JORDAN-LIE INNER IDEALS OF FINITE DIMENSIONAL ASSOCIATIVE ALGEBRAS

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Any associative ring A becomes a Lie ring $A^{(-)}$ under $[x,y] = xy - yx$. Let $A^{(1)} = [A,A]$ be the derived subalgebra of $A^{(-)}$ and let Z be its center. In the early 1950s Herstein initiated a study of Lie ideals of A in case of a simple ring. In particular, he showed that in that case $A^{(1)}/Z$ is simple, $A^{(1)}$ generates A and $A^{(1)}$ is perfect (i.e. $[A^{(1)}, A^{(1)}] = A^{(1)}$), except if A is of characteristic 2 and is of dimension 4 over its center. Over the years, his work was generalized in various directions, on the one hand, to the setting of prime and semiprime rings, and, on the other hand, to Lie structures other than Lie ideals. However, little is known about the Lie structure of non-semiprime rings.

In this work we study Lie structure of finite dimensional associative algebras over an algebraically closed field of characteristic $p \neq 0$. Let A be such an algebra. It is natural to suppose that A is 1-perfect, i.e. has no ideals of codimesnion 1. Indeed, if A has a chain of ideals with 1-dimensional subquotients then $A^{(-)}$ is solvable and there is little correlation between ideals of $A^{(-)}$ and those of A. Suppose that A is 1-perfect. Then we prove that the Lie algebra $A^{(1)}$ is perfect and $A = A^{(1)}A^{(1)} + A^{(1)}$. Moreover, we show that most Lie ideals of $A^{(-)}$ and $A^{(1)}$ are induced by the ideals of A.

We also describe Jordan-Lie inner ideals of $A^{(-)}$ and $A^{(1)}$. Recall that a subspace B of a Lie algebra L is an *inner ideal* of L if $[B,[B,L]] \subseteq B$. An inner ideal B of $A^{(-)}$ is said to be *Jordan-Lie* if $B^2 = 0$ (in that case B is also an inner ideal of the Jordan algebra $A^{(+)}$).

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