

**Are Prices a Good Signal of the Degree of Product Differentiation in Oligopoly
Markets with Asymmetric Information?**

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Abstract

This article analyses price competition in a two-period duopoly model in which only one firm knows the degree of substitutability between products. Using a Hotelling model, we analyse the informed firm's incentive to reveal its private information throughout its price set in period 1. In this setting, the price set by the informed firm only reveals the degree of product differentiation in period 1 when the prior probability of closer substitutes is sufficiently high and the discount factor is sufficiently low. Finally, we find that firms differentiate their products as far as possible under asymmetric information.

Keywords: Asymmetric information; degree of substitutability between products; demand uncertainty; signalling game; undefeated equilibrium.

JEL Classification: D43, D82, L13.

I. INTRODUCTION

II.1 Motivation

The consumers' perceived degree of substitutability between products sold by main competitors is one of the variables which best represents consumers' preferences and the level of competition in a particular market. For example, when consumers consider that the products sold by two rivals are perfect substitutes, the firms will have to fiercely compete with each other to attract the customers by setting lower prices, whereas they can set higher prices when products are highly differentiated. Hence, each firm has to assess the consumers' perceived degree of differentiation between its products and those sold by its competitors.

Nowadays, there is a huge variety of products on offer and consumers use new technologies, friendship networks, and other tools to decide which product they want to buy. Consequently, customers are highly influenced by social networks, marketing campaigns, and other technological stimuli when they buy their products. Thus, the increased volatility of consumers' preferences makes it much more difficult for firms to estimate the cross-price elasticity of different products. However, there are usually some sellers whose knowledge of the market is better than that of others. In fact, many firms measure consumers' preferences by employing community managers, surveying targeted groups of consumers, gathering customers' information from different online sources, and using other strategies. Obviously, there are significant differences in information costs and in the quality of the estimations obtained by different firms, and thus some firms will have more information on the market demand than others.

This article analyses a two-period Hotelling model with linear transportation costs in which one firm knows the consumers' perceived degree of substitutability between products, whereas the other firm does not. Moreover, both firms compete à la Bertrand in both periods

and observe the prices set in the first period before choosing their prices in the second one. In this setting, the uninformed firm will use the price set by its informed rival in period 1 in order to infer the unknown parameter in period 2. Under these assumptions, we determine the conditions under which the informed firm will use its price set in period 1 to reveal the degree of product differentiation to its rival.

In particular, we prove that in period 1 the informed firm will conceal its information on the market demand parameter from its rival by setting its optimal price for products that are highly differentiated, whenever the firm is sufficiently patient and the probability of closer substitutes is sufficiently low. However, if the informed firm is sufficiently impatient or the probability of closer substitutes is sufficiently high, the price set by the informed firm in period 1 will reveal the degree of product differentiation. We demonstrate that these results will also be obtained when a Stackelberg model is used in which the informed firm chooses its price in each period after the uninformed one. However, the price set by the informed firm in period 1 will always reveal the degree of product differentiation when the informed firm chooses its price before the uninformed firm.

According to these results, when products are close substitutes, the informed firm conceals this information from its rival whenever the prior probability of closer substitutes is sufficiently low and firms are sufficiently patient. Then, we might expect that this opportunity of the informed firm to conceal its information on a high degree of substitutability between products will decrease its incentives to differentiate its product from its rival. Nevertheless, using a Hotelling model with quadratic transportation costs, we endogenize firms' location decisions and obtain that they try to differentiate their products as much as possible even with this type of incentives.

II.2 Related literature and contribution of the model

Previous theoretical research has analysed firms' decisions in oligopolistic markets in which none of the firms can observe some parameters of the market demand curve. In particular, some studies have investigated firms' capacity to manipulate market outcomes in order to distort their rival's information (e.g., Riordan, 1985; Mirman, Samuelson, and Urbano, 1993; Bernhardt and Taub, 2015). This is referred to as signal jamming. Other studies have investigated firms' ability to learn about an unknown parameter of the demand curve through experimentation (Harrington, 1992, 1995; Aghion, Espinosa, and Jullien, 1993; Keller and Rady, 2003). This is referred to as learning through experimentation. For example, Aghion, Espinosa, and Jullien (1993) also used the same two-period Hotelling model as ours, but they assume that firms' information is symmetric. As in our model, the parameter that firms cannot observe is the degree of substitutability between products. Like us, Aghion, Espinosa, and Jullien (1993) assume that the degree of substitutability between products can take only two values. Since these authors assume that each firm can observe its own volume of sales in period 1 before choosing its price in period 2, imperfect information will give firms incentives to increase price dispersion in the market to learn the degree of product differentiation. Unlike this model, we assume that information on the degree of product differentiation is asymmetric and that each firm cannot observe its own volume of sales in period 1 before setting its price in period 2. This assumption may be plausible in many real markets. For example, hotels and airlines have to set their prices for the next season before knowing their quantity sold in the current season. Likewise, the fashion industry may provide an additional example of future pricing decisions without knowing the quantity sold in the previous period. In fact, designers usually present their new proposals for the next year in advance and they have to set their prices for the next year without knowing their quantity sold this year. Furthermore, home builders may start a second phase of construction for

which they have to set a price before knowing the number of homes of the first phase they will sell.

In all the previous articles, all the firms in the market cannot observe a particular parameter of the market demand curve, but none of them have better information than others. However, in some real markets, some firms might have informational advantages. The main contribution of our model to previous theoretical literature is that we introduce asymmetric information to analyse the effects of demand uncertainty on firms' decisions in a duopoly game. Finally, Brandão and Pinho (2015) used the same Hotelling's model as ours and also assume that one firm knows the transportation cost whereas the other does not. They only model a one-shot game and analyse firms' incentives to exchange its private information. However, these authors found that there is no contract that induces the informed firm to reveal its private information in their one-period game, whereas we found that the informed firm may use its price set in period 1 to reveal the transportation cost in a two-period model. Additionally, as most studies that use a signalling game in industrial organization, we have to deal with the presence of multiple sequential equilibria. For this reason, we will use the criterion developed by Mailath, Okuno-Fujiwara, and Postlewaite (1993) to find the unique equilibrium of our model. They called the resulting equilibrium "undefeated equilibrium".

This article is organized as follows. The next section describes the model. The third section determines the conditions under which a pooling or a separating equilibrium arises and proves the uniqueness of our equilibrium outcomes. Sections 4 and 5 analyse the robustness of our results under different assumptions, and section 6 summarizes the main conclusions. The appendix includes the proofs of some propositions obtained.

