Public debt frontiers: The Greek case*

Abstract

This paper attempts to quantify the maximum amount of debt that a government can sustain by itself, i.e., the limits to public indebtedness. Using a Dynamic General Equilibrium model where the government is fully characterized, we compute the steady state inverse relationship between the public debt to output ratio and the size of the government, measured as the total public expenditures to output ratio. This line is the budget constraint of a government in steady state. Calibration of the model for the Greek economy to fiscal targets reveals that, for the period just before the current recession, i.e. 2002-2006, the debt to GDP ratio was very close to the calculate limits which depends dramatically on the interest rates. However, short after de financial crisis of 2008, sustained deficits drove the Greek economy to a point where the Greek Government crossed the debt limit where the country could only meet its debt obligations only if international investors were willing to lend. We conclude that the hight initial level of debts previous the crisis together with the rise in interest rates were the causes of the posterior debt crisis.

JEL Classification: H5; H6.

Keywords: Debt limit, fiscal policy, government expenditure, public debt sustainability, Dynamic General Equilibrium models.

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1 Introduction

One of the many debates caused by the recent international financial crisis has focused the attention of economists and policy makers on the sovereign debt sustainability. As a result, a controversy about the causes and the cures of debt crisis, which is still hitting some countries of the Euro Area with particular intensity, has emerged. Some of the proposed solutions for the European debt crisis shows that the perceived origins of this crisis can be found in i) A crisis of imbalances, caused by the weak competitiveness of peripheral Europe, and ii) A fiscal crisis, due to either direct fiscal indiscipline in the cases of Portugal and Greece, and irresponsible financial policies that triggered excessive fiscal guarantees, as in the cases of Ireland and Spain.

However, in this paper we claim that Greece was before the crisis a country that could be compared to other eurozone member states in all fiscal dimensions: Public spending over GDP, expenditure structure, average tax rates, number of public employees, etc., and therefore we argue that the current debt crisis hitting the Greek economy is not due to past fiscal indiscipline or to initial inherently unsustainable debt levels but a consequence of government attitude towards the crisis, together with a spike in the interest rates faced by the Greek bond which finally triggered the Greek public financial disaster.

To support this claim, we attempt to quantify the maximum amount of debt that a government can sustain by itself, its sovereign debt limit. Beyond that limit the government risks the possibility of a self-fulfilling crisis. These crises arise when lenders think that a government will not repay its debt. If lenders think a government will not repay, they do not lend. If a government cannot roll over the portion of its debt becoming due within a period, it may choose to default even though it would not default if the lenders do lend. This is the idea in Cole and Kehoe (1996, 2000). The maximum level of debt that can be sustained if lenders do not lend is much lower than the maximum that can be sustained if they do lend. We use a Dynamic General Equilibrium (DGE) model to compute those limits, to show that Greece was well below the critical thresholds.

Conesa and Kehoe (2015) show that governments with low debt can choose to run this debt up to levels where they risk crises if their country is unlucky enough to be in a recession period after period. This is the idea of gambling for redemption, where a fixed and exogenous probability of a recovery entices governments to gamble with the expenditure-debt policy, risking a default if period after period the recovery does not occur and a sunspot realization scares international lenders away.

We compute the debt limits where the Cole-Conesa-Kehoe kind of arguments are more likely to explain what has occurred to Greece.

We define an equilibrium where the government roll over its debt and another equilibrium where it cannot do so, and calculate the welfare of the consumers at
each equilibrium. If debts levels are too large, it can be optimal for the government to decide to default on its debts, and this decision implicitly defines what it is a sustainable debt level. We show that using a diagram where two key ratios of fiscal data (the debt/output and the government spending/output) are plotted together, is useful to asses how close an economy is from the default decision. For this diagram, we compute the steady state relationship between the public debt/output ratio and the size of the government, measured as the total public expenditures/output ratio. This line is called the government budget constraint. Along this line the economy has to generate enough primary fiscal surpluses to finance current government expenditures plus the interest service of its debts, rolling over the existing debt with zero deficits. Therefore, the budget constraint provides the maximum level of sustainable debt with rolling over. The debt frontier provides a picture dividing the long term sustainable region from the long term unsustainable region for any given level of public expenditure to GDP ratio. At the right of the debt frontier, all traders know that the economy cannot last for too long: Bad news and a recovery that never arrives configure a situation where rolling might not be possible, and where the government has to decide whether or not to default.

In our model, if the government decides to default, it has to face a TFP penalty interpreted as an economic dislocation induced by the default. If on the contrary, the government decides not to default, then the economy has to generate enough fiscal primary surpluses to pay back any maturing bond until the debt is canceled. These are the debt limits that we compute with the model.

In this paper we compute the debt to GDP threshold where the government decides to default, and we show that above the computed limit, the government will choose to default with probability one if lenders decide not to lend. To this end, we construct a DGE model, calibrated for the Greek economy, where the role of the government affects a large variety of fiscal policies on both sides of the government budget restriction: revenues and expenditures. In our model, total government spending is divided into several variables: public consumption of goods and services; public investment in physical capital; a public wage bill; transfer payments to households; and interest payments of public debt. As we will show in this paper, the amount of total debt issued is not independent from the spending policies, as different shares of total government spending have different effects on fiscal income: for example, spending in social transfers does not improve productivity of private factors, whereas increasing public investment does. Therefore, the amount of sustainable debt varies across policies, and this is the ultimate reason why we construct a model with a very detailed public sector. On the other hand, public revenues are raised by taxation and new debt issuance, but taxable income varies across policies, and so will the debt needed to balance public accounts. We consider the existence of five taxes: consumption tax, labor income tax, capital income tax, corporate tax and a social security tax. Additionally, we include the fiscal funding of the social
security system of the economy as a pay-as-you-go system. This rich public sector modeling is justified because we want to show that Greece was not so different from the rest of euro area countries\textsuperscript{1} and that we have to reject fiscal indiscipline as the fundamental cause of the Greek default in favor of an alternative theory such as gambling for redemption.

We have chosen Greece for our study because it was the first country under the Euro currency union to lose its triple A rating on government bonds, and the country has faced strong pressure to consolidate the budget, to finally default. We carefully calibrate the model to fiscal targets to reach the conclusion that Greece was well inside the sustainable debt to GDP ratio when the crisis hit. Then, the government decided not to respond with an immediate reduction in government spending. On the contrary, government spending smoothly kept increasing. The government consumption to GDP ratio increased as a consequence, rapidly driving the economy beyond the line we draw and into the region where any additional bad news could scare investors away. In the meantime, the recovery didn’t happen, or the bad news arrived before the recovery, and the crisis unfolded. Imposing a reasonable default penalty to TFP, we find that the debt to GDP ratio of Greece by 2010 was such that the Greek government would have chosen to default if international lenders decide not to lend, provoking a self-fulfilling crisis. We conclude that a gambling for redemption attitude rather than fiscal indiscipline is behind the Greek debt crisis drama.

The structure of the rest of the paper is as follows. Section 2 presents the model. Section 3 discusses the calibration exercise. The main results from the calibrated model to the Greek economy are shown in Section 4. Finally, Section 5 concludes.

\section{The model}

We develop a general equilibrium model where the government affects private decisions in a number of ways. We consider the role of taxes, public consumption of goods and services, public investment in public capital, public labor markets and

\textsuperscript{1}Even taking into account the suspicion about some creative debt accounting carried out by the Greek Government in order to meet the Maastricht criteria to join the Euro Zone we find that the debt to GDP ratio limit was still well above given the expenditures to GDP ratio for the years before the crisis. The suspicious creative accounting was probably more important to act as a coordinating sunspot variable than the effects on actual levels of debt. The Treaty on the European Union was signed on February 7, 1992 by the members of the European Community in Maastricht, Netherlands. The Treaty led to the creation of the Euro, and established a set of rules imposing control over inflation, public debt and the public deficit, exchange rate stability and the convergence of interest rates. With regard to public finances it imposed an annual limit of 3\% in the ratio of government deficit to GDP, and a 60\% of gross government debt prior to the entry in the European Monetary Union.
The government displays a high degree of disaggregation in both expenditures and fiscal income sides. On the expenditure side, we distinguish four components: public consumption of goods and services; public investment in capital; public wage bill; and transfers. On the fiscal income side, we consider four income taxes (consumption tax, labor income tax, capital income tax and corporate tax) plus revenues from the social security tax. Firms are represented by a CES production function nested within a standard Cobb-Douglas. The production of the final output requires four factors: labor services and capital, both private and public. Finally, consumers are modeled in a standard way, but including public goods in the utility function and splitting worked hours between the private and the public labor sectors.

