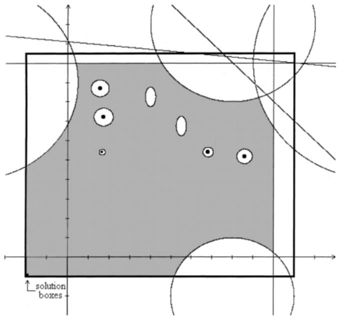

Fig. 3.7. An example of a feasible set in grey with the cities located in the solid black spots.

<u>Objective function</u>

$$\min z_g \tag{3.11}$$

<u>Subject to</u>

$$z_g = \sum_{i=1}^{n} w_i \cdot rp(P_i, x) \tag{3.12}$$

$$rp(P_i, x) = \frac{1}{1 + \exp(\alpha_i + \beta_i \cdot d(P_i, x))} \tag{3.13}$$

$$x \in U \tag{3.14}$$

The lower the value of α_i, the higher the repulsion of the inhabitants to the location of the facility near their city or its outlaying areas, and the higher the value of β_i, the faster is the change in the opinion from considering a distance non-acceptable to acceptable.

3.4.5 Model 5. Generic model with weights

Carrizosa presents a model in which one minimizes the total damage caused by the facility to the demand points. This model is the weighted version of the generic model. Besides the weights, model 5 differs because it has no influence radius involved. All demand points in the feasible region are considered. Constraint 3.1 is therefore replaced by constraint (3.15) and constraint (3.2) is deleted.

$$z_e = \sum_{i=1}^{n} \frac{w_i}{d(P_i, x)^{L_i}} \tag{3.15}$$

4. Algorithms for solving multimodal location problems

4.1 Introduction

In this chapter algorithms are presented that are tested on their ability of finding all optima of an obnoxious single facility location model. The algorithms described are:

1. Grid Search
2. Controlled Random Search
3. Genetic Algorithm
4. Multistart
5. MILP Branch-and-bound

For each algorithm the pseudo-code is given.

4.2 Grid Search

One of the simplest approaches to find an approximation of all optima is doing a grid search. Low dimensional problems can be solved by laying a grid on top of the feasible area (see Figure 4.1). At each grid point the function value is measured. A drawback of this method is that it is possible to miss an optimum. Another drawback is the relative high number of evaluations. The method requires K points to be evaluated. K is ((range of x/ grid size of x) + 1) * ((range of y/ grid size of y) + 1) (Hendrix, 2009).
This being said, here Grid Search is used to get an indication of the number of existing optima. The results of Grid Search are used as reference for the outcomes of the other algorithms.

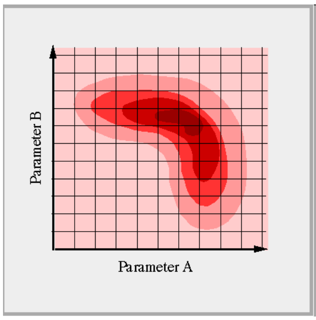

Fig. 4.1. Example of a grid over a feasible area (Berkeley Lab, 2009)

The Grid Search algorithm

(a) Generate grid points uniformly over the feasible area, with a certain mesh size.
(b) Evaluate all points on the grid.
(c) If a point in the grid has a higher function value than the 8 neighbour grid points, this is regarded an approximation of an optimum, when maximizing (see Table 4).

Table 4. In the red circle an optimum of Grid Search is shown.

4.470	4.564	3.996
4.515	5.178	4.702
4.773	5.054	4.565

4.3 Controlled Random Search

Controlled Random Search (CRS) is the method of Price (1979). It is not very popular by researchers because no analytical property can be derived. Still, the method is often used, because it is easy to implement. It was one of the first algorithms which used population points (Hendrix, 2009).

CRS starts with an initial set P of N points sampled uniformly in the feasible area. At every iteration new trial points are generated and replace the worst point in N if they are better. The algorithm stops when all function values are close enough, that is closer than a given accuracy value α. ([Price 1977], influenced by [Becker and Lago 1970]).

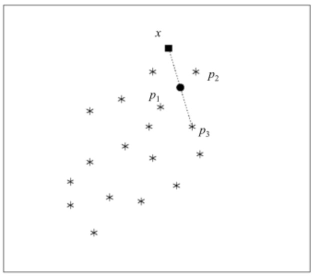

Fig. 4.2. Generation of a trial point by CRS (Hendrix, 2009)

The controlled random search algorithm

1. Generate a population P, of N points uniformly over the feasible area
2. Evaluate all points of population P

Iteration process

3. Determine the worst point in P
4. Select random parent points from P
5. Generate a trial point from these parent points
   If the trial point is feasible, go to the next step, if not, start a new iteration
6. Evaluate the trial point
   If the trial point is better than the worst point of P go to the next step, if not start a new iteration
7. Replace the worst point of P by the trial point
8. Stop if the stopping criterion is met, this is when best and worst point differ less in function value than α.

4.4 Genetic Algorithm

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce children for the next generation (see Figure 4.3). Over successive generations, the population "evolves" toward an optimal solution (Matlab, 2008).

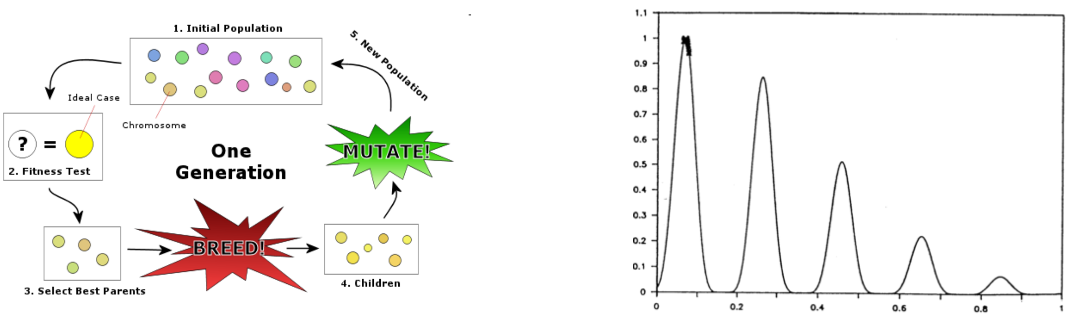

Fig. 4.3. The modification of the population (see left: Nitrogen, 2009) which leads to convergence to an optimal solution (see right: Singh, 2006)

The Genetic Algorithm

1. Obtain a population of parents. This is the first generation.
2. Produce a new generation
   2.1. *Evaluation*: evaluate the fitness of the individuals
   2.2. *Parent selection*: select pairs of parents. The probability of a chromosome being selected as a parent depends on its fitness.
   2.3. *Crossover*: produce one or two new individuals from each pair of parents.
   2.4. *Mutation*: some genes of the offspring are modified randomly.
   2.5. *Population selection*: a new generation is selected replacing some or all of the original population by an identical number of offspring.

Repeat step 2 until some termination criteria apply (Lemmen- Gerdessen, 2007).

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4.5 Multistart

A method for generating random starting points and doing local searches, requires much less function evaluations than a plain grid search and is more effective in looking for solutions. This method is called Multistart. A drawback of Multistart is that there is no guarantee to find the global optimum. If after some calculation time no solution is found, it is not certain that there doesn't exist one. For this thesis the most important is to find the total number of optima, finding the exact global optimum is of less importance.
Multistart uses random generated points as a starting point for the local optimization procedure. Several starting points occur in the same region of attraction which causes a lack of efficiency (Hendrix, 2009). Multistart can work with several local solvers.

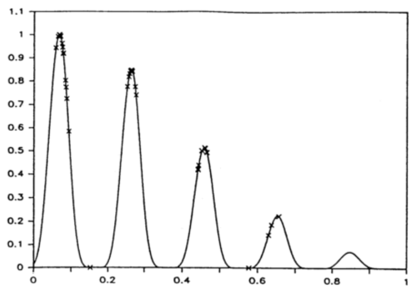


Fig. 4.4. Starting points climbing to closest optima (Singh, 2006)

The Multistart Algorithm

1. Generate a set of N starting points uniformly over the feasible area
2. The solver creates a new convergence point for all N points
3. Evaluate all new points

4.6 MILP Branch-and-Bound

In this section the Branch-and- Bound algorithm is described because Maximin model 3 is an Mixed Integer Programming model that is solved in GAMS, which is a modelling system for mathematical programming and optimization. All algorithms in GAMS for solving MIPs are LP based Branch-and-Bound algorithms (GAMS, 2009). The Branch-and-Bound algorithm is not tested on being the most suitable algorithm for solving multimodal problems because the output parameters are not comparable with those of the other algorithms. The algorithm is only used for gaining insight in the characteristics of Maximin model 3.