II. THE MODEL

To start this section, we introduce the Hotelling's model used. Assume that there is a line of unit length (say 1 mile) along which consumers are uniformly distributed and this market is supplied by two risk-neutral firms: firms 1 and 2. Firm 1 is located at the west end of town whereas firm 2 is located at the east end and they cannot change their locations because they find it prohibitively expensive to do so. We define a consumer location in this market to be that consumer's most preferred product or style. Thus, "consumer x " is located distance x from the left-hand end of the market, where distance may be geographic in a spatial model or measured in terms of characteristics in a more general product differentiation sense. Although consumers differ regarding which variant or location of the good is the best or ideal product, they are identical in that they assign the same reservation price V for their most preferred product. Each consumer is assumed to buy at most one unit of the product. As usual, if consumer x purchases a good that is not her ideal product, she will incur a utility loss or a cost equal to Rx when she consumes good 1, and a cost equal to $R(1-x)$ when she consumes good 2, where $R = \{L, H\}$, and $H > L > 0$. In this set-up, R is the transportation cost and can take only two values. This parameter represents the degree of product differentiation because a consumer who is located closer to one firm than to another would find it more costly to substitute the product from the closest supplier with one from a supplier further away when the transportation cost is higher. Then, if a consumer buys good 1 at price p_t^1 in period t , she will enjoy consumer surplus $V - p_t^1 - Rx$ and if she buys good 2 at price p_t^2 in period t , she will enjoy consumer surplus $V - p_t^2 - R(1-x)$. Of course, she will purchase the good that offers her the greatest consumer surplus provided that this is greater than zero. Both firms compete for customers by simultaneously setting prices in a stable market in two consecutive periods (i.e., we assume that R does not change over time). Each firm has a constant unit cost of production equal to c . It is assumed that V is substantially greater than the unit cost of production c . Moreover, the discount factor, which is δ , is the same for both firms and is common knowledge.

When the entire market is served, there will be a marginal consumer, x^m , who is indifferent to buying from firm 1 or firm 2 (i.e., she enjoys the same consumer surplus either way).

Algebraically, it means that for consumer x^m

$$V - p_t^1 - R x^m = V - p_t^2 - R(1 - x^m) \quad (1)$$

This equation may be solved to find the location of the marginal consumer. This location is

$$x_t^m(p_t^1, p_t^2) = \frac{p_t^2 - p_t^1 + R}{2R} \quad (2)$$

At any set of prices, p_t^1 and p_t^2 , all the consumers to the left of x_t^m buy from firm 1 in period t . All those to the right of x_t^m buy from firm 2 in period t . Thus, x_t^m is the fraction of the market that buys from firm 1 and $(1 - x_t^m)$ is the fraction that buys from firm 2. If the total number of consumers is N and they are uniformly distributed over the market space, the demand function facing firm 1 in period t at any combination, (p_t^1, p_t^2) in which the entire market is served is

$$D_t^1(p_t^1, p_t^2) = \frac{(p_t^2 - p_t^1 + R)}{2R} N + \varepsilon_t \quad (3)$$

where ε_t is a specific random shock and its expected value is equal to zero. Similarly, firm 2's demand function in period t is

$$D_t^2(p_t^1, p_t^2) = \frac{(p_t^1 - p_t^2 + R)}{2R} N - \varepsilon_t \quad (4)$$

The timing of the information is as follows.

Firms' information in period 1. Before choosing prices in the first period, it is assumed that firm 1 (the informed one) can observe R , but firm 2 (the uninformed one) cannot observe the true value of this parameter. It is common knowledge that the prior probability of R being equal to L is ϕ .

Firms' information in period 2. Before choosing its price in period 2, each firm observes the prices chosen in period 1, p_1^1, p_1^2 ; however, neither of the firms can observe the quantities sold in that period¹. Using this information, firm 2 will try to update its knowledge of the demand parameter, R , but firm 1 could conceal the true value of R by choosing a suboptimal price in period 1 because firm 2 cannot observe its quantity sold in period 1.

The next sections show the sequential equilibria of our game as defined by Kreps and Wilson (1982). A sequential equilibrium for our signalling game will be a triple of strategies and beliefs that satisfy the following conditions:

1. Sequential rationality: For each R , the informed firm should set a price in period 1 and a price in period 2 that maximize its total profit given the prices set in periods 1 and 2 by the uninformed firm. Additionally, the uninformed firm should set a price in period 1 and a price in period 2 that maximize its total profit given its posterior beliefs about the unknown parameter and the prices set by its rival.
2. Consistency: In the equilibrium path the uninformed firm's beliefs about the unknown parameter should be calculated using Bayes's rule.

As there can be multiple sequential equilibria, we will choose the unique undefeated equilibrium of our model.

III. POOLING VERSUS SEPARATING EQUILIBRIUM

III.1 Main results

¹ Since each firm cannot observe its own quantity sold in period 1 before setting its price in period 2, the specification of the distribution function of ε_t is irrelevant. Nevertheless, if we assume that each firm can observe its own volume of sales in period 1 before setting its price in period 2 and that the specific demand shock has a uniform distribution, as usual in models of learning through experimentation, and that the support of that demand shock is sufficiently large, the results obtained are the same as those presented in the next section. The proof of this result is available from the authors upon request.

In this subsection, we only derive two potential candidates for equilibria in pure strategies (i.e., a separating and a pooling equilibrium), whereas in the next subsection we will prove that one of them will be the unique undefeated equilibrium depending on the values of the parameters of the model. In a separating equilibrium, the informed firm's decision in period 1 will reveal the true value of the unknown parameter. This will occur if the informed firm prefers setting an optimal price in period 1 for each value of R to a suboptimal price to conceal its information on the demand parameter.