2.1 The Government

First, we describe the elements present in the government budget constraint:

$$G_t + R^B_t B_t + \Delta D_t = T_t + R^D_t D_t + CBT_t + \Delta B_t$$

Equation (1) says that all cash outlays (including transfer payments to households) - for non-interest total government spending ($G_t$), interest payments of total government debt ($R^B_t$ times $B_t$), and new purchases of financial assets ($\Delta D_t$) - must be funded by some combination of tax receipts ($T_t$), interest earnings on government assets ($R^D_t$ times $D_t$), transfers from the central bank ($CBT_t$), and new debt issuance ($\Delta B_t$).

For Euro zone countries, transfers from the central bank are zero, and direct purchases of government bonds are precluded by the Treaty (i.e. $CBT_t = 0$). If we denote by $B_t$ the net position of the government, we can also set financial purchases to zero (i.e. $D_t = 0$).

2.1.1 Government spending

Non-interest total government spending is defined as:

$$G_t = C_{g,t} + (1 + \tau^s_t)W_{g,t}L_{g,t} + I_{g,t} + Z_t$$

where $C_{g,t}$ is public consumption of goods and services, $I_{g,t}$ is public investment, $W_{g,t}L_{g,t}$ is the wage bill for public employees, $\tau^s_t$ is a social security tax, and $Z_t$ are transfer payments to households, such as welfare, social security or unemployment benefit payments. Public investments accrue into the public structures stock, $K_{g,t}$.

We assume the following accumulation process for the public capital:

$$K_{g,t} = (1 - \delta_{K_g})K_{g,t-1} + I_{g,t}$$
which is analogous to the private capital accumulation process, and where $\delta_{K_{g}}$ is the public physical capital depreciation rate.

Next, we need to specify the government spending structure at the time of calibration. This spending structure implies the selection of i) a certain level of public spending and ii) its distribution among the different components. The level of government spending in the long run, given a certain amount of fiscal revenues, depends on the target levels for the public deficit and public debt. While the Maastricht Treaty establishes limits together with sanctions for deficit and debt sinners, these limits have only been respected to enter into the monetary union, but never after that date. Therefore, we do not consider the Maastricht criteria to be binding for these two variables.

The distribution among the different components of public spending is as follows:\(^2\)

$$C_{g,t} = \theta_1 G_t$$

$$I_{g,t} = \theta_2 G_t$$

$$(1 + \tau_t^{ss})W_{g,t}L_{g,t} = \theta_3 G_t$$

$$Z_t = \theta_4 G_t$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 1$. We assume that public spending on goods and services are constant proportions of total output and these proportions are kept constant all along the exercise, that is, the government’s income and expenditure sides are fully parametrized. Appendix B reports the results of a sensitivity analysis where both public spending components and taxes rates are changed.

2.1.2 Tax revenues

The government obtains resources from the economy by taxing consumption and income from labor, capital and profits, whose effective average tax rates are denoted by $\tau_t^c$, $\tau_t^l$, $\tau_t^k$, $\tau_t^\Pi$, respectively. Additionally, we consider a pay-as-you-go social security system and thus we include the social security tax, $\tau_t^{ss}$. The government budget in each period is given by,

$$T_t = \tau_t^c C_{p,t} + \tau_t^l (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k (R_t - \delta_{K_p})K_{p,t-1} + \tau_t^{ss} (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^\Pi \Pi_t$$

\(^2\)This split of the government expenditures can be thought of as the result of maximization of preferences of the form $U_{g}(C_{g,t}, I_{g,t}, W_{g,t}L_{g,t}, Z_t) = \log C_{g,t} + \log I_{g,t} + \log L_{g,t} + \log Z_t$, subj ect to a budget constraint where the Government can spend $G_t$.
where $C_{p,t}$ is private consumption, $W_{p,t}$ is private sector wages, $L_{p,t}$ is private labor, $R_t$ is the rental rate of private capital, $\delta_{K_p}$ is the depreciation rate of private capital, $K_{p,t}$ is private capital stock, and $\Pi_t$ are profits to be defined later.

### 2.1.3 The government identity

As we previously argued the government budget constraint can be written as:

$$G_t + R^B_t B_t = T_t + B_{t+1} - B_t$$

with the meaning that non financial spending, plus servicing the existing government debt must be financed through taxes plus new debt. Putting together all the elements defined above, the government budget constraint can be written as:

$$C_{g,t} + (1 + \tau^s_t) W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1 + R^B_t) B_t$$

$$= \tau_i^C C_{p,t} + \tau_i^T (W_{p,t}L_{p,t} + W_{g,t}L_{g,t})$$

$$+ \tau_k^B (R_t - \delta_{K_p}) K_{p,t-1} + \tau^s_t (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_i^\Pi_t + B_{t+1} \tag{4}$$

or, collecting uses and resources:

$$C_{g,t} + W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1 + R^B_t) B_t$$

$$= \tau_i^C C_{p,t} + \tau_i^T (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_k^B (R_t - \delta_{K_p}) K_{p,t-1}$$

$$+ \tau_i^s W_{p,t}L_{p,t} + \tau_i^\Pi_t + B_{t+1} \tag{5}$$

[Insert here Figure 1]

### 2.1.4 Default

We have described up to this point a fully parametrized government. We say it is parametrized in the sense that all those decisions (tax code, expenditure proportions, public wages, wage premium, and public labor supply) were taken once and for all time. The only decision the government undertakes at any moment is whether to honor its debt obligations or, on the contrary, to default. This decision is registered by a binary variable $z = \{0, 1\}$ that takes the value $z = 0$ if the government defaults in the current period or if it has ever defaulted in the past, and it takes the value $z = 1$ if the government decides to honor its debt obligations in the current period.

The decision function used by the government to determine whether to pay or to default is the utility function of the consumers. In this way we assume that the Government is benevolent at the moment of taking a crucial decision for the entire economy. Since the value of $z$ affects the value of other variables in equilibrium, we will postpone the definition of equilibrium until the model is completely specified.
2.2 Trade unions

The public labor market is modeled following the work of Fernández de Córdoba, Pérez and Torres (2012). The purpose of the mechanism described in this section is to distort the labor market to prevent wages equalization between the private and the public sector. An analysis of the public labor market among OECD countries show that the public wage bill is a source of major differences among these economies. Our analysis shows that government interventions in the wage setting of public wages can have a significant effect not only on the wage bill, but also in the growth path of the economy affecting the income shares of private inputs, having therefore a long-term effect on the debt budget constraint and the debt limits we want to calculate.

We have chosen a mechanism where the government has preferences over the number of public workers and their pay. To provide an objective function for the government defined over wages and employment, we follow a standard text-book approach (for example see Oswald, Grout and Ulph, 19843) and pose an objective function for the government as the solution of a game between a public sector union that cares about the wages of public-sector employees, \( W_g,t \), and a government that cares about the level of public employment, \( L_g,t \), given its budget constraint. Thus, the government agrees with the public sector union to maximize the following objective function subject to a budget constraint:

\[
\max \left[ \omega W_{g,t}^\rho + (1 - \omega)L_{g,t}^\rho \right]^{1/\rho}
\]

where \( \omega \) is the weight given to wages and \( \rho \) is a negative parameter indicating the curvature of the trade-off between the elements present in the objective function of the government. If \( \omega \) is close to zero, then the main goal of the government is to maximize public employment (benevolent government preference), whereas if \( \omega \) is close to one, the main goal of the government is to maximize public wages (public sector union’s preferred option).

Note that expression (6) encompasses the different approaches found in the literature. On the one hand, it takes into account the fact that public employment and wages are determined in an environment different to the private sector. The government itself can increase the number of public employees or can increase public wages subject to the budgetary constraint. On the other hand, it takes into account the fact that trade unions are more important in the public labor sector than in the private sector (see for instance Blanchflower, 1996).