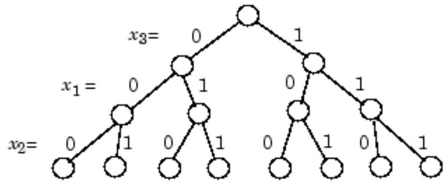


Fig. 4.5. An example of an branch-and-bound tree with three variables: x_1, x_2 and x_3 (Matlab, 2009)

The MILP based Branch-and-Bound algorithm

Branch-and-Bound is a deterministic algorithm based on search trees (Redondo, 2008), see Figure 4.5. The algorithm searches for an optimal solution to the binary integer programming problem by solving a series of LP-relaxation problems, in which the binary integer requirement on the variables is replaced by the weaker constraint $0 \leq x \leq 1$.
The algorithm (e.g. minimization):

1. The stored minimum value is infinite
   P is an empty set
   Solve the LP relaxation, without the binary constraints
   Stop if:
   a. If the optimal solution satisfies all binary constraints: optimum solution is found
   b. If the problem is infeasible or unbounded
2. Branching: select variable x_j with a value that does not satisfy binary constraint, i.e. $0<x_j<1$
   Create LP-sub-problems
   - $x_j =0$
   - $x_j =1$

   Add this problems to the set P
3. Select a sub-problem from set P and solve it. Remove the problem from P.
4. If the optimum of the sub-problem is an improvement on the stored minimum value, but the corresponding x does not satisfy binary constraints, go to step 1 (branching).
   Sub-problem is totally solved in the following three situations:
   - Bounding: If the optimum of the sub-problem is an improvement on the stored minimum value, and the corresponding x satisfies all binary constraints: this optimum, with the corresponding x is the new stored minimum value, go to step 4.
   - If the optimum of the sub-problem is not an improvement on the stored minimum value, go to step 4.
   - If the sub-problem is infeasible, go to step 4
5. If set P is not empty, go to step 2.
   If set P is empty, stop: all sub-problems are solved.
   If the stored minimum value is infinite, the problem has no binary solution
   Else, the solution is the stored minimum value, with the corresponding x

(Lappeenranta University of Technology, 2009).

5. Solvers

In this chapter two of the solvers are described. These solvers are Fmincon, that is used by Multistart and CPLEX which is used as default solver for MIP problems in GAMS. The code of the CRS solver can be found in Appendix G.1.2.. For the Genetic Algorithm the standard code in Matlab is used.

5.1 Fmincon (Multistart)

The Multistart algorithm can work with various local optimization solvers. For this thesis fmincon is selected. Fmincon is a local solver which tries to find the minimum of constrained nonlinear multivariable functions. The advantage of using fmincon is that it is able to find global and local optima.
A drawback is that the solver sometimes converges to another solution which is not optimal. This means that the extra convergence points do not correspond to an optimum solution of the optimisation problem. Such a point is shown in the following example.
For this example the feasible area is [0,2]x[0,2], where one demand point is located, at (1,1). When the generic Maximin model is used, the optima are the vertices (0,0), (2,0), (2,2) and (0,2), see Figure 5. Fmincon finds an extra solution in point (2,1), which is depicted in Figure 5.

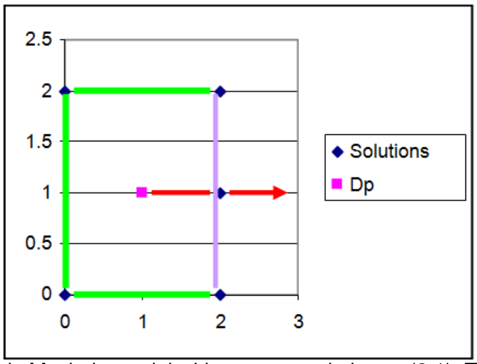


Fig 5. The results of the generic Maximin model with an extra solution at (2,1). This point is a Karush-Kuhn-Tucker point, and not an optimum.

Point (2,1) is a Karush-Kuhn-Tucker (KKT) point i.e. the first order optimality conditions apply. A KKT point is a point where the gradient is a positive combination of the gradients of the binding constraints. The gradient is represented by the red arrow (Figure 5). The direction of the gradient of this point is in exactly the same direction as the gradient perpendicular to the binding constraint (the purple line in Figure 5), which makes it a KKT point. Although this point is a KKT point, it is not a local maximum because a small step into the feasible direction along the constraint generates a higher objective function value.
KKT points that are solutions but not optima appear because fmincon uses gradient information for optimization. By using Multistart, there is a chance of having a starting point on the line from the demand point to the KKT point (red line in Figure 5). All points on that line will move to the KKT point, because fmincon uses the gradient which points in that direction. The consequence is that it is possible that fmincon finds more convergence points than the number of optima.

5.2 CPLEX

GAMS Version 22.9 is used to solve maximin Model 3, the Mixed Integer Programming model. ILOG CPLEX Version 11.2 is used as the default MIP solver operating under GAMS Version 22.9. CPLEX uses the Branch-and-Bound technique to solve MIP problems as explained in Section 4.6.

6. Test cases

In this chapter we describe 3 test cases and one illustration case. In order to be able to test algorithms on their ability of finding optima, three test cases are developed.
The first Test case is a random case. This case is interesting because there is no knowledge in advance on the location and number of optima. Test case 2 and 3 are extreme cases. The two extreme cases are designed in a symmetric way for simplicity and the optima are known. The knowledge on the number and location of the optima is used to get more insights in how the algorithms work. Test case 3 only has global optima, while test case 2 has global and local optima. The coordinates of the demand points and the weights can be found in Appendix C. Finally an illustration case is designed, called Andalucía. Andalucía is an area in the south of Spain which is used as feasible area with the main cities as demand points. The case is used to illustrate the characteristics of the obnoxious single facility models of Chapter 3.

6.1 Test Case 1

Test case 1 is an example based on a case of Saamaño (2006). It has 21 randomly generated demand points that have randomly generated weights attached to them and a feasible area of [0,30] x [0,30].

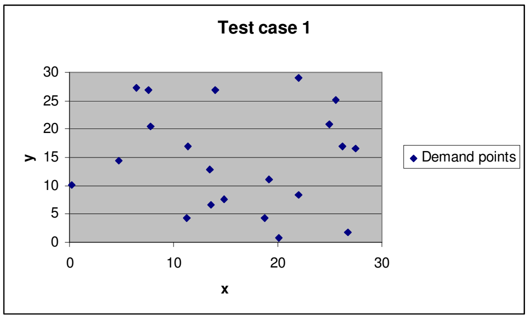


Fig. 6.1. Test case 1

6.2 Test Case 2

Test case 2 has four demand points in feasible area [0,6] x [0,6].

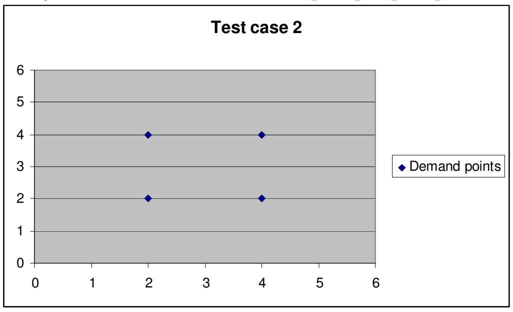


Fig. 6.2. Test case 2

6.3 Test Case 3

Test case 3 has one demand point in feasible area [0,2] x [0,2].

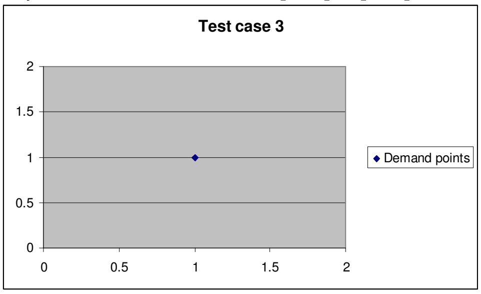


Fig. 6.3. Test case 3

Figure 6.4 shows the location and number of global and local optima of test cases 2 and 3. In the graph of test case 2, the red dots indicate the global optima for the Maximin, Maxisum and Minisum model. The light blue dot in the middle gives the local optimum for the Minisum model. The local optima of the Maximin model are the light blue dot and the green triangles, together this are 5 local optima. The Maxisum model has no local optima in this test case.

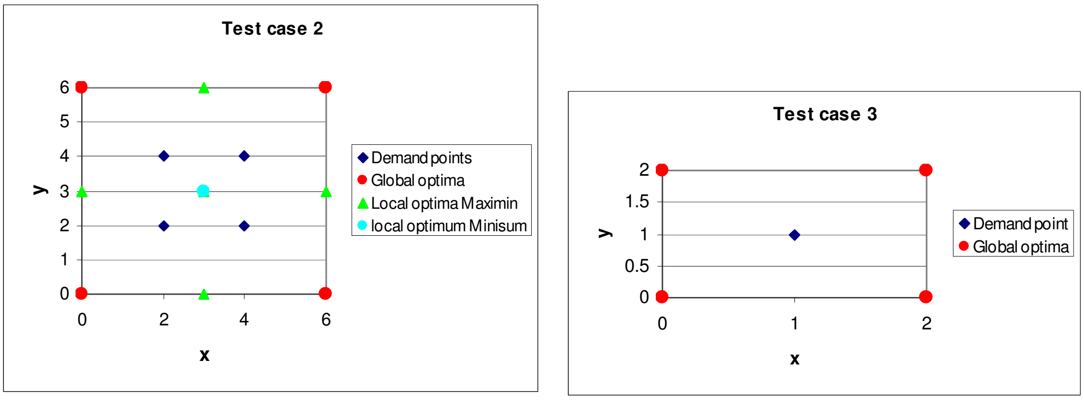


Fig. 6.4 Global and local optima of test cases 2 and 3

Test case 3 has four global optima, the red dots in the vertices, for the Maximin, Maxisum and the Minisum model.

6.4 Illustration Case Andalucía

In the illustration case the Andalucía region is considered. Andalusia is represented by a pentagonal region, defined by the red lines (see Figure 6.5). The demand points (blue dots in Figure 6.5) are cities within the feasible area that have more than 22.500 inhabitants. The weights attached to the cities are proportional to their population size. The x-axe expresses the longitudes and the y-axe the latitude. Coordinate (0,0) represents earth coordinates (-8.2, 35.57). The data of this case can be found in Appendix D.