To understand the way in which we build our proposed separating equilibrium, let's imagine that both firms compete with each other *à la* Bertrand in a one-period game. In that case, we could obtain the Bayesian-Nash equilibrium of this simple game with asymmetric information and the prices set by both firms would be:

$$p_t^{2B} = c + \frac{LH}{R} \quad (5)$$

and

$$p_t^{1B}(R) = c + \frac{R\hat{R}+LH}{2\hat{R}} \quad (6)$$

where $\hat{R} = \phi H + (1 - \phi)L$. We propose a separating equilibrium in which both firms set the prices given in equations (5) and (6) in the first period. Then, $p_1^{1B}(R)$ and p_1^{2B} will denote these firms' decisions in period 1. The price chosen by firm 1 depends on the transportation cost, R , and thus, it will reveal the true value of this parameter. In particular, if we substitute L for R in equation (6), we obtain the optimal price set by the informed firm when the transportation cost is low. Similarly, when $R=H$, equation (6) shows the optimal price set by firm 1 in period 1 if the transportation cost is high. Nevertheless, equation (5) shows that the uninformed firm's optimal price cannot depend on R , because this firm sets its optimal price given its prior information. In this separating equilibrium, the uninformed firm will observe

the price set by its rival in period 1 and learn the true value of the transportation cost in period 2 and both firms will compete with full information. Thus, the optimal prices set by both firms in period 2 will be $p_2^{1Bf}(R) = p_2^{2Bf}(R) = c + R$, which are the same as the optimal prices chosen by firms in a standard Hotelling's model with complete information. To finish the description of our proposed separating equilibrium we need to specify the uninformed firm's posterior beliefs in period 2 after observing the price chosen by its rival in period 1. The next function defines the posterior probability of a low transportation cost, $\hat{\phi}$:

$$\hat{\phi} = \begin{cases} 0 & \text{if } p_1^1 \geq p_1^{1B}(H) \\ 1 & \text{if } p_1^1 < p_1^{1B}(H) \end{cases} \quad (7)$$

It means that the uninformed firm will believe that the transportation cost is high whenever the price set by its rival in period 1 is greater than or equal to the optimal price for a high transportation cost. Likewise, if the informed firm sets a lower price, the uninformed one will believe that the transportation cost is low.

Now, we will describe our proposed pooling equilibrium, which will consist of the following strategies. In the first period, the informed firm will set its optimal price in a one-period game for a high transportation cost irrespective of the true value of the unknown parameter (i.e., the price shown in equation (6) when $R=H$). However, as equation (5) shows, the uninformed firm will set its optimal price given its prior information. Now, as the price set by firm 1 in the first period does not depend on the true value of R , it will keep firm 2 uninformed in period 2 and once again, equation (5) shows the price set by this uninformed firm in period 2 in this pooling equilibrium, $p_2^{2B} = p_1^{2B}$, because the posterior probabilities are the same as the priors. Finally, the informed firm will take advantage of its better information in period 2 and equation (6) shows the price set by firm 1 given the true value

of the unknown parameter, $p_2^{1B}(R)$. The following equation describes the posterior probability function in this pooling equilibrium:

$$\hat{\phi} = \begin{cases} \phi & \text{if } p_1^1 \geq p_1^{1B}(H) \\ 1 & \text{if } p_1^1 < p_1^{1B}(H) \end{cases} \quad (8)$$

To understand the intuition of this function, we need to recall that the discount factor is common knowledge in this model. Then, when the transportation cost is low, the informed firm will have to decide whether to set a high price in period 1 in order to make its rival believe that the transportation cost is high or to set its optimal price for a low transportation cost. If the informed firm sets a high price in period 1, it will assume a cost because it knows that its rival will undercut it. Nonetheless, the uninformed firm will be less competitive in the second period and the informed one will take advantage of its better information obtaining more profits. Thus, the informed firm will find it profitable to set a higher price in period 1 whenever the discount factor is sufficiently high so that the discounted additional profit obtained in period 2 is greater than the cost assumed in period 1. Since the uninformed firm knows the discount factor, it will know whether it is optimal for its rival to hide its private information or not. For this reason, when the discount factor is sufficiently high and firm 1 sets a high price, firm 2 will not know whether the true transportation cost is high or low and will continue having the same information as at the beginning of period 1. Finally, when firm 1 sets a lower price, firm 2 will believe that the transportation cost is low.

By construction, both our pooling and separating equilibria are sequential equilibria and now we will derive the conditions under which the informed firm prefers concealing its private information to revealing it when the transportation cost is low, that is, we will derive the conditions under which the pooling equilibrium will arise.

First, we obtain the informed firm's expected profit in period 1 if it sets its optimal price for a high transportation cost ($R = H$), when it is actually low (i.e., $R = L$):

$$E\{\pi_1^1[p_1^{1B}(H), p_1^{2B}|R = L]\} = \frac{(H\hat{R}+LH)[LH+\hat{R}(2L-H)]}{8\hat{R}^2L}N \quad (9)$$

In this case, the informed firm conceals its private information on R from its rival, but it will take advantage of its better information in period 2 by undercutting its rival. Then, its expected profit will be:

$$E\{\pi_2^1[p_1^{1B}(H), p_1^{2B}, p_2^{1B}(L), p_2^{2B}|R = L]\} = \frac{(LH+L\hat{R})^2}{8L\hat{R}^2}N \quad (10)$$

Similarly, in our separating equilibrium, when $R = L$ and the informed firm chooses its optimal price for a low transportation cost in period 1, its expected profit in that period is:

$$E\{\pi_1^1[p_1^{1B}(L), p_1^{2B}|R = L]\} = \frac{(LH+L\hat{R})^2}{8L\hat{R}^2}N \quad (11)$$

In this case, the uninformed firm will know that the true value of R is L and both firms will compete with full knowledge in period 2 and then, their expected profits will be:

$$E\{\pi_2^1[p_1^{1B}(L), p_1^{2B}, p_2^{1Bf}(L), p_2^{2Bf}(L)|R = L]\} =$$

$$E\{\pi_2^2[p_1^{1B}(L), p_1^{2B}, p_2^{1Bf}(L), p_2^{2Bf}(L)|R = L]\} = \frac{L}{2}N \quad (12)$$

Thus, the informed firm sets its optimal price for a high transportation cost in period 1 even if the true value of this cost is low whenever it is more profitable for the informed firm to do this, rather than revealing the true value of the unknown parameter by setting its optimal price for a low transportation cost. In other words, a pooling equilibrium arises provided that:

$$E\{\pi_1^1[p_1^{1B}(H), p_1^{2B}|R = L]\} + \delta E\{\pi_2^1[p_1^{1B}(H), p_1^{2B}, p_2^{1B}(L), p_2^{2B}|R = L]\} >$$

$$E\{\pi_1^1[p_1^{1B}(L), p_1^{2B}|R = L]\} + \delta E\{\pi_2^1[p_1^{1B}(L), p_1^{2B}, p_2^{1Bf}(L), p_2^{2Bf}(L)|R = L]\} \quad (13)$$