As defined previously, the government wage bill is defined as:

\[
\theta_3 G_t = (1 + \tau_{t}^{ss})W_{g,t}L_{g,t}
\]

3On related grounds Ardagna (2007) and Forni and Giordano (2003) consider the wage bill of the government, employment and wages, separately as arguments of the objective function of the government or the public sector union.
Maximizing the government objective function subject to the government budget constraint is to find critical values for the auxiliary Lagrangian function:

\[
L_g (\cdot) = \max \left[ \omega W_{g,t}^\rho + (1 - \omega) L_{g,t}^\rho \right]^{1/\rho} + \xi (\theta_3 G_t - (1 + \tau_t^{ss})W_{g,t} L_{g,t})
\]

That provides, upon differentiation, the first order necessary conditions:

\[
\frac{\partial L_g (\cdot)}{\partial W_{g,t}} = \left[ \omega W_{g,t}^\rho + (1 - \omega) L_{g,t}^\rho \right]^{1/\rho - 1} \omega W_{g,t}^{\rho-1} - \xi (1 + \tau_t^{ss})L_{g,t} = 0
\]

\[
\frac{\partial L_g (\cdot)}{\partial L_{g,t}} = \left[ \omega W_{g,t}^\rho + (1 - \omega) L_{g,t}^\rho \right]^{1/\rho - 1} (1 - \omega) L_{g,t}^{\rho-1} - \xi (1 + \tau_t^{ss})W_{g,t} = 0
\]

Dividing orderly:

\[\omega W_{g,t}^\rho = (1 - \omega) L_{g,t}^\rho \quad (8)\]

Combining this expression with equation (7) we obtain that public wages and employment are equal to:

\[
W_{g,t} = \left( \frac{\omega}{1 - \omega} \right) -1/2^\rho \left[ \frac{\theta_3 G_t}{(1 + \tau_t^{ss})} \right]^{1/2}
\]

\[
L_{g,t} = \left( \frac{\omega}{1 - \omega} \right)^{1/2^\rho} \left[ \frac{\theta_3 G_t}{(1 + \tau_t^{ss})} \right]^{1/2}, \text{ if } W_{g,t} > W_{p,t} \quad (9)
\]

This distribution of the public resources depends on government preferences. However, private and public sectors are competing for the same labor input and as a consequence there is a relationship between public sector and private sector wages inducing a wage premium. The wage premium is implicit in equation (10) and it is part of the solution of the governments problem. This wage premium ensures the government that it’s demand for labor will always be satisfied. This relationship will become clearer once we present the household’s problem.

### 2.3 Firms

The problem of the firm is to find optimal values for the utilization of labor and capital in the presence of public inputs. The representative firm operates a CES production function nested within a standard Cobb-Douglas production function, and thus this technology exhibits constant returns to scale. The production of final output, \(Y\), requires labor services, \(L\) and capital, \(K\), both private and public. Goods and factors markets are assumed to be perfectly competitive. The firm rents capital and hires labor to maximize period profits, taking factor prices and public labor and capital as given. The technology is given by:

\[
Y_t = A_t(z) K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g)/\eta}
\]
where $Y_t$ is aggregate output, $A_t(z)$ is a measure of total-factor productivity that depends on the variable $z = \{0, 1\}$ that indicates if the government has defaulted its debt obligations in the past, with $A_t(1) > A_t(0)$, indicating that a default produces a once and for all reduction in TFP.\textsuperscript{4} The size of the default penalty is important for the government, because its size will be crucial to determine the debt frontier where the crisis zone is defined. Nevertheless, we postpone the determination of its size until we solve the model.

The parameters $0 < \alpha_p < 1$ and $0 < \alpha_g < 1$ are private and public capital share of output respectively, $(\alpha_p < \alpha_g < 1)$ measures the weight of public employment relative to private employment and $\psi = 1/(1 - \eta)$ is a measure of the elasticity of substitution between public and private labor inputs.

If we assume final output to be the unit of account, profits are defined as:

$$\Pi_t = A_t(z) K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g)/\eta} - (1 + \tau^{ss}_t) W_{p,t} L_{p,t} - R_t K_{p,t-1}$$

(12)

Under the assumptions that private workers are paid their marginal productivity, we get:

$$(1 + \tau^{ss}_t) W_{p,t} = \mu(1 - \alpha_p - \alpha_g) A_t(z) K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g-\eta)/\eta} L_{p,t}^{-1}$$

$$R_t = \alpha_p A_t(z) K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g)/\eta} L_{p,t}$$

From the above equations, it is found that private factor incomes are:

$$(1 + \tau^{ss}_t) W_{p,t} L_{p,t} = \frac{\mu(1 - \alpha_p - \alpha_g) A_t(z) K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g-\eta)/\eta} L_{p,t}^\eta}{\mu L_{p,t}^\eta + (1 - \mu) L_{g,t}^\eta} Y_t$$

(13)

$$R_t K_{p,t-1} = \alpha_p Y_t$$

(14)

The aggregate production function has four productive factors. However, the two public factors have no market price. The government does not usually charge a price that covers the full cost of the services provided with the contribution of public factors. This implies that those rents generated by public factors are not assigned

\textsuperscript{4}There are several interpretations for this reaction of TFP to a default. The interpretation of a default penalty seems the most natural, indicating a loss of confidence in trade, or an increase in the uncertainty of the economy. More attractive is the interpretation of the crowding out of investment that is produced once the debt sold to national banks does not return to the economy in the form of credits to the private sector after a default.
to public factors. As public factors are paid by the government, there is a positive profit, \( \Pi_t \), which turns out to be:

\[
\Pi_t = Y_t - R_t K_{p,t-1} - (1 + \tau^{*}_t)W_{p,t}L_{p,t} > 0
\]

Substituting private factor incomes given by expressions (13) and (14) yields:\(^5\)

\[
\Pi_t = \left[ 1 - \alpha_p - \frac{\mu(1 - \alpha_p - \alpha_g)L_{g,t}}{\mu L_{p,t} + (1 - \mu)L_{g,t}} \right] Y_t > 0
\]

We assume that profits are paid out to households given that they are the owners of the firm.

### 2.4 Households

In our model economy, the decisions made by consumers are represented by a stand-in consumer with a period utility where consumption can be decomposed into two components:

\[
U(C_t, L_t) = U(C_{p,t}, C_{g,t}, L_t)
\]

where \( C_{p,t} \) is private consumption and \( C_{g,t} \) is consumption of the same private good provided by the government to the consumer. We assume that households obtain utility from the public spending in good and services. In particular, we assume that:

\[
C_t = C_{p,t} + \pi C_{g,t} \quad \text{with} \quad \pi \in (0, 1]
\]

Households’ preferences are given by the following instantaneous utility function:

\[
U(C_t, N_t \overline{H} - L_t) = \gamma \log C_t + (1 - \gamma) \log(N_t \overline{H} - L_t)
\]

Leisure is \( N_t \overline{H} - L_t \), where \( \overline{H} \) is total time endowment and it is calculated as the number of effective hours in the week times the number of weeks in a year times population in the age of taking labor-leisure decisions, \( N_t \), minus the aggregated number of hours worked in a year, \( L_t \). The parameter \( \gamma \) (0 \( \gamma \) 1) is the fraction of private consumption on total private income. Households consume final goods and supply labor to the private and the public sectors,

\[
L_t = L_{p,t} + L_{g,t}
\]

where \( L_t \) is the aggregate level of employment, \( L_{p,t} \) is private employment and \( L_{g,t} \) is public employment. Public employment is chosen by the government and thus it is exogenous to the households as a quantity constraint. At an aggregate level,

\(^5\)See appendix A.2 for the derivation of this expression.
the household can only choose the supply of private labor, \( L_{p,t} = L_t - L_{g,t} \). Recall that public employment demand is fully covered by the household, provided that \( W_{g,t} > W_{p,t} \).

The budget constraint faced by the stand-in consumer is:

\[
(1 + c_t)C_{p,t} + K_{p,t} - K_{p,t-1} = (1 - \tau_t)W_{p,t}L_{p,t} + W_{g,t}L_{g,t} + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} + Z_t + (1 - \tau_t^n)\Pi_t \tag{19}
\]

where \( K_{p,t} \) is private capital stock, \( W_{p,t} \) is private compensation per employee, \( W_{g,t} \) is public compensation per employee, \( R_t \) is the rental rate of capital, \( \delta_{K_p} \) is the capital depreciation rate which is modeled as tax deductible, \( Z_t \) are lump sum transfers and entitlements, and \( \Pi_t \) denotes profits from firms, as defined previously. The budget constraint states that consumption and investment in physical capital, cannot exceed the sum of labor and capital rental incomes and profits net of taxes.

Private capital holdings evolve according to:

\[
K_{p,t} = (1 - \delta_{K_p})K_{p,t-1} + I_{p,t} \tag{20}
\]

where \( I_{p,t} \) is household’s gross investment.

The consumer maximizes the value of her lifetime utility given by:

\[
\max_{\{C_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log(C_{p,t} + \pi C_{g,t}) + (1 - \gamma) \log(N_t \overline{H} - L_{p,t} - L_{g,t}) \right] \tag{21}
\]

subject to the budget constraint, where \((K_{p0}, K_{g0})\) and the paths of public employment and taxes are given, and where \( \beta \in (0, 1) \), is the consumer’s discount factor.