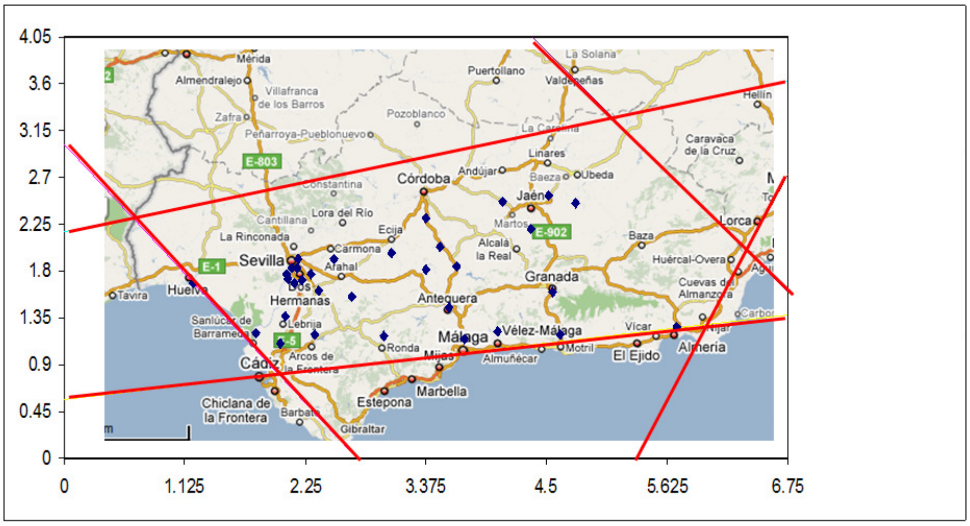


Fig. 6.5. Map of Andalusia, a region of Spain, where the feasible area is depicted by the red lines (Google Maps, 2008).

The picture in Figure 6.5 is used illustrate the results of the models of Chapter 3: the optimal location for an obnoxious single facility. The data of case Andalucía is used to illustrate the characteristics of the obnoxious single facility models.

7. Results Algorithm Testing

7.1 Introduction

The algorithms described in Chapter 4 are tested for the Maximin, Maxisum and the Minisum problem. The used algorithm codes that are used in Matlab can be found in Appendix G.1.1. to G.1.4.. All three models use data of test case 1 (21 DP, [0,30]x[0,30]), test case 2 (4 DP, [0,6]x[0,6]) and test case 3 (1 DP, [0,2]x[0,2]), see Chapter 6. For the Minisum problem, a parameter G is introduced. G is the influence radius of the obnoxious facility. If a demand point is outside this radius, the obnoxious facility does not give damage to this demand point. Test case 1 has an influence radius of G=15, test cases 2 and 3 have a influence radius of G=5.

First a grid search is performed for all models on all test cases. The number of optima that result from the grid search is set as a reference for the other algorithms. The algorithm that has a number of optima that is closest to the results of the grid search is considered to be the most suitable algorithm for finding all optima.

The stochastic algorithms are run 100 times, with default settings. After each run, an optima counter algorithm is used to determine the number of optima that are found. The pseudo-code of the optima counter algorithm used is as follows:

Optima counter algorithm

0. Take the first outcome and save it

Iteration process

1. Compare the next outcome with all saved outcomes
   If the Euclidean distance between the next outcome and each of the saved outcomes is bigger than ε, add the new outcome to the list of saved outcomes.
   If not, start a new iteration.
2. Stop if there is no next outcome.
3. Count the number of saved outcomes = number of optima found.

The computational experiments are conducted on a Intel core 2 duo personal computer with 1024 MB random access memory (RAM) (notebookcheck, 2009). Microsoft Office Excel 2003 SP3, part of the Microsoft Office Professional Edition 2003, is used to make illustrations. Matlab stands for matrix laboratory. It is an interactive system whose basic data element is an array that does not require dimensioning. Version 7.6.0 (R2008b) of Matlab is used.

The performance indicators that are measured are:

1. **Minimum**: the minimum optimum function value that is found in 100 runs
2. **Maximum**: the maximum optimum function value that is found in 100 runs
3. **Mean**: the mean of the optimum function value found in 100 runs

- **Variance**: the variance of the optimum function value over 100 runs
- **Function E.**: the average number of function evaluations per run
- **# optima**: the average number of optima over 100 runs
- **Min opt**: the minimum number of optima found by a run, per 100 runs
- **Max opt**: the maximum number of optima found by a run, per100 runs
- **Global opt.**: the number of global optima found in 100 runs
- **Non global opt.**: the number of optima in a run that are not global optima, per 100 runs

7.2 Results Grid Search

Implementation

The algorithm is executed one time. The stepsize setting is 0.5. The used M-files are in Appendix G.1.1..

Results Maximin model

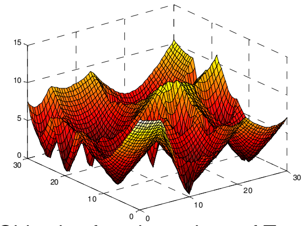


Fig. 7.1. Objective function values of Test case 1

Table 7.1. Results Grid Search Maximin model

Maximin Test case	Maximum	Function E.	Global opt.	Non global opt.	#opt
1	12.04	3721	1	25	26
2	2.83	169	4	5	9
3	1.41	25	4	0	4

Results Maxisum model

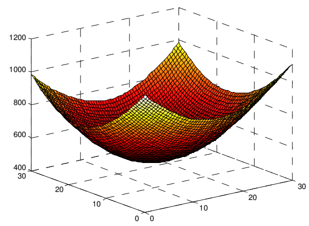


Fig. 7.2. Objective function values of Test case 1

Table 7.2. Results Grid Search Maxisum model

Maxisum Test case	Maximum	Function E.	Global opt.	Non global opt.	#opt
1	1114.8	3721	1	3	4
2	17.43	169	4	0	4
3	1.41	25	4	0	4

Results Minisum

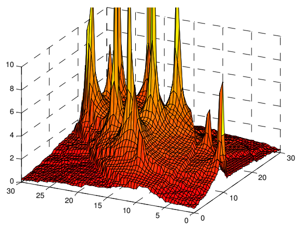


Fig. 7.3. Objective function values of Test case 1

Table 7.3. Results Grid Search Minisum model

Minisum Test case	Maximum	Function E.	Global opt.	Non global opt.	#opt
1	0.23	3721	1	16	17
2	0.80	169	4	1	5
3	0.71	25	4	0	4

Explanation
With mesh size 0.5 it is possible to find all optima of test cases 2 and 3. From this results it is assumed that the number of optima found in test case 1 is a good approximation of the number of existing optima. It is not sure that it is the exact amount of existing optima because it has to be taken into account that there is a possibility of missing a needle point between 4 evaluated points. Another possibility is that an optimum is found that is not a real optimum: in the grid the so called optimum is higher than its eight neighbours, but not higher than a not evaluated point in between. Therefore an optimum found can be on the slope of another optimum.

7.3 Results Controlled Random Search

Implementation
Population size N = 50
Stop criterion α = 0.05
Number of iterations i = 10000
The used M-files are in Appendix G.1.2.

Table 7.4. Results Controlled Random Search Algorithm

Maximin								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	11.95	12.04	12.03	0.00	1176.84	1	1	1
2	2.80	2.83	2.82	0.00	722.42	4	4	4
3	1.37	1.41	1.40	0.00	410.89	4	4	4

Maxisum								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	1114.71	1114.77	1114.75	0.00	1713.90	1	1	1
2	17.39	17.43	17.42	0.00	927.98	3	4	3.99
3	1.38	1.41	1.40	0.00	410.82	4	4	4

Minisum								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	0.28	0.35	0.28	0.00	972.24	1	2	1.01
2	0.64	1.00	0.88	0.00	3045.74	1	5	3.86
3	0.71	0.74	0.72	0.00	391.39	4	4	4

Explanation
As can be seen in Table 7.4, Controlled Random Search succeeds in finding the global optima of all test cases. The variance of the found optimum function value is approximately zero so it is not hard for the algorithm to find these optima. Controlled Random Search does not find more optima than the global ones, so the algorithm is not effective in detecting local non global optima.

7.4 Results Genetic Algorithm

Implementation
The M-file was generated in Matlab with default settings, except for the population size. The solver is called ´ga- Genetic Algorithm´.
Population size N = 50
The used M-files are in Appendix G.1.3.

Table 7.5. Results Genetic Algorithm

Maximin

Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	10.61	12.04	11.53	0.42	2600.5	1	1	1
2	2.83	2.83	2.83	0.00	2602.5	1	1	1
3	1.41	1.41	1.41	0.00	2604	1	1	1

Maxisum

Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	983.70	1114.76	1104.93	927.02	1066.8	1	1	1
2	17.42	17.43	17.43	0.00	1044.4	1	1	1
3	1.41	1.41	1.41	0.00	1048	1	1	1

Minisum

Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	0.28	1.37	0.39	0.05	1052.4	1	1	1
2	0.80	0.80	0.80	0.00	1042.8	1	1	1
3	0.71	0.71	0.71	0.00	1042.6	1	1	1

Explanation
Genetic Algorithm is able to find one optimum. The end population converges to one point. This point is not always the same. For test cases 2 and 3 the four different global optima are found, over the 100 runs, but not during one individual run.
For test case 1, the number of different optima differs per model. The Maximin and the Minisum models resulted in 5 different optima over 100 runs, while the Maxisum model gave only one optimum. The variance of the optimum function value found is low for the test cases 2 and 3 for all models. It is more difficult for the algorithm to find the global optimum of test case1. Genetic Algorithm is not capable of finding local optima, therefore it is not effective in that sense.