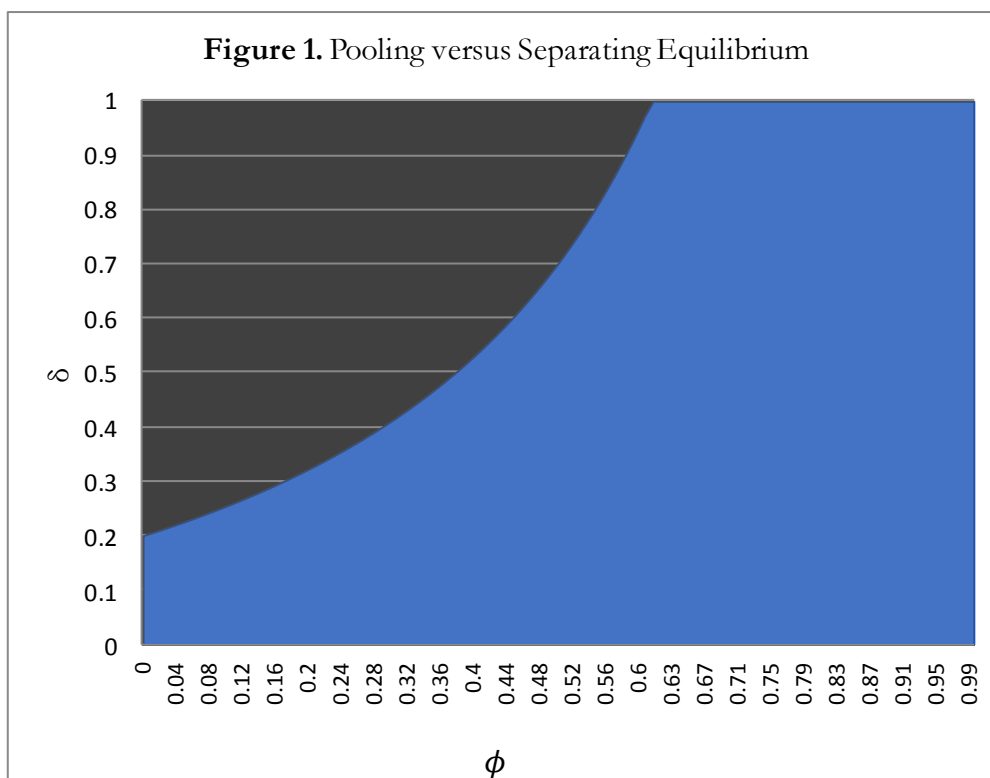
This inequality allows us to obtain the next proposition, which addresses the market conditions under which a pooling or a separating equilibrium arises.

Proposition 1. Our proposed pooling equilibrium will arise provided that the prior probability of a high transportation cost is sufficiently high and firms are sufficiently patient. Otherwise, a separating equilibrium will arise.

The proof of this proposition can be found in the Appendix and Figure 1 shows the regions in which there will be a pooling or a separating equilibrium, depending on ϕ and δ , when $L=1$ and $H=2$. The grey shaded area represents the values of ϕ and δ for which we obtain a pooling equilibrium, whereas the blue shaded area represents the values of ϕ and δ for which a separating equilibrium arises.

To understand this proposition, imagine that the transportation cost is low. In that case, in period 1, the informed firm will have to choose whether to conceal its private information by setting a high price, or to reveal that information by setting a low price. Therefore, concealing information on the transportation cost imposes a cost on the informed firm in period 1 because it knows that its rival will undercut it. However, the informed firm will face a less competitive rival in period 2 and will make a greater profit. Thus, it will be profitable for the informed firm to conceal its private information whenever the additional profit obtained in period 2 is greater than the cost incurred in period 1, that is, whenever the discount factor is sufficiently high. When the prior probability of a high transportation cost is greater (i.e., when ϕ is lower in Figure 1), the uninformed firm will set a higher price in the first period and so the informed firm's cost of concealing its information in period 1 will be lower. Therefore, the minimum discount factor for which firm 1's additional profit obtained in period 2 from taking advantage of its private information exceeds its cost, is lower as shown in Figure 1. Likewise, when the prior probability of a low transportation cost is sufficiently high, the uninformed firm will fiercely compete in period 1 and so the informed

firm's cost of setting a high price to conceal its information from its rival will be so high, that the informed firm will always prefer to reveal its information in period 1, in which case a separating equilibrium will always arise as shown by Figure 1.



III.2 Uniqueness of the undefeated equilibrium

Since it is well-known that there are multiple sequential equilibria in signalling games, previous literature has proposed different criteria to restrict the off-the-equilibrium-path beliefs in order to obtain unique equilibria in the most relevant economic models². Nevertheless, Mailath, Okuno-Fujiwara, and Postlewaite (1993) questioned previous refinements of sequential equilibria and suggested that a sequential equilibrium could be tested in the following way. Consider a message that is not sent in the equilibrium and suppose that there is an alternative sequential equilibrium in which some set of types of

² For example, the intuitive criterion was proposed by Cho and Kreps (1987), whereas Banks and Sobel (1987) suggested two different criteria that they call divinity and universal divinity.

informed player choose the given message and prefer the alternative equilibrium to the proposed equilibrium. The test requires that the uninformed player's beliefs at that action in the original equilibrium to be consistent with this set. Mailath et al. (1993) stated that if the beliefs are not consistent, the second equilibrium defeats the proposed equilibrium, in which case they only choose the equilibria that are *undefeated* by any other equilibrium.

These authors showed that the undefeated equilibrium coincides with the lexicographically undominated equilibrium in monotonic signalling games, which are games in which the uninformed player prefers to interact with the highest possible type of the informed player³ and the informed player prefers that the uninformed player believes that he or she is the highest possible type. Since our game is monotonic, our undefeated equilibrium will coincide with the lexicographically undominated equilibrium of our model, which can be obtained as follows. Imagine that there is an equilibrium for which there is a message which is out of that equilibrium path for a particular type of informed player, but this type sends that message at a second equilibrium and is better off at the second equilibrium than at the first one. If those types of the informed player who are higher than the deviating type are at the second equilibrium, at least as well as they are at the first, we say that the second equilibrium lexicographically dominates the first one. Likewise, if we find a type of informed player who is better off by sending his or her message specified at the second equilibrium than by sending his or her message at the first, but all the higher types of informed player are sending the same message at the second equilibrium than at the first, we also conclude that the second equilibrium lexicographically dominates the first one regardless of whether those higher types are better off or worse off at the second equilibrium. Thus, an equilibrium is a lexicographically maximum sequential equilibrium (undefeated equilibrium) whenever there

³ In the classical model of the labor market, workers are the informed players because they know their productivity, whereas employers do not. In this example, we can arrange the types of the informed player according to his or her productivity. In our model, we consider that the informed firm has a greater type when the transportation cost is high than when it is low.

is no other equilibrium that lexicographically dominates it. We are going to use this concept to prove that in the blue shaded area of Figure 1 our proposed separating equilibrium will be the unique undefeated equilibrium of this game, whereas our pooling equilibrium will be the unique undefeated equilibrium in the grey shaded area.