The Lagrangian auxiliary function is: The first order conditions for the consumer maximization problem are:

\[
\frac{\partial L}{\partial C_{p,t}} = \gamma \frac{1}{C_{p,t} + \pi C_{g,t}} - \lambda_t(1 + \tau_t^t) = 0 \tag{22}
\]

\[
\frac{\partial L}{\partial L_{p,t}} = -(1 - \gamma) \frac{1}{N_t \overline{H} - L_{p,t} - L_{g,t}} + \lambda_t(1 - \tau_t^t)W_{p,t} = 0 \tag{23}
\]

\[
\frac{\partial L}{\partial K_{p,t}} = \beta^{t+1} \left[ \lambda_{t+1} \left( 1 + (1 - \tau_{t+1}^k)(R_{t+1} - \delta_{K_p}) \right) \right] - \lambda_t \beta^t = 0 \tag{24}
\]

plus the budget constraint and a transversality condition stating that the today-value of long distant future values of assets are zero.

This formulation implies that the wage-setting process in the private sector is totally different to that of the public sector. Whereas in the private sector wages are determined in terms of their marginal products, in the public sector a given
amount from the government’s budget constraint is distributed between public wages and public employment. Note that the above expressions imply that the consumer can only choose the supply of private labor, given that public labor is determined inelastically by the government at a wage that includes a positive premium that guarantees that all public labor demand is covered by the consumer at any market wage \( W_{p,t} \).

### 2.5 International investors

The rest of the world for this economy is modeled as a single international banker whose objective is to maximize the discounted dividend \( x_t \) obtained from the asset holdings of government bonds. The discount factor is \( \beta \), identical to the consumer’s discounting parameter. Purchases of government bonds are denoted by \( b_t \). Of course, supply and demand are equal at all times, so \( B_t = b_t \).

\[
\max_{x_t} \sum_{t=0}^{\infty} \beta^t x_t \\
\text{s.t. } b_{t+1} - b_t + x_t = w^I + R^B b_t
\]

Where \( w^I \) is a constant endowment.

From the above problem we obtain

\[
\beta(1 + R^B_t) = 1 \tag{25}
\]

Walras’s Law is satisfied at all times.\(^6\) From equations (24) and (25) we obtain a non arbitrage steady state condition

\[
(1 - \tau^k)(R - \delta_K) = R^B
\]

The net real return to capital has to equate the real return of the government bond, including any risk premium.

### 3 Calibration

In this section we calibrate the model for the Greek economy to a number of targets. We select this economy as our case study given that it represents a benchmark for studying the causes of a debt crisis, as it was the first country under the European Monetary Union to lose its triple A rating on government bonds and to adopt a financial program. Figure 1 plots the evolution of the public debt/GDP ratio (left scale) and total public expenditure/GDP ratio (right scale) for the period 2002-2011.

\(^6\)See Appendix A.1 for a proof.
Both ratios remain almost constant for the period 2002-2006. As these figures are central in our analysis, we choose the average values of the macroeconomic variables for the Greek economy for this period as the targets for our calibration of the model economy. We choose \( z = 1 \) (no default), as the target for the calibration of total factor productivity.

To calibrate this economy some parameter values are computed from ratios taken from the national accounts, other parameters are taken from the set of equilibrium conditions while the labor factors technological parameters of the nested CES production function are estimated using OLS. The ratio \( G/Y \), takes values from the interval \( G/Y \in [0.4, 0.6] \). For any given vector that defines the fiscal policy \((\tau^k, \tau^l, \tau^c, \tau^\pi, \theta_1, \theta_2, \theta_3, \theta_4)\), we calculate the steady state values for prices and quantities that satisfy the set of first order conditions and the market clearing equations described above, for each value of \( G/Y \).

First, the parameters of the model are calibrated to replicate the following targets taken from the Greek economy: total output at the calibration point (years 2002-2006), is set to \( Y = 100 \). From OECD statistics we obtain the ratio of total government expenditures \((G/Y = 0.4504)\), total (both public and private) investment \((I/Y = 0.2236)\), and total consumption \((C/Y = 0.7764)\), as well as the fraction of total labor force actually employed \((L/H = 0.5750)\). Notice that public and private consumption plus total investment is total GDP. The reason for this is that our measure of \( G \) adds to public consumption all transfers to the consumer such as public education, public health, transfers for the unemployed and the public wage bill. Public investment was about 15% of total investment, which yields a value of \( \theta_2 = 0.0745 \), while total public consumption was about 19% of GDP. The value taken from National Accounts yield a value for \( \theta_1 = 0.4467 \). Computing private and public capital depreciation rates is a difficult task, since it involves computing what types of investments are done, and what is the depreciation rate for each of them. Due to its intrinsic difficulty, we use the estimates of Mas et al. (2010) for the Spanish economy, which implies that \( \delta_{K_p} = 0.08 \) and \( \delta_{K_g} = 0.04 \), and hence, we compute steady state values for capital as \( \hat{K}_p = I_p/\delta_{K_p} = 237.5750 \) and \( \hat{K}_g = I_g/\delta_{K_g} = 83.8500 \). The depreciation rate for public capital is lower than for private capital given their different composition. These calculations imply that public capital stock represents around 26% of total capital stock for the calibrated economy and that total capital stock is 3.2 times total output.

The values for effective average tax rates \((\tau^c, \tau^l, \tau^k, \tau^\pi, \tau^{sa})\) are taken from Boscá et al. (2012), who use the methodology developed by Mendoza et al. (1994). The nominal return of the Greek bond at the calibration period was \( R^B = 0.041 \). Equation (25), provides a steady state relationship between \( R^B \) and \( \beta \). The value we obtain is \( \beta = 0.96 \). Once we have this value, equation (24) together with the value of \( \delta_{K_p} \) and \( \tau^k = 0.1640 \) delivers \( R = 0.12094 \). From equation (14) and the value for public capital we obtain a value for \( \alpha_p = 0.3061 \).
Next, equate the marginal products of public and private capital to get:

\[
R_{p,t} = \alpha_p A_t(z)K_{p,t-1}^{\alpha_p - 1}K_{g,t-1}^{\alpha_g} \left[ \mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta \right]^{\frac{1 - \alpha_p - \alpha_g}{\eta}}
\]

\[
R_{g,t} = \alpha_g A_t(z)K_{p,t-1}^{\alpha_p - 1}K_{g,t-1}^{\alpha_g} \left[ \mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta \right]^{\frac{1 - \alpha_p - \alpha_g}{\eta}}
\]

\[
\frac{R_{p,t}}{R_{g,t}} = \frac{\alpha_p}{\alpha_g} \frac{K_{p,t-1}}{K_{p,t}}
\]

If we assume that the real return to public capital is equal to the real return to private capital, we can compute a value for \( \alpha_g = \alpha_p (K_g/K_p) = 0.1082 \). With the OECD data series on public sector labor and wages for Greece, we obtain the ratio of public labor to private labor in 2002-2006, \( L_g/L_p = 0.2399 \), while the wage premium for the same years was about, \( W_g/W_p = 1.4 \).

Comparing Greece’s labor market with Europe (Figures 2 and 3), we observe the same trend of a decrease in the participation of public labor in total employment, with an increase in the wage premium. The dissimilarity that might have fiscal consequences relates to the level of the wage premium. While in Europe in 2002-2006 the average wage premium was about 1.25, we find a value of 1.4 for 2002 for Greece. The weight of public employment relative to private employment and the elasticity of substitution between public and private labor inputs are estimated econometrically. From the production function, we can obtain the ratio of public wages to private wages as:

\[
\frac{W_{g,t}}{W_{p,t}} = \frac{1 - \mu}{\mu} \left( \frac{L_{g,t}^{\eta - 1}}{L_{p,t}^{\eta - 1}} \right)
\]  

(26)

From this expression we get, taking logs

\[
\log \left( \frac{W_{g,t}}{W_{p,t}} \right) = \log \left( \frac{1 - \mu}{\mu} \right) + (\eta - 1) \log \left( \frac{L_{g,t}}{L_{p,t}} \right)
\]

(27)

and estimate by OLS.\(^8\) From the estimation for Greece we obtain the values for \( \mu = 0.6008 \) and \( \eta = 0.4326 \). Results from the estimation are represented in the lower panel of Figures 2 and 3, where it can be observed that the fit of the estimated model is quite good. When we estimate the coefficients of equation (27) we find values of \( \eta \) and \( \mu \) that imply that a wage premium is being paid by the government to public workers.