7.5 Results Multistart

Implementation
Number of starting points N = 50
Local optimizer= fmincon
The used M-files are in Appendix G.1.4.

Table 7.6. Results Multistart Algorithm

Maximin								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	12.04	12.04	12.04	0.00	3481.33	7	14	10.53
2	2.83	2.83	2.83	0.00	1233.13	6	9	8.23
3	1.41	1.41	1.41	0.00	599.28	4	4	4

Maxisum								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	1114.77	1114.77	1114.77	0.00	537.03	4	4	4
2	17.43	17.43	17.43	0.00	594.30	4	4	4
3	1.41	1.41	1.41	0.00	602.67	4	5	4.05

Minisum								
Test case	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
1	0.28	0.28	0.28	0.00	2270.35	8	17	12.23
2	0.80	0.80	0.80	0.00	315.45	3	8	5.06
3	0.71	0.71	0.71	0.00	530.35	4	5	4.01

Explanation
The local optimizer, fmincon, gives the following warning:´Trust-region-reflective method does not currently solve this type of problem, using active-set (line search) instead´. One of the algorithms that is used by fmincon is the ´trust region reflective´ algorithm and it can accept a user-supplied Hessian as the final output of the objective function. Since this algorithm has only bounds or linear constraints, the Hessian of the Lagrangian is the same as the Hessian of the objective function. In this research no Hessian is supplied, so instead of the ´trust region reflective method´ fmincon used another algorithm: the ´active-set´. The active-set algorithm does not accept a user-supplied Hessian. It computes a quasi-Newton approximation to the Hessian of the Lagrangian. (Matlab, 2009).

The local solver of the Multistart algorithm is able to find all global optima. The global optima are easy to find by the solver as the variance is approximately zero for all test cases for all models. Besides global optima, other optima are found. From this can be concluded that fmincon is able to find local optima.
Fmincon did not find all local optima of test case 1 for the Maximin and the Minisum model. This test case has probably many optima with a small region of attraction. In the results can be observed that the solver converges to more solutions than there actually are optima in test case 3 of the Maxisum model and test cases 2 and 3 of the Minisum model. These points are Karush- Kuhn- Tucker points as explained as in Section 5.1.
Besides the chance of hitting a KKT point and although it is not possible to find all local optima with the fmincon solver in Multistart, like in test case 1 of the Maximin model and the Minisum model, it is the most promising algorithm. It is the only tested algorithm that is able to find local optima and it gives the correct ratio of optima found between the different test cases and models.

7.7 Conclusion Results Algorithms

Effectiveness

Three algorithms are tested on finding all optima, with Grid Search (the blue bar in Figure 7.4) as a reference algorithm. The Controlled Random Search algorithm (dark green bar in Figure 7.4) is able to find all global optima but it is not effective in finding local optima. The global optima are hit by the genetic algorithm (light green bar in Figure 7.4), but it only converges to one global optimum per run. The Genetic algorithm is not capable of finding local optima, so it is not effective in that sense. The last tested algorithm, Multistart, detects global and local optima (yellow bar in Figure 7.4). The algorithm thus is effective and suitable for testing the selected obnoxious facility location models from literature on their number of optima.

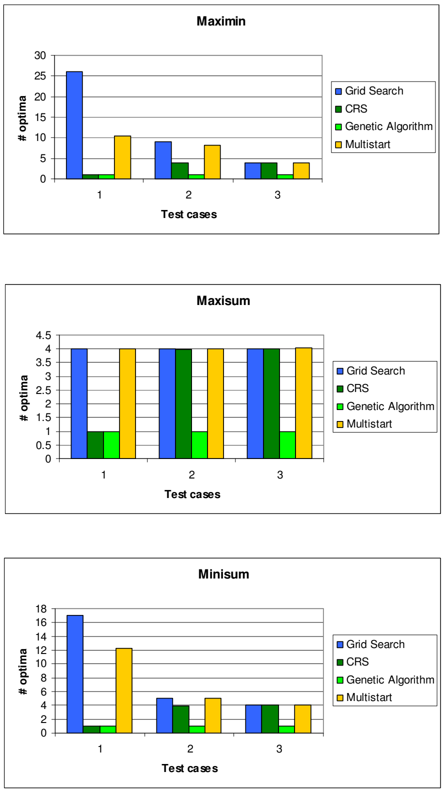


Fig. 7.4. Number of optima found per algorithm per test case, for the Maximin, Maxisum and the Minisum model.

Efficiency

In Table 7.7 the least number of function evaluations per test case per model are marked red. As can be seen there is not one most efficient algorithm for all test cases and models.

Table 7.7. Results function evaluations for the algorithms

Maximin	CRS	GA	MS
Test case	Function E.	Function E.	Function E.
1	1176.84	2600.5	3481.33
2	722.42	2602.5	1233.13
3	410.89	2604	599.28
Maxisum			
Test case	Function E.	Function E.	Function E.
1	1713.90	1066.8	537.03
2	927.98	1044.4	594.30
3	410.82	1048	602.67
Minisum			
Test case	Function E.	Function E.	Function E.
1	972.24	1052.4	2270.35
2	3045.74	1042.8	315.45
3	391.39	1042.6	530.35

The Controlled Random Search Algorithm is most efficient on function evaluations for the generic maximin model for all test cases. Multistart is the most efficient algorithm in solving the

Maxisum model for test case 1 and 2. For the Minisum model no conclusion can be drawn from the number of function evaluations. Genetic Algorithm is in no situation the most efficient algorithm. Although Controlled Random Search is in 6 out of 9 times the most efficient algorithm, it is not chosen for testing the obnoxious facility location models because it is not effective in detecting local non global optima.

8.Analysis Models

8.1 Introduction

In the following sections the selected obnoxious facility location models from literature that are described in Chapter 3 are evaluated. The models are evaluated with the illustration case Andalucía (see Section 6.4) on how hard they are to solve assuming that having more optima is a measure for with being harder to solve.

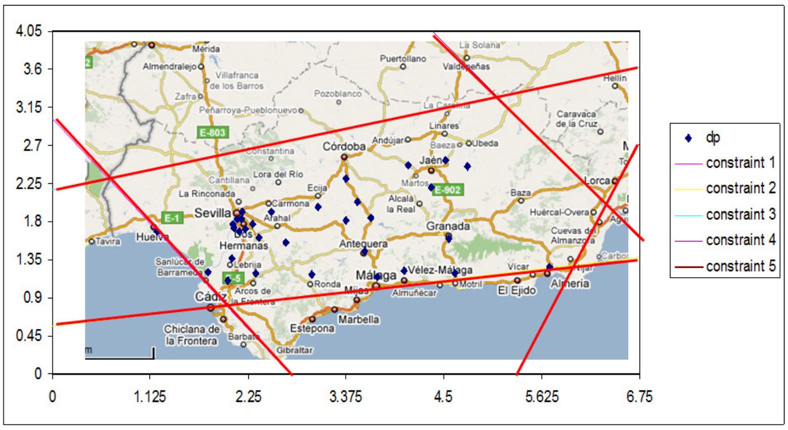


Fig. 8.1. Illustration Case Andalucía

The computational experiments are conducted on a Intel core 2 duo personal computer with 1024 MB random access memory (RAM) (notebookcheck, 2009).
Microsoft Office Excel 2003 SP3, part of the Microsoft Office Professional Edition 2003, is used to make illustrations. Matlab stands for matrix laboratory. It is an interactive system whose basic data element is an array that does not require dimensioning. Version 7.6.0 (R2008b) of Matlab is used for all models, except for Maximin model 3, the Mixed Integer Programming model. Model 3 could not be solved with Matlab, so it is solved with GAMS Version 22.9. GAMS stands for General Algebraic Modeling System, which is a modeling system for mathematical programming and optimization. It consists of a language compiler and a stable of integrated high-performance solvers.

The distance metric is Euclidean, but in some articles the rectilinear distance was mentioned as well, so for the models of these articles the distance is both measured Euclidean and rectilinear. These models are:

4. Maximin : Model 0, 5, 7, 8
5. Maxisum: Model 0, 1, 2
6. Minisum: Model 0

Minisum model 0 is the generic minisum model, which is the only model that makes use of an influence radius. If a demand point is in this radius, the obnoxious facility has effect on the demand point. Since other models do not have an influence radius, it is complex to compare. For this reason, the influence radius is set on G= 25, so that every demand point in the feasible area is for certain under influence of the obnoxious facility, like in all the other models.

Maximin Model 3 is a Mixed Integer Programming model and therefore it could not be tested in Matlab. The model is solved with GAMS (see Appendix E). Due to license restrictions, a maximum of 50 discrete variables, only 25 of the 32 cities of the case Andalucía could be used as input. So the number of demand points is different from the demand points in the other models. Besides differences in input, there are differences in output. The local solver used in GAMS, CPLEX, does not give the number of local optima, only the global optimum. Instead of the number of function evaluations, CPU time and number of iterations are presented as output. It can be concluded that the input and output of Maximin model 3 is not equal to that of the other models and thus can not be compared. However, the output of the model is presented in Table 8.2 in Section 8.2, Results of the Maximin models, and the global optimum of model 3 is drawn in Figure 8.2.

Each model solved with Matlab, is run 100 times. After each run the optima counter algorithm is used to determine the number of optima that are found. The pseudo-code of the optima counter algorithm used is given in Section 7.1.

Performance indicators

The characteristics of the models are determined by the ability of an algorithm to find the global optimum and this is measured by the number of times that Multistart finds the global optimum (see # Global optima). The second indicator for analysing the models is the variance: a low variance means that it is easy for Multistart to find the global optima (see Variance). Finally the number of optima (see # optima) is an indicator. If the model has few optima, it is considered to be easier to solve.