To simplify the exposition, we will call the informed firm which knows that the transportation cost is low the “low-cost firm”, whereas the informed firm which knows that the transportation cost is high will be the “high-cost firm”. First, we will prove that our proposed separating equilibrium lexicographically dominates our proposed pooling equilibrium in the blue shaded area. As shown in the previous subsection, the low-cost firm is better off by choosing its strategy specified by the separating equilibrium in the blue shaded area, whereas the high-cost firm is choosing the same price in period 1 (its message) at both equilibria. Thus, the separating equilibrium dominates the pooling one in that area. Similarly, the low-cost firm is better off by choosing its strategy specified by the pooling equilibrium in the grey shaded area, whereas the other type of informed firm is choosing the same message (price in period 1) at both equilibria. Therefore, the pooling equilibrium dominates the separating one in the grey shaded area.

It is easy to prove that there cannot be a separating equilibrium that is different from ours⁴. In this model, an equilibrium will be separating whenever each type of informed firm sets a different price in period 1 and the posterior probability of a low transportation cost is either equal to zero or one. Then, another separating equilibrium might differ from ours because the prices chosen in period 1 by each type of informed firm are different or because the posterior probabilities are different. Obviously, a different set of prices chosen by each type of informed firm in period 1 cannot be an equilibrium when the posterior probabilities are

⁴ Since the uninformed firm’s profit function is strictly concave in relation to its chosen price, we can ensure that this firm will never randomize and then we only have to focus on pure strategy equilibria.

the same as in our separating equilibrium. In fact, it would mean that some type of informed firm would be choosing a suboptimum strategy given its rival's decisions. However, there could be an alternative separating equilibrium in which the low-cost firm chooses a different price in period 1 pretending to be a high-cost firm. If that strategy is successful, the uninformed firm will assume that the posterior probability of a low transportation cost is equal to zero when it observes the price chosen by the low-cost firm in that alternative equilibrium. Since this alternative equilibrium is a separating equilibrium, when the uninformed firm observes the price chosen by the high-cost firm, the posterior probability of a low transportation cost is equal to 1. If it were the case, the high-cost firm will be better off at our proposed separating equilibrium where the uninformed firm learns the true value of the transportation cost. Thus, we have proven that any alternative separating equilibrium would be dominated by ours.

Finally, we prove that any alternative pooling equilibrium would be dominated by our proposed pooling equilibrium. A set of strategies and beliefs would form a pooling equilibrium in this set-up whenever both types of informed firm set the same price in period 1 and the posterior probabilities at that equilibrium are the same as the priors. For example, imagine that there were an alternative pooling equilibrium in which both types of informed firm set a higher price in period 1 than the price we obtained. In this case, both types of informed firm would be better off at our pooling equilibrium. In particular, if the high-cost firm sets a higher price in period 1 than in the proposed equilibrium, it will be setting a suboptimal price and then, its profit in period 1 will be lower, whereas its profit in period 2 will be the same. Likewise, if the low-cost firm sets a higher price in period 1 than in the proposed equilibrium, it will be assuming a higher cost in period 1 to conceal its private information. However, the informed firm's profit in period 2 cannot be greater than that obtained in our proposed pooling equilibrium because the latter is the maximum possible profit it can obtain at a pooling equilibrium given its rival's decision. Thus, another pooling

equilibrium in which both types of informed firm set a higher price in period 1 would be defeated by our proposed pooling equilibrium. Similarly, imagine that there were an alternative pooling equilibrium in which both types of informed firm set a lower price in period 1 than that obtained in our pooling equilibrium. If the high-cost firm sets a lower price in period 1 than in our proposed equilibrium, this type of informed firm will be setting a suboptimal price in the first period, but it will not be able to increase its profit in period 2 because it is already obtaining its maximum possible profit in that period in our proposed pooling equilibrium. As there is no a higher type of informed firm than the high-cost firm in this model, it means that our proposed pooling equilibrium dominates the alternative one.

Now, putting together all these arguments, we have proven that any other separating equilibrium would be defeated by ours, that any other pooling equilibrium would be defeated by ours and that our proposed pooling equilibrium dominates the separating one in the grey shaded area, whereas the opposite occurs in the blue shaded area. Then, there will be a unique undefeated equilibrium of our game, which is the pooling equilibrium in the grey shaded area and the separating one in the blue shaded area. We will use the same arguments to choose our unique equilibria in the next sections, but we will not repeat them again.

IV. STACKELBERG MODEL

We now analyse the effects of a change in the timing of firms' decisions on the equilibria obtained. We consider a Stackelberg model in which one firm chooses its price before its rival. It is easy to see that there will be no pooling equilibria if the informed firm moves first. In this case, the informed firm will not find it profitable to conceal its private information in the first period because the first mover will not be able to exploit its knowledge in the next period. In particular, even if the uninformed firm did not know the degree of product differentiation in the second period, the informed competitor would not be able to get one step ahead of its rival in that period by undercutting it because the uninformed firm chooses

its price after observing its rival's decision. For this reason, we focus on a model on which the uninformed firm chooses its price before its rival in each period.