We set a total labor endowment of \( H = 100 \), and from the OECD labor statistics we have \( L = 57.500 \) for the year 2002. This number, plus the public to private labor

\[^7\text{See appendix A.2 for a derivation of equation (26).}\]

\[^8\text{The estimation procedure is explained in Fernández-de-Córdoba, Pérez and Torres (2012).}\]
ratio yield the corresponding values for $L_p$, and $L_g$. The production function gets fully calibrated computing $A(1)$ as a residual:

$$ A(1) = \frac{Y}{K_p^{\alpha_p} K_g^{\alpha_g} [\mu L_p^\eta + (1 - \mu)L_g^\eta]^{(1-n_p-n_g)}} = 1.6135 $$

Fix $\rho = -1$, in the government public sector objective function, and compute the value for $\omega$ as

$$ \omega = \frac{1}{1 + \left(\frac{W_g}{T_g}\right)^\rho} = 0.0794 $$

Finally, we compute $\gamma$ as:

$$ \gamma = \frac{C_p + \pi C_g}{C_p + \pi C_g + (H - L_p - L_g)W_p \frac{1 - \tau}{1 + \tau_h}} = 0.8384 $$

The public wage bill of Greece $\theta_3$, is obtained as the public wage bill over total government expenditures $\theta_3 = (1 + \tau^s)W_g L_g / G = 0.3213$. Finally, putting together the different fractions of government expenditures we obtain as a residual the value of $\theta_4 = 1 - \theta_1 - \theta_2 - \theta_3 = 0.1572$ as total transfers to consumers.

We collect the parameter values in two tables. The first table (Table 1) contains values taken from National Accounts, average effective tax rates, depreciation rates and the wage premium. Table 2 shows the set of parameters that we calibrate using the equilibrium conditions from the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t^{by}$</td>
<td>Nominal return of a Greek bond</td>
<td>0.041</td>
</tr>
<tr>
<td>$Y$</td>
<td>Total GDP</td>
<td>100</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Ratio total public spending/output</td>
<td>0.4504</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>Ratio total investment/output</td>
<td>0.2336</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>Ratio total consumption/output</td>
<td>0.7764</td>
</tr>
<tr>
<td>$L/H$</td>
<td>Employment/labor endowment</td>
<td>0.5750</td>
</tr>
<tr>
<td>$L_g/L_p$</td>
<td>Public labor/private labor</td>
<td>0.2399</td>
</tr>
<tr>
<td>$W_g/W_p$</td>
<td>Public sector wage premium</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Table 2: Government parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>θ₁</td>
<td>Ratio public consumption/total government expenditure</td>
<td>0.4467</td>
<td>0.4024</td>
<td></td>
</tr>
<tr>
<td>θ₂</td>
<td>Ratio public investment/total government expenditure</td>
<td>0.0745</td>
<td>0.0432</td>
<td></td>
</tr>
<tr>
<td>θ₃</td>
<td>Ratio public wage bill/total government expenditure</td>
<td>0.3213</td>
<td>0.1712</td>
<td></td>
</tr>
<tr>
<td>θ₄</td>
<td>Ratio public transfers/total government expenditure</td>
<td>0.1575</td>
<td>0.3832</td>
<td></td>
</tr>
<tr>
<td>τ₁</td>
<td>Labor income tax rate</td>
<td>0.4100</td>
<td>0.3810</td>
<td></td>
</tr>
<tr>
<td>τᵏ</td>
<td>Capital income tax rate</td>
<td>0.1640</td>
<td>0.1810</td>
<td></td>
</tr>
<tr>
<td>τₚₛₛ</td>
<td>Social security contribution</td>
<td>0.3560</td>
<td>0.3390</td>
<td></td>
</tr>
<tr>
<td>τₚ</td>
<td>Profit tax rate</td>
<td>0.2500</td>
<td>0.3870</td>
<td></td>
</tr>
<tr>
<td>τᶜ</td>
<td>Consumption tax rate</td>
<td>0.1480</td>
<td>0.1240</td>
<td></td>
</tr>
<tr>
<td>B/Y</td>
<td>Ratio public debt/output</td>
<td>1.0882</td>
<td>0.6560</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Fraction of debt refinanced every period</td>
<td>7</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Calibration of the economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>Discount factor</td>
<td>0.9606</td>
<td>0.9606</td>
<td></td>
</tr>
<tr>
<td>δₖₚ</td>
<td>Private capital depreciation rate</td>
<td>0.0800</td>
<td>0.0800</td>
<td></td>
</tr>
<tr>
<td>δₖ₉</td>
<td>Public capital depreciation rate</td>
<td>0.0400</td>
<td>0.0400</td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>Private-Private labor elasticity of substitution</td>
<td>0.4326</td>
<td>0.5762</td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>Private employment weight</td>
<td>0.6008</td>
<td>0.6440</td>
<td></td>
</tr>
<tr>
<td>αₚ</td>
<td>Private capital income share</td>
<td>0.3066</td>
<td>0.2556</td>
<td></td>
</tr>
<tr>
<td>αₙ</td>
<td>Labor share</td>
<td>0.5852</td>
<td>0.6026</td>
<td></td>
</tr>
<tr>
<td>α₉</td>
<td>Public capital technical parameter</td>
<td>0.1082</td>
<td>0.1418</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>TFP</td>
<td>1.6135</td>
<td>1.6422</td>
<td></td>
</tr>
<tr>
<td>γ</td>
<td>Consumption-leisure preferences</td>
<td>0.8956</td>
<td>0.8792</td>
<td></td>
</tr>
</tbody>
</table>

Greek figures for taxes, fiscal revenues, total government spending and its distribution are not so different from the figures for the rest of countries in the euro area. The tax menu is very similar to countries such as Germany. Fiscal revenues (including social security contributions) to GDP ratio for Greece is in the line of the rest euro area countries and even higher than countries like Ireland. Furthermore, government spending to GDP ratio was about 45% for Greece compared to the 47% for Germany or 53% for France, and public to private labor ratio is around 24% for Greece compared to about 32% for France.
4 Equilibrium, debt frontier and default

Given the calibrated values for the parameters for the Greek economy, we compute the steady state of the model. In our framework, private agents' decisions are not only affected by the tax menu, but also by the composition of public spending. The composition of public spending is critical for output and, thus, also for fiscal revenues. As a consequence, the amount of total debt is not independent of the spending policies, since different shares of total government spending have different effects on fiscal income: for example spending in social transfers does not improve productivity of private factors, whereas increasing public investment does. The effects of these different policies imply that the amount of sustainable debt varies across policies. This is an important property of our theoretical framework.

The questions that we want to respond is: Given a tax menu, given the government expenditure and its distribution and given a perpetual default penalty imposed on TFP if the government decides to default, What is the level of debt that leaves the government indifferent between honoring and defaulting the debt? Honoring the debt implies to pay large sums that could otherwise be used in providing goods to the consumer, investing in public capital, paying to public workers or transferring income to the poor. Defaulting implies the opposite, but in turn, the country faces a once and for all penalty for defaulting its debt.

The answer to this question comes from the decision problem taken by a benevolent government that maximizes the utility of the consumer given by equation (15). This problem is:

\[
\max U(C_{p,t} + C_{g,t}, L_t) + \beta E_t V(B_{t+1}, z_t)
\]

\[s.t. \ G_t + z R_{t} B_t = T_t + \Delta B_t \]

\[(28)\]

\[z = 0 \ or \ z = 1, \ but \ z = 0 \ if \ z_{-1} = 0\]

With the model economy parametrized to replicate the size of the government for the period 2002-2006 we proceed to define a steady state where the economy can roll over the existing debt as follows.

**Definition of steady state with rolling over** (\(z = 1\)): An equilibrium for this economy is a vector of prices \((W^*_g, W^*_p, R^*_g, R^*_p, R^B)\), a vector of input quantities \((L^*_g, L^*_p, K^*_g, K^*_p)\), and a vector of private consumption and investment \((C^*_p, I^*_p)\) such that for a given fiscal policy summarized by a collection of taxes \((\tau_c, \tau_l, \tau_k, \tau_{ss}, \tau_{x})\) and expenditure proportions \((\theta_1, \theta_2, \theta_3, \theta_4)\), induces a vector of public consumption, investment, transfers, and debt services \((C^*_g, I^*_g, Z^*, R^B)\), such that the optimization problems of the household, the firm, and the government are satisfied in a way
that the resources constraints are satisfied and all markets clear with TFP given by $A(1)$.

This steady state induces a level of welfare for the consumer given by

$$U^* = \frac{1}{1-\beta} U(C_p^*, C_g^*, L^*)$$

(29)

We can compute one steady state with rolling over for every ratio $G/Y$ and build what we call the "debt frontier", defined as the sustainable debt limit for each level of public expenditure. Sustainable debt limit here stands for a level where fiscal income is sufficient to cover current government expenditures and the service of debt. This notion of sustainable debt limit coincides formally with the steady state level of debt (with constant bond yields) for the model we have presented.