The efficiency of solving the models with Multistart is measured by the average number of function evaluations that is needed to converge per run, over 100 runs (see # Function E.).

To calculate the above described indicators to measure how hard a model is to solve, the following performance indicators are measured:

1. **# Global opt**: the number of times that the optimum is found per 100 runs
7. **Minimum**: the minimum optimum function value that is found in 100 runs
8. **Maximum**: the maximum optimum function value that is found in 100 runs
9. **Mean**: the mean of the optimum function value found in 100 runs
- **Variance**: the variance of the optimum function value over 100 runs
- **Function E.**: the average number of function evaluations per run
- **Min opt:** the minimum number of optima found by a run, per 100 runs
- **Max opt**: the maximum number of optima found by a run, per100 runs
- **# optima**: the average number of optima over 100 runs

Performance indicators for Maximin model 3:

10. **Global optimum**: function value of the global optimum
11. **x1, x2**: the x- and y- coordinate of the global optimum
12. **CPU**: Central Processing Unit, in seconds
13. **DP**: Demand Points
14. **# iterations**: number of iterations needed to find the global optimum

8.2 Results Maximin models

In Figure 8.2 the global optima of the Maximin models are shown. The global optima of all models are situated on constraint 4. This appears to be the location where the minimum distance from the obnoxious facility to the closest demand point is at the maximum.

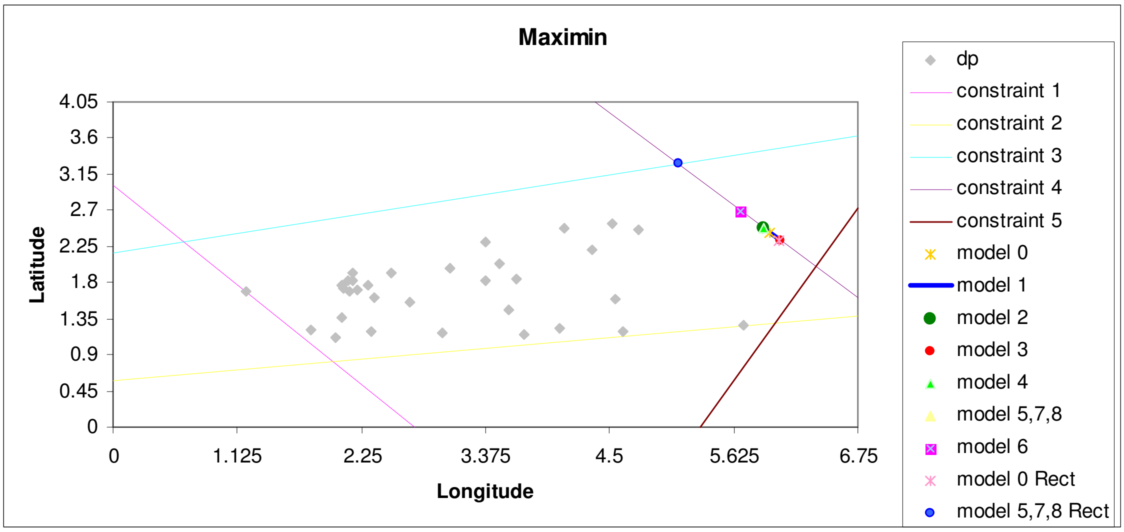


Figure 8.2. Location of the global optimum for each Maximin model

The algorithm finds the global optimum of all models in approximately 100% of the runs (see Table 8.1), the variances give the same indication. Of the models where the Euclidean distance metric is used, it can be said that on average model 1, the obnoxious linesegment model, has the most optima, and it needs the most function evaluations. Models 5,7,8, the weighted generic maximin model, has on average the most optima if distance is measured rectilinearly. Model 6, the model that minimizes the maximum damage to one demand point, gives five optima as result, which is the smallest number of optima of the Maximin models. This result corresponds with the least number of needed function evaluations. The models measured with rectilinear distances have on average more optima than the same models measured with Euclidean distances.

Implementation
Number of starting points N=50
Local optimizer = fmincon
Matlab files: Appendix G.2.1.
Matlab Results: Appendix H.2.

Table 8.1. Result Maximin models

Euclidean

Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	100	1.19	1.19	1.19	0.00	2831.33	6	16	10.45
1	98	1.01	1.12	1.11	0.00	9727.52	16	29	23.16
2	100	1.09	1.09	1.09	0.00	2590.56	7	17	11.52
4	100	1.09	1.09	1.09	0.00	2554.06	5	14	9.42
5,7,8	100	0.48	0.48	0.48	0.00	3769.46	8	16	11.46
6	97	1.46	1.51	1.46	0.00	1186.19	3	8	5.08

Rectilinear

Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	97	1.28	1.40	1.40	0.00	2150.57	12	23	16.69
5,7,8	100	0.62	0.62	0.62	0.00	2400.87	19	33	24.69

The output of GAMS of model 3 is given in Table 8.2 (Appendix F). The global optimum is approximately on the same location as the other Maximin models.

Implementation
Global optimizer = CPLEX
GAMS file: Appendix E.
GAMS Result: Appendix F.

Table 8.2. Results Maximin model 3

CPLEX

Model	Global opt	x1	x2	CPU	DP	#iterations
3	1.40	6.04	2.33	0.016 s	25	882

8.3 Results Maxisum models

The weighted and generic Maxisum models have the same optimum (Figure 8.3). They are both trying to maximize the distance from the obnoxious facility to all demand points.

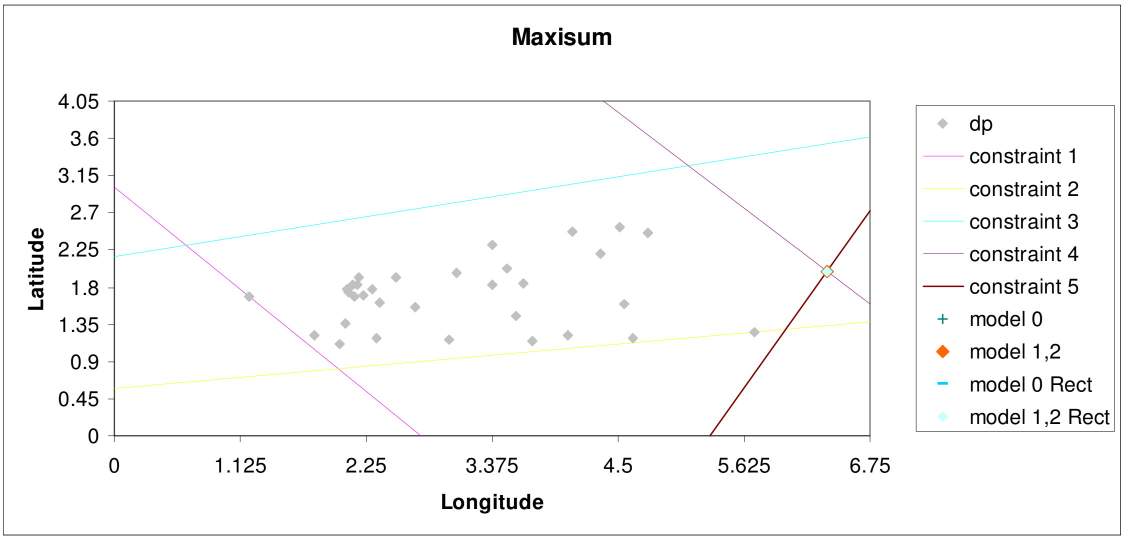


Fig. 8.3. Location the global optimum for each Maxisum model

The Maxisum models have a 100% score in resulting in one global optimum. They only have 3 optima on average with the Euclidean distance metric. The same models measured with the rectilinear distance metric have an average of 4 optima with the generic model and an average of 5 optima with the weighted model (Table 8.3).

Implementation

Number of starting points N=50
Local optimizer = fmincon
Matlab files: Appendix G.2.2.
Matlab Results: Appendix H.2.

Table 8.3. Results Maxisum Euclidean and rectilinear

Euclidean									
Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	100	107.32	107.32	107.32	0.00	523.26	2	4	2.97
1,2	100	80.68	80.68	80.68	0.00	524.04	2	3	2.99

Rectilinear									
Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	100	119.12	119.12	119.12	0.00	497.97	3	5	4.02
1,2	100	88.80	88.80	88.80	0.00	465.00	4	5	4.98

8.4 Results Minisum models

The Minisum models all have the same optimal location, except for the wind dispersion models (Minisum model 1,2 and 3). The aim is to minimize damage to all demand points, so the optimal locations for an obnoxious facility of the wind dispersion models are remarkable since they are close to demand points.