Proceeding in the same way as in the previous section, firstly, we obtain the optimal prices set by both firms in a one-shot Stackelberg game. Now, the uninformed firm's optimal price in period t , if both firms played in that period as in a one-period Stackelberg game, would be:

$$p_t^{2s} = c + \frac{3LH}{2\hat{R}} \quad (14)$$

Whereas the informed firm's optimal price would be:

$$p_t^{1s}(R) = c + \frac{3LH+2R\hat{R}}{4\hat{R}} \quad (15)$$

Once again, this equation shows the optimal price set by firm 1 in period 1 in our proposed separating equilibrium, which depends on the true value of the transportation cost. Similarly, equation (14) presents the optimal price set by firm 2 in period 1 in the separating equilibrium. In period 2, both firms will set its optimal prices in a one-shot game with complete information, $p_2^{1sf}(R), p_2^{2sf}(R)$. Then, the strategies that define our proposed separating equilibrium include all those prices set by both firms in both periods. Additionally, the function that defines the posterior probability of a low transportation cost, $\hat{\phi}$, is similar to that considered in the Bertrand set-up:

$$\hat{\phi} = \begin{cases} 0 & \text{if } p_1^1 \geq p_1^{1s}(H) \\ 1 & \text{if } p_1^1 < p_1^{1s}(H) \end{cases} \quad (16)$$

Now we can describe our proposed pooling equilibrium in this Stackelberg set-up. Using equation (15), if we substitute H for R , we obtain the optimal price set by both types of informed firm in period 1. In this equilibrium, the uninformed firm will set the same price in period 1 as in the separating equilibrium (i.e., the price given by equation (14)). In period

2, the uninformed firm will set the same price as in period 1 because the posterior probabilities of the unknown parameter coincide with the prior probabilities; however, the informed firm will take advantage of its better information by setting its optimal price given the true value of R . Thus, the informed firm's optimal price of our pooling equilibrium in period 2 for each value of R is also shown by equation (15). Finally, the function that defines the posterior probability of a low transportation cost is:

$$\hat{\phi} = \begin{cases} \phi & \text{if } p_1^1 \geq p_1^{1s}(H) \\ 1 & \text{if } p_1^1 < p_1^{1s}(H) \end{cases} \quad (17)$$

Now, as in the previous section, we will derive the conditions under which the low-cost firm prefers pretending to be a high-cost firm to revealing its information. For example, if the transportation cost is low, the expected profit obtained by firm 1 in period 1 at the pooling equilibrium is:

$$E\{\pi_1^{1s}[p_1^{1s}(H), p_1^{2s}|R = L]\} = \frac{H(3L+2\hat{R})[H(3L-2\hat{R})+4L\hat{R}]}{32\hat{R}^2L} N \quad (18)$$

In this pooling equilibrium, the uninformed firm will remain uninformed in period 2 and choose the same price as in period 1. Therefore, the informed firm will take advantage of its better information in the second period by undercutting its rival when the transportation cost is low and its expected profit is:

$$E\{\pi_2^{1s}[p_1^{1s}(H), p_1^{2s}, p_2^{1s}(L), p_2^{2s}|R = L]\} = \frac{L(3H+2\hat{R})^2}{32\hat{R}^2} N \quad (19)$$

Equation (19) also shows the expected profit the informed firm would obtain in period 1 if it chose to reveal its private information when the transportation cost is low; that is,

$$E\{\pi_1^{1s}[p_1^{1s}(L), p_1^{2s}|R = L]\} = E\{\pi_2^{1s}[p_1^{1s}(H), p_1^{2s}, p_2^{1s}(L), p_2^{2s}|R = L]\} \quad (20)$$

However, if firm 1 sets its optimal price in period 1 for each value of the transportation cost revealing to its rival its private information, both firms will compete with full information in

the second period and the informed firm's expected profit in that period when the transportation cost is low will be⁵:

$$E\{\pi_2^{1s}[p_1^{1s}(L), p_1^{2s}, p_2^{1sf}(L), p_2^{2sf}(L)|R = L]\} = \frac{25L}{32}N \quad (21)$$

Thus, when the transportation cost is low, the informed firm would hide its private information in period 1, whenever:

$$E\{\pi_1^{1s}[p_1^{1s}(H), p_1^{2s}|R = L]\} + \delta E\{\pi_2^{1s}[p_1^{1s}(H), p_1^{2s}, p_2^{1s}(L), p_2^{2s}|R = L]\} > \\ E\{\pi_1^{1s}[p_1^{1s}(L), p_1^{2s}|R = L]\} + \delta E\{\pi_2^{1s}[p_1^{1s}(L), p_1^{2s}, p_2^{1sf}(L), p_2^{2sf}(L)|R = L]\} \quad (22)$$

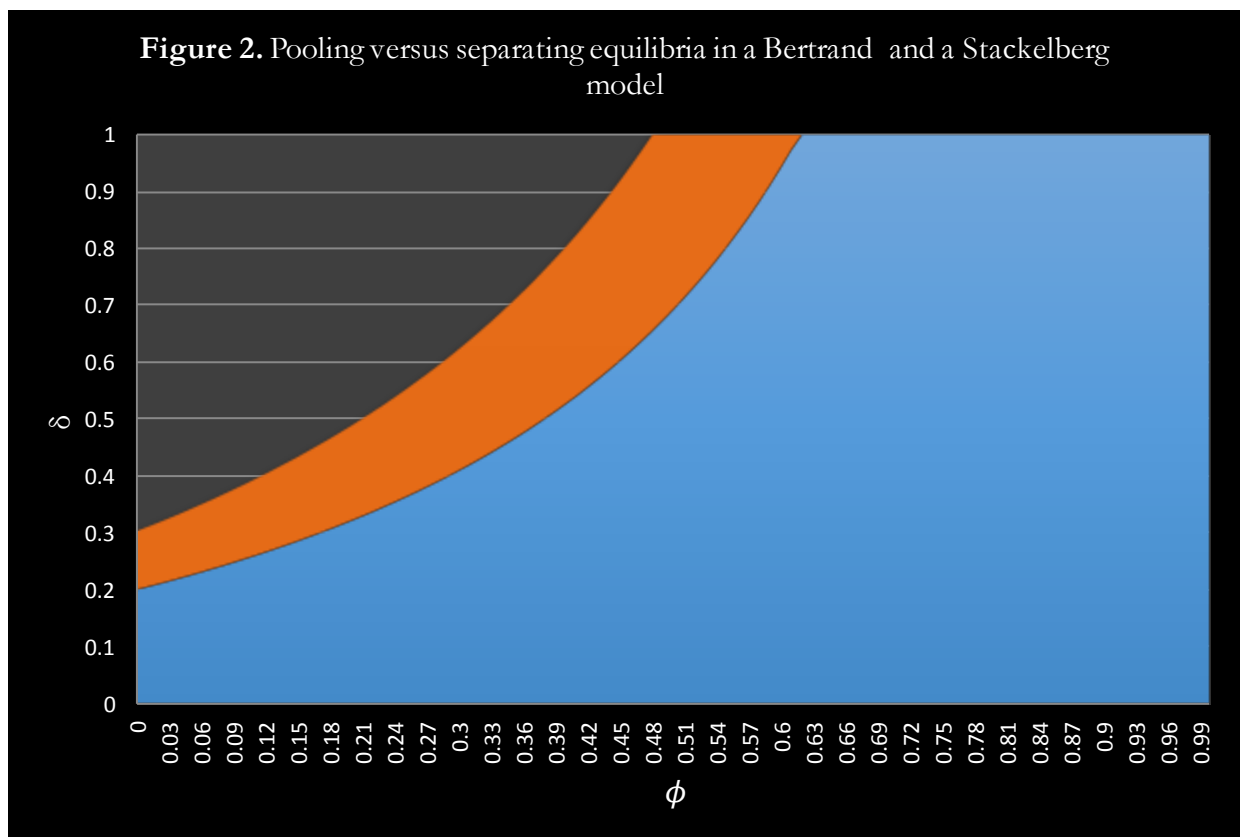
The next proposition compares the new conditions under which a pooling equilibrium arises in this Stackelberg model to those obtained in the Bertrand model.