[Insert here Figure 4]

From the model we obtain a numerical representation of the trade-off between public debt long-run sustainable limit and government size measured as the total government spending to GDP ratio. A larger government size, given a constant level of public revenues, corresponds to a lower long-run sustainable level of public debt. The debt frontier is the relationship between public expenditure to GDP ratio, $G/Y$, and total debt to GDP ratio, $B/Y$, implied by the government budget constraint. Above the curve, we have all pairs where given the ratio $G/Y$, the amount of endogenous fiscal revenues are not enough to cover the services of total debt, $R^B$. Below the curve, we have all data pairs where fiscal revenues suffice to cover the given $G/Y$ ratio and services the outstanding debt. Figure 4 above, shows that the ratios of public expenditures and total debt where very far from the debt limit, calculated with the real return of bonds set at 1% for the period 2002-2006. Figure 4 also plots the actual values of $G/Y$ and $B/Y$ ratios for the period 2002-2006. These ratios, remained almost constant for the period 2002-2006 at a value of total public spending/GDP of 45% and a public debt/GDP of around 100%. The intuition behind this result is simple. In our model, public debt is modeled as if bond markets were infinitely liquid and thus, any maturing bond can always be rolled over at the given rate in the steady state. In this context, the long term sustainable amount of debt depends on both public revenues and expenditures and on the public bond interest rate. The sustainable debt limit is increasing in public revenues and decreasing in public expenditure and bond interest rate. A negative shock to output will reduce both the public income/output ratio and the public expenditure/output ratio, driving the economy toward the long-run unsustainable
debt area on one hand, and reducing the long-run sustainable amount of debt on the other hand.

The strong negative shock to output suffered by the Greek economy from 2007 onwards is reflected in Figure 5 as an increase in the public-income/output ratio and the public-expenditure/output ratio, driving the economy toward the long-run unsustainable debt area on one hand, and reducing the long-run sustainable amount of debt on the other hand. Simple inspection of the figures suggests that the Greek government decided to run large deficits while waiting for a recovery to come. After 12 quarters running deficits and piling debt, the yields of the Greek debt rose to a nominal yield of 12% in December 2010. In Figure 5 we plot two debt frontiers. The steeper one is the original computed frontier for a real return of bonds of 1%, consistent with the steady state. The other is a debt frontier using a real return of 9%, but keeping constant the rest of parameters values consistent with the steady state (that is, without recalibrating our model economy). By 2009, Greek’s fiscal ratios crossed the debt frontier. They were located in a place where fiscal revenues would be insufficient to service the existing debt at the new yields.

[Insert here Figure 5]

The debt level defined by the debt frontier becomes relevant when we consider the likelihood of a sun-spot variable capable to coordinate international investors scaring them away from buying bonds. The information obtained from a debt frontier crossing is that such situation has to be reverted. If no signals are produced, a self-fulling crisis can be started at any moment. Moreover, at any time this fully parametrized government can take the decision of defaulting on its debts, and thus a different steady state is induced by the decision of defaulting. The steady state for this economy after default is as follows.

**Definition of steady state under default** (\(z = 0\)): An equilibrium for this economy is a vector of prices \((W^d_g, W^d_p, R^d_p, R^d_g)\), a vector of input quantities \((L^d_g, L^d_p, K^d_g, K^d_p)\), and a vector of private consumption and investment \((C^d_p, I^d_p)\) such that for a given fiscal policy summarized by a collection of taxes \((\tau_c, \tau_l, \tau_k, \tau_{ss}, \tau_x)\) and expenditure proportions \((\theta_1, \theta_2, \theta_3, \theta_4)\), induce a vector of public consumption, investment, and transfers \((C^d_g, I^d, Z^d)\), such that the optimization problems of the household, the firm, and the government are satisfied in a way that the resources constraints are satisfied and all markets clear with TFP given by \(A(0)\), where \(A(0) = 0.95 \times A(1)\).

In this case, the long term welfare level attained by the consumer is given by

\[
U^d = \frac{1}{1 - \beta} U(C^d_p, C^d_g, L^d) \tag{30}
\]
Notice that the scenario with default introduces two modifications in the definition of equilibrium. First, the production function with government default is given by

\[ Y_t = A_t(0)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L^n_{p,t} + (1 - \mu)L^n_{g,t}]^{(1-\alpha_p-\alpha_g)\frac{n}{\eta}} \]  

(31)

We choose a permanent default penalty of 5% as in Cole and Kehoe (1996) and Conesa and Kehoe (2015). Therefore TFP after a default is set to be 0.95 of the calibrated value for TFP in the scenario where the government honors its debt, and reported in Table 2. This results in

\[ A_t(0) = 1.9234. \]

The second modification occurs in the government restriction which is simplified to be \( G_t = T_t \) under default. We can compute the equilibrium path for every possible value of \((B/Y, G/Y)\) and to obtain the associated utility, denoted by \( U^d(B_0/Y_0, G_0/Y_0) \).

However, in our context, the level of sustainable debt indicated by the debt frontier is not the only relevant debt level. There is also a debt level where the government can honor its debt independently of what international investors and market yields do. For example, if international lenders do not lend, and yields rise, the government could decide not to roll over, but instead to pay back any maturing bond until all outstanding debt gets canceled. If the government decides to pay back all outstanding debt, then a constant fraction of the total debt has to be paid out every year until all bonds mature. We follow Chatterjee and Eyigungor (2012) considering that the fraction of debt that has to be repaid corresponds to the average maturity of debt. If in period \( k \) the government decides to repay, and the average maturity of debt is \( \sigma \), then the government budget constraint is

\[ G_k + \frac{1}{\sigma}B_k = T_k \]

\[ : = : \]

\[ G_{k+\sigma-1} + \frac{1}{\sigma}B_{k+\sigma-1} = T_{k+\sigma-1} \]

\[ G_{k+\sigma+j} = T_{k+\sigma+j}, \quad j = 0, 1, 2 \ldots \]

We use the value of \( \sigma = 4 \), which corresponds to the average maturity on the Greek debt in the year 2010 (see Figure 7), to calculate the equilibrium paths starting at any point as we did with the default path, to obtain the associated level of utility, denoted by \( U^{rb}(\frac{R_0}{\sigma_0}, \frac{C_0}{\sigma_0}) \).

So for a given level of \((G/Y)\), we can vary the ratio \((B/Y)\), and compute the utility level associated to each pair. Comparing these two utility levels with that of an economy where it is always possible to roll over new debt, we construct Figure 6.

In Figure 6, the ratio \((G/Y)\) is fixed to the level where default occurred in 2010, and the debt to GDP ratio level varies from 0% to 350%. Two thresholds appear.
The debt to GDP that makes it preferable for the government to default if lenders decide not to lend, and the debt to GDP ratio where the government decides to default even if international lenders were ready to lend.

At the point where the utility of paying back equates the utility of default we have a threshold of debt. We find that this threshold is consistent with the debt frontier we have calculated. For a level of around $B/Y = 150\%$ at the long-term unsustainable area defined by the debt frontier, we have that Greece would have preferred (as she did) to default if lenders decide not to lend (as it happened). The second threshold in Figure 6 is found near a point were $B/Y = 300\%$. At this point, the government would choose to default even if international lenders are willing to lend and they do not expect a default.

[Insert here Figure 6]

From these pictures, we conclude that the current financial crisis affecting Greece has to be explained by an approach not directly linked to the fundamentals of the economy, as a carefully calibrated neoclassical growth model shows. Prior to the crisis, the Greek economy was well inside the long-run sustainable debt area with a public budget carrying with it a constant level of public debt/GDP ratio. Nevertheless, the crisis rapidly deteriorated output and public revenues, driving the Greek economy to a situation where it was vulnerable to the realization of a sunspot capable of coordinating investors to require more yield to buy debt. Once yields rise, the frontier rapidly moves downwards to the left, leaving the Greek economy in a situation that cannot be sustained in the long-run. But increasing yields, and decreasing maturities forced a situation where a default was preferable than paying back, with the resulting default.

One can argue that the initial value of public debt was too high (around 100\% of GDP) and that a lower level of public debt would have increased the strength of the Greek economy to cope with the crisis and remain in the long-run sustainable area. However, looking to the evolution of the Greek economy from 2007 onwards, an initial lower level of public debt does not guarantee that it would have avoided the debt crisis, given the evolution of Public Expenditures to GDP. From Figure 5 it is clear that small reductions in the public expenditure to GDP ratio induce large increases in the debt to GDP ratio. The immediate implication is that reductions of expenditure above the expected decrease in GDP, together with an increase in fiscal revenues from increased taxation should be enough to guarantee the solvency of the Greek State. Conversely, increases in the public expenditures to GDP ratio deteriorates the credit position very rapidly. The data shows that the swing to the right in the expenditures to GDP ratio from 2006 to 2009 was too large.
5 Conclusions

This paper develops a DGE model in which the government is fully characterized in both income and spending sides. Calibration of the model for the Greek economy provides evidence in favor of a gambling for redemption attitude towards the crisis, as in Conesa and Kehoe (2015) and Arellano, Conesa and Kehoe (2012). The gambling for redemption hypothesis can explain quite well the path of the Greek economy from 2007 onwards. As Conesa and Kehoe (2015) point out, countries that are in deep recessions have the incentive to cut government spending very slowly and increase the public debt, gambling that a recovery in the economy will lead to larger fiscal revenues. This argument is consistent with the recent experience of Greece during the period 2007-2009. Nevertheless, the debt-sustainability problem emerges when the recession is prolonged as indeed was the case. In this situation, government revenues never recover and the gamble for redemption cannot be maintained indefinitely, forcing the default. The main consequence we extract from our analysis is that the Greek government gambled for redemption and lost the bet. Period by period for three consecutive years, the global economy deteriorated, fiscal revenues never recovered, and suddenly astronomical bond yields indicated that the game was over. When a recession is exceptionally long lasting, gambling for redemption is a bad choice.