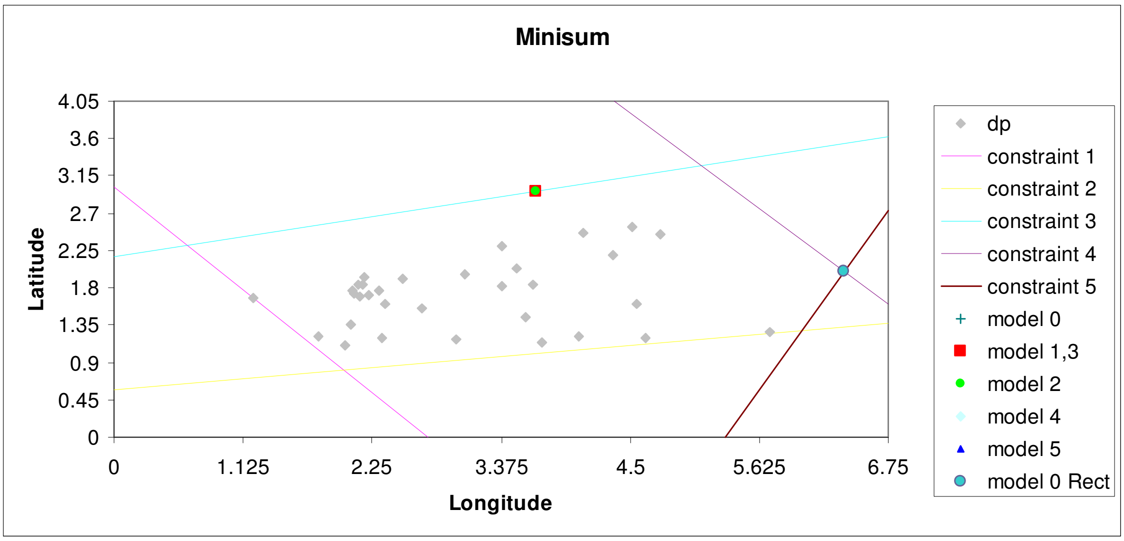


Figure 8.4. Location the global optimum of each Minisum model

Just like in the Maximin and the Maxisum models, Multistart does not have difficulty finding the global optimum of the Minisum models. The wind dispersion models, models 1,2 and 3, have most optima on average, together with the highest number of function evaluations. The global repulsion model, model 4, has the smallest number of optima. Model 0 is measured with Euclidean as well as with rectilinear distances. On average the rectilinear measured model has more optima (Table 8.4).

Implementation

Number of starting points N=50
Local optimizer = fmincon
Matlab files: Appendix G.2.3.
Matlab Results: Appendix H.2.

Table 8.4. Results Minisum models

Euclidean

Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	100	4.60	4.60	4.60	0.00	505.93	3	6	4.34
1,3	97	0.00	0.00	0.00	0.00	1475.70	2	9	5.66
2	100	0.00	0.00	0.00	0.00	1488.60	3	10	6.09
4	100	9.17	9.17	9.17	0.00	521.94	3	4	3.99
5	100	9.41	9.41	9.41	0.00	502.16	3	6	4.59

Rectilinear

Model	#Global opt	Minimum	Maximum	Mean	Variance	Function E.	min opt	max opt	# optima
0	100	3.24	3.24	3.24	0.00	544.19	4	7	5.03

8.5 Conclusion Results models

For the illustration case Andalucía Multistart finds for all models the global optimum. From the low variances can be concluded that the global optima are easy to find. Next to a global optimum, all models have local non global optima. The Maximin models have on average most optima, the Maxisum the least.

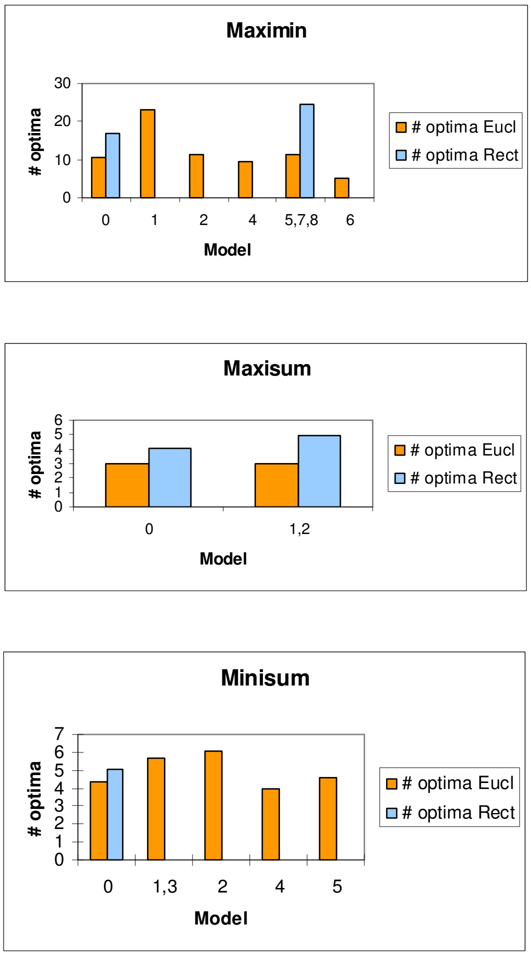


Fig. 8.5. Overview of the number of optima per model, measured with Euclidean and rectilinear distances.

The models that are measured with the Euclidean distances (the orange bars in Figure 8.5) have on average less optima than the same models measured with the rectilinear distance metric (the blue bars in Figure 8.5). This difference in number of optima can be explained by the phenomenon described in the following section.

8.6 Discussion Results distance metrics

In this section a possible explanation is given for the difference in number of optima that are found when measuring with different distance metrics, being the Euclidean and the rectilinear distance metric. The contour graphs are generated in Matlab using a grid search with a stepsize of 0.2. The grid search is performed for the generic Maximin, the generic Maxisum and the generic Minisum model. The coordinate of the single demand point is (2.71, 2.26). When two demand points are located, the coordinates of the second demand point are (3.96, 1.01).

Maximin

By using a contour plot, a possible explanation for the difference in optima between the models measured with the Euclidean and rectilinear distance metric is found (Appendix G.2.1.1.). In the graph a) of Figure 8.6 can be seen that the contour of the rectilinear measured function value coincides with the left border of the feasible area. The slope of this border is -1, which is equivalent to the slope of the bottom leftside of the rectilinear distance norm. All points on this border thus have the same function value. The whole linesegment is the set of optimum solutions.

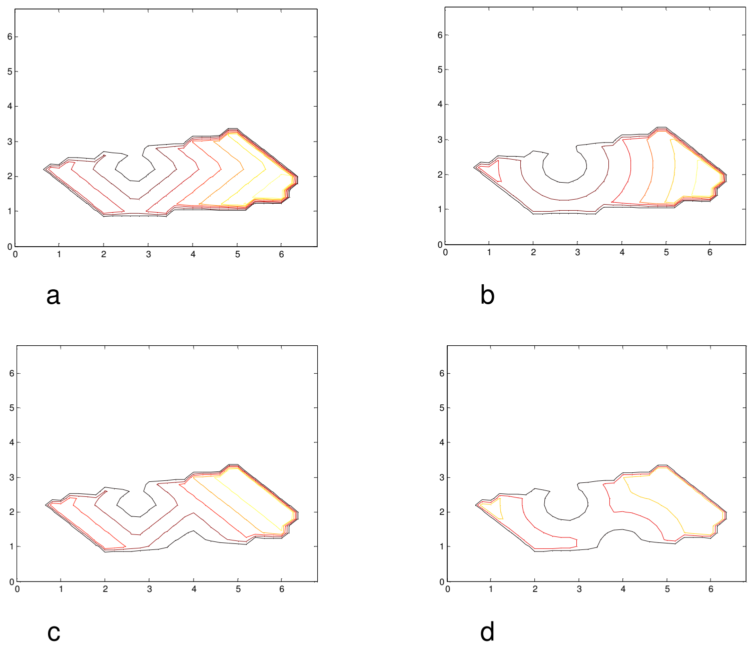
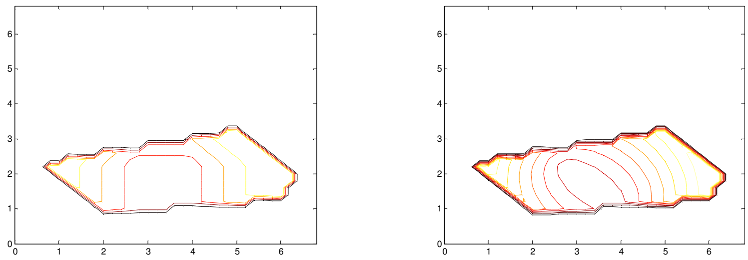


Fig. 8.6. Contour plots of the generic Maximin model, illustration case Andalucía. In graph a) and c) are 1 respectively 2 demand points measured in rectilinear distance. Graph b) and d) represent 1 respectively 2 demand points measured with Euclidean distances

If fmincon converges to a point on this linesegment, fmincon recognize this point as optimal. Multistart can have more than one starting point hitting this line, on different places of the linesegment. The optima counter algorithm, see Section 7.1, counts all these hits as different optima, while actually it are points of the set of optimum solutions.
If two demand points are drawn in the feasible area, instead of one, another linesegment that is an optimum appears between these demand points, see graph c) of Figure 8.6. On this linesegment fmincon is able to find an optimum at infinitely many locations. The optima counter algorithm does not distinguish between these locations. And thus every time a starting point converges to the linesegment, an optimum is counted, while it is actually the same optimum. This phenomenon can be an explanation for the appearance of more optima of the same Maximin model measured with rectilinear instead of Euclidean distances in the Andalucía illustration case. This is besides the chance of hitting KKT points.

Maxisum
The graph of the Maxisum contours with one demand point is the same as for the Maximin model. In both cases the aim is to maximize the distance to that single demand point.
When there are 2 demand points in the feasible area, the Maxisum contour changes in shape, in comparison with the Maximin model contour.
As can be seen in Figure 8.7, graph a), the contour of the rectilinear norm does not coincide with the border of the feasible area. Next to this, graph a) and b) show that there does not exists an optimal point or linesegment in between the two demand points. The extra optima found of the Maxisum models that are measured with rectilinear distances can not be explained from this graphs. The used Grid Search Matlab files are in Appendix G.2.2.1..