Proposition 2. The parameter space in which a pooling equilibrium arises in the Stackelberg model is wider than in the Bertrand model; that is, for each prior probability of a low transportation cost, in the Stackelberg model the informed firm needs to be less patient to give rise to a pooling equilibrium.

Although the proof of this proposition can be found in the Appendix, Figure 2 compares the regions where there will be a pooling or a separating equilibrium in the Bertrand and the Stackelberg model when $L=1$ and $H=2$. The orange and grey shaded areas represent the values of ϕ and δ for which we obtain a pooling equilibrium in the Stackelberg model, whereas the pooling equilibrium in the Bertrand model only arises in the grey shaded area. Thus, the orange shaded area represents the additional region in which we obtain a pooling equilibrium in the Stackelberg set up. Imagine that the true transportation cost is low. Since the informed firm has the advantage of moving after its rival in each period, the informed

⁵ $p_2^{1sf}(L), p_2^{2sf}(L)$ are the optimal prices set by both firms in a one-period Stackelberg game with full information when the transportation cost is low.

firm will not have to set a so high price in period 1 to hide its private information. Thus, the informed firm's cost of concealing its information in period 1 will be lower. Furthermore, the informed firm's additional profit in period 2 from hiding its information will be greater for any value of the prior probability of a low transportation cost because it has the advantage of being the second mover. For these reasons, it will be profitable for the informed competitor to conceal its information on the unknown parameter even if the discount factor is lower.



V. QUADRATIC TRANSPORT COSTS

As D'Aspremont, Gabszewicz, and Thisse (1979) showed, it is not possible to analyse firms' location decisions in a Hotelling model with linear transport costs. The reason for this is that no equilibrium price solution exists when both sellers are not far enough from each other.

Thus, we consider a Hotelling model with quadratic transport costs in the same way as D'Aspremont et al. (1979); however, we assume that one competitor knows the true cost of transport, whereas the other one remains uninformed. We use this model to demonstrate the same tendency to maximize product differentiation as that shown by D'Aspremont et al. (1979).

Firstly, we outline the model developed by D'Aspremont et al. (1979) and provide some notation. On a line of length l , the two firms sell a homogeneous product with a constant marginal production cost equal to c . The informed firm, (firm 1), is located at distance a from the left end of the line, whereas the uninformed firm, (firm 2), is located at distance b from the right end of the line, where $a + b \leq l$; $a \geq 0$ and $b \geq 0$. Once again, customers are evenly distributed along the line, and each customer consumes a single unit of this commodity per period of time irrespective of its price. As in the model of linear transportation costs, a customer will buy from the seller who quotes the least delivered price; that is, the mill price plus transportation cost (disutility of consuming a product that is not his or her ideal). Nevertheless, the transportation cost is a quadratic function of the distance. Under these assumptions, an easy computation leads to the following expressions for the demand functions:

$$D_t^1(p_t^1, p_t^2) = \begin{cases} a + \frac{p_t^2 - p_t^1}{2R(l-a-b)} + \frac{l-a-b}{2} + \varepsilon_t, & \text{if } 0 \leq a + \frac{p_t^2 - p_t^1}{2R(l-a-b)} + \frac{l-a-b}{2} + \varepsilon_t \leq l \\ l, & \text{if } a + \frac{p_t^2 - p_t^1}{2R(l-a-b)} + \frac{l-a-b}{2} + \varepsilon_t > l \\ 0, & \text{if } a + \frac{p_t^2 - p_t^1}{2R(l-a-b)} + \frac{l-a-b}{2} + \varepsilon_t < 0 \end{cases} \quad (23)$$

$$D_t^2(p_t^1, p_t^2) = \begin{cases} b + \frac{p_t^1 - p_t^2}{2R(l-a-b)} + \frac{l-a-b}{2} - \varepsilon_t, & \text{if } 0 \leq b + \frac{p_t^1 - p_t^2}{2R(l-a-b)} + \frac{l-a-b}{2} - \varepsilon_t \leq l \\ l, & \text{if } b + \frac{p_t^1 - p_t^2}{2R(l-a-b)} + \frac{l-a-b}{2} - \varepsilon_t > l \\ 0, & \text{if } b + \frac{p_t^1 - p_t^2}{2R(l-a-b)} + \frac{l-a-b}{2} - \varepsilon_t < 0 \end{cases} \quad (24)$$

Once again, ε_t is a specific random shock and firms 1 and 2 know that its expected value is equal to zero. In line with D'Aspremont, Gabszewicz and Thisse (1979), we assume that both firms set their prices simultaneously in each period. Under these assumptions, we obtain a similar result to the previous ones, that is, if both competitors are located at different points on the street, a pooling equilibrium will arise provided that the prior probability of a high transportation cost is sufficiently high and firms are sufficiently patient. Otherwise, the separating equilibrium will arise⁶.

The main advantage of this specification of the transportation costs is that it allows us to analyse firms' location decisions as D'Aspremont, Gabszewicz and Thisse (1979) did. Using a one-period model, they showed that firms prefer to locate as far as possible from each other when they have perfect information on the transportation cost. Our last proposition shows that this statement is also true under the assumptions of our model.

Proposition 3. Under asymmetric information on the transportation cost, each firm will locate as far as possible from each other irrespective of the type of equilibrium obtained.