However, the historically observed frequency of the cycle can entice governments to gamble for redemption with the hope that the next expected expansion will dissolve past fiscal deficits. This implies that the gambling for redemption attitude towards a crisis can be the product of our past statistical knowledge of the cycle. It is reasonable, as we argue, and also optimal as Conesa and Kehoe demonstrate, to gamble for redemption when purely statistically based policies are put in place. Once the economic policy that emerges from a gambling for redemption strategy is proved incorrect by reality, some structural adjustments have to be put in place.

The table in Appendix B shows that policies oriented to increase productivity, together with a fiscal package that includes increases in VAT, labor taxes and corporate taxes, plus a re-structuring of public expenditures increasing public investment, at the expense of transfers, can be effective to solve a debt crisis. The proposed combination of increasing by 10% the following vector of policy instruments \((\tau_k, \tau_l, \tau_x, \theta_3)\) would depress output by \(-2.37\%\), it would depress private consumption and public consumption by \(-2.30\%\) and \(-2.37\%\) respectively, and it would depress total investment by \(-4.98\%\), but it would rise the debt ceiling by \(24.47\%\).
Appendix A.1: Walras’ Law

Take the budget constraint faced by the consumer:

\[(1 + \tau^c_t)C_{p,t} + K_{p,t} - K_{p,t-1} = (1 - \tau^l_t)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau^k_t)(R_t - \delta)K_{p,t-1} + Z_t + \Pi_t\]

And substitute the value of

\[Z_t = G_t - C_{g,t} - (1 + \tau^{ss}_t)W_{g,t}L_{g,t} - I_{g,t}\]

to obtain:

\[(1 + \tau^c_t)C_{p,t} + I_{gt} + K_{p,t} - K_{p,t-1} = (1 - \tau^l_t)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau^k_t)(R_{p,t} - \delta)K_{p,t-1} + G_t - C_{g,t} - (1 + \tau^{ss}_t)W_{g,t}L_{g,t} + \Pi_t\]

Or,

\[C_{pt} + C_{g,t} + I_{gt} + I_{pt} = -\tau^c_tC_{p,t} + (1 - \tau^l_t)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_{p,t}K_{p,t-1} - \tau^k_t(R_{p,t} - \delta K_p)K_{p,t-1} + G_t - C_{g,t} - (1 + \tau^{ss}_t)W_{g,t}L_{g,t} + \Pi_t\]

But, the government identity establishes the following relation:

\[(1 + R^B_t)B_t - B_{t+1} = T_t - G_t\]

Direct substitution yields

\[C_{pt} + C_{g,t} + I_{gt} + I_{pt} - T_t - B_{t+1} + (1 + R^B_t)B_t = -\tau^c_tC_{p,t} + (1 - \tau^l_t)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_{p,t}K_{p,t-1} - \tau^k_t(R_{p,t} - \delta K_p)K_{p,t-1} - (1 + \tau^{ss}_t)W_{g,t}L_{g,t} + \Pi_t\]

Government fiscal income is given by:

\[T_t = \tau^c_tC_{p,t} + \tau^l_t(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau^k_t(R_{p,t} - \delta K_p)K_{p,t-1} + \tau^{ss}_t(W_{p,t}L_{p,t} + W_{g,t}L_{g,t})\]

Substitution and elimination drives to:
\[ C_{pt} + C_{g,t} + I_{gt} + I_{pt} - B_{t+1} + (1 + R^B_t)B_t = W_{p,t}L_{p,t} + R_tK_{p,t^{-1}} + \Pi_t + \tau_t^{ss}W_{p,t}L_{p,t} \]

From the definition of profits we find that,
\[ \Pi_t = Y_t - (1 + \tau_t^{ss})W_{p,t}L_{p,t} - R_{p,t}K_{p,t} \]

Substitution yields:
\[ C_{pt} + C_{g,t} + I_{gt} + I_{pt} = Y_t + B_{t+1} - (1 + R^B_t)B_t \]

Which implies that all uses come from all available resources from an open economy. Therefore, Walras’ Law is satisfied at all times.

**Appendix A.2: Positive profits**

In a private economy where the government supply capital and labor with market pricing, the firm would have a profit function as:
\[ \Pi_t = Y_t - (1 + \tau_t^{ss})(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) - R_{p,t}(K_{p,t^{-1}} + K_{g,t^{-1}}) \]

where
\[ Y_t = A_t(z)K_{p,t^{-1}}^{\alpha_p}K_{g,t^{-1}}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}] \]

Under the assumptions that private factors are paid their marginal productivity, we get:
\[ (1 + \tau_t^{ss})W_{p,t} = \mu(1 - \alpha_p - \alpha_g)A_t(z)K_{p,t^{-1}}^{\alpha_p}K_{g,t^{-1}}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}] \frac{(1-\alpha_p-\alpha_g)}{\eta} L_{p,t}^{\eta-1} \tag{A.2.1} \]
\[ (1 + \tau_t^{ss})W_{g,t} = (1 - \mu)(1 - \alpha_p - \alpha_g)A_t(z)K_{p,t^{-1}}^{\alpha_p}K_{g,t^{-1}}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}] \frac{(1-\alpha_p-\alpha_g)}{\eta} L_{g,t}^{\eta-1} \tag{A.2.2} \]

\[ R_{p,t} = \alpha_pA_t(z)K_{p,t^{-1}}^{\alpha_p-1}K_{g,t^{-1}}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}] \frac{(1-\alpha_p-\alpha_g)}{\eta} \]
\[ R_{g,t} = \alpha_gA_t(z)K_{p,t^{-1}}^{\alpha_p-1}K_{g,t^{-1}}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}] \frac{(1-\alpha_p-\alpha_g)}{\eta} \]
Division of equation (A.2.1) by (A.2.2) yields equation (26) of Section 3. From the above equations we can obtain all income shares as:

\[
(1 + \tau_t^{ss})W_{p,t}L_{p,t} = \mu(1 - \alpha_p - \alpha_g)A_t(z)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}}L_{p,t}^\eta \\
= \frac{\mu(1 - \alpha_p - \alpha_g)L_{p,t}^\eta}{\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta}Y_t
\]

\[
(1 + \tau_t^{ss})W_{g,t}L_{g,t} = (1 - \mu)(1 - \alpha_p - \alpha_g)A_t(z)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}}L_{g,t}^\eta \\
= \frac{(1 - \mu)(1 - \alpha_p - \alpha_g)L_{g,t}^\eta}{\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta}Y_t
\]

and

\[
R_{p,t}K_{p,t-1} = \alpha_p Y_t
\]

\[
R_{g,t}K_{g,t-1} = \alpha_g Y_t
\]

Profits are zero because of the homogeneity of the production function:

\[
\bar{\Pi}_t = Y_t - \left(1 - (1 - \alpha_p - \alpha_g - \alpha_p - \alpha_g)\right)Y_t = 0
\]

If, on the contrary, the government pays public factor through taxes as it is assumed in the paper, then there are positive profits which can be calculated as the difference between total output and the rents paid to the private factors:

\[
\Pi_t = Y_t - R_{p,t}K_{p,t-1} - (1 + \tau_t^{ss})W_{p,t}L_{p,t} > 0
\]

Substituting private factor incomes yields:

\[
\Pi_t = \left[1 - \alpha_p - \frac{\mu(1 - \alpha_p - \alpha_g)L_{g,t}^\eta}{\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta}\right]Y_t > 0
\]
Appendix A.3: Equilibrium conditions