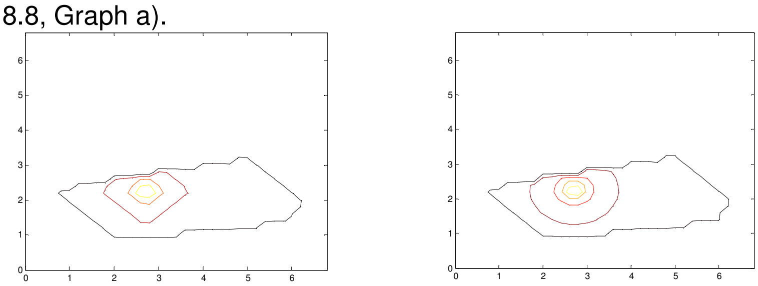


a b

Figure 8.7. Contour plots of the generic Maxisum model, illustration case Andalucía. In graph a) 2 demand points are measured with rectilinear distances. Graph b) shows two demand points and Euclidean distances.

Minisum

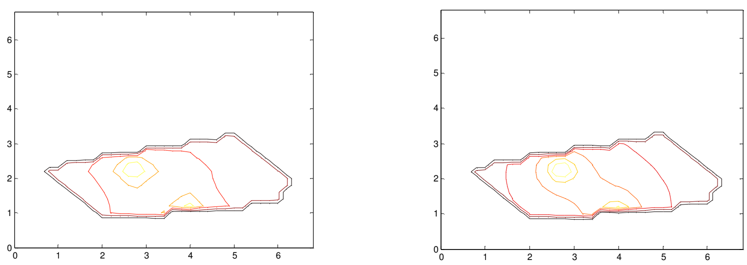
If there is only one demand point in the feasible area, the contours of the Minisum model have a similar shape as the Maximin model contours. Figure 8.8 illustrates the contour plots for the Minisum model (Appendix G.2.3.1.). Although the shape is similar, these graphs are not identical to the Maximin graph due to aim of the models. The similar shape of the rectilinear norm implies that there is again an optimum in the shape of a linesegment for the Minisum model (Figure 8.8, Graph a).



a b

Fig. 8.8. Contour plots of the generic Minisum model, illustration case Andalucía. In graph a) one demand point is measured with rectilinear distances. Graph b) shows one demand point and Euclidean distances.

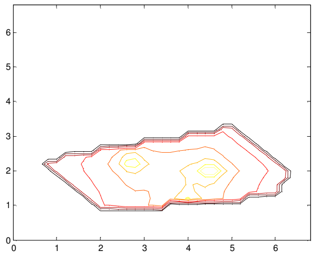
Figure 8.9 shows the feasible area with two demand points. It seems that in graph c) an optimal linesegement exists in between the two demand points. However, the shape of the norm has changed in a way that there will not be a optimal linesegment at the border of the feasible area.



c d

Fig. 8.9. Contour plots of the generic Minisum model, illustration case Andalucía. In graph c) shows 2 demand points measured with rectilinear distances. Graph d) shows 2 demand points and Euclidean distances.

In order to gain more insight in the way the norm is behaving, Figure 8.10 is designed, with three demand points. The shape of the norm has changed even more than in the graph c). The graph of the combination of three demand points (see graph e) illustrates that there is no optimal linesegment between the demand points. It is possible that an optimal point in between the three demand points exist, but this can not be concluded from graph e). The extra optima of the Minisum models measured with rectilinear distances can not be explained from the contour graphs.



e

Fig. 8.10. Contour plot of the generic Minisum model, illustration case Andalucía. Graph e) shows 3 demand points measured with the rectilinear distance metric.

9. Conclusions

Obnoxious single facility location models that are often referred to in literature are the Maximin, the Maxisum and the Minisum model. In the literature search in Scopus 8 articles are Maximin models, 2 Maxisum and 5 articles are on Minisum models.

Out of the tested algorithms, Grid Search, Controlled Random Search, Genetic Algorithm and Multistart, the latter one is the most appropriate for investigating obnoxious single facility models on their multimodality. Multistart is the only tested algorithm that is able to find local optima besides the global ones.

All models have global and local optima. Multistart does not have problems in finding the global optimum of the models.

Maximin models are the most difficult to solve in the sense that they have on average the most optima. When bounds on the feasible area are more complex, the number of optima rises.

Maxisum models are most easy to solve as they have few optima, which can be found in the extreme points of the feasible area.

Three of the five Minisum models are wind dispersion models. These models have unexpected locations found for the location of an obnoxious facility. The wind models are very sensitive for the input data. The data used for this models is not real data of Andalucía and this could be a reason for the unexpected outcomes of the models.

The generic model with weights has slightly more optima for all models than the unweighted generic model and thus is harder to solve.

For the models tested with the illustration case Andalucía with the rectilinear distance metric, on average more optima are found than for the same models measured with Euclidean distances. It can not be concluded that models measured with rectilinear distances have more optima than the same models measured with Euclidean distances. As illustrated in Section 8.6, it is possible that the reason for more optima with rectilinear distances is the shape of the feasible area of the Andalucía illustration case.

10. Discussion

- The literature search is executed at the University of Almeria, in Scopus, a digital scientific database, via the digital library of Wageningen University. The consequence is that not all articles that are found in Scopus could be used. Some of the articles were only available in hard copy in the library of Wageningen University and are therefore not included in this research. Only articles on obnoxious single facility location models that are available on the internet are included.

- The test cases are compared to real world situations relatively small. This may have consequences on the results of the solution methods. For the test cases used in this research, Multistart worked very well but it is possible that it is not the best algorithm to solve bigger problems.

- The algorithms are tested with their default settings. Tuning of the algorithms can lead to better results on finding global and local optima.

11. Recommendations for future research

The research can be extended to

15. Semi- obnoxious facility location models
16. (semi-) obnoxious multi facility location models
17. (semi-) obnoxious multi objective facility location models
- a discreet feasible area or a feasible area on a network

Although Multistart is a suitable algorithm for testing multimodality, it is interesting to investigate the performance of other methods for finding all optima, like other multimodal methods, for example based on niching.

The Mixed Inter Programming model, Maximin model 3, is only tested with 25 demand points due to licence restrictions of GAMS. To be able to compare the model with the other models, an equal number of demand points, 32, should be used as input.

Minisum models 1,2 and 3, also referred to as the wind models, are implemented with wind data that are not of Andalucía. With the data that are used as input for this research, the models have unexpected results for the optimal location. By changing the input data to realistic data for Andalucía it is possible that the models give better optimal solutions.

To exclude the influence of illustration case Andalucía on the results of the difference in optima when measuring with the Euclidean or rectilinear norm, another illustration case should be used.

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Appendix A. Results of Scopus search on ¨obnoxious facility model¨ and ¨undesirable facility model¨

= article of search I is the same as article x of search II

Search I:
Scopus ¨obnoxious facility¨ model Hits:42, no selection

1 **Median problems with positive and negative weights on cycles and cacti**
Burkard, R.E., Hatzl, J. 2008 Journal of Combinatorial Optimization, pp. 1-20. Article in Press

2 **Locating a semi-obnoxious facility with expropriation**
Berman, O., Wang, Q. 2008 Computers and Operations Research 35 (2), pp. 392-403

3 **The p-maxian problem on a tree**
Burkard, R.E., Fathali, J., Taghizadeh Kakhki, H. 2007 Operations Research Letters 35 (3), pp. 331-335

4 **A polynomial method for the pos/neg weighted 3-median problem on a tree**
Burkard, R.E., Fathali, J. 2007 Mathematical Methods of Operations Research 65 (2), pp. 229-238

5 **Locating semi-obnoxious facilities with expropriation: Minisum criterion**
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6 **A study of location problem and vehicle routing problem for the obnoxious facility**
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10 **Locating two obnoxious facilities using the weighted maximin criterion**
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11 **A new bound and an O(mn) algorithm for the undesirable 1-median problem (maxian)**
on networks Colebrook, M., Gutiérrez, J., Sicilia, J. 2005 Computers and Operations Research 32 (2), pp. 309-325 3

12 **A lattice covering model for evaluating existing service facilities**
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14 **Discrete facility location and routing of obnoxious activities**
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15 **Efficient Location for a Semi-Obnoxious Facility**
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34 **A branch-and-bound algorithm for the location of facilities causing atmospheric pollution**
Karkazis, J., Papadimitriou, C. 1992 European Journal of Operational Research 58 (3), pp. 363-373
35 **Location of facilities producing airborne pollution**
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36 **The problem of locating facilities causing airborne pollution revisited**
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37 **The discrete p-Maxian location problem**
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38 **Analytical models for locating undesirable facilities**
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39 **EFFICIENT COMPUTATIONAL PROCEDURE FOR THE RECTILINEAR MAXIMIN LOCATION PROBLEM.**
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40 **Location of multiple obnoxious facilities**
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41 **Comments on Orloff's net accessibility model.**
Church, R.L., Roberts, K.L., Bell, T.L. 1979 Geographical Analysis 11 (1), pp. 89-93

42 **LOCATING AN OBNOXIOUS FACILITY ON A NETWORK.**
Church, Richard L., Garfinkel, Robert S. 1978 Transportation Science 12 (2), pp. 107-118

Search II:
Scopus ¨undesirable facility¨ model Hits: 106, selection:
LIMIT-TO(SUBJAREA, "DECI") OR LIMIT-TO(SUBJAREA, "MATH") OR LIMIT-TO(SUBJAREA, "BUSI") OR LIMIT-TO(SUBJAREA, "COMP") OR LIMIT-TO(SUBJAREA, "ECON") OR LIMIT-TO(SUBJAREA, "MULT")
Hits: 38