Here, we will only sketch the proof of this proposition because it is similar to that developed by D'Aspremont et al. (1979). In a one-period game, the result is exactly the same as theirs even if there is asymmetric information. The only difference is that, with asymmetric information, the uninformed firm will compete in the market as if the transportation cost is equal to its expected value. However, firm's location decisions at equilibrium will be the same as those with perfect information and the proof is the same as that presented by D'Aspremont et al. (1979). In our separating equilibrium, each firm behaves as in a one-period game with asymmetric information in period 1 and as in a one-period game with perfect information in period 2. Therefore, the maximum differentiation principle will also

⁶The proof of this proposition is similar to that of the previous propositions and is available from the authors upon request.

be obtained when the separating equilibrium arises because in this case, both firms compete with each other as in a one-shot game.

When our pooling equilibrium arises, there are two cases. First, when the true value of the transportation cost is high, once again, each firm will compete with each other in each period as in a one-period game with asymmetric information. Thus, we obtain the same result as that shown by D'Aspremont et al. (1979). Second, when the true transportation cost is low, the low-cost firm may have incentives to conceal its information in period 1 in order to make a greater profit in period 2. This is the only case in which firm's location optimal decisions might change because of those incentives. In fact, when the informed firm locates closer to its rival, its cost of concealing its information by setting a high price in period 1 increases because both products become closer substitutes. Nevertheless, in period 2, the informed competitor will be able to make more profits by undercutting its rival when both firms locate closer to each other. Therefore, the low-cost firm's incentives to conceal its information in period 1 when it locates closer to its rival will depend on the magnitude of those effects. In our model, the parameter that summarizes both types of effects is the minimum discount factor to obtain a pooling equilibrium, which is:

$$\underline{\delta}^Q(\hat{R}) = \frac{9\hat{R}^2(H-L)^2(l+a-b)^2}{L^2(3l+b-a)[H^2(3l+b-a)+6H\hat{R}(l+a-b)-\hat{R}^2(9l+5a-5b)]} \quad (25)$$

Since this minimum discount factor increases with a , we confirm the maximal differentiation principle obtained by D'Aspremont, Gabszewicz and Thisse (1979) even in the region of the pooling equilibrium.

VI. CONCLUSIONS

This article analyses price competition in a two-period duopoly model in which only one firm does not know the degree of substitutability between products, whereas the other competitor is completely informed about all the demand parameters. Using a Hotelling model, we obtain

the conditions under which the price set by the informed firm may be a good signal of the degree of product differentiation in the market. The price set by the informed firm will reveal its private information on the unknown parameter whenever the prior probability of closer substitutes is sufficiently high and the discount factor is sufficiently low. Similar results are obtained in a Stackelberg model in which the informed firm sets its price after its rival in each period, but the price set by the informed firm will always reveal its private information in that model when the informed firm is the first mover. Finally, we confirm the maximal differentiation principle obtained by D'Aspremont, Gabszewicz and Thisse (1979) in our two-period model with asymmetric information.

Previous literature on signal jamming has shown that firms set lower prices in markets with demand uncertainty than in markets with perfect information in order to fool their rivals into thinking that the demand is lower than it really is (Riordan, 1985). Additionally, several studies on learning by experimentation have found that in markets for closer substitutes firms' prices are more dispersed with demand uncertainty than with perfect information, whereas in markets for highly differentiated products price dispersion between firms is lower with demand uncertainty than without it (see for example, Harrington, 1992). Unlike the results obtained in some models of signal jamming, we found that asymmetric information on the degree of substitutability between products might make informed firms set higher prices than in a market with perfect information. Furthermore, in contrast to the literature on learning by experimentation, in our model with asymmetric information price dispersion is higher with demand uncertainty than without it regardless of the degree of product differentiation in the market. Thus, information asymmetries may significantly change firms' pricing policies.

Finally, a recent research developed by Brandão and Pinho (2015) used the same Hotelling's model as ours and also assume that one firm knows the transportation cost whereas the other

does not. However, they only model a one-shot game. In our two-period model, the low cost firm will set a higher price in period 1 than in their one-shot model under the conditions that give rise to our pooling equilibrium. Hence, the introduction of dynamics in the model with asymmetric information also affects firms' decisions in oligopolistic markets.

APPENDIX

Proof of Proposition 1. Using equation (13), we obtain the minimum value of δ such that the informed firm that knows that the transportation cost is low does not want to deviate from the pooling equilibrium, which is:

$$\underline{\delta}^B(\hat{R}) = \frac{\hat{R}^2(H-L)^2}{L^2(H^2+2H\hat{R}-3\hat{R}^2)} \quad (\text{A.1})$$

Thus, when the discount factor is greater than the previous threshold, the pooling equilibrium will arise. Otherwise, the separating equilibrium will be obtained. From equation (A.1), we observe that the threshold obtained depends on \hat{R} , which depends on the prior probability of a low transportation cost, ϕ . We obtain that

$$\frac{\partial \underline{\delta}^B(\hat{R})}{\partial \phi} = \frac{2H\hat{R}(H+\hat{R})(H-L)^3}{L^2(H^2+2H\hat{R}-3\hat{R}^2)^2} \quad (\text{A.2})$$

Since this derivative is greater than zero, the threshold increases with ϕ . Additionally, from equation (A.1), we obtain that $\lim_{\phi \rightarrow 1^-} \underline{\delta}^B(\hat{R}) = +\infty$ and $\lim_{\phi \rightarrow 0^+} \underline{\delta}^B(\hat{R}) < 1$. Thus, when the prior probability of a low transportation cost is too high, the pooling equilibrium will never arise. Nevertheless, when the prior probability of a low transportation cost is sufficiently low, the pooling equilibrium will arise whenever the informed firm is sufficiently patient; that is, when $\delta > \underline{\delta}^B(\hat{R})$. Thus, proposition 1 has been proven.

Proof of Proposition 2. Using (22), we find that the pooling equilibrium in the Stackelberg model will arise whenever the discount factor is greater than the following value:

$$\underline{\delta}^S(\hat{R}) = \frac{4\hat{R}^2(H-L)^2}{3L^2(3H^2+4H\hat{R}-7\hat{R}^2)} \quad (\text{A.3})$$

Then, subtracting (A.3) from (A.1), the difference between the Bertrand and the Stackelberg thresholds is:

$$\underline{\delta}^B(\hat{R}) - \underline{\delta}^S(\hat{R}) = \frac{\hat{R}^2(H-L)^2(5H^2+4H\hat{R}-9\hat{R}^2)}{3L^2(H^2+2H\hat{R}-3\hat{R}^2)(3H^2+4H\hat{R}-7\hat{R}^2)} \quad (\text{A.4})$$

Since this difference is greater than zero for any value of ϕ , proposition 2 has been proven.

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