The collection of the model’s first order conditions, market clearing and resource constraints are:

\[ \frac{\gamma}{C_{p,t} + \pi C_{g,t}} - \lambda_t(1 + \tau_t^e) = 0 \]  \hspace{0.5cm} (A.3.1a)

\[ \frac{1 - \gamma}{N_t H - L_{p,t} - L_{g,t}} - \lambda_t(1 - \tau_t^l)W_{p,t} = 0 \]  \hspace{0.5cm} (A.3.1b)

\[ \beta \left[ \lambda_{t+1} \left( 1 + (1 - \tau_{t+1}^k)(R_{t+1} - \delta_{Kp}) \right) \right] - \lambda_t = 0 \]  \hspace{0.5cm} (A.3.2)

\[ \lambda_{t-1} - \beta \lambda_t(1 + R_t^B) = 0 \]  \hspace{0.5cm} (A.3.3)

\[ Y_t - A_t(z)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g)\eta} = 0 \]  \hspace{0.5cm} (A.3.4)

\[ R_{p,t} - \alpha_p A_t(z)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g)\eta} = 0 \]  \hspace{0.5cm} (A.3.5)

\[ (1 + \tau_{t}^{ss})W_{p,t} - \mu(1 - \alpha_p - \alpha_g)A_t(z)K_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{(1-\alpha_p-\alpha_g-\eta)\eta}L_{p,t}^{\eta-1} = 0 \]  \hspace{0.5cm} (A.3.6)

\[ \Pi_t = \left[ \alpha_g + \frac{(1 - \mu)(1 - \alpha_p - \alpha_g)L_{g,t}^\eta}{[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]} \right] Y_t = 0 \]  \hspace{0.5cm} (A.3.7)

\[ K_{p,t} - ((1 - \delta_{Kp})K_{p,t-1} + I_{p,t}) = 0 \]  \hspace{0.5cm} (A.3.8)

\[ K_{g,t} - ((1 - \delta_{Kg})K_{g,t-1} + I_{g,t}) = 0 \]  \hspace{0.5cm} (A.3.9)

\[ G_t - (C_{g,t} + (1 + \tau_{t}^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t) = 0 \]  \hspace{0.5cm} (A.3.10)

\[ C_{g,t} - \theta_1 G_t = 0 \]  \hspace{0.5cm} (A.3.11)
\[ I_{g,t} - \theta_2 G_t = 0 \]  
(A.3.12)

\[ (1 + \tau_t^{ss}) W_{g,t} L_{g,t} - \theta_3 G_t = 0 \]  
(A.3.13)

\[ Z_t - \theta_4 G_t = 0 \]  
(A.3.14)

\[ W_{g,t} - \left( \frac{\omega}{1 - \omega} \right)^{-1/\theta} \left[ \frac{\theta_3 G_t}{(1 + \tau_t^{ss})} \right]^{1/2} = 0 \]  
(A.3.15)

\[ L_{g,t} - \left( \frac{\omega}{1 - \omega} \right)^{1/2\theta} \left[ \frac{\theta_3 G_t}{(1 + \tau_t^{s})} \right]^{1/2} = 0 \]  
(A.3.16)

\[ T_t - \left( \tau_t^l C_{p,t} + \tau_t^l (W_{p,t} L_{p,t} + W_{g,t} L_{g,t}) + \tau_t^k (R_t - \delta K_p) K_{p,t-1} + \tau_t^{ss} (W_{p,t} L_{p,t} + W_{g,t} L_{g,t}) + \tau_t^k R^B_t B_t(z) + \tau_t^\Pi \Pi_t \right) = 0 \]  
(A.3.17)

\[ G_t + (1 + R^B_t) B_t(z) - (T_t + B_{t+1}(z)) = 0 \]  
(A.3.18)

\[ L_t - L_{p,t} - L_{g,t} = 0 \]  
(A.3.19)

This set of conditions fully characterizes a unique solution for any given policy vector. The set of equations of the model is completed with the budget constraint of the consumer and the following transversality conditions:

\[ \lim_{t \to \infty} \beta^t \lambda_t K_t = 0 \]

\[ \lim_{t \to \infty} (1 + R^B_t)^{-t} B_t(z) = 0 \]

The first transversality condition means that the present value of future capital, \( K_t \), must be going to zero. The second transversality condition requires a zero limit of future government debt discounted at the bond rate.

From this set of equations we can define the set of equilibria used by the government to take the default choice. Given a history of default \( z = \{0, 1\} \) equations (A.3.4), (A.3.5), (A.3.6), (A.3.17) and (A.3.18) are conveniently modified to accommodate the equilibrium with rolling over and the equilibrium under default.
Appendix B: Sensitivity Analysis

The results shown in the paper relate interest rates to the ratios $G_t/Y_t$ and $B_t/Y_t$. We have seen during the crisis enormous variations in the yields that the Greek bond had to pay to be attractive in the markets. In the calibration period 2002-2006, we observe a steady relation in the ratio $G/Y \approx 0.45$, and $B/Y \approx 100\%$. The implication is that when the yield of the bond increases by a factor of four, the expenditure made by the government in any other area has to decrease by a similar amount, and we know how extremely difficult this is. The result is that an enormous jump in the debt frontier has to take place.

In this appendix we analyze the sensitivity of the calibrated model to changes in some key parameters. Table B.1 shows the percentage change in the relevant variables given an increase of 10% in the parameters of the first row. Several interesting results emerge from this sensitivity analysis. Overall, this exercise shows the robustness of the model. As expected, a rise in Total Factor Productivity increases output, consumption and investment in the same amount. Additionally, the sustainable debt level increases by 14%, showing that public debt sustainability is also very sensible to productivity shocks.

An increase in taxes has a negative impact on all macroeconomic variables but on the sustainable debt level. From our model specification, a higher level of public revenues, given a particular government size, allows to cover a higher amount of debt services. The higher impact came from the labor income tax and consumption tax.

The reaction of our model economy to changes in total government spending composition is of interest. A rise in the proportion of public consumption ($\theta_1$) does not affect output and investment, reducing private consumption and raising public consumption by the same amount. Nevertheless, this policy change reduces the long-run sustainability debt limit by around 1%. A rise in public investment ($\theta_2$) has a positive impact on all macroeconomic variables, raising the long-run debt limit by 0.82%. The positive impact of a rise in public wage bill ($\theta_3$) on output, consumption and investment is easily explained, as more public employment is added to the aggregate production function, in spite of a fall in private employment. At the same time, the residual parameter $\theta_4$ is reduced by the same amount. Therefore public accounts remain unchanged while more factors are placed into the production function. The table also shows that a change in the composition of wages and public employment has mild effects on the economy. A reduction in civil servants’ compensations increases the debt ceiling by just 0.31% at the cost of −0.7% decrease in output.

Table B.1: Sensitivity Analysis
We complete our sensitivity analysis with a variation of the yield. Figure 8 shows how the frontier moves inwards as a consequence of an increase in the yield of the Greek bond. We represent the frontier for a 4%, 5% and a 7% yields, recalibrating the other parameters values of the model economy to the new interest rate. Notice that the effective spreads of the Greek bond with respect to the German Bund were much larger. Since the very beginning of the negotiations of the details of the rescue package for Greece by April 2010, the spreads skyrocketed due to a number of reasons. One of those reasons is discussed in Chamley and Pinto (2011). They argue that the seniority of the new bonds issued to finance the rescue program would disincentive other private investors from buying Greek bonds. However, we agree with Arellano, Conesa and Kehoe (2012) in saying that the rescue package was an effective mechanism to provide liquidity to the Greek State at a controlled yield. Figure 8 shows that the fiscal ratios displayed by the Greek economy prior to the crisis were sustainable at the yield of 5%, that is, the real return of the rescue package bond was consistent with a long-term sustainability of the Greek State prior to the unfolding of events that drove Greece to the current crisis. Nevertheless, the pre-crisis figures were unsustainable at the yield of 7%.

![Insert here Figure 8](image)

### Appendix C: Data Sources

The frequency of the data is annual for the period 2002-2011. The model is calibrated using data for the sub-period 2002-2006, which is selected as the steady state for our model economy. GDP, government expenditure, public debt, private consumption,
private investment, public investment and public consumption are taken from the OECD Statistics data base and Eurostat. Data on capital stock are taken from the EU-KLEMS database.

Public and private compensation of employees and public and private employment are taken from OECD Economic Outlook database December 2007 Issue, for the period 1960-2006. Public wage bill is calculated as total final public compensation of employees.

Effective average tax rates are taken from Boscá et al (2012), who use the methodology developed by Mendoza et al. (1994), and from OECD Revenue Statistics.


References


Figure 1: Public expenditures (G) and debt (B), as a proportion of GDP (Y).
Figure 2: Estimation of labor technological parameters
Figure 3: Same fit for Europe
Figure 4: The debt frontier before the crisis
Figure 5: The debt frontier after gambling
Figure 6: Default thresholds in normal times
Figure 7: Public debt average maturity (Source: Greek Public Debt Management Agency)
Public expenditure to GDP ratio: \( G_t/Y_t \)
Total debt to GDP ratio: \( B_t/Y_t \)

Long term sustainable debt area
Long term unsustainable debt area

Figure 8: Sensitivity Analysis