1 **Mixed integer programming-based solution procedure for single-facility location with maximin of rectilinear distance**
Nadirler, D., Karasakal, E. 2008 Journal of the Operational Research Society 59 (4), pp. 563-570

2 **Server-rack air flow and heat transfer interactions in data centers**
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4 **Fuzzy facility location problem with preference of candidate sites**
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5 **Microwave production of carbon fiber reinforced plastics | [Herstellung von Kohlenstofffaserverstärkten Kunststoffbauteilen mit Hilfe von Mikrowellen]**
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Appendix B. Selection of Obnoxious Single Facility Models

Legenda

[orange]	=	Article in press (filled in on basis of title and/or abstract)
[red]	=	Not available by WUR (and/or not on-line)(filled in on basis of title and/or abstract) (and/or does not contain a model)
[green]	=	Selection criteria for the thesis
[blue]	=	Satisfy all criteria
*	=	Same article in both searches

SCOPUS 3-11-2008
Search for: ¨obnoxious facility¨ model **in** Article Title, Abstract, Keywords
Date Range
Published All years to Present
Results: 42

Article #	**Obnox**	**Semi-**	**Other**	**# facilities to be located**		**Feasible region facility**			**Distance measure**		**Objective**					
(time ord)		**obnox**	**Facility**	p = 1	p > 1	discreet	continuous	network	Euclidean	Rectilinear	maximin	maxisum	minisum	other	single	multi
1			1		1									1		
2		1		1				1						1		1
3	1				1			1				1				1
4		1			1			1								1
5		1		1	1		1	1		1				1		1
6	1			1				1								1
7		1							1					1		1
*8	1			1			1		1		1	1		1	1	
9		1		1			1		1	1			1	1	1	1
10	1				1		1			1	1				1	
*11	1			1				1				1			1	
12			1				1		1					1	1	
*13		1		1			1		1		1		1			1
14		1		1		1		1						1	1	

Article # (time ord)	Obnox	Semi-obnox	Other Facility	# facilities to be located		Feasible region facility			Distance measure		Objective					
				p = 1	p > 1	discreet	continuous	network	Euclidean	Rectilinear	maximin	maxisum	minisum	other	single	multi
15		1		1			1			1	1		1			1
16	1			1			1		1		1					
*17		1		1			1	1	1	1				1		1
18	1			1				1				1		1	1	
*19		1		1				1						1		1
*20	1			1				1			1			1	1	
21	1			1	1		1		1					1	1	
22	1			1				1			1	1				1
23	1			1				1			1		1			1
24	1			1			1	1	1					1	1	
25	1				1			1						1	1	
26																
*27		1		1			1			1	1		1			1
28	1		1	1				1						1	1	
29		1		1			1			1				1		1
30	1			1	1			1	1		1	1			1	
31	1			1			1		1		1				1	
32																
33																
34	1			1			1		1				1	1	1	
35	1			1			1		1				1	1	1	
36	1			1			1		1				1	1	1	
37	1			1	1	1			1		1	1			1	
*38	1			1	1		1	1	1	1	1	1		1	1	1
*39	1			1		1				1	1				1	
*40	1				1						1			1		
41																
42																

Search for: ¨undesirable facility¨ model **in** Article Title, Abstract, Keywords
Date Range
Published All years to Present
Results: 106
LIMIT-TO(SUBJAREA, "DECI") OR LIMIT-TO(SUBJAREA, "MATH") OR LIMIT-TO(SUBJAREA, "BUSI") OR LIMIT-TO(SUBJAREA, "COMP")
OR LIMIT-TO(SUBJAREA, "ECON") OR LIMIT-TO(SUBJAREA, "MULT")
Results: 38

Article #	**Obnox**	**Semi-**	**Other**	**# facilities to be located**		**Feasible region**			**Distance measure**		**Objective**					
(time ord)		**obnox**	**Facility**	p = 1	p > 1	discreet	continuous	network	Euclidean	Rectilinear	maximin	maxisum	minisum	other	single	multi
1	1			1			1			1	1			1	1	
2																
3																
4		1		1			1			1				1		1
5																
6	1			1			1		1	1	1			1	1	
7	1				1			1				1		1		1
8	1			1			1		1		1	1		1	1	
9	1			1				1				1			1	
10																
11			1	1				1			1			1	1	
12																
13		1		1			1		1		1		1			1
14																
15		1		1			1	1	1	1				1		1
16	1			1				1			1			1	1	
17		1		1				1						1		1
18																

Article # (time ord)	Obnox	Semi-obnox	Other Facility	# facilities to be located p = 1	p > 1	Feasible region discreet	continuous	network	Distance measure Euclidean	Rectilinear	Objective maximin	maxisum	minisum	other	single	multi
19																
20	1			1			1		1				1		1	
21		1		1			1			1	1		1			1
22	1			1			1		1					1	1	
23	1			1			1		1		1		1		1	
24																
25		1		1	1		1		1					1		1
26																
27	1			1			1		1					1	1	
28	1				1	1			1		1				1	
29	1			1	1		1	1	1	1	1	1		1	1	1
30	1			1		1				1	1				1	
31																
32	1			1			1		1		1				1	
33	1				1						1			1		
34																
35	1			1			1		1		1				1	
36																
37																
38																

Appendix C. Data of Test Cases 1, 2 and 3

1. Demand points and weights Test Case 1. Feasible area = [0,30] x [0,30]

P	30*rand	30*rand	4*rand round up
i	x_1	x_2	w_i
1	25.62	25.11	3
2	0.23	10.04	2
3	13.54	6.7	1
4	22.02	8.45	1
5	24.98	20.81	1
6	22.04	28.96	3
7	11.29	4.19	1
8	20.15	0.77	2
9	13.52	12.88	3
10	7.79	20.44	4
11	18.7	4.29	3
12	11.32	16.89	3
13	4.73	14.46	1
14	13.96	26.82	2
15	14.85	7.55	3
16	7.58	26.9	1
17	19.13	11.03	3
18	26.73	1.84	2
19	27.49	16.62	4
20	26.18	16.91	2
21	6.38	27.23	1

2. Demand points Test Case 2. Feasible area = [0,6] x [0,6]

P		
i	x_1	x_2
1	2	4
2	2	2
3	4	2
4	4	4

3. Demand point Test Case 3. Feasible area = [0,2] x [0,2].

P		
i	x_1	x_2
1	1	1

Appendix D. Data of Illustration Case Andalucía

1 a. Demand points of Illustration Case Andalucía.

P i		Inhabitants	w_i	Longitude	Latitude	x_1	x_2
1	**Almeria**	173338	3	-2.43	36.83	5.71	1.26
2	Arcos de la Frontera	28369	1	-5.81	36.76	2.33	1.19
3	Jerez de la Frontera	202678	3	-6.13	36.68	2.01	1.11
4	Sanlucar de Barrameda	61908	2	-6.36	36.78	1.78	1.21
5	**Cordoba**	314805	3	-4.77	37.88	3.37	2.31
6	Lucena	37660	1	-4.49	37.41	3.65	1.84
7	Montilla	23235	1	-4.64	37.6	3.5	2.03
8	Puente Genil	27720	1	-4.77	37.39	3.37	1.82
9	**Granada**	240522	3	-3.59	37.17	4.55	1.6
10	Motril	51928	2	-3.52	36.76	4.62	1.19
11	**Huelva**	145150	3	-6.94	37.25	1.20	1.68
12	Andujar	37920	1	-4.06	38.04	4.08	2.47
13	**Jaen**	112921	3	-3.8	37.77	4.34	2.2
14	Linares	57800	2	-3.63	38.1	4.51	2.53
15	Ubeda	32971	1	-3.38	38.02	4.76	2.45
16	Antequera	41197	1	-4.56	37.02	3.58	1.45
17	**Malaga**	535686	3	-4.42	36.72	3.72	1.15
18	Ronda	34470	1	-5.16	36.74	2.98	1.17
19	Velez-Malaga	57457	2	-4.1	36.79	4.04	1.22
20	Alcala de Guardaira	58351	2	-5.84	37.34	2.3	1.77
21	Camas	25109	1	-6.02	37.4	2.12	1.83
22	Carmona	25932	1	-5.63	37.48	2.51	1.91
23	Coria del Rio	24288	1	-6.06	37.3	2.08	1.73
24	Dos Hermanas	103282	3	-5.93	37.28	2.21	1.71
25	Ecija	37900	1	-5.09	37.54	3.05	1.97
26	Lebrija	24450	1	-6.08	36.93	2.06	1.36
27	Mairena del Aljarafe	36232	1	-6.07	37.34	2.07	1.77
28	Moron de la Frontera	27786	1	-5.46	37.13	2.68	1.56
29	Los Palacios y villafranca	33461	1	-6	37.26	2.14	1.69
30	La rinconada	29759	1	-5.97	37.49	2.17	1.92
31	**Seville**	704114	3	-5.98	37.4	2.16	1.83
32	Utrera	45947	1	-5.78	37.18	2.36	1.61

The cities that correspond with the yellow marked numbers are left out for Maximin model 3, the Mixed Integer Programming model. These are the cities with the least inhabitants from the list.

1 b. Bounds on the feasible region for the Illustration Case Andalucía.

A * x ≤ b

```
A = [-1.10311     -1
   0.118788     -1
   -0.21495      1
   1.027         1
   1.918172     -1];

b =[-3.01636
   -0.57273
   2.17636
   8.53588357
   10.21766131];